Circles

Single Correct Answer Type

1. Let $C_1 : x^2 + y^2 = 1$; C_2: $(x - 10)^2 + y^2 = 1$ and C_3: $x^2 + y^2 - 10x - 42y + 457 = 0$ be three circles. A circle C has been drawn to touch circles C_1 and C_2 externally and C_3 internally. Now circles C_1, C_2 and C_3 start rolling on the circumference of circle C in anticlockwise direction with constant speed. The centroid of the triangle formed by joining the centres of rolling circles C_1, C_2 and C_3 lies on

(A) $x^2 + y^2 - 12x - 22y + 144 = 0$ (B) $x^2 + y^2 - 10x - 24y + 144 = 0$

(C) $x^2 + y^2 - 8x - 20y + 64 = 0$ (D) $x^2 + y^2 - 4x - 2y - 4 = 0$

Key. B

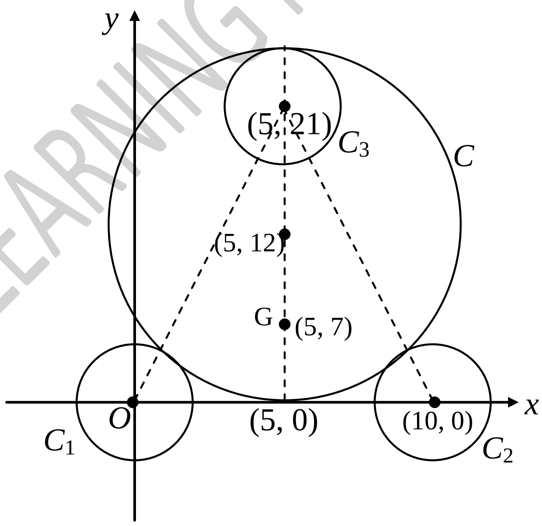
Sol.

The equation of circle C is

$$(x - 5)^2 + (y - 12)^2 = 12^2$$

This circle also touches x–axis at (5, 0).

From the geometry, centroid lies on the circle $(x - 5)^2 + (y - 12)^2 = 5^2$.



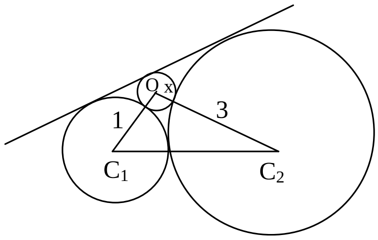
2. The circles $x^2 + y^2 - 6x + 6y + 17 = 0$ and $x^2 + y^2 - 6x - 2y + 1 = 0$, a common exterior tangent is drawn thus forming a curvilinear triangle. The radius of the circle inscribed in this triangle is

(A) $\frac{3}{2}\left(2+\sqrt{3}\right)$ (B) $\frac{1}{2}\left(2-\sqrt{3}\right)$ (C) $\frac{1}{2}\left(2+\sqrt{3}\right)$ (D) $\frac{3}{2}\left(2-\sqrt{3}\right)$

Key. D

Sol. The given circles are touching each other externally.

x = $\frac{3}{(1+\sqrt{3})^2} = \frac{3}{2}\left(2-\sqrt{3}\right)$



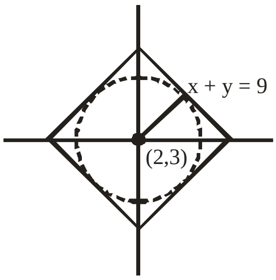
3. Equation of circle touching the line $|x - 2| + |y - 3| = 4$ will be

(A) $(x - 2)^2 + (y - 3)^2 = 12$ (B) $(x - 2)^2 + (y - 3)^2 = 4$

(C) $(x - 2)^2 + (y - 3)^2 = 10$ (D) $(x - 2)^2 + (y - 3)^2 = 8$

Key. D

Sol. PERPENDICULAR distance from centre to tangent = radius



$$r = \frac{|2+3-9|}{\sqrt{2}} = \frac{4}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

Equation of circle is $(x - 2)^2 + (y - 3)^2 = 8$

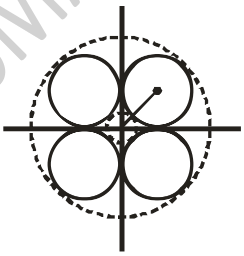
4. The equation of four circles are $(x \pm a)^2 + (y \pm a)^2 = a^2$. The radius of a circle touching all the four circles is

(A) $(\sqrt{2}-1)a$ or $(\sqrt{2}+1)a$ (B) $\sqrt{2}a$ or $2\sqrt{2}a$

(C) $(2-\sqrt{2})a$ or $(2+\sqrt{2})a$ (D) None of these

Key. A

Sol. Radius of smallest circle is



$$r + a = a\sqrt{2}$$
$$r = a\sqrt{2} - a$$

Another circle $\Rightarrow$ $r = a\sqrt{2} + a$

5. If two distinct chords, drawn from the point (p, q) on the circle $x^2 + y^2 = px + qy$ (where $(pq \neq 0)$ are bisected by the x-axis, then

A. $p^2 = q^2$ B. $p^2 = 8q^2$ C. $p^2 < 8q^2$ D. $p^2 > 8q^2$

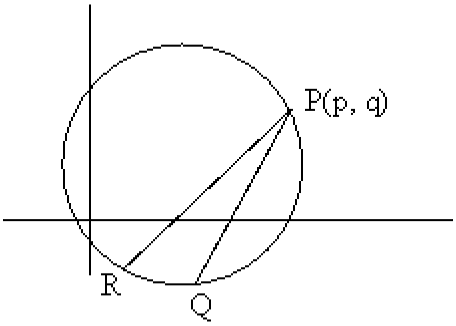
Key. D

Sol. Let PQ be a chord of the given circle passing through P(p, q) ad the coordinates of Q be (x, y). Since PQ is bisected by the x-axis, the mid-point of PQ lies on the x-axis which gives y = -q

Now Q lies on the circle $x^2 + y^2 px - qy = 0$

So $x^2 + q^2 - px + q^2 = 0$

$\Rightarrow x^2 - px + 2q^2 = 0$



Which gives two values of x and hence the coordinates of two points Q and R (say), so that the chords PQ and PR are bisected by x-axis. If the chords PQ and PR are distinct, the roots of (i) are real distinct.

$\Rightarrow$ the discriminant $p^2 - 8q^2 > 0 \Rightarrow p^2 > 8q^2$

6. C_1 and C_2 are circles of unit radius with centres at (0, 0) and (1, 0) respectively. C_3 is a circle of unit radius, passes through the centres of the circles C_1 and C_2 and have its centre above x-axis. Equation of the common tangent to C_1 and C_3 which does not pass through C_2 is

A. $x - \sqrt{3}y + 2 = 0$ B. $\sqrt{3}x - y + 2 = 0$ C. $\sqrt{3}x - y - 2 = 0$ D. $x + \sqrt{3}y + 2 = 0$

Key. B

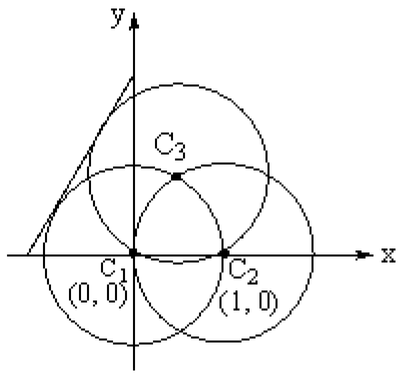
Sol. Equation of any circle through (0, 0) and (1, 0)

$$(x-0)(x-1) + (y-0)(y-0) + \lambda \begin{vmatrix} x & y & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 0$$

$\Rightarrow x^2 + y^2 - x + \lambda y = 0$

If it represents C_3, its radius =1

$\Rightarrow 1 = (1/4) + (\lambda^2 / 4) \Rightarrow \lambda = \pm\sqrt{3}$



As the centre of C_3, lies above the x-axis, we take $\lambda = -\sqrt{3}$ and thus an equation of C_3 is $x^2 + y^2 - x - \sqrt{3}y = 0$. Since C_1 and C_2 interest and are of unit radius, their common tangents are parallel to the joining their centres (0, 0) and $(1/2, \sqrt{3}/2)$.

So, let the equation of a common tangents be $\sqrt{3}x - y + 2 = 0$

It will touch C_1, if $\left|\dfrac{k}{\sqrt{3+1}}\right| = 1 \Rightarrow k = \pm 2$

From the figure, we observe that the required tangent makes positive intercept on the y-axis and negative on the x-axis and hence its equation to $\sqrt{3}x - y + 2 = 0$

7. The equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of distinct non parallel lines. If constant c is changed as new constant k then new equation represents

A. Pair of lines B. Parabola C. Ellipse D. Hyperbola

Key. D

Sol. Since c is changed as k, $\Delta \neq 0$ and $h^2 > ab$

$\therefore$ new equation represents hyperbola

8. A circle of radius '5' touches the coordinate axes in the first quadrant. If the circle makes one complete roll on x-axis along the positive direction, then its equation in new position is

1) $x^2 + y^2 - 10(2\pi + 1)x - 10y + 100\pi^2 + 100\pi + 25 = 0$

2) $x^2 + y^2 + 10(2\pi + 1)x - 10y + 100\pi^2 + 100\pi + 25 = 0$

3) $x^2 + y^2 - 10(2\pi + 1)x + 10y + 100\pi^2 + 100\pi + 25 = 0$

4) $x^2 + y^2 + 10(2\pi + 1)x + 10y + 100\pi^2 + 100\pi + 25 = 0$

Key. 1

Sol. $c = (5,5)$ and $(5 + 10\pi, 5)$

$(x - 5 - 10\pi)^2 + (y - 5)^2 = 5^2$

9. From origin, chords are drawn to the circle $x^2+y^2-2y=0$. The locus of the middle points of these chords is

1) $x^2+y^2-y=0$
2) $x^2+y^2-x=0$
3) $x^2+y^2-2x=0$
4) $x^2+y^2-x-y=0$

Key. 1

Sol. $T=S_1$

i.e., $xx_1+yy_1-(y+y_1)=x_1^2+y_1^2-2y_1$

Passes through $(0,0)$

$\therefore x^2+y^2-y=0$

10. Circles are drawn through the point (2,0) to cut intercept of length '5' units on the x-axis. If their centres lie in the first quadrant then their equation is

1) $x^2+y^2-9x+2ky+14=0, k>0$
2) $3x^2+3y^2+27x-2ky+42=0, k>0$
3) $x^2+y^2-9x-2ky+14=0, k>0$
4) $x^2+y^2-2ky-9y+14=0, k>0$

Key. 3

Sol. $c=\left(\frac{9}{2},k\right)$

$$\left(x-\frac{9}{2}\right)^2+(y-k)^2=\frac{25}{4}+k^2$$

$$(or) x^2+y^2-9x-2ky+14=0$$

11. A line meets the coordinate axes in A and B. If a circle is circumscribed about the ΔAOB. If m, n are the distances of the tangent to the circle at the origin from the points A and B respectively, The diameter of the circle is

1) $m(m+n)$
2) $m+n$
3) $n(m+n)$
4) $2(m+n)$

Key. 2

Sol. $m=A(a,o)on(1)=\frac{a^2}{\sqrt{a^2+b^2}}$

$$n = (o,b)\, on\ (1) = \frac{b^2}{\sqrt{a^2+b^2}}$$
$$d = \sqrt{a^2+b^2} = m+n$$

12. The equation of the circle passing through the point $(2,-1)$ and having two diameters along the pair of lines $2x^2+6y^2-x+y-7xy-1=0$ is

1) $x^2+y^2+10x+6y-19=0$
2) $x^2+y^2+10x-6y+19=0$
3) $x^2+y^2+10x+6y+19=0$
4) $x^2+y^2-10x+6y+19=0$

Key. 1

Sol. $2x^2+6y^2-x+y-7xy-1=0$
$x--2y-1=0\ and \rightarrow (1)$
$2x-3y+1=0 \rightarrow (2)$
$Cetre\ (-5,-3)$
$\therefore x^2+y^2+10x+6y-19=0$

13. If from any point on the circle $x^2+y^2=a^2$, tangents are drawn to the circle $x^2+y^2=b^2 (a>b)$ then the angle between tangents is

1) $\sin^{-1}\left(b/a\right)$
2) $2\sin^{-1}\left(a/b\right)$
3) $2\sin^{-1}\left(b/a\right)$
4) $\sin^{-1}\left(a/b\right)$

Key. 3

Sol. $\sin\theta = \frac{b}{a} \Rightarrow \theta = \sin^{-1}\frac{b}{a}$

$Angle\ between\ the\ 2\theta = 2\sin^{-1}\frac{b}{a}$

14. An equilateral triangle has two vertices $(-2,0)$ and $(2,0)$ and its third vertex lies below the x-axis, The equation of the circumcircle of the triangle is

1) $\sqrt{3}\left(x^2+y^2\right)-4y+4\sqrt{3}=0$
2) $\sqrt{3}\left(x^2+y^2\right)-4y-4\sqrt{3}=0$
3) $\sqrt{3}\left(x^2+y^2\right)+4y+4\sqrt{3}=0$
4) $\sqrt{3}\left(x^2+y^2\right)+4y-4\sqrt{3}=0$

Key. 4

Sol. $Vertex\ A\left(0,-\sqrt{12}\right)$

$Centroid\ G\left(0,\frac{-2}{\sqrt{3}}\right)$

$Circum\ \ radius=\sqrt{4+\frac{4}{3}}=\frac{4}{\sqrt{3}}$

$\therefore \sqrt{3}\left(x^2+y^2\right)+4y-4\sqrt{3}=0$

15. The coordinates of two points on the circle $x^2+y^2-12x-16y+75=0,$ one nearest to the origin and the other farthest from it, are

1) $(3,4),(9,12)$
2) $(3,2),(9,12)$
3) $(-3,4),(9,12)$
4) $(3,4),(9,-12)$

Key. 1

Sol.

