Straight lines

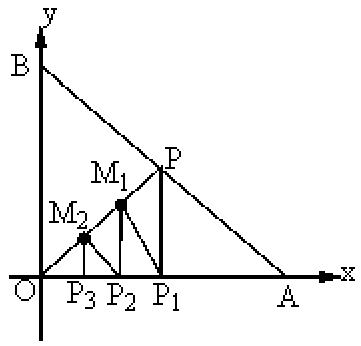
Single Correct Answer Type

1. The line x + y = 1 meets x-axis at A and y-axis at B. P is the mid-point of AB. P_1 is the foot of the perpendicular from P to OA; M_1 is that from P_1 to OP; P_2 is that from M_1 to OA; M_2 is that from P_2 to OP; P_3 is that from M_2 to OA and so on. If P_n denotes the nth foot of the perpendicular on OA from M_{n-1}, then $OP_n =$

A. $\frac{1}{2}$ B. $\frac{1}{2^n}$ C. $\frac{1}{2^{n/2}}$ D. $\frac{1}{\sqrt{2}}$

Key. B

Sol. x + y = 1 meets x-axis at A(1, 0) and y-axis at B(0, 1).



The coordinates of P are (1/2, 1/2) and PP_1 is perpendicular to OA.

$\Rightarrow OP_1 = P_1P = 1/2$

Equation of line OP is y = x

We have $(OM_{n-1})^2 = (OP_n)^2 + (P_nM_{n-1})^2 = 2(OP_n)^2 = 2P_n^2$ (say)

Also, $(OP_{n-1})^2 = (OM_{n-1})^2 + (P_{n-1}M_{n-1})^2$

$$(OP_{n-1})^2 = (OM_{n-1})^2 + (P_{n-1}M_{n-1})^2 = 2p_n^2 + \frac{1}{2}p_{n-1}^2$$

$$\Rightarrow p_n^2 = \frac{1}{4}p_{n-1}^2 \Rightarrow p_n = \frac{1}{2}p_{n-1}$$

$$\therefore OP_n = p_n = \frac{1}{2}p_{n-1} = \frac{1}{2^2}p_{n-2} = ... = \frac{1}{2^{n-1}}p_1 = \frac{1}{2^n}$$

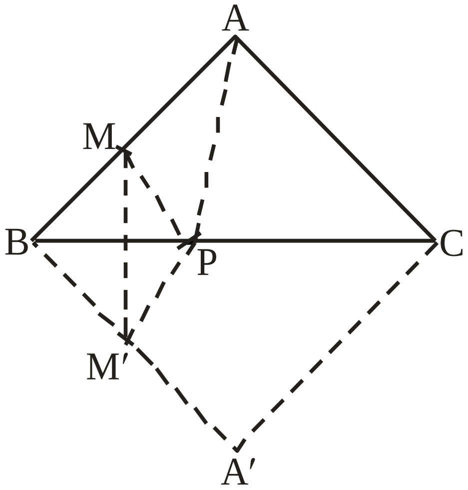
2. M is the mid point of side AB of equilateral triangle ABC. P is a point on BC such that AP + PM is minimum. If AB = 20 then AP + PM is

(A) $10\sqrt{7}$ (B) $10\sqrt{3}$

(C) $10\sqrt{5}$ (D) 10

Key. A

Sol. Take the reflection of ΔABC in BC.



PM = PM′

PA + PM = PA + PM′ it is minimum when M′ PA lies in a line

Now apply cosine rule in triangle ABM′

We will get $AM' = 10\sqrt{7}$

3. The algebraic sum of distances of the line ax + by + 2 = 0 from (1, 2), (2, 1) and (3, 5) is zero and the lines bx – ay + 4 = 0 and 3x + 4y + 5 = 0 cut the co-ordinate axes at concyclic points then

(a) $a+b=-\frac{2}{7}$

(b) area of the triangle formed by the line ax + by + 2 = 0 with coordinate axes is $\frac{14}{5}$.

(c) line ax + by + 3 = 0 always passes through the point $(-1,1)$

(d) max {a, b} = $\frac{5}{7}$

Key. C

Sol. Line always passes through the point $\left(2,\frac{8}{3}\right)$ hence 6a + 8b + 6 = 0 $\Rightarrow$ 3a + 4b + 3 = 0

bx – ay + 4 = 0 and 3x + 4y + 5 = 0 are concyclic.

So, $m_1m_2 = 1$

$\frac{b}{a}.-\frac{3}{4}=1 \Rightarrow$ 4a + 3b = 0

Solving a = 9/7, b = −12/7

4. The algebraic sum of distances of the line ax + by + 2 = 0 from (1, 2), (2, 1) and (3, 5) is zero and the lines bx – ay + 4 = 0 and 3x + 4y + 5 = 0 cut the co-ordinate axes at concyclic points then

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So, $m_1m_2 = 1$

$\frac{b}{a}.-\frac{3}{4}=1 \Rightarrow$ 4a + 3b = 0

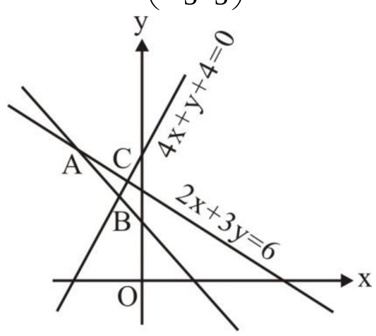
Solving a = 9/7, b = –12/7

5. The orthocentre of the triangle formed by the lines x + y = 1, 2x + 3y = 6 and 4x – y + 4 = 0 lies in

(A) I quadrant (B) II quadrant (C) III quadrant (D) IV quadrant

Key. A

Sol. Coordinates of A and B are (–3, 4) and $\left(-\frac{3}{5},\frac{8}{5}\right)$ if orthocentre p (h, k)



Then, (slope of PA) × (slope of BC) = –1

$\frac{k-4}{h+3}\times 4=-1$

$\Rightarrow$ 4k – 16 = –h –3

$\Rightarrow$ h + 4k = 13 ... (i)

and slope of PB × slope of AC = – 1

$\Rightarrow \quad \dfrac{k-\frac{8}{5}}{h+\frac{3}{5}} \times -\dfrac{2}{3} = -1$

$\Rightarrow \quad \dfrac{5k-8}{5h+3} \times \dfrac{2}{3} = 1$

$\Rightarrow$ 10k – 16 = 15th + 9

15th – 10k + 25 = 0

3h – 2k + 5 = 0 ... (ii)

Solivng Eqs. (i) and (ii), we get $h = \frac{3}{7}, k = \frac{22}{7}$

Hence, orthocentre lies in I quadrant.

6. A, B, C are three points on the curve $xy - x - y - 3 = 0$ which are not collinear. D, E, F are foot of perpendiculars from vertices A, B, C to the sides BC, CA and AB of ΔABC respectively. If (α, α) is incentre of ΔDEF then $'\alpha'$ can be

A) 1 B) 2 C) 3 D) 4

Key. C

Sol. Incentre of ΔDEF is ortho-centre of ΔABC. But in a rectangular hyperbola & ortho-centre lies on hyperbola $\Rightarrow \alpha^2 - 2\alpha - 3 = 0 \Rightarrow (\alpha-3)(\alpha+1) = 0 \Rightarrow \alpha = 3$

7. The reflection of the curve $xy = 1$ in the line $y = 2x$ is the curve $12x^2 + r\,xy + s\,y^2 + t = 0$ then the value of $'r'$ is

A) – 7 B) 25 C) – 175 D) 90

Key : A

HINT : The reflection of (α, β) in the line $y = 2x$ is

$(\alpha_1, \beta_1) = \left(\dfrac{4\beta - 3\alpha}{5}, \dfrac{4\alpha + 3\beta}{5}\right) = \alpha_1\beta_1 = 1$

$\Rightarrow 12\alpha^2 - 7\alpha\beta - 12\beta^2 + 25 = 0$

8. The line x + y = 1 meets x-axis at A and y-axis at B. P is the mid-point of AB. P_1 is the foot of the perpendicular from P to OA; M_1 is that from P_1 to OP; P_2 is that from M_1 to OA and so on. If P_n denotes the nth foot of the perpendicular on OA from M_{n-1}, then OP_n =

(a) 1/2 (b) $1/2^n$ (c) $1/2^{n/2}$ (d) $1/\sqrt{2}$

Key: b

Hint:

$x + y = 1$ meets x-axis at A(1, 0) and y-axis at B(0, 1).

The ordinates of P are (1/2, 1/2) and PP_1 is perpendicular to OA.

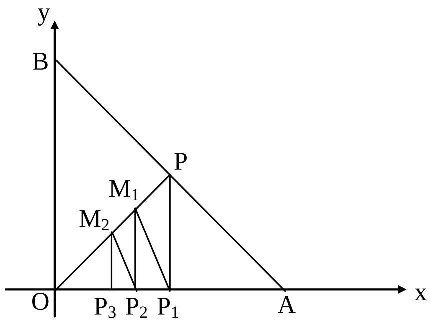
$\Rightarrow \quad OP_1 = P_1P = 1/2$

Equation of the line OP is y = x.

We have

$(OM_{n-1})^2 = (OP_n)^2 + (P_nM_{n-1})^2$

$= 2(OP_n)^2 = 2p_n^2$ (say)



Also, $(OP_{n-1})^2 = (OM_{n-1})^2 + (P_{n-1}M_{n-1})^2 = 2p_n^2 + 2p_n^2$

$\Rightarrow \; p_n^2 = \frac{1}{4}p_{n-1}^2 \Rightarrow p_n = \frac{1}{2}p_{n-1}$

$\therefore \; OP_n = p_n = \frac{1}{2}p_{n-1} = \frac{1}{2^2}p_{n-2} = ... = \frac{1}{2^{n-1}}p_1 = \frac{1}{2^n}$

9. A line passes through (2, 0). The slope of the line, for which its intercept between $y = x - 1$ and $y = -x + 1$ subtends a right angle at the origin, is/are

(A) $\sqrt{3}$ (B) $-\sqrt{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) $-\frac{1}{\sqrt{3}}$

Key. C,D

Sol. The joined equation of straight line $y = x - 1$ and $y = -x + 1$ is

$$(x - y - 1)(x + y - 1) = 0$$

$\Rightarrow \quad x^2 - y^2 - 2x + 1 = 0$ (1)

Let equation of line passes through (2, 0) is

$y = m(x - 2)$ (2)

By homogenizing equation (1) with help of line (2) is

$$x^2 - y^2 - 2x\left(\frac{mx - y}{2m}\right) + \left(\frac{mx - y}{2m}\right)^2 = 0$$

$\because$ coefficient of x^2 + coefficient of $y^2 = 0$

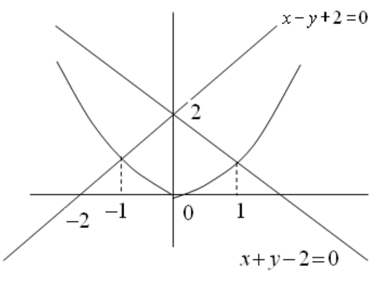
$\Rightarrow \quad m = \pm\frac{1}{\sqrt{3}}$

10. The complete set of values of $'a'$ for which the point (a, a^2), $a \in R$ lies inside the triangle formed by the lines $x - y + 2 = 0$, $x + y = 2$ and $x - axis$ is

(A) $(-2, 2)$ (B) $(-1, 1)$ (C) $(0, 2)$ (D) $(-2, 0)$

KEY : B

HINT :



$\left(a,a^2\right)$ lies of $y=x^2$

$a-a^2-2=0 \quad a=-1,2$

$a+a^2-2=0 \quad a=1,-2$

11. The values of k for which lines $kx+2y+2=0,\ 2x+ky+3=0,\ 3x+3y+k=0$ are concurrent

a) $\{2,3,5\}$ b) $\{2,3,-5\}$ c) $\{3,-5\}$ d) $\{-5\}$

Key: C

Hint: Three non parallel lines are concurrent if $\Delta=0$

$\begin{vmatrix} k & 2 & 2 \\ 2 & k & 3 \\ 3 & 3 & k \end{vmatrix}=0 \Rightarrow k=2,3,-5$ But for k= 2, first two lines are parallel.

12. A straight line passes through the point of intersection $x-2y-2=0$ and $2x-by-6=0$ and the origin then the complete set of values of b for which the acute angle between this line and y = 0 is less than 45^o

(A) $(-\infty,4)\cup(7,\infty)$ (B) $(-\infty,5)\cup(7,\infty)$

(C) $(-\infty,4)\cup(5,7)\cup(7,\infty)$ (D) $(-\infty,4)\cup(4,5)\cup(7,\infty)$

Key: D

Hint: As line passes through the point of intersection of $x-2y-2=0$ and $2x-by-6=0$

It can be represented as $\lambda(x-2y-2)+(2x-by-6)=0$

As it passes through the origin

$-2\lambda-6=0$

$\lambda=-3$

$\therefore$ equation of the line is $-x+(6-b)y=0$

Its slope is $\frac{1}{6-b}$

As its angle with y = 0 is less than $\frac{\pi}{4}$

$\therefore -1<\frac{1}{6-b}<1$

$\Rightarrow 6-b>1 \text{ or } <-1 \Rightarrow b<5 \text{ or } b>7$

But $b\neq 4$ (as the lines intersect)

$\therefore b\in(-\infty,4)\cup(4,5)\cup(7,\infty)$

13. Equation of angle bisector of the lines 3x - 4y + 1 = 0 and 12x + 5y - 3 = 0 containing the point (1, 2) is

(A) 3x + 11y - 4 = 0 (B) 99x - 27y – 2 = 0

(C) 3x + 11y + 4 = 0 (D) 99x + 27y - 2 = 0

Key: B

Hint: Since 3 × 1 - 4 × 2 + 1 and 12 × 1 + 5 × 2 - 3 are of the opposite sign, so required angle bisector is given by

$$\frac{3x-4y+1}{5}=-\left(\frac{12+5y-3}{13}\right)$$

14. Let S be the set of all values of α such that the points $(\alpha,6)$, (-5, 0) and (5, 0) form an isosceles triangle. Then the value of $\sum_{\alpha\in S}\alpha^2$ is

(A) 356 (B) 18 C) 178 (D) 338

Key: A

Hint α can take 5 values :0,3,-3,13.-13

15. If the orthocenter and circumcentre of a triangle are $(0,0)$ and $(3,6)$ respectively then the centroid of the triangle is

(A) $(1,2)$ (B) $(2,4)$ (C) $\left(\frac{2}{3},\frac{4}{3}\right)$ (D) $\left(\frac{1}{3},\frac{2}{3}\right)$

Key: B

Hint In any triangle centroid divides the line joining orthocenter and circumcentre internally in the ratio 2 : 1.

So, centroid is $(2,4)$.

16. The line $L_1 \equiv 4x+3y-12=0$ intersects the $x-$axis and $y-$axis at A and B respectively. A variable line perpendicular to L_1 intersect the x and $y-$axes at P and Q respectively. The locus of the circumcentre of triangle ABQ is

A) $6x-8y+7=0$ B) $6x+8y-25=0$ C) $8x-6y+7=0$ D) $14x-12y+3=0$

Key. A

Sol. clearly circumcentre of triangle ABQ will lie on perpendicular bisector of line AB, which is $6x-8y+7=0$

17. If the area of the rhombus enclosed by the lines $lx\pm my\pm n=0$ be 2square units, then

A) $l,2m,n$ are in G.P B) l,n,m are in G.P

C) $lm=n$ D) $ln=m$

Key. B

Sol. By solving the sides of the rhombus, the vertices are

$$\left(0,\frac{-n}{m}\right),\left(\frac{-n}{l},0\right)\left(0,\frac{n}{m}\right),\left(\frac{n}{l},0\right)$$

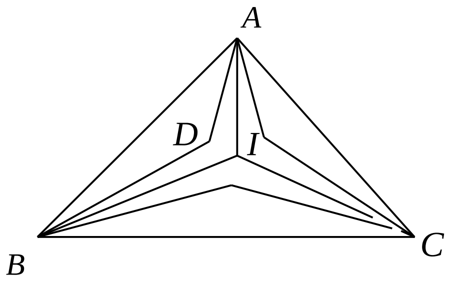
$\therefore$ The area = $\frac{1}{2}\left(\frac{2n}{m}\right)\left(\frac{2n}{l}\right)=2\Rightarrow n^2=lm$

18. If P is a point which moves inside an equilateral triangle of side length 'a' such that it is nearer to any angular bisector of the triangle than to any of its sides, then the area of the region in which P lies is __________ sq units

A) $a^2\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$ B) $\frac{\sqrt{3}a^2}{2}\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$ C) $\sqrt{3}a^2\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$ D) a^2

Key. B

Sol. Shaded area is the region traced by P, its area = $\Delta ABC - 3\Delta ABD$



$$=\frac{\sqrt{3}}{4}a^2-\frac{3}{2}a\times\frac{a}{2}\tan 15^0$$

$$= =\frac{\sqrt{3}}{2}a^2\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$$

19. In ΔABC orthocentre is (6,10) circumcentre is (2,3) and equation of side $\overline{BC}$ is 2x+y=17.Then the radius of the circumcircle of ΔABC is

a) 4 b) 5 c)2 d) 3

Key: B

Hint Image of orthocenter of ΔABC w.r.t. $\overline{BC}$ lies on the circle.

