Trigonometric Equations

Single Correct Answer Type

1. If $2\tan^2 x - 5\sec x = 1$ for exactly 7 distinct values of $x \in \left[0, \frac{n\pi}{2}\right], n \in N$ then the greatest value of n is

A. 13 B. 17 C. 19 D. 15

KEY. D

SOL. $\sec x = 3 \Rightarrow \cos x = \frac{1}{3}$

Which gives two values of x in each of $[0, 2\pi], (2\pi, 4\pi], (4\pi, 6\pi]$ and one value in $6\pi + \frac{3\pi}{2} = 15\frac{\pi}{2}$

$\therefore$ greatest value of n = 15

2. Let $S = \{a \in N, a \le 100\}$. If the equation $[Tan^2 x] - Tanx - a = 0$ has real roots then number of elements in S is (where [ ] is step function).

A. 10 B. 8 C. 9 D. 0

KEY. C

SOL. Given equation is true only when Tan x is an integer $Tanx = \frac{1 \pm \sqrt{4a+1}}{2} \Rightarrow 4a+1$ is perfect square and $4a+1 \le 401$

3. If $\sin^2(\theta - \alpha)\cos\alpha = \cos^2(\theta - \alpha)\sin\alpha = m\sin\alpha\cos\alpha$ then

A. $|m| \le \frac{1}{\sqrt{2}}$ B. $|m| \ge \frac{1}{\sqrt{2}}$ C. $|m| \ge 1$ D. $|m| \le 1$

KEY. B

SOL. $\frac{\sin^2(\theta-\alpha)}{\sin\alpha} = \frac{\cos^2(\theta-\alpha)}{\cos\alpha} = m = \frac{1}{\sin\alpha + \cos\alpha} \Rightarrow |m| \ge \frac{1}{\sqrt{2}}$

4. The number of values of y in $[-2\pi, 2\pi]$ satisfying the equation $|\sin 2x| + |\cos 2x| = |\sin y|$ is

A. 1 B. 2 C. 3 D. 4

KEY. D

SOL. $1 \le |\sin 2x| + |\cos 2x| \le \sqrt{2}$ and $|\sin y| \le 1 \Rightarrow \sin y = \pm 1 \Rightarrow y = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}$

5. Let $\theta \in [0, 4\pi]$ satisfying the equation $(\sin\theta + 2)(\sin\theta + 3)(\sin\theta + 4) = 6$. If the sum of all values of θ is $K\pi$ then value of K is

A. 6 B. 5 C. 4 D. 2

KEY. B

SOL. $\sin\theta = -1 \Rightarrow \theta = \frac{3\pi}{2}, \frac{7\pi}{2}$

$\therefore K = 5$

6. If $\tan\left(\frac{\pi}{4}+\frac{y}{2}\right) = \tan^3\left(\frac{\pi}{4}+\frac{x}{2}\right)$, then $\sin x\left(\frac{3+\sin^2 x}{1+3\sin^2 x}\right)$ equals

(A) cos y (B) sin y

(C) sin 2y (D) 0

Key. B

Sol. $\frac{1+\tan\frac{y}{2}}{1-\tan\frac{y}{2}} = \left(\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right)^3$

Square both sides, we get

$\frac{1+\sin y}{1-\sin y} = \frac{(1+\sin x)^3}{(1-\sin x)^3}$

Using componendo and dividendo

$\frac{2\sin y}{2} = \frac{(3+\sin^2 x)}{1+3\sin^2 x}\sin x$

7. The number of solutions of the equation $16\left(\sin^5 x+\cos^5 x\right) = 11\left(\sin x+\cos x\right)$ in the interval $\left[0, 2\pi\right]$ is

(A) 6 (B) 7

(C) 8

(D) 9

KEY : A

HINT: $16\ (\sin^5 x+\cos^5 x)-11(\sin x+\cos x)=0$

$\Rightarrow (\sin+\cos x)\{16(\sin^4 x-\sin^3 x\cos\ x+\sin^2 x\cos^2 x-\sin\ x\cos^3 x+\cos^4 x)-11\}=0$

$\Rightarrow (\sin x+\cos x)\{16(1-\sin^2 x\cos^2 x-\sin\ x\cos x)-11\}=$

$\Rightarrow (\sin x+\cos x)\ (4\sin x\cos x-1)\ (4\sin x\cos x+5)=0$

$As\ \ 4\sin x\cos x+5 \neq 0$, WE HAVE

$\sin x+\cos x = 0, 4\sin x\cos x-1=0$

THE REQUIRED VALUES ARE $\pi/12, 5\pi/12, 9\pi/12, 13\pi/12, 17\pi/12, 21\pi/12, -$ THEY ARE 6 SOLUTIONS ON $\left[0, 2\pi\right]$

8. Sum of integral values of n such that sin x $\left(2\sin x+\cos x\right)=n,$ has at least one real solution is

(A) 3 (B) 1 (C) 2 (D) 0

Key : A

Hint : $2\sin^2 x+\frac{2\sin x\cos x}{2}=n$

$\sin 2x-2\cos x=2n-2$

$-\sqrt{5}\le 2n-2\le\sqrt{5}$

$\Rightarrow\ 1-\frac{\sqrt{5}}{2}\le n\le 1+\frac{\sqrt{5}}{2}$

$\therefore$ (A)

9. The equation 2x = (2n + 1) π $\left(1-\cos x\right)$, (where n is a positive integer)

(A) has infinitely many real roots (B) has exactly one real root

(C) has exactly 2n + 2 real roots (D) has exactly 2n + 3 real roots

Key : C

Hint : $\sin^2\left(\frac{x}{2}\right)=\frac{x}{\left(2n+1\right)\pi}$

the graph of $\sin^2\left(\frac{x}{2}\right)$ will be above the x–axis and will be meeting the x-axis at 0, 2π, 4π, . . . etc. It will attain maximum values at odd multiples of π lie. π, 3π, . . . (2n + 1)π. The last point after which graph of $y=\frac{x}{\left(2n+1\right)\pi}$ will stop cutting will be $\left(2n+1\right)\pi$.

Total intersection = $2\left(n+1\right)$

10. If sin A = sin B and cos A = cos B, A > B, then

(a) sin (1/2) (A – B) = 0 (b) sin (1/2) (A + B) = 0

(c) cos (1/2) (A – B) = 0 (d) cos (1/2) (A + B) = 0

Key: a

Hint: sin A = sin B $\Rightarrow$ sin A – sin B = 0

$\Rightarrow\ 2\sin\frac{A-B}{2}\cos\frac{A+B}{2}=0$ (1)

and cos A = cos B $\Rightarrow$ cos B – Cos A = 0

$\Rightarrow\ 2\sin\frac{A+B}{2}\sin\frac{A-B}{2}=0$ (2)

Equations (1) and (2) are simultaneously true if sin (1/2) (A – B) = 0, while the other factors sin (1/2) (A+ B) and cos (1/2) (A + B) cannot both be zero simultaneously.

11. Number of solutions of the equation

tan x + sec x = 2 cos x

lying in the interval [0, 2π] is

(a) 0 (b) 1 (c) 2 (d) 3

Key: c

Hint: The given equation can be written as

$\dfrac{1+\sin x}{\cos x}=2\cos x$

$\Rightarrow$ 1 + sin x = 2 $\cos^2x$ = 2(1 – $\sin^2x$)

$\Rightarrow$ 2 $\sin^2x$ + sin x – 1 = 0

$\Rightarrow$ (1 + sin x)(2 sin x – 1) = 0

$\Rightarrow$ sin x = –1 or 1/2

Now sin x = –1 $\Rightarrow$ tan x and sec x not defined. Sin x = 1/2 $\Rightarrow$ x= $\pi/6$ or $5\pi/6$.

$\therefore$ The required number of solution is 2.

12. The number of solutions of the pair of equations $2\sin^2\theta-\cos 2\theta=0, 2\cos^2\theta-3\sin\theta=0$ in the interval $[0,2\pi]$ is

A. 0 B. 1 C. 2 D. 4

$\therefore$ KEY.C

$\therefore$ SOL. $$2\sin^2\theta-\cos 2\theta=0\Rightarrow 1-\cos 2\theta=0\Rightarrow \cos 2\theta=\frac{1}{2}$$

$\therefore$ $2\theta=\frac{\pi}{3},\frac{5\pi}{3},\frac{7\pi}{3},\frac{11\pi}{3}\Rightarrow \theta=\frac{\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6}$

$\therefore$ $2\cos^2\theta-3\sin\theta=0$

$\therefore$ $(2\sin\theta-1)(\sin\theta+2)=0\Rightarrow \sin\theta=\frac{1}{2}$

$\therefore$ $\theta=\frac{\pi}{6},\frac{5\pi}{6}\Rightarrow No.of\ solutions=2$

13. $\left|x^2\sin x+\cos^2 x\,e^x+\ln^2 x\right|<x^2\left|\sin x\right|+\cos^2 x\,e^x+\ln^2 x$ true for $x\in$

(A) $(-\pi, 0)$ (B) $\left(0, \frac{\pi}{2}\right)$

(C) $\left(\frac{\pi}{2}, \pi\right)$ (D) $\left(2n\pi, (2n+1)\pi\right) n\in N$

Key. A

Sol. $|a+b+c|<|a|+|b|+|c|$

If a, b, c do not have same sign.

So $x^2\sin x<0$

If $x\in(-\pi, 0)$

14. The number of solutions of $\sin^2x\cos^2x = 1 + \cos^2x\sin^4x$ in the interval $[0, 2\pi]$ is

(A) 0 (B) 1

(C) 2 (D) 3

Key. A

Sol. $\sin^2x\cos^2x\,(1-\sin^2x) = 1$

$\sin^2x\cos^4x = 1$

No values of x for L.H.S. = R.H.S.

15. If $\log_{0.5}\sin x = 1-\log_{0.5}\cos x$ then number of values of $x\in[-2\pi, 2\pi]$ is

(A) 1 (B) 2
(C) 3 (D) 4

Key. D

Sol. $\log_{0.5} \sin x = 1 - \log_{0.5} \cos x$

$x \in [-2\pi, 2\pi]$

$\sin x > 0$ and $\cos x > 0$

$\sin x \cos x = \frac{1}{2}$

$\sin 2x = 1 \quad 2x \in [-4\pi, 4\pi]$

$\Rightarrow$ 4 solutions

16. If $f(x) = \frac{\sin 3x}{\sin x}, x \neq n\pi$ then the range of values of f(x) for real values of x is

(A) [–1, 3] (B) (–∞, –1)
(C) (3, ∞) (D) [–1, 3)

Key. D

Sol. $3 - 4\sin^2 x = y$

$\therefore \quad \sin^2 x = \frac{3-y}{4}$. But $0 < \sin^2 x \leq 1$ $(\because \sin x = 0 \Rightarrow x = n\pi)$

$\therefore$ $0 < \frac{3-y}{4} \leq 1$ or $0 < 3 - y \leq 4$.