$c=(6,8),\ \ radius=5=AC$

$oc=\sqrt{36+64}=10$

$OA=5$

$\because OA:AC=5:5=1:1$

$A\ is\ midpo\text{int}\ 07\ OC$

$i.e.,\ (3:4)$

$Coordinate\ B\ be\ (h,k)$

$'c'\ is\ the\ midpo\text{int}\ 07\ AB$

$\therefore h=9,\ k=12$

$\therefore\ \ B(9,12)$

16. Two distinct chords drawn from the point (p,q) on the circle $x^2+y^2=px+qy$, where $pq\neq 0$, are bisected by the x-axis. Then

1) $|p|=|q|$
2) $p^2=8q^2$
3) $p^2<8q^2$
4) $p^2>8q^2$

Key. 4

Sol. $y=-q\ and\ \ x^2+y^2-px+qy=0$

Disc > 0

17. The centre of a circle of radius $4\sqrt{5}$ lies on the line $y = x$ and satisfies the inequality $3x + 6y > 10$. If the line $x + 2y = 3$ is a tangent to the circle, then the equation of the circle is

1) $\left(x - \frac{23}{3}\right)^2 + \left(y - \frac{23}{3}\right)^2 = 80$ 2) $\left(x + \frac{17}{3}\right)^2 + \left(y + \frac{17}{3}\right)^2 = 80$

3) $\left(x + \frac{23}{3}\right)^2 + \left(y - \frac{23}{3}\right)^2 = 80$ 4) $\left(x - \frac{17}{3}\right)^2 + \left(y - \frac{17}{3}\right)^2 = 80$

Key. 1

Sol. $c = (a, a)$

$radius = 4\sqrt{5} = length of the \perp \ from (a, a) to the line$

$i.e., \ \frac{|a + 2(a) - 3|}{\sqrt{4+1}} = \pm 4\sqrt{5} \Rightarrow a = \frac{23}{3}, \frac{-17}{3}$

$\therefore Centre \left(\frac{23}{3}, \frac{23}{5}\right) or \left(\frac{-17}{3}, \frac{-17}{3}\right)$

$3x + 6y > 10$

$C = \left(\frac{23}{3}, \frac{23}{3}\right)$

$\therefore \left(x - \frac{23}{3}\right)^2 + \left(y - \frac{23}{3}\right)^2 = 80$

18. The equation to the circle which is such that the lengths of the tangents to it from the points (1,0), (2,0) and (3,2) are $1, \sqrt{7}, \sqrt{2}$ respectively is

1) $2x^2 + 2y^2 + 6x + 17y + 6 = 0$ 2) $2x^2 + 2y^2 + 6x - 17y - 6 = 0$

3) $x^2 + y^2 + 6x + 15y + 5 = 0$ 4) $x^2 + y^2 + 6x - 15y - 5 = 0$

Key. 2

Sol. Let S=0 be the required circle

Apply $\sqrt{S_{11}}$

19. If the equations of four circles are $\left(x \pm 4\right)^2 + \left(y \pm 4\right)^2 = 4^2$ then the radius of the smallest circle touching all the four circles is

1) $4\left(\sqrt{2} + 1\right)$ 2) $4\left(\sqrt{2} - 1\right)$ 3) $2\left(\sqrt{2} - 1\right)$ 4) $\sqrt{2} - 1$

Key. 2

Sol. $r=\sqrt{4^2+4^2}-4$ $i.e., 4\sqrt{2}-4$.

20. Let L_1 be a straight line passing through the origin and L_2 be the straight line x+y=1. If the intercepts made by the circles $x^2+y^2-x+3y=0$ on L_1 and L_2 are equal then which of the following equations can represent L_1?

1) $x+y=0$ 2) $x-y=0$

3) $7x-y=0$ 4) $x-7y=0$

Key. 2

Sol. $c\left(\frac{1}{2},\frac{-3}{2}\right)$

$$\frac{\left|\frac{1}{2}-\frac{3}{2}^{-1}\right|}{\sqrt{1+1}}=\frac{\left|\frac{m}{2}-\frac{3}{2}\right|}{\sqrt{1+m^2}}\Rightarrow m=1,\frac{-1}{7}$$

$two\ chords\ are\ \ y=x\ and\ 7x+y=0$

21. Let $A_0A_1A_2A_3A_4A_5$ be a regular hexogon inscribed in a circle of unit radius. Then the product of the lengths of the line segments A_0A_1, A_0A_2 and A_0A_4 is

1) $\frac{3}{4}$ 2) $3\sqrt{3}$ 3) 3 4) $\frac{3\sqrt{3}}{2}$

Key. 3

Sol. $A_0A_1=1$

$A_0A_1=\sqrt{3}$

$Similarly,\ A_0\ A_4=\sqrt{3}$

$\therefore (A_0A_1)\ (A_0A_2)\ (A_0A_4)=3$

22. The ΔPQR is inscribed in the circle $x^2+y^2=25$. If Q and R have coordinates (3,4) and (-4,3) respectively, then $\underline{|QPR}$ is equal to

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{6}$

Key. 3

Sol. $m_1=slope\ of\ \ OP=\frac{4}{3}\ and\ m_2\ slope\ of\ \ OQ=\frac{-3}{4}$

$\Rightarrow m_1\ m_2=-1$

$\angle QOP=\Pi/2$

$Thus\ \angle QPR=\Pi/4$

23. A variable circle passes through the fixed point A(p,q) and touches x-axis. The locus of the other end of the diameter through A is

1) $(y-p)^2 = 4qx$ 2) $(x-q)^2 = 4py$

3) $(x-p)^2 = 4qy$ 4) $(y-q)^2 = 4px$

Key. 3

Sol. $(x-p)(x-\alpha)+(y-g)(y-\beta)=0 \quad (or)$

$x^2+y^2-(p+\alpha)x-(g+\beta)y+p\alpha+g\beta=0 \rightarrow (1)$

$Put\ y=0,\ we\ get\ x^2-(p+\alpha)x+p\alpha+g\beta=0 \rightarrow (2)$

$\therefore \Rightarrow Locus\ of\ B\ (\alpha,\beta)\ is\ (p-x)^2=4gy$

$(x-p)^2=4gy$

24. The locus of the mid point of the chord of the circle $x^2+y^2-2x-2y-2=0$ which makes an angle of 120^0 at the centre is

1) $x^2+y^2-2x-2y+1=0$ 2) $x^2+y^2+x+y+1=0$

3) $x^2+y^2-2x-2y-1=0$ 4) $x^2+y^2+x-y-1=0$

Key. 1

Sol. $Centre\ (1,1)\ and\ radius\ = 2 = OB$

$In\ \nabla OBP\ = 30^o$

$\therefore \sin 30^o\ \frac{op}{2}\ or\ op=1$

$\sin ce\ \ op=1$

$\Rightarrow x^2+y^2-2x-2y+1=0$

25. The chord of contact of tangents from a point 'P' to a circle passes through Q. If l_1 and l_2 are the lengths of the tangents from P and Q to the circle, then PQ is equal to

1) $\frac{l_1+l_2}{2}$ 2) $\frac{l_1-l_2}{2}$

3) $\sqrt{l_1^2+l_2^2}$ 4) $\sqrt{l_1^2-l_2^2}$

Key. 3

Sol. $P=(x_1,y_1)\ and\ Q=(x_2,y_2)$

$p=(x_1,y_1)\ to\ the\ given\ circle\ is\ xx_1+yy_1=a^2$

$Since\ it\ passes\ through\ Q\ (x_2,y_2)$

$\therefore xx_1+yy_1=a^2 \rightarrow (1)$

$Now,\ l_1=\sqrt{x_1^2+y_1^2-a^2},\ l_2=\sqrt{x_2^2+y_2^2-a^2}$

$and\ PQ = \sqrt{l_1^2 + l_2^2}$

26. If a chord of a the circle $x^2 + y^2 = 32$ makes equal intercepts of length l on the Co-ordinate axes, then

1) $|l| < 8$
2) $|l| < 16$
3) $|l| > 8$
4) $|l| > 16$

Key. 1

Sol. $Centre\ (0,0),$

$$radius \left|\frac{l}{\sqrt{2}}\right| < \sqrt{32} \Rightarrow |l| < 8$$

27. If the chord of contact of tangents from 3 points A,B,C to the circle $x^2 + y^2 = a^2$ are concurrent, then A,B,C will

1) be concyclic
2) Be collinear
3) Form the vertices of triangle
4) None of these

Key. 2

Sol. $xx_1 + yy_1 = a^2,\ xx_2 + yy_2 = a^2$

$and\ xx_3 + yy_3 = a^2$

$These\ lines\ will\ be\ conurrent$

$$\begin{vmatrix} x_1 & y_1 & -a^2 \\ x_2 & y_2 & -a^2 \\ x_3 & y_3 & -a^2 \end{vmatrix} = 0 \quad \begin{vmatrix} x_1 & y_1 & -1 \\ x_2 & y_2 & -1 \\ x_3 & y_3 & -1 \end{vmatrix} = 0$$

$Which\ is\ the\ condition\ to\ the\ collinearity\ of\ A,B,C.$

28. If the line passing through P=(8,3) meets the circle $S \equiv x^2 + y^2 - 8x - 10y + 26 = 0$ at A,B then PA.PB=

1) 5
2) 14
3) 4
4) 24

Key. 1

Sol. $PA.PB = |S_{11}|$

29. (a, b) is the mid point of the chord $\overline{AB}$ of the circle $x^2+y^2=r^2$. The tangent at A, B meet at C. then area of $\Delta ABC=$

1) $\frac{\left(a^2+b^2+r^2\right)^{\frac{3}{2}}}{\sqrt{a^2+b^2}}$ 2) $\frac{\left(r^2-a^2-b^2\right)^{\frac{3}{2}}}{\sqrt{a^2+b^2}}$

3) $\frac{\left(a^2-b^2-r^2\right)^{\frac{3}{2}}}{\sqrt{a^2+b^2}}$ 4) $\frac{\left(a^2-b^2+r^2\right)^{\frac{3}{2}}}{\sqrt{a^2+b^2}}$

Key. 2

Sol. Equation of the chord AB having (a,b)

$as\ M.P.\ S_1=S_{11}\Rightarrow ax+by-(a^2+b^2)=0$

$chord\ \ length=2\sqrt{r^2-a^2-b^2}$

$c=\left(\frac{-ar^2}{a^2+b^2},\frac{br^2}{a^2+b^2}\right)$

$h=\frac{r^2-a^2-b^2}{\sqrt{a^2+b^2}}$

Area = ½ x b x h

30. The length and the midpoint of the chord $4x-3y+5=0$ w.r.t circle $x^2+y^2-2x+4y-20=0$ is

1) $8,\left(-\frac{7}{5},-\frac{1}{5}\right)$ 2) $18,\left(\frac{7}{5},\frac{1}{5}\right)$

3) $10,\left(-\frac{17}{5},-\frac{11}{5}\right)$ 4) $28,\left(-\frac{7}{5},-\frac{8}{5}\right)$

Key. 1

Sol. $2\sqrt{r^2-d^2}\ =8$.

$M.P=\left({}^{-7}\!/\!_{5},{}^{-1}\!/\!_{5}\right)$

31. A variable circle passes through the fixed point (2, 0) and touches the y-axis then the locus of its centre is

1) a parabola 2) a circle

3) an ellipse 4) a hyperbola

Key. 1

Sol. $Circle\ (x-x_1)^2+(y-y_1)^2=x_1^{\ 2}$

$y^2=4(x-1)\ Parabola$

32. If the lengths of the tangents from the point (1, 2) to the circles $x^2+y^2+x+y-4=0$ and $3x^2+3y^2-x-y-\lambda=0$ are in the ratio 4 : 3 then $\lambda=$

1) $\frac{23}{4}$ 2) $\frac{21}{4}$ 3) $\frac{17}{4}$ 4) $\frac{19}{4}$

Key. 2

Sol. $\frac{\sqrt{s_{11}}}{\sqrt{s_{11}^1}}=4/3$

33. If a tangent drawn from the point (4, 0) to the circle $x^2+y^2=8$ touches it at a point A in the first quadrant, then the coordinates of another point B on the circle such that AB = 4 are

1) $(2,-2)\ or\ (-2,2)$ 2) $(1,-2)\ or\ (-2,1)$

3) $(-1,1)\ or\ (1,-1)$ 4) $(3,-2)\ or\ (-3,2)$

Key. 1

Sol. equation of tangent through (4,0)

x+y-4=0

Point of contact = (2,2)

$AB=4\ \Rightarrow\ B=(2+4\cos\theta,\ 2+4\sin\theta)$

$\theta=\pi,\ \ \theta=\frac{3\pi}{2}$

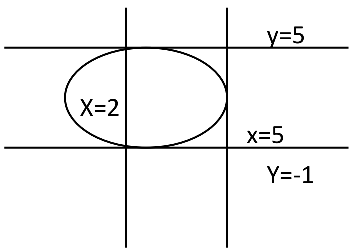
$(-2,2)\ \ (2,-2)$

34. The number of points common to the circle $x^2+y^2-4x-4y=1$ and to the sides of the rectangle formed by $x=2, x=5, y=-1,\ \ and\ \ y=5$ is

1) 5 2) 1 3) 2 4) 3

Key. 4

Sol. (3) points



35. A rectangle ABCD is inscribed in a circle with a diameter lying along the line $3y = x + 10$. If $A = (-6, 7)$, $B = (4, 7)$ then the area of rectangle is

1) 80 2) 40 3) 160 4) 20

Key. 1

Sol. Area $= \pi r^2$

$r = \frac{\sqrt{17}}{4}$

36. Let ABCD be a quadrilateral with area 18, with side AB parallel to CD and AB = 2CD. Let AD be perpendicular to AB and CD. If a circle is drawn inside the quadrilateral ABCD touching all the sides, then its radius is

1) 3 2) 2 3) $\frac{3}{2}$ 4) 1

Key. 2

Sol. A(0,0); B(2a,o); C(a, 2r); D(0, 2r)
Equation of ABCD = ½ (2a+a) x 2r = 18
r=2

37. If OA and OB are two equal chords of the circle $x^2 + y^2 - 2x + 4y = 0$ perpendicular to each other and passing through the origin O, the slopes of OA and OB are the roots of the equation

1) $3m^2 + 8m - 3 = 0$ 2) $3m^2 - 8m - 3 = 0$

3) $8m^2 - 3m - 8 = 0$ 4) $8m^2 + 3m - 8 = 0$

Key. 2

Sol. equation of chords y-mx = 0
my + x = 0

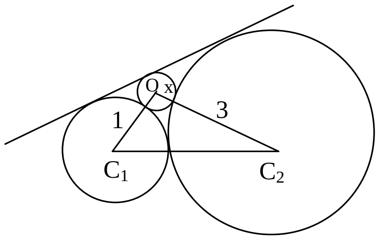
38. The circles $x^2 + y^2 - 6x + 6y + 17 = 0$ and $x^2 + y^2 - 6x - 2y + 1 = 0$, a common exterior tangent is drawn thus forming a curvilinear triangle. The radius of the circle inscribed in this triangle is

(A) $\frac{3}{2}\left(2+\sqrt{3}\right)$ (B) $\frac{1}{2}\left(2-\sqrt{3}\right)$ (C) $\frac{1}{2}\left(2+\sqrt{3}\right)$ (D) $\frac{3}{2}\left(2-\sqrt{3}\right)$

Key. D

Sol. The given circles are touching each other externally.

x = $\frac{3}{(1+\sqrt{3})^2}=\frac{3}{2}\left(2-\sqrt{3}\right)$



39.