20. The area of the triangle formed by the line x + y = 3 and the angular bisectors of pair of straight lines $x^2-y^2+2y=1$ is

A. 8 sq.units B. 6 sq.units C. 4 sq.units D. 2 sq.units

Key. D

Sol. $x^2-(y-1)^2=0$ is given pair of lines

Vertices are (0,1),(0,3),(2,1) ,

Angular bisector is $x(y-1)=0$

Area = 2 sq.units

21. Let $O(0,0), P(3,4), Q(6,0)$ be the verticals of triangle OPQ .The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The point S is such that OS = PS = QS.Then RS =

A. $\frac{13}{16}$ B. $\frac{11}{12}$ C. $\frac{13}{24}$ D. $\frac{11}{24}$

Key. D

Sol. R is centroid .S is circumcentre . $R=\left(3,\frac{4}{3}\right), S=\left(3,\frac{7}{8}\right)$

RS= $\frac{11}{24}$

22. An equilateral triangle has its centroid at origin and one side is $x+y=1$. The equations of the others sides are

A. $y+1=(2\pm\sqrt{3})(x+1)$

B. $y+1=(2\pm\sqrt{3})x, y+1=(3\pm\sqrt{3})x$

C. $y+1=(3\pm\sqrt{3})(x-1), y+1=\sqrt{3}x$

D. $y\pm 1=(3\pm\sqrt{3})(x-1), y+1=\frac{\sqrt{3}-1}{\sqrt{3}+1}(x+1)$

Key. A

Sol. Third vertex 'A' lies on x – y =0 and in III quadrent

Perpendicular distance from (0,0) to x + y =1 is $\frac{1}{\sqrt{2}}$

$\therefore AO=\sqrt{2} \Rightarrow A(-1,-1)$

If m is the slope of other side,

$\tan 60^0 = \left|\frac{m+1}{1-m}\right|$

$\Rightarrow m = 2\pm\sqrt{3}$

23. Triangle is formed by the lines x + y = 0, x – y = 0 and $lx+my=1$. If l and m vary subject to the condition $l^2+m^2=1$, then the locus of its circumcentre is

(A) $(x^2-y^2)^2=x^2+y^2$ (B) $(x^2+y^2)^2=(x^2-y^2)$

(C) $(x^2+y^2)^2=4x^2y^2$ (D) $(x^2-y^2)^2=(x^2+y^2)^2$

Key. A

Sol. Circumcentre of the triangle formed by the given lines is given by

$$\left(\frac{l}{l^2-m^2}, \frac{m}{l^2-m^2}\right)$$

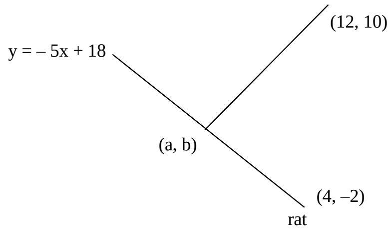
Hence the locus of this point is

$(x^2-y^2)^2 = x^2+y^2$

24. A piece of cheese is located at (12, 10) in a coordinate plane. A mouse is at (4, –2) and is running up the line y = –5x + 18. At the point (a, b), the mouse starts getting farther from the cheese rather than closer to it. The value of (a + b) is

(A) 6 (B) 10 (C) 18 (D) 14

Key. B



Sol.

a = 2, b = 8
a + b = 10

25. $A(3x_1, 3y_1)$, $B(3x_2, 3y_2)$, $C(3x_3, 3y_3)$ are vertices of a triangle with orthocentre H at $(x_1 + x_2 + x_3, y_1 + y_2 + y_3)$ then the $\angle ABC$

A. $\frac{\pi}{2}$ B. $\frac{\pi}{3}$ C. $\frac{\pi}{6}$ D. $\frac{\pi}{4}$

KEY. B

SOL. Centroid $G = \left(\frac{3x_1 + 3x_2 + 3x_3}{3}, \frac{3y_1 + 3y_2 + 3y_3}{3}\right) = (x_1 + x_2 + x_3, y_1 + y_2 + y_3) = H$

$\therefore \angle ABC = \pi / 3$

26. The area of the triangle with vertices $(a,b),(x_1, y_1),(x_2, y_2)$ where a, x_1, x_2 are in G.P. with common ratio ‘r’ and b, y_1, y_2 are in G.P with common ratio ’s’ is

A. $|ab(r-1)(s-1)(s-r)|$

B. $\frac{1}{2}|ab(r+1)(s+1)(s-r)|$

C. $\frac{1}{2}|ab(s-1)(r-1)(s-r)|$

D. $\frac{1}{2}abrs$

KEY. C

SOL. a, x_1, x_2 are in GP with C.R is ‘r’, b, y_1, y_2 are in G.P with C.R is s, $x_1 = ar, x_2 = r^2$,
$y_1 = bs, x_2 = bs^2$

27. If h denote the A.M, k denote G.M of the intercepts made on axes by the lines passing through (1, 1) then (h, k) lies on

A. $y^2 = 2x$ B. $y^2 = 4x$ C. $y = 2x$ D. $x + y = 2xy$

KEY. A

SOL. $a = x -$ intercept, b = *y* - intercept

2h = a + b, $k^2 = ab$

$\frac{x}{a} + \frac{y}{b} = 1$, substitute (1, 1)

$\frac{1}{a} + \frac{1}{b} = 1$

$a + b = ab$

$2h = k^2 \Rightarrow y^2 = 2x$

28. A straight rod of length 3*l* units slides with its ends A, B always on the x and y axes respectively then the locus of centroid of ΔOAB is

A. $x^2+y^2=3l^2$ B. $x^2+y^2=l^2$ C. $x^2+y^2=4l^2$ D. $x^2+y^2=2l^2$

KEY. B

SOL. Let OA= a, OB = b, AB = 3l

A = (a, 0), b = (0, b)

Let G(x, y) = $\left(\frac{a}{3},\frac{b}{3}\right)$, a = 3x, b = 3y

$a^2+b^2=9l^2 \Rightarrow x^2+y^2=l^2$

29. By translation of axes the equation $xy-x+2y-6=0$ changed as XY =c then c =

A. 4 B. 5 C. 6 D. 7

KEY. A

SOL. New origin $(x_1,y_1)=\left(\frac{-f}{h},\frac{-g}{h}\right)=(-2,1)$

Transformed equation of $xy-x+2y+6=0$ is xy = 4

= C = 4

30. A line has intercepts a, b on axes when the axes are rotated through an angle α, the line makes equal intercepts on axes then $\tan\alpha$ =

A. $\frac{a+b}{a-b}$ B. $\frac{a-b}{a+b}$ C. $\frac{a}{b}$ D. $\frac{b}{a}$

KEY. B

SOL. Equation of the lime $\frac{x}{a}+\frac{y}{b}=1$

Transformed eqution is $\frac{1}{a}(x\cos\alpha-y\sin\alpha)+\frac{1}{b}(x\sin\alpha+y\cos\alpha)=1$

Intercepts are equal

x - coefficient $\equiv y$ - coefficient

$\therefore \tan\alpha=\frac{a-b}{a+b}$

31. In a ΔABC, the coordinates of B are (0,0) AB=2, $\angle ABC=\frac{\pi}{3}$ and the mid point of BC is (2,0). The centroid of triangle is

1) $\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$ 2) $\left(\frac{5}{3},\frac{1}{\sqrt{3}}\right)$ 3) $\left(\frac{4+\sqrt{3}}{3},\frac{1}{\sqrt{3}}\right)$ 4) $\left(\frac{4-\sqrt{3}}{3},\frac{1}{\sqrt{3}}\right)$

Key. 2

Sol. Let $A(h,k)$ then $\cos 60^0=\frac{h}{2}\Rightarrow h=1$

$$\sin 60^0=\frac{k}{2}\Rightarrow k=\sqrt{3}$$

$\therefore A(1,\sqrt{3})$

$\therefore$ centroid = $\left(\frac{5}{3},\frac{\sqrt{3}}{3}\right)$

32. A point moves in the XY- plane such that the sum of its distances form two mutually perpendicular lines is always equal to 3. The area enclosed by the locus of the point is .

1) 18 Sq. Units 2) 9/2 Sq. Units 3) 9 Sq. Units 4) 27 Sq. Units

Key. 1

Sol. Given $|x|+|y|=3$

Required area = $\frac{2c^2}{|ab|}=9\times 2$ =18 S.U

33. If the point (a,a) falls between the lines $|x+y|=2$, then

1) $|a|=2$ 2) $|a|=1$ 3) $|a|<1$ 4) $|a|<\frac{1}{2}$

Key. 3

Sol. Origin and (a, a) lies on the same side of the given lines $\therefore |a|<1$

34. A ray travelling along the line $3x-4y=5$ after being reflected from a line $'l'$ travels along the line $5x+12y=13$. Then the equation of the line $'l'$ is

1) $x+8y=0$ 2) $x-8y=0$ 3) $32x+4y+65=0$ 4) $32x-4y+65=0$

Key. 2

Sol. The line $'l'$ can be any one of the bisectors of the angles between the lines $3x-4y=5$ and $5x+12y=13$

$\therefore$ Angular bisectors, $\frac{3x-4y-5}{5}=\pm\left[\frac{5x+12y-13}{13}\right]$

$\Rightarrow x-8y=0, 32x+4y-65=0$

35. The values of m for which the system of equations $3x+my=m$ and $2x-5y=20$ has a solution satisfy the conditions $x>0, y>0$ are given by the set

1) $\left\{m: m<\frac{-13}{2}\right\}$ 2) $\left\{m: m>\frac{17}{2}\right\}$

3) $\left\{m : m < \frac{-13}{2} \ or \ m > \frac{17}{2}\right\}$ 4) $\left\{m : m > 30 \ or \ m < \frac{-15}{2}\right\}$

Key. 4

Sol. Solve the equations $x = \frac{25m}{2m+15}, y = \frac{2m-60}{2m+15}$

But $x > o, y > 0 \Leftrightarrow 25m > 0, 2m+15 > 0, 2m-60 > 0$

$\Leftrightarrow m > 30$ or $m < \frac{-15}{2}$

36. $A_1, A_2.....A_n$ are points on the line y=x lying in the positive quadrant such that $OA_n = nOA_{n-1}$ O being the origin. If $OA_1 = 1$ and the coordinates of A_n are $\left(2520\sqrt{2}, 2520\sqrt{2}\right)$, then n=

1) 5 2) 6 3) 7 4) 8

Key. 3

Sol. We have, $OA_n = n.OA_{n-1} = n(n-1).OA_{n-2} = ---$

$\therefore OA_n = \frac{n!}{\sqrt{2}}$

$\Rightarrow \sqrt{2}(2520\sqrt{2}) = n! \Rightarrow n! = 5040$

$\Rightarrow n = 7$

37. M is the mid point of side AB of an equilateral triangle ABC. P is a point on BC such that AP + PM is minimum. If AB = 20 then AP + PM is

(A) $10\sqrt{7}$ (B) $10\sqrt{3}$

(C) $10\sqrt{5}$ (D) 10

Key. A

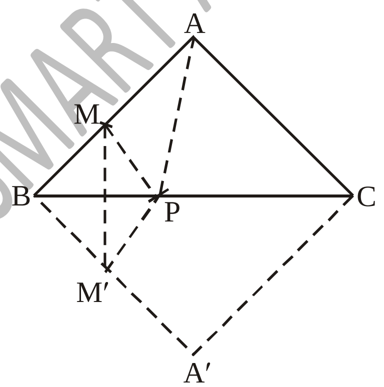
Sol. Take the reflection of ΔABC in BC.

PM = PM′

PA + PM = PA + PM′ it is minimum when M′ PA lies in a line

Now apply cosine rule in triangle ABM′

We will get $AM' = 10\sqrt{7}$



38. All points inside the triangle formed by A(1, 3), B(5, 6), C(–1, 2) will satisfy

(A) 2x + 2y ≤ 0 (B) 2x + y + 1 ≥ 0

(C) 2x + 3y – 12 ≥ 0 (D) –2x + 11 ≤ 0

Key. B

Sol. $L_1 \equiv 2x + 2y = 0$

$L_1(1, 3) > 0$ so a is wrong

$L_2 \equiv 2x + y + 1 = 0$

$L_2(1, 3) > 0$

$L_2(5, 6) > 0 \quad \Rightarrow \quad$ b is ture

$L_3(-1, 2) > 0$

39. Let P(1, 1), Q (2, 4), R(α, β) be the vertices of the triangle PQR. The point S(2, 2) inside the triangle PQR is such that

Area (ΔPQS) = Area (ΔPSR) = Area (Δ RSQ), then (α, β) =

(A) (2, 3) (B) (2, 5/2)

(C) (3, 1) (D) (5/2, 2)

Key. C

Sol. Here S must be cenroid of ΔPQR

$\Rightarrow \frac{1+2+\alpha}{3} = 2 \;\&\; \frac{1+4+\beta}{3} = 2$

$\Rightarrow \alpha = 3 \;\&\; \beta = 1$.

40. A system of line is given as $y = m_i x + c_i$, where m_i can take any value out of 0, 1, –1 and when m_i is positive then c_i can be 1 or –1 when m_i equal 0, c_i can be 0 or 1 and when m_i equals – 1, c_i can take 0 or 2. Then the area enclosed by all these straight line is

(A) $\frac{3}{\sqrt{2}}(\sqrt{2}-1)$ sq. units (B) $\frac{3}{\sqrt{2}}$ sq. units

(C) $\frac{3}{2}$ sq. untis (D) $\frac{3}{4}$ sq. units

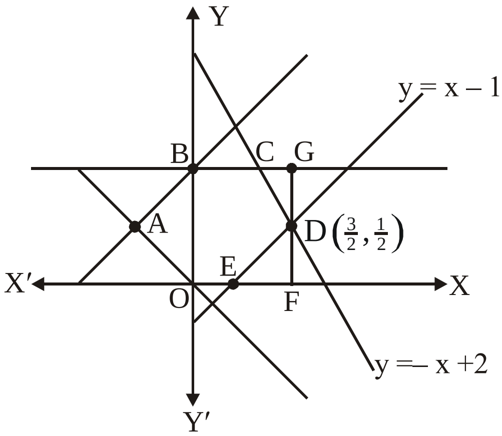
Key. C

Sol. Lines are y = 1, y = 0, y = –x, y = –x + 2, y = x +1, y = x – 1

Area of OABCDE

= area of OBGF

$= \frac{3}{2} \times 1 = \frac{3}{2}$ units.



41. Point A lies on y = x and point B on y = mx so that length AB = 4 units then value of m for which locus of mid point of AB represents a circle is

(A) m = 0 (B) m = –1
(C) m = 2 (D) m = –2

Key. B

SOL. LET CO-ORDINATES OF $A(X_1, X_1)$ AND $B(X_2, MX_2)$.
CLEARLY $(X_1 - X_2)^2 + (X_1 - MX_2)^2 = 16$
LET MID POINT OF P(H, K)
$\Rightarrow$ $X_1 + X_2 = 2H$ AND $X_1 + MX_2 = 2K$
$\Rightarrow$ $(X_1 - X_2)^2 + 4X_1X_2 = 4H^2$ AND $(X_1 - MX_2)^2 + 4MX_1X_2 = 4K^2$
$(X_1 - X_2)^2 + (X_1 - MX_2)^2 = 4H^2 + 4K^2 = 16$
when m = –1

42. The joint equation of two altitudes of an equilateral triangle is
$\left(\sqrt{3}x - y + 8 - 4\sqrt{3}\right)\left(\sqrt{3}x + y - 12 - 4\sqrt{3}\right) = 0$. The equation of the third altitude is

a) $\sqrt{3}x = 2 - 4\sqrt{3}$ b) $y - 10 = 0$ c) $\sqrt{3}x = 2 + 4\sqrt{3}$ d) $y + 10 = 0$

Key. B

Sol. The required altitude will be the bisector of obtuse angle between the 2 given altitudes

43. A line $x + 2y = 4$ is translated by 3 units, away from origin and then rotated by 30^0 in clock wise sense about the point where shifted line cuts x-axis . If m is the slope of line in new position then [m] where [.] denotes GIF, is

a) -1 b) -2 c) -3 d) -4

Key. A

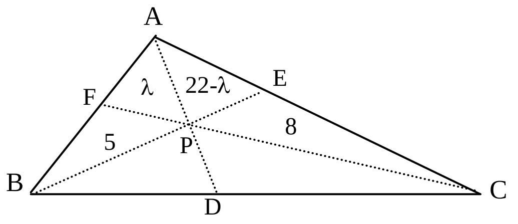
Sol. The required line is at a distance of 3 units from given line and parallel to it . Hence it is
$x + 2y - 4 - 3\sqrt{5} = 0,$ cuts x-axis at $C(4 + 3\sqrt{5}, 0)$ with slope $\tan\theta = \frac{-1}{2}$ .After rotation
about C by 30^0, slope becomes $m = \tan(\theta - 30^0) = \frac{-(2+\sqrt{3})}{2\sqrt{3}-1} = \frac{-(4+3\sqrt{3})}{11} \Rightarrow [m] = -1$

44. In a triangle ABC, E and F are points on AC and AB respectively. The lines BE and CF intersect at P. If area (BPF) = 5. area (PFAE) = 22, and area (CPE) = 8, then area (BPC) is

(A) 22 (B) 16
(C) 10 (D) not uniquely decidable

Key. C

Sol.



