Inverse

Single Correct Answer Type

1. If x, y, z are natural numbers such that $\cot^{-1} x + cot^{-1} y = \cot^{-1} z$ then the number of ordered triplets (x,y,z) that satisfy the equation is _______

A) 0 B) 1 C) 2 D) Infinite solutions

Key. D

Sol. $x = 1 : y = 1$ is not a solution of the given equation

Suppose $(x, y) \neq (1,1)$

Then $\dfrac{\frac{1}{x}+\frac{1}{y}}{1-\frac{1}{xy}} = \dfrac{1}{z}$ $\Rightarrow x(z-y) = -(1+yz)$ $\Rightarrow x = -\dfrac{(1+yz)}{z-y}$

If $y = n+1 : z = n$ then $x = n^2 + n + 1$

All such numbers are solutions

2. The number of solutions of the equation $\tan^{-1}\left(\dfrac{x}{1-x^2}\right) + \tan^{-1}\left(\dfrac{1}{x^3}\right) = \dfrac{3\pi}{4}$ belonging to the interval $(0,1)$ is

A) 0 B) 1 C) 2 D) 3

Key: A

Hint: $\because x \in (0,1) \dfrac{x}{1-x} > 0; \dfrac{1}{x^3} > 0 \,\&\, \dfrac{x}{1-x^2}.\dfrac{1}{x^3} > 1$

$$\therefore \tan^{-1}\frac{x}{1-x^2} + \tan^{-1}\frac{1}{x^3} = \pi + \tan^{-1}\left(\frac{\frac{x}{1-x^2}+\frac{1}{x^3}}{1-\frac{x}{1-x^2}.\frac{1}{x^3}}\right) = \pi + \tan^{-1}\left(\frac{-1}{x}\right) = \frac{3\pi}{4}$$

$\Rightarrow x = 1(\text{not possible})$

3. Number of common points for the curves $\mathrm{y} = \sin^{-1}(2\mathrm{x}) + \tan^{-1}\left(\dfrac{1}{[2\mathrm{x}]}\right) + 2$ and $\mathrm{y} = \cos^{-1}(2\mathrm{x}+5) + 1$ is (where [.] denotes greatest integer function)

(A) 0 (B) 1 (C) 3 (D) 4

Key: A

Hint: As domain of first function is $\left[-\dfrac{1}{2}, 0\right) \cup \left\{\dfrac{1}{2}\right\}$ and domain of 2^{nd} function $[-3,-2]$ there is no common point

4. If $\sin^{-1}\left(\sin\dfrac{33\pi}{7}\right) + \cos^{-1}\left(\cos\dfrac{46\pi}{7}\right) + \tan^{-1}\left(-\tan\dfrac{13\pi}{8}\right) + \cot^{-1}\left(\cot\left(-\dfrac{19\pi}{8}\right)\right) = \dfrac{\mathrm{a}\pi}{\mathrm{b}}$

Where a and b are in their lowest form, then (a + b) equal to

(A) 17 (B) 20 (C) 23 (D)None of these

Key: B

Hint: On solving, we get $\frac{a\pi}{b} = \frac{13\pi}{7} \Rightarrow 13+7=20$.

5. If 0 < x < 1, the number of solutions of the equation
$\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}3x$ is

(a) 0 (b) 1 (c) 2 (d) 3

Key: b

Hint: The given equation can be written as
$\tan^{-1}(x-1)+\tan^{-1}(x+1)=\tan^{-1}3x-\tan^{-1}x$

$\Rightarrow \frac{2x}{2-x^2}=\frac{2x}{1+3x^2} \Rightarrow x+3x^3=2x-x^3$

$\Rightarrow 4x^3-x=0 \Rightarrow x(4x^2-1)=0$

$\Rightarrow$ x = 0, $x=\pm\frac{1}{2}$. Thus x = $\frac{1}{2}$

6. If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=3\pi$ then $x^2+y^2+z^2-xy-yz-zx$ equals to

A) 0 B) 1 C) 2 D) 3

Key. A

Sol. Since maximum value of $\cos^{-1}x$ is π

Therefore, $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=3\pi \Rightarrow \cos^{-1}x=\cos^{-1}y=\cos^{-1}z=\pi$

$\Rightarrow x=y=z=-1 \Rightarrow x^2+y^2+z^2-xy-yz-zx=0$

7. $\sum_{K=1}^{\infty} Tan^{-1}\left(\frac{1}{2K^2}\right)=\theta$, then $Tan\theta$ = _______

(1) 0 (2) 1 (3) $\sqrt{3}$ (4) ∞

Key. 2

Sol. $Tan^{-1}\left[\frac{(2K+1)-(2K-1)}{1+(2K+1)(2K-1)}\right]=Tan^{-1}(2K+1)-Tan^{-1}(2K-1)$

Expanding Σ we get, $Tan^{-1}1$

8. If $x\geq 1$, then $2Tan^{-1}x+Sin^{-1}\left(\frac{2x}{1+x^2}\right)=$ __________

(1) $4Tan^{-1}x$ (2) 0 (3) $\frac{\pi}{2}$ (4) π

Key. 4

Sol. When $x\geq 1$; $Sin^{-1}\frac{2x}{1+x^2}=\pi-2Tan^{-1}x$

9. $Cos^{-1}\ Cos(12)-Sin^{-1}(Sin12)$ = _________

(1) 0 (2) π (3) $8\pi-24$ (4) $8\pi+24$

Key. 3

Sol. $Cos^{-1}Cos(4\pi-12)-Sin^{-1}Sin(12-4\pi)$

10. If $Sin^{-1}\left(x-\frac{x^2}{2}+\frac{x^3}{4}.....\infty\right)+Cos^{-1}\left(x^2-\frac{x^4}{2}+\frac{x^6}{4}.....\infty\right)=\frac{\pi}{2}$ for $0<|x|<\sqrt{2}$.

Then x = ________

(1) $\frac{1}{2}$ (2) 1 (3) $-\frac{1}{2}$ (4) -1

Key. 2

Sol. $Sin^{-1}x+Cos^{-1}x=\frac{\pi}{2}$

$\therefore x-\frac{x^2}{2}+\frac{x^3}{4}.....\infty=x^2-\frac{x^4}{2}+\frac{x^6}{4}.....\infty$

11. The number of solutions of equation $Sin^{-1}\left(|x^2-1|\right)+Cos^{-1}\left(|2x^2-5|\right)=\frac{\pi}{2}$ is ______

(1) 1 (2) 0 (3) 3 (4) 2

Key. 4

Sol. $|x^2-1|=|2x^2-5|\Rightarrow x=\pm\sqrt{2}$

12. The number of real solutions of $Tan^{-1}\sqrt{x(x+1)}+Sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$ is _______

(1) 0 (2) 1 (3) 2 (4) infinite

Key. 3

Sol. $Cos^{-1}\frac{1}{\sqrt{x^2+x+1}}+Sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}\Rightarrow x^2+x+1=1$

13. $\sin^{-1}(\sin 5)>x^2-4x$ holds if

A) $x<2-\sqrt{9-2\pi}$ B) $-1<x<5$

C) $x\in(-\infty,-1)\cup(5,\infty)$ D) $x\in(2-\sqrt{9-2\pi},2+\sqrt{9-2\pi})$

Key. D

Sol. $\because \frac{3\pi}{2}<5<\frac{5\pi}{2}$ $\therefore \sin^{-1}(\sin 5)=5-2\pi$

Given $\sin^{-1}(\sin 5)>x^2-4x\Rightarrow x^2-4x<5-2\pi$

$\Rightarrow x^2-4x+(2\pi-5)<0$

Roots of $x^2-4x+2\pi-5=0$ are $2\pm\sqrt{9-2\pi}$

$\therefore x^2-4x+2\pi-5<0$

$\Rightarrow 2-\sqrt{9-2\pi}<x<2+\sqrt{9-2\pi}$

14. The value of $\sin\left(2\tan^{-1}\frac{1}{3}\right)+\cos\left(\tan^{-1}2\sqrt{2}\right)$ is

1) $\frac{14}{15}$ 2) $-\frac{14}{15}$ 3) $\frac{13}{12}$ 4) $\frac{12}{13}$

Key. 1

Sol. $\sin\left(\tan^{-1}\frac{3}{4}\right)+\cos\left(\tan^{-1}2\sqrt{2}\right)=\sin\left(\sin^{-1}\frac{3}{5}\right)+\cos\left(\cos^{-1}\frac{1}{3}\right)=\frac{3}{5}+\frac{1}{3}=\frac{14}{15}$

15. The number of real solutions of $\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$ is

1) zero 2) one 3) two 4) infinite

Key. 3

Sol. $x(x+1)\geq 0$ and $x^2+x+1\leq 1\Rightarrow x(x+1)\leq 0$

$\therefore x=0$ or – 1

16. If $\sin^{-1}\left(\sin\left(\frac{2x^2+4}{1+x^2}\right)\right)<\pi-3$ then x satisfies

1) $|x|>1$ 2) $|x|<1$ 3) $|x|\leq 1$ 4) $|x|\geq 1$

Key. 2

Sol. $\sin^{-1}\left(\sin\left(\pi-\left(2+\frac{2}{1+x^2}\right)\right)\right)<\pi-3\Rightarrow\pi-2-\frac{2}{1+x^2}<\pi-3$

$\Rightarrow|x|<1$

17. The value of 'a' for which $ax^2+\sin^{-1}\left(x^2-2x+2\right)+\cos^{-1}\left(x^2-2x+2\right)=0$ has a real solution is

1) $\frac{\pi}{2}$ 2) $-\frac{\pi}{2}$ 3) $\frac{2}{\pi}$ 4) $-\frac{2}{\pi}$

Key. 2

Sol. Clearly x = 1 is the only solution $\Rightarrow a+\frac{\pi}{2}=0\Rightarrow a=-\frac{\pi}{2}$

18. Given $0\leq x\leq\frac{1}{2}$ then the value of $\tan\left[\sin^{-1}\left\{\frac{x}{\sqrt{2}}+\frac{\sqrt{1-x^2}}{\sqrt{2}}\right\}-\sin^{-1}x\right]$ is

1) – 1 2) 1 3) $\frac{1}{\sqrt{3}}$ 4) $\sqrt{3}$

Key. 2

Sol. Put $x=\sin\theta\Rightarrow\tan\left[\sin^{-1}\left(\sin\left(\theta+\frac{\pi}{4}\right)\right)-\theta\right]=1$

19. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{3\pi}{2}$ then $\frac{\sum_{r=1}^{2}\left(x^{100r}+y^{103r}\right)}{\sum x^{201}y^{201}}=$

1) 0 2) 2 3) 4 4) $\frac{4}{3}$

Key. 4

Sol. x = y = z = 1

$\therefore$ Required value = $\frac{4}{3}$

20. Sum to infinite terms of the series

$\cot^{-1}\left(1^2+\frac{3}{4}\right)+\cot^{-1}\left(2^2+\frac{3}{4}\right)+\cot^{-1}\left(3^2+\frac{3}{4}\right)+.........$ is

1) $\frac{\pi}{4}$ 2) $\tan^{-1}2$ 3) $\tan^{-1}3$ 4) $\cot^{-1}3$

Key. 2

Sol.

$$T_n=\cot^{-1}\left(n^2+\frac{3}{4}\right)=\tan^{-1}\left(\frac{1}{n^2+\frac{3}{4}}\right)=\tan^{-1}\left(n+\frac{1}{2}\right)-\tan^{-1}\left(n-\frac{1}{2}\right)$$

$$S_n=\sum_{n=1}^{n}t_n=\tan^{-1}\left(n+\frac{1}{2}\right)-\tan^{-1}\frac{1}{2}\Rightarrow S_\infty=\frac{\pi}{2}-\tan^{-1}\frac{1}{2}=\cot^{-1}\frac{1}{2}=\text{Tan}^{-1}2$$

21. If α is the only real root of the equation $x^3+bx^2+cx+1=0$ (b<c) then the value of $\tan^{-1}\alpha+\tan^{-1}\left(\frac{1}{\alpha}\right)$ is

1) $\frac{\pi}{2}$ 2) $-\frac{\pi}{2}$

3) 0 4) can not be determined

Key. 2

Sol. $f(0)=1,\ f(-1)=b-c<0\Rightarrow\alpha\in(-1,0)\Rightarrow\alpha<0$

$$\therefore\tan^{-1}(\alpha)+\tan^{-1}\left(\frac{1}{\alpha}\right)=\tan^{-1}(\alpha)-\pi+\cot^{-1}\alpha=-\pi+\frac{\pi}{2}=-\frac{\pi}{2}$$

22. If the mapping f(x) = ax + b, a>0 maps [- 1,1] onto [0, 2] then $\cot\left[\cot^{-1}7+\cot^{-1}8+\cot^{-1}18\right]$ is equal to

1) f(-1) 2) f(0) 3) f(1) - 1 4) f(1) + 1

Key. 4

Sol. $f(-1)=0,\ f(1)=2\Rightarrow -a+b=0$

$$a+b=2$$

$\Rightarrow b=1,\ a=1\Rightarrow f(x)=x+1$

$$\cot\left[\text{Tan}^{-1}\frac{1}{7}+\text{Tan}^{-1}\frac{1}{8}+\text{Tan}^{-1}\frac{1}{18}\right]=\cot\left[\text{Tan}^{-1}\left(\frac{15}{55}\right)+\text{Tan}^{-1}\left(\frac{1}{18}\right)\right]$$

$$=\cot\left[\text{Tan}^{-1}\left(\frac{65}{195}\right)\right]=\cot\left[\text{Tan}^{-1}\left(\frac{1}{3}\right)\right]=3=f(2)$$

23. If $\cot^{-1}\left(\frac{n}{\pi}\right)>\frac{\pi}{6},\ n\in N$ then the maximum value of n is

1) 1 2) 5 3) 9 4) 6

Key. 2

Sol. $\frac{n}{\pi}<\cot\left(\frac{\pi}{6}\right)<\sqrt{3}\Rightarrow n<\sqrt{3}\pi<5.34$

24. The number of solutions for the equation $\cos^{-1}(1-x)+m\cos^{-1}x=\frac{n\pi}{2}$ where m > 0, n $\leq$ 0 is

1) 0 2) 1 3) 2 4) Infinite

Key. 1

Sol. L.H.S is always +ve and R.H.S is negative.
There is no solution.