40.

41. ABCD is a square of side 1 unit. A circle passes through vertices A,B of the square and the remaining two vertices of the square lie out side the circle. The length of the tangent drawn to the circle from vertex D is 2 units. The radius of the circle is

A) $\sqrt{5}$ B) $\frac{1}{2}\sqrt{10}$ C) $\frac{1}{3}\sqrt{12}$ D) $\sqrt{8}$

Key. B

Sol. Let $A=(0,1)$, $B=(0,0)$, $C=(1,0)$, $D=(1,1)$

Family of circles passing through A,B is $x^2+y^2-y+\lambda x=0$ $\sqrt{1+\lambda}=2 \Rightarrow \lambda=3$

42. The equation of circum-circle of a ΔABC is $x^2+y^2+3x+y-6=0.$

If $A=(1,-2)$, $B=(-3,2)$ and the vertex C varies then the locus of ortho-centre of ΔABC is a

A) Straight line B) Circle C) Parabola D) Ellipse

Key. B

Sol. Equation of circum-circle is $\left(x+\frac{3}{2}\right)^2+\left(y+\frac{1}{2}\right)^2=\frac{17}{2}$

$C=\left(\frac{-3}{2}+\sqrt{\frac{17}{2}}\cos\theta, \frac{-1}{2}+\sqrt{\frac{17}{2}}\sin\theta\right)$

Circum centre of ΔABC is $\left(\frac{-3}{2},\frac{-1}{2}\right)$ Centroid can be obtained.

In a triangle centroid, circum centre and ortho centre are collinear.

43. The line $y = mx$ intersects the circle $x^2 + y^2 - 2x - 2y = 0$ and $x^2 + y^2 + 6x - 8y = 0$ at point A and B (points being other than origin). The range of $'m'$ such that origin divides AB internally is

A) $-1 < m < \frac{3}{4}$ B) $m > \frac{4}{3}$ *or* $m < -2$ C) $-2 < m < \frac{4}{3}$ D) $m > -1$

Key. A

Sol. The tangents at the origin to C_1 and C_2 are $x + y = 0$, $3x - 4y = 0$ respectively. Slope of the tangents are $-1, \frac{3}{4}$ respectively thus if $-1 < m < \frac{3}{4}$, then origin divides AB internally.

44. The equation of the smallest circle passing through the intersection of $x^2 + y^2 - 2x - 4y - 4 = 0$ and the line $x + y - 4 = 0$ is

(A) $x^2 + y^2 - 3x - 5y - 8 = 0$ (B) $x^2 + y^2 - x - 3y = 0$

(C) $x^2 + y^2 - 3x - 5y = 0$ (D) $x^2 + y^2 - x - 3y - 8 = 0$

Key. C

Sol. Conceptual

45. Three distinct points A, B and C are given in the two-dimensional coordinate plane such that the ratio of the distance of any one of them from (2,-1) to its distance from (-1, 5) is 1 : 2. Then the centre of the circle passing through A,B and C is

a) (1,1) b) (5, - 7)

c) (3,-3) d)(4,-8)

Key: C

Hint The circle ABC is the circle described on the join of (1,1) and (5, -7) as diameter.

46. Point A lies on y = x and point B on y = mx so that length AB = 4 units. Then value of 'm' for which locus of mid point of AB represents a circle is

a) m = 0 b) m = –1 c) m = 2 d) m = –2

Key: B

Hint Let co-ordinates of A(x_1, x_1) and B(x_2, mx_2).

Clearly $(x_1 - x_2)^2 + (x_1 - mx_2)^2 = 16$

Let mid point of P(h, k)

$\Rightarrow$ $x_1 + x_2 = 2h$ and $x_1 + mx_2 = 2k$

$\Rightarrow$ $(x_1 - x_2)^2 + 4x_1x_2 = 4h^2$ and

$$\left(x_1 - mx_2\right)^2 + 4mx_1x_2 = 4k^2$$

$$\left(x_1 - x_2\right)^2 + \left(x_1 - mx_2\right)^2 = 4h^2 + 4k^2 = 16$$

when m = –1

47. Equation of circle inscribed in |x – a| + |y – b| = 1 is

(A) $(x + a)^2 + (y + b)^2 = 2$ (B) $(x - a)^2 + (y - b)^2 = \frac{1}{2}$

(C) $(x - a)^2 + (y - b)^2 = \frac{1}{\sqrt{2}}$ (D) $(x - a)^2 + (y - b)^2 = 1$

KEY : B

HINT: Radius of the required circle is $\frac{1}{\sqrt{2}}$ and centre is (a, b)

Hence equation is $(x - a)^2 + (y - b)^2 = \frac{1}{2}$

48. Let L = 0 be a common normal to the circle $x^2 + y^2 - 2\alpha x - 36 = 0$ and the curve $S : (1+x)^y + e^{xy} = y$ drawn at a point x = 0 on S, then the radius of the circle is

A) 10 B) 5 C) 8 D) 12

Key: A

Hint: at x = 0 y = 2 $y'(0) = 4$

Equation of Normal is $x + 4y = 8$ $(\alpha,\ 0)$ lies on normal $\Rightarrow$ $\alpha = 8$

49. $x^2 + y^2 + 6x + 8y = 0$ and $x^2 + y^2 - 4x - 6y - 12 = 0$ are the equation of the two circles. Equation of one of their common tangent is

(A) $7x - 5y - 1 - 5\sqrt{74} = 0$ (B) $7x - 5y - 1 + 5\sqrt{74} = 0$

(C) $7x - 5y + 1 - 5\sqrt{74} = 0$ (D) $5x - 7y + 1 - 5\sqrt{74} = 0$

Key : C

Hint: Both the circles have radius = 5 and they intersect each other, therefore their common tangent is parallel to the line joining their centres.

Equation of the line joining their centre is 7x – 5y + 1 = 0.

$\therefore$ Equation of the common tangent is 7x – 5y = c

$\therefore \left|\frac{c+1}{\sqrt{74}}\right| = 5 \Rightarrow c = \pm 5\sqrt{74} - 1$

$\therefore$ Equation is $7x - 5y + 1 \pm 5\sqrt{74} = 0$.

Let each of the circles

$S_1 \equiv x^2 + y^2 + 4y - 1 = 0$

$S_2 \equiv x^2 + y^2 + 6x + y + 8 = 0$

$S_3 \equiv x^2 + y^2 - 4x - 4y - 37 = 0$

Touches the other two. Let P_1, P_2, P_3 be the point of contact of S_1 and S_2, S_2 and S_3, S_3 and S_1 respectively. Let T be the point of concurrence of the tangents at P_1, P_2, P_3 to the circles. C_1, C_2, C_3 are the centres of S_1, S_2, S_3 respectively.

50. P and Q are any two points on the circle $x^2 + y^2 = 4$ such that PQ is a diameter. If α *and* β are the lengths of perpendicular from P and Q on x + y = 1 then the maximum value of $\alpha\beta$ is

a) $\frac{1}{2}$ b) $\frac{7}{2}$ c) 1 d) 2

Key: B

$$P(2\cos\theta, 2\sin\theta), Q(-2\cos\theta, -2\sin\theta)$$

Hint: $$\alpha\beta = \frac{|2\cos\theta + 2\sin\theta - 1||-2\cos\theta - 2\sin\theta - 1|}{2}$$

$$= \frac{\left|4(\cos\theta + \sin\theta)^2 - 1\right|}{2} \le \frac{7}{2}$$

51. The equation of chord of the circle $x^2 + y^2 - 6x - 4y - 12 = 0$ which passes through the origin such that origin divides it in the ratio 3 : 2 is

(A) y + x = 0, 7y + 17x = 0 (B) y + 3x = 0, 7y + 3x = 0

(C) 4x + y = 0, 9y + 8x = 0 (D) y + 3x = 7, y + 3x = 0

Key: A

Hint:

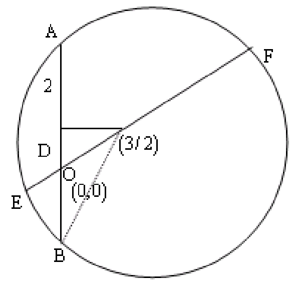
Let AO = 2x, BO = 3x

Now, AO. BO = OE. OF

$X = \sqrt{2}$

Now, D is mid point of chord AB

$AD = DB = \frac{5}{\sqrt{2}}$



Equation of AB is y = mx

$$\Rightarrow \frac{|3m-2|}{\sqrt{1+m^2}} = \frac{5}{\sqrt{2}} \Rightarrow m = -1, -17/7$$

Equation of AB is $y = -x$ and $y = -\frac{17}{7}x$

52. If two circles $(x-1)^2 + (y-3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then

(A) $2 < r < 8$ (B) $r < 2$

(C) $r = 2$ (D) $r > 2$

Key : A

Sol : Centres and radii of the given circles are C_1 (1,3), r_1 = r and $C_2 = (4, -1), r_2 = 3$ respectively since circles intersect in two distinct points, then

$|r_1 - r_2| < C_1C_2 < r_1 + r_2$

$\Rightarrow \ |r-3| < 5 < r+3$ ....(i)

from last two relations, $r > 2$

from firs two relations

$|r-3| < 5$

$\Rightarrow -5 < r-3 < 5$

$\Rightarrow -2 < r < 8$ ....(ii)

from eqs. (i) and (ii), we get $2 < r < 8$

53. (L-1)From a point P outside a circle with centre at C, tangents PA and PB are drawn such that $\frac{1}{(CA)^2} + \frac{1}{(PA)^2} = \frac{1}{16}$, then the length of chord AB is

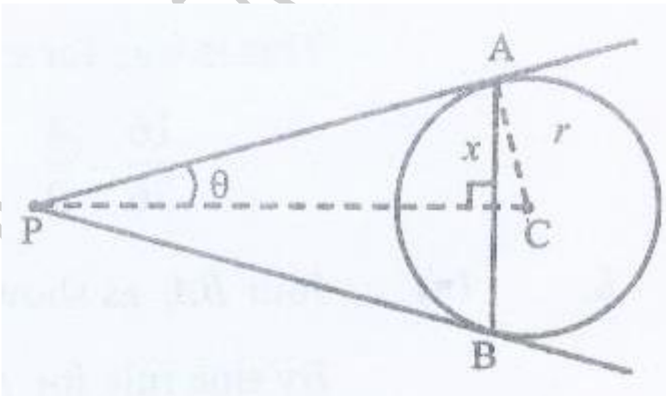
a) 8 b) 12

c) 16 d) none of these

Key : a

Sol : $\tan\theta = \frac{r}{PA}$

Given $\frac{1}{r^2} + \frac{1}{PA^2} = \frac{1}{16}$



$\Rightarrow \frac{\cot^2\theta + 1}{(PA)^2} = \frac{1}{16}$

$\Rightarrow PA\sin\theta = 4 = x \Rightarrow 2x = 8$

54. (L-II)Tangents PT_1 and PT_2 are drawn from a point P to the circle $x^2 + y^2 = a^2$. If the point P lies on the line px + qy + r = 0, then the locus of the centre of circumcircle of the triangle PT_1T_2 is

a) px + qy = r

b) $(x-p)^2+(y-q)^2=r^2$

c) $px+qy=\frac{r}{2}$

d) $2px+2py+r=0$

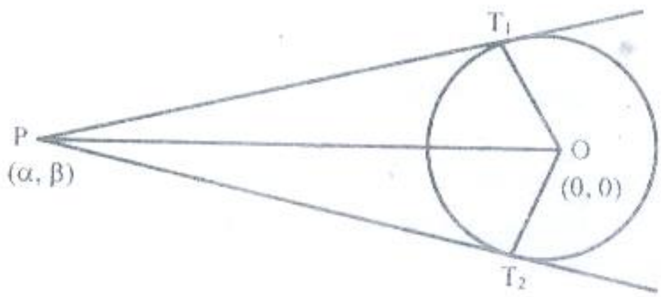
Key : d

Sol : P, T_2, O, T_1 are concylic points with PO as diameter

$\Rightarrow$ The circumcentre of ΔPT_1T_2 is $\left(\frac{\alpha}{2},\frac{\beta}{2}\right)$

New (α,β) lies on px + qy + r = 0

$\Rightarrow$ Locus of $\left(\frac{\alpha}{2},\frac{\beta}{2}\right)$ is 2px + 2qy + r = 0



55. (L-1)The circle $x^2+y^2=1$ cuts the x-axis at P and Q. Another circle with centre at Q and variable radius intersects to first circle at R above the X-axis and the line segment PQ at S. The maximum area of the triangle QSR is

a) $\frac{2}{9}$

b) $\frac{5\sqrt{2}}{7}$

c) $\frac{4\sqrt{3}}{9}$

d) $\frac{\sqrt{2}}{13}$

Key : c

Sol : Q is (-1, 0)

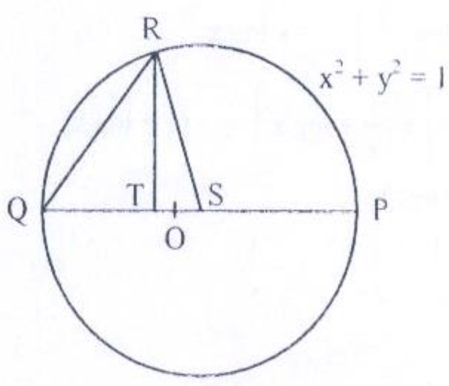
The circle with centre at Q and variable radius r has the equation

$(x+1)^2+y^2=r^2$

This circle meets the line segment QP at S where QS = r

It meets the circle $x^2+y^2=1$ at $R\left(\frac{r^2-2}{2},\frac{r}{2}\sqrt{4-r^2}\right)$ found by solving the equations of the two circles simultaneously.