Let area of ΔPBC = x

$\Rightarrow$ $\frac{x}{5+\lambda} = \frac{8}{22-\lambda}$ and $\frac{x}{30-\lambda} = \frac{5}{\lambda}$

$\Rightarrow$ $\frac{\lambda+5}{30-\lambda} = \frac{(22-\lambda)5}{8\lambda}$

$\Rightarrow$ $8\lambda^2 + 40\lambda = 5(\lambda^2 - 52\lambda + 660)$

$\Rightarrow$ $\lambda^2 + 100\lambda - 1100 = 0$

$\Rightarrow$ $(\lambda + 110)(\lambda - 10) = 0$ $\Rightarrow$ $\lambda = 10$

$\Rightarrow$ x = $\frac{(30-\lambda)5}{\lambda}$ = $\frac{(30-10)\times 5}{10}$ = 10 square units.

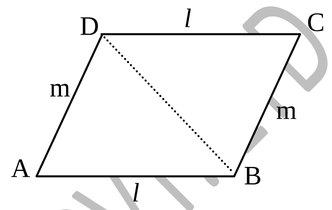
Ans. (C) 10 square units.

45. The perimeter of a parallelogram is 40. All the sides of the parallelogram are natural numbers, and at least one of its diagonals is a natural number. The number of noncongruent parallelograms so formed is

(A) 10 (B) 30
(C) 60 (D) 100

Key. D

Sol.



Let BD be integer and $l \geq m$

$2(l + m) = 40$

$\Rightarrow l + m = 20$

Possible values of m = 1, 2, 3, ..., 10

Note in any triplet of *l*, m, BD if atleast one is different parallelogram will be noncongruent

Now $l - m < BD < l + m$ (triangle inequality)

$\Rightarrow 20 - 2m < BD < 20$

$\Rightarrow$ No. of possible values of BD for a given 'm' is $20 - (20 - 2m) - 1 = 2m - 1$

$\Rightarrow$ Total no. of noncongruent parallelogram $= \sum_{m=1}^{10}(2m-1) = 10^2 = 100$

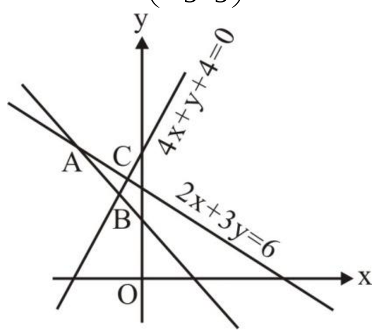
Ans. (D) 100

46. The orthocentre of the triangle formed by the lines x + y = 1, 2x + 3y = 6 and 4x – y + 4 = 0 lies in

(A) I quadrant (B) II quadrant (C) III quadrant (D) IV quadrant

Key. A

Sol. Coordinates of A and B are (–3, 4) and $\left(-\frac{3}{5}, \frac{8}{5}\right)$ if orthocentre p (h, k)



Then, (slope of PA) × (slope of BC) = –1

$\frac{k-4}{h+3} \times 4 = -1$

$\Rightarrow \quad 4k - 16 = -h - 3$

$\Rightarrow \quad h + 4k = 13 \quad \ldots (i)$

and slope of PB × slope of AC = – 1

$$\Rightarrow \quad \frac{k-\frac{8}{5}}{h+\frac{3}{5}} \times -\frac{2}{3} = -1$$

$$\Rightarrow \quad \frac{5k-8}{5h+3} \times \frac{2}{3} = 1$$

$\Rightarrow$ 10k – 16 = 15th + 9

15th – 10k + 25 = 0

3h – 2k + 5 = 0 ... (ii)

Solivng Eqs. (i) and (ii), we get $h=\frac{3}{7}, k=\frac{22}{7}$

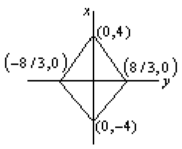
Hence, orthocentre lies in I quadrant.

47. If $f(x+y)=f(x)f(y)\ \forall x,y \in R$ and $f(1)=2$, then area enclosed by $3|x|+2|y| \le 8$ is (in sq.units)

A) $f(4)$ B) $\frac{1}{2}f(6)$ C) $\frac{1}{3}f(6)$ D) $\frac{1}{3}f(5)$

Key. C

Sol.



Area = $= 4\times\frac{1}{2}\times\frac{8}{3}\times 4 = \frac{64}{3} = \frac{2^6}{3}$

$f(x)=2^x$

48. $9x^2+2hxy+4y^2+6x+2fy-3=0$ represents two parallel lines then

a) h = 6, f = 2 b) h = -6, f = 2 c) h = 6, f = -2 d) none

Key. A

Sol. Since the given equation represents a pair of parallel lines, we have $h^2=ab \Rightarrow h = \pm 6$

Condition for pair of lines $\begin{vmatrix} 9 & h & 3 \\ h & 4 & f \\ 3 & f & -3 \end{vmatrix} = 0$

$\Rightarrow 108 \pm 36f - 9f^2 - 144 = 0$

$\Rightarrow$ f = 2 & h = 6

$\Rightarrow$ f = -2, h = -6

Straight lines

Multiple Correct Answer Type

1. The triangle formed by the lines $x+y=0,\ 3x+y-4=0$ and $x+3y-4=0$ is

A) isosceles B) scalene C) acute angled D) obtuse angled

Key. A,D

Sol. Conceptual

2. Given pair of lines $2x^2+5xy+2y^2+4x+5y+a=0$ and the line $L: bx+y+5=0$ then

A) $a=2$

B) $a=-2$

C) There exist no circle which touches the pair of lines and the line L if $b=5$

D) There exist no circle which touches the pair of lines and the line L if $b=-5$

Key. A,C

Sol. For the equation to be pair of lines $\Delta=0$ then $a=2$

If the three lines are concurrent, no circle exist then $b=5$

3. If the ortho–centre of an isosceles triangle lies on the in–circle of the triangle then

A) The base angle of the triangle is $\cos^{-1}\frac{2}{3}$

B) The triangle is acute

C) The base angle of the triangle is $\tan^{-1}\frac{\sqrt{5}}{2}$

D) If S, I are the circumcentre and in–centre and R is circum–radius then $\frac{SI}{R}=\frac{1}{3}$

Key. A,B,C,D

Sol. Let ABC be the triangle in which $AB=AC$. Let I, P respectively be the incentre and the ortho–centre of the triangle.

$AI = r\,\mathrm{cosec}\frac{A}{2},\ AP = 2R\cos A$

$r\,\mathrm{cosec}\frac{A}{2} = 2R\cos A + r$

4. P is a point inside a ΔABC of area K (K>0) .The lengths of perpendiculars drawn to the sides BC,CA,AB of lengths a,b,c are respectively P_1, P_2, P_3. $\frac{a}{P_1}+\frac{b}{P_2}+\frac{c}{P_3}$ is minimum when

(A) P is incentre of ΔABC (B) P is equidistant to all the 3 sides

(C) $P_1 = P_2 = P_3 = \frac{2K}{a+b+c}$ (D) P is orthocentre of ΔABC

KEY : ABCD

HINT : given $\frac{1}{2}(aP_1 + bP_2 + cP_3) = K \Rightarrow y = \frac{a}{P_1} + \frac{b}{P_2} + \frac{c}{P_3}$ is minimum.

when $y = \frac{1}{2K}(aP_1 + bP_2 + cP_3)\left(\frac{a}{P_1} + \frac{b}{P_2} + \frac{c}{P_3}\right)$ is minimum.

but , $y = \frac{1}{2K}\left(a^2 + b^2 + c^2 + ab\left(\frac{P_1}{P_2} + \frac{P_2}{P_1}\right) + bc\left(\frac{P_2}{P_3} + \frac{P_3}{P_2}\right) + ab\left(\frac{P_1}{P_3} + \frac{P_3}{P_1}\right)\right)$

$\geq \frac{1}{2K}\left(a^2 + b^2 + c^2 + 2ab + 2bc + 2ac\right)$

$\Rightarrow y \geq \frac{(a+b+c)^2}{2K}$ *when* $\frac{P_1}{P_2} = \frac{P_2}{P_1} = \frac{P_2}{P_3} = \frac{P_3}{P_2} = \frac{P_1}{P_3} = \frac{P_3}{P_1} = 1$

$\Rightarrow i.e, when\, P_1 = P_2 = P_3$

$\therefore P\ is\ \ incentre\ \ of\ \ \Delta ABC$

5. In ΔABC, P is any point inside a triangle such that area of ΔBPC, ΔAPC, ΔAPB are equal. Line AP cut BC at M, area of ΔPMC is 5 sq. unit then area of ΔABC is

(A) 20 sq.units (B) 25 sq.units

(C) 30 sq.units (D) 10 sq.units

Key: A, B, C

Hint: P is centroid of ΔABC

$\therefore$ area of ΔABC = $6 \times 5 = 30$ sq.units

6. Equations $(b-c)x + (c-a)y + (a-b) = 0$ and $(b^3 - c^3)x + (c^3 - a^3)y + (a^3 - b^3) = 0$ will represent the same line if

(A) b = c (B) c = a

(C) a = b (D) a + b + c = 0

Key. A,B,C,D

Sol. The two lines will be identical if there exists some real number k, such that

$b^3 - c^3 = k(b-c), c^3 - a^3 = k(c-a)$

and $a^3 - b^3 = k(a-b)$

$\Rightarrow$ b – c = 0

or $b^2 + c^2 + bc = k,$

$c - a = 0$

or $c^2 + a^2 + ca = k$

and $a - b = 0$

or $a^2 + b^2 + ab = k$

that is, b = c or c = a or a = b

next $b^2 + c^2 + bc = c^2 + a^2 + ca$

$\Rightarrow$ $b^2 - a^2 = c(a-b)$

Hence, a = b

or a + b + c = 0

7. Let $u = ax + by + a\sqrt[3]{b} = 0$, $v = bx - ay + b\sqrt[3]{a} = 0$, $a, b \in R$ be two straight lines. The equation of the bisectors of the angle formed by $L_1 \equiv (\tan\theta_1)u - (\tan\theta_2)v = 0$ and $L_2 \equiv (\tan\theta_1)u + (\tan\theta_2)v = 0$ for $\theta_1, \theta_2 \in \left(0, \frac{\pi}{2}\right)$ is

A) $u = 0$ B) $(\tan\theta_2)u + (\tan\theta_1)v = 0$

C) $(\tan\theta_2)u - (\tan\theta_1)v = 0$ D) $v = 0$

Key. A,D

Sol. Note that lines $u = 0, v = 0$ are perpendicular . Make the co-ordinate axes coincide with $u = 0, v = 0$. Now the lines $L_1 \equiv 0, L_2 \equiv 0$ are equally inclined with $u - v$ axes

$\therefore$ $u = 0, v = 0$ are bisectors.

8. Equations of bisectors of angles between intersecting lines $\frac{x-3}{\cos\theta} = \frac{y+5}{\sin\theta}, \frac{x-3}{\cos\phi} = \frac{y+5}{\sin\phi}$ are $\frac{x-3}{\cos\alpha} = \frac{y+5}{\sin\alpha}$ *and* $\frac{x-3}{\beta} = \frac{y+5}{\gamma}$ then

A. $\alpha = \frac{\theta+\phi}{2}$ B. $\beta^2 + \gamma^2 = 1$ C. $Tan\alpha = \frac{-\beta}{\gamma}$ D. $Tan\alpha = \frac{\beta}{\gamma}$

Key. A,B,C

Sol. Inclinations of two lines are θ and ϕ

$\therefore$ Inclination of angle bisector is $\frac{\theta+\phi}{2}$

$\alpha = \frac{\theta+\phi}{2}$ and $Tan\alpha \times \frac{\gamma}{\beta} = -1$

$\Rightarrow Tan\alpha = \frac{-\beta}{\gamma}$

$\therefore \beta = -\sin\alpha, \gamma = \cos\alpha$

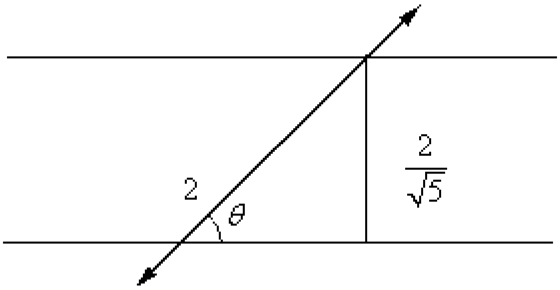
$\Rightarrow \beta^2 + \gamma^2 = 1$

9. The equation of the line passing through (2,3) and making an intercept of 2 units between the lines $y + 2x = 5$ *and* $y + 2x = 3$ is

A. $5x - 4y + 2 = 0$ B. $3x + 4y = 18$ C. x = 2 D. y = 3

Key. B,C

Sol. $\tan\theta = \frac{1}{2}$



Slope of required line =m

Slope of given lines =-2

$$\left|\frac{m+2}{1+2m}\right| = \frac{1}{2} \Rightarrow m = \frac{-5}{4} \text{ or } \infty$$

The lines are 5x + 4y – 22 = 0, x -2 =0

10. The equation of the diagonal of the square formed by the pairs of lines $xy+4x-3y-12=0$ and $xy-3x+4y-12=0$ is

A. $x-y=0$ B. $x+y+1=0$ C. $x+y=0$ D. $x-y+1=0$

Key. A,B

Sol. $(x-3)(y+4)=0,(x+4)(y-3)=0$

The vertices are $A=(-4,-4), B=(-4,3), C=(3,3), D=(-4,3)$

Diagonal AC is x = y ,Diagonal BD is x + y + 1 = 0

11. Under rotation of axes through θ, $x\cos\alpha + y\sin\alpha = P$ changes to $X\cos\beta + Y\sin\beta = P$ then

A. $\cos\beta = \cos(\alpha-\theta)$ B. $\cos\alpha = \cos(\beta-\theta)$ C. $\sin\beta = \sin(\alpha-\theta)$ D. $\sin\alpha = \sin(\beta-\theta)$

KEY. A,C

SOL. $x\cos\alpha + y\sin\alpha = P$

Axis rotated through angle 'θ'

Transformed equation

$$\cos\alpha(x\cos\theta - y\sin\theta) + \sin\alpha(x\sin\theta + y\cos\theta) = P$$

$$x\cos(\alpha-\theta) + y\sin\alpha(\alpha+\theta) = P \Rightarrow x\cos\beta + y\sin\beta = P$$

$$\cos\beta = \cos(\alpha-\theta),\ \sin\beta = \sin(\alpha-\theta)$$

12. (3, 2), (-4, 1) and (-5, 8) are vertices of triangle then

A. orthocentre is (4, 1) B. orthocentre is (-4, 1) C. circumcentre is (-1,5) D. circumcentre is (3, 2)

KEY. B,C

SOL. (3, 2) (-4, 1) (-5, 8) form a right angle triangle at (-4, 1)
Orthocnetre is (-4, 1), circumcentre is mid point of (3, 2) (-5, 2). If (-1, 5)

13. The point A divides the join of P = (-5, 1) and Q = (3, 5) in the ratio k : 1. The values of k for which the area of ΔABC. Where B = (1, 5), C = (7, -2) is equal to 2 sq. Units are

A. 7 B. 4 C. 30/4 D. 31 / 9

KEY. A,D

SOL. Dividing point of P(-5, 1), Q(3, 5) in the ration k:1 is

$$A=\left(\frac{3k-5}{k+1},\frac{5k+1}{k+1}\right)\text{, B = (1, 5), C = (7, -2)}$$

Area of triangle ABC = 2

$$\therefore 9k^2-94k+217=0$$

$$(k-7)(9k-31)=0$$

$$k=7,\frac{31}{9}$$

14. The angle through which the co-ordinate axes be rotated so that xy-term in the equation $5x^2+4\sqrt{3}xy+9y^2=0$ may be missing is

A. $\frac{\pi}{6}$ B. $\frac{-\pi}{6}$ C. $\frac{\pi}{4}$ D. $\frac{\pi}{3}$

KEY. B,D

SOL. $\theta=\frac{1}{2}\tan^{-1}\left(\frac{2h}{a-b}\right)=\frac{1}{2}\tan^{-1}\left(\frac{4\sqrt{3}}{-4}\right)=\tan^{-1}(-\sqrt{3})$

$$\theta=\frac{1}{2}\left(\pi-\frac{\pi}{3}\right),\frac{1}{2}\left(-\frac{\pi}{3}\right)$$

$$\theta=\frac{\pi}{3},-\frac{\pi}{6}$$

15. Sides of a rhombus are parallel to the lines x + y – 1 = 0 and 7x – y – 5 = 0. It is given that diagonals of the rhombus intersect at (1, 3) and one vertex 'A' of the rhombus lies on the line y = 2x. Then the coordinates of the vertex A are

(A) $\left(\frac{8}{5},\frac{16}{5}\right)$ (B) $\left(\frac{7}{15},\frac{14}{15}\right)$ (C) $\left(\frac{6}{5},\frac{12}{5}\right)$ (D) $\left(\frac{4}{15},\frac{8}{15}\right)$

Key. A,C

Sol. It is clear that diagonals of the rhombus will be parallel to the bisectors of the given lines and will pass through (1, 3). Equations of bisectors of the given lines are

$$\frac{x+y-1}{\sqrt{2}}=\pm\left(\frac{7x-y-5}{5\sqrt{2}}\right)$$

Or, $2x - 6y = 0$, $6x + 2y = 5$

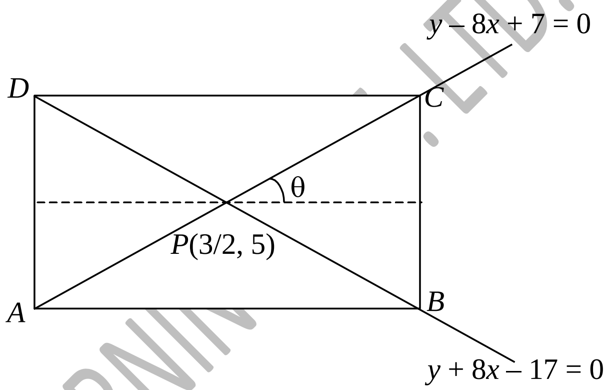
Therefore, the equations of diagonals are $x - 3y + 8 = 0$ and $3x + y - 6 = 0$. Thus the required vertex will be the point where these lines meet the line $y = 2x$. Solving these lines we get possible coordinates as $\left(\frac{8}{5}, \frac{16}{5}\right)$ and $\left(\frac{6}{5}, \frac{12}{5}\right)$.