25. The value of $\sin^{-1}\left[\cot\left(\sin^{-1}\sqrt{\frac{2-\sqrt{3}}{4}}+\cos^{-1}\frac{\sqrt{12}}{4}+\sec^{-1}\sqrt{2}\right)\right]$ is

1) 0 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{6}$ 4) $\frac{\pi}{2}$

Key. 1

Sol. $\sin^{-1}\left[\cot\left(\sin^{-1}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)+\cos^{-1}\frac{\sqrt{3}}{2}+\sec^{-1}\sqrt{2}\right)\right]=\sin^{-1}[0]=0$

26. Number of solutions of the equation $\left(\tan^{-1}x\right)^2+\left(\cot^{-1}x\right)^2=\frac{5\pi^2}{8}$ is

1) Infinite 2) 1 3) 0 4) 2

Key. 2

Sol. $\frac{\pi^2}{4}-2\tan^{-1}x\left(\frac{\pi}{2}-\tan^{-1}x\right)=\frac{5\pi^2}{8}\Rightarrow\tan^{-1}x=\frac{-\pi}{4},\frac{3\pi}{4}$

27. The value of $\tan^2\left(\sec^{-1}3\right)+\cot^2\left(\operatorname{cosec}^{-1}4\right)$ is

1) 20 2) 21 3) 23 4) 25

Key. 3

Sol. Let $\sec^{-1}3=\alpha,\ \operatorname{cosec}^{-1}4=\beta\Rightarrow\tan^2\alpha+\cot^2\beta=9-1+16-1=23$

28. If $x=\sin\left(2\tan^{-1}2\right),\ y=\sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right)$ then

1) x = 1 – y 2) x^2 = 1 – y 3) x^2 = 1 + y 4) y^2 = 1 – x

Key. 4

Sol. Let $\tan^{-1}2=\alpha\Rightarrow x=\sin 2\alpha=\dfrac{4}{5}$

$$y=\sin\left(\frac{\beta}{2}\right)=\sqrt{\frac{1-\cos\beta}{2}}=\sqrt{\frac{1-\frac{3}{5}}{2}}=\frac{1}{\sqrt{5}}$$

29. If $\cos^{-1}(\frac{x}{3})+\cos^{-1}(\frac{y}{2})=\frac{\theta}{2}$ then $4x^2-12xy\cos\frac{\theta}{2}+9y^2=$

1) 36 2) 36 – 36 cos θ 3) 18 – 18 cos θ 4) 18 + 18 cos θ

Key. 3

Sol. $\cos\frac{\theta}{2}=\frac{xy}{6}-\sqrt{\left(1-\frac{x^2}{9}\right)\left(1-\frac{y^2}{4}\right)}\Rightarrow\left(\cos\frac{\theta}{2}-\frac{xy}{6}\right)^2=1-\frac{x^2}{9}-\frac{y^2}{4}+\frac{x^2y^2}{36}$

30. The value of $\sin\left(\cot^{-1}(\cos(\tan^{-1}x))\right)$ is equal to

1) $\sqrt{\dfrac{x^2+1}{x^2+2}}$ 2) $\sqrt{\dfrac{x+2}{x^2+1}}$

3) $\sqrt{\dfrac{x^2-1}{x^2+2}}$ 4) $\sqrt{\dfrac{x^2+2}{x^2+1}}$

Key. 1

Sol. $\sin\left(\sin^{-1}\left(\frac{1}{\sqrt{1+\cos^2\theta}}\right)\right)=\frac{1}{\sqrt{1+\cos^2\theta}}=\frac{\sec\theta}{\sqrt{1+\sec^2\theta}}=\frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}$

31. If $\sin^{-1}\left(a-\frac{a^2}{3}+\frac{a^3}{9}-..........\infty\right)+\cos^{-1}\left(1+b+b^2+.........\infty\right)=\frac{\pi}{2}$ then

1) a = -3, b = 1 2) a = 1, b = $\dfrac{-1}{3}$

3) $a=\dfrac{1}{6},b=\dfrac{1}{2}$ 4) $a=\dfrac{1}{6},b=\dfrac{1}{3}$

Key. 2

Sol. $\dfrac{a}{1+\frac{a}{3}}=\dfrac{1}{1-b}\Rightarrow\dfrac{3a}{a+3}=\dfrac{1}{1-b}$

There are infinitely many solutions but option ‘b’ satisfies.

32. The value of $\cos^{-1}(\cos 10)=$

1) 10 2) $4\pi-10$ 3) $2\pi+10$ 4) $2\pi-10$

Key. 2

Sol. $\cos^{-1}(\cos 10) = \cos^{-1}(\cos(4\pi - 10)) = 4\pi - 10$

33. The sum to infinite terms of the series $\operatorname{cosec}^{-1}\sqrt{10} + \operatorname{cosec}^{-1}\sqrt{50} + \operatorname{cosec}^{-1}\sqrt{170} + \ldots\ldots\ldots + \operatorname{cosec}^{-1}\sqrt{(n^2+1)(n^2+2n+2)}$ is

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{6}$

Key. 1

Sol. Given series = $\cot^{-1}(3) + \cot^{-1}(7) + \cot^{-1}(13) + \cot^{-1}(21) + \ldots\ldots\ldots$

$$= \operatorname{Tan}^{-1}\left(\frac{1}{3}\right) + \operatorname{Tan}^{-1}\left(\frac{1}{7}\right) + \operatorname{Tan}^{-1}\left(\frac{1}{13}\right) + \operatorname{Tan}^{-1}\left(\frac{1}{21}\right) + \ldots\ldots\ldots$$

$$= \sum_{n=1}^{\infty} \operatorname{Tan}^{-1}\left(\frac{1}{1+n(n+1)}\right) = \frac{\pi}{4}$$

34. Indicate the relation which is true?

1) $\tan\left|\tan^{-1} x\right| = |x|$ 2) $\cot\left|\cot^{-1} x\right| = |x|$

3) $\tan^{-1}|\tan x| = |x|$ 4) $\sin\left|\sin^{-1} x\right| = |x|$

Key. 1

Sol. $\left|\tan^{-1} x\right| = \tan^{-1} x$ if $= 0 \le x < \frac{\pi}{2}$

$= -\tan^{-1} x$ if $-\frac{\pi}{2} < x < 0$

but $\tan^{-1}|x| = \tan^{-1} x$ if x > 0

$= \tan^{-1} x$ if x < 0

$\therefore \left|\tan^{-1} x\right| = \tan^{-1}|x| \Rightarrow \tan\left(\tan^{-1}(|x|)\right) = |x|$

35. Identify the pair of functions which are not identical

1) $y = \tan(\cos^{-1} x); y = \frac{\sqrt{1-x^2}}{x}$ 2) $y = \tan(\cot^{-1} x); y = \frac{1}{x}$

3) $y = \sin(\text{arc tanx}); y = \frac{x}{\sqrt{1+x^2}}$ 4) $y = \cos(\tan^{-1}\text{x}); y = \sin(\tan^{-1} x)$

Key. 4

Sol. Conceptual

36. If $\cot^{-1}\sqrt{\cos\alpha} - \tan^{-1}\sqrt{\cos\alpha} = x$ then sinx =

1) $\tan^2\left(\frac{\alpha}{2}\right)$ 2) $\cot^2\left(\frac{\alpha}{2}\right)$ 3) $\tan\alpha$ 4) $\cot\left(\frac{\alpha}{2}\right)$

Key. 1

Sol. Let $\cot^{-1}\sqrt{\cos\alpha} = \theta \Rightarrow \sqrt{\cos\alpha} = \cot\theta$

$$\theta - \tan^{-1}(\cot\theta) = x \Rightarrow \theta - \tan^{-1}\left(\tan\left(\frac{\pi}{2}-\theta\right)\right) = x$$

$$\Rightarrow 2\theta - \frac{\pi}{2} = x$$

$$\Rightarrow \sin x = -\cos 2\theta = -\frac{1-\frac{1}{\cos\alpha}}{1+\frac{1}{\cos\alpha}} = \tan^2\left(\frac{\alpha}{2}\right)$$

37. If $x \ge 1$ then $2\tan^{-1}x + \sin^{-1}\left(\frac{2x}{1+x^2}\right) =$

1) $4\tan^{-1}x$ 2) 0 3) $\frac{\pi}{2}$ 4) π

Key. 4

Sol. If $x \ge 1$ then $2\tan^{-1}x = \pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \tan^{-1}\left(\frac{2x}{x^2-1}\right)$$

38. The root of the equation $\tan^{-1}\left(\frac{x-1}{x+1}\right) + \tan^{-1}\left(\frac{2x-1}{2x+1}\right) = \tan^{-1}\left(\frac{23}{36}\right)$ is

1) $-\frac{3}{8}$ 2) $-\frac{1}{2}$ 3) $\frac{3}{4}$ 4) $\frac{4}{3}$

Key. 4

Sol. $\frac{2x^2-1}{3x} = \frac{23}{36} \Rightarrow x = \frac{4}{3}, \frac{-3}{8}$ but x can not be negative.

39. Find maximum value of x for which $2\tan^{-1}x + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is independent of x.

Key. 0

Sol. Let x = tanθ $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$2\theta + \cos^{-1}\cos 2\theta$

(i) $0 \le 2\theta < \frac{\pi}{2}.2$

$\cos^{-1}\cos 2\theta = 2\theta$

So given expression = 4θ = $4\tan^{-1}x$

(ii) $-\frac{\pi}{2} < \theta \le 0 \quad \Rightarrow -\pi < 2\theta \le 0$

$\cos^{-1}\cos 2\theta = -2\theta$

So, given expression becomes independent of θ.

For $-\frac{\pi}{2} < \theta \le 0 \quad \Rightarrow -\infty < x \le 0$

40. $\sin^{-1}|\cos x| - \cos^{-1}|\sin x| = a$ has at least one solution iff $a \in$

a) $\{0\}$ b) $\left[0, \frac{\pi}{2}\right]$ c) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ d) $(0, \pi)$

Key. A

Sol. $\sin^{-1}|\cos x| - \cos^{-1}|\sin x| = \frac{\pi}{2} - \cos^{-1}|\cos x| - \frac{\pi}{2} + \sin^{-1}|\sin x| = a$

$\Rightarrow \sin^{-1}|\sin x| - \cos^{-1}|\cos x| = a$

$\Rightarrow a = 0 \forall x$

41. If x_1, x_2, x_3, x_4 are the roots of the equation $x^4 - x^3 \sin 2\beta + x^2 \cos 2\beta - x\cos\beta - \sin\beta = 0$, then $\sum_{i=1}^{4} \tan^{-1} x_i$ is equal to

a) β b) $\frac{\pi}{2} - \beta$ c) $\pi - \beta$ d) $-\beta$

Key. B

Sol. $\sum_{i=1}^{4} \tan^{-1} x_i = \tan^{-1}\left(\frac{\sum x_1 - \sum x_1 x_2 x_3}{1 - \sum x_1 x_2 + \pi x_i}\right) = \tan^{-1}\left(\frac{\sin 2\beta - \cos\beta}{1 - \cos 2\beta - \sin\beta}\right)$

$= \tan^{-1}\left\{\frac{\cos\beta(2\sin\beta - 1)}{\sin\beta(2\sin\beta - 1)}\right\} = \tan^{-1}\left(\tan\left(\frac{\pi}{2} - \beta\right)\right) = \frac{\pi}{2} - \beta$

42. The least and greatest values of $(\sin^{-1}x)^3 + (\cos^{-1}x)^3$ are respectively : $x \in [-1, 1]$

a) $-\frac{\pi}{2}, \frac{\pi}{2}$ b) $-\frac{\pi^3}{8}, \frac{\pi^3}{8}$ c) $\frac{\pi^3}{32}, \frac{7\pi^3}{8}$ d) $\frac{\pi^3}{32}, \frac{\pi^3}{8}$

Key. C

Sol. $(\sin^{-1}x + \cos^{-1}x)^3 - 3\sin^{-1}x\cos^{-1}x(\sin^{-1}x + \cos^{-1}x)$

$= \frac{\pi^3}{8} - \frac{3\pi}{2}\sin^{-1}x\left(\frac{\pi}{2} - \sin^{-1}x\right)$

$= \frac{\pi^3}{8} + \frac{3\pi}{2}\left[\left(\sin^{-1}x - \frac{\pi}{4}\right)^2\right] - \frac{3\pi^3}{32} = \frac{\pi^3}{32} + \frac{3\pi}{2}\left(\sin^{-1}x - \frac{\pi}{4}\right)^2 \left(\left(\sin^{-1}x - \frac{\pi}{4}\right)^2 \le \left(\frac{3\pi}{4}\right)^2\right)$

L.V.= $\frac{\Pi^3}{32}$ & $G.V = \frac{7\Pi^3}{8}$

43. The area bounded by the identity curve in the first quadrant, y = 0 and $x = \sin^{-1}\left(a^4+1\right)+\cos^{-1}\left(a^4+1\right)-\tan^{-1}\left(a^4+1\right)$ is

a) $\frac{\pi^2}{8}-\frac{a^2}{4}$ b) $\frac{\pi^2}{32}+\frac{a^2}{4}$ c) $\frac{\pi^2}{16}+\frac{a^2}{2}$ d) $\frac{\pi^2}{8}-\frac{a^2}{4}$

Key. B

Sol. x is defined if $a^4 = 0 \Rightarrow a = 0$.