A = area of the triangle QSR



$$= \frac{1}{2} QS \times RT$$

$$= \frac{1}{2} r \left(\frac{r}{2} \sqrt{4 - r^2} \right)$$ since RT is the y coordinate of R

$$\frac{dA}{dr} = \frac{1}{4} \left\{ 2r\sqrt{4 - r^2} + \frac{r^2(-r)}{\sqrt{4 - r^2}} \right\} = \frac{\left\{ 2r\left(4 - r^2\right) - 4^3 \right\}}{4\sqrt{4 - r^2}} = \frac{8r - 3r^3}{4\sqrt{4 - r^2}}$$

$$\frac{dA}{dr} = 0 \text{ when } r\left(8 - 3r^2\right) = 0 \text{ giving } r = \sqrt{\frac{8}{3}}$$

$$\frac{d^2A}{dr^2} = \frac{4\sqrt{4 - r^2}\left(8 - 9r^2\right) - \left(8r - 3r^3\right)\frac{(-r)^4}{\sqrt{4 - r^2}}}{16\left(4 - r^2\right)}, \text{ where, } r = \sqrt{\frac{8}{3}}, \frac{d^2A}{dr^2} < 0$$

Hence A is maximum when r = $\sqrt{\frac{8}{3}}$ and the maximum area =

$$\frac{8}{4 \times 3}\sqrt{4 - \frac{8}{3}} = \frac{16}{12\sqrt{3}} = \frac{4}{3\sqrt{3}} = \frac{4\sqrt{3}}{9}$$

56. (L-I1)A ray of light incident at the point (-2, -1) gets reflected from the tangent at (0, -1) to the circle $x^2 + y^2 = 1$. The reflected ray touches the circle. The equation of the line along which the incident ray moved is

a) 4x – 3y + 11 = 0

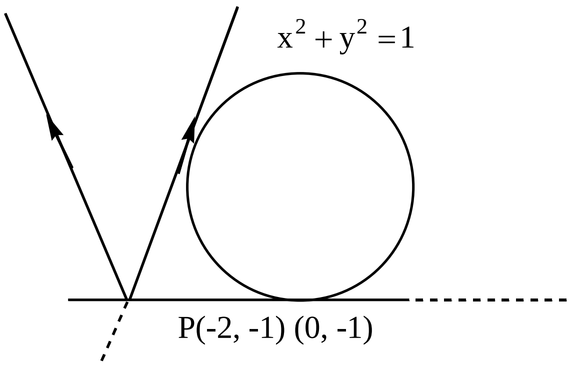
b) 4x + 3y + 11 = 0

c) 3x + 4y + 11 = 0

d) 4x + 3y + 7 = 0

Key : b

Sol : Any line through $(-2, -1)$ is y + 1 = m(x + 2)



It touches the circle, if $\left|\frac{2m-1}{\sqrt{1+m^2}}\right| = 1$

$\Rightarrow m = 0, \frac{4}{3}$

$\therefore$ Equation of PB is $y+1 = \frac{4}{3}(x+2)$

$\Rightarrow 4x - 3y + 5 = 0$

A point of PB is (-5, -2)

Its image by the line y = -1 is (-5, -3)

Hence, equation of incident ray PP' is

$y - 3 = \frac{3+1}{-5+2}(x+5)$

$4x + 3y + 11 = 0$

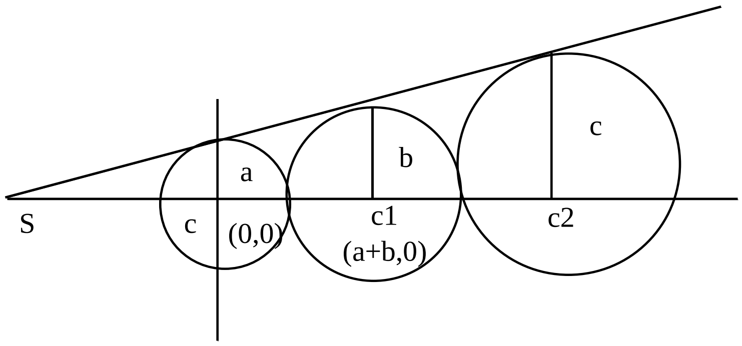
57. (L-I1)A circle C_1 of radius b touches the circle $x^2 + y^2 = a^2$ externally and has its centre on the positive x-axis; another circle C_2 of radius c touches the circle C_1 externally and has its centre on the positive x-axis. Given a < b < c, then the three circles have a common tangent if a, b, c are in

a) A.P. b) G.P.

c) H.P. d) none of these

Key : b

Sol : Similitude point wrt 0^4 s C and C_1 = Similitude point wrt – c_1c_2 the weget a, b, c and 14 G.



58. The locus of the centre of a circle which touches the circle $x^2 + y^2 - 6x - 6y + 14 = 0$ externally and also the y-axis is given by

a) $x^2 - 6y - 7y + 14 = 0$ b) $x^2 - 10x - 6y + 14 = 0$

c) $y^2 - 6x - 10y + 14 = 0$ d) $y^2 - 10x - 6y + 14 = 0$

Key ; d

Sol : Let (x_1, y_1) be the centre. Since it touches y – axis its radius is $|x_1|$ Also it touches the given circle externally

$\therefore \sqrt{(x_1 - 3)^2 + (y_1 - 3)^2} = |x_1| + 2$ Squaring we get

$\cancel{x_1^2} + y_1^2 - 6x_1 - 6y_1 + 18 = \cancel{x_1^2} + 4x_1 + 4$

$\Rightarrow y_1^2 - 10x_1 - 6y + 14 = 0$

59. There is a system of circles, in which two pairs of circles have neither same nor parallel radical axis. If the number of radical axis of system is same as the number of radical centres, then number of circles in the system is

a) 4   b) 5   c) 6   d) 10

Key : b

Sol : Given $n_{c_2} = n_{c_3} \Rightarrow n = 3 + 2 = 5$

60. A line cuts the x-axis at A(4, 0) and the y-axis at B(0, 8). A variable line PQ is drawn perpendicular to AB cutting the x-axis in P and the y-axis in Q. If AQ and BP intersect at R, find the locus of R.

a) $x^2 + y^2 - 2x - 4y = 0$ b) $x^2 + y^2 + 2x + 4y = 0$

c) $x^2 + y^2 - 2x + 4y = 0$ d) $x^2 + y^2 - 4x - 8y = 0$

Key : d

Sol : Locus of R is a circle on AB as diameter ie $x^2 + y^2 - 4x - 8y = 0$

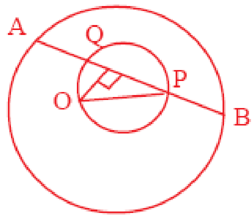
61. Let P, $P \neq 0$ be any point inside a circle with centre at O. Draw a circle with diameter $\overline{OP}$. The point Q ($\neq p$) is any point on this circle. Extend $\overline{PQ}$ to meet the larger circle at A and B then which of the following statements is true

I) Q, P are points of trisection of AB
II) Q is mid point of AB
III) OA, OQ, OP, OB are in H.P

a) only I
b) only II
c) only II and II
d) all the three

Key : b

Sol : Angle in the semicircle is 90^0. Q is the midpoint of AB



62. From a point P outside a circle with centre at C, tangents PA and PB are drawn such that $\frac{1}{(CA)^2}+\frac{1}{(PA)^2}=\frac{1}{16}$, then the length of chord AB is

(A) 8
(B) 12
(C) 16
(D) none of these

Key : A

Hint : $\tan\theta=\frac{r}{PA}$

Given $\frac{1}{r^2}+\frac{1}{PA^2}=\frac{1}{16}$

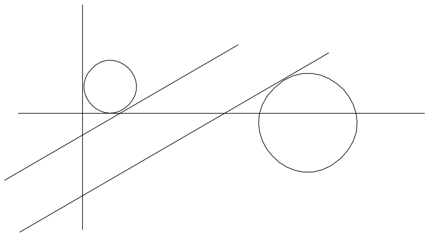
$\Rightarrow \frac{\cot^2\theta+1}{(PA)^2}=\frac{1}{16}$

$\Rightarrow (PA)\sin\theta=4=x\Rightarrow 2x=8$

63. If a variable line y = 2x + p lies between the circles $x^2 + y^2 - 2x - 2y + 1 = 0$ and $x^2 + y^2 - 16x - 2y + 61 = 0$ without intersecting or touching either circles, then number of integral values of p is

a) 9
b) 8
c) 7
d) 6

Key : c

sol : 

$\sin a - \sqrt{5} - 1 < P < 2\sqrt{5} - 15$

Integral value of P = 7

64. The locus of the centre of the circle which touches the y-axis and also touches the circle $(x+1)^2 + y^2 = 1$ externally is

A) $\{(x,y) \mid x^2 = 4y\} \cup \{(x,y) \mid y \le 0\}$ B) $\{(x,y) \mid y^2 = 4x\} \cup \{(x,y) \mid x \le 0\}$

C) $\{(x,y) \mid x^2 + 4y = 0\} \cup \{(x,y) \mid y \ge 0\}$ D) $\{(x,y) \mid y^2 + 4x = 0\} \cup \{(x,y) \mid x \ge 0\}$

Key. D

Sol. Let $P(x_1, y_1)$ be the centre of the touching $(x+1)^2 + y^2 = 1$ externally and touching y-axis

$\therefore 1 - x_1 = (x_1+1)^2 + y_1^2 \Rightarrow y_1^2 + 4x_1 = 0$

Also every circle with centre on positive x-axis and touching y-axis at origin satisfy the condition.

65. Three circles with centres at A, B, C intersect orthogonally. The point of intersection of the common chords is

A) Orthocentre of ΔABC B) Circumcentre of ΔABC

C) Incentre of ΔABC D) Centroid of ΔABC

Key. A

Sol. Common chord of two intersecting circles is $\perp^r$ to line of centres

66. The length of the common chord of the circles which are touching both the coordinate axes and passing through (2, 3) is

A) 3/2 B) 2/3 C) 2 D) $\sqrt{2}$

Key. D

Sol. y=x is the line joining the centres of the two circles.

67. A ray of light incident at the point (3, 1) gets reflected from the tangent at (0, 1) to the circle $x^2 + y^2 = 1$. The reflected ray touches the circle. The equation of the line containing the reflected ray is

A) $3x + 4y - 13 = 0$ B) $4x - 3y - 13 = 0$

c) $3x - 4y + 13 = 0$ D) $4x - 3y - 10 = 0$

Key. A

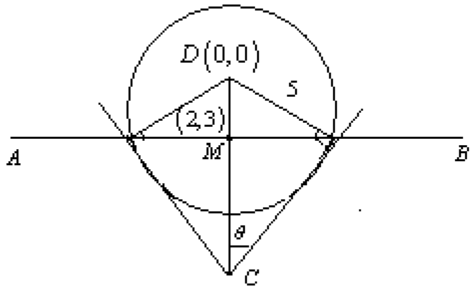
Sol. Angle of incidence is equal to angle of reflection.

68. AB is a chord of the circle $x^2+y^2=25$. The tangents to the circle at A and B intersect at C. If $(2,3)$ is the mid point of AB, then the area of quadrilateral OACB is

A) $\frac{50}{\sqrt{3}}$ B) $50\sqrt{\frac{3}{13}}$ C) $50\sqrt{3}$ D) $\frac{50}{\sqrt{13}}$

Key. B

Sol. From omB, $\cos(90- q)= \frac{\sqrt{13}}{5}$



Þ $\sin q = \sqrt{\frac{13}{5}}$

Þ $\cot q = \frac{2\sqrt{3}}{\sqrt{13}}$

Area of quad $OACB = 2´ \frac{1}{2}´ OB´ BC$

$= 5´ 5\cot q = 25´ \frac{2\sqrt{3}}{\sqrt{13}} = 50\sqrt{\frac{3}{13}}$

69. P(3,2) is a point on the circle $x^2+y^2=13$. Two points A, B are on the circle such that $PA=PB=\sqrt{5}$. The equation of chord AB is

A) $4x-6y+21=0$ B) $6x+4y-21=0$ C) $4x+6y-21=0$ D) $6x+4y+21=0$

Key. B

Sol. AB is common chord of $x^2+y^2=13$ and circle having centre at p and radius $\sqrt{5}$.