17. If $2^{\sqrt{\sin^2 x - 2\sin x + 5}} \frac{1}{4^{\sin^2 y}} \leq 1$, then the ordered pair (x, y) is equal to (m, n ∈ I)

(A) $x = (4n+1)\frac{\pi}{2}, y = (2m+1)\frac{\pi}{2}$ (B) $x = 2n\pi, y = 2m\pi$

(C) $x = (2n+1)\frac{\pi}{2}, y = (2m+1)\frac{\pi}{2}$ (D) $x = n\pi, y = m\pi$

Key. A

Sol. $\sin^2 x - 2\sin x + 5 = (\sin x - 1)^2 + 4 \geq 4$

$\therefore \quad 2^{\sqrt{\sin^2 x - 2\sin x + 5}} \geq 2^2 = 4$

and $\sin^2 y \leq 1 \Rightarrow \frac{1}{4^{\sin^2 y}} \geq \frac{1}{4}$

$\therefore$ LHS ≥ 1 and according to question LHS ≤ 1, so therefore, LHS = 1 for which

$\therefore \sin^2 x - 2\sin x + 5 = 4$

$(\sin x - 1)^2 = 0$

$\sin x = 1 \Rightarrow x = (2n+1)\frac{\pi}{2}$

and $\operatorname{cosec}^2 y = 1$, $\sin^2 y = 1$ or $\cos y = 0$

$y = (2m+1)\frac{\pi}{2}$

18. The number of solutions of the equation $\left|\sin\frac{\pi}{2}(1-x) + \cos\frac{\pi}{2}(1-x)\right| = \sqrt{\left|\log_e |x|^3 + 1\right|}$ is /are.

a) 4 b) 6 c) 8 d)10

Key. B

Sol. Simplify, sin $\pi \quad x = \left|\left(\log|x|\right)^3\right|$ By graph, we get 6 solutions.

19. $\sin^2 x \cos^2 x = 1 + \cos^2 x \sin^4 x$

Number of solutions in the interval [0, 2π] is

(A) 0 (B) 1

(C) 2 (D) 3

Key. A

Sol. $\sin^2 x \cos^2 x(1 - \sin^2 x) = 1$

$\sin^2 x \cos^4 x = 1$

No values of x possible.

20. If $0 < x < 1000$ and $\left[\frac{x}{2}\right] + \left[\frac{x}{3}\right] + \left[\frac{x}{5}\right] = \frac{31}{30}x$ where [.] GIF; the number of possible values of *x* is

A) 34 B) 33 C) 32 D) 35

Key. B

Sol. LHS is integer

∴ RHS must be integer for which *x* is multiple of 30

$x = 30, 60, 90, \ldots.., 990$

No. of possible values = 33

21. A set of values of x, satisfying the equation $\cos^2\left(\frac{1}{2}px\right) + \cos^2\left(\frac{1}{2}qx\right) = 1$ form an Arithmetic progression with common difference

a) $\frac{2}{p+q}$ b) $\frac{2}{p-q}$ c) $\frac{\pi}{p+q}$ d) none

Key. D

Sol. 1 + cos px + 1 + cos qx = 2

$$\Rightarrow \cos\left(\frac{p+q}{2}\right)x \cos\left(\frac{p-q}{2}\right)x = 0$$

$$\Rightarrow x = \frac{(2n+1)\pi}{p+q} \text{ or } \frac{(2n+1)\pi}{p-q}$$

for n = 0, $\pm 1, \pm 2, \ldots\ldots\ldots$

forms an AP with common difference $\frac{2\pi}{p+q}$ or $\frac{2\pi}{p-q}$

22. If $\tan\beta = 2\sin\alpha.\sin\gamma.\operatorname{cosec}(\alpha+\gamma)$, then $\cot\alpha, \cot\beta, \cot\gamma$ are in

A) A.P. B) G.P. C) H.P. D) none of these

Key. A

Sol. $\tan\beta = 2\sin\alpha.\sin\gamma.\operatorname{cosec}(\alpha+\gamma) = \dfrac{2\sin\alpha\sin\gamma}{\sin(\alpha+\gamma)}$

$\cot\beta = \dfrac{\sin(\alpha+\gamma)}{2\sin\alpha\sin\gamma}$

i.e. $2\cot\beta = \dfrac{\sin\alpha\cos\gamma + \cos\alpha\sin\gamma}{\sin\alpha\sin\gamma} = \cot\alpha + \cot\gamma$

23. The value of $\cos^2 10° - \cos 10°\cos 50° + \cos^2 50° =$

A) $\frac{4}{5}$ B) $\frac{1}{3}$ C) $\frac{3}{4}$ D) 3

Key. C

Sol. $\cos^2 10° - \cos 10°\cos 50° + \cos^2 50° =$

$= \frac{1}{2}\left[1 + \cos 20° - (\cos 60° + \cos 40°) + (1 + \cos 100°)\right]$

$= \frac{1}{2}\left[1 + \cos 20° - \frac{1}{2} - \cos 40° + 1 - \cos 80°\right] = \frac{1}{2}\left[\frac{3}{2} + \cos 20° - (2\cos 60°\cos 20°)\right] = \frac{3}{4}$

24. If $\cos^6\alpha + \sin^6\alpha + k\sin^2 2\alpha = 1 \ \forall\alpha \in (0, \pi/2)$, then k is

A) $\frac{3}{4}$ B) $\frac{1}{4}$ C) $\frac{1}{3}$ D) $\frac{1}{8}$

Key. A

Sol. The given condition can be written

$(\cos^2\alpha + \sin^2\alpha)^3 - 3\sin^2\alpha\cos^2\alpha(\cos^2\alpha + \sin^2\alpha) + k\sin^2 2\alpha = 1$

$\Rightarrow \quad \left(-\frac{3}{4}\right)\sin^2 2\alpha + k\sin^2 2\alpha = 0$,

Showing that $k = \frac{3}{4}$.

25. The most general solution of the equations $\tan\theta = -1, \cos\theta = \frac{1}{\sqrt{2}}$ is

A) $n\pi + 7\pi/4$ B) $n\pi + (-1)^n \frac{7\pi}{4}$ C) $2n\pi + \frac{7\pi}{4}$ D) none of these

Key. C

Sol. We have $\tan\theta = -1$ and $\cos\theta = \frac{1}{\sqrt{2}}$

The value of θ lying between $\frac{3\pi}{2}$ and 2π and satisfying these two is $\frac{7\pi}{4}$. Therefore the most general solution is $\theta = 2n\pi + 7\pi/4$ where $n \in Z$

26. $\dfrac{\sin^3\theta-\cos^3\theta}{\sin\theta-\cos\theta}-\dfrac{\cos\theta}{\sqrt{1+\cot^2\theta}}-2\tan\theta\cot\theta=-1$ if:

A) $\theta\in\left(0,\dfrac{\pi}{2}\right)$ B) $\theta\in\left(\dfrac{\pi}{2},\pi\right)$ C) $\theta\in\left(\pi,\dfrac{3\pi}{2}\right)$ D) $\theta\in\left(\dfrac{3\pi}{2},2\pi\right)$

Key. B

Sol. note: $\sin\theta\neq\cos\theta$

$\Rightarrow \quad \theta\notin\left(0,\dfrac{\pi}{2}\right)\cup\left(\pi,\dfrac{3\pi}{2}\right);\ \theta\neq\dfrac{\pi}{2},\dfrac{3\pi}{2},\ 0,\pi,\ 2\pi$

And equality holds if $\theta\in\left(\dfrac{\pi}{2},\pi\right)$

27. The least positive values of x satisfy the equation $8^{1+|\cos x|+\cos^2 x+|\cos^3 x|+...\infty=4^3}$ will be (where |cos x| < 1)

A) $\dfrac{\pi}{3}$ B) $\dfrac{2\pi}{3}$ C) $\dfrac{\pi}{4}$ D) none of these

Key. A

Sol. $1+|\cos x|+\cos^2 x$..........

$=\dfrac{1}{1-|\cos x|} \quad \Rightarrow \quad \dfrac{1}{8^{1-|\cos x|}=4^3}$

$\Rightarrow \quad 2^{\frac{3}{1-|\cos x|}}=2^6 \quad \Rightarrow \quad \dfrac{3}{1-|\cos x|}=6 \quad \Rightarrow \quad 1-|\cos x|=\dfrac{1}{2}$

$|\cos x|=\dfrac{1}{2} \quad \Rightarrow \quad \cos x=\pm\dfrac{1}{2}$

$\cos x=\dfrac{1}{2}$ will give least positive value of x

$x=\dfrac{\pi}{3}$ Ans.

28. Value of $\dfrac{3+\cot 80^\circ\cot 20^\circ}{\cot 80^\circ+\cot 20^\circ}$ is equal to

A) $\sqrt{2}\ \cos x$ B) $-\sqrt{2}\ \cos x$ C) $\sqrt{2}\ \sin x$ D) $-\sqrt{2}\ \sin x$

Key. B

Sol. $\dfrac{3+\dfrac{\cos 80^\circ\cos 20^\circ}{\sin 80^\circ\sin 20^\circ}}{\dfrac{\cos 80^\circ}{\sin 80^\circ}+\dfrac{\cos 20^\circ}{\sin 20^\circ}}=\dfrac{2\sin 80^\circ\sin 20^\circ+\cos 80^\circ\cos 20^\circ+\sin 80^\circ\sin 20^\circ}{\sin 20^\circ\cos 80^\circ+\cos 20^\circ\sin 80^\circ}$

$=\dfrac{\cos 60^\circ-\cos 100^\circ+\cos 60^\circ}{\sin 100^\circ}=\dfrac{1-\cos 100^\circ}{\sin 100}=\tan 50^\circ$

29. If $\sin x+\cos x=\sqrt{2}\cos x$, then cosx – sin x is equal to

A) $\sqrt{2}\ \cos x$ B) $-\sqrt{2}\ \cos x$ C) $\sqrt{2}\ \sin x$ D) $-\sqrt{2}\ \sin x$

Key. C

Sol. $\cos x+\sin x=\sqrt{2}\cos x$

$\sin x=\left(\sqrt{2}-1\right)\cos x$

$$\cos x=\frac{1}{\left(\sqrt{2}-1\right)}\sin x$$

$$\cos x=\left(\sqrt{2}+1\right)\sin x$$

$$\cos x-\sin x=\sqrt{2}\sin x$$

30. If $\frac{\sin x}{a}=\frac{\cos x}{b}=\frac{\tan x}{c}=k$, then $bc+\frac{1}{ck}+\frac{ak}{1+bk}$ is equal to

A) $k\left(a+\frac{1}{a}\right)$ B) $\frac{1}{k}\left(a+\frac{1}{a}\right)$ C) $\frac{1}{k^2}$ D) $\frac{a}{k}$

Key. B

Sol. $\frac{\cos x\tan x}{k^2}+\frac{1}{\tan x}+\frac{\sin x}{1+\cos x}=\frac{\sin x}{k^2}+\frac{\cos x\left(1+\cos x\right)+\sin^2 x}{\sin x\left(1+\cos x\right)}=\frac{a}{k}+\frac{1}{\sin x}=\frac{a}{k}+\frac{1}{ak}$

31. In a triangle ABC, if $\sin A\cos B=\frac{1}{4}$ and 3 tan A = tan B, then $\cot^2 A$

A) 2 B) 3 C) 4 D) 5

Key. B

Sol. 3 sin A cos B = sin B cos A

$$\cos A\sin B=\frac{3}{4}$$

$$\sin\left(A+B\right)=1 \quad\Rightarrow\quad C=\frac{\pi}{2},\ B=\frac{\pi}{2}-A$$

$$3\tan A=\tan\left(\frac{\pi}{2}-A\right)$$

$$3=\cot^2 A$$

32. If $\Delta=\begin{vmatrix}1 & 3\cos\theta & 1\\ \sin\theta & 1 & 3\cos\theta\\ 1 & \sin\theta & 1\end{vmatrix}$, then maximum value of is

A) 1 B) 9 C) 16 D) none of these

Key. D

Sol. $\Delta=\begin{vmatrix}1 & 3\cos\theta & 1\\ \sin\theta & 1 & 3\cos\theta\\ 1 & \sin\theta & 1\end{vmatrix}$ applying $R_3\to R_3-R_1$

$$=\begin{vmatrix}1 & 3\cos\theta & 1\\ \sin\theta & 1 & 3\cos\theta\\ 1 & \sin\theta-3\cos\theta & 1\end{vmatrix}=-\left(\sin\theta-3\cos\theta\right)\left(3\cos\theta-\sin\theta\right)=\left(3\cos\theta-\sin\theta\right)^2$$