$\Rightarrow x = \frac{\pi}{2}+0-\frac{\pi}{4}=\frac{\pi}{4}$.

Area = $\frac{1}{2}\frac{\pi}{4}\frac{\pi}{4}=\frac{\pi^2}{32}$ sq. units

44. Let $I_1 = \frac{\pi^2}{16}+\sqrt{2}, I_2 = \left(\tan^{-1}\left(\frac{1}{e}\right)\right)^2+\frac{2e}{\sqrt{e^2+1}}$, $I_3 = \left(\tan^{-1}e\right)^2+\frac{2}{\sqrt{e^2+1}}$, then which of the following is true.

a) $I_1 < I_2 < I_3$ b) $I_2 < I_1 < I_3$ c) $I_1 < I_3 < I_2$ d) $I_3 < I_2 < I_1$

Key. B

Sol. Consider the function $f(x) = \left(\tan^{-1}x\right)^2+\frac{2}{\sqrt{x^2+1}}$ then $f^1(x) > 0 \ \forall \ x \in (0,\infty)$

$\Rightarrow f\left(\frac{1}{e}\right) < f(1) < f(e) \Rightarrow I_2 < I_1 < I_3$

45. If 0 < x < 1, then $\sqrt{1+x^2}$ $[\{x \cos(\cot^{-1}x) + \sin(\cot^{-1}x\}^2 - 1]^{1/2}$ =

(A) $\frac{x}{\sqrt{1+x^2}}$ (B) x

(C) $x\sqrt{1+x^2}$ (D) $\sqrt{1+x^2}$

Key. C

Sol. Exp. $= \sqrt{1+x^2}\left[\left\{x.\cos\left(\cos^{-1}\frac{x}{\sqrt{1+x^2}}\right)+\sin\left(\sin^{-1}\frac{1}{1+x^2}\right)\right\}^2-1\right]^{1/2}$

$= \sqrt{1+x^2}\left[\left\{x.\frac{x}{\sqrt{1+x^2}}+\frac{1}{\sqrt{1+x^2}}\right\}^2-1\right]^{1/2}$

$= \sqrt{1+x^2}[1+x^2-1]^{1/2} = x\sqrt{1+x^2}$

46. The value of 'a' for which $ax^2+\sin^{-1}(x^2-2x+2)+\cos^{-1}(x^2-2x+2)=0$ has a real solution , is

a) $-\frac{2}{\pi}$ b) $\frac{2}{\pi}$ c) $-\frac{\pi}{2}$ d) $\frac{\pi}{2}$

Key. C

Sol. Here, $x^2 - 2x + 2 = (x-1)^2 + 1 \ge 1$

But $-1 \le \left(x^2 - 2x + 2\right) \le 1$

Which is possible only when $x^2 - 2x + 2 = 1$

$\therefore x = 1$

Then, $a(1)^2 + \sin^{-1}(1) + \cos^{-1}(1) = 0$

$\Rightarrow a + \frac{\pi}{2} + 0 = 0$

$\therefore a = -\frac{\pi}{2}$

47. The value of tan $\left\{\left(\cos^{-1}\left(-\frac{2}{7}\right) - \frac{\pi}{2}\right)\right\}$ is

a) $\frac{2}{3\sqrt{5}}$ b) $\frac{2}{3}$ c) $\frac{1}{\sqrt{5}}$ d) $\frac{4}{\sqrt{5}}$

Key. A

Sol. $\tan\left\{\left(\cos^{-1}\left(-\frac{2}{7}\right) - \frac{\pi}{2}\right)\right\} = \tan\left\{\pi - \cos^{-1}\left(\frac{2}{7}\right) - \frac{\pi}{2}\right\}$

$= \tan\left\{\frac{\pi}{2} - \cos^{-1}\left(\frac{2}{7}\right)\right\} = \tan\left\{\sin^{-1}\left(\frac{2}{7}\right)\right\} = \tan\tan^{-1}\left(\frac{2}{3\sqrt{5}}\right) = \frac{2}{3\sqrt{5}}$

48. The principal value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is

a) π b) $\frac{\pi}{2}$ c) $\frac{\pi}{3}$ d) $\frac{4\pi}{3}$

Key. A

Sol. $\because \cos^{-1}\left(\cos\left(\frac{2\pi}{3}\right)\right) + \sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) = \frac{2\pi}{3} + \left(\pi - \frac{2\pi}{3}\right) = \pi$

49. The range of the function $f(x) = 2\tan^{-1}\left(\frac{1+x}{1-x}\right) + \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is

A) $\{\pi\}$ B) $\{-\pi, \pi\}$ C) $(-\pi, \pi]$ D) $[-\pi, \pi]$

Key. D

Sol. $$f(x)=\begin{cases}\pi+4Tan^{-1}x & ,x<0\\ \pi & ,0\le x<1\\ -\pi & ,x>1\end{cases}$$

50. If domain of the function $f\left(x\right)=\cos^{-1}\frac{x^3}{2}+\sqrt{\sin^{-1}\log_2\left(x+1\right)}$ is [a,b], then a + 2b =

(A) 0 (B) 1 (C) 2 (D) 3

Key. C

Sol. $-1\le\frac{x^3}{2}\le1$ $\quad 0\le\log_2\left(x+1\right)\le1$

$\Rightarrow-\sqrt[3]{2}\le x\sqrt[3]{2}\Rightarrow0\le x\le1$

51. If $y=\cot^{-1}\left(\sqrt{\cos2\theta}\right)-\tan^{-1}\left(\sqrt{\cos2\theta}\right)$ then $\sin y$ =

a) $\tan^2\theta$ b) $\cot^2\theta$

c) $\tan\theta$ d) $\cot\theta$

Key. A

Sol. $\frac{\pi}{2}-y=2\tan^{-1}\left(\sqrt{\cos^2\theta}\right)\Rightarrow\sin y=\cos\left(\frac{\pi}{2}-y\right)=\tan^2\theta$

52. If $\alpha,\beta>0$ then $\frac{\alpha^3}{2}\cos ec^2\left(\frac{1}{2}\tan^{-1}\frac{\alpha}{\beta}\right)+\frac{\beta^3}{2}\sec^2\left(\frac{1}{2}\tan^{-1}\frac{\beta}{\alpha}\right)=$

a) $\left(\alpha-\beta\right)\left(\alpha^2+\beta^2\right)$ b) $\left(\alpha+\beta\right)\left(\alpha^2-\beta^2\right)$ c) $\left(\alpha+\beta\right)\left(\alpha^2+\beta^2\right)$ d) 0

Key. C

Sol. Let $\frac{1}{2}\tan^{-1}\left(\frac{\alpha}{\beta}\right)=\theta;\frac{1}{2}\tan^{-1}\left(\frac{\beta}{\alpha}\right)=\frac{\pi}{4}-\theta$

G.E = $$\frac{\alpha^3}{2\sin^2\theta}+\frac{\beta^2}{2\cos^2\left(\frac{\pi}{4}-\theta\right)}=\frac{\alpha^3}{1-\cos^2\theta}+\frac{\beta^3}{1+\sin^2\theta}$$

$$=\frac{\alpha^3\left(\sqrt{\alpha^2+\beta^2}\right)}{\sqrt{\alpha^2+\beta^2-\beta}}+\frac{\beta^3\sqrt{\alpha^2+\beta^2}}{\sqrt{\alpha^2+\beta^2+\alpha}}=\left(\alpha+\beta\right)\left(\alpha^2+\beta^2\right)$$

53. The value of the expression $\sin^{-1}\left(\sin\frac{22\pi}{7}\right)+\cos^{-1}\left(\cos\frac{5\pi}{3}\right)+\tan^{-1}\left(\tan\frac{5\pi}{7}\right)+\sin^{-1}\left(\cos2\right)$ is

A) $\frac{17\pi}{42}-2$ B) -2 C) $\frac{-\pi}{21}-2$ D) none of these

Key. A

Sol. $\sin^{-1}\sin\left(\frac{22\pi}{7}\right)=\sin^{-1}\sin\left(3\pi+\frac{\pi}{7}\right)=-\frac{\pi}{7}$

$\cos^{-1}\cos\left(\frac{5\pi}{3}\right)=\cos^{-1}\cos\left(2\pi-\frac{\pi}{3}\right)=\frac{\pi}{3}$

$\tan^{-1}\tan\left(\frac{5\pi}{7}\right)=\tan^{-1}\tan\left(\pi-\frac{2\pi}{7}\right)=-\frac{2\pi}{7}$

$\sin^{-1}\cos(2)=\frac{\pi}{2}-\cos^{-1}\cos 2=\frac{\pi}{2}-2$

$\therefore$ Required Value $=-\frac{\pi}{7}+\frac{\pi}{3}-\frac{2\pi}{7}+\frac{\pi}{2}-2=\frac{(-18+35)\pi}{42}-2=\frac{17\pi}{42}-2$

54. If $\sin^{-1}(x-1)+\cos^{-1}(x-3)+\tan^{-1}\left(\frac{x}{2-x^2}\right)=\cos^{-1}k+\pi$, then the value of K =

A) 1 B) $-\frac{1}{\sqrt{2}}$ C) $\frac{1}{\sqrt{2}}$ D) none of these

Key. C

Sol. $\sin^{-1}(x-1) \Rightarrow -1\le x-1\le 1 \Rightarrow 0\le x\le 2$

$\cos^{-1}(x-3) \Rightarrow -1\le x-3\le 1 \Rightarrow 2\le x\le 4$

$\tan^{-1}\left(\frac{x}{2-x^2}\right) \Rightarrow x\in R,\ x\ne\sqrt{2},-\sqrt{2}$

$\therefore \quad x=2$

$\sin^{-1}(2-1)+\cos^{-1}(2-3)+\tan^{-1}\frac{2}{2-4}=\cos^{-1}k+\pi$

$\Rightarrow \quad \sin^{-1}1+\cos^{-1}(-1)+\tan^{-1}(-1)=\cos^{-1}k+\pi$

$\frac{\pi}{2}+\pi-\frac{\pi}{4}=\cos^{-1}k+\pi$

$\Rightarrow \quad \cos^{-1}K=\frac{\pi}{4} \quad \Rightarrow \quad K=\frac{1}{\sqrt{2}}$

55. If $\sin^{-1}\sin(5)>x^2-4x$, then the number of possible integral values of x is

A) 1 B) 2 C) 3 D) none of these

Key. C

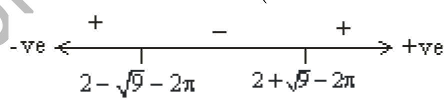
Sol. $\sin^{-1}\sin 5=\sin^{-1}\sin(5-2\pi)=5-2\pi \quad \left(\text{As}-\frac{\pi}{2}\le 5-2\pi\le\frac{\pi}{2}\right)$

$\therefore \quad \sin^{-1}\sin 5>x^2-4x$

$\Rightarrow \quad 5-2\pi>x^2-4x$

$\Rightarrow \quad x^2-4x+2\pi-5<0$

Sign sum of $\left(x^2-4x+2\pi-5\right)$



$2-\sqrt{9-2\pi}<x<2+\sqrt{9-2\pi}$

Integral values of x are 1, 2, 3

Number of integral value of x = 3

56. If $x\in[-1,0)$, then $\cos^{-1}\left(2x^2-1\right)-2\sin^{-1}x=$

A) $-\frac{\pi}{2}$ B) π C) $\frac{3\pi}{2}$ D) -2π

Key. B

Sol. $\cos^{-1}\left(2x^2-1\right)=2\pi-2\cos^{-1}x$ (as x < 0)

$\cos^{-1}\left(2x^2-1\right)-2\sin^{-1}x=2\pi-2\cos^{-1}x-2\sin^{-1}x$

$=2\pi-2\left(\cos^{-1}x+\sin^{-1}x\right)$

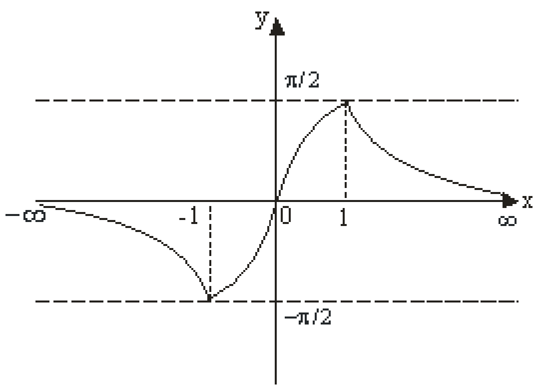
$=2\pi-2\frac{\pi}{2}=\pi$

57. α_1 and α_2 satisfies $\sin^{-1}\frac{2x}{1+x^2}=\tan^{-1}\frac{2x}{1-x^2}$ and $\left|\alpha_1-\alpha_2\right|<K$, for all α_1 and α_2 then k is equal to