16. Equations of the diagonals of a rectangle are $y + 8x - 17 = 0$ and $y - 8x + 7 = 0$. If the area of the rectangle is 8 sq. units, then the equation of the sides of the rectangle is/are

(A) $x = 1$ (B) $x + y = 1$ (C) $y = 9$ (D) $x - 2y = 3$

Key. A,C

Sol.



The intersection point of the given diagonals is $P \equiv \left(\frac{3}{2}, 5\right)$

Equation of angular bisectors of the diagonals are

$$\frac{y+8x-17}{\sqrt{65}} = \pm\frac{y-8x+7}{\sqrt{65}}$$

$\Rightarrow$ $x = \frac{3}{2}$ and $y = 5$

Let length of *BC* be *a* and that of *CD* be *b*

Then $\tan\theta = \frac{a/2}{b/2} = \frac{a}{b} = 8$.

Also $ab = 8$

$\Rightarrow$ $a = 8$, $b = 1$.

So equations of sides are $y = 1$, $y = 9$, $x = 1$ and $x = 2$.

17. The lines $(m-2)x + (2m-5)y = 0$; $(m-1)x + (m^2-7)y - 5 = 0$ and $x + y - 1 = 0$ are

a) concurrent for three value of 'm' b) concurrent for one value of 'm'

c) concurrent for no value of 'm' d) are parallel for m = 3

Key. C,D

Sol. $$\Delta = \begin{vmatrix} m-2 & 2m-5 & 0 \\ m-1 & m^2-7 & -5 \\ 1 & 1 & -1 \end{vmatrix} = 0 \qquad \Rightarrow (m-3)(m^2-m+2) = 0$$

For m = 3, the lines become parallel

18. If the points $(-2,1), (3,4), (a^2, a)$ lie on the same side of the straight line $6x - 7y - 3 = 0$ then *a* can lie in

A) $(0,1)$ B) $\left(\frac{6}{5}, \frac{7}{5}\right)$ C) $(2,4)$ D) $(-1,1)$

Key. A,B

Sol. $6a^2 - 7a - 3 < 0 \Rightarrow a \in \left(-\frac{1}{3}, \frac{3}{2}\right)$

19. If the straight line $3x+4y=24$ intersect the axes at A and B and the straight line $4x+3y=24$ intersect the axes at C and D then points A, B, C, D lie on

(a) the circle (b) the parabola(c) an ellipse (d) the hyperbola

Key. A,B,C,D

Sol. Equation of the curve passing through all four points A, B, C, D can be written as

$(3x+4y-24)(4x+3y+-24)+\lambda xy=0$

Now for different values of λ we will get different curves.

20. If $x^2+2hxy+y^2=0 \ (h\neq 1)$ represents the equations of the straight lines through the origin which make an angle α with the straight line $y+x=0$ then

a) $\sec 2\alpha = h$ b) $\cos\alpha = \sqrt{\frac{l+h}{2h}}$ c) $m_1+m_2=-2\sec 2\alpha$ d) $\cot\alpha=\sqrt{\frac{h+1}{h-1}}$

Key. A,B,C,D

Sol. Let $x^2+2hxy+y^2=0$ be given by

$y=m_1x$ & $y=m_2x$

$m_1+m_2=-2h$

slope of y +x = -1

$\tan\alpha=\left|\frac{m_1+1}{1-m_1}\right|$, $\tan\alpha=\left|\frac{m_2+1}{1-m_2}\right|$

$\tan\alpha=\frac{m_1+1}{1-m_1}$ & $\tan\alpha=-\left(\frac{m_2+1}{1-m_2}\right)$

(for +ve signs, in both gives the same value but $m_1\neq m_2$).

$\Rightarrow m_1=\frac{\tan\alpha-1}{\tan\alpha+1}, m_2=\frac{\tan\alpha+1}{\tan\alpha-1}$

$m_1+m_2=-2\sec 2\alpha$

$\Rightarrow h=\sec 2\alpha$

$\cos 2\alpha=\frac{1}{h}$

$2\cos^2\alpha-1=\frac{1}{h}$

$$\Rightarrow \cos\alpha=\left(\frac{1+h}{2h}\right)^{\frac{1}{2}} \Rightarrow \cot\alpha=\sqrt{\frac{h+1}{h-1}}$$

21. If $6a^2-3b^2-c^2+7ab-ac+4bc=0$ then the family of lines $ax+by+c=0, |a|+|b|\neq 0$ is concurrent at

a) $(-2,-3)$ b) $(3,-1)$

c) $(2,3)$ d) $(-3,1)$

Key. A,B

Sol. $(2a+3b-c)(3a-b+c)=0$

$\Rightarrow -2a-3b+c=0 \; or \; 3a-b+c=0$

Straight lines

Assertion Reasoning Type

A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct Explanation for Statement-1

B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

C) Statement-1 is True, Statement-2 is False

D) Statement-1 is False, Statement-2 is True

1. Statement 1: Consider the point $A(0,1)$ and $B(2,0)$ and $'P'$ be a point on the line $4x+3y+9=0,$ then coordinates of $'P'$ such that $|PA-PB|$ is maximum is $\left(\frac{-12}{5},\frac{17}{5}\right)$

Statement 2: $|PA-PB|\le|AB|$

Key. D

Sol. Equation of AB is $y-1=\frac{0-1}{2-0}(x-0)\Rightarrow x+2y-2=0$

$|PA-PB|\le|AB|$

Thus $|PA-PB|$ is maximum when A,B and P are collinear.

2. Statement-1:The points $(-17,21),(15,-11),(71,-67)$ are collinear.

Statement-2 : Given three points $(x_1,y_1),(x_2,y_2)\,and\,(x_3,y_3)$, if the value of the determinant $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ equals zero, then the points are collinear.

KEY : A

HINT: CONCEPTUAL

3. Statement-1: A chord y = mx + c of the curve $3x^2 - y^2 - 2x + 4y = 0$, which passes through the point (1, –2), subtend a right angle at the origin.

Statement-2: Lines represented by the equation

$(3c + 2m)x^2 - 2(1 + 2m)xy + (4 - c)y^2 = 0$ are perpendicular if c + m + 2 = 0.

Key: a

Hint: Statement-2 is true as the sum of the coefficients of x^2 and y^2 = 3c + 2m + 4 – c = 0 $\Rightarrow$ c + m + 2 = 0 so the lines are perpendicular if c + m + 2 = 0.

In statement-1, let the equation of the chord be y = mx + c, then equation of the pair of lines joining the origin to the points of intersection of the chord and the curve is

$$3x^2 - y^2 - 2x\left(\frac{y-mx}{c}\right) + 4y\left(\frac{y-mx}{c}\right) = 0$$

$\Rightarrow$ $(3c + 2m)x^2 - 2(1 + 2m)xy + (4 - c)y^2 = 0$

which are at right angles if c + m + 2 = 0 (using statement-2) and since the line y = mx + c passes through (1, –2), c + m + 2 =0. So statement-1 is also true.

4. Statement 1: The image of the line $2x-y-1=0$ with respect to $3x-2y+4=0$ is $22x-19y+77=0$

Statement 2: The image of the line $lx+my+n=0$ with respect to the line $ax+by+c=0$ is $\left(a^2+b^2\right)\left(lx+my+n\right)-2\left(la+mb\right)\left(ax+by+c\right)=0$

1) Statement -1 is true , statement -2 is true, statement – 2 is a correct explanation for statement – 1
2) Statement -1 is true , statement -2 is true, statement – 2 is not a correct explanation for statement – 1
3) Statement -1 is true , statement -2 is false
4) Statement -1 is false , statement -2 is true

Key. 1

Sol. Conceptual

5. Statement – 1 : If algebraic sum of perpendicular distances from (–2, 0), (3, 1) and (4, 2) to the line ax + by + c = 0 is zero then line must pass through $\left(\frac{5}{3},1\right)$

Because

Statement – 2 : If algebraic sum of perpendicular distances from $A_i(x_i, y_i)$ i = 1, 2, 3, ….n to ax + by +c = 0 is zero then line must pass through centroid of polygon having vertices at (x_i, y_i).

Key. A

Sol. $\sum_{i=1}^{n}\frac{a_0x_i+by_i+c}{\sqrt{a^2+b^2}}=0$

$$a\left(\sum x_i\right)+b\left(\sum y_i\right)+nc=0$$

$$a\frac{\sum_{i=0}^{n}x_i}{n}+b\frac{\sum_{i=0}^{n}y_i}{n}+c=0$$

6. Statement I : The point of intersection of the lines joining A(2,3), B(-1,2) and C(-2,1),D(3,4)is an internal point of $\overline{AB}$

Statement II : A(2,3),B(-1,2) are on opposite sides of the line through C(-2,1) and D(3,4)

Key. A

Sol. The line through C,D is $3x-5y+11=0.\ L_A=2>0, L_B=-3<0.$

7. Statement - 1: The image of the curve $x^2=4y$ in the line $x+y=2$ is

$$(y-2)^2+4(x-2)=0.$$

Statement - 2: $x^2=4y$ is symmetric with respect to the line $x+y=2$.

Key. C

Sol. $P(2t,t^2)$. Find locus of image of P w.r.t the line $x+y=2$.

8. **STATEMENT-1:** The vertices of a triangle are $A\left(x_1, x_1\tan\theta_1\right)$, $B\left(x_2, x_2\tan\theta_2\right)$ and $C\left(x_3, x_3\tan\theta_3\right)$. If the circumcentre of the triangle ABC coincides with origin and orthocentre $H\left(x^1, y^1\right)$ then $\frac{y^{|}}{x^{|}}=\frac{\sin\theta_1+\sin\theta_2+\sin\theta_3}{\cos\theta_1+\cos\theta_2+\cos\theta_3}$

STATEMENT-2: In a triangle circumcentre, centroid and orthocentre are collinear.

Key. A

Sol. since circumcentre is origin and OA = OB = OC = r

OA = $x_1^2+x_1^2\sin^2\theta_1=x_1\sec\theta_1$

$\therefore x_1=r\cos\theta_1$

similarly, $A\left(r\cos\theta_1, r\sin\theta_1\right)$, $B\left(r\cos\theta_2, r\sin\theta_2\right)$, $C\left(r\cos\theta_3, r\sin\theta_3\right)$

cicumcentre (o), centroid (G), and orthocentre (H) are collinear

$\Rightarrow$ slope OH = slope GO

$$\Rightarrow \frac{y^2-0}{x^2-0}=\frac{\left(y\ coordinate\ of\ G\right)-0}{\left(x\ coordinate\ of\ G\right)-0}$$

$$\Rightarrow \frac{y^1}{x^1}=\frac{\sin\theta_1+\sin\theta_2+\sin\theta_2}{\cos\theta_1+\cos\theta_2+\cos\theta_3}$$

Straight lines

Comprehension Type

Passage – 1:

In a ΔPQR, with $PQ = r$, $QR = p$, $PR = q$ the cosine values of the angles are given by

$\cos P = \dfrac{q^2 + r^2 - p^2}{2qr}$; $\cos Q = \dfrac{p^2 + r^2 - q^2}{2pr}$; $\cos R = \dfrac{p^2 + q^2 - r^2}{2pq}$, and the area of ΔPQR is

$\Delta = \dfrac{1}{2} pq \sin R = \dfrac{1}{2} qr \sin P = \dfrac{1}{2} pr \sin Q$. Let ABCD be a parallelogram whose diagonal equations are $AC \equiv x + 2y - 3 = 0$; $BD \equiv 2x + y - 3 = 0$. If $AC = 4 units$, and area of ABCD = 8 sq units, and $\angle BPC$ is acute where P is point of intersection of diagonals AC, BD, then

1. The length of other diagonal BD is

A) $\dfrac{10}{3}$ B) 2

C) $\dfrac{20}{3}$ D) $\dfrac{11\sqrt{2}}{3}$

Key. C

2. The length of side AB is equal to

A) $\dfrac{2\sqrt{58}}{3}$ B) $\dfrac{4\sqrt{58}}{3}$ C) $\dfrac{\sqrt{58}}{3}$ D) $\dfrac{4}{3}\sqrt{58}$

Key. A

3. The length of BC is equal to

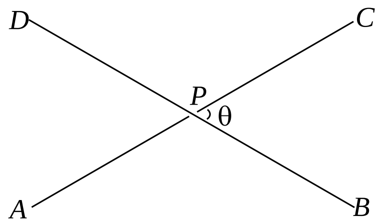
A) $\dfrac{4\sqrt{10}}{3}$ B) $\dfrac{2\sqrt{10}}{3}$ C) $\dfrac{8\sqrt{10}}{3}$ D) $\dfrac{\sqrt{10}}{3}$

Key. B

Sol. 1, 2 & 3

Let P be the point of inter section

$$\tan\theta = \left|\frac{\frac{-1}{2} + 2}{1 + 1}\right| = \frac{3}{4} \Rightarrow \sin\theta = 3/5$$



area of $\Delta CPB = \dfrac{1}{2} PCPB \sin\theta = 2 \rightarrow PB = \dfrac{10}{3}$

$$\Rightarrow BD = \frac{20}{3}$$

$$\cos(\pi-\theta)=\frac{-4}{5}=\frac{4+\frac{100}{9}-AB^2}{2\times 2\times\frac{10}{3}}\Rightarrow AB=\frac{2\sqrt{58}}{3}$$

again from $\Delta CPB,\quad BC=\frac{2\sqrt{10}}{3}$

Passage – 2:

Let the equations of two straight lines L_1, L_2 be respectively be $x-5=\frac{y-3}{5}=\frac{z-15}{2}$ and $\frac{x}{2}=\frac{y+1}{5}=\frac{z+6}{3}$. A,B are two distinct points on the x – axis such that two straight lines l_1, l_2 both perpendicular to the x – axis (l_1 through A, l_2 through B) are drawn so as to intersect both L_1, L_2.

4. Direction ratios of one of the lines l_1, l_2 are

A) $(0,3,1)$ B) $(0,4,-3)$ C) $(0,5,-2)$ D) $(0,2,3)$

5. If θ is the acute angle between the lines l_1, l_2 and $\cos\theta=\frac{\lambda}{5\sqrt{794}}$ then $\lambda=$

A) 42 B) 53 C) 61 D) 64

6. The shortest distance between the lines l_1, l_2 is

A) $\frac{105}{4}$ B) $\frac{127}{5}$ C) $\frac{119}{6}$ D) $\frac{128}{7}$

Sol. 4,5,6. (B,C,B)

Let $(t,0,0)$ be a point on the x axis through which a straight line L is drawn perpendicular to the x axis and intersecting both the lines L_1, L_2. D.R's of L may be taken as $(0,1,\lambda)$

L and L_1 are coplanar $\Rightarrow \begin{vmatrix} 5-t & 3 & 15 \\ 1 & 5 & 2 \\ 0 & 1 & \lambda \end{vmatrix}=0$

L and L_1 are coplanar $\Rightarrow \begin{vmatrix} t & 1 & 6 \\ 2 & 5 & 3 \\ 0 & 1 & \lambda \end{vmatrix}=0$

Solving we get $(\lambda, t) = \left(\frac{-3}{4}, 2\right)$ *or* $\left(\frac{13}{25}, \frac{137}{5}\right)$

Passage – 3:

ABC is a triangle right angled at A with vertices A,B,C in the anti-clockwise sense in that order. $A=(1,2),\ B=(-3,1)$ and vertex C lies on the X – axis. BCEF is a square with vertices B,C,E,F in the clockwise sense in that order. ACD is an equilateral triangle with vertices A,C,D in the anti-clockwise sense in that order.

7. Slope of AF is

A) $\frac{7}{10}$ B) $\frac{7}{9}$ C) $\frac{9}{10}$ D) $\frac{11}{10}$

8. The abscissa of centroid of ΔBCE is

A) -1 B) $\frac{-1}{2}$ C) $\frac{-1}{3}$ D) $\frac{-2}{3}$

9. If $D=(\alpha,\beta)$ then $(4\beta-4)^2 =$

A) 2 B) 3 C) 4 D) 5

Sol. 21,22,23 (D,C,B)

$$C=\left(\frac{3}{2},0\right),\ F=\left(-4,\frac{-7}{2}\right),\ E=\left(\frac{1}{2},\frac{-9}{2}\right),\ D=\left(\frac{5+4\sqrt{3}}{4},\frac{4+\sqrt{3}}{4}\right)$$

Passage – 4:

ABCD is a parallelogram whose side lengths are $a\ \&\ b\ (a \neq b)$. The angular bisectors of interior angles are drawn to intersect one another to form quadrilateral. Let $'\alpha'$ be one angle of parallelogram.