Now, $-\sqrt{9+1}\le 3\cos\theta-\sin\theta\le\sqrt{9+1}$ $\Rightarrow$ $\left(3\cos\theta-\sin\theta\right)^2\le 10$

33. If $\frac{\sin^3\theta-\cos^3\theta}{\sin\theta-\cos\theta}-\frac{\cos\theta}{\sqrt{\left(1+\cot^2\theta\right)}}-2\tan\theta\cot\theta=-1,\ \theta\in\left[0,2\pi\right]$, then

A) $\theta\in\left(0,\frac{\pi}{2}\right)-\left\{\frac{\pi}{4}\right\}$ B) $\theta\in\left(\frac{\pi}{2},\pi\right)-\left\{\frac{3\pi}{4}\right\}$ C) $\theta\in\left(\pi,\frac{3\pi}{2}\right)-\left\{\frac{5\pi}{4}\right\}$ D) $\theta\in\left(0,\pi\right)-\left\{\frac{\pi}{4},\frac{\pi}{2}\right\}$

Key. D

Sol. Since $\sin\theta-\cos\theta\neq 0$

$\tan\theta\neq 1$

$\therefore \quad \theta\neq\frac{\pi}{4},\frac{5\pi}{4}$

Now $\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta-|\sin\theta|\cos\theta-2\tan\theta\cot\theta=-1$

$\Rightarrow \quad 1+\cos\theta(\sin\theta-|\sin\theta|)-2=-1 \quad \Rightarrow \quad \cos\theta(\sin\theta-|\sin\theta|)=0$

$\therefore \quad \theta\in(0,\pi)\left\{\frac{\pi}{4},\frac{\pi}{2}\right\}$

34. In the interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ the equation $\log_{\sin}(\cos 2\theta)=2$ has

A) no solution B) a unique solution C) two solutions D) infinitely many solutions

Key. B

Sol. $\therefore \quad -\frac{\pi}{2}\leq\theta\leq\frac{\pi}{2} \quad \Rightarrow \quad -1\leq\sin\theta\leq 1$

Here $0<\sin\theta<1 \quad \Rightarrow \quad \log_{\sin\theta}\cos 2\theta=2$

$\cos 2\theta=\sin^2\theta \quad \Rightarrow \quad \log_{\sin\theta}\cos 2\theta=2$

$3\sin^2\theta=1 \quad \Rightarrow \quad \sin^2\theta=\frac{1}{3}$

$\therefore \quad \sin\theta=\frac{1}{\sqrt{3}}\{\therefore \quad 0<\sin\theta<1\}$ a unique solution

35. If $\sin 2A=\frac{1}{2}$ and $\sin 2B=-\frac{1}{2}$, then which one of the following is false

A) sin (A + B) may be 0 B) cos (A – B) may be zero

C) sin(A + B) or cos (A – B) is zero D) sin (A + B) = 0

Key. D

Sol. $\sin 2A=\frac{1}{2},\sin 2B=-\frac{1}{2}$

sin2A + sin 2B = 0

2sin(A + B) cos (A – B) = 0

Sin(A + B) = 0 or cos(A – B) = 0

36. If $\sin 2\beta$ is the G.M. between $\sin\alpha$ and $\cos\beta$, then $\cos 4\beta$ is equal to

A) $2\sin^2\left(\frac{\pi}{4}-\alpha\right)$ B) $2\cos^2\left(\frac{\pi}{4}-\alpha\right)$ C) $2\cos^2\left(\frac{\pi}{2}+\alpha\right)$ D) $2\sin^2\left(\frac{\pi}{4}+\alpha\right)$

Key. A

Sol. $\sin 2\beta=\sqrt{\sin\alpha.\cos\alpha}$

$\cos 4\beta=1-2\sin^2 2\beta=1-2\sin\alpha.\cos\alpha=(\sin\alpha-\cos\alpha)^2=2\sin^2\left(\alpha-\frac{\pi}{4}\right)$

Or $=2\sin^2\left(\frac{\pi}{4}-\alpha\right)$

37. Number of ordered pairs (a, x) satisfying the equation $\sec^2(a+2)x + a^2 - 1 = 0; -\pi < x < \pi$ is

A) a = - 3 and b = 1 B) a = 1 and $b = -\frac{1}{3}$ C) $a = \frac{1}{6}$ and $b = \frac{1}{2}$ D) none of these

Key. C

Sol. Given equation $\sec^2(a+2)x + a^2 - 1 = 0$

$\Rightarrow \quad \tan^2(a+2)x + a^2 = 0 \quad \Rightarrow \quad \tan^2(a+2)x = 0$ and a = 0

$\Rightarrow \quad \tan^2 2x = 0 \quad \Rightarrow \quad \tan^2 2x = 0 \quad \Rightarrow \quad x = 0, \frac{\pi}{2}, \frac{\pi}{2}$

$\therefore \quad (0,0), (0, \pi/2), (0, -\pi/2)$ are ordered pairs satisfying the equation

38. In a parallelogram ABCD $\left|\overrightarrow{AB}\right| = a, \left|\overrightarrow{AD}\right| = b$ and $\left|\overrightarrow{AC}\right| = c$. Then has the value

A) $\frac{3a^2 + b^2 - c^2}{2}$ B) $\frac{a^2 + 3b^2 - c^2}{2}$ C) $\frac{a^2 - b^2 + 3c^2}{2}$ D) $\frac{a^2 + 3b^2 + c^2}{2}$

Key. A

Sol. $\therefore \quad \overrightarrow{DB} = \overrightarrow{DA} + \overrightarrow{AB}$ or $\overrightarrow{DA} = \overrightarrow{DB} - \overrightarrow{AB}$

$\therefore \quad \left(\overrightarrow{DA}\right)^2 = \left(\overrightarrow{DB}\right)^2 + \left(\overrightarrow{AB}\right)^2 - 2\overrightarrow{DB}.\overrightarrow{AB}$

In parallelogram $2(a^2 + 2b^2) = c^2 + DB^2$

$\therefore \quad (DB)^2 = 2a^2 + 2b^2 - c^2$

From (i) $\Rightarrow \quad b^2 = 2a^2 + 2b^2 - c^2 + a^2 - 2\overrightarrow{AB}, \overrightarrow{DB}$

$\therefore \quad \overrightarrow{AB}.\overrightarrow{DB} = \frac{3a^2 + b^2 - c^2}{2}$

39. Let $\vec{a}(x) = (\sin x)\hat{i} + (\cos x)\hat{j}$ and $\vec{b}(x) = (\cos 2x)\hat{i} + (\sin 2x)\hat{j}$ be two variable vectors $(x \in R)$, then $\vec{a}(x)$ and $\vec{b}(x)$ are

A) collinear for unique value of x B) perpendicular for infinitely many values of x

C) zero vectors for unique value of x D) none of these

Key. B

Sol. If $\vec{a}(x)$ and $\vec{b}(x)$ are perpendicular then $\vec{a}.\vec{b} = 0$

$\Rightarrow$ sin x cos 2x + cos sin 2x = 0

Sin (3x) = 0 sin0

$x = \frac{n\pi}{3}$

For infinitely many value of x.

40. If are two vectors, such that $\vec{a}\,\vec{b}$ and < 0 and $\left|\vec{a}\,\vec{b}\right| = \left|\vec{a} \times \vec{b}\right|$, then angle between vectors is

A) π B) $\frac{7\pi}{4}$ C) $\frac{\pi}{4}$ D) $\frac{3\pi}{4}$

Key. D

Sol. $\left|\vec{a}.\vec{b}\right| = \left|\vec{a}\times\vec{b}\right|$

$\left|\vec{a}\right| = \left|\vec{b}\right|\cos\theta = \left|\vec{a}\right|\left|\vec{b}\right|\left|\sin\theta\right|$ (where θ is angle between $\vec{a}$ and $\vec{b}$)

$\Rightarrow \quad |\cos\theta| \quad \Rightarrow \quad |\sin\pi|$

$\Rightarrow \quad \theta = \dfrac{\pi}{4}$ or $\dfrac{3\pi}{4}$ $(\text{as} \le \theta \le \pi)$

But $\vec{a}.\vec{b} < 0$ $\quad \theta = \dfrac{3\pi}{4}$

41. If $\hat{a}, \hat{b}$ and $\hat{c}$ are three unit vectors, such that $\hat{a}+\hat{b}+\hat{c}$ is also a unit vector and 1, 2 and 3 are angles between the vectors $\hat{c}\ \hat{a}, \hat{b}; \hat{b}, \hat{c}$ and $\hat{c}, \hat{a}$ respectively, then among

A) all are acute angles

B) all are right angles

C) at least one is obtuse angle

D) none of these

Key. C

Sol. Given condition $\left(\vec{a}+\vec{b}+\vec{c}\right).\left(\vec{a}+\vec{b}+\vec{c}\right)=1$

$\left|a^2\right| + \left|b^2\right| + \left|c^2\right| + 2|a||b|\cos\theta + 2|b||c|\cos\theta_2 + 2|c||a|\cos\theta_3 = 1$

$\Rightarrow \quad \cos\theta_1 + \cos\theta_2 + \cos\theta_3 = -1$

$\Rightarrow$ one of 1, 2 and 3 should be obtuse angle

Trigonometric Equations

Multiple Correct Answer Type

1. In which of the following sets the inequality $\sin^6 x + \cos^6 x > \frac{5}{8}$ holds good

A. $\left(-\frac{\pi}{8}, \frac{\pi}{8}\right)$ B. $\left(\frac{3\pi}{8}, \frac{5\pi}{8}\right)$ C. $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ D. $\left(\frac{7\pi}{8}, \frac{9\pi}{8}\right)$

KEY. A,B,D

SOL. $1 - 3\sin^2 x \cos^2 x > \frac{5}{8}$

$\cos 4x > 0$

$4x \in \left(2n\pi - \frac{\pi}{2}, 2n\pi + \frac{\pi}{2}\right)$

2. If $\left(\cos^2 x + \frac{1}{\cos^2 x}\right)(1+\tan^2 2y)(3+\sin 3z) = 4$ then

A. x may be a multiple of π

B. x can not be an even multiple of π

C. z can be a multiple of π

D. y can be multiple of $\frac{\pi}{2}$

KEY. A,D

SOL. $\left(\cos^2 x + \frac{1}{\cos^2 x}\right) \ge 2, (1+\tan^2 2y) \ge 1,\ 2 \le 3+\sin 3z \le 4$

So the only possibility is

$\cos^2 x + \frac{1}{\cos^2 x} = 2, 1+\tan^2 2y = 1,\ (3+\sin 3z) = 2$

$\therefore \cos x = \pm 1 \quad \tan 2y = 0 \quad \sin 3z = -1 \quad x = m\pi \quad y = \frac{n\pi}{2}$

$z = (4p-1)\frac{\pi}{6}$

$m, n, p \in I$

3. Let x, y, z be real numbers with $x \ge y \ge z \ge \frac{\pi}{12}$ such that $x+y+z = \frac{\pi}{2}$ and let $P = \cos x . \sin y . \cos z$ then

A) Minimum value of P is $\frac{1}{8}$

B) Minimum value of P is $\frac{1}{4}$

C) Maximum value of P is $\frac{2+\sqrt{3}}{4}$ D) Maximum value of P is $\frac{2+\sqrt{3}}{8}$

Key. A,D

Sol. $P=\frac{1}{2}\cos x\left[\sin(y+z)+\sin(y-z)\right]\geq\frac{1}{2}\cos x.\sin(y+z)=\frac{1}{2}\cos^2 x$