A) 1 B) 3/2 C) 2 D) none of these

Key. C

Sol. Graph $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

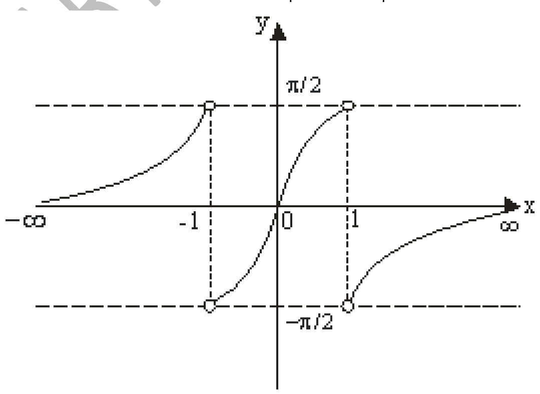


Graph of $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$

From graph one can say

$-1<x<1 \Rightarrow -1<\alpha_1<1$

$-1<\alpha_2<1 \Rightarrow \left|\alpha_1-\alpha_2\right|<2$



58. The solution of the inequality $\log_{1/2}\sin^{-1}x>\log_{1/2}\cos^{-1}x$ is

A) $x \in \left[0, \frac{1}{\sqrt{2}}\right]$ B) $x \in \left(\frac{1}{\sqrt{2}}, 1\right]$ C) $x \in \left(0, \frac{1}{\sqrt{2}}\right)$ D) None of these

Key. C

Sol. $\log_{1/2} \sin^{-1} x > \log_{1/2} \cos^{-1} x$

$\Leftrightarrow \cos^{-1} x > \sin^{-1} x, \quad 0 < x < 1$

$\Leftrightarrow \cos^{-1} x > \frac{\pi}{2} - \cos^{-1} x, \ 0 < x < 1$

$\Leftrightarrow \cos^{-1} x > \frac{\pi}{4}, \ 0 < x < 1$

$\Leftrightarrow 0 < x < \frac{1}{\sqrt{2}}$

59. The value of $\tan\left[\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$ is

A) $\frac{6}{17}$ B) $\frac{7}{16}$ C) $\frac{16}{7}$ D) none of these

Key. D

Sol. Since $\cos^{-1}\left(\frac{4}{5}\right) = \tan^{-1}\left(\frac{3}{4}\right)$

$\therefore \quad \tan\left[\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right] = \tan\left[\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\right] = \frac{\frac{3}{4} + \frac{2}{3}}{1 - \frac{3}{4} \cdot \frac{2}{3}} = \frac{17}{6}$

60. If $\tan^{-1}\left(\sqrt{\cos\alpha + 1}\right) + \cot^{-1}\left(\sqrt{\cos\alpha + 1}\right) = \mu$ (where $\alpha \neq n\pi + \frac{\pi}{2}, n \in I$), then $\sin\mu$ is equal to

A) $\tan^2 \alpha$ B) $\tan 2\alpha$ C) $\sec^2 \alpha - \tan^2 \alpha$ D) $\cos^2 \frac{\alpha}{2}$

Key. C

Sol. Since $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}, \ \forall x \in R$,

$\therefore \quad \mu = \frac{\pi}{2} \quad \Rightarrow \quad \sin\mu = 1 = \sec^2 \alpha - \tan^2 \alpha$

61. Range of the function $f(x) = \cos^{-1}(-\{x\})$, where {.} is fractional part function, is

A) $\left(\frac{\pi}{2}, \pi\right)$ B) $\left(\frac{\pi}{2}, \pi\right]$ C) $\left[\frac{\pi}{2}, \pi\right)$ D) $\left(0, \frac{\pi}{2}\right]$

Key. C

Sol. $\therefore \quad \frac{\pi}{2} \le \cos^{-1}(-x\{x\}) < \pi$

$\therefore$ the range is $\left[\frac{\pi}{2}, \pi\right)$

62. The sum of solutions of the equation $2\sin^{-1}\sqrt{x^2 + x + 1} + \cos^{-1}\sqrt{x^2 + x} = \frac{3\pi}{2}$ is

A) 0 B) – 1 C) 1 D) 2

Key. B

Sol. $0 \le x^2 + x + 1 \le 1$ and $0 \le x^2 + x \le 1$

$\therefore$ x = - 1, 0

For x = - 1

L.H.S $= 2\sin^{-1}1 + \cos^{-1}0 = \frac{3\pi}{2}$

$\therefore$ x = - 1 is a solution

for x = 0 L.H.S $= 2\sin^{-1}1 + \cos^{-1}0 = \frac{3\pi}{2}$

$\therefore$ x = 0 is a solution

$\therefore$ sum of the solution = - 1

63. If $\sin^{-1}a + \sin^{-1}b + \sin^{-1}c = \pi$, then the value of $a\sqrt{(1-a^2)} + b\sqrt{(1-b^2)} + c\sqrt{(1-c^2)}$ will be

A) 2abc B) abc C) $\frac{1}{2}$abc D) $\frac{1}{3}$abc

Key. A

Sol. Let $\sin^{-1} a = A$,

$\sin^{-1} b = B$

$\sin^{-1} c = C$

$\therefore$ sin A = a, sin B = b, sin C = c

and A + B + C = π, then

sin 2A + sin 2B + 2C = 4 sin A sin B sin C

Now $a\sqrt{(1-a^2)} + b\sqrt{(1-c^2)} + c\sqrt{(1-c^2)}$ = sin A cos A + sin B cos B + sin C cos C

$= \frac{1}{2}$ [sin 2A + sin 2B + sin 2C] = 2 sin A sin B sin C = 2abc

64. If $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$, then x =

A) $\frac{3\pi}{4}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{3}$ D) None of these

Key. B

Sol. We have $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$

$\Rightarrow \tan(2\tan^{-1}\cos x) = 2 \operatorname{cosec} x$

$\Rightarrow \frac{2\cos x}{1-\cos^2 x} = 2\operatorname{cosec} x \quad \Rightarrow \quad \frac{2\cos x}{\sin^2 x} = 2 \operatorname{cosec} x$

$\Rightarrow \sin x = \cos x \quad \Rightarrow \quad x = \frac{\pi}{4}$.

65. $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right] =$

A) $\frac{\pi}{4} + \frac{1}{2}\cos^{-1}x$ B) $\frac{\pi}{4} + \cos^{-1}x^2$ C) $\frac{\pi}{4} - \cos^{-1}x^2$ D) $\frac{\pi}{4} - \frac{1}{2}\cos^{-1}x^2$

Key. A

Sol. $\tan^{-1}\left(\dfrac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$

$= \tan^{-1}\left(\dfrac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}\right)$ (putting $x^2 = \cos 2\theta \quad \Rightarrow \quad \theta = \frac{1}{2}\cos^{-1} x^2$)

$= \tan^{-1}\left(\dfrac{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta-\sqrt{2}\sin\theta}\right) = \tan^{-1}\left(\dfrac{1+\tan\theta}{1-\tan\theta}\right) = \tan^{-1}\tan\left(\dfrac{\pi}{4}+\theta\right) = \dfrac{\pi}{4}+\dfrac{1}{2}\cos^{-1} x$

66. If x_1, x_2, x_3, x_4 are roots of the equation $x^4 - x^3\sin 2\beta + x^2\cos 2\beta - x\cos\beta - \sin\beta = 0$, then $\tan^{-1}x_1 + \tan^{-1}x_2 + \tan^{-1}x_3 + \tan^{-1}x_4 =$

A) β B) $\dfrac{\pi}{2}-\beta$ C) $\pi-\beta$ D) $-\beta$

Key. B

Sol. We have

$\Sigma x_1 = \sin 2\beta,\ \Sigma x_1x_2 = \cos 2\beta, \Sigma x_1x_2x_3 = \cos\beta$

and $x_1x_2x_3x_4 = -\sin\beta$

$\therefore \quad \tan^{-1}\left(\dfrac{\Sigma x_1 - \Sigma x_1x_2x_3}{1-\Sigma x_1x_2 + x_1x_2x_3x_4}\right) = \tan^{-1}\left(\dfrac{\sin 2\beta - \cos\beta}{1-\cos 2\beta - \sin\beta}\right)$

$= \tan^{-1}\left(\dfrac{(2\sin\beta-1)\cos\beta}{\sin\beta(2\sin\beta-1)}\right) = \tan^{-1}(\cot\beta) = \tan^{-1}\left[\tan\left(\dfrac{\pi}{2}-\beta\right)\right] = \dfrac{\pi}{2}-\beta$.

Inverse

Multiple Correct Answer Type

1. The value of x satisfying $\sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x$ are

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2

Key. A,B

Sol. $\sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x$

$\Rightarrow \quad \frac{\pi}{2} - \cos^{-1}x + \frac{\pi}{2} - \cos^{-1}(1-x) = \cos^{-1}x$

$\Rightarrow \quad 2\cos^{-1}x = \pi - \cos^{-1}(1-x)$

$\Rightarrow \quad \cos^{-1}(2x^2-1) = \cos^{-1}(x-1)$

$\Rightarrow \quad 2x^2 - 1 = x - 1$

$\Rightarrow \quad x(2x - 1) = 0$

$\Rightarrow \therefore \quad x = 0, \frac{1}{2}$

2. $\sin^{-1}(x^2+2x+2) + \tan^{-1}(x^2-3x-k^2) > \frac{\pi}{2}$ for $k \in$

(A) $(-1, 0)$ (B) $(0,1)$ (C) $(1,2)$ (D) $(0,2)$

Key: A, B, C, D

Hint: It is satisfied for only x = -1

$\pi/2 + \tan^{-1}(x^2-3x-k^2) > \pi/2 \quad for \quad x = -1$

$\Rightarrow k^2 - 4 < 0$.

3. Which of the following are true

a) $\tan^{-1}\frac{1}{3} = \frac{1}{2}\sin^{-1}\frac{3}{5}$

b) $\tan^{-1}\frac{1}{3} = \frac{\pi}{4} - \cot^{-1}2$

c) $\tan^{-1}\frac{1}{3} = \frac{\pi}{4} - \frac{1}{2}\sin^{-1}\frac{4}{5}$

d) $\tan^{-1}\frac{1}{3} = \frac{\pi}{2} - \cot^{-1}3$

Key: A, B, C

Hint:

$$2\tan^{-1}\frac{1}{3} = \tan^{-1}\frac{3}{4} = \sin^{-1}\frac{3}{5} = \frac{\pi}{2} - \cos^{-1}\frac{3}{5} = \frac{\pi}{2} - \sin^{-1}\frac{4}{5}$$

$$\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2} = \frac{\pi}{4} \Rightarrow \tan^{-1}\frac{1}{3} = \frac{\pi}{4} - \tan^{-1}\frac{1}{2} = \frac{\pi}{4} - \cot^{-1}2$$

4. The value (s) of x satisfying the equation $\sin^{-1}|\sin x| = \sqrt{\sin^{-1}|\sin x|}$ is/are given by (n is any integer)

A) $n\pi$ B) $n\pi+1$ C) $n\pi-1$ D) $2n\pi+1$

Key. A,B,C,D

Sol. The solution of $y=\sqrt{y}$ is y = 0 or y = 1

If $\sin^{-1}|\sin x| = 1 \Rightarrow x = 1$ or $\pi-1$ (in the interval $(0,\pi)$)

But $y=\sin^{-1}|\sin x|$ is periodic with period π, so $x = n\pi+1$ or $n\pi-1$

Again if $\sin^{-1}|\sin x| = 0 \Rightarrow x = n\pi$

5. If $S_n = \cot^{-1}3+\cot^{-1}7+\cot^{-1}13+\cot^{-1}21+...$ n terms then

A. $S_{10} = \tan^{-1}\frac{5}{6}$ B. $S_{\infty} = \frac{\pi}{4}$ C. $S_6 = \sin^{-1}\left(\frac{4}{5}\right)$ D. $S_{20} = \cot^{-1}(1.1)$

KEY. A,B,D

SOL. $S_n = \tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{13}+...+\tan^{-1}\frac{1}{1+n(n+1)}$

$$=\tan^{-1}2-\tan^{-1}1+\tan^{-1}3-\tan^{-1}2+...+\tan^{-1}(n+1)-\tan^{-1}n$$

$$=\tan^{-1}(n+1)-tan^{-1}(1)=\tan^{-1}\left(\frac{n}{n+2}\right)$$

$$S_{10} = tan^{-1}\left(\frac{10}{12}\right)=\tan^{-1}\left(\frac{5}{6}\right)$$

$$S_{\infty}=\tan^{-1}(1)=\frac{\pi}{4}$$

$$S_6=\tan^{-1}\left(\frac{6}{8}\right)=\tan^{-1}\left(\frac{3}{4}\right)=\sin^{-1}\left(\frac{3}{5}\right)$$

$$S_{20}=\tan^{-1}\left(\frac{20}{22}\right)=\tan^{-1}\left(\frac{10}{11}\right)=\cot^{-1}\left(\frac{11}{10}\right)$$

$\Rightarrow a,b,d$

6. $2\cot^{-1}7+\cos^{-1}\frac{3}{5}$ is equal to

a) $\cot^{-1}\left(\frac{44}{117}\right)$ b) $\cos ec^{-1}\left(\frac{125}{117}\right)$

c) $\tan^{-1}\left(\frac{4}{117}\right)$ d) $\cos^{-1}\left(\frac{44}{125}\right)$

Key. A,B,D

Sol. $2\cot^{-1}7+\cos^{-1}\frac{3}{5}=\cos^{-1}\left(\frac{44}{125}\right)$

7. If $0<x<1,\ Tan^{-1}\frac{\sqrt{1-x^2}}{1+x}$ is equal to

a) $\frac{1}{2}\cos^{-1}x$ b) $\cos^{-1}\sqrt{\frac{1+x}{2}}$ c) $\sin^{-1}\sqrt{\frac{1-x}{2}}$ d) none

Key. A,B,C

Sol. Conceptual

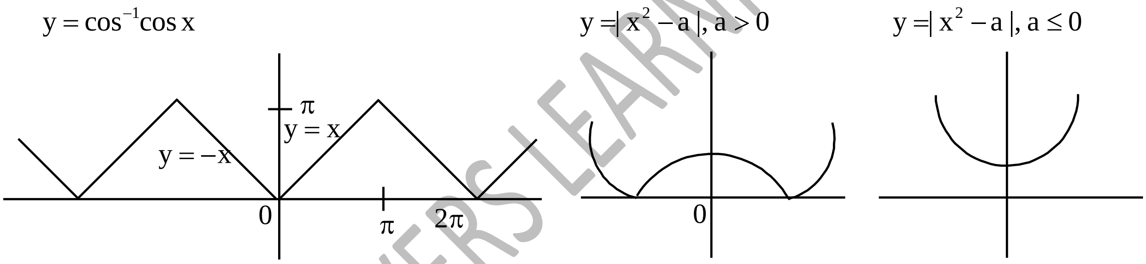
8. For the equation $|x^2 - a| = \cos^{-1}\cos x$ to have some solution the value of 'a' can be

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$

(C) $-\frac{1}{2}$ (D) $-\frac{1}{4}$

Key. A,B,D

Sol.