10. The area of the quadrilateral formed by the angular bisectors is

A) $\frac{1}{2}(a-b)^2 \sin\frac{\alpha}{2}$ B) $\frac{1}{2}(a-b)^2 \sin\alpha$

C) $\frac{1}{2}(a-b)^2 \cos\frac{\alpha}{2}$ D) $\frac{1}{2}(a-b)^2 \cos\alpha$

Key. B

Sol. The quadrilateral formed by angular bisectors is a rectangle. Whose sides are

$\left|(a-b)\right|\sin\frac{\alpha}{2}, \left|(a-b)\right|\cos\frac{\alpha}{2}$

$S = ab\sin\alpha$

$Q = \frac{1}{2}(a-b)^2\sin\alpha$

$\frac{S}{Q} = \frac{2ab}{(a-b)^2} \Rightarrow \frac{a}{b} = \frac{S+Q+\sqrt{Q^2+2QS}}{S}$

11. If $'S'$ is the area of the given parallelogram and Q is the area of the quadrilateral formed by the angular bisectors then ratio of the larger side to smaller side of the parallelogram is

A) $\frac{(S+Q)}{S}$ B) $\frac{S+Q+\sqrt{2QS}}{S}$ C) $\frac{S+Q+\sqrt{Q^2+2QS}}{S}$ D) $\frac{S+Q+\sqrt{Q^2-2QS}}{S}$

Key. C

12. The sides of the quadrilateral formed by the angular bisectors where $(a>b)$

A) $(a-b)\sin\frac{\alpha}{2}, (a-b)\cos\frac{\alpha}{2}$ B) $(a+b)\sin\frac{\alpha}{2}, (a+b)\cos\frac{\alpha}{2}$

C) $(a-b)\sin\alpha, (a-b)\cos\alpha$ D) $(a+b)\sin\alpha, (a+b)\cos\alpha$

Key. A

Passage – 5:

The sides of a triangle ABC satisfy the relations a +b-c =2 and 2ab-c^2 = 4 and f(x) = ax^2+bx+c.

13. Area of the triangle ABC in square units, is

a) $\sqrt{3}$ b) $\frac{\sqrt{3}}{4}$ c) $\frac{9\sqrt{3}}{4}$ d) $4\sqrt{3}$

14. If $x \in [0,1]$ then maximum value of f(x) is

a) 3/2 b) 2 c) 3 d) 6

15. The radius of the circle escribed opposite to the angle A is

a) 1 b) $\sqrt{3}$ c) $\frac{\sqrt{3}}{2}$ d) $\frac{1}{\sqrt{3}}$

Sol. 22. Ans. (a)

23. Ans. (d)

24. Ans. (b)

. $a+b-c=2$ and $2ab-c^2=4$

$\therefore a = b = c$

Passage – 6:

A man at the crossing of two roads $x-2y-4=0$, $2x-y-4=0$ starts walking along the bisector of the acute angle between the roads and after covering a distance $\sqrt{2}$ units reaches the bank of a straight river at right angles to his path L .

16. Find the equation of bank

(a) $x+y=0$ (b) $x-y=0$ (c) $x+y+2=0$ (d) $3x-4y+5=0$

17. origin lies in

(a) acute angle bisector (b) obtuse angle bisector

(c) cannot be said (d) None

18. Find the co-ordinates of the point where his path meets the bank

(a) $\left(\frac{4}{3},\frac{2}{3}\right)$ (b) $\left(\frac{4}{3},\frac{-4}{3}\right)$ (c) $\left(\frac{1}{2},\frac{-1}{2}\right)$ (d) $\left(\frac{1}{3},\frac{-7}{3}\right)$

Sol. 16. (C) Writing the equations of the roads such that constant terms are positive
-x + 2y + 4 = 0 , -2x + y + 4 = 0
$a_1 a_2 + b_1 b_2 = 4 > 0$. Origin lies in the obtuse angle.

17. (B) Bisectors of acute angle is $\frac{-x+2y+4}{\sqrt{(-1)^2+4}} = -\frac{-2x+y+4}{\sqrt{(-2)^2+1}}$

3x – 3y – 8 = 0.

18. (D) Solving x + y +2 = 0
3x – 3y – 8 = 0

P $\left(\frac{1}{3},\frac{-7}{3}\right)$

Passage – 7:

The vertex A of triangle ABC is $(3,-1)$. The equation of median BE and angular bisector CF are $6x+10y-59=0$ $x-4y+10=0$ and. Then

19. The equation of AB must be

a) $x+y=2$ b) $18x+13y=41$ c) $23x+y=70$ d) $x+4y=0$

20. Slope of the side BC must be

a) $\frac{1}{7}$ b) $\frac{1}{9}$ c) $\frac{-2}{9}$ d) None of these

21. The length of the side AC must be

a) $\sqrt{83}$ b) $\sqrt{85}$ c) $\sqrt{71}$ d) None of these

18,19,20 (B,C,A)

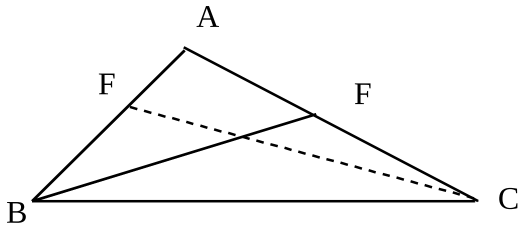


Image of A with respective CF lies an BC

Let C = (4K-10,K)

Mid point of AC lies on BE find K.

Passage – 8:

A straight line $'L'$ is drawn through the origin and parallel to the tangent to the curve $f(x,y)=0$ at an arbitrary point $'M'$ on the curve. $'P'$ is the point of intersection of the line $'L'$ and the straight line parallel to the Y–axis and passing through the point $'M'$.

22. If $f(x,y)\equiv y-\log_b x$ then the locus of $'P'$ is a

A) Straight line B) Parabola

C) Circle D) Central conic

Key. A

Sol. Let $M=(x,y)$. Then equation of $'L'$ is $Y=\left(\frac{dy}{dx}\right)X$

$\therefore$ Coordinates of $'P'$ are $\left(x, x\frac{dy}{dx}\right)$

23. If $f(x,y)\equiv y^2-4ax$ then the locus of $'P'$ is a

A) Straight line B) Parabola

C) Circle D) Central conic

Key. B

24. If $f(x,y)\equiv y-\sqrt{a^2-x^2}+a\,ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$ then the locus of $'P'$ is a

A) Straight line B) Parabola

C) Circle D) Central conic

Key. C

Passage – 9:

A straight line L is drawn through the origin and parallel to the tangent to the curve $f(x, y) = 0$ at an arbitrary point M on the curve. P is the point of intersection of the line L and the straight line parallel to the Y-axis and passing through the point M.

25. If $f(x, y) \equiv y - \log_b x$ then the locus of P is a

A) straight line B) parabola C) circle D) central conic

26. If $f(x, y) \equiv y^2 - 4ax$ then the locus of P is a

A) straight line B) parabola C) circle D) central conic

27. If $f(x, y) \equiv y - \sqrt{a^2 - x^2} + a \ln \frac{a + \sqrt{a^2 - x^2}}{x}$ then the locus of P is a

A) straight line B) parabola C) circle D) central conic

KEY : A-B-C

HINT

Question nos: 25 – 27

Let $M = (x, y)$. Then equation of L is $Y = \left(\frac{dy}{dx}\right) X$

$\therefore$ coordinates of P are $\left(x, x\frac{dy}{dx}\right)$

Passage – 10:

Let $A\left(\frac{1}{2}, 0\right), B\left(\frac{3}{2}, 0\right), C\left(\frac{5}{2}, 0\right)$ be the given points and P be a point satisfying

max $\left(PA + PB,\ PB + PC\right) < 2$.

28. P lies inside

(A) A circle and an ellipse (B) A circle and a hyperbola

(C) A parabola and an ellipse (D) None of these

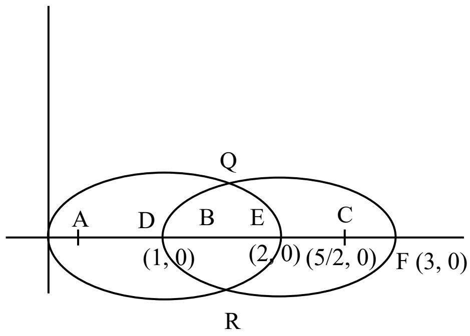
Key: D

Hint: Passage – 1 (33 to 35)

Equation of ellipse satisfying $PA + PB = 2$ with major

$axis = 2\ is\ \frac{(x-1)^2}{1} + \frac{y^2}{3/4} = 1$

Similarly the equation of the ellipse satisfying $PB + PC = 2 is\ \frac{(x-2)^2}{1} + \frac{y^2}{3/4} = 1$



Required region is DQERD $y=\frac{\sqrt{3}}{2}\sqrt{1-(x-2)^2}$

Required area $=4\int_1^{3/2} ydx=2\sqrt{3}\int_1^{3/2}\sqrt{1-(x-2)^2}dx$

$=\sqrt{3}\left(\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right)$

29. The locus of P is symmetric about

(A) Origin

(B) The line y = x

(C) The line $2x-3=0$

(D) None of these

Key: C

30. The area of the region of the point P is

(A) $\sqrt{2}\left(\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right)$

(B) $\sqrt{3}\left(\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right)$

(C) $2\left(\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right)$

(D) None of these

Key: B

Passage – 31:

Let ABC be a triangle in which AD is the angular bisector of $\angle$A.

Then D divides BC in the ratio of sides containing the angle i.e., AB : AC

If length of BC is x, then

BD = $\left(\frac{AB}{AB+AC}\right)x$ and DC = $\left(\frac{AC}{AB+AC}\right)x$ and

(i) if PA= PB then P lies on perpendicular bisector of the line joining points A and B.

(ii) if P is equidistant from two non–parallel lines $\Rightarrow$ P lies on angular bisector of given two lines.

For a square ABCD having vertices at A(0, 0), B(2, 0), C(0, 2) and D(2, 2).

Answer the following questions :

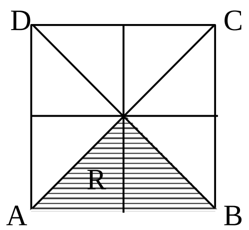
31. Let $d(P, AB)$ represent the distance of point P from side AB. Then the area of region R consisting of all points P inside the square satisfying d $(P, AB) \le \min\{d(P, BC), d(P, CD), d(P, DA)\}$ is

(A) $\frac{1}{2}$ sq units. (B) 1 sq units.

(C) 2 sq units. (D) 4 sq units.

KEY : B

SOL : $A = \frac{1}{4} \times 4 = 1$



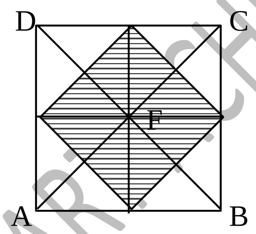
32. Let the centre of the square of F. Then area of region R consisting of all points P inside the square satisfying PF ≤ min{PA, PB, PC, PD} is

(A) $\frac{1}{2}$ (B) 1

(C) 2 (D) $\frac{1}{4}$ sq. units

KEY : C

SOL :



$= \frac{1}{2} \times 4 = 2$

33. The area of the region R consisting of all points P inside the square such that distance of P from nearest diagonal is less than distance of P from any of the sides is

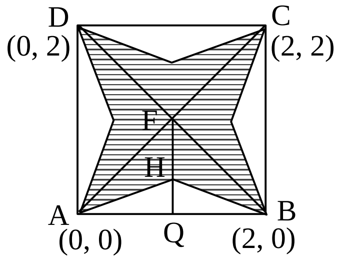
(A) $2\sqrt{2}$ sq. units (B) $2\left(2\sqrt{2}\right)$ sq. units

(C) $4\left(2-\sqrt{2}\right)$ sq. units (D) $\frac{2-\sqrt{2}}{2}$ sq. units

KEY : C)

SOL : (C)

According to the given situation, region R is shown in figure.



$AF = \sqrt{2}$, $AQ = 1$

Since AH is the angle bisector,

$\therefore$ FH : HQ = $\sqrt{2}:1$

$\therefore$ HQ = $\frac{1}{1+\sqrt{2}}$

$\therefore$ Area of AHB = $\frac{1}{2} \times 2 \times \frac{1}{1+\sqrt{2}} = \sqrt{2}-1$

Passage – 32:

A curve C which is not a straight line lies in the first quadrant. The tangent at any point on C meets the positive directions of the coordinate axes at the points A,B. Let $'d'$ be the minimum distance of the curve C from the origin O.

34. If $OA + OB = 1$ then $d =$

A) $\frac{1}{2\sqrt{2}}$ B) $\frac{1}{2}$ C) $\frac{1}{\sqrt{2}}$ D) $\sqrt{2}$

35. If $OA.OB = 4$ then $d =$

A) $\frac{1}{2\sqrt{2}}$ B) $\frac{1}{2}$ C) $\frac{1}{\sqrt{2}}$ D) $\sqrt{2}$

36. If $AB = 1$ then $d =$

A) $\frac{1}{2\sqrt{2}}$ B) $\frac{1}{2}$ C) $\frac{1}{\sqrt{2}}$ D) $\sqrt{2}$

Key. A, D, B

Sol.

34. (A) Equation of tangent at (x, y) is $Y - y = p(X - x)$

Where $p = \frac{dy}{dx}$. Then $OA = x - \frac{y}{p}$ and $OB = y - px$ $OA + OB = 1 \Rightarrow y = px + \frac{p}{p-1}$

35. (D) $OA.OB = 4 \Rightarrow y = px + 2\sqrt{-p}$

36. (B) $AB = 1 \Rightarrow y = px - \dfrac{p}{\sqrt{1+p^2}}$

Passage – 33:

The equations of two equal sides AB and AC of an isosceles triangle ABC are $x + y = 5$ and $7x - y = 3$.If the area of triangle ABC is 5 units then answer the following

37. Which of the following cannot represent BC

A. $3y - x = 21$ **B.** $y + 3x = 7$ **C.** $3y - x = 1$ **D.** $y + 3x = 2$

Key. B

38. If $\left(\frac{1}{2}, \frac{1}{2}\right)$ is one of the possible positions of C then which of the following cannot be possible position of B

A. $\left(\frac{3}{2}, \frac{15}{2}\right)$ **B.** $\left(\frac{-3}{2}, \frac{13}{2}\right)$ **C.** $\left(\frac{7}{2}, \frac{3}{2}\right)$ **D.** None

Key. A

Sol. 37. equation AB is $x + y = 5$, AC is $7x - y = 3$

$$A = (1,4), \sin A = \frac{4}{5}, Area = \frac{1}{2} AB^2 \sin A = 5$$

$$\therefore AB = AC = \frac{5}{\sqrt{2}} = r$$

If α, β are inclintations of AB,AC

$$\sin\alpha = \frac{1}{\sqrt{2}}, \cos\alpha = \frac{-1}{\sqrt{2}}, \sin\beta = \frac{7}{5\sqrt{2}}, \cos\beta = \frac{1}{5\sqrt{2}}$$

$$B = (1 \pm \frac{5}{\sqrt{2}} \cos\alpha, 4 \pm \frac{5}{\sqrt{2}} \sin\alpha); C = (1 \pm \frac{5}{\sqrt{2}} \cos\beta, 4 \pm \frac{5}{\sqrt{2}} \sin\beta)$$

$$B = \left(\frac{7}{2}, \frac{3}{2}\right) or \left(\frac{-3}{2}, \frac{13}{2}\right); C = \left(\frac{1}{2}, \frac{1}{2}\right) or \left(\frac{3}{2}, \frac{15}{2}\right)$$

$$\therefore B \neq \left(\frac{3}{2}, \frac{15}{2}\right)$$

$y + 3x = 7$ can not be the equal of BC

38. equation AB is $x + y = 5$, AC is $7x - y = 3$

$A=(1,4),\sin A=\frac{4}{5}, Area=\frac{1}{2}AB^2\sin A=5$

$\therefore AB=AC=\frac{5}{\sqrt{2}}=r$

If α,β are inclintations of AB,AC

$\sin\alpha=\frac{1}{\sqrt{2}},\cos\alpha=\frac{-1}{\sqrt{2}},\sin\beta=\frac{7}{5\sqrt{2}},\cos\beta=\frac{1}{5\sqrt{2}}$

$B=(1\pm\frac{5}{\sqrt{2}}\cos\alpha,4\pm\frac{5}{\sqrt{2}}\sin\alpha);C=(1\pm\frac{5}{\sqrt{2}}\cos\beta,4\pm\frac{5}{\sqrt{2}}\sin\beta)$

$B=\left(\frac{7}{2},\frac{3}{2}\right)\ or\ \left(\frac{-3}{2},\frac{13}{2}\right);C=\left(\frac{1}{2},\frac{1}{2}\right)\ or\ \left(\frac{3}{2},\frac{15}{2}\right)$

$\therefore B\neq\left(\frac{3}{2},\frac{15}{2}\right)$

$y+3x=7$ can not be the equal of BC

Passage – 34:

: Consider a triangle PQR with co-ordinates of its vertices as P(-8, 5), Q(-15, - 19), and R (1, -7).