But $x=\frac{\pi}{2}-(y+z)\leq\frac{\pi}{2}-2.\frac{\pi}{2}=\frac{\pi}{3}$ $\therefore$ $P\geq\frac{1}{8}$

Again, $P=\frac{1}{2}\cos z\left[\left(\sin(x+y)-\sin(x-y)\right)\right]$

$P\leq\frac{1}{2}\cos^2 z=\frac{1+\cos 2z}{4}$ $P\leq\frac{2+\sqrt{3}}{8}$

4. If $(1 + k)\tan^2x - 4\tan x - 1 + k = 0$ has real roots $\tan x_1$ and $\tan x_2$, then

(A) $k^2\leq 5$ (B) $\tan(x_1+x_2)=2$

(C) for k = 2, $x_1 = \frac{\pi}{4}$ (D) for k = 1, $x_1 = 0$

KEY : A, B, C, D

HINT : $(1 + k)\tan^2x - 4\tan x - 1 + k = 0$ (1)

Since, roots are real, we have

$(-4)^2-4(1+k)(-1+k)\geq 0$

$\Rightarrow\ 16-4(k^2-1)\geq 0\Rightarrow k^2\leq 5$

We have, $\tan x_1+\tan x_2=\frac{-4}{1+k}=\frac{4}{1+k}$

And $\tan x_1.\tan x_2=\frac{-1+k}{1+k}$

$\therefore$ $\tan(x_1+x_2)=\frac{\frac{4}{1+k}}{1-\left(\frac{-1+k}{1+k}\right)}=\frac{4}{2}=2$

For k = 2, equation (1) $\Rightarrow$ $3\tan^2x - 4\tan x + 1 = 0$

$\Rightarrow$ $\tan x = 1, \frac{1}{3}$ $\therefore$ $x_1 = \frac{\pi}{4}, x_2=\tan^{-1}\frac{1}{3}$

For k = 1, equation (1) $\Rightarrow$ $2\tan^2x - 4\tan x = 0$

$\Rightarrow$ $\tan x = 0, 2$ $\Rightarrow$ $x_1 = 0, x_2 = \tan^{-1}2$

5. The solution set of $|\sin x|\leq|\cos 2x|$ contains

(A) $\bigcup_{n\in I}\left\{\left[n\pi-\frac{\pi}{6}, n\pi+\frac{\pi}{6}\right]\right\}$ (B) $\bigcup_{n\in I}\left\{n\pi+\frac{\pi}{2}\right\}$

(C) $\bigcup_{n\in I}\left\{\left[n\pi-\frac{\pi}{8}, n\pi+\frac{\pi}{8}\right]\right\}$ (D) $\bigcup_{n\in I}\left\{\left[n\pi-\frac{\pi}{4}, n\pi+\frac{\pi}{4}\right]\right\}$

KEY : A,B,C

HINT: $|\cos 2x|^2 \ge |\sin x|^2 \Rightarrow \cos^2 2x \ge \frac{1-\cos 2x}{2} \Rightarrow (\cos 2x + 1)(2\cos 2x - 1) \ge 0$

$\Rightarrow$ either $\cos 2x = -1$ or $\cos 2x \ge \frac{1}{2}$

$\Rightarrow$ either $2x \in 2n\pi + \pi$ or $\left[2n\pi-\frac{\pi}{3}, 2n\pi+\frac{\pi}{3}\right], n \in I$.

6. For $0<\theta<\frac{\pi}{2}$, the solution(s) of $\sum_{m=1}^{6}\operatorname{cosec}\left(\theta+\frac{(m-1)\pi}{4}\right)\operatorname{cosec}\left(\theta+\frac{m\pi}{4}\right)=4\sqrt{2}$ is/are

A. $\frac{\pi}{4}$ B. $\frac{\pi}{6}$ C. $\frac{\pi}{12}$ D. $\frac{5\pi}{12}$

KEY. C,D

SOL. $\sum_{m=1}^{6}\frac{\sin\left[\left(\theta+\frac{m\pi}{4}\right)-\left(\theta+\frac{(m-1)\pi}{4}\right)\right]}{\left[\sin\theta+\frac{(m-1)\pi}{4}.\sin\left(\theta+\frac{m\pi}{4}\right)\right]}=4$

$\Rightarrow \cot\theta-\cot\left(\theta+\frac{\pi}{4}\right)+\cot\left(\theta+\frac{\pi}{4}\right)-\cot\left(\theta+\frac{2\pi}{4}\right)+...+\cot\left(\theta+\frac{5\pi}{4}\right)-\cot\left(\theta+\frac{6\pi}{4}\right)=4$

$\cot\theta+\tan\theta=4$

$\theta=\frac{\pi}{12}\, or\, \frac{5\pi}{12}$

7. Let $\theta,\phi\in[0,2\pi]$ be such that

$2\cos\theta(1-\sin\phi)=\sin^2\theta\left(\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)\cos\phi-1, \tan(2\pi-\theta)>0$ and

$-1<\sin\theta<\frac{-\sqrt{3}}{2}$ then ϕ cannot satisfy

A. $0<\phi<\frac{\pi}{2}$ B. $\frac{\pi}{2}<\phi<\frac{4\pi}{3}$ C. $\frac{4\pi}{3}<\phi<\frac{3\pi}{2}$ D. $\frac{3\pi}{2}<\phi<2\pi$

KEY. A,C,D

SOL. $\tan(2\pi-\theta)>0, -1<\sin\theta<\frac{-\sqrt{3}}{2}\theta\in[0,2\pi]$

$\frac{3\pi}{2}<\theta<\frac{5\pi}{3}$

$2\cos\theta(1-\sin\phi)=\sin^2\theta\left(\frac{2}{\sin\theta}\right)\cos\phi-1$

$2\cos\theta(1-\sin\phi)=2\sin\theta\cos\phi-1$

$2\cos\theta+1=2\sin(\theta+\phi)$

$\theta\in\left(\frac{3\pi}{2},\frac{5\pi}{3}\right)\Rightarrow 2\cos\theta+1\in(1,2)$

$1<2\sin(\theta+\phi)<2$

$\frac{1}{2}<\sin(\theta+\phi)<1$

$as\ \ \theta+\phi\in[0,4\pi]$

$\theta+\phi\in\left(\frac{\pi}{6},\frac{5\pi}{6}\right)\ or\ \theta+\phi\in\left(\frac{13\pi}{6},\frac{17\pi}{6}\right)$

$\Rightarrow\frac{\pi}{6}-\theta<\phi<\frac{5\pi}{6}-\theta\ or\ \frac{13\pi}{6}-\theta<\phi<\frac{17\pi}{6}-\theta$

$\phi\in\left(\frac{-3\pi}{2},\frac{-2\pi}{3}\right)\cup\left(\frac{2\pi}{3},\frac{7\pi}{6}\right)\quad Q\ \theta\in\left(\frac{3\pi}{2},\frac{5\pi}{3}\right)$

8. $\sin\theta+\sqrt{3}\cos\theta=6x-x^2-11,\ 0\le\theta\le4\pi,\ x\in R$ holds for

A. no value of x and θ

B. one value of x and two values of θ

C. two values of x and two values of θ

D. two pairs of values of (x,θ)

Key. B,D

Sol. Since $\sin\theta+\sqrt{3}\cos\theta=6x-x^2-11$

$0\le\theta\le4\pi$

$\Rightarrow\quad \frac{1}{2}\sin\theta+\frac{\sqrt{3}}{2}\cos\theta=\frac{6x-x^2-11}{2}$

$\Rightarrow\quad \sin\left(\theta+\frac{\pi}{3}\right)=\frac{6x-x^2-11}{2}$

$Q\ \ -1\le\sin\left(\theta+\frac{\pi}{3}\right)\le1$

$\Rightarrow\quad -1\le\frac{6x-x^2-11}{2}\le1$

$\Rightarrow\quad -2\le6x-x^2-11\le2$

Caser I : If $6x-x^2-11\le2$

$\Rightarrow\quad x^2-+6x+13\ge0$

$\Rightarrow\quad (x-3)^2+4\ge0$, which is always true.

Case II : If $-2\le6x-x^2-11$

$\Rightarrow\quad x^2-6x+9\le0$

$\Rightarrow\quad (x-3)^2\le0$

Which is possible only, when x – 3 = 0

$\therefore \quad x = 3$

From Eq. (1), $\sin\left(\theta + \frac{\pi}{3}\right) = -1$

$\theta + \frac{\pi}{3} = 2n\pi - \frac{\pi}{2}$

$\Rightarrow \theta = 2n\pi - \frac{5\pi}{6}$

For n = 1, 2

$\theta = \frac{7\pi}{6}, \frac{19\pi}{6} \quad (\because 0 \le \theta \le 4\pi)$

or $(x,\theta) = \left(3, \frac{7\pi}{6}\right), \left(3, \frac{19\pi}{6}\right)$

9. The solutions of the system of equations $\sin x \sin y = \frac{\sqrt{3}}{4}$, $\cos x \cos y = \frac{\sqrt{3}}{4}$ are

A. $x_1 = \frac{\pi}{3} + \frac{\pi}{2}(2n+k);\ n, k \in I$

B. $y_1 = \frac{\pi}{6} + \frac{\pi}{2}(k-2n); n, k \in I$

C. $x_2 = \frac{\pi}{6} + \frac{\pi}{2}(2n+k); n, k \in I$

D. $y_2 = \frac{\pi}{3} + \frac{\pi}{2}(k-2n); n, k \in I$

Key. A,B,C,D

Sol. $\sin x \sin y = \frac{\sqrt{3}}{4}$ and $\cos x \cos y = \frac{\sqrt{3}}{4}$

Then, $\cos x \cos y + \sin x \sin y = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad \cos(x-y) = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad x - y = 2n\pi \pm \frac{\pi}{6},\ n \in I$ …..(i)

and $\cos x \cos y - \sin x \sin y = 0$

cos(x + y) = 0

$x + y = k\pi + \frac{\pi}{2}, k \in I$ ….(ii)

From Eqs. (i) and (ii),

$2x = 2n\pi + k\pi \pm \frac{\pi}{6} + \frac{\pi}{2}$

$\Rightarrow \quad x = \frac{\pi}{2}(2n+k) \pm \frac{\pi}{12} + \frac{\pi}{4}$

$\therefore \quad x_1 = \frac{\pi}{2}(2n+k) + \frac{\pi}{3}$

and $x_2 = \frac{\pi}{2}(2n+k)+\frac{\pi}{6}$

$\therefore \quad y_1 = \frac{\pi}{6}+\frac{\pi}{2}(k-2n)$

and $\quad y_2 = \frac{\pi}{3}+\frac{\pi}{2}(k-2n)$

10. The expression (cos3θ + sin3θ) + (2 sin 2θ – 3) (sinθ – cosθ) is positive for all θ ∈ R in

(A) $\left(2n\pi-\frac{3\pi}{4}, 2n\pi+\frac{\pi}{4}\right), n\in I$ (B) $\left(2n\pi-\frac{\pi}{2}, 2n\pi+\frac{\pi}{6}\right), n\in I$

(C) $\left(2n\pi-\frac{\pi}{3}, 2n\pi+\frac{\pi}{3}\right), n\in I$ (D) $\left(2n\pi-\frac{\pi}{4}, 2n\pi+\frac{3\pi}{4}\right), n\in I$