For a > 0, there is always a solution. For a ≤ 0, the solution exists for all a ∈ [b, 0] where b is the number when $y = |x^2 - b|$ touches $y = \cos^{-1}\cos x$.

So, $y_1 = y_2$ and $y_1' = y_2'$ $\Rightarrow x = x^2 - b$ and $1 = 2x \Rightarrow b = -\frac{1}{4}$. So finally $a\in\left[-\frac{1}{4},\infty\right)$.

9. If $f(x)=\sin^{-1}x+\cos^{-1}x$ then $\frac{\pi}{2}$ is equal to

a) $f\left(-\frac{1}{2}\right)$ b) $f\left(k^2-2k+3\right)\quad K\in R$

c) $f\left(\frac{1}{1+k^2}\right)\quad K\in R$ d) $f(-2)$

Key. A,C

Sol. $f(x)=\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}\ \forall -1\le x\le 1$

$\because \frac{1}{2}\in[-1,1]\Rightarrow f\left(-\frac{1}{2}\right)=\frac{\pi}{2}$

$0<\frac{1}{1+K^2}\le 1\forall K\in R\Rightarrow f\left(\frac{1}{1+K^2}\right)=\frac{\pi}{2}$

$K^2-2k+3=(K-1)^2+2\ge 2\,\forall\in R$

(d) does not hold (d) does not hold clearly $-2\in[-1]$

10. If the equation $\sin^{-1}(x^2+x+1)+\cos^{-1}(\lambda x+1)=\frac{\pi}{2}$ has exactly two distinct solutions then the integral value (s) of λ can be

a) 0 b) 1 c) -1 d) 2

Key. A

Sol. $x^2+x+1=\lambda x-1,-1\le x^2+x+1\le 1$

$x=0$ or $x=\lambda-1$ $x^2+x\le 0, x^2+x+\alpha>0$

$\Rightarrow -1\le x\le 0$ & $\forall x\in R$

$\because x=0$ is one solution

$-1\le x<0\Rightarrow -1\le\lambda-1<0\Rightarrow 0\le\lambda<1$

11. If α is a real number for which $f(x)=l\,n\cos^{-1}x$ is defined, then a possible value of $[\alpha]$ is (where [.] denotes greatest integer function).

A) 0 B) 1 C) – 1 D) – 2

Key. A,C

Sol. Domain of f(x) = ln $\cos^{-1}x$

is $x\in[-1,1)$

$\therefore$ $[\alpha]=-1$ or 0

12. Which of the following is a rational number:

A) $\sin\left(\tan^{-1}3+\tan^{-1}\frac{1}{3}\right)$ B) $\cos\left(\frac{\pi}{2}-\sin^{-1}\frac{3}{4}\right)$

C) $\log_2\left(\sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)\right)$ D) $\tan\left(\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}\right)$

Key. A,B,C

Sol. (A) $\sin\left(\tan^{-1}3+\tan^{-1}\frac{1}{3}\right)=\sin\frac{\pi}{2}=1$

(B) $\cos\left(\frac{\pi}{2}-\sin^{-1}\frac{3}{4}\right)=\cos\left(\cos^{-1}\frac{3}{4}\right)=\frac{3}{4}$

(C) $\sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$

Let $\sin^{-1}\frac{\sqrt{63}}{8}=\theta$

so $\sin\theta=\frac{\sqrt{63}}{8}$ if $\cos\theta=\frac{1}{8}$

we have $\cos\frac{\theta}{2}=\sqrt{\frac{1+\cos\theta}{2}}=\frac{3}{4}$

$$\sin\frac{\theta}{4} = \sqrt{\frac{1-\cos\frac{\theta}{2}}{2}} = \frac{1}{2\sqrt{2}}$$

Now $\log_2 \sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right) = \log_2 \frac{1}{2\sqrt{2}} = -\frac{3}{2}$

(D) $\cos^{-1}\frac{\sqrt{5}}{3} = 0$

$\cos\theta = \frac{\sqrt{5}}{3}$

$\therefore \quad \tan\frac{\theta}{2} = \frac{3-\sqrt{5}}{2}$ which is irrational

Inverse

Assertion Reasoning Type

A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct Explanation for Statement-1

B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

C) Statement-1 is True, Statement-2 is False

D) Statement-1 is False, Statement-2 is True

1. Statement 1: $\sum_{k=1}^{\infty} \cot^{-1} 2k^2 = \frac{\pi}{4}$

Statement 2: $\tan^{-1}\frac{1}{2k^2} = \tan^{-1}(2k+1) - \tan^{-1}(2k-1),\ k \in N$

Key. A

Sol. Conceptual

2. Statement 1: $\sum_{k=1}^{\infty} \cot^{-1} 2k^2 = \frac{\pi}{4}$

Statement 2: $\tan^{-1}\frac{1}{2k^2} = \tan^{-1}(2k+1) - \tan^{-1}(2k-1),\ k \in N$

Key. A

Sol. Conceptual

3. STATEMENT-1

The number of solutions of the equation $\sin x + \cos x = \sin^{-1} x + \cos^{-1} x$ is zero.

STATEMENT–2

$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}\ \forall x \in [-1, 1]$ and the maximum value of $(\sin x + \cos x)$ is $\sqrt{2}$ for $x \in R$.

Key: A

Hint: The maximum value of $(\sin x + \cos x)$ is $\sqrt{2}$ while $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}\ \forall\ x \in [-1, 1]$ so the equation $\sin x + \cos x = \sin^{-1} x + \cos^{-1} x$ has no solution

4. STATEMENT- 1: Range of the function f(x) = $\tan^{-1}$ ($\sqrt{x^2 - 6x + 12}$) is $\left[\frac{\pi}{3}, \frac{\pi}{2}\right)$

STATEMENT – 2: Range of f(x) = $\tan^{-1}$x is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Key: B

Hint: Conceptual

5. Statement I : $\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{1}{7}\right)=\frac{\pi}{4}$

Statement II : For x>0, y>0 $\tan^{-1}\left(\frac{x}{y}\right)+\tan^{-1}\left(\frac{y-x}{y+x}\right)=\frac{\pi}{4}$

1) Statement I & II are correct. Statement II is correct explonation of statement I.
2) Statement I & II are correct. Statement II is not explaining statement I
3) Statement I is correct, statement II is wrong.
4) Statement I is wrong, statement II is correct.

Key. 1

Sol. Conceptual

6. Statement I : The range of $\sin^{-1}x+\cos^{-1}x+\tan^{-1}x$ is $\left[\frac{\pi}{4},\frac{3\pi}{4}\right]$

Statement II : $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2} \quad \forall x\in R$

1) Statement I & II are correct. Statement II is correct explonation of statement I.
2) Statement I & II are correct. Statement II is not explaining statement I
3) Statement I is correct, statement II is wrong.
4) Statement I is wrong, statement II is correct.

Key. 3

Sol. Conceptual

7. Statement I : If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$ then $x^2+y^2+z^2-2xyz=1$

Statement II : If $A+B+C=\pi$ then $\cos^2 A+\cos^2 B+\cos^2 C=1-2\cos A\cos B\cos C$

1) Statement I & II are correct. Statement II is correct explonation of statement I.
2) Statement I & II are correct. Statement II is not explaining statement I
3) Statement I is correct, statement II is wrong.
4) Statement I is wrong, statement II is correct.

Key. 4

Sol. Conceptual

8. Statement I : If a,b,c are +ve then $\tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}}+\tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}}+\tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}=\pi$

Statement II : $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$ if x>0, y>0

1) Statement I & II are correct. Statement II is correct explonation of statement I.
2) Statement I & II are correct. Statement II is not explaining statement I
3) Statement I is correct, statement II is wrong.
4) Statement I is wrong, statement II is correct.

Key. 3

Sol. Conceptual

9. Statement I : The domain of the function $f(x)=\cos^{-1}\left(\log_2\left(x^2+5x+8\right)\right)$ is [-3, -2]

Statement II : $\cos^{-1}x$ is positive when $x\in[0,1)$

1) Statement I & II are correct. Statement II is correct explonation of statement I.
2) Statement I & II are correct. Statement II is not explaining statement I
3) Statement I is correct, statement II is wrong.
4) Statement I is wrong, statement II is correct.

Key. 2

Sol. Conceptual

10. Assertion: $\cot^{-1}3+\cot^{-1}7+\cot^{-1}13+..........\infty=\cot^{-1}2.$

Reason: $\sum_{r=1}^{n}\tan^{-1}\left(\frac{x_r-x_{r-1}}{1+x_r x_{r-1}}\right)=\tan^{-1}x_n-\tan^{-1}x_0\,\forall n\in N.$

Key. D

Sol. $\lim_{n\to\infty}\sum_{n=1}^{n}\cot^{-1}\left(1+n+n^2\right)=\lim_{n\to\infty}\sum_{n=1}^{n}\left[\tan^{-1}(n+1)-\tan^{-1}n\right]=\tan^{-1}\infty-\frac{\pi}{4}=\frac{\pi}{2}$

11. Assertion : If $\sum_{i=1}^{2n}\sin^{-1}x_i=n\pi, n\in N$ then $\sum_{i=1}^{n}x_i=\sum_{i=1}^{n}x_i^2=\sum_{i=1}^{n}x_i^3$

Reason: $-\frac{\pi}{2}\le\sin^{-1}x\le\frac{\pi}{2},\forall x\in[-1,1]$

Key. A

Sol. $\sum_{i=1}^{2n}\sin^{-1}x_i=n\pi$ si possible if $x_1=x_2=....x_{2n}=1$

$\sum_{i=1}^{n}x_i=1+1+1+......n\text{ terms=n}$

$\sum_{i=1}^{n}x_i^2=1^2+1^2+1^2+......n\text{ terms=n}$

$\sum_{i=1}^{n}x_i^3=1^3+1^3+1^3+......n\text{ terms=n}$

12. STATEMENT–1

If $\alpha\in(-\pi/2, 0)$ then the value of $2\tan^{-1}(\text{cosec}\alpha)+\tan^{-1}(2\sin\alpha\sec^2\alpha)$ is $-\pi$.

because

STATEMENT–2

If x < 0, $\tan^{-1}x+\tan^{-1}\frac{1}{x}=-\frac{\pi}{2}$

Key. A

Sol. Let $\sin\alpha=t\in(-1,0)$

So, $2\tan^{-1}\left(\frac{1}{t}\right)+\tan^{-1}\left(\frac{2t}{1-t^2}\right)=2\left[\tan^{-1}\left(\frac{1}{t}\right)+\tan^{-1}(t)\right]=2\left(-\frac{\pi}{2}\right)=-\pi$

13. STATEMENT – 1

$$\sin^{-1}2x+\sin^{-1}3x=\frac{\pi}{3}\Rightarrow x=\sqrt{\left(\frac{3}{76}\right)} \text{ only}$$

STATEMENT – 2

Sum of two negative angles cannot be positive

Key. A

Sol. $\because \sin^{-1}2x+\sin^{-1}3x=\frac{\pi}{3}$

$$\Rightarrow \frac{\pi}{2}-\cos^{-1}2x+\frac{\pi}{2}-\cos^{-1}3x=\frac{\pi}{3}$$

$$\Rightarrow \cos^{-1}2x+\cos^{-1}3x=\frac{2\pi}{3}$$

$$\Rightarrow \cos^{-1}\left\{6x^2-\sqrt{1-(2x)^2}\sqrt{1-(3x)^2}\right\}=\frac{2\pi}{3}$$

$$\Rightarrow 6x^2-\sqrt{\left(1-13x^2+36x^4\right)}=-\frac{1}{2}$$

$$\Rightarrow \left(6x^2+\frac{1}{2}\right)^2=1-13x^2+36x^4$$

$$\Rightarrow 19x^2=\frac{3}{4}$$

14. **STATEMENT-1 :** $\sec^{-1}5<\tan^{-1}(7)$

STATEMENT-2 : $\sec^{-1}x<\tan^{-1}x$, if $x\ge 1$ and $\sec^{-1}x>\tan^{-1}x$ if $x\le -1$ and

$\tan^{-1}x_1>\tan^{-1}x_2$ if $x_1>x_2$

Key. A

Sol. $\tan^{-1}x$ is an increasing function

$\sec^{-1}x=\tan^{-1}x\left(\sqrt{x^2-1}\right)$, for $x\ge 1$

$\sec^{-1}x<\tan^{-1}x$ if $\tan^{-1}\left(\sqrt{x^2-1}\right)<\tan^{-1}x$

if $\sqrt{x^2-1}<x$

if $x^2-1<x^2$ which is true

15. Statement – 1: $\operatorname{cosec}^{-1}\left(\frac{1}{2}+\frac{1}{\sqrt{2}}\right)>\sec^{-1}\left(\frac{1}{2}+\frac{1}{\sqrt{2}}\right)$

Statement – 2: $\operatorname{cosec}^{-1}x>\sec^{-1}x$ if $1\le x<\sqrt{2}$

Key. A

Sol. $\operatorname{cosec}^{-1}x>\sec^{-1}x$

$\operatorname{cosec}^{-1}x>\frac{\pi}{2}\operatorname{cosec}^{-1}x$

$\text{cosec}^{-1}x > \frac{\pi}{4}$

$1 \le x < \sqrt{2}$ and $\left(\frac{1}{2}+\frac{1}{\sqrt{2}}\right) \in \left[1, \sqrt{2}\right)$

Statement 2 is true and explains statement 1

16. Let $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Statement – 1: $f'(2) = -\frac{2}{5}$

Statement – 2: $\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \pi - 2\tan^{-1}x, \ \forall x > 1$

Key. A

Sol. $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \pi - 2\tan^{-1}x, x \ge 1$

$f'(x) = -\frac{2}{1+x^2} \Rightarrow \quad f'(2) = -\frac{2}{5}$

Statement – 1 is statement – 2 is True; Statement-2 is a correct explanation for statement-1.