70. The point $([P+1],[P])$, (where [.] denotes the greatest integer function) lying inside the region bounded by the circle $x^2+y^2-2x-15=0$ and $x^2+y^2-2x-7=0$, then

a) $P\in[-1,0)\cup[0,1)\cup[1,2)$ b) $P\in[-1,2)-\{0,1\}$

c) $P\in(-1,2)$ d) $P\notin R$

Key. D

Sol. $x^2+y^2-2x-15=0 \Rightarrow [P]^2<8$

$x^2+y^2-2x-7=0 \Rightarrow 4<[P]^2$

71. The locus of centre of a circle which touches externally the circle $x^2+y^2-6x-6y+14=0$ and also touches the y – axis, is given by the equation

a) $x^2-6x-10y+14=0$ b) $x^2-10x-6y+14=0$

c) $y^2-6x-10y+14=0$ d) $y^2-10x-6y+14=0$

Key. D

Sol. Conceptual

72. The points A, B are the feet of O(0, 0) on $x-2y+1=0$, $2x-y-1=0$ respectively then the circum radius of the ΔOAB

a) 2 b) 1 c) $\sqrt{2}$ d) $1/\sqrt{2}$

Key. D

Sol. Point of meet $=(1,1)P$

$\therefore \underline{|OAP} = 90^0 = \underline{|OBP}$

$\therefore$ diameter = OP

73. A circle cuts x – axis at $A(a,0)$, $B(b,0)$ and y – axis at $C(0,c), D(0,2)$ then the orthocentre of the $\Delta ABC =$

a) $(2,0)$ b) $(-2,0)$

c) $(0,2)$ d) $(0,-2)$

Key. D

Sol. $O.C.(ABC) = \text{Image}$ of D, w.r.t $\overline{AB}$

74. The locus of the image of the point (2, 3) with respect to the line $(x-2y+3)+\lambda(2x-3y+4)=0 \ (\lambda \in R)$

a) $x^2+y^2-2x-4y+4=0$ b) $x^2+y^2+2x-4y+4=0$

c) $x^2+y^2-3x-4y-4=0$ d) $x^2+y^2-2x-4y+3=0$

Key. D

Sol. $(1,2)$ lie on both the lines and locus is $(h-1)^2+(k-2)^2=(2-1)^2+(3-2)^2$

75. ABCD is a rectangle . A circle passing through C touches AB,AD at M,N respectively . If the area of rectangle ABCD is K^2 units $(k>0)$ then $\perp^r$ distance from C to MN is

a) $2K$ b) K c) $\frac{K}{2}$ d) $4K$

Key. B

Sol. Taking AB,AD along axes and centre of the circle as E(h,h) we get M (h,0) N(0,h) and equation of MN as $x+y=h$. If $C=(\alpha,\beta)$ then given $\alpha\beta=k^2$ and also

$(\alpha-h)^2+(\beta-h)^2=h^2$, $\perp^r$ distance C to MN is $\frac{|\alpha+\beta-h|}{\sqrt{2}}=k$

$\therefore(\alpha+\beta-h)^2=\alpha^2+\beta^2-2h(\alpha+\beta)+2\alpha\beta=2k^2$

76. The number of integer values of λ for which the variable line $3x+4y=\lambda$ lies completely outside of circles $x^2+y^2-2x-2y+1=0$ and $x^2+y^2-18x-2y+78=0$ without meeting either circle, is

a) 8 b) 10 c) 12 d) 6

Key. A

Sol. The line given does not meet the circles if $(C_1=(1,1), C_2=(9,1)$

$$\frac{|3+4-\lambda|}{5}>1 \text{ and } \frac{|27+4-\lambda|}{5}>2$$

$$\Rightarrow |7-\lambda|>5 \& |31-\lambda|>10$$

$$But\ 7-\lambda<0 \text{ and } 31-\lambda>0.$$

Hence $\lambda>12 \& \lambda<21$

77. The curves $C_1: y=x^2-3$; $C_2: y=kx^2, k<1$ intersect each other at two different points. The tangent drawn to C_2, at one of the points of intersection $A=(a,y_1)(a>0)$ meets C_1 again at $B(1,y_2)$. $(y_1 \neq y_2)$. Then value of $a=$ ___?

a) 4 b) 3 c) 2 d) 1

Key. B

Sol. solving

$$C_1 \& C_2 \Rightarrow A\left(\sqrt{\frac{3}{1-k}}, \frac{3k}{1-k}\right)=(a, ka^2) \equiv (a, a^2-3).$$

$$\text{tan gent } l \text{ to } C_2 \text{ at A is } y+a^2-3=2kx -----(1)$$

$$\Rightarrow B=(1,-2) \ (A \neq 1).$$

$$\text{from expression (1) } -2+a^2-3=2a\left(1-\frac{3}{a^2}\right).$$

$$\Rightarrow a=3, a=-2, a=1$$

$$\therefore a=3$$

78 Let A(1, 2), B(3, 4) be two points and C(x, y) be a point such that area of ΔABC is 3 sq.units and $(x-1)(x-3)+(y-2)(y-4)=0$. Then maximum number of positions of C, in the xy plane is

a) 2 b) 4 c)8 d) no such C exist

Key. D

Sol. (x,y) lies on the circle ,with AB as a diameter . Area

$$(\Delta ABC)=3$$

$$\Rightarrow \left(\tfrac{1}{2}\right)(AB)(\text{altitude})=3.$$

$$\Rightarrow \text{altitude }=\frac{3}{\sqrt{2}} \Rightarrow \text{no such "C" exists}$$

79. The equation of the smallest circle passing through the intersection of $x^2+y^2-2x-4y-4=0$ and the line $x+y-4=0$ is

(A) $x^2+y^2-3x-5y-8=0$ (B) $x^2+y^2-x-3y=0$

(C) $x^2+y^2-3x-5y=0$ (D) $x^2+y^2-x-3y-8=0$

Key. C

Sol. Family of circles passing through circle S = 0 and line L = 0 will be S + λ L = 0

$$x^2+y^2-2x-4y-4+\lambda(x+y-4)=0 \qquad \text{….. (1)}$$

For smallest circle line x + y – 4 = 0 will become the diameter for (1)

80. Equation of a straight line meeting the circle $x^2+y^2=100$ in two points, each point is at a distance of 4 units from the point (8, 6) on the circle, is

(A) $4x+3y-50=0$ (B) $4x+3y-100=0$ (C) $4x+3y-46=0$ (D) $4x+3y-16=0$

Key. C

Sol.

$S_1=x^2+y^2=100$

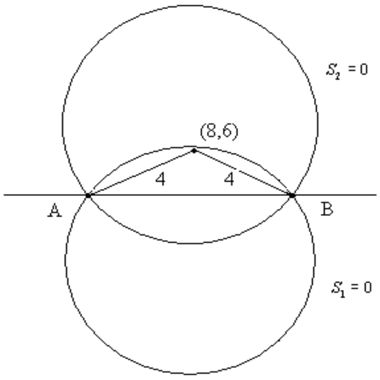
equation of circle centred at (8,6) & radius 4 units is

$(x-8)^2+(y-6)^2=16$

required line AB is the common chord of $S_1=0$ & $S_2=0$, is

$S_1-S_2=0$

$4x+3y-46=0$



81. The locus of the middle points of the chords of the circle of radius r which subtend an angle $\pi/4$ at any point on the circumference of the circle is a concentric circle with radius equal to

A. $r/2$ B. $2r/3$ C. $r/\sqrt{2}$ D. $r/\sqrt{3}$

Key. C

Sol. Equation of the circle be $x^2+y^2=r^2$. The chord which substends an angle $\pi/4$ at the circumference will subtend a right angle at the centre. Chord joining (r, 0) and (0, r) substends a right angle at the centre so(h, k) is $x^2+y^2=r^2/2$.

82. Two distinct chords drawn from the point (p,q) on the circle $x^2+y^2=px+qy$ where $pq\neq 0$, are bisected by the x-axis then

1) $|p|=|q|$ 2) $p^2=8q^2$ 3) $p^2<8q^2$ 4) $p^2>8q^2$

Key. 4

Sol. Let $A(p,q)$. Let $P(k,o)$ bisects the chord $\overline{AB}$

Then $B(2k-p,-q)$ lies on the circle

$\Rightarrow (2k-p)^2+q^2=p(2k-p)+q(-q)$

$\Rightarrow 4k^2+p^2-4kp+q^2=2kp-p^2-q^2$

$\Rightarrow 2k^2-3kp+(p^2+q^2)=0$

$b^2-4ac>0 \Rightarrow 9p^2-8(p^2+q^2)>0$

$\Rightarrow p^2 > 8q^2$

83. The sum of the radii of inscribed and circumscribed circle of 'n' sided regular polygon of side 'a' is

1) $\frac{4}{a}\cot\left(\frac{\pi}{2n}\right)$ 2) $a\cot\left(\frac{\pi}{2n}\right)$ 3) $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$ 4) $2a\cot\left(\frac{\pi}{2n}\right)$

Key. 3

Sol. Circumradius, $R = \frac{a}{2}.\operatorname{cosec}\frac{\pi}{n}$

In radius, $r = \frac{a}{2}.\cot\frac{\pi}{n}$

Now, $R + r = \frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$

84. B and C are points on the circle $x^2 + y^2 = a^2$. A point A(b,c) lies on the circle such that AB=AC=d. Then the equation of $\overrightarrow{BC}$ is

1) $bx + ay = a^2 - d^2$ 2) $bx + ay = d^2 - a^2$

3) $bx + cy = 2a^2 - d^2$ 4) $2(bx + cy) = 2a^2 - d^2$

Key. 4

Sol. Equation of the circle with centre at $A(b,c)$ and radius d is $(x-b)^2 + (y-c)^2 = d^2$

$\Rightarrow x^2 + y^2 - 2bx - 2cy + b^2 + c^2 - d^2 = 0$ --------- (1)

$\therefore$ $\overline{BC}$ is Radical axis of (1) and $x^2 + y^2 = a^2$

$\therefore$ $\overline{BC}$ is $2bx + 2cy - b^2 - c^2 + d^2 - a^2 = 0$

But, $b^2 + c^2 = a^2 \rightarrow 2bx + 2cy = 2a^2 - d^2$

85. The locus of poles of the line $lx + my + n = 0$ w.r.t the circle passing through $(-a,0),(a,0)$ is

1) $lx^2 - mxy + nx + a^2l = 0$ 2) $lx^2 - mxy + ny + a^2l = 0$

3) $ly^2 - mxy + nx + a^2l = 0$ 4) $ly^2 - mxy + ny + a^2l = 0$

Key. 3

Sol. Equation of the circle passing through $A(-a,0)$ $B(a,0)$ is $x^2 + y^2 - a^2 + 2\lambda(y) = 0$

$\Rightarrow x^2 + y^2 + 2\lambda y - a^2 = 0$

Polar of $P(x_1, y_1)$ is $xx_1 + yy_1 + \lambda(y + y_1) - a^2 = 0$ ------------- (1)

Given Polar, $lx + my + n = 0$ ------------- (2)

Compare (1) & (2), eliminate λ, we get $ly^2 - mxy + nx + a^2l = 0$

86. If two circles which pass through the points $(0, a)$ and $(0, -a)$ cut each other orthogonally and touch the straight line $y = mx + c$, then

A) $c^2 = a^2(1+m^2)$ B) $c^2 = a^2|1-m^2|$

C) $c^2 = a^2(2+m^2)$ D) $c^2 = 2a^2(1+m^2)$

Key. C

Sol. Equation of a family of circles through $(0, a)$ and $(0, -a)$ is $x^2 + y^2 + 2\lambda ax - a^2 = 0.$ If two members are for $\lambda = \lambda_1$ and $\lambda = \lambda_2$ then since they intersect orthogonally

$2\lambda_1\lambda_2 a^2 = -2a^2 \Rightarrow \lambda_1\lambda_2 = -1$

Since the two circles touch the line $y = mx + c$

$$\left[\frac{-\lambda am + c}{\sqrt{1+m^2}}\right]^2 = \lambda^2 a^2 + a^2$$

$$\Rightarrow a^2\lambda^2 + 2mca\lambda - c^2 + a^2(1+m^2) = 0$$

$$\Rightarrow a^2(1+m)^2 - c^2 = -a^2 \Rightarrow c^2 = (2+m^2)a^2$$

87. Equation of a circle through the origin and belonging to the co-axial system, of which the limiting points are $(1,2),(4,3)$ is

A) $x^2 + y^2 - 2x + 4y = 0$ B) $x^2 + y^2 - 8x - 6y = 0$

C) $2x^2 + 2y^2 - x - 7y = 0$ D) $x^2 + y^2 - 6x - 10y = 0$

Key. C

Sol. Since the limiting points of a system of coaxial circles are the point circles (radius being zero), two members of the system are

$(x-1)^2 + (y-2)^2 = 0 \Rightarrow x^2 + y^2 - 2x - 4y + 5 = 0$ and

$(x-4)^2 + (y-3)^2 = 0 \Rightarrow x^2 + y^2 - 8x - 6y + 25 = 0$

The co-axial system of circles with these as members is

$x^2 + y^2 - 2x - 4y + 5 + \lambda(x^2 + y^2 - 8x - 6y + 25) = 0$

It passes through the origin if $5 + 25\lambda = 0$

or $\lambda = -(1/5)$

which gives the equation of the required circle as

$5(x^2 + y^2 - 2x - 4y + 5) - (x^2 + y^2 - 8x - 6y + 25) = 0$

$\Rightarrow 4x^2 + 4y^2 - 2x - 14y = 0$

$\Rightarrow 2x^2 + 2y^2 - x - 7y = 0.$

88. Circle are drawn to cut two circles $x^2 + y^2 + 6x + 5 = 0$ and $x^2 + y^2 - 6y + 5 = 0$ orthogonally. All such circles will pass through the fixed points.

A) $(1,-1)$ only B) $(2,-2)$ and $(0,0)$ C) $(-1,1)$ and $(-2,2)$ D) $(1,-1)$ and $(2,-2)$

Key. C

Sol. The radical axis of the given circles is $x+y=0.$ Let P $(\lambda,-\lambda)$ be any point on the above radical axis.

The length of the tangent drawn from P to any of the given circles is $l=\sqrt{\lambda^2+\lambda^2+6\lambda+5}$

A circle having centre at P and radius equal to l will be orthogonal to both the given circles.

Equation of such a circle, is $(x-\lambda)^2+(y+\lambda)^2=l^2=\lambda^2+\lambda^2+6\lambda+5$

i.e. $x^2+y^2+2\lambda^2-2\lambda x+2\lambda y=2\lambda^2+6\lambda+5$

i.e. $(x^2+y^2-5)-2\lambda(x-y+3)=0$

which represents a family of circles passing through the intersection points of

$x^2+y^2-5=0....(i)$ and $x-y+3=0...(ii)$

Eliminating y we get

$x=-1,-2$ and the corresponding y=1, 2

Hence, the required points are $(-1,1)$ and $(-2,2)$.