Key. A,B

Sol. $4(\cos\theta - \sin\theta)(\cos^2\theta + \sin\theta\cos\theta + \sin^2\theta - \sin\theta\cos\theta)$

$$= -4\sqrt{2}\sin\left(\theta-\frac{\pi}{4}\right) > 0$$

$\Rightarrow \quad \sin\left(\theta-\frac{\pi}{4}\right)$ is – ve

$\Rightarrow \quad (2n-1)\pi < \theta - \frac{\pi}{4} < 2n\pi,\ n\in I$

11. Values of x∈(–π, π) satisfying the equation $(\sqrt{3}\sin x+\cos x)^{\sqrt{\sqrt{3}\sin 2x-\cos 2x+2}} = 4$ are,

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$

(C) $-\frac{\pi}{3}$ (D) $-\frac{2\pi}{3}$

Key. A,D

Sol. The given equation is $(\sqrt{3}\sin x+\cos x)^{\sqrt{\sqrt{3}\sin 2x-\cos 2x+2}} = 4$

$\Rightarrow \quad \left[2\sin\left(x+\frac{\pi}{6}\right)\right]^{\sqrt{3\sin^2 x+\cos^2 x+2\sqrt{3}\sin x\cos x}} = 4$

$\Rightarrow \quad \left[2\sin\left(x+\frac{\pi}{6}\right)\right]^{\left|2\sin\left(x+\frac{\pi}{6}\right)\right|} = 4$

Hence, $2\sin\left(x+\frac{\pi}{6}\right) = \pm 2 \Rightarrow \sin\left(x+\frac{\pi}{6}\right) = \pm 1$

$\Rightarrow \quad x+\frac{\pi}{6} = 2n\pi \pm \frac{\pi}{2};\ x = 2n\pi \pm \frac{\pi}{2} - \frac{\pi}{6}$

also $\quad x\in(-\pi,\pi),\ x = \frac{\pi}{3}\ \&\ x = \frac{-2\pi}{3}$

12. If $(1 + \tan^2 2x)\left(\cos^2 y + \frac{1}{\cos^2 y}\right)(3+\sin 3z) = 4$, then

(A) x is an integral multiple of $\pi/2$

(B) y is an integral multiple of π

(C) $z = \frac{2n\pi}{3} + \frac{\pi}{2}$, $n \in I$

(D) z is multiple of π

Key. A,B,C

Sol. L.H.S. ≥ 4

so $\tan 2x = 0$, $\cos y = \pm 1$

and $\sin 3z = -1$

13. The solution set of $|\sin x| \leq |\cos 2x|$ contains

(A) $\bigcup_{n\in I}\left\{\left[n\pi - \frac{\pi}{6}, n\pi + \frac{\pi}{6}\right]\right\}$

(B) $\bigcup_{n\in I}\left\{n\pi + \frac{\pi}{2}\right\}$

(C) $\bigcup_{n\in I}\left\{\left[n\pi - \frac{\pi}{8}, n\pi + \frac{\pi}{8}\right]\right\}$

(D) $\bigcup_{n\in I}\left\{\left[n\pi - \frac{\pi}{4}, n\pi + \frac{\pi}{4}\right]\right\}$

Key. A,B,C

Sol. $|\cos 2x|^2 \geq |\sin x|^2 \Rightarrow \cos^2 2x \geq \frac{1-\cos 2x}{2} \Rightarrow (\cos 2x + 1)(2\cos 2x - 1) \geq 0$

$\Rightarrow$ either $\cos 2x = -1$ or $\cos 2x \geq \frac{1}{2} \Rightarrow$ either

$2x \in 2n\pi + \pi$ or $\left[2n\pi - \frac{\pi}{3}, 2n\pi + \frac{\pi}{3}\right], n \in I$.

14. Which of following functions have the maximum value unity?

A) $\sin^2 x - \cos^2 x$

B) $\sqrt{\frac{6}{5}}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{3}}\cos x\right)$

C) $\cos^6 + \sin^6 x$

D) $\cos^2 x + \sin^4 x$

Key. A,B,C,D

Sol. $\sqrt{\frac{6}{5}}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{3}}\cos x\right) = \frac{\sqrt{3}}{\sqrt{5}}\sin x + \frac{\sqrt{2}}{\sqrt{5}}\cos x$

$= \sin x \,.\, \sin\phi + \cos x \cos\phi$ where $\sin\phi = \sqrt{\frac{3}{5}}, \cos\phi = \sqrt{\frac{2}{5}}$

$= \cos(x - \phi) \leq 1$

$\cos^6 x + \sin^6 x = (\cos^2 x)^3 + (\sin^2 x)^3$

$= 1 - 3\sin^2 x\cos^2 x = 1 - \frac{3}{4}(\sin 2x)^2$

$= \leq 1$

$\cos^2 x + \sin^4 x = 1 - \frac{(\sin 2x)^2}{4} \leq 1$

15. If $\cos\beta$ is geometric mean between $\sin\alpha$ and $\cos\alpha$, where $0<\alpha,\ \beta<\frac{\pi}{2}$, then $\cos 2\beta =$

A) $-2\sin^2\left(\frac{\pi}{4}-\alpha\right)$ B) $-2\cos^2\left(\frac{\pi}{4}+\alpha\right)$ C) $2\sin^2\left(\frac{\pi}{4}+\alpha\right)$ D) $2\cos^2\left(\frac{\pi}{4}-\alpha\right)$

Key. A,B

Sol. $2\sin\alpha\cos\alpha = 2\cos^2\beta$

$\sin 2\alpha = 1+\cos 2\beta$

$\therefore \quad \cos 2\beta = -(1-\sin 2\alpha)$

$$= -\left(1-\cos\left(\frac{\pi}{2}-2\alpha\right)\right) = -2\sin^2\left(\frac{\pi}{4}-\alpha\right) = -2\cos^2\left(\frac{\pi}{4}+\alpha\right)$$

Trigonometric Equations

Assertion Reasoning Type

A) Statement – 1 is True, Statement – 2 is True; Statement-2 is a correct explanation for Statement – 1.
B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is NOT a correct explanation for Statement – 1
C) Statement – 1 is True, Statement – 2 is False
D) Statement – 1 is False, Statement – 2 is True

1. STATEMENT- 1
The only solution of the equation $\sin^{2009}x + \cos^{2010}x = 1$ is $x = m\pi$, $m \in I$.
STATEMENT 2
$-1 \le \sin x, \cos x \le 1$ and $\sin^2 x + \cos^2 x = 1$

Key: D
Hint: Conceptual

2. Statement – 1: The number of integral values of λ, for which the equation 7cosx + 5 sin x = $2\lambda + 1$ has a solution, is 8
Statement – 2: $a\cos\theta + b\sin\theta = c$ has atleast one solution if $|c| < \sqrt{a^2 + b^2}$

Key. C
Sol. $7\cos x + 5\sin x = 2\lambda + 1$

$|2\lambda + 1| \le \sqrt{49 + 25}$

$\Rightarrow \quad |2\lambda + 1| \le \sqrt{74}$

$-\sqrt{74} \le 2\lambda + 1 \le \sqrt{74}$

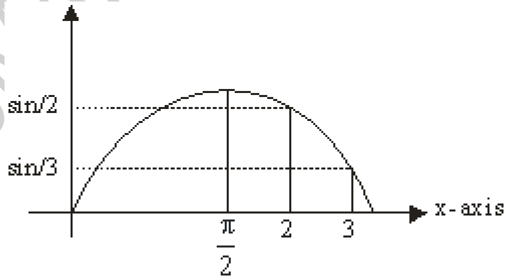
$-8.6 \le 2\lambda \le 7.6 - 4.8 \le \lambda \le 3.8$

$\lambda = -4, -3, -2, -1, 0, 1, 2, 3$

$a\cos\theta + b\sin\theta = c$ has no solution if $|c| > \sqrt{a^2 + b^2}$

3. Statement – 1: sin 2 > sin 3
Statement – 1: If $x, y \in \left(\frac{\pi}{2}, \pi\right), x < y$, then sin x > sin y

Key. A
Sol.



4. Let $\alpha, \beta, \gamma > 0$ and $\alpha + \beta + \gamma = \frac{\pi}{2}$.

Statement – 1: $\left|\tan\alpha\tan\beta - \frac{a!}{6}\right| + \left|\tan\beta\tan\gamma - \frac{b!}{2}\right| + \left|\tan\gamma\tan\alpha - \frac{c!}{3}\right| \le 0$, where n! = 1.2 …. n, then $\tan\alpha\tan\beta, \tan\beta\ \tan\gamma, \tan\gamma\ \tan\alpha$ are in A.P.

Statement – 2: $\tan\alpha\tan\beta + \tan\beta\tan\gamma + \tan\gamma\tan\alpha = 1$

Key. D

Sol. Statement – 2 $\alpha + \beta = \frac{\pi}{2} - \gamma$

$$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{1}{\tan\gamma}$$

$\Rightarrow$ $\Sigma\tan\alpha\tan\beta = 1$

$\therefore$ Statement -2 is true.

Statement – 1 $\tan\alpha\tan\beta = \frac{a!}{6}$, $\tan\beta\tan\gamma = \frac{b!}{2}$ and $\tan\alpha\tan\gamma = \frac{c!}{3}$

$$\frac{a!}{6} + \frac{b!}{2} + \frac{c!}{3} = 1$$

$\Rightarrow$ a! = 1 b! = 1 c! = 1

$\Rightarrow$ $\tan\alpha\tan\beta, \tan\gamma\ \tan\alpha$ and $\tan\beta\ \tan\gamma$ are in A.P.

$\therefore$ statement – 1 is false

Trigonometric Equations

Comprehension Type

Passage – 1

If the curves $y = f(x)$ and $y = g(x)$ intersects at n different points then $f(x) = g(x)$ is said to have 'n' solutions

1. Number of solutions of $|\cos x| = 2[x]$ (where [ ] is step function) is

A. 0 B. 1 C. 2 D. Infinite

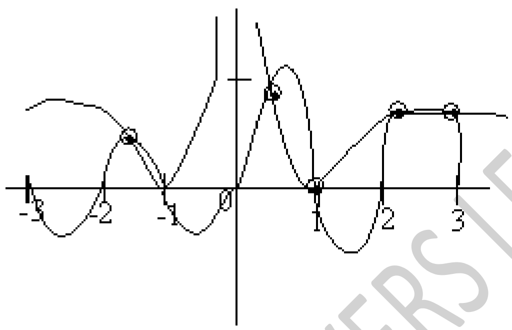
KEY. A

SOL. Using graph, no.of solutions =0

2. The number of solutions of $\sin \pi x = |\log_e |x||$ is

A. 0 B. 6 C. 4 D. 8

KEY. B



SOL.

3. The number of solutions of $|\cos x| = \sin x, 0 \le x \le 4\pi$

A. 4 B. 8 C. 6 D. 2

KEY. A

SOL. Using graph, no.of solutions =4

Passage – 2:

If curve of y = f(x) and y = g(x) intersects at n different points x = x_1, x_2, x_3 ..... Then equation f(x) = g(x) is

said to have n solutions

4. Number of solutions of $|\cos x| = 2[x]$ is (where [x] is integral part of x)

a) 0 b) 1 c) 2 d) infinite

5. The number of solutions of $\mathrm{Sin}\pi x = |\log_e |x||$ is

a) 0 b) 6 c) 4 d) 8

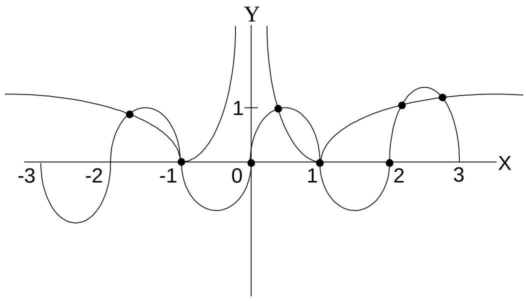
6. Number of solutions of the equation $\sin^5 x - \cos^5 x = \frac{1}{\cos x} - \frac{1}{\sin x} (\sin x \ne \cos x)$ is

a) 0 b) 1 c) 2 d) infinite

KEY : A-B-A

HINT

4.