17. Statement – 1: $\sin^{-1}\left(\frac{1}{\sqrt{e}}\right) > \tan^{-1}\left(\frac{1}{\sqrt{\pi}}\right)$

Statement – 2: $\sin^{-1}x > \tan^{-1}y$ for $x > y, \ \forall \ x, y \in (0,1)$

Key. A

Sol. $\sin^{-1}x = \tan^{-1}\frac{x}{\sqrt{1-x^2}} > \tan^{-1}x > \tan^{-1}y \qquad \left\{ Q \qquad x > y, \frac{x}{\sqrt{1-x^2}} > x \right\}$

$\therefore$ Statement -2 is true

$e < \pi$

$\frac{1}{\sqrt{e}} > \frac{1}{\sqrt{\pi}}$

by statement – 2

$\sin^{-1}\left(\frac{1}{\sqrt{e}}\right) > \tan^{-1}\left(\frac{1}{\sqrt{e}}\right) > \tan^{-1}\left(\frac{1}{\sqrt{\pi}}\right)$

Statement – 1 is true

Inverse

Comprehension Type

Paragraph – 1

Consider the equations $\cos^{-1}x+\left(\sin^{-1}y\right)^2=\dfrac{a\pi^2}{4}$ $-(1)$

$\&\cos^{-1}x.\left(\sin^{-1}y\right)^2=\dfrac{\pi^4}{16}$ $-(2)$

1. The complete set of values of a for which the equation (1) holds good is

a) $\left(0,\dfrac{4}{\pi}+1\right)$ b) $\left[0,\dfrac{4}{\pi}+1\right]$ c) R d) ϕ

Key. B

2. The set of values of a for which $(1)\&(2)$ posses solution is

a) $(-\infty,-2]\cup[2,\infty)$ b) $(-2,2)$ c) $\left[2,\dfrac{4}{\pi}+1\right]$ d) R

Key. C

3. The values of x and y such that system $(1)\&(2)$ posses solutions for integral values of 'a':

a) $\left(\cos\dfrac{\pi^2}{4},\dfrac{1}{2}\right)$ b) $\left(\cos\dfrac{\pi^2}{4},-\dfrac{1}{2}\right)$

c) $\left(\cos\dfrac{\pi^2}{4},\pm1\right)$ d) $x\in R, y\in R$

Key. C

Sol. 1,2,3. $0\le\cos^{-1}x\le\pi$ & $0\le\left(\sin^{-1}y\right)^2\le\dfrac{\pi^2}{4}$

$\therefore 0\le\dfrac{\Pi^2}{4}\le\Pi+\dfrac{\Pi^2}{4}\Rightarrow 0\le a\le\dfrac{4}{\pi}+1$

From $(1)\&(2)$ $16\left(\cos^{-1}x\right)^2-4a\pi^2\left(\cos^{-1}x\right)+\pi^4=0$ & $\cos^{-1}x\in R; D\ge 0$

$\Rightarrow a^2\ge4\Rightarrow|a|\ge2$

$\therefore a\in\left[2,\dfrac{4}{\pi}+1\right]$

If a is an integer then $a=2$

For $a=2$

$16\left(\cos^{-1}x\right)^2-8\pi^2\cos^{-1}x+\pi^4=0$

$\left(4\cos^{-1}x-\pi^2\right)^2=0\Rightarrow\cos^{-1}x=\dfrac{\pi^2}{4}\Rightarrow x=\cos\dfrac{\pi^2}{4}$

$\dfrac{\pi^2}{4}\left(\sin^{-1}y\right)^2=\dfrac{\pi^2}{16}\Rightarrow\left(\sin^{-1}y\right)^2=\dfrac{\pi^2}{4}\Rightarrow\sin^{-1}y=\pm\dfrac{\pi}{2}$ then $y=\pm1$

Paragraph – 2

If $f(x)=\sin^{-1}\left(\left[\dfrac{x-1}{2-x}\right]\right)$ where [.] denotes the greatest integer function.

4. The domain of f(x) is

(a) (–∞, –2) (b) [1, 5/3)

(c) (5/3, 2) (d) (–∞, 5/3)

Key. D

5. The range of f(x) is

(a) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ (b) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\sim\{0\}$

(c) $\left\{-\frac{\pi}{2},\frac{\pi}{2}\right\}$ (d) $\left\{-\frac{\pi}{2},0,\frac{\pi}{2}\right\}$

Key. D

6. Let n be an non-negative integer, then solution of the equation $\left[\frac{x-1}{2-x}\right]=n$ is

(a) $\left[\frac{2n+1}{n+1},\frac{2n+3}{n+2}\right)$ (b) $\left[\frac{2n+1}{n+1},2\right)$

(c) $\left(-\infty,\frac{2n+3}{n+2}\right)\cup(2,\infty)$ (d) $\left(-\infty,\frac{2n+3}{n+2}\right)$

Key. A

Sol. 4. There domain of f consists of those real numbers x ≠ 2 for which $\left[\frac{x-1}{2-x}\right]$ lies on the interval [–1, 1]

$\Rightarrow\quad -1\le\frac{x-1}{2-x}<2$

$\frac{1}{x-2}\le 0\ \&\ \frac{3x-5}{x-2}>0$

$\Rightarrow$ domain $\left(-\infty,\frac{5}{3}\right)$

5. For $\sin\left(-\infty,\frac{5}{3}\right)$, the $\left[\frac{x-1}{2-x}\right]$ lies in [–1, 1] and thus takes the values – 1, 0 and 1 atmost

Therefore range of f(x) is $\left\{-\frac{\pi}{2},0,\frac{\pi}{2}\right\}$

6. $\left[\frac{x-1}{2-x}\right]=n$

$\Rightarrow\quad n\le\frac{x-1}{2-x}<n+1$

$\Rightarrow\quad \frac{(n+1)x-(2n+1)}{x-2}\le 0\ \&\ \frac{(n+2)x-(2n+3)}{x-2}>0$

$\Rightarrow\quad x\in\left[\frac{2n+1}{n+1},\frac{2n+3}{n+2}\right)$

Paragraph – 3

$$\sum_{r=1}^{n}\tan^{-1}\left(\frac{x_r-x_{r-1}}{1+x_{r-1}x_r}\right)=\sum_{r=1}^{n}\left(\tan^{-1}x_r-\tan^{-1}x_{r-1}\right)=\tan^{-1}x_n-\tan^{-1}x_0,\forall n\in N$$

7. The sum to infinite terms of the series $\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{2}{9}\right)+......+\tan^{-1}\left(\frac{2^{n-1}}{1+2^{2n-1}}\right)+.......to\,\infty$ is

(A) $\frac{\pi}{4}$
B) $\frac{\pi}{2}$
(C) π
(D) none of these

Key. A

8. The value of $\cos ec^{-1}\sqrt{5}+\cos ec^{-1}\sqrt{65}+\cos ec^{-1}\sqrt{(325)}+.....to\,\infty$

(A) π
(B) $\frac{3\pi}{4}$
(C) $\frac{\pi}{2}$
(D) $\frac{\pi}{4}$

Key. D

9. The sum to infinite terms of the series

$$\cot^{-1}\left(2^2+\frac{1}{2}\right)+\cot^{-1}\left(2^3+\frac{1}{2^2}\right)+\cot^{-1}\left(2^4+\frac{1}{2^3}\right)+.....is$$

(A) $\frac{\pi}{4}$
(B) $\frac{\pi}{2}$
(C) $\cot^{-1}2$
(D) $-\cot^{-1}2$

Key. C

Sol. 7. $\lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(\frac{2^{r-1}}{1+2^{2r-1}}\right)$

$= \lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(\frac{2^r-2^{r-1}}{1+2^r.2^{r-1}}\right)$

$= \tan^{-1}2^{\infty}-\tan^{-1}1$

$= \frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}$

8. $\cos ec^{-1}\sqrt{5}+\cos ec^{-1}\sqrt{65}+\cos ec^{-1}\sqrt{(325)}+...\infty$

$= \cot^{-1}2+\cot^{-1}8+\cot^{-1}18+...\infty$

$= \lim_{n\to\infty}\sum_{r=1}^{n}\cot^{-1}\left(2r^2\right)$

$= \lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(\frac{2}{4r^2}\right)$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(\frac{2}{1+4r^2-1}\right)$$

$$=\tan^{-1}\infty-\tan^{-1}1$$

9. $$\lim_{n\to\infty}\sum_{r=1}^{n}\cot^{-1}\left(2^{r+1}+\frac{1}{2^r}\right)$$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(\frac{2^r}{1+2^r.2^{r+1}}\right)$$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(\frac{2^{r+1}-2^r}{1+2^r.2^{r+1}}\right)$$

$$=\tan^{-1}\infty-\tan^{-1}2$$

$$=\cot^{-1}2$$

Paragraph – 4

It is given that $A=\left(\tan^{-1}x\right)^3+\left(\cot^{-1}x\right)^3$ where $x>0$ and $B=\left(\cos^{-1}t\right)^2+\left(\sin^{-1}t\right)^2$ where $t\in\left[0,\frac{1}{\sqrt{2}}\right]$, and $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$ for $-1\le x\le 1$ and $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$ for all $x\in R$

10. The interval in which A lies is

A) $\left[\frac{\pi^3}{7},\frac{\pi^3}{2}\right)$ B) $\left[\frac{\pi^3}{32},\frac{\pi^3}{8}\right)$ C) $\left[\frac{\pi^3}{40},\frac{\pi^3}{10}\right)$ D) none of these

Key. B

11. The maximum value of B is

A) $\frac{\pi^2}{8}$ B) $\frac{\pi^2}{16}$ C) $\frac{\pi^2}{4}$ D) none of these

Key. C

12. If least value of A is λ and maximum value of B is μ, then $\cot^{-1}\cot\left(\frac{\lambda-\mu\pi}{\mu}\right)=$

A) $\frac{\pi}{8}$ B) $-\frac{\pi}{8}$ C) $\frac{7\pi}{8}$ D) $-\frac{7\pi}{8}$

Key. A

Sol. 10.

$$A=\left(\tan^{-1}x\right)^3+\left(\cot^{-1}x\right)^3$$

$$A=\left(\tan^{-1}x+\cot^{-1}x\right)^3-3\tan^{-1}x\cot^{-1}x\left(\tan^{-1}x+\cot^{-1}x\right)$$

$$\Rightarrow\quad A=\left(\frac{\pi}{2}\right)^3-3\tan^{-1}x\cot^{-1}x.\frac{\pi}{2}$$

$$\Rightarrow\quad A=\frac{\pi^3}{8}-\frac{3\pi}{2}\tan^{-1}x\left(\frac{\pi}{2}-\tan^{-1}x\right)$$

$$\Rightarrow\quad A=\frac{\pi^3}{32}+\frac{3\pi}{2}\left(\tan^{-1}x-\frac{\pi}{4}\right)^2$$

as $x>0$

$$\frac{\pi^3}{32} \le A < \frac{\pi^3}{8}$$

11. $B = \left(\sin^{-1} t\right)^2 + \left(\cos^{-1} t\right)^2$

$$B = \left(\sin^{-1} t + \cos^{-1} t\right)^2 - 2\sin^{-1} t \cos^{-1} t$$

$$B = \frac{\pi^2}{4} - 2\sin^{-1} t\left(\frac{\pi}{2} - \sin^{-1} t\right)$$

$$B = \frac{\pi^2}{8} + 2\left(\sin^{-1} t - \frac{\pi}{4}\right)^2$$

$$B^{max} = \frac{\pi^2}{8} + 2.\frac{\pi^2}{16} = \frac{\pi^2}{4}$$

12. $\lambda = \frac{\pi^3}{32}$ $\qquad \mu = \frac{\pi^2}{4}$

$$\frac{\lambda}{\mu} = \frac{\pi}{8}$$

$$\frac{\lambda - \mu\pi}{\mu} = \frac{\pi}{8} - \pi = \frac{-7\pi}{8}$$

$$\cot^{-1}\cot\left(\frac{\lambda - \mu\pi}{\mu}\right) = \cot^{-1}\cot\left(-\frac{7\pi}{8}\right) = \frac{\pi}{8}$$