89. If one circle of a co-axal system is $x^2+y^2+2gx+2fy+c=0$ and one limiting point is (a,b) then equation of the radical axis will be

A) $(g+a)x+(f+b)y+c-a^2-b^2=0$

B) $2(g+a)x+2(f+b)y+c-a^2-b^2=0$

C) $2gx+2fy+c-a^2-b^2=0$ D) None of these

Key. B

Sol. Given circle $S_1\equiv x^2+y^2+2gx+2fy+c=0...(i)$

$\therefore$ Equation of the second circle is $(x-a)^2+(y-b)^2=0$

$S_2\equiv x^2+y^2-2ax-2by+a^2+b^2=0...(ii)$

From (i) and (ii), equation radical axis is $S_1-S_2=0$

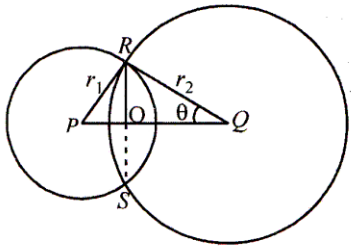
$\Rightarrow 2(g+a)x+2(f+b)y+c-a^2-b^2=0$

90. The circles having radii r_1 and r_2 intersect orthogonally. The length of their common chord is

A) $\dfrac{2r_1r_2}{\sqrt{r_1^2+r_2^2}}$ B) $\dfrac{2r_1^2+r_2^2}{\sqrt{r_1^2+r_2^2}}$ C) $\dfrac{r_1r_2}{\sqrt{r_1^2+r_2^2}}$ D) $\dfrac{2r_2^2+r_1^2}{\sqrt{r_1^2+r_2^2}}$

Key. A

Sol.



Let the centres of the circles be P and Q which intersect orthogonally at the point R, then $\angle PRQ = 90^0$

Let $\angle PQR = \theta$ then $\angle QPR = 90^0 - \theta$

$\therefore RO = r_2 \sin\left(90^0 - \theta\right) = r_1 \sin\theta$

$\Rightarrow \sin\theta = \dfrac{RO}{r_1}$ and $\cos\theta = \dfrac{RO}{r_2}$

$\Rightarrow RO^2\left(\dfrac{1}{r_1^2} + \dfrac{1}{r_2^2}\right) = 1 \Rightarrow RO = \dfrac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$

$\therefore$ Length of common chord $RS = 2RO = \dfrac{2r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$

91. Radical centre of the three circles $x^2 + y^2 = 9,\ x^2 + y^2 - 2x - 2y = 5,\ x^2 + y^2 + 4x + 6y = 19$ lies on the line $y = mx$ if m is equal to

A) -1 B) $-2/3$ C) $-3/4$ D) 1

Key. D

Sol. The radical centre is the point of intersection of $2x + 2y = 4$ and $4x + 6y = 10$ i.e. $(1,1)$ which lies on $y = mx$ if $m = 1$.

92. If $\dfrac{x}{a} + \dfrac{y}{b} = 1$ touches the circle $x^2 + y^2 = r^2$ then $\left(\dfrac{1}{a}, \dfrac{1}{b}\right)$ lie on

(A) straight line (B) circle

(C) parabola (D) ellipse

Key. B

Sol. Let $\alpha = \dfrac{1}{a},\ \beta = \dfrac{1}{b}$

$x\alpha + y\beta - 1 = 0$ touches $x^2 + y^2 = r^2$

$\Rightarrow \left|\dfrac{-1}{\sqrt{\alpha^2 + \beta^2}}\right| = r$

$\Rightarrow \alpha^2 + \beta^2 = \dfrac{1}{r^2}$

$\Rightarrow \alpha, \beta$ lies on $x^2 + y^2 = \dfrac{1}{r^2}$

93. The value of 'c' for which the sets $\{(x, y) : x^2 + y^2 + 2x - 1 \le 0\} \cap \{(x, y) : x - y + c \ge 0\}$ contain only one point.

(A) – 1 only (B) 3 only

(C) both – 1 and 3 (D) 2

Key. A

Sol.

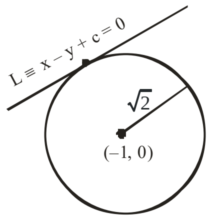
$$\frac{|-1+c|}{\sqrt{2}} = \sqrt{2}$$

c = 3, –1

L(–1, 0) > 0 when c = 3

< 0 when c = –1

$\Rightarrow$ c = –1



94. A circle of radius 'r' is inscribed in a square. The mid points of sides of square are joined to form a new square. The mid point of sides of resulting square are again joined so that a new square was obtained and so on. Then radius of circle inscribed in n^{th} square is

(a) $\left(2^{\frac{1-n}{2}}\right)r$ (b) $\left(2^{\frac{n-1}{2}}\right)r$

(c) $\left(2^{\frac{3-3n}{2}}\right)r$ (d) $\left(2^{-\frac{n}{2}}\right)r$

Key. A

SOL. CLEARLY RADIUS OF 2^{ND} CIRCLE $= \frac{\sqrt{r^2+r^2}}{2} = \frac{r}{\sqrt{2}}$

AND THIRD CIRCLE $= \frac{r}{2}$

$\Rightarrow$ radius of *n*th circle $= \frac{r}{2^{\left(\frac{n-1}{2}\right)}}$

95. A variable circle touches the line y = x and passes through (0, 0). The common chord of the above circle and the circle $x^2 + y^2 + 6x + 8y - 7 = 0$ will pass through

(a) (0, 0) (b) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

(c) $\left(\frac{1}{2}, -\frac{1}{2}\right)$ (d) $\left(\frac{1}{2}, \frac{1}{2}\right)$

Key. D

SOL. EQUATION OF FAMILY OF CIRCLE TOUCHING

X = Y AT (0, 0) IS $X^2 + Y^2 + \lambda(X - Y) = 0$

REQUIRED COMMON CHORD $\equiv 6X + 8Y - 7 - \lambda(X - Y) = 0$

always passes through $\left(\frac{1}{2}, \frac{1}{2}\right)$

96. A circle passes through the points (2, 2) and (9, 9) and touches the *x*–axis. The *x*–cordinate of the point of contact is

(A) –2 or 2 (B) –4 or 4

(C) –6 or 6 (D) –9 or 9

Key. C

Sol. Any circle through (2, 2) and (9, 9) is

$$(x-2)(x-9)+(y-2)(y-9)+\lambda(y-x)=0 \qquad (1)$$

For the point of intersection with x–axis, we put y = 0 in (1), to get

$$(x-2)(x-9)+18-\lambda x=0$$

$D = 0 \Rightarrow (11+\lambda)^2 - 4 \times 36 \Rightarrow \lambda = -23, 1$

97. From a fixed point on the circle $x^2 + y^2 = a^2$, two tangents are drawn to the circle $x^2 + y^2 = b^2$ $(a > b)$. If the chord of contact touches a variable circle passing through origin, then the locus of the centre of the variable circle is

(A) a circle (B) a parabola

(C) an ellipse (D) a hyperbola

Key. B

Sol. The centre of the variable circle is always equidistant from the given chord of contact and the origin, its locus is a parabola.

98. If $9 + f''(x) + f'(x) = x^2 + f^2(x)$ be the differential equation of a curve and let P be the point of minima then the number of tangents which can be drawn from P to the circle $x^2 + y^2 = 9$ is

(A) 2 (B) 1

(C) 0 (D) either 1 or 2

Key. A

Sol. At the point of minima $f'(x) = 0$, $f''(x) > 0$

$\Rightarrow f''(x) = -9 + x^2 + f^2(x) > 0 \Rightarrow x^2 + y^2 - 9 > 0 \Rightarrow$ point P(x, f(x)) lies outside $x^2 + y^2 = 9$

$\Rightarrow$ two tangents are possible.

99. A point P lies inside the circles $x^2+y^2-4=0$ and $x^2+y^2-8x+7=0$. The point P starts moving under the conditions that its path encloses greatest possible area and it is at a fixed distance from any arbitrarily chosen fixed point in its region. The locus of P is

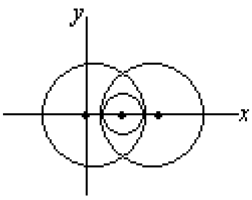
A) $4x^2+4y^2-12x+1=0$ B) $4x^2+4y^2+12x-1=0$

C) $x^2+y^2-3x-2=0$ D) $x^2+y^2-3x+2=0$

Key. D

Sol. For the point P to enclose greatest area, the arbitrarily chosen point should be $\left(\frac{3}{2},0\right)$ and P should move in a circle of radius ½ . Locus of P is a circle of radius ½ .

$$\left(x-\frac{3}{2}\right)^2+(y-0)^2=\frac{1}{4} \Rightarrow x^2+y^2-3x+2=0.$$



100. A circle of unit radius touches positive co-ordinate axes at A & B respectively. A variable line passing through origin intersects the circle in two points D and E. If the area of ΔDEB is maximum, then the reciprocal of the square of the slope of the line is

a) $\frac{1}{3}$ b) 3 c) $\frac{1}{2}$ d) 2

Key. B

Sol. Let 'm' the slope of the line (m > 0)

$$\Delta = \frac{\sqrt{2}\sqrt{m}}{m^2+1} \quad \Delta_{max} \Rightarrow m^2 = \frac{1}{3}$$

101. ABCD is a rectangle . A circle passing through C touches AB,AD at M,N respectively . If the area of rectangle ABCD is K^2 units $(k>0)$ then $\perp^r$ distance from C to MN is

a) $2K$ b) K c) $\frac{K}{2}$ d) $4K$

Key. B

Sol. Taking AB,AD along axes and centre of the circle as E(h,h) we get M (h,0) N(0,h) and equation of MN as $x+y=h.$ If $C=(\alpha,\beta)$ then given $\alpha\beta = k^2$ and also

$(\alpha-h)^2+(\beta-h)^2=h^2,\ \perp^r$ distance C to MN is $\frac{|\alpha+\beta-h|}{\sqrt{2}}=k$

$\therefore (\alpha+\beta-h)^2=\alpha^2+\beta^2-2h(\alpha+\beta)+2\alpha\beta=2k^2$

102. The number of integer values of λ for which the variable line $3x+4y=\lambda$ lies completely outside of circles $x^2+y^2-2x-2y+1=0$ and $x^2+y^2-18x-2y+78=0$ without meeting either circle, is

a) 8 b) 10 c) 12 d) 6

Key. A

Sol. The line given does not meet the circles if $(C_1=(1,1),C_2=(9,1)$

$$\frac{|3+4-\lambda|}{5}>1 \text{ and } \frac{|27+4-\lambda|}{5}>2$$

$$\Rightarrow |7-\lambda|>5 \& |31-\lambda|>10$$

$$But\ 7-\lambda<0 \ and\ 31-\lambda>0.$$

Hence $\lambda>12 \& \lambda<21$

103. All chords of the curve $x^2+y^2-10x-4y+4=0$, which make a right angle at (8,-2) pass through

a) (2,5) b) (-2,-5) c) (-5,-2) d) (5,2)

Key. D

Sol.(8,-2) lies on the circle $(x-5)^2+(y-2)^2=25$ and a chord making a right angle at (8,-2) must be a diameter of the circle .So they all pass through the centre (5,2).

104. The locus of the centre of the circle which touches the y-axis and also touches the circle $(x+1)^2 + y^2 = 1$ externally is

A) $\{(x,y) \mid x^2 = 4y\} \cup \{(x,y) \mid y \le 0\}$
B) $\{(x,y) \mid y^2 = 4x\} \cup \{(x,y) \mid x \le 0\}$
C) $\{(x,y) \mid x^2 + 4y = 0\} \cup \{(x,y) \mid y \ge 0\}$
D) $\{(x,y) \mid y^2 + 4x = 0\} \cup \{(x,y) \mid x \ge 0\}$

Key. D

Sol. Let $P(x_1, y_1)$ be the centre of the touching $(x+1)^2 + y^2 = 1$ externally and touching y-axis

$\therefore\ 1 - x_1 = (x_1+1)^2 + y_1^{\,2} \Rightarrow\ y_1^{\,2} + 4x_1 = 0$

Also every circle with centre on positive x-axis and touching y-axis at origin satisfy the condition.

105. Three circles with centres at A, B, C intersect orthogonally. The point of intersection of the common chords is

A) Orthocentre of ΔABC
B) Circumcentre of ΔABC
C) Incentre of ΔABC
D) Centroid of ΔABC

Key. A

Sol. Common chord of two intersecting circles is $\perp^r$ to line of centres

106. The length of the common chord of the circles which are touching both the coordinate axes and passing through (2, 3) is

A) 3/2 B) 2/3 C) 2 D) $\sqrt{2}$

Key. D

Sol. y=x is the line joining the centres of the two circles.

107. A ray of light incident at the point (3, 1) gets reflected from the tangent at (0, 1) to the circle $x^2 + y^2 = 1$. The reflected ray touches the circle. The equation of the line containing the reflected ray is

A) $3x + 4y - 13 = 0$
B) $4x - 3y - 13 = 0$
c) $3x - 4y + 13 = 0$
D) $4x - 3y - 10 = 0$

Key. A

Sol. Angle of incidence is equal to angle of reflection.

108. AB is a chord of the circle $x^2 + y^2 = 25$. The tangents to the circle at A and B intersect at C. If $(2,3)$ is the mid point of AB, then the area of quadrilateral OACB is

A) $\frac{50}{\sqrt{3}}$ B) $50\sqrt{\frac{3}{13}}$ C) $50\sqrt{3}$ D) $\frac{50}{\sqrt{13}}$

Key. B

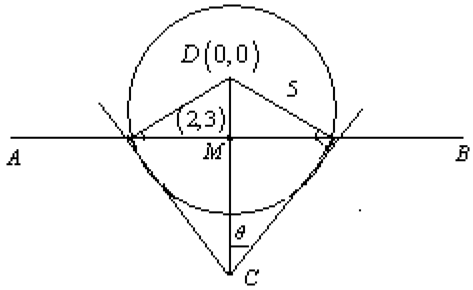
Sol. From omB, $\cos(90 - \theta) = \frac{\sqrt{13}}{5}$

$\Rightarrow\ \sin\theta = \sqrt{\frac{13}{5}}$

$\Rightarrow\ \cot\theta = \frac{2\sqrt{3}}{\sqrt{13}}$

Area of quad $OACB = 2\times \frac{1}{2}\times OB\times BC$

$= 5\times 5\cot q = 25\times \frac{2\sqrt{3}}{\sqrt{13}} = 50\sqrt{\frac{3}{13}}$



109. P(3,2) is a point on the circle $x^2+y^2=13$. Two points A, B are on the circle such that $PA=PB=\sqrt{5}$. The equation of chord AB is

A) $4x-6y+21=0$ B) $6x+4y-21=0$

C) $4x+6y-21=0$ D) $6x+4y+21=0$

Key. B

Sol. AB is common chord of $x^2+y^2=13$ and circle having centre at p and radius $\sqrt{5}$.