5. $\sin^5 x - \cos^5 x = \dfrac{\sin x - \cos x}{\sin x \cos x}$

$\Rightarrow \quad \sin x \cos x \left[\dfrac{\sin^5 x - \cos^5 x}{\sin x - \cos x} \right] = 1$

$\sin 2x \left[\sin^4 x + \sin^3 x \cos x + \sin^2 x \cos^2 x + \sin x \cos^3 x + \cos^4 x \right] = 2$

$\left(\sin 2x - 2\right)^2 \left(\sin 2x + 2\right) = 0$

$\sin 2x = \pm 2$

no solution

Passage – 3:

If θ is an angle one measured in radian and $\theta \in [0, 2\pi]$, then $r\theta$ is length of arc AB, of circle of radius r_1 subtending angle θ at the centre O, of the circle. Area of sector OAB is $\frac{1}{2} r^2 \theta$

7. The angle between minute hand and hour hand of a clock at “half past 4” equals

A) 42° B) 43° C) 44° D) none of these

Key. D

Sol. Angle subtended by two consecutive marks at centre = 30°

Hence at “half past 4”, the angle is 45°

8. The wheel of a train is 1 meter in diameter and it makes 5 revolutions per second. Then the speed of the train is approaximately equal to

A) 57 km/hr B) 66 km/hr C) 68 km/hr D) 42.6 km/hr.

Key. A

Sol. Distance covered in 1 second $= 5\left(2\pi, \frac{1}{2}\right) = 5\pi m$

Distance covered in 1 hour $= \dfrac{5\pi}{1000} \times 60 \times 60 = 56.52$

9. Two lines drawn through a point on the circumference of a circle divide the circle into three regions of equal area. Then the angle θ between the lines is given by

A) $3\theta + 3\sin\theta = \pi$ B) $6\theta + 3\sin\theta = \pi$ C) $2\theta + \sin\theta = \pi$ D) $\theta + \sin\theta = \pi/2$

Key. A

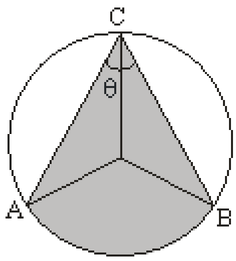
Sol. Area of region $ABC = \dfrac{\pi r^2}{3}$

Area of $OAB = \frac{1}{2} r^2 .\ 2\theta = r^2\theta$

Area of $\Delta OAC = \frac{1}{2}r^2 \sin\theta = \text{Area or } \Delta OBC$

$\therefore \quad \frac{1}{2}r^2\sin\theta + \frac{1}{2}r^2\sin\theta + r^2\theta = \frac{\pi r^2}{3}$

$\Rightarrow \quad 3\sin\theta + 3\theta = \pi$



Passage – 4:

Given $\cos 2^m\theta \cos 2^{m+1}\theta ....\cos 2^n\theta = \frac{\sin 2^{n+1}\theta}{2^{n-m+1}\sin 2^m\theta}$, where $2^m\theta \neq k\pi,\ n, m, k \in I$ Solve the following:

10. $\sin\frac{9\pi}{14}.\sin\frac{11\pi}{14}\sin\frac{13\pi}{14} =$

A) $\frac{1}{64}$ B) $-\frac{1}{64}$ C) $\frac{1}{8}$ D) $-\frac{1}{8}$

Key. C

Sol. $\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14} = \sin\frac{5\pi}{14}\sin\frac{3\pi}{14}.\sin\frac{\pi}{14}$

$= \cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{3\pi}{7} = -\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}$

$= -\frac{\sin\frac{8\pi}{7}}{8\sin\frac{\pi}{7}} = \frac{1}{8}$

11. $\cos 2^3\frac{\pi}{10}\cos 2^4\frac{\pi}{10}\cos 2^5\frac{\pi}{10}...\cos 2^{10}\frac{\pi}{10} =$

A) $\frac{1}{128}$ B) $\frac{1}{256}$ C) $\frac{1}{512}\sin\frac{\pi}{10}$ D) $\frac{\sqrt{5}-1}{512}\sin\frac{3\pi}{10}$

Key. B

Sol. $\cos 2^3\frac{\pi}{10}\cos 2^4\frac{\pi}{10}.\ \cos 2^5\frac{\pi}{10}.....\cos 2^{10}\frac{\pi}{10}$

$= \frac{\sin 2^{11}\frac{\pi}{10}}{256\sin 2^3\frac{\pi}{10}} = \frac{1}{256}$

12. $\cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cos\frac{3\pi}{11}...\frac{3\pi}{11}...\cos\frac{11\pi}{11} =$

A) $-\frac{1}{32}$ B) $\frac{1}{512}$ C) $\frac{1}{1024}$ D) $-\frac{1}{2048}$

Key. C

Sol. $\cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cos\frac{3\pi}{11}.....\cos\frac{11\pi}{11}=\left(\cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cos\frac{3\pi}{11}\cos\frac{4\pi}{11}\cos\frac{5\pi}{11}\right)^2$

$$=\left(\cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cos\frac{4\pi}{11}\cos\frac{8\pi}{11}\cos\frac{5\pi}{11}\right)^2$$

$$=\left(\frac{\sin 16\frac{\pi}{11}}{16\sin\frac{\pi}{11}}.\cos\frac{5\pi}{11}\right)^2=\left(\frac{2\sin\frac{5\pi}{11}\cos\frac{5\pi}{11}}{32\sin\frac{\pi}{11}}\right)^2=\frac{1}{1024}$$

Trigonometric Equations

Integer Answer Type

1. The number of ordered pairs (x,y) where $x, y \in [0,10]$ satisfying $\left(\sqrt{\sin^2 x - \sin x + \frac{1}{2}}\right).2^{\sec^2 y} \le 1$ is 2K then K =

KEY. 8

SOL. $\sqrt{\sin^2 x - \sin x + \frac{1}{2}} = \sqrt{(\sin x - \frac{1}{2})^2 + \frac{1}{4}} \ge \frac{1}{2}$ and

$(\sec^2 y) \ge 1, 2^{\sec^2 y} \ge 2$

It is possible only when $\sin x = \frac{1}{2}, \sec^2 y = 1$

$\therefore x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

$y = 0, \pi, 2\pi, 3\pi$

No. of ordered pairs = 16

2. The A.M. of the solutions of the equation $4\cos^3 x - 4\cos^2 x - \cos(\pi + x) - 1 = 0$ in the interval $(0, 315)$ is $(17K\pi)$ then K =

KEY. 3

SOL. $(4\cos^2 x + 1)(\cos x - 1) = 0 \Rightarrow \cos x = 1$

$x = 2\pi, 4\pi, 6\pi, ...100\pi$

$A.M. = \frac{2(\pi + 2\pi + ...+ 50\pi)}{50} = 51\pi$

3. The no.of values of $x \in [0, 4\pi]$ satisfying $|\sqrt{3}\cos x - \sin x| \ge 2$ is

KEY. 4

SOL. Since the maximum value of $\sqrt{3}\cos x - \sin x$ is 2

$|\sqrt{3}\cos x - \sin x| = 2$ only $\cos(x + \frac{\pi}{6}) = \pm 1$

$x = \frac{5\pi}{6}, \frac{11\pi}{6}, \frac{17\pi}{6}, \frac{23\pi}{6}$

4. If $\theta \in [0, 5\pi]$ and $r \in R$ such that $2\sin\theta = r^4 - 2r^2 + 3$ then the maximum no.of values of the pair (r, θ) is ______

KEY. 6

SOL. $2\sin\theta = (r^2 - 1)^2 + 2$

This is possible only $\sin\theta = 1, r^2 = 1, r = \pm 1$

$\theta = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}$

No.of values of the pair = 6.

5. If $[\sin x] + \left[\frac{x}{2\pi}\right] + \left[\frac{2x}{5\pi}\right] = \frac{9x}{10\pi}$ when the number of solutions in the interval (30, 40) is (where [.] is GIF)

Key. 1

Sol. $[\sin x] = \frac{x}{2\pi} - \left[\frac{x}{2\pi}\right] + \frac{2x}{5\pi} - \left[\frac{2x}{5\pi}\right]$ No. of solutions = 1.

6. Let 'k' be sum of all 'x' in the interval $[0, 2\pi]$ such that $3\cot^2 x + 8\cot x + 3 = 0$. Then the value of $\frac{k}{\pi}$ is

Key. 5

Sol. $\cot x = u : 3u^2 + 8u + 3 = 0$

Both roots are real and product of roots $= 1$

But $\cot x$ bijection in $(0, \pi)$. Let x_1, x_2 are roots such that $0 < x_1 x_2 < \pi$.

But $\cot x_1, \cot x_2$ are both negative.

$\therefore \frac{\pi}{2} < x_1, x_2 < \pi$

But $\pi < x_1 + x_2 < 2\pi$

$\cot x_1 . \cot x_2 = 1$

$\cot x_1 . \cot\left(\frac{3\pi}{2} - x_1\right) = 1$

$\therefore x_1 + x_2 = \frac{3\pi}{2}$ similarly $x_3 + x_4 = \frac{7\pi}{2}$

$\therefore k = 5\pi$

7. The number of solutions of equation $8\left[x^2 - x\right] + 4[x] = 13 + 12[\sin x]$ is (here [.] represents greatest integer less than or equal to 'x')

Key. 0

Sol. $\because R.H.S$ is always odd.

While $L.H.S$ is always even.

8. Let x be in radians with $0 < x < \frac{\pi}{2}$. If $\sin(2\sin x) = \cos(2\cos x)$; then $\tan x + \cot x$ can be written as $\frac{a}{\pi^c - b}$ where $a, b, c \in N$. Then the value of $\left(\frac{a+b+c}{25}\right)$ is

Key: 2

Hint: $\sin(2\sin x) = \sin\left(\frac{\pi}{2} - 2\cos x\right)$

$\sin x + \cos x = \frac{\pi}{4}$

s.o.b.s

$1 + \sin 2x = \frac{\pi^2}{16}$

$\sin 2x = \frac{\pi^2 - 16}{16}$

$\therefore \tan x + \cot x = \frac{2}{\sin 2x} = \frac{2 \times 16}{\pi^2 - 16} = \frac{32}{\pi^2 - 16}$

$\therefore a = 32,\ b = 16,\ c = 2$

$\frac{a+b+c}{25} = 2$

9. If a is irrational then number of solutions of the equation $1 + \sin^2 ax = \cos x$

1) 0 2) 2 3) 1 4) infinite

KEY : 3

HINT: CONCEPUAL

10. Find the number of pairs (x, y) satisfying the equation $\sin x + \sin y = \sin(x + y)$ and $|x| + |y| = 1$.

Key. 6

Sol. The first equation can be written as

$2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x+y}{2}\right)$

or $2\sin\left(\frac{x+y}{2}\right)\left\{\cos\left(\frac{x-y}{2}\right) - \cos\left(\frac{x+y}{2}\right)\right\} = 0$

or $2\sin\left(\frac{x+y}{2}\right)2\sin\left(\frac{x}{2}\right)\sin\left(\frac{y}{2}\right) = 0$