Paragraph – 5

Given that $\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \begin{cases} 2\tan^{-1} x & , \quad |x| < 1 \\ -\pi + 2\tan^{-1} x & , \quad x > 1 \\ \pi + 2\tan^{-1} x & , \quad x < -1 \end{cases}$

$$\sin^{-1}\frac{2x}{1+x^2} = \begin{cases} 2\tan^{-1} x & \text{is} \quad |x| \le 1 \\ \pi - 2\tan^{-1} x & \text{is} \quad x > 1 \\ -\left(\pi + 2\tan^{-1} x\right) & \text{is} \quad x < -1 \end{cases}$$

And $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ for $-1 \le x \le 1$

13. $\sin^{-1}\left(\frac{4x}{x^2+4}\right) + 2\tan^{-1}\left(-\frac{x}{2}\right)$ is independent from x then

A) $x \in [-3, 4]$ B) $x \in [-2, 2]$ C) $x \in [-1, 1]$ D) $x \in [1, \infty]$

Key. B

14. If $\cos^{-1}\frac{6x}{1+9x^2} = -\frac{\pi}{2} + 2\tan^{-1} 3x$, then $x \in$

A) $\left(\frac{1}{3}, \infty\right)$ B) $(-1, \infty)$ C) $(-\infty, -1)$ D) none of these

Key. A

15. If $(x-1)(x^2+1) > 0$, then $\sin\left(\frac{1}{2}\tan^{-1}\frac{2x}{1-x^2} - \tan^{-1} x\right) =$

A) 1 B) $\frac{1}{\sqrt{2}}$ C) – 1 D) none of these

Key. C

Sol. 13. $\sin^{-1}\left(\frac{4x}{x^2+4}\right)+2\tan^{-1}\left(-\frac{x}{2}\right)=\sin^{-1}\left(\frac{2.\frac{x}{2}}{\left(\frac{x}{2}\right)^2+1}\right)-2\tan^{-1}\frac{x}{2}=2\tan^{-1}\frac{x}{2}-2\tan^{-1}\frac{x}{2}=0$

Here $\left|\frac{x}{2}\right|\le 1$

$|x|\le 2 \quad \Rightarrow \quad -2x\le x\le 2$

14. $\cos^{-1}\frac{6x}{1+9x^2}=-\frac{\pi}{2}+2\tan^{-1}3x$

$\Rightarrow \quad \frac{\pi}{2}-\sin^{-1}\frac{6x}{1+9x^2}=-\frac{\pi}{2}+2\tan^{-1}3x$

$\Rightarrow \quad \sin^{-1}\frac{6x}{1+9x^2}=\pi-2\tan^{-1}3x \quad \Rightarrow \quad \sin^{-1}\frac{2.3x}{1+(3x)^2}=\pi-2\tan^{-1}3x$

Above is true when 3x > 1 $\quad \Rightarrow \quad x>\frac{1}{3}$

$x\in\left(\frac{1}{3},\infty\right)$

15. $(x-1)(x^2+1)>0$

$\Rightarrow \quad x>1$

$\therefore \quad \sin\left[\frac{1}{2}\tan^{-1}\left(\frac{2x}{1-x^2}\right)-\tan^{-1}x\right]=\sin\left[\frac{1}{2}\left(-\pi+2\tan^{-1}x\right)-\tan^{-1}x\right]=\sin\left(-\frac{\pi}{2}\right)=-1$

Inverse

Integer Answer Type

1. The number of solutions of the equation $\sin^{-1}\frac{1+x^2}{2x}=\frac{\pi}{2}\sec(x-1)$ are

Key: 1

Hint: $\left|\frac{1+x^2}{2x}\right|\le 1 \Rightarrow |x|=1 \Rightarrow x=\pm 1$

But $x=-1$ will not satisfy the equation.

2. If $x\in[0,4\pi], y\in[0,4\pi]$, then number of ordered pairs (x, y) which satisfy the equation $S\,\text{in}^{-1}\sin x+C\,\text{os}^{-1}\cos y=\frac{3\pi}{2}$ are ……………………

Key: 4

Hint: $\sin^{-1}\sin x\le\frac{\pi}{2},\cos^{-1}\cos y\le\pi$ hence sinx = 1 , cos y = -1

Ordered pairs $\left(\frac{\pi}{2},\pi\right),\left(\frac{\pi}{2},3\pi\right),\left(\frac{5\pi}{2},\pi\right),\left(\frac{5\pi}{2},3\pi\right)$ are the solution.

3. If the equation $\sin^{-1}(x^2+x+1)+\cos^{-1}(ax+1)=\frac{\pi}{2}$ has exactly two solutions then the only possible integral value of a is

Key. 1

Sol. The given equation holds if $x^2+x+1=ax+1$ and $-1\le x^2+x+1\le 1$

$\Rightarrow x(x+1+a)=0$ and $-1\le x\le 0$

$\Rightarrow x=0$ or $a-1$ and $-1\le x\le 0$

$\therefore x=0$ is one solution and for another different solution $-1\le a-1<0$

$\Rightarrow 0\le a\le 1$. So only integral value a can have is 0.

4. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{3\pi}{2}$ and f is a function

Which satisfies $f(1)=2,\ f(p+q)=f(p).f(q)\forall p,q\in R,$ then $f\left(\frac{x+y+z}{xyz}\right)$ equals to

Key. 8

Sol. $-\frac{\pi}{2}<\sin^{-1}x\le\frac{\pi}{2}$

$\therefore \sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{3\pi}{2}$

$\Leftrightarrow \sin^{-1}x=\sin^{-1}y=\sin^{-1}z=\frac{\pi}{2}\Leftrightarrow x=y=z=1$

Also $f(p+q)=f(p).f(q)\forall p.q\in R$ .....(1)

Given $f(1)=2$

From (1), $f(1+1)=f(1).f(1)\Rightarrow f(2)=2^2=4$ ....(2)

From (2), $f(2+1)=f(2).f(1)=2^2.2=2^3=8$

Now given expression = $f(3)=8$

The given relation is possible when

$$a-\frac{a^2}{3}+\frac{a^3}{9}+...=1+b+b^2+...$$

Also

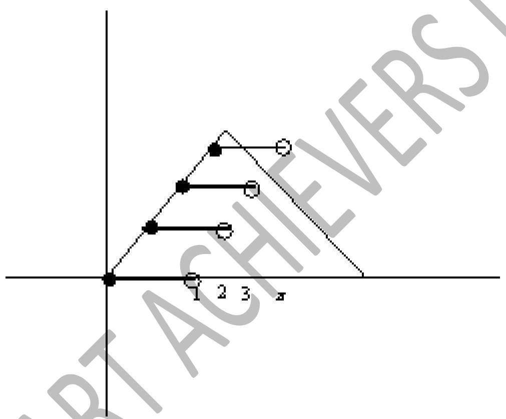
$$-1\le a-\frac{a^2}{3}+\frac{a^3}{9}+....\le 1\ \&\ -1\le 1+b+b^2+....\le 1$$

$$\Rightarrow |b|<1\Rightarrow |a|<3 \text{ and } \frac{a}{1+\frac{a}{3}}=\frac{1}{1-b}$$

$\Rightarrow \frac{3a}{a+3}=\frac{1}{1-b}$, there are infinitely many solution. But in given options it is satisfied only when $a=1$ and $b=\frac{1}{3}$

5. The number of Real solutions of $\cos^{-1}(\cos x)=[x]$ where [.] denotes greatest integer function is________

KEY. 5

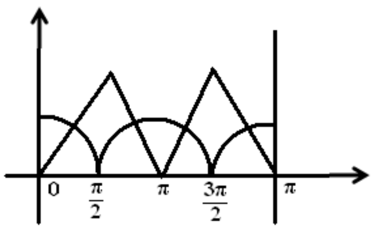


SOL.
from graph no.of solutions =5

6. The number of solutions of the equation $\left|\cos^{-1}x\right|=\sin^{-1}\left|\sin x\right|$ in $\left[0,2\pi\right]$ is

Key. 4

Sol.



7. The number of ordered pairs (x, y) satisfying the system of equations

$\left(\cos^{-1} x\right)^2 + \sin^{-1} y = 1$ and $\cos^{-1} x + \left(\sin^{-1} y\right)^2 = 1$ is (are)

Key. 3

Sol. Let $a = \cos^{-1} x, b = \sin^{-1} y$

$a^2 + b = 1$ & $a + b^2 = 1 \Rightarrow a = b$ or $a = 1, b = o$ or $a = 1, b = 0$

If $a^2 b \Rightarrow a^2 + a - 10 \Rightarrow a = \frac{-1 \pm \sqrt{5}}{2}$

$a \in (o, \pi) b \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right) \Rightarrow a = b = \frac{\sqrt{5}-1}{2} \Rightarrow (x, y) = \left(\cos^{-1}\left(\frac{\sqrt{5}-1}{2}\right), \sin^{-1}\left(\frac{\sqrt{5}-1}{2}\right)\right)$

$a = 1, b = 0 \Rightarrow (x, y) = \left(\cos^{1}, 0\right)$

$a = 0, b = 1 \Rightarrow (x, y) = (1, \sin 1)$

8. Least value of n for which

$(n-2)x^2 + 8x + n + 4 > \sin^{-1} \sin(12) + \cos^{-1} \cos(12), \ \forall \ x \in R$ where $n \in N$

Ans. 5

Sol. $(n-2)x^2 + 8x + n + 4 > 0 \ \forall \ x \in R$

$\sin^{-1} \sin(12) = -(4\pi - 12)$

$\cos^{-1} \cos(12) = 4\pi - 12$

9. The number of solutions of the equation $\sin^{-1}\left(\frac{ax}{c}\right) + \sin^{-1}\left(\frac{bx}{c}\right) = \sin^{-1} x$ where

$a^2 + b^2 = c^2, a, b, c$ are positive real numbers, is (are)

Key. 2

Sol. $\sin^{-1} \frac{ax}{c} = \sin^{-1} x - \sin^{-1}\left(\frac{bx}{c}\right)$

$\Rightarrow \frac{ax}{c} = x\sqrt{-\frac{b^2 x^2}{c^2}} - \sqrt{1 - x^2}\left(\frac{bx}{c}\right)$

$\Rightarrow x = 0$ or $a = \sqrt{c^2 - b^2 x^2} - b\sqrt{1 - x^2}$

$x = 0$ or $\sqrt{1 - x^2} = 0$

10. The value of $\tan\left[\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{2}{9} + \tan^{-1}\frac{4}{33} + ... + \tan^{-1}\left(\frac{2^{n-1}}{1 + 2^{2n-1}}\right) + ...\infty\right]$ is`

Key. 1

Sol. $Tn = \tan^{-1}\left[\frac{2^{n-1}}{1 + 2^{2n-1}}\right] = \tan^{-1}\left[\frac{2^n - 2^{n-1}}{1 + 2^n . 2^{n-1}}\right]$

$= \tan\left(2^n\right) - \tan^{-1}\left(2^{n-1}\right)$

11. If $S = \sum_{r=1}^{50} \tan^{-1}\left(\frac{2r}{2 + r^2 + r^4}\right) = \tan^{-1}\left(\frac{\lambda}{\mu}\right)$, where $\left(\frac{\lambda}{\mu}\right)$ is in simplest form, then $\mu - \lambda =$

Key. 1

Sol. $S=\sum_{r=1}^{50}\left(\tan^{-1}(1+r+r^2)-\tan^{-1}(1-r+r^2)\right)$

12. Simplify the following: $\tan^{-1}\left[\frac{3\sin 2\alpha}{5+3\cos 2\alpha}\right]+\tan^{-1}\left[\frac{\tan\alpha}{4}\right]$ where $-\frac{\pi}{2}<\alpha<\frac{\pi}{2}$.

Key. α

Sol. $\tan^{-1}\left[\frac{3\sin 2\alpha}{5+3\cos 2\alpha}\right]+\tan^{-1}\left[\frac{\tan\alpha}{4}\right]=\tan^{-1}\left(\frac{6\tan\alpha}{8+2\tan^2\alpha}\right)+\tan^{-1}\left(\frac{\tan\alpha}{4}\right)$

$$=\tan^{-1}\left(\frac{\frac{3\tan\alpha}{4+\tan^2\alpha}+\frac{\tan\alpha}{4}}{1-\frac{3\tan^2\alpha}{16+4\tan^2\alpha}}\right)\qquad\left\{\because\ \frac{3\tan^2\alpha}{16+4\tan^2\alpha}<1\right\}$$

$=\tan^{-1}(\tan\alpha)=\alpha$ Ans.

13. Solve for x, if $\left(\tan^{-1}x\right)^2+\left(\cot^{-1}x\right)^2=\frac{5\pi^2}{8}$.

Key. $x=-1$

Sol.

$$\left(\tan^{-1}x\right)^2+\left(\cot^{-1}x\right)^2=\left(\tan^{-1}x+\cot^{-1}x\right)^2-2\tan^{-1}x\left(\frac{\pi}{2}-\tan^{-1}x\right)=\frac{\pi^2}{4}-\pi\tan^{-1}x+2\left(\tan^{-1}x\right)^2=\frac{5\pi^2}{8}$$

$\therefore$ $\tan^{-1}x=\frac{2\pi}{3},-\frac{\pi}{4}$

$\tan^{-1}x=-\frac{\pi}{4}$ $\left\{\tan^{-1}x\neq\frac{2\pi}{3}\right\}$

$\therefore$ $x=-1$ is the solution

Inverse

Matrix-Match Type

1.