110. The range of a for which eight distinct points can be found on the curve $|x|+|y|=1$ such that from each point two mutually perpendicular tangents can be drawn to the circle $x^2+y^2=a^2$ is

a) 1 < a < 2 b) $\frac{1}{2}<a<1$ c) $\frac{1}{\sqrt{2}}<a<1$ d) $\frac{1}{2}<a<\frac{1}{\sqrt{2}}$

Key. D

Sol. Director circle $x^2+y^2=2a^2$ must cut square formed by $|x|+|y|=1$ at 8 points

Min radius = OE

Max radius = OA

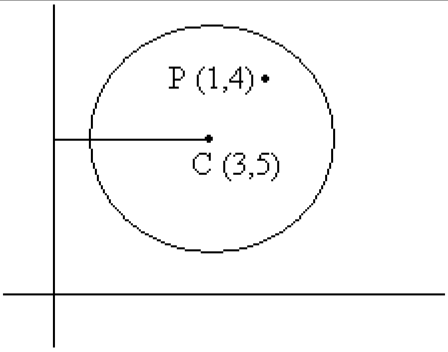
$\therefore \frac{1}{\sqrt{2}}<\sqrt{2}a<1$

111. The point (1,4) lies inside the circle $x^2+y^2-6x-10y+p=0$ which does not touch or intersects the coordinate axes then

a) 0 < p < 29 b) 25 < p < 29 c) 9 < p < 25 d) 9 < p < 29

Key. B

Sol.



CP < r < 3

$CP^2 < r < 9$

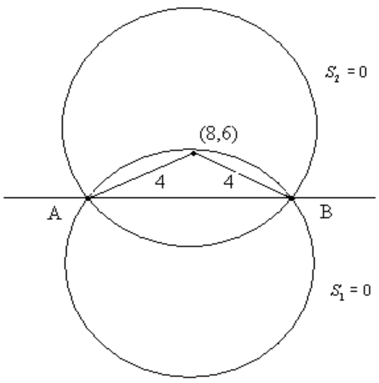
25 < P < 29

112. Equation of a straight line meeting the circle $x^2 + y^2 = 100$ in two points, each point at a distance of 4 from the point (8,6) on the circle, is

a) $4x+3y-50=0$ b) $4x+3y-100=0$ c) $4x+3y-46=0$ d) $4x+3y-16=0$

Key. C

Sol.



$S_1 = x^2 + y^2 = 100$

equation of circle centred at (8,6) & radius 4 units is $(x-8)^2 + (y-6)^2 = 16$

required line AB is the common chord of $S_1 = 0$ & $S_2 = 0$, is $S_1 - S_2 = 0$

$$4x+3y-46=0$$

113. Minimum radius of circle which in orthogonal with both the circles $x^2 + y^2 - 12x + 35 = 0$ and $x^2 + y^2 + 4x + 3 = 0$ is

a) 4 b) 3 c) $\sqrt{15}$ d) 1

Key. C

Sol. equation of the radical axis of two given circles is -16x + 32 = 0

$\Rightarrow$ x = 2

and it intersect the line joining the centres is y = 0 at the point (2,0)

$\therefore$ required radius is $\sqrt{4-24+35}=\sqrt{4+8+3}$

$$=\sqrt{15}$$

114. If $f(x+y)=f(x).f(y)$ for all x and y, $f(1)=2$ and $\alpha_n=f(n), n\in N$, then the equation of the circle having (α_1,α_2) and (α_3,α_4) as the ends of its one diameter is

A) $(x-2)(x-8)+(y-4)(y-16)=0$ B) $(x-4)(x-8)+(y-2)(y-16)=0$

C) $(x-2)(x-16)+(y-4)(y-8)=0$ D) $(x-6)(x-8)+(y-5)(y-6)=0$

Key. A

Sol. $f(x+y)=f(x).f(y)$, Q $f(1)=2$

Put $x=y=1 \Rightarrow f(2)=2^2 \Rightarrow f(n)=2^n$

Hence required circle is $(x-2)(x-8)+(y-4)(y-16)=0$

115. ABCD is a square of side 1 unit. A circle passes through vertices A, B of the square and the remaining two vertices of the square lie out side the circle. The length of the tangent drawn to the circle from vertex D is 2 units. The radius of the circle is

A) $\sqrt{5}$ B) $\frac{1}{2}\sqrt{10}$ C) $\frac{1}{3}\sqrt{12}$ D) $\sqrt{8}$

Key. B

Sol. Let $A=(0,1)$, $B=(0,0)$, $C=(1,0)$, $D=(1,1)$.

Family of circles passing through A, B is $x^2+y^2-y+\lambda x=0$.

$\sqrt{1+\lambda}=2 \Rightarrow \lambda=3$

116. The point A lies on the circle $(x+3)^2+(y-4)^2=r^2$. Two chords of lengths 13 and 15 are drawn to the circle through A such that the distance between the mid points of these chords is 7. Then r =

A) $\frac{45}{4}$ B) $\frac{70}{9}$ C) $\frac{32}{3}$ D) $\frac{65}{8}$

Key. D

Sol. r is the circumradius of the triangle whose sides are a = 13, b = 15, c = 14 . $r=\frac{abc}{4\Delta}$

117. The equation of circumcircle of a Δ ABC is $x^2+y^2+3x+y-6=0$. If $A=(1,-2)$, $B=(-3,2)$ and the vertex C varies then the locus of orthocenter of Δ ABC is a

A) Straight line B) Circle C) Parabola D) Ellipse

Key. B

Sol. Equation of circumcircle is $\left(x+\frac{3}{2}\right)^2+\left(y+\frac{1}{2}\right)^2=\frac{17}{2}$

$$C=\left(-\frac{3}{2}+\sqrt{\frac{17}{2}}\cos\theta,-\frac{1}{2}+\sqrt{\frac{17}{2}}\sin\theta\right)$$

Circum centre of ΔABC is $\left(-\frac{3}{2},-\frac{1}{2}\right)$

Centroid can be obtained.

In a triangle centroid, circum centre and ortho centre are collinear.

Circles

Multiple Correct Answer Type

1. The circles $x^2 + y^2 + 2x + 4y - 20 = 0$ & $x^2 + y^2 + 6x - 8y + 10 = 0$

(A) are such that the number of common tangents on them is 2

(B) are such that the length of their common tangent is $5\ (12/5)^{1/4}$

(C) are not orthogonal

(D) are such that the length of their common chord is 5.

Key. A,B

Sol. Given circles intersects at two distinct points.

2. The circles $x^2 + y^2 + 2x + 4y - 20 = 0$ & $x^2 + y^2 + 6x - 8y + 10 = 0$

(A) are such that the number of common tangents on them is 2

(B) are such that the length of their common tangent is $5\ (12/5)^{1/4}$

(C) are not orthogonal

(D) are such that the length of their common chord is 5.

Key. A,B

Sol. Given circles intersects at two distinct points.

3. If $m(x-2)+\sqrt{1-m^2}.y=3$, is tangent to a circle for all $m \in [-1,1]$ then the radius of the circle.

Key. 3

Sol. $(x-2)\cos\theta + y\sin\theta = 3$ is tangent to the circle $(x-2)^2 + y^2 = 3^2$

4. If $16l^2+9m^2=24lm+6l+8m+1$ and S be the equation of circle having $lx+my+1=0$ is tangent then

A. equation of director circle of S is $x^2+y^2-6x-8y-25=0$

B. radius of circle is 5

C. perpendicular distance from centre of S to x - y + 1 = 0 is $\sqrt{2}$

D. equation of circle S is $x^2+y^2+6x+8y=0$

Key. A,B

Sol. $16l^2+9m^2=24lm+6l+8m+1$

$\Rightarrow 25(l^2+m^2)=9l^2+16m^2+24lm+6l+8m+1=(3l+4m+1)^2$

$\Rightarrow \{\frac{l(3)+m(4)+1}{\sqrt{l^2+m^2}}\}^2=5$

Centre = 93,4), radius = 5

5. (L-1I)A point P(x, y) is called a lattice point if $x, y \in I$ (set of integers). Then the total number of lattice points in the interior of the circle $x^2+y^2=a^2, a \neq 0$ cannot be

a) 1996 b) 1998

c) 1999 d) 2001

Key : a, b, c

Sol : Given circle is $x^2+y^2=a^2$ …… (1)

Clearly (0, 0) will belong to the interior of circle (1)

Also other points interior to circle (1) will have the co-ordinates of the form $(\pm\alpha, 0), (0, \pm\alpha)$ where $\alpha^2 < a^2$ and $(\pm\alpha, \pm\beta)$ and $(\pm\beta, \pm\alpha)$, where $\alpha^2+\beta^2 < a^2$ and $\alpha, \beta \in I$

$\therefore$ Number of lattice points in the interior of the circle will be of the form 1 + 4k + 84,

Where k, r = 0, 1, 2, …..

$\therefore$ Number of such points must be of the form 4m + I, where m = 0, 1, 2,……

6. (L-1I)Consider the circle $x^2+y^2-8x-18y+93=0$ with centre 'C' and point P(2, 5) outside it. From the point P, a pair of tangents PQ and PR are drawn to the circle with S as the midpoint of QR. The line joining P to C intersects the given circle at A and B. Which of the following hold(s) good ?

a) CP is the arithmetic mean of AP and BP

b) PR is the geometric mean of PS and PC

c) PS is the harmonic mean of PA and PB

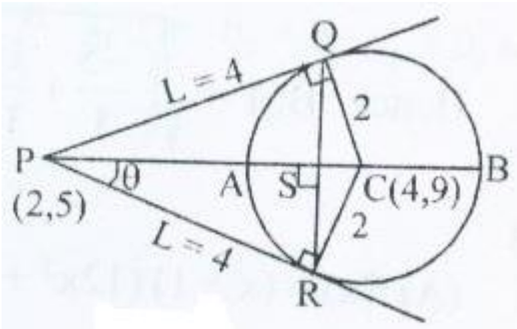
d) The angle between the two tangents from P is $\tan^{-1}\left(\frac{3}{4}\right)$

Key : a, b, c

Sol : Radius = $\sqrt{16+81-93}=2$

$CP=\sqrt{20};\ AP=\sqrt{20}-2;\ BP=\sqrt{20}+2$

$\Rightarrow CP=\dfrac{AP+BP}{2}\Rightarrow$ A is correct



Now, let $L=PR=\sqrt{(PC)^2-r^2}=\sqrt{20-4}=4=PQ;\ \tan\theta=\dfrac{2}{4}=\dfrac{1}{2}$

Also $\cos\theta=\dfrac{PS}{PR}\Rightarrow PS=PR\cos\theta=4.\left(\dfrac{2}{\sqrt{5}}\right)=\dfrac{8}{\sqrt{5}}$

Harmonic Mean between PA and PB = $\dfrac{2\left(\sqrt{20}-2\right)\left(\sqrt{20}+2\right)}{2\sqrt{20}}=\dfrac{16}{2\sqrt{5}}=\dfrac{8}{\sqrt{5}}=PS\Rightarrow$ C is correct

Hence $(PS)(PC)=\left(\dfrac{8}{\sqrt{5}}\right)\left(\sqrt{20}\right)=16=(PR)^2$

$\Rightarrow$ PR is the Geometric Mean of PS and PC $\Rightarrow$ B is correct

Now angle between the two tangents is 2θ, then $\left(\text{As } \tan\theta=\dfrac{1}{2}\right)$

$$2\tan^{-1}\left(\frac{1}{2}\right)=\tan^{-1}\left(\frac{2.\frac{1}{2}}{1-\frac{1}{4}}\right)=\tan^{-1}\left(\frac{4}{3}\right)\Rightarrow \text{D is incorrect}$$

7. (L-1)Q is any point on the circle $x^2+y^2=9$. QN is perpendicular from Q on the x-axis. Locus of the point of trisection of QN is

a) $4x^2+9y^2=36$

b) $9x^2+4y^2=36$

c) $9x^2+y^2=9$

d) $x^2+9y^2=9$

Key : a, d

Sol : Let $Q=(3\cos\theta,3\sin\theta),N=(3\cos\theta,o)$

Point of trisection are $(3\cos\theta, \sin\theta), (3\cos\theta, 2\sin\theta)$

Lotus is $\frac{x^2}{9} + y^2 = 1; \frac{x^2}{9} + \frac{y^2}{4} = 1$

8. (L-1) $\left(1, 2\sqrt{2}\right)$ is a point on circle $x^2 + y^2 = 9$ locate the points on the given circle which are at 2 unit distance from $\left(1, 2\sqrt{2}\right)$

A) $\left(-1, 2\sqrt{2}\right)$ B) $\left(2\sqrt{2}, 1\right)$ C) $\left(\frac{23}{9}, \frac{10\sqrt{2}}{9}\right)$ D) $\left(\frac{23}{9}, \frac{10}{9}\right)$

Key : A, C

Hint : $\frac{x_1 - 1}{\cos\theta} = \frac{y_1 - 2\sqrt{2}}{\sin\theta} = 2$

$x_1^2 + y_1^2 = 9$

On solving

$\cos\theta = \frac{7}{9} \text{ or } -1$

Then get the point $\left(-1, 2\sqrt{2}\right), \left(\frac{23}{9}, \frac{10\sqrt{2}}{9}\right)$

9. A line L_1 intersects *x* and *y* axes at P and Q respectively. Another line L_2 perpendicular to L_1 cuts the x and y axes at R and S respectively. The locus of the point of intersection of the lines PS and QR is a circle passing through

A) origin B) P C) Q D) R

Key. A,B,C

Sol. S is orthocentre of ΔPQR.