39. The distance between the orthocenter and circumcentre of the triangle PQR

A. 25 / 2 **B.** 29 / 2 **C.** 37 / 2 **D.** 51 / 2

KEY. A

SOL. P, Q, R are vertices of a right angle triangle P(-8, 5) Q(-15, -19) and R(1, -7)

$\angle PRQ=90^0$

Orthocentre = R (1, -7)

Circumcentre = mid point PQ

$S=\left(\frac{-23}{2},-7\right),\ RS=\left|1+\frac{23}{2}\right|=25/2$

Circum diameter = PQ = 25

Circum radius = 25/2

Radius of nine point circle = $\frac{25/2}{2} = \frac{25}{4}$ (1/2 circum radius)

40. Radius of nine point circle is

A. 25 **B.** 25 / 2 **C.** 25 / 4 **D.** 25 / 8

KEY. C

SOL. P, Q, R are vertices of a right angle triangle P(-8, 5) Q(-15, -19) and R(1, -7)

$\angle PRQ = 90^0$

Orthocentre = R (1, -7)

Circumcentre = mid point PQ

$S = \left(\frac{-23}{2}, -7\right),\ RS = \left|1 + \frac{23}{2}\right| = 25/2$

Circum diameter = PQ = 25

Circum radius = 25/2

Radius of nine point circle = $\frac{25/2}{2} = \frac{25}{4}$ (1/2 circum radius)

Passage – 35:

A(4, 0), B(-4, 0) are two points then the locus of P such that

41. PA + PB = 10 is

A. $\frac{x^2}{25} + \frac{y^2}{16} = 1$ **B.** $\frac{x^2}{25} + \frac{y^2}{9} = 1$ **C.** $\frac{x^2}{16} + \frac{y^2}{9} = 1$ **D.** $\frac{x^2}{9} + \frac{y^2}{25} = 1$

KEY. B

42. PA = PB is

A. x = 0 **B.** y = 0 **C.** x – y = 0 **D.** x + y = 0

KEY. A

43. Area of triangle PAB is 4sq units is

A. x + y – 1 = 0 **B.** x – 1 = 0 **C.** y – 1 = 0 **D.** x – y + 1 =0

KEY. C

SOL. 41. A(4,0), B(-4, 0) are two point t

Let P = (x, y)

PA + PB = 10 = 2a

a = 5

(h, k) = (4, 0), (-h, k) = (-4, 0)

Equation of locus is

$$\frac{x^2}{a^2}+\frac{(y-k)^2}{a^2-h^2}=1$$

$$\frac{x^2}{25}+\frac{y^2}{25-16}=1,\ \frac{x^2}{25}+\frac{y^2}{9}=1$$

42. PA = PB $\Rightarrow$ $PA^2 = PB^2$

$$(x-y)^2+y^2=(x+y)^2+y^2$$

x = 0

43. Area of triangle PAB = 4

$$\frac{1}{2}|x(0)+4(0-y)-4(y-0)|=4$$

$$|-4y-4y|=8$$

8y = 8

y = 1, y-1 = 0

Passage – 36:

Let ABCD is a square with sides of unit length. Points E and F are taken on sides AB and AD respectively so that AE = AF. Let P be a point inside the square ABCD.

44. The maximum possible area of quadrilateral CDFE is

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{5}{8}$ (D) $\frac{3}{8}$

Key. C

45. The value of $(PA)^2 - (PB)^2 + (PC)^2 - (PD)^2$ is equal to

(A) 3 (B) 2 (C) 1 (D) 0

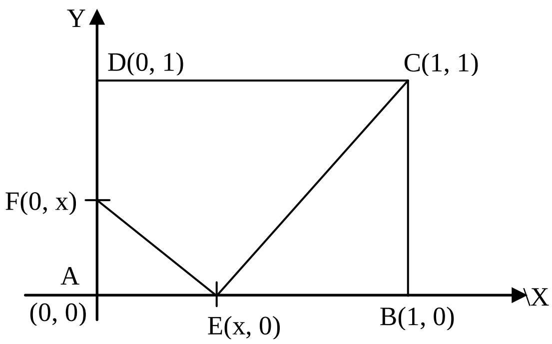
Key. D

46. Let a line passing through point A divides the square ABCD into two parts so that area of one portion is double the other, then the length of portion of line inside the square is

(A) $\frac{\sqrt{10}}{3}$ (B) $\frac{\sqrt{13}}{3}$ (C) $\frac{\sqrt{11}}{3}$ (D) $\frac{2}{\sqrt{3}}$

Key. B

Sol. 44. (C)

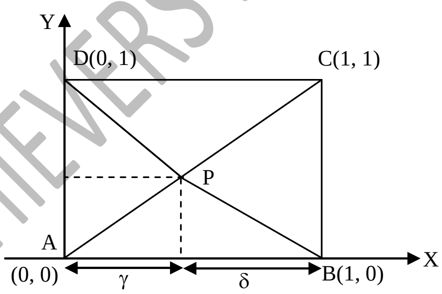


Area of CDFE, A = $1-\frac{1}{2}x^2-\frac{1}{2}(1-x)$

= $\frac{2-x^2-1+x}{2}=\frac{1+x-x^2}{2}$

A_{max} = $\frac{1+\frac{1}{2}-\frac{1}{4}}{2}=\frac{5}{8}$ at x = $\frac{1}{2}$

45. (D)



$(PA)^2 - (PB)^2 + (PC)^2 - (PD)^2 = (\alpha^2+\gamma^2)-(\alpha^2+\delta^2)+(\delta^2+\beta^2)-(\gamma^2+\beta^2)$

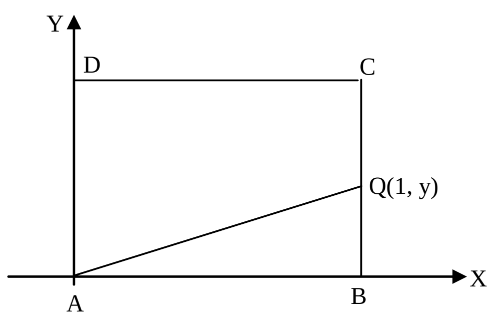
46. (B)

$\frac{1}{2}y(1) = \frac{1}{3}(1)$

y = $\frac{2}{3}$

LAQ =

$\sqrt{(1)^2+\left(\frac{2}{3}\right)^2}=\frac{\sqrt{13}}{3}$



Passage – 37:

ABC is an isosceles triangle with AB = AC = 5 and BC = 6. Let P be a point inside the triangle ABC such that the distance from P to the base BC equals the geometric mean of the distance to the sides AB and AC.

47. The locus of the point P is

(A) a semi circle (B) a minor arc of a circle
(C) a major arc of circle (D) a complete circle

Key. B

48. The minimum distance of the point A from the locus of the point P is

(A) $\frac{5}{2}$ (B) $\frac{3}{2}$
(C) 2 (D) none of these

Key. A

49. If the tangents to the locus at B and C intersect at point P, then the area of the triangle PBC is

(A) 10 (B) 12
(C) 14 (D) 18

Key. B

Sol. 47. (B)

Let the triangle ABC has vertices A(0, 4), B(–3, 0) and C(3, 0).

Let the point P be (α, β).

Equation of line AC is 4; x + 3y – 12 = 0 and the equation of line AB is 4x – 3y + 12 = 0

$\Rightarrow \ |\beta| = \sqrt{\frac{|(4\alpha+3\beta-12)(-4\alpha+3\beta-12)|}{\sqrt{25}\times\sqrt{25}}}$

$\Rightarrow \ 2(\alpha^2 + \beta^2) + 9\beta - 18 = 0$ (1)

Since point P lies inside triangle ABC its locus is the minor arc of circle.

48. (A)

The circle (1) cuts the y–axis at

$R\left(0, \frac{3}{2}\right)$ and S(0, –6)

Hence, minimum distance of A from the locus OS

$AR = 4 - \frac{3}{2} = \frac{5}{2}$.

49. (B)

Equation of the tangents to (1) at C is obtained by

T = 0

$\Rightarrow$ The tangent is $2(3x + 0y) + \frac{9}{2}y - 18 = 0$

$\Rightarrow \ 4x + 3y - 12 = 0$

which is same as line AC. Hence tangents at B and C intersect at A.

$\Rightarrow$ Area of the triangle PBC is $\frac{1}{2}$. 6 . 4 = 12.

Passage – 38:

Let O(0, 0), A(2, 0) and B$\left(1, \frac{1}{\sqrt{3}}\right)$ be the vertices of a triangle. Let R be the region consisting of all these points P inside triangle OAB which satisfy $d(P, OA) \le \min\{d(P, OB), d(P, AB)\}$ where d denotes the perpendicular distance from the point to the corresponding line.

50. For any point P the maximum value of $d(P, OB) =$

(a) 2 (b) 1 (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

Key. B

51. Area of the region R is

(a) $2+\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $4-\sqrt{3}$ (d) $2-\sqrt{3}$

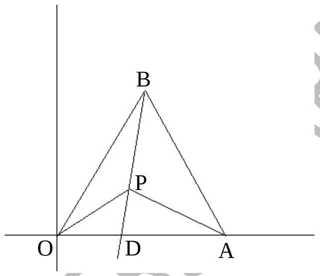
Key. D

52. Area of the region inside the triangle OAB except the region R is

(a) $\frac{2(2-\sqrt{3})}{\sqrt{3}}$ (b) $\frac{2(2+\sqrt{3})}{\sqrt{3}}$ (c) $2\sqrt{3}$ (d) $\frac{2}{\sqrt{3}}$

Key. A

Sol. 50-52.



$d(P, OA) \le d(P, OB)$ when $d(P, OB) \le d(P, AB)$
$d(P, OA) \le d(P, AB)$ when $d(P, OB) \ge d(P, AB)$
Use angular bisector concept
R = area of triangle OPD + Area of triangle APD

Passage – 39:

Let y = f(x) be the given continous function let (x_1, y_1) be any point on it. As (x_1, y_1) is moving on the curve corresponding to each position of (x_1, y_1). There is a image point with respect to the straight line $ax+by+c=0\,(a,b,c \in R)$ then the locus of image points is called image of the curve y = f(x) with respect to the given line $ax+by+c=0$. Then attempt the following questions.

53. Image of the curve $y = 1+\frac{1}{x^2}$ w.r.t the line $x+y+2=0$ is

a) $(x-2)^2(y+3)+1=0$ b) $(x+3)(y+2)^2+1=0$

c) $(x-3)(y-3)^2+1=0$ d) none

Key. B

54. Image of the line $2x+3y=1$ w.r.t the line $3x+2y+1=0$ is

a) $46x+9y+37=0$ b) $46x-9y+37=0$ c) $46x-9y-37=0$ d) $46x+9y-37=0$

Key. A

55. Image of the curve $x^2+2xy+2y^2=1$ w.r.t to the line $x+y-2=0$ is

a) $2x^2+2xy-y^2-12x-8y-19=0$ b) $2x^2-2xy+y^2-12x-8y-19=0$

c) $2x^2+2xy+y^2-12x-8y+19=0$ d) $2x^2-2xy-y^2-12x+8y-19=0$

Key. C

Sol. 53. $\frac{x-x_1}{1}=\frac{y-y_1}{1}=\frac{-2(x_1+y_1+2)}{2}$

$$x=x_1+\frac{-2(x_1+y_1+2)}{2}$$

$$y=y_1+\frac{-2(x_1+y_1+2)}{2}$$

$$\therefore y-\frac{2(x_1+y_1+2)}{2}=1+\frac{1}{\left(x-\frac{2(x_1+y_1+2)}{2}\right)^2}$$

$$\Rightarrow (x+3)(y+2)^2+1=0$$

54. $\frac{x-x_1}{3}=\frac{y-y_1}{2}=\frac{-2(3x_1+2y_1+1)}{13}$

$$x=x_1+\frac{-6(3x_1+2y_1+1)}{13}$$

$$y=y_1+\frac{-4(3x_1+2y_1+1)}{13}$$

$$\therefore 2\left\{x-6\left(\frac{3x+2y+1}{13}\right)\right\}+3\left\{y-4\left(\frac{3x+2y+1}{13}\right)\right\}=1$$

$$\Rightarrow 46x+9y+37=0$$

55. $\frac{x-x_1}{1}=\frac{y-y_1}{1}=\frac{-2(x_1+y_1-2)}{2}$

$$x=x_1+\frac{-2(x_1+y_1-2)}{2}$$

$$y=y_1+\frac{-2(x_1+y_1-2)}{2}$$

$$\therefore \left\{x-2\frac{(x+y-2)}{2}\right\}^2+2\left\{x-\frac{2(x+y-2)}{2}\right\}\left\{y-\frac{2(x+y-2)2}{2}\right\}+2\left\{y-\frac{2(x+y-2)}{2}\right\}^2=1$$

$$\Rightarrow 2x^2+2xy+y^2-12x-8y+19=0$$

Passage – 40:

Let us consider the situation when axes are inclined at an angle 'w'. If co-ordinates of a point P are (x_1, y_1) then PN = x_1, PM = y_1 where PM is parallel to y-axis and PN is parallel to x-axis equation of straight line, through P and makes an angle θ with x-axis is

$$y-y_1=\frac{\sin\theta}{\sin(w-\theta)}(x-x_1),$$

because $R\theta=y-y_1$, $P\theta=x-x_1$ & $\frac{P\theta}{\sin(w-\theta)}=\frac{R\theta}{\sin\theta}$

This can be written

$$y-y_1=m(x-x_1)$$

Where $m=\frac{\sin\theta}{\sin(w-\theta)}$

If slope of line is m, then angle of inclination of line with x-axis is given by

$$\tan\theta=\frac{m\sin w}{1+m\cos w}.$$

56. The axes being inclined at an angle of 60^0, then inclination of the line y = 2x + 5 with the x-axis is

a) 30^0 b) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ c) $\tan^{-1}(2)$ d) 60^0

Key. B

57. The axes being inclined at an angle of 60^0, then angle between the two straight lines y = 2x + 5 and 2y + x + 7 = 0

a) 90^0 b) $\tan\left(\frac{5}{3}\right)$ c) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ d) $\tan^{-1}\left(\frac{5}{\sqrt{3}}\right)$

Key. D

58. The axes being inclined at an angle of 30^0 then equation of straight line which makes an angle of 60^0 with the positive direction of x-axis and having x-intercept equal to 2, is

a) $y-\sqrt{3}x=2\sqrt{3}$ b) $\sqrt{3}y=x$ c) $y+\sqrt{3}x=2\sqrt{3}$ d) $y+2x=0$

Key. C

Sol. 56. w = 60^0, m = 2

$$\tan\theta = \frac{m\sin w}{1+m\cos w} = \frac{\sqrt{3}}{2}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

57. w = 60^0, m_1 = 2, m_2 = $\frac{-1}{2}$

$$\tan\theta_1 = \frac{m_1\sin w}{1+\cos w} = \frac{\sqrt{3}}{2}$$

$$\tan\theta_2 = -\frac{1}{\sqrt{3}}$$

angle between the lines $\tan\phi = \left|\frac{\tan\theta_1 - \tan\theta_2}{1+\tan\theta_1 + \tan\theta_2}\right|$

$$= \tan^{-1}\left(\frac{5}{\sqrt{3}}\right)$$

58. $m = \frac{\sin 60^0}{\sin\left(30^0 - 60^0\right)} = -\sqrt{3}$

$\therefore$ equation of the line

$$y - 0 = -\sqrt{3}\left(x-2\right)$$

$$y + \sqrt{3}x = 2\sqrt{3}$$

Passage – 41:

ABC is a triangle right angled at A with vertices A, B, C in the anti clockwise sense in that order. A = (1, 2), B = $(-3,1)$ and vertex C lies on the X-axis. BCEF is a square with vertices B, C, E, F in the clock wise sense in that order. ACD is an equilateral triangle with vertices A, C, D in the anti clockwise sense in that order.

59. Slope of AF is

A) $\frac{7}{10}$ B) $\frac{7}{9}$ C) $\frac{9}{10}$ D) $\frac{11}{10}$

Key. D

60. The abscissa of centroid of Δ BCE is

A) -1 B) $\frac{-1}{2}$ C) $\frac{-1}{3}$ D) $\frac{-2}{3}$

Key. C

61. If $D = (\alpha, \beta)$ then $(4\beta - 4)^2$ =

A) 2 B) 3 C) 4 D) 5

Key. B

Sol. **(59 – 61)**

$$C=\left(\frac{3}{2},0\right),\ F=\left(-4,\frac{-7}{2}\right),\ E=\left(\frac{1}{2},\frac{-9}{2}\right),\ D=\left(\frac{5+4\sqrt{3}}{4},\frac{4+\sqrt{3}}{4}\right)$$

Straight lines

Integer Answer Type

1. A point P(x, y) moves is such a way that [x + y + 1] = [x] (where [.] greatest integer function) and $x \in (0, 2)$. Then the area representing all the possible positions of P equals

Key. 2

Sol.

If $x \in (0, 1)$

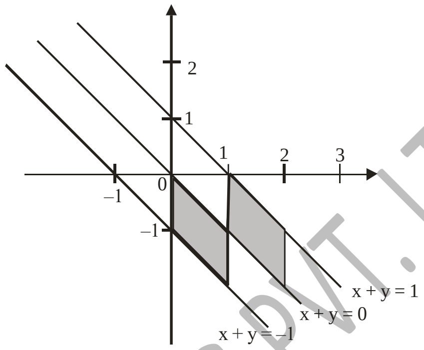
Then $-1 \le x + y < 0$

And if $x \in [1, 2)$

$0 \le x + y < 1$

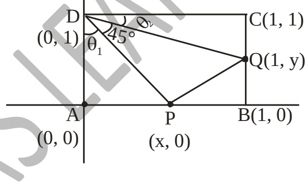
Required area

$= 4\left(\frac{1}{2}.1.\sqrt{2}\sin\frac{\pi}{4}\right) = 2$ sq. units



2. ABCD is a square of side length 1 unit. P and Q are points on AB and BC such that $\angle PDQ = 45°$. Find the perimeter of ΔPBQ.

Key. 2



Sol.