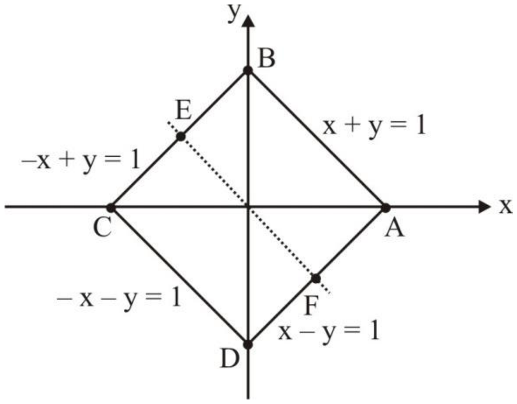
Either $\frac{x+y}{2} = n\pi$

or $\frac{x}{2} = n\pi$ or $\frac{y}{2} = n\pi$

Either x + y = 2nπ or x = 2nπ

Or $y = 2n\pi$

$\therefore$ $|x| + |y| = 1$



$\therefore$ $|x| \le 1$

and $|y| \le 1$

Hence, $x + y = 0$

or $x = 0$ or $y = 0$ clearly $y = 0$ cuts the curve $|x|+|y|=1$ at A, C, $x = 0$, cuts the curve $|x|+|y|=1$ at B, D and $x + y = 0$ cuts the curve at E, F, hence 6 solutions are possible

11. The positive integer value of n > 3 satisfying the equation $\frac{1}{\sin\frac{\pi}{n}} = \frac{1}{\sin\frac{2\pi}{n}} + \frac{1}{\sin\frac{3\pi}{n}}$ is

____

KEY. 7

SOL. $\frac{1}{\sin\theta} = \frac{1}{\sin 2\theta} + \frac{1}{\sin 3\theta}$

$\frac{1}{\sin\theta} = \frac{\sin 3\theta + \sin 2\theta}{\sin 2\theta \sin 3\theta}$

$\sin 2\theta \sin 3\theta = \sin\theta(\sin 3\theta + \sin 2\theta)$

$2\sin\theta\cos\theta\sin 3\theta = \sin\theta(\sin 3\theta + \sin 2\theta)$

$\sin 4\theta + \sin 2\theta = \sin 3\theta + \sin 2\theta$

$4\theta = \pi - 3\theta$

$7\theta = \pi$

$7.\frac{\pi}{n} = \pi$

$n = 7$

12. Set $a, b \in [-\pi, \pi]$ such that $\cos(a-b) = 1$ and $\cos(a+b) = \frac{1}{e}$. The number of pairs (a, b) satisfying the above system of equation is

Key. 4

Sol. $\because \cos(a-b) = \cos 0$

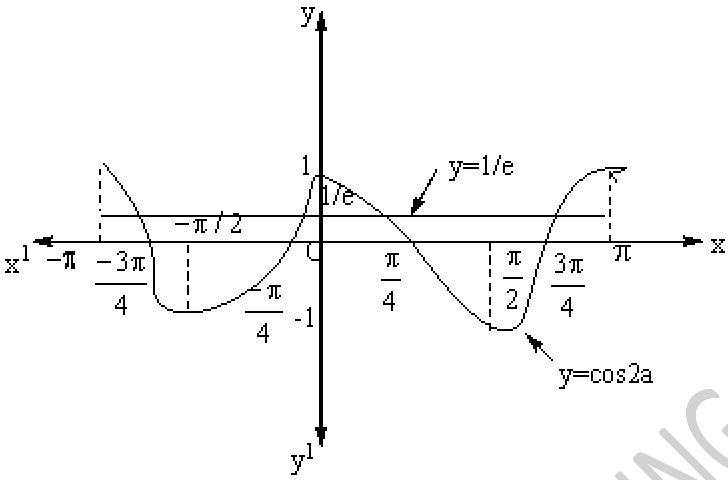
$\therefore$ $a - b = 2n\pi, n \in I$

$a - b = -2\pi, 0, 2\pi$

$\therefore \quad a+b=2\pi+2a, 2a, 2a-2\pi$

$\therefore \quad \cos(a+b)=\cos 2a, \cos 2a, \cos 2a=\frac{1}{e}$

$y=\cos 2a=\frac{1}{e}$



Hence, number of solutions is 4.

13. $2\cot^2 x-5\cos ec x$ is equal to 1 for exactly 7 distinct values of $x\in[0, n\pi]$, then the greatest value of *n* is ....

Key. 7

Sol. $2\cot^2 x-5\cos ec x=1$

$\Rightarrow \quad 2\cos ec^2 x-5\cos ec x-3=0$

$\Rightarrow \quad 2(\cos ec x+1)(\cos ec x-3)=0$

$\cos ec\, x=-\frac{1}{2}, \cos ec x=3$

$\cos ec\, x=3$ gives the solution in 1st and 2nd quadrant, while $\cos ec\, x=-\frac{1}{2}$ gives no solution. So, in $[0, 2\pi]$, we get only two solutions. In $[0, 6\pi]$, we get 6 solution and between 6π and 7π, we get the seventh solution. Hence, n =7

14. The number of solutions of the equation $\sin^5 x-\cos^5 x=\frac{1}{\cos x}-\frac{1}{\sin x}(\sin x\neq\cos x)$ is

Key. 0

Sol. Given that

$$\sin^5 x-\cos^5 x=\frac{\sin x-\cos x}{\sin x\cos x}$$

$$\sin x \cos x\left[\frac{\sin^5 x - \cos^5 x}{\sin x - \cos x}\right] = 1$$

$$\sin x \cos x\{\sin^4 x + \sin^3 x \cos x + \sin^2 x \cos^2 x + \sin x \cos x(\sin^2 x + \cos^2 x)\} = 1$$

$$\sin x \cos x\{(\sin^2 x + \cos^2 x)^2 - \sin^2 x \cos^2 x + \sin x \cos x\} = 1$$

$$\Rightarrow \frac{1}{2}\sin 2x\left[1 - \frac{1}{4}\sin^2 2x + \frac{1}{2}\sin 2x\right] = 1$$

$$\sin 2x(\sin^2 2x - 2\sin 2x - 4) = -8$$

$$\sin^3 2x - 2\sin^2 x - 4\sin 2x + 8 = 0$$

$$(\sin 2x - 2)^2(\sin 2x + 2) = 0$$

$\Rightarrow \quad \sin 2x = \pm 2$ which is impossible

15. The number of solutions of x, which satisfy the equation $l\text{og}_{|\sin x|}(1 + \cos x) = 2$. when $x \in [0, 2\pi]$ is

Key. 0

Sol. $\log_{|\sin x|}(1 + \cos x) = 2 \Rightarrow 1 + \cos x = |\sin x|^2$

$\Rightarrow \quad 1 + \cos x = 1 - \cos^2 x \Rightarrow \cos x(1 + \cos x) = 0$

But $(1 + \cos x) \neq 0 \Rightarrow \cos x = 0, \Rightarrow \sin x = 1$.

But sinx = 1 is not possible because the base of log can not be 1. Hence no solution.

16. If $x, y \in [0, 10]$, then the number of solutions (x, y) of the inequation $3^{\sec^2 x - 1}\sqrt{9y^2 - 6y + 2} \leq 1$ is

Key. 4

Sol. $3^{\tan^2 x}.\sqrt{(3y-1)^2 + 1} \leq 1$

$3^{\tan^2 x} \geq 1$ and $\sqrt{(3y-1)^2 + 1} \geq 1$ $\tan^2 x = 0$, $y = 1/3$ $x = 0, \pi, 2\pi, 3\pi$

17. If a triangle ABC, prove that sin 10A + sin 10B + sin 10C = 4sin 5A sin 5B sin 5C.

Sol. LHS = 2 sin 5 (A + B) cos 5(A – B) + 2 sin 5C cos 5C.

$= 2\sin(5\pi - 5C)$ cos 5(A – B) + 2sin 5C cos 5C

= 2 sin 5C [cos (5A – 5B) + cos 5C]

= 2 sin 5C [cos (5A – 5B) + cos 5 $(\pi - \overline{AB})$]

= 2 sin 5c [cos (5A – 5B) – cos (5A + 5B)]

= 4 sin 5A sin 5B sin 5C = RHS.

18. If $\tan\beta = \dfrac{\tan\alpha + \tan\gamma}{1 + \tan\alpha\tan\gamma}$, prove that $\sin 2\beta = \dfrac{\sin 2\alpha + \sin 2\gamma}{1 + \sin 2\alpha \sin 2\gamma}$

Sol. $\dfrac{\tan\alpha + \tan\gamma}{1 + \tan\alpha\tan\gamma} = \dfrac{\sin(\alpha + \gamma)}{\cos(\alpha - \gamma)}$

$$\therefore \qquad \sin 2\beta = \frac{2\tan\beta}{1+\tan^2\beta} = \frac{2\dfrac{\sin(\alpha+\gamma)}{\cos(\alpha-\gamma)}}{1+\dfrac{\sin^2(\alpha+\gamma)}{\cos^2(\alpha-\gamma)}}$$

$$= \frac{2\sin(\alpha+\gamma)\cos(\alpha-\gamma)}{\cos^2(\alpha-\gamma)+\sin^2(\alpha+\gamma)}$$

$$= \frac{\sin 2\alpha + \sin 2\gamma}{\dfrac{1+\cos 2(\alpha-\gamma)}{2}+\dfrac{1-\cos 2(\alpha+\gamma)}{2}}$$

$$= \frac{\sin 2\alpha + \sin 2\gamma}{1+\dfrac{1}{2}\left(\cos 2(\alpha-\gamma)-\cos 2(\alpha+\gamma)\right)}$$

$$= \frac{\sin^2\alpha + \sin\gamma}{1+\sin 2\alpha \sin 2\gamma}$$

= RHS

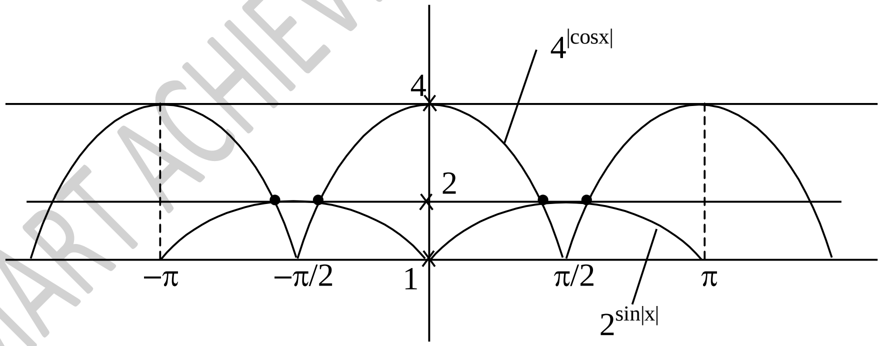
19. Solve the equation $\cos^{n+1} x - \sin^{n+1} x = 1$, where n is an odd natural number.

Sol. The given equation $\cos^{n+1} x - \sin^{n+1} x = 1$, where n + 1 an even integer.
Since LHS $\le$ 1 and RHS $\ge$ 1
$\Rightarrow \quad \cos^{n+1} x = 1 + \sin^{n+1} x = 1 \quad \Rightarrow \quad \sin x = 0 \quad \Rightarrow \; x = n\pi$

20. Number of solutions of $2^{\sin|x|} = 4^{|\cos x|}$ in $[-\pi, \pi]$ is equal to

Key. 4

Sol. Number of solution of the equation is the number of intersection points of graphs $2^{\sin|x|}$ and $4^{|\cos x|}$ in $[-\pi,\pi]$
-



There are 4 intersection points in $[-\pi,\pi]$.