Column-I	Column-II
A) $\sin\left(2Tan^{-1}\frac{3}{4}\right)$	P) $\frac{14}{15}$
B) $\cos\left(2Tan^{-1}\frac{1}{7}\right)$	Q) $\frac{3}{5}$
C) $\sin\left(4Tan^{-1}\frac{1}{3}\right)$	R) $\frac{2}{\sqrt{13}}$
D) $\cos 2\left(Tan^{-1}\frac{1}{4}+Tan^{-1}\frac{2}{9}\right)$	S) 1
	T) $\frac{24}{25}$

Key. A-t:B-t;C-t; D-q

Sol. Conceptual

2. Let $t_1=\left(\sin^{-1}x\right)^{\sin^{-1}x}, t_2=\left(\sin^{-1}x\right)^{\cos^{-1}x}$ $t_3=\left(\cos^{-1}x\right)^{\sin^{-1}x}, t_4=\left(\cos^{-1}x\right)^{\cos^{-1}x}$

Match the items of Column I with that of Column II

	Column-I		Column-II
(A)	$x\in\left(0, \cos 1\right)$	(p)	$t_1>t_2>t_4>t_3$
(B)	$x\in\left(\cos 1, \frac{1}{\sqrt{2}}\right)$	(q)	$t_4>t_3>t_1>t_2$
(C)	$x\in\left(\frac{1}{\sqrt{2}}, \sin 1\right)$	(r)	$t_2>t_1>t_4>t_3$
(D)	$x\in\left(\sin 1, 1\right)$	(s)	$t_3>t_4>t_1>t_2$

Key $A\to Q; B\to S; C\to R; D\to P$

Hint : (A) In $0 < x < \cos 1$ we have $\cos^{-1}x > \sin^{-1}x$.

Also $\cos^{-1}x > 1$ and $\sin^{-1}x < 1$

The greatest is $\left(\cos^{-1}x\right)^{\cos^{-1}x} = t_4$ and least is $\left(\sin^{-1}x\right)^{\cos^{-1}x} = t_2$ and $\left(\sin^{-1}x\right)^{\sin^{-1}x} < = t_2 \left(\cos^{-1}x\right)^{\sin^{-1}x}$

$t_1 < t_3$

So, $t_4 > t_3 > t_1 > t_2$

(B) Similarly in $\cos 1 < x < \frac{1}{\sqrt{2}}$ $\cos^{-1}x > \sin^{-1}x$ and both are less than 1

So, greatest is t_3 and least is t_2 and $t_4 > t_1$

Hence, $t_3 > t_4 > t_1 > t_2$

(C) For $\frac{1}{\sqrt{2}} < x < \sin 1$

We have, $1 > \sin^{-1}x > \cos^{-1}x$

So, greatest is t_2 and the least is t_3, also $t_1 > t_4$

Hence, $t_2 > t_1 > t_4 > t_3$

(D) For $\sin 1 < x < 1$, we have $\sin^{-1}x > 1 > \cos^{-1}x$

So, the greatest is t_1 and the least is t_3 and $t_2 > t_4$.

3. Column – I | Column - II

Column – I	Column - II
A) If the equation $x^2 + 4 + 3\sin(ax+b) - 2x = 0$ has at least One real solution, where $a, b \in [0, 2\pi]$ then $\sin(a+b)$ can be equal to	P) -1
B) If $\sin^{-1}x \le \cos^{-1}x$ then x can be equal to	Q) 0
C) The number of the ordered pairs (x, y) satisfying $\lvert y \rvert = \cos x$ and $y = \sin^{-1}(\sin x)$, where $-2\pi \le x \le 3\pi$ is equal to	R) 1
D) If $n \in N$ and the set of equation $\cos^{-1}x + (\sin^{-1}y)^2 = \frac{n\pi^2}{4}$ and $(\sin^{-1}y)^2 - \cos^{-1}x = \frac{\pi^2}{16}$ is consistent, then n can be equal to	S) 5

Key. A-P, B-PQ, C-S, D-R

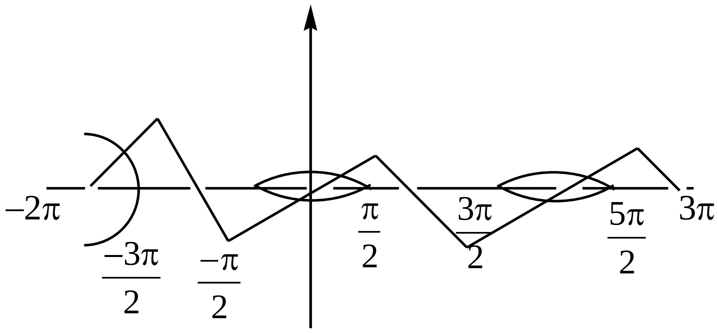
Sol. $x^2 + 4 + 3\sin(ax+b) - 2x = 0 \Rightarrow (x-1)^2 + 3\{1 + \sin(ax+b)\} = 0$

The above equation holds if and only if x = 1 and $\sin(ax+b) = -1 \Rightarrow \sin(a+b) = -1$

$\cos^{-1}x \ge \sin^{-1}x \Rightarrow \frac{\pi}{2} - \sin^{-1}x \ge \sin^{-1}x$

or $\sin^{-1}x \le \frac{\pi}{4} \Rightarrow -1 \le x \le \frac{1}{\sqrt{2}}$

The graphs of $\lvert y \rvert = \cos x$ and intersect at five points in $[-2\pi, 3\pi]$



Adding we get, $(\sin^{-1} y)^2 = \frac{4n+1}{32}\pi^2$

$\Rightarrow 0 \le \frac{4n+1}{32}\pi^2 \le \frac{\pi^2}{4} \Rightarrow -\frac{1}{4} \le n \le \frac{7}{4}$

Also, $\cos^{-1} x = \frac{4n-1}{32}x^2$

$\Rightarrow\ 0 \le \frac{4n-1}{32}\pi^2 \le \pi \Rightarrow \frac{1}{4} \le \frac{8}{\pi} + 1$

Hence, $n = 1$

4. Match the following:

Column-I Column-II

a) No. of solutions of the equations $y = |\sin x|$ and $y = \cos^{-1}(\cos x)$ for $x \in [-4\pi, 4\pi]$ p) 1

b) If $\sin^{-1} x \le \cos^{-1} x,$ then x can be equal to q) 0

c) The number of the ordered pair (x, y) satisfying $|y| = \cos x$ and $y = \sin^{-1}(\sin x),$ where $-2\pi \le x \le 3\pi$ r) 4

d) If $n \in N$ and the set of equations $\cos^{-1} x + (\sin^{-1} y)^2 = \frac{n\pi^2}{4}$ and $(\sin^{-1} y)^2 - \cos^{-1} x = \frac{\pi^2}{16}$ is consistent, then n can be equal to. s) 5

Key. A $\rightarrow$ S; B $\rightarrow$ Q; C $\rightarrow$ S; D $\rightarrow$ P

Sol. (d) plot both graphs, there five points of intersection.

(b) $\frac{\pi}{2} - \sin^{-1} x \ge \sin^{-1} x$

(c) plot both graphs.

(d) Adding, $(\sin^{-1} y)^2 = \frac{4n+1}{32}\pi^2$

$\Rightarrow 0 \le \frac{4n+1}{32}\pi^2 \le \frac{\pi^2}{4}$

$\Rightarrow -\frac{1}{4} \le n \le \frac{7}{4}$

$\cos^{-1} x = \frac{4n-1}{32}\pi^2$

$\Rightarrow 0 \le \frac{4n-1}{32}\pi^2 \le \pi$

$\Rightarrow \frac{1}{4} \le n \le \frac{8}{\pi} + 1$

$\therefore n = 1.$

5. [.] epresents greatest integer function in parts (A), (B) and (C)

Column – I		Column – II	
(A)	If $f(x) = \sin^{-1} x$ and $\lim\limits_{x\to\frac{1^+}{2}} f(3x-4x^3) = a - 3\lim\limits_{x\to\frac{1^+}{2}}$ f(x) then [a] =	(p)	2
(B)	If $f(x) = \tan^{-1}$ g(x) where $g(x) = \frac{3x-x^3}{1-3x^2}$ and $\lim\limits_{h\to 0}\frac{f(a+3h)-f(a)}{3h} = \frac{3}{1+a^2}$, when $\frac{-1}{3} < a < \frac{1}{\sqrt{3}}$, then find $\left[\lim\limits_{h\to 0}\frac{f\left(\frac{1}{2}+6h\right)-f\left(\frac{1}{2}\right)}{6h}\right] =$		
(C)	If $\cos^{-1}(4x^3-3x) = a + b\cos^{-1} x$ for $-1 < x < \frac{-1}{2}$, then [a + b + 2] =	(r)	4
(D)	If $f(x) = \cos^{-1}(4x^3-3x)$ and $\lim\limits_{x\to\frac{1^+}{2}} f'(x) = a$ and $\lim\limits_{x\to\frac{1^-}{2}} f'(x) = b$, then a + b – 3 =	(s)	- 2
		(t)	-3

Key. A $\to$ q; B $\to$ p; C $\to$ s; D $\to$ t

Sol. (A) $\sin^{-1}(3x-4x^3) = \pi - 3\sin^{-1} x$ if $\frac{1}{x} < x < 1$

$\therefore \quad \lim\limits_{x\to\frac{1^+}{2}} f(3x-4x^3) = \lim\limits_{x\to\frac{1^+}{2}}(\pi - 3\sin^{-1} x) = \pi - 3\lim\limits_{x\to\frac{1^+}{2}} \sin^{-1} x$

$\therefore \quad a = \pi$

$\therefore \quad [a] = 3$

(B) $f(x) = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) = 3\tan^{-1} x$, when $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

If $-\frac{1}{\sqrt{3}} < a < \frac{1}{\sqrt{3}}$, then $\lim\limits_{h\to 0}\frac{f(a+3h)-f(a)}{3h} = \frac{3}{1+a^2} \quad \Rightarrow \quad f'(a) = \frac{3}{1+a^2}$

$= \lim\limits_{h\to 0}\frac{f\left(\frac{1}{2}+6h\right)-f\left(\frac{1}{2}\right)}{6h} = f'(1/2) = \frac{12}{5}$

$\therefore$ required value = 2

(C) $\cos^{-1}(4x^3-3x) = \cos^{-1}(\cos 3\theta) = 3\theta - 2\pi \qquad \{\because\ 2\pi/3 < \theta < \pi\}$

$= -2\pi + 3\cos^{-1} x$

$\therefore \quad [a+b+2] = [-2\pi+3+2] = -2$

(D) $f(x) = \cos^{-1}(4x^3-3x) = \cos^{-1}(\cos 3\theta) = \begin{cases} 3\theta, & 0 < \theta < \frac{\pi}{3} \\ 2\pi - 3\theta, & \frac{\pi}{3} < \theta < \frac{\pi}{2} \end{cases} = \begin{cases} 3\cos^{-1} x, & \frac{1}{2} < x < 1 \\ 2\pi - 3\cos^{-1} x, & 0 < x < \frac{1}{2} \end{cases}$

$$\therefore \quad f'(x)=\begin{cases}\dfrac{-3}{\sqrt{1-x^2}}, & \dfrac{1}{2}<x<1\\[2mm] \dfrac{3}{\sqrt{1-x^2}}, & 0<x<\dfrac{1}{2}\end{cases}$$

$$a=\lim_{x\to\frac{1}{2}^+} f'(x)=-2\sqrt{3}$$

$$b=\lim_{x\to\frac{1}{2}^-} f'(x)=2\sqrt{3}$$

$\therefore \quad$ a + b – 3 = - 3

6. Match the following: -

Column – I		Column – II	
(A)	Absolute difference of greatest and least value of $\sqrt{2}\,(\sin 2x-\cos 2x)$	(p)	$\frac{\pi}{4}$
(B)	Absolute difference of greatest and least value of $x^2-4x+3,\ x\in[1,3]$, is		$\frac{\pi}{6}$
(C)	Greatest value of $\tan^{-1}\frac{1-x}{1+x}, x\in[0,1]$, is	(r)	4
(D)	Absolute difference of greatest and least value of $\cos^{-1}x^2, x\in\left[-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right]$, is	(s)	1
		(t)	$\frac{\pi}{3}$

Key. A $\to$ r; B $\to$ s; C $\to$ p; D $\to$ q

Sol. (A) The difference = 2 – (- 2) = 4

(B) Let $f(x)=x^2-4x+3$

$f'(x)=2x-4=0 \quad \Rightarrow \quad x=2$

f(1) = 0, f(2) = - 1, f(3) = 0

$\therefore$ |greatest value – least value| = 1

(C) $\tan^{-1}\frac{1-x}{1+x}=\tan^{-1}1-\tan^{-1}x$

$\therefore \quad$ greatest value $=\frac{\pi}{4}$

(D) $\therefore \quad$ greatest value $=\frac{\pi}{2}$, least value $=\frac{\pi}{3}$

$\therefore \quad$ difference $=\frac{\pi}{6}$

7. Match the following: -

Column – I		Column – II	
(A)	Value of x satisfying $\sin^{-1}x-\cos^{-1}x=\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is	(p)	< 0
(B)	Value of $\sec^2\left(\tan^{-1}2\right)+\operatorname{cosec}^2\left(\cot^{-1}3\right)$ is		< 1
(C)	Greatest value of x, satisfying $\sin^{-1}x-\cos^{-1}x=\sin^{-1}(3x-2)$ is	(r)	≥ 1

(D)	Value of $\sin^{-1}(\sin 5)$ is	(s)	> 0
		(t)	> 2

Key. A $\rightarrow$ q,s; B $\rightarrow$ r,s,t; C $\rightarrow$ r,s; D $\rightarrow$ p,q

Sol. Conceptual