CLASS : CC (Advanced) **Limit, Continuity, Derivability & MOD** **TEST-15**

M.M.: 72 **Time: 60 Min**

PART-A
[SINGLE CORRECT CHOICE TYPE]

Q.1 to Q.8 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct. **[8 × 3 = 24]**

Q.1 Let $f(x, y) \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ such that $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ and $h^2 > ab$. If $f(2, 1) = f(1, 3) = f(3, 6) = 0$ and $\left.\frac{dy}{dx}\right|_{(2,1)} = \left.\frac{dy}{dx}\right|_{(3,6)}$ then $\left.\frac{dy}{dx}\right|_{(0,0)}$ is equal to

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) -2 (D) 3

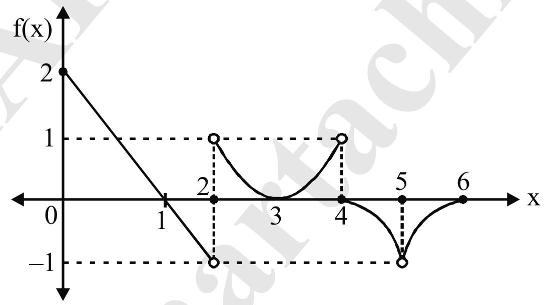
Q.2 $\lim_{x\to\infty} \frac{2x - \ln^2 x}{\sqrt{x}\ln x + x}$ is equal to

(A) $\frac{1}{2}$ (B) -1 (C) 1 (D) 2

Q.3 If $f : [1, \infty) \to [\sqrt{2}, \infty]$ be a function defined by $f(x) = \sqrt{x^2 - 2x + 3}$ and g be the inverse function of f then derivative of $g\left(f^2(x)\right)$ at $x = 3$ is

(A) $\frac{6}{\sqrt{34}}$ (B) $\frac{\sqrt{34}}{6}$ (C) $\frac{24}{\sqrt{34}}$ (D) $\frac{12}{\sqrt{34}}$

Q.4 Let graph of a function $y = f(x)$ is shown in the figure given below for $x \in [0, 6]$.



The number of points of discontinuity of the function $y = [|f(|x|)|]$ in $[-6, 6]$ are

(A) 0 (B) 2 (C) 3 (D) 4

[**Note :** [k] denotes greatest integer less than or equal to k.]

Q.5 If f and g are two functions with $g(x) = x - \frac{1}{x}$ and $fog(x) = x^3 - \frac{1}{x^3}$, then f '(x) is

(A) $3x^2 + 3$ (B) $x^2 - \frac{1}{x^2}$ (C) $1 + \frac{1}{x^2}$ (D) $3x^2 + \frac{3}{x^4}$

Q.6 Let $f : (-10\pi, 10\pi) \to (-10\pi, 10\pi)$, $f(x) = x - \sin x$ be a function, and g be the inverse of f. The number of points where $g'(x)$ does not exist, is

(A) 8 (B) 9 (C) 18 (D) 19

Q.7 Let $f(x) = \text{minimum}\left(\cos x, \frac{1}{2}, \{\sin x\}\right), 0 < x < 2\pi$. Let $x = x_i$, $i = 1, 2, 3, \ldots n$ be the points where f(x) is non-differentiable if $\sum_{i=1}^{n} x_i = \frac{\lambda\pi}{6}$ then value of λ is

[**Note:** {k} denotes the fractional part function of k.]

(A) 11 (B) 13 (C) 19 (D) 23

Q.8 Let $f(x) = \begin{cases} \dfrac{e^{x^2} - \frac{2}{\pi}\sin^{-1}\sqrt{1-x}}{\ln\left(1+\sqrt{x}\right)}; & x \in (0, 1) \\ K; & x \le 0 \end{cases}$

If f(x) is continuous at $x = 0$, then the value of K is

(A) $1 + \frac{2}{\pi}$ (B) $1 - \frac{2}{\pi}$ (C) $\frac{2}{\pi}$ (D) $\frac{-2}{\pi}$

[PARAGRAPH TYPE]

Q.9 to Q.12 has four choices (A), (B), (C), (D) out of which **ONE OR MORE** may be correct. **[4 × 3 = 12]**

Paragraph for question nos. 9 & 10

Let f be a monic quadratic trinomial such that $g(x) = \begin{cases} \dfrac{e^{\sqrt{x - \sin x}} - f(6)\sqrt{x^3} - e^{f(2)}}{(\tan x - x)^{1/2}}; & x > 0 \\ \dfrac{1}{\sqrt{2}}; & x = 0 \\ \dfrac{1}{\sqrt{2}}\left[\dfrac{\sin 2x}{x}\right]; & x < 0 \end{cases}$

is continuous at $x = 0$.

[**Note :** [k] denotes greatest integer less than or equal to k.]

Q.9 The least value of f(x) is

(A) –6 (B) –4 (C) 1 (D) 2

Q.10 The possible value of $(a + b)$ for which $h(x) = \left| f(|x| + a) + b \right|$ is derivable for all $x \in R$ is

(A) 0 (B) 7 (C) 8 (D) 12

Paragraph for question nos. 11 & 12

Let f be a differentiable function which satisfies the relation

$$f(x+y) = f(x) + f(y) - \frac{1}{x-1} - \frac{1}{y-1} + \frac{1}{x+y-1} + 1 \ \forall\ x, y > 1 \text{ and } \lim_{h\to 0}\frac{f(2+h)-2}{h} = 0.$$

Q.11 Identify which of the following statement(s) is (are) correct?