Trigonometric Equations

Matrix-Match Type

1. Match the following Column- I (Equations) with Column – II (No:of solutions)

Column I	ColumnII
A. $\sin\left(\frac{\pi x}{2\sqrt{3}}\right) = x^2 - 2\sqrt{3}x + 4$	P. 4
B. $\sin^4 x = 1 + \tan^8 x$	Q. 1
C. $\cos 2x = \lvert \sin x \rvert, x \in \left(-\frac{\pi}{2}, \pi\right)$	R. 3
D. If m and n (>m) are positive integers, the no.of solutions of the equation $n\lvert \sin x \rvert = m \lvert \cos x \rvert$ in $[0, 2\pi]$ is	S. 0

KEY. A – Q; B – S; C – R; D – P

SOL. (A) LHS $= \sin\left(\frac{\pi x}{2\sqrt{3}}\right) \le 1$

$RHS = (x - \sqrt{3})^2 + 1 \ge 1$

LHS = RHS = 1 at $x = \sqrt{3}$ only

(B) $\sin^4 x \le 1, 1 + \tan^8 x \ge 1$

L.H.S = R.H.S = 1

$\sin^4 x = 1, \tan^8 x = 0$

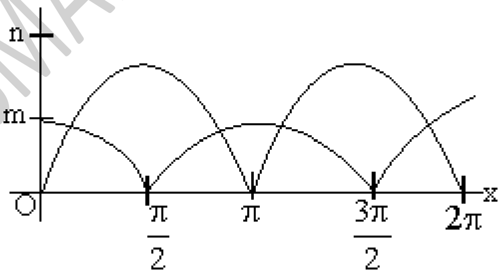
It is not possible.

(C) If sinx > 0 if sinx < 0

$2\sin^2 x + \sin x - 1 = 0$ $2\sin^2 x - \sin x - 1 = 0$

$\sin x = \frac{1}{2}, \; x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\sin x = -\frac{1}{2}, \; x = -\frac{\pi}{6}$

(D) $y = n\lvert \sin x \rvert, y = m \lvert \cos x \rvert$



No. of point of intersections = 4

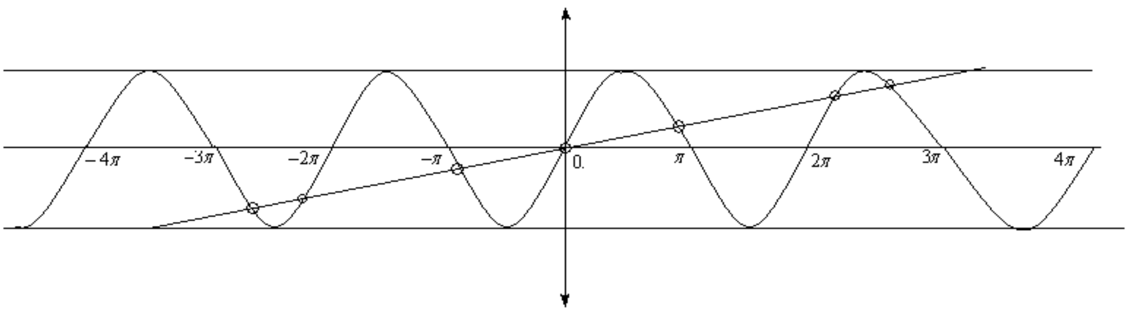
2. Match the following

Column-I	Column-II
A) Number of solutions of $\sin x = \dfrac{x}{10}$ is	p) 1
B) Number of ordered pairs (x,y) satisfying $\lvert x\rvert + \lvert y\rvert = 2$ $\sin\left(\dfrac{\pi x^2}{3}\right) = 1$ is	q) 4
C) Number of solution of the equation $\sin\left(\dfrac{\pi x}{2\sqrt{3}}\right) = x^2 - 2\sqrt{3}x + 4$	r) 7
D) The number of ordered pairs (x,y) satisfying the equation $\sin x + \sin y = \sin(x+y)$ and $\lvert x\rvert + \lvert y\rvert = 1$ is	s) 6

Key: A) r B) q C) p D) s

Hint (A)

.

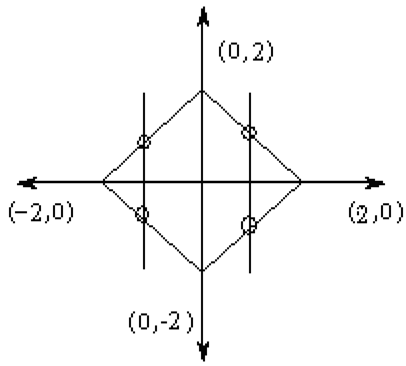


(B)

$$\frac{\pi x^2}{3} = (4n+1)\frac{\pi}{2} \qquad n \in z$$

$$x^2 = \frac{3}{2}(4n+1) \qquad n \in z$$

$$x = \pm\sqrt{\frac{3}{2}}$$



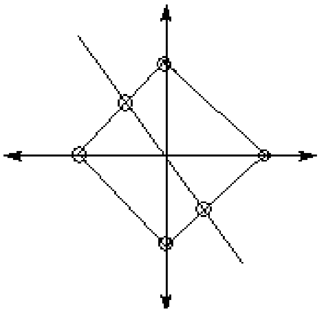
(C)

$$\sin\frac{\pi x}{2\sqrt{3}} = (x-\sqrt{3})^2 + 1$$

$$x = \sqrt{3}$$

(D) $2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = 2\sin\frac{x+y}{2}.\cos\frac{x+y}{2}$

$x+y = 2n\pi, x = 2m\pi, y = 2k\pi$



3. Match the statements/expressions in Column I with the open intervals in Column II

	Column I		Column II
(A)	The number of solutions of the equation tanx + secx = 2 cos x in the interval [0, 2π] is	(p)	4
(B)	The number of real solution of the equation $x^{2\log_x(x+3)} = 16$ is	(q)	$\frac{\pi}{2}$
(C)	In any triangle the area A ≤ $\frac{b^2+c^2}{\lambda}$, then best possible numerical quantity λ is	(r)	2
(D)	If the four roots of the equation |sin θ| = k, between [0, 2π] are in A.P., then the common difference of A.P. is	(s)	0
		(t)	$\frac{\pi}{3}$

Key. (A–r), (B –s), (C –p), (D –q)

Sol. (A) $1 + \sin x = 2\cos^2 x$, $\cos x \neq 0$

$1 + \sin x = 2 - 2\sin^2 x$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$\sin x = \frac{1}{2}, -1$

$\sin x \neq 1$ (as $\cos x \neq 0$)

∴ it has two solutions.

(B) $x^{2\log_x(x+3)} = 16$

$x \neq 1$, $x > 0$

$(x+3)^2 = 16$

$x + 3 = \pm 4$

$x = 1, -7$

So, no solution

(C) $\frac{1}{2}bc\sin A \leq \frac{b^2+c^2}{\lambda}$

$\Rightarrow \frac{1}{2}\lambda bc\sin A \leq b^2 + c^2$

bc$\left[\frac{1}{2}\lambda \sin A - 2\right] \le (b-c)^2$

since sinA ≤ 1, the above inequality will always be satisfied if λ = 4.

(D) if sin θ = $\pm\frac{1}{\sqrt{2}}$

values of θ arc $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ which are in A.P.

4. Match the following: -

Column – I		Column – II	
(A)	The number of real roots of the equation $\cos^7 x + \sin^4 x = 1$ in $(-\pi, \pi)$ is	(p)	1
(B)	The value of $\sqrt{3}\,\text{cosec}\,20° - \sec 20°$ is	(q)	4
(C)	$4\cos 36° - 4\cos 72° + 4\sin 18°.\cos 36°$ equals	(r)	0
(D)	He number of values of $x \in [-2\pi, 2\pi]$, which satisfy cosecx = 1 + cotx	(s)	3
		(t)	2

Key. A → s; B → q; C → s; D → t

Sol. (A) $\cos^7 x + \sin^4 x = 1$

$\cos^7 x = (1 + \sin^2 x)\cos^2 x$

$\Rightarrow \quad \cos x = 0 \;$ or $\; \cos^5 x = 1 + \sin^2 x$

$\cos x = 0 \quad \Rightarrow \quad x = -\frac{\pi}{2}, \frac{\pi}{2}$; $\cos^5 x = 1 + \sin^2 x \quad \Rightarrow \quad x = 0$ ($\because$ LHS ≤ 1 and RHS ≥ 1)

$\therefore \quad x = -\frac{\pi}{2}, 0, \frac{\pi}{2}$.

(B) $\sqrt{3}\,\text{cosec}\,20° - \sec 20°$

$= \frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°} = \frac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20° \cos 20°}$

$= \frac{2\left(\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°\right)}{\sin 20° \cos 20°} = \frac{4\cos 50°}{\sin 40°} = 4$

(C) 4cos36° - 4cos 72° + 4sin18° . cos 36°

$= 4\left(\frac{\sqrt{5}+1}{4}\right) - 4\left(\frac{\sqrt{5}-1}{4}\right) + 4\left(\frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4}\right)$

$= \sqrt{5}+1-\sqrt{5}+1+1=3$

(D) cosex = 1 + cotx

$\frac{1}{\sin x}=\frac{\sin x+\cos x}{\sin x} \quad \Rightarrow \quad \sin x+\cos x=1 \text{ and } \sin x \neq 0$

$\cos\left(x-\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$

$\Rightarrow \quad x-\frac{\pi}{4}=-2\pi+\frac{\pi}{4}, \frac{\pi}{4} \qquad \left(Q \quad x-\frac{\pi}{4}\in\left[-2\pi-\frac{\pi}{4}, 2\pi-\frac{\pi}{4}\right]\right)$

$\Rightarrow \quad x=-\frac{3\pi}{2}, \frac{\pi}{2}$

5. Match the following: -

	Column – I		**Column – II**
(A)	The number of solutions of the equation $\lvert\cot x\rvert=\cot x+\frac{1}{\sin x}(0<x<\pi)$ is	(p)	No solution
(B)	If $\sin\theta+\sin\phi=\frac{1}{2}$ and $\cos\theta+\cos\phi=2$, then value of $\cot\left(\frac{\theta+\phi}{2}\right)$ is	(q)	$\frac{1}{3}$
(C)	The value of $\sin^2\alpha+\sin\left(\frac{\pi}{3}-\alpha\right)\sin\left(\frac{\pi}{3}-\alpha\right)$ is	(r)	1
(D)	If $\tan\theta=3\tan\phi$, then maximum value of $\tan^2(\theta-\phi)$ is	(s)	2
		(t)	4

Key. A $\to$ r; B $\to$ t; C $\to$ p; D $\to$ q

Sol. (A) $\lvert\cot x\rvert=\cot x+\frac{1}{\sin x}$

If $0<x<\frac{\pi}{2}\Rightarrow \cot x>0$

So $\cot x=\cot x+\frac{1}{\sin x}\Rightarrow\frac{1}{\sin x}=0$ no solution

If $\frac{\pi}{2}<\cot x<\pi, -\cot x=\cot x+\frac{1}{\sin x}$

$\frac{2\cos x}{\sin x}+\frac{1}{\sin x}=0$

1 + 2 cos x = 0 and sin x $\neq$ 0 $\quad\Rightarrow\quad x=\frac{2\pi}{3}$

(B) $\text{since } \sin\phi+\sin\theta=\frac{1}{2}$ and $\cos\theta+\cos\phi=2$ has no solution

So

(C) $\sin^2\alpha + \sin\left(\frac{\pi}{3} - \alpha\right).\sin\left(\frac{\pi}{3} + \alpha\right)$

$= \sin^2\alpha + \sin^2\frac{\pi}{3} - \sin^2\alpha = \frac{3}{4}$

(D) $\tan\theta = 3\tan\phi$

$\tan(\theta - \phi) = \frac{\tan\theta - \tan\phi}{1 + \tan\theta\tan\phi} = \frac{2\tan\phi}{1 + 3an^2\phi}$

$= \frac{2}{\cot\phi + 3\tan\phi}$. Max if $\tan\phi > 0$

$\frac{\cot\phi + 3\tan\phi}{2} \geq \sqrt{3}$ (using AM $\geq$ GM)

$\Rightarrow \quad (\cot\phi + 3\tan\phi)^2 \geq 12$

$\Rightarrow \quad \tan^2(\theta - \phi) \leq \frac{1}{3}$