10. The equation of a circle C_1 is $x^2 + y^2 = 4$. The locus of point of intersection of perpendicular tangents to the circle is the curve C_2 and the locus of midpoints of the chords of the circle subtending a right angle at the origin is the curve C_3. Then

A) C_2 and C_3 are circles with same centre

B) area enclosed by C_3 is 2π

C) The angle between the tangents to C_3 from any point on C_1 is $\pi/3$.

D) Only one line touches C_1, C_2, C_3

Key. A,B

Sol. C_2 is $x^2 + y^2 = 8$ and C_3 is $x^2 + y^2 = 2$

11. If the line $3x - 4y - \lambda = 0$ touches the circle $x^2 + y^2 - 4x - 8y - 5 = 0$ at (a, b), then $\lambda + a + b$ is equal to

a) 20 b) 22 c) -30 d) -28

Key. A,D

Sol. Tangent $\Rightarrow \lambda = 15, -35$

$\lambda = 15 \Rightarrow (a, b) = (5, 0)$

$\lambda = -35 \Rightarrow (a, b) = (-1, 8)$

12. Tangents are drawn to the circle $x^2 + y^2 = 32$ from a point A lying on the x – axis. The tangents cut the y – axis at points B and C, then the possible coordinates of A such that the area of ΔABC is minimum, are

a) $(8\sqrt{2}, 0)$ b) $(8, 0)$ c) $(-8, 0)$ d) $(-8\sqrt{2}, 0)$

Key. B,C

Sol. Conceptual

13. Consider the circles

$$C_1 \equiv x^2 + y^2 - 2x - 4y - 4 = 0 \text{ and}$$

$$C_2 \equiv x^2 + y^2 + 2x + 4y + 4 = 0$$

And the line $L \equiv x + 2y + 2 = 0$, then

a) L is a direct common tangent of C_1 and C_2

b) L is a transverse common tangent of C_1 and C_2

c) L is the radical axis of C_1 and C_2

d) L is perpendicular to the line joining centres of C_1 and C_2

Key. C,D

Sol. Conceptual

14. Let $C_1 = x^2 + y^2 = r_1^2, C_2 = x^2 + y^2 = r_2^2 \ (r_1 < r_2)$ be two circles. Let A be the fixed point $(r_1, 0)$ on C_1 and B be a variable point on C_2. Let the line BA meet the circle C_2 again at E. Then,

a) The maximum value of BE is $2r_2$

b) The minimum value of BE is $2\sqrt{r_2^2 - r_1^2}$

c) If O is origin, then, the best possible lower bound for $OA^2 + OB^2 + BE^2$ is, $5r_2^2 - 3r_1^2$

d) If O is origin, then, the best possible upper bound for $OA^2 + OB^2 + BE^2$ is, $r_1^2 + 5r_2^2$

Key. A,B,C,D

Sol. $(BE)_{\max} =$ diameter of circle $C_2 = 2r_2$

$\left(BE\right)_{\min} = 2\sqrt{r_2^2 - r_1^2}$

$\left(OA^2 + OB^2 + BE^2\right)_{\min}$ is , $r_1^2 + r_2^2 + 4r_2^2 - 4r_1^2 = 5r_2^2 - 3r_1^2$

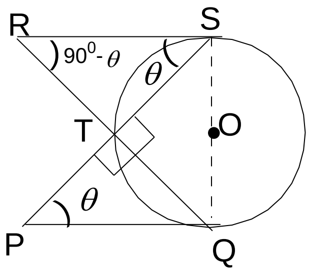
$\left(OA^2 + OB^2 + BE^2\right)_{\max}$ is , $r_1^2 + r_2^2 + 4r_2^2 = r_1^2 + 5r_2^2$

15. If Q, S are two points on the circle $x^2 + y^2 = 4$ such that the tangents QP, SR are parallel. If PS, QR intersect at T then $\left(\frac{QT}{PQ}\right)^2 + \left(\frac{ST}{RS}\right)^2 + PQ.RS \neq$

a) 5 b) 10 c) 16 d) 17

Key. A,B,C

Sol.



$\underline{|QTS} = \underline{|PTQ} = \frac{\pi}{2}$ as QS is diameter

$\frac{QT}{PQ} = \sin\theta, \frac{ST}{RS} = \cos\theta, \frac{4}{PQ} = \tan\theta, \frac{4}{RS} = \cot\theta$

$\therefore \left(\frac{QT}{PQ}\right)^2 + \left(\frac{ST}{RS}\right)^2 + PQ.RS = \sin^2\theta + \cos^2\theta + \left(4\cot\theta\right)\left(4\tan\theta\right)$

16. C_1, C_2 are two circles of radii $a, b(a < b)$ touching both the coordinate axes and have their centres in the first quadrant. Then the true statements among the following are

A) If C_1, C_2 touch each other then $\frac{b}{a} = 3 + 2\sqrt{2}$

B) If C_1, C_2 are orthogonal then $\frac{b}{a} = 2 + \sqrt{3}$

C) If C_1, C_2 intersect in such a way that their common chord has maximum length then $\frac{b}{a} = 3$

D) If C_2 passes through centre of C_1 then $\frac{b}{a} = 2 + \sqrt{2}$

Key. A,B,C,D

Sol. Equation of C_1 is $(x-a)^2 + (y-a)^2 = a^2$

Equation of C_2 is $(x-b)^2 + (y-b)^2 = b^2$ If a variable tangent to the circle

17. In a variable $\Delta ABC,$ the base BC is fixed and $\angle BAC = \alpha$ (a constant)

A) The locus of centroid of ΔABC lies on a circle

B) The locus of incentre of ΔABC lies on a circle

C) The locus of ortho–centre of ΔABC lies on a circle

D) The locus of ex-centre opposite to $'A'$ lies on a circle

Key. A,B,C,D

Sol. $\because \angle A = \alpha$ a constant. If $'I'$ is in-centre of ΔABC

$BIC = 90^0 + \frac{\alpha}{2}$ which is also fixed chord.

Hence $'I'$ lies on a fixed circle of which BC is a fixed chord

$\because \angle A = \alpha$ If $'H'$ is orthocentre $\angle BHC = 180^0 - \alpha$ which is fixed.

Hence, $'H'$ lies on a circle of which BC is fixed chord.

$\therefore \angle A = \alpha, \angle HGK = \alpha$ where H, K are points of trisection of base BC which are fixed.

$\therefore$ The fixed line segment H, K subtends a constant angle α at a variable point G.

Hence, locus of centroid is also lies on circle.

18. C_1, C_2 are two circles of radii $a, b (a < b)$ touching both the coordinate axes and have their centres in the first quadrant. Then the true statements among the following are

A) If C_1, C_2 touch each other then $\frac{b}{a} = 3 + 2\sqrt{2}$ B) If C_1, C_2 are orthogonal then $\frac{b}{a} = 2 + \sqrt{3}$

C) If C_1, C_2 intersect in such a way that their common chord has maximum length then $\frac{b}{a} = 3$

D) If C_2 passes through centre of C_1 then $\frac{b}{a} = 2 + \sqrt{2}$

Key: A,B,C,D

Hint Equation of C_1 is $(x-a)^2 + (y-a)^2 = a^2$

Equation of C_2 is $(x-b)^2 + (y-b)^2 = b^2$

19. A circle is inscribed in a trapezium in which one of the non-parallel sides is perpendicular to the two parallel sides. Then

(A) the diameter of the inscribed circle is the geometric mean of the lengths of the parallel sides

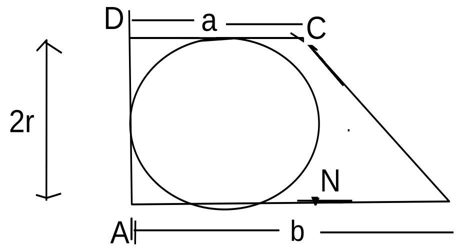
(B) the diameter of the inscribed circle is the harmonic mean of the lengths of the parallel sides

(C) the area of the trapezium is the area of the rectangle having lengths of its sides as the lengths of the parallel sides of the trapezium

(D) the area of the trapezium is half the area of the rectangle having lengths of its sides as the lengths of the parallel sides of the trapezium

KEY : B,C

HINT:. $DC + AB = AD + CB \Rightarrow CB = a + b - 2r$



The triangle CNB gives

$$(2r)^2 + (b-a)^2 = (a+b-2r)^2$$

Which gives $r = \dfrac{ab}{a+b} \Rightarrow 2r = \dfrac{2ab}{a+b}$

Area = $\dfrac{1}{2}(a+b)2r = ab$.

20. If A and B are two points on the circle $x^2 + y^2 - 4x + 6y - 3 = 0$ which are farthest and nearest respectively from the point (7,2) then

(A) $A \equiv \left(2 - 2\sqrt{2}, -3 - 2\sqrt{2}\right)$ (B) $A \equiv \left(2 + 2\sqrt{2}, -3 + 2\sqrt{2}\right)$

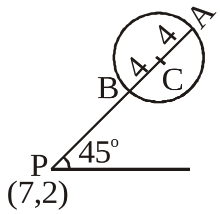
(C) $B \equiv \left(2 + 2\sqrt{2}, -3 + 2\sqrt{2}\right)$ (D) $B \equiv \left(2 - 2\sqrt{2}, -3 - 2\sqrt{2}\right)$

Key ; A, C

Sol : Slope of PC = $\dfrac{-3-2}{2-7} = 1$

if $\tan\theta = 1 Q\ \theta = 45^\circ$

Equation of PA is $\dfrac{x-7}{1/\sqrt{2}} = \dfrac{y-2}{1/\sqrt{2}} = r$



$\therefore \left(7 + \dfrac{r}{\sqrt{2}}, 2 + \dfrac{r}{\sqrt{2}}\right)$ lie on circle,

then, $\left(7 + \dfrac{r}{\sqrt{2}}\right)^2 + \left(2 + \dfrac{r}{\sqrt{2}}\right)^2 - 4\left(7 + \dfrac{r}{\sqrt{2}}\right) + t\left(2 + \dfrac{r}{\sqrt{2}}\right) - 3 = 0$

$\Rightarrow r^2 + 10\sqrt{2}r + 34 = 0$

$\therefore \quad r = -5\sqrt{2} \pm 4$

$\therefore$ Point $\left(7 + \dfrac{-5\sqrt{2} \pm 4}{\sqrt{2}}, 2 + \dfrac{-5\sqrt{2} \pm 4}{\sqrt{2}}\right)$

$\Rightarrow \left(2 \pm 2\sqrt{2}, -3 \pm 2\sqrt{2}\right)$

taking + ve sign for A, - ve sign for B.

21. Two circles $x^2+y^2+px+py-7=0$ and $x^2+y^2-10x+2py+1=0$ will cut orthogonally if the value of p is

A) -2 B) -3 C) 2 D) 3

Key. C,D

Sol. The given circles will cut orthogonally if $2\left(\frac{p}{2}\right)\left(\frac{-10}{2}\right)+2\left(\frac{p}{2}\right)\left(\frac{2p}{2}\right)=-7+1$

or $p^2-5p+6=0$ of if $p=2$ or 3.

22. If two circles $x^2+y^2-6x-12y+1=0$ and $x^2+y^2-4x-2y-11=0$ cut a third circle orthogonally then the radical axis of the two circles passes through

A) $(1,1)$ B) $(0,6)$

C) centre of the third circle

D) mid-point of the line joining the centres of the given circles.

Key. A,C

Sol. Radical axis of the given circles is $x+5y-6=0$ which passes through (1,1)

Let the given circles intersect the circle

$x^2+y^2+2gx+2fy+c=0$ orthogonally then $2g(-3)+2f(-6)=c+1$

$2g(-2)+2f(-1)=c-11$

$\Rightarrow 2g(-1)+2f(-5)=12 \Rightarrow -g-5f-6=0$

$\Rightarrow$ the radical axis passes through the centre $(-g,-f)$ of the third circle.

Verify it does not pass through the mid-point of the line joining the centres.

23. The equation of a circle C_1 is $x^2+y^2=4$. The locus of the intersection of orthogonal tangents to the circle is the curve C_2 and the locus of the intersection of perpendicular tangents to the curve C_2 is the curve C_3 then

A) C_3 is a circle B) The area enclosed by the curve C_3 is 8π

C) C_2 and C_3 are circles with the same centre D) None of the above

Key. A,C

Sol. $\because C_2$ is the director circle of C_1

$\therefore$ Equation of C_2 is

$x^2+y^2=2(2)^2=8$

Again C_3 is the director circle of C_2, Hence the equation of C_3 is

$x^2+y^2=2(8)=16$

24. Consider the circles $C_1 \equiv x^2+y^2-2x-4y-4=0$ and $C_2 \equiv x^2+y^2+2x+4y+4=0$ and the line $L \equiv x+2y+2=0$, then

A) L is the radical axis of C_1 and C_2 B) L is the common tangent of C_1 and C_2

C) L is the common chord of C_1 and C_2 D) L is perpendicular to the joining centers of C_1 and C_2

Key. A,C,D

Sol. $C_1 \equiv x^2+y^2-2x-4y-4=0$ ....(i)

And $C_2 = x^2+y^2+2x+4y+4=0$ ....(ii)

$\therefore$ Radical axis is $C_1 - C_2 = 0$

$\Rightarrow -4x-8y-8=0$

Or $x+2y+2=0$ which is $L=0$

(a) option is correct

centre and radius of $C_1 = 0$ are $(1,2)$ and 3.

$\because$ length of $\perp$ from $(1,2)$ on $L=0$

is $\dfrac{|1+4+2|}{\sqrt{1+4}} = \dfrac{7}{\sqrt{5}} \neq$ radius

$\therefore$ (b) option is wrong

L is also the common chord of C_1 and C_2

$\therefore$ (c) option is correct.

$\because$ centres of $C_1 = 0$ and $C_2 = 0$ are $(1,2)$ and $(-1,-2)$

$\therefore$ slope of line joining centres of circles $C_1 = 0$ & $C_2 = 0$ is

$\dfrac{-2-2}{-1-1} = \dfrac{4}{2} = 2 = m_1$ (say)

And slope of $L=0$ is $-\dfrac{1}{2} = m_2$ (say)

$\therefore m_1 m_2 = -1$

Hence, L is perpendicular to the line joining centres of C_1 and C_2

$\therefore$ (