(A) $f''(2) = 2$

(B) least value of $f(x)$ is 3

(C) $f'(2) + f(2) = 2$

(D) $\left.\frac{d}{dx}(f(x))\right|_{x=f(2)} = 2$

Q.12 If (x, θ) is the ordered pair satisfying the equation $f(x) = e^{\ln(2-\sin^2\theta)}$ where $\theta \in [0, 2\pi]$, then possible value(s) of $[x+\theta]$ is(are)

(A) 2 (B) 5 (C) 6 (D) 8

[**Note :** [k] denotes greatest integer value less than or equal to k.]

[MULTIPLE CORRECT CHOICE TYPE]

Q.13 to Q.16 has four choices (A), (B), (C), (D) out of which **ONE OR MORE** may be correct. **[4 × 4 = 16]**

Q.13 Consider, $f(x) = \sin^{-1}\left(\frac{2e^x}{1+e^{2x}}\right) - \cos^{-1}\left(\frac{e^{2x}-1}{e^{2x}+1}\right), x \in R$. Identify which of the following statement(s) is(are) correct?

(A) Range of $f(x)$ is $(-\pi, 0]$

(B) Range of $f(x)$ is $(-\pi, \pi)$

(C) Number of points of discontinuity of $[f(x)]$ is 4.

(D) Number of points of discontinuity $[f(-|x|)]$ is 7.

[**Note :** [k] denotes greatest integer less than or equal to k.]

Q.14 Consider, $y^3 + ay^2x^3 + byx^6 - cx^9 = 0$, a, b, c $\in$ R. If the possible values of $\frac{d^3y}{dx^3}$ are -6, 12 and -18, then

(A) $a + b = -3$ (B) $b + c = -6$ (C) $a + b + c = 3$ (D) $a + b + c = -13$

Q.15 If $a, b \in \{0, 1, 2\}$ and n_1, n_2, n_3 and n_4 denote number of ordered pairs (a, b) for which $\lim_{x\to 0}\frac{e^{ax}-x-1}{(\sin x)^b}$ exists finitely and has the value equal to l_1, l_2, l_3 and l_4 respectively then

(A) $\sum_{i=1}^{4} n_i = 7$ (B) $\sum_{i=1}^{4} n_i = 6$ (C) $\sum_{i=1}^{4} l_i = \frac{1}{2}$ (D) $\sum_{i=1}^{4} l_i = \frac{5}{2}$

Q.16 If $2\sin^{-1}x_0 + \tan^{-1}x_0 = \frac{5\pi}{\sqrt{-x_0^2 + 2x_0 + 15}}$, then the possible value(s) of $\frac{dy}{dx}$ to the curve $y = \frac{2x}{y+x}$ at $x = x_0$ is(are)

(A) $\frac{1}{3}$ (B) $\frac{-4}{3}$ (C) -2 (D) $\frac{1}{2}$

PART-D

[INTEGER TYPE]

Q.1 to Q.4 are "Integer Type" questions. (The answer to each of the questions **are upto 4 digits**) **[$4 \times 5 = 20$]**

Q.1 Let $f(x) = \begin{cases} \sin^{-1}\left(\frac{2x}{1+x^2}\right), & x \le 0 \\ [3x], & 0 < x < k \\ (x-\alpha)^2 + \beta, & x \ge k \end{cases}$

where [p] denotes greatest integer less than or equal to p and $k \in N$. If $f(x)$ is non-derivable at exactly 7 points, then find the value of $(\alpha + \beta + k)$.

Q.2 If $\lim\limits_{x \to 0} \frac{\left(\int_0^x (1-\cos t)\,dt\right)\left(\int_0^x (2-\cos 2t)\,dt\right)\left(\int_0^x (3-\cos 3t)\,dt\right)\ldots\ldots\left(\int_0^x (n-\cos nt)\,dt\right)}{x^m}$

exists and has the value equal to 20, where $m, n \in N$, then find the value of n.

Q.3 Let f be a differentiable function satisfying $\log_2(f(3x)) = x + \log_2(3f(x))\ \forall\ x \in R$ and $f'(0) = 1$. Find the value of $[f(3)]$.
[**Note :** [k] denotes greatest integer less than or equal to k.]

Q.4 Let $f(x)$ be a differentiable function such that
$2f(x+y) + f(x-y) = 3f(x) + 3f(y) + 2xy\ \forall\ x, y \in R$ and $f'(0) = 0$ then find the value of $\left(f(5) + f'(5)\right)$.

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ANSWER KEY
PART-A

Q.1	D	Q.2	D	Q.3	C	Q.4	C	Q.5	A
Q.6	B	Q.7	D	Q.8	C	Q.9	B	Q.10	CD
Q.11	AC	Q.12	ABD	Q.13	ACD	Q.14	AC	Q.15	AC
Q.16	AB								

PART-D

Q.1	9	Q.2	6	Q.3	8	Q.4	0035