$\tan\theta_1 = x$ and $\tan\theta_2 = 1 - y$

Since, $\theta_1 + \theta_2 = 45°$

$\Rightarrow \frac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1 \tan\theta_2} = 1$

$\Rightarrow \frac{x + (1 - y)}{1 - x(1 - y)} = 1 \Rightarrow y = \frac{2x}{1 + x}$ ...(i)

Now, Perimeter$= 1 - x + y + \sqrt{(1 - x)^2 + y^2}$

By using (i), we get

Perimeter=2

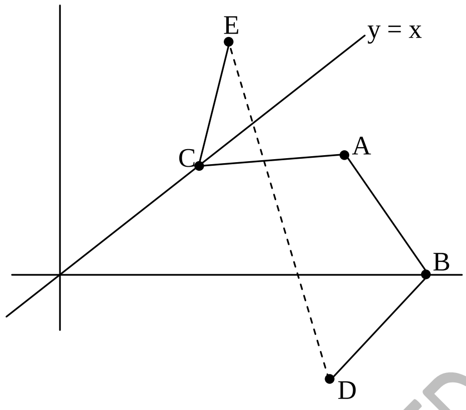
3. Given a point (2, 1). If the minimum perimeter of a triangle with one vertex at (2, 1), one on the x-axis, and one on the line y = x, is k, then [k] is equal to (where [ ] denotes the greatest integer function)

Key. 3

Sol.

Let, $D = (2, -1)$ be the reflection of A in x-axis, and let $E = (1, 2)$ be the reflection in the line y = x. Then AB = BD and AC = CE, so the perimeter of ABC is

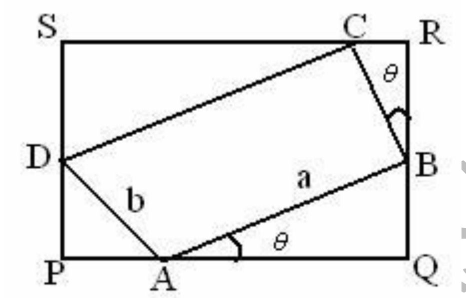
DB + BC + CE $\geq$ DE = $\sqrt{1+9} = \sqrt{10}$



4. ABCD and PQRS are two variable rectangles, such that A, B, C and D lie on PQ, QR, RS and SP respectively and perimeter $'x'$ of $ABCD$ is constant. If the maximum area of $PQRS$ is 32, then $\frac{x}{4} =$

Key. 4

Sol.



$2(a+b) = x$ (a constant)

Area of $PQRS = (b\sin\theta + a\cos\theta)(a\sin\theta + b\cos\theta)$

$= ab + \frac{a^2+b^2}{2}.\sin 2\theta \leq \frac{(a+b)^2}{2} = \frac{x^2}{8}$

$\therefore \frac{x^2}{8} = 32 \Rightarrow x = 16$

5. The no of points on the line 3x + 4y = 5 which are at a distance of $\sec^2\theta + 2\operatorname{cosec}^2\theta, \theta \in R$ from the point (1,3) is

Key. 2

Sol. $\sec^2\theta + 2\operatorname{cosec}^2\theta \geq (\sqrt{2}+1)^2$

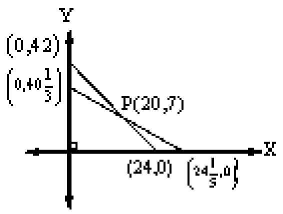
Perpendicular distance from $(1,3)$ to the line $3x + 4y = 5$ is 2

$\therefore$ No. of lines =2

6. The area of the triangular region in the first quadrant bounded on the left by the y-axis, bounded above by the line 7x + 4y = 168 and bounded below by the line 5x + 3y = 121 is A, then the value of 3A/10 is

Key. 5

Sol. The given lines 7x + 4y = 168 and 5x + 3y = 121 intersect at P(20, 7)



$\therefore$ Area of shaded region

$$A = \frac{1}{2}(42 - 40\frac{1}{3})20$$

$$= \frac{1}{2}(\frac{5}{3})20 = \frac{50}{3}$$ (square units)

7. Consider a ΔOAB formed by the point $O(0,0)$, $A(2,0)$, $B(1,\sqrt{3})$, $P(x,y)$ be any arbitrary interior point of ΔOAB moving in such way that $d(P,OA)+d(P,AB)+d(P,OB)=\sqrt{3}$ where $d(P,OA)$, $d(P,AB)$, $d(P,OB)$ represents perpendicular distances of P from the sides OA, AB & OB respectively. If area of the region representing all possible positions of P is 'k' then $k\sqrt{3}=$

Key. 3

Sol. $\Delta OAB = \Delta OPA + \Delta OPB + \Delta PAB = \frac{\sqrt{3}}{4}.4$

Since, the triangle is an equilateral Δ

$\therefore \sqrt{3} = \frac{1}{2}.2\left(d(OA)+d(P,OB)+d(P,AB)\right)$

$\therefore$ For all positions of P $d(P,OA)+d(P,OB)+d(P,AB)=\sqrt{3}$

$\therefore k=\sqrt{3} \Rightarrow \sqrt{3}k=3$

8. In a triangle ABC, AB is parallel to y – axis, BC is parallel to x – axis, centroid is at (2, 1). If median through C is x – y = 1, then the slope of median through A is ……..

Key: 4

Hint: Let B (a, b) , C (c, b) , A (a, d)

Then D, (mid point of BC) is $\left(\frac{a+c}{2}, b\right)$

E, (mid point of AB) is $\left(a, \frac{b+d}{2}\right)$

Given slope of CE = 1 $\Rightarrow \frac{b-\frac{b+d}{2}}{c-a}=1 \Rightarrow \frac{(b-d)}{c-a}=2$

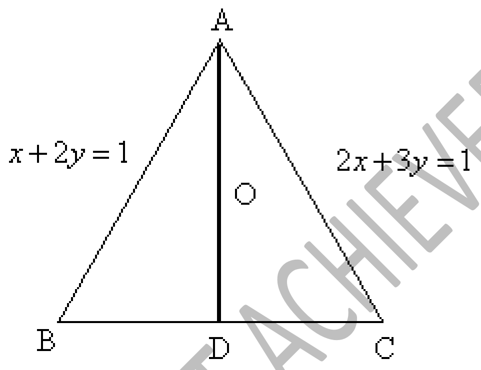
Slope of AD = $\frac{b-d}{\frac{a+c}{2}-a}=2\frac{(b-d)}{c-a}=4$

9. If the orthocentre of the triangle formed by $2x+3y-1=0, x+2y-1=0, ax+by-1=0$ is at the origin then $\frac{b-a}{4}=$

Key. 4

Sol. Solving $2x+3y=1, x+2y=1, A=(-1,1)$

Orthocentre =(0,0) $\Rightarrow$ slope if altitude AD =-1



Equation of BC is x – y = k

Solving $x-y=k, x+2y=1$

$B=\left(\frac{1+2k}{3}, \frac{1-k}{3}\right)$

Slope of OB $=\frac{1-k}{1+2k}$, slope of AC = -2/3

$\therefore \frac{1-k}{1+2k}=\frac{3}{2} \Rightarrow k=\frac{-1}{8}$

$equ\ of\ \ BC\ is\ x-y+\frac{1}{8}=0$

$\Rightarrow -8x+8y-1=0$

$\Rightarrow a=-8, b=8$

10. The area of the rhombus ABCD is 24. The equation of the diagonal BD is 4x+3y+2=0 and $A=(3,2)$. The length of the side of the rhombus is

Key. 5

Sol. Let AC,BD intersect at P

$AP=\frac{12+6+2}{\sqrt{16+9}}=4$

Area if $\Delta ABD = AP\times BP=\frac{24}{2}=12$

BP=3

$AB=\sqrt{AP^2+BP^2}=5$

11. In triangle ABC the equation of alltitudes AM and BN are $x+5y-3=0, x+y-k=0$ .If the altitude CL is given by $3x-y-1=0$, then k =

Key. 1

Sol. Solving the altitudes AM,CL

Orthocenter=$\left(\frac{1}{2},\frac{1}{2}\right)$ lies on x + y – k = 0

k =1

12. The co-ordinate axes are rotated through an angle θ about the origin in anticlockwise sense. If the equation $2x^2+3xy-6x+2y-4=0$ change to $aX^2+2hXY+bY^2+2gX+2fY+c=0$ then (a + b) is equal to

Key. 2

Sol. $x=x\cos\theta-y\sin\theta,\ y=x\cos\theta+y\sin\theta$

Substitute x and y in the equation

$a=2\cos^2\theta+\cos\theta\sin\theta,\ b=2\sin^2\theta-\cos\theta\sin\theta$

$a+b=2$

13. Origin is shifted to (1, 2) then the equation $y^2 - 8x - 4y + 12 = 0$ changes as $y^2 = 4ax$. Then *a* =

KEY. 2

SOL. Transformed Equation is

$$(y+2)^2 - 8(x+1) - 4(y+2) + 12 = 0$$

$$y^2 = 8x \Rightarrow y^2 = 4ax$$

a = 2

14. If the sum of the squares of the sides of triangle is 16 then the sum of the squares of the medians is 3k. Then k is

KEY. 4

SOL. $AB^2 + BC^2 + CA^2 = \frac{4}{3}(AD^2 + BE^2 + CF^2)$

$16 = \frac{4}{3}(3k)$, k = 4

15. The centroid of the triangle formed by (a, b) ,(b, c), (c,a) is the origin and $a^3 + b^3 + c^3 = kabc$ then k is

KEY. 3

SOL. $G = \left(\frac{a+b+c}{3}, \frac{a+b+c}{3}\right) = (0,0)$

$$a+b+c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$$

K = 3

16. If A = (0, 4), B = (0, -4) and $|PA - PB| = 6$ and the locus of P is $7y^2 - 9x^2 = 9k$. Then k is

KEY. 7

SOL. A=(0, 4) = (h, k), B=(0, -4) = (h, -k)

$$|PA - PB| = 6 = 2a$$

$a = 3$

Equation of locus is

$$\frac{(x-h)^2}{(a^2 - k^2)} + \frac{y^2}{a^2} = 1$$

$$\frac{x^2}{9-16} + \frac{y^2}{9} = 1$$

$7y^2 - 9x^2 = 63$, $9k = 63$, $k = 7$

17. The locus of the point which is collinear with the points (3, 4) and (-4, 3) is ax + by + c = 0 then a + 3b + c =

KEY. 5

SOL. P(x, y), A(3, 4), B(-4, 3)

Area of triangle PAB = 0

$$\frac{1}{2}|x(4-3)+3(3-y)-4(y-4)|=0$$

$$x+9-3y-4y+16=0$$

$$x-7y+25=0 \Rightarrow ax+by+c=0$$

$$a+3b+c=1-21+25=5$$

18. When (0, 0) is shifted to (3, -3) then the points P(5, 5), Q(-2, 4) and R (7, -7) are changes as A, B, C and if the area of triangle ABC is k then $\frac{86}{k}$ is

KEY. 2

SOL. P(5, 5), Q(-2, 4), R(7, -7)

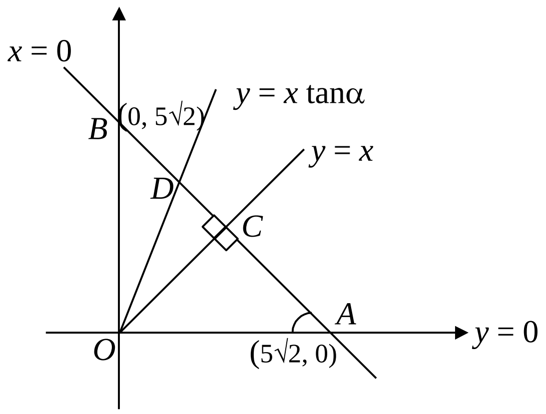
Area of triangle PQR= Area of triangle ABC

$k = 43$, $\frac{86}{k} = \frac{86}{43} = 2$

19. The intercepts made on the line $x + y = 5\sqrt{2}$ by the lines $y = x\tan\theta$; $\theta = 0, \frac{\pi}{4}, \alpha, \frac{\pi}{2}$ are in A.P. Then $\tan\alpha =$

Key. 5

Sol.



Since AC, CD and DB are in A.P. and let their lengths be taken as $a - d$, a and $a + d$ respectively.

$\therefore \quad a - d + a + a + d = AB = 10$

$\therefore \quad a = \frac{10}{3}$

In ΔOAC

$OA^2 = OC^2 + AC^2$

$50 = OC^2 + OC^2$

$\therefore \quad OC = 5$

In ΔOCD

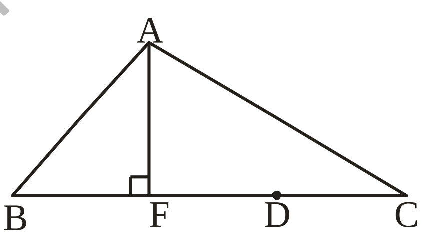
$$\tan(\alpha - 45) = \frac{CD}{OC}$$

$$\frac{\tan\alpha - 1}{\tan\alpha + 1} = \frac{2}{3} \quad \Rightarrow \quad \alpha = \tan^{-1}5$$

20. Consider a triangle ABC with BC = 3. Choose a point D on BC such that BD = 2. Find the value of $AB^2 + 2AC^2 - 3AD^2$.

Key. 6

Sol.



Drop the perpendicular from A to BC and let F be its foot. Further more, suppose BF = x and AF = y.

Then, by Pythagorean theorem,

$AB^2 = AF^2 + BF^2 = x^2 + y^2$

$AC^2 = CF^2 + AF^2 = (3 - x)^2 + y^2$

$AD^2 = DF^2 + AF^2 = (2 - x)^2 + y^2$

$\Rightarrow AB^2 + 2AC^2 - 3AD^2 = 6$

21. A triangle ABC is given with A(1,-2), B(-4,3), C(3,2). Find sum of all possible coordinates of 'P' in the plane of the ΔABC such that area of ΔPAB = ΔPBC = ΔPAC.

Key. 4

Sol. Centroid is the obvious point, Construct 3 parallograms by taking any two sides of the triangle as adjacent sides.

22. If the lines $ax + by + p = 0, x\cos\alpha + y\sin\alpha - p = 0\,(p \neq 0)$ and $x\sin\alpha - y\cos = 0$ are concurrent and the first two lines include an angle $\frac{\pi}{4}$, then $a^2 + b^2$ is equal to

Key. 2

Sol. Since the lines are concurrent

$$\Rightarrow \begin{vmatrix} a & b & p \\ \cos\alpha & \sin\alpha & -P \\ \sin\alpha & -\cos\alpha & 0 \end{vmatrix} = 0 \Rightarrow 1 + a\cos\alpha + b\sin\alpha = 0$$

$$\cos\frac{\pi}{4} = \frac{|a\cos\alpha + b\sin\alpha|}{\sqrt{a^2 + b^2}} \Rightarrow \sqrt{a^2 + b^2} = \sqrt{2} \Rightarrow a^2 + b^2 = 2$$

Straight lines

Matrix-Match Type

1. Match the following Column-I with Column-II

Column I	Column II
A. The distance between the lines $(x+7y)^2+4\sqrt{2}(x+7y)-42=0$ is	P. 2
B. If the sum of the distance of a point from two perpendicular lines in a plane is 1, then its locus is |x| + |y| = k, where k is equal to	Q. 7
C. If 6x + 6y + m = 0 is acute angle bisector of line x + 2y + 4 = 0 and4x + 2y – 1 = 0, then m is equal to	R. 3
D. Area of the triangle formed by the lines $y^2-9xy+18x^2=0$ and y = 6 is	S. 1

Key. A-P; B-S; C-Q; D-R;

Sol. A) $(x+7y)^2+7\sqrt{2}(x+7y)-42=0$

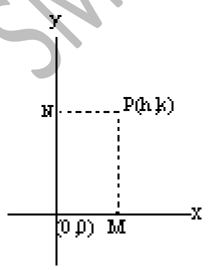
$$\Rightarrow (x+y)[x+7y+7\sqrt{2}]-3\sqrt{2}(x+y)-42=0$$

$$\Rightarrow (x+y)[x+7y+7\sqrt{2}]-3\sqrt{2}(x-7y+7\sqrt{2})=0$$

$$\Rightarrow (x+7y+7\sqrt{2})(x+7y-3\sqrt{2})=0$$

$$x+7y+7\sqrt{2}=0 \text{ and } x+7y-3\sqrt{2}=0$$

$$\Rightarrow d=\left|\frac{7\sqrt{2}+3\sqrt{2}}{\sqrt{1+49}}\right|=\frac{10\sqrt{2}}{\sqrt{50}}=2$$



B)

Let two perpendicular lines are coordinate axes.

Then, PM + PN = 1

$\Rightarrow h+k=1$

Hence, the locus is x + y =1

But if the point lies in other quadrants also, then $|x|+|y|=1$. Hence, value of k is 1.

C) Angle bisector between the lines x + 2y +4 = 0 and 4x + 2y – 1 = 0

$$\frac{x+2y+4}{\sqrt{1+4}}=\pm\frac{(-4x+2y+1)}{\sqrt{16+4}}$$

$$\Rightarrow x+2y+4=\pm\frac{(-4x+2y+1)}{2}$$

$$\Rightarrow 2(x+2y+4)=\pm(-4x-2y+1)$$

Since $AA^{'}+BB^{'}<0$, so +ve sign gives acute angle bisector. Hence,

$$2x+4y+8=-4x-2y+1$$

$$\Rightarrow 6x+6y+7=0$$

$$\Rightarrow m=7$$

D) We have,

$$y^2-9xy+18x^2=0$$

Or $y^2-16xy-3xy+18x^2=0$

$$\Rightarrow y(y-6x)-3x(y-6x)=0$$

$\Rightarrow (y-3x)=0$ and $y-6x=0$

The third line is y = 6. Therefore, area of the triangle formed by these lines,

$$=\frac{1}{2}\begin{vmatrix}0 & 0 & 1\\ 1 & 6 & 1\\ 2 & 6 & 1\end{vmatrix}$$

$$=\frac{1}{2}|6-12|\ =3units^2$$

2. Column – I Column – II

A) The value of k for which $4x^2+8xy+ky^2=9$ is the equation of a pair of straight lines,is	P) 3
B) If the sum of the slopes of the lines given by $x^2-2Cxy-7y^2=0$ is four times their product, then the value of C is	Q) -3
C) If the gradient of one of the lines $x^2+hxy+2y^2=0$ is twice that of the other, then h =	R) 2
D) If the lines $ax^2+2hxy+by^2=0$ are equally inclined to the lines $ax^2+2hxy+by^2+\lambda(x^2+y^2)=0$ then the value of λ can be	S) 4

Key. A – s; B – r,s; C – q; D - q

Sol. A) The equation represents pair of lines if $(4)(k)(-9)-(-9)(4)^2=0$
$\Rightarrow k=4$

B) $m_1+m_2=4m_1m_2 \Rightarrow \frac{-2h}{b}=\frac{4a}{b} \Rightarrow \frac{-2(-c)}{-7}=\frac{4\times 1}{-7} \Rightarrow C=2$

C) $2m_2+m_2=\frac{-h}{2}, 2m_2^2=\frac{1}{2} \Rightarrow 2\left(\frac{-h}{6}\right)^2=\frac{1}{2} \Rightarrow h^2=9 \Rightarrow h=\pm 3$

D) The angular bisectors of $ax^2+2hxy+by^2+\lambda(x^2+y^2)=0$ is

$h(x^2-y^2)-(a-b)xy=0$

Which are angular bisectors of $ax^2+2hxy+by^2=0$. The two pairs are equally inclined for any λ

3. Match the following:

Let ABC be a isosceles triangle with $AB=AC$. If AB lies along $x+y=10$ and AC lies along $7x-y=30$ and area of triangle be 20 sq. units.

	Column -I		Column -II
(A)	Coordinate of point B cannot be	(p)	$(10,0)$
(B)	Coordinate of point C cannot be	(q)	$(4,-2)$
(C)	Centroid of ΔABC cannot be	(r)	$\left(\frac{-5}{2},\frac{5}{2}\right)$
(D)	Circumcentre of ΔABC cannot be	(s)	$\left(3,\frac{13}{3}\right)$
		(t)	$(0,10)$

Key. A$\rightarrow$q,r, s; B$\rightarrow$p,r,s,t; C$\rightarrow$p,q,r,t; D$\rightarrow$p,q,s,t

Sol. $\sin A = \frac{4}{5}, AB = AC = 5\sqrt{2}$

So, point B will be $(0,10),(10,0)$ and point C will be $(4,-2),(6,12)$

Centroid will be $\left(3,\frac{13}{3}\right),....$

Circumcentre will be $\left(\frac{-5}{2},\frac{5}{2}\right)$

4. Match the following :

Column I		Column II	
(A)	The equation of the straight lines passing through the point (2,1) and parallel to the lines represented by $2x^2 - y^2 - xy + 9x - 3y + 10 = 0$	(p)	$x^2 - y^2 - 24xy = 0$
(B)	The equation of the pair of straight lines joining the origin to the points of intersection of the line $3x + 4y - 5 = 0$ and the curve $2x^2 + 3y^2 = 5$ is	(q)	$x^2 - \frac{xy}{2} - \frac{y^2}{2} - \frac{7x}{2} + 2y + \frac{5}{2} = 0$
(C)	The equation of the ellipse whose one focus is $(2,1)$, the corresponding directrix is $2x - y + 3 = 0$ and the eccentricity is $\frac{1}{\sqrt{2}}$ is	(r)	$x^2 + 4xy + y^2 - 32x - 32y + 154 = 0$
(D)	The equation of the hyperbola for which the eccentricity is the square of the eccentricity of a rectangular hyperbola, one focus is (2,2) and the corresponding directrix is $x + y = 9$ is	(s)	$6x^2 + 4xy + 9y^2 - 52x - 14y + 41 = 0$

Key. (A – q),
(B – p),
(C – s),
(D – r)
(D – p)

Sol. (A) Since $2x^2 - xy - y^2 = (2x + y)(x - y)$

So the lines parallel to $2x + y = 0$ and $x - y = 0$ passing through $(2,1)$ is $2x + y - 5 = 0$ and $x - y - 1 = 0$

Their combined equation is $(2x + y - 5)(x - y - 1) = 0$

i.e. $2x^2 - y^2 - xy - 7x + 4y + 5 = 0$

i.e. $x^2 - \frac{y^2}{2} - \frac{xy}{2} - \frac{7}{2}x + 2y + \frac{5}{2} = 0$

(B) Using the concept of homogenisation we can find the required pair of lines as $2x^2 + 3y^2 = 5\left(\frac{3x+4y}{5}\right)^2$

i.e $10x^2 + 15y^2 = 9x^2 + 16y^2 + 24xy$

i.e $x^2 - y^2 - 24xy = 0$

(C) $PS^2 = e^2.PM^2$ (using focus-directrix property)

$\Rightarrow (x-2)^2 + (y-1)^2 = \frac{1}{2} \times \frac{(2x-y+3)^2}{5}$

$\Rightarrow 6x^2 + 9y^2 + 4xy - 52x - 14y + 41 = 0$

(D) $PS^2 = e^2.PM^2$ (Using focus-directrix property)

$\Rightarrow (x-2)^2 + (y-2)^2 = 4 \times \frac{(x+y-9)^2}{2} \quad \left(\because e = \left(\sqrt{2}\right)^2 = 2\right)$

$\Rightarrow x^2 + y^2 + 4xy - 32x - 32y + 154 = 0$

5. Let ABC be a isosceles triangle with AB = AC. If AB lies along x + y = 10 and AC lies along 7x - y = 30 and area of triangle be 20 sq. unit

Column I		Column II	
(A)	Coordinate of point B	(p)	(10, 0)
(B)	Coordinate of point C	(q)	(4, -2)
(C)	Centroid of ΔABC	(r)	$\left(\frac{-5}{2}, \frac{5}{2}\right)$
(D)	Circumcentre of ΔABC	(s)	$\left(3, \frac{13}{3}\right)$

Key: (A-p), (B-q), (C-s), (D-r)

Hint:

6. If $y = m_i x + \frac{1}{m_i}$ $(i = 1, 2, 3)$ represent three straight lines whose slopes are the roots of the equation $2m^3 - 3m^2 - 3m + 2 = 0$, then

Column I		Column II	
(A)	Algebraic sum of the intercepts made by the lines on x-axis	(P)	$\left(4\sqrt{2} + 9\sqrt{5}\right)/4$
(B)	Algebraic sum of the intercepts made by the lines on y-axis	(Q)	*3/2*

(C)	Sum of the distances of the lines from the origin	(R)	$-21/4$
(D)	Sum of the lengths of the lines intercepted between the coordinate axes	(S)	$\left(5\sqrt{2}+9\sqrt{5}\right)/10$

Key: A-R, B-Q, C- S, D-P

Hint: A-R, B-Q, C- S, D-P

Solving the equation $2m^3 - 3m^2 - 3m + 2 = 0$ we get, $2(m^3 + 1) - 3m(m + 1) = 0$

$\Rightarrow (m + 1)(2m^2 - 5m + 2) = 0$

$\Rightarrow (m + 1)(2m - 1)(m - 2) = 0 \Rightarrow m = -1, 1/2$ or 2.

Equation of the given lines can be written as $m_i^2 x - m_i y = -1$.

(A) Algebraic sum of the intercepts made by the lines on x-axis

$$= -\sum \frac{1}{m_i^2} = -\left[1+\frac{1}{4}+4\right] = -\frac{21}{4}$$

(B) Algebraic sum of the intercepts made by the lines on y-axis

$$= = \sum \frac{1}{m_i} = -1+2+\frac{1}{2} = \frac{3}{2}$$

(C) Let p_i denote the perpendicular distance of the line from the origin

then $p_i = \left|\frac{-1/m_i}{\sqrt{1+m_i^2}}\right| \Rightarrow \sum p_i = \frac{1}{\sqrt{1+1}} + \frac{2}{\sqrt{1+(1/4)}} + \frac{1/2}{\sqrt{1+4}}$

$$= \frac{1}{\sqrt{2}} + \frac{4}{\sqrt{5}} + \frac{1}{2\sqrt{5}} = \frac{1}{10}\left(5\sqrt{2}+9\sqrt{5}\right)$$

(D) l_i = length of the line intercepted between the coordinates are

$$= \sqrt{\left(\frac{1}{m_i^2}\right)^2 + \left(\frac{1}{m_i}\right)^2}$$

$$\sum l_i = \sqrt{1+1} + \sqrt{16+4} + \sqrt{\frac{1}{16}+\frac{1}{4}} = \sqrt{2} + 2\sqrt{5} + \frac{\sqrt{5}}{4} = \left(4\sqrt{2}+9\sqrt{5}\right)/4$$

7.

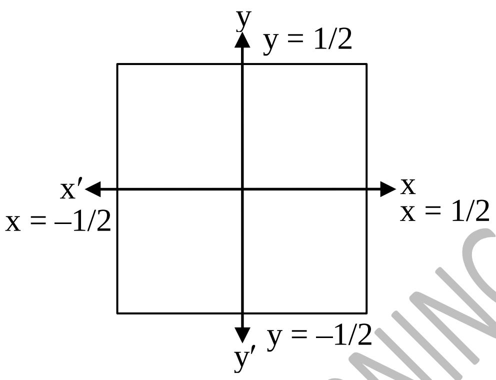
	Column I		Column II
(A)	The area bounded by the curve max. {|x|, |y|} = ½ is	(P)	0
(B)	If the point (a, a) lies between the lines |x + y| = 6, then [|a|] is (where [.] denotes the greatest integer function)	(Q)	1
(C)	Number of non–zero integral values of b for which the origin and the point (1, 1) lies on the same side of the st. line $a^2x + aby + 1 = 0$ for all $a \in R \sim \{0\}$ is	(R)	2

(D) If (α, α^2) lies inside triangle formed by the lines $2x + 3y - 1 = 0$, $x + 2y - 3 = 0$, $5x - 6y - 1 = 0$, then $[\alpha]$ is (where [.] denotes the greatest integer function) (S) –2

Key : A→ Q, B→ P, Q, R; C→ R, D→ P, S

Sol : (A) Q max $\{|x|, |y|\} = 1/2$

$$\begin{cases} |x| = 1/2 & \text{if} \quad |y| < 1/2 \\ |y| = 1/2 & \text{if} \quad |x| < 1/2 \end{cases}$$



∴ Required area = 1 × 1 = 1 sq unit

(B) The line y = x cuts the lines |x + y| = 6 i.e,

$$x + y = \pm 6$$

at $x = \pm 3$, y =

or (–3, –3) and (3, 3)

then $-3 < a < 3$

∴ $0 \le |a| < 3$

∴ $[|a|] = 0, 1, 2$

(C) Since (0, 0) and (1, 1) lie on the same side.

So, $a^2 + ab + 1 > 0$

Q Coefficient of a^2 is > 0

∴ $D < 0$

$b^2 - 4 < 0$

or $-2 < b < 2$

⇒ $b = -1, 0, 1$

∴ Number of non-zero values of b are 2 (b = –1 & b = 1)

(D) Let P (α, α^2) first draw the exact diagram of Δ ABC.

The point P (α, α^2) move on the curve $y = x^2$ for all α.

Now intersection of $y = x^2$ and $2x + 3y - 1 = 0$

or $2x + 3x^2 - 1 = 0$

∴ $x = -1, x = \dfrac{1}{3}$

Let intersection points D ≡ (–1, 1) and $E \equiv \left(\dfrac{1}{3}, \dfrac{1}{9}\right)$

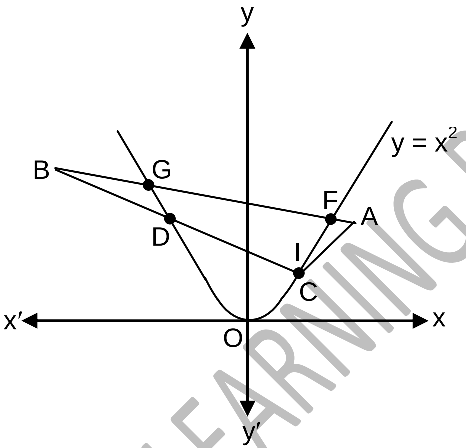
intersection of $y = x^2$ and $x + 2y - 3 = 0$

or $x + 2x^2 - 3 = 0$

$\therefore$ $x = 1, x = -3/2$

Let intersection points $F \equiv (1, 1)$

and $G \equiv \left(-\frac{3}{2}, \frac{9}{4}\right)$ and intersection of $y = x^2$



and $5x - 6y - 1 = 0$

or $5x - 6x^2 - 1 = 0$

$\therefore$ $x = \frac{1}{3}, x = \frac{1}{2}$

Let intersection points

$$H \equiv \left(\frac{1}{3}, \frac{1}{9}\right) \text{ and } I \equiv \left(\frac{1}{2}, \frac{1}{4}\right)$$

Thus , the points on the curve $y = x^2$ whose x-coordinates lies between $-3/2$ and -1 and $\frac{1}{2}$ and 1 lies within the triangle ABC.

Hence, $-\frac{3}{2} < \alpha < -1$ and $\frac{1}{2} < \alpha < 1$

i.e $\alpha \in \left(-\frac{3}{2}, -1\right) \cup \left(\frac{1}{2}, 1\right)$

$\therefore$ $[\alpha] = -2, 0$

8. If $y = m_i x + \frac{1}{m_i}, (i = 1, 2, 3)$ represent three Straight lines whose slopes roots of the equation $2m^3 - 3m^2 - 3m + 2 = 0$, then

	Column I		Column II

(A)	Algebraic sum of the intercepts made by the lines on x-axis is,	p)	$\frac{4\sqrt{2}+9\sqrt{5}}{4}$
(B)	Algebraic sum of the intercepts made by the lines on y-axis is,	(q)	$\frac{3}{2}$
(C)	Sum of the distances of the lines from origin is	(r)	$\frac{-21}{4}$
(D)	Sum of the lengths of the lines intercepted between the coordinate axes is	(s)	$\frac{5\sqrt{2}+9\sqrt{5}}{10}$
		(t)	0

Key. $A-r$
$B-q$
$C-s$
$D-p$

Sol. M = -1, ½, 2

a) $\sum \frac{-1}{M_i^2} = \frac{-21}{4}$

b) $\sum \frac{1}{M_i} = \frac{3}{2}$

c) $\sum \left| \frac{-1/M_i}{\sqrt{1+M_i^2}} \right| = \frac{5\sqrt{2}+9\sqrt{5}}{10}$

d) $\sum \sqrt{\left(\frac{1}{M_i^2}\right)^2 + \left(\frac{1}{M_i}\right)^2} = \frac{4\sqrt{2}+9\sqrt{5}}{10}$ 40. B- p,q,r,s

C-p,q,r

D- q,r,s,t

The other vertices of the triangle are (5,2$\sqrt{5}$) and $\left(5,-2\sqrt{5}\right)$

Therefore , the centroid is $\left(\frac{10}{3},0\right)$;

the circumcenter is $\left(\frac{9}{2},0\right)$

and the incenter is (3,0).

9. Consider the following linear equations in x and y

$ax + by + c = 0$

$bx + cy + a = 0$

$cx + ay + b = 0$

Match the condition in column I with statement in column II

	Column – I		Column – II
(A)	$a + b + c = 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(P)	Lines are identical
(B)	$a + b + c = 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(Q)	Lines represent the whole of the xy plane
(C)	$a + b + c \neq 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(R)	Lines are different and passing through a fixed point
(D)	$a + b + c \neq 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(S)	Lines are sides of a triangle

Key. A – q, B – r, C – p, D – s

Sol. Conceptual

10. The given consistent second degree equation $(a_1x + b_1y + c_1)(a_2x + b_2y + c_2) = k$ where $a_1b_1c_1a_2b_2c_2k \neq 0$ represents some member given in column–II under the condition given in column–I. Match them accordingly

Column – I	Column – II
(A) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	(p) a parabola
(B) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	(q) a hyperbola
(C) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	(r) a pair of lines
(D) $a_1 a_2 + b_1b_2 = 0$	(s) a circle

Key. (A – r); (B – r); (C – q); (D – q)

Sol. $(a_1x + b_1y + c_1)(a_2x + b_2y + c_2) = 0$ represents a pair of lines,
its $\Delta = 0$. Now for $(a_1x + b_1y + c_1)(a_2 x + b_2y + c_2) - k = 0$,
$\Delta' = \Delta - abk + kh^2 = k(h^2 - ab)$

If $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ then $h^2 = ab$, so $\Delta' = 0$

If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ then $h^2 > ab$, so, $\Delta' \neq 0$.

(A) a pair of parallel lines as $\Delta' = 0$, $h^2 = ab$

(B) a pair of parallel lines as $\Delta' = 0$, $h^2 = ab$

(C) a hyperbola as $\Delta' \neq 0$, $h^2 > ab$

(D) a rectangular hyperbola as $\Delta' \neq 0$, $a + b = 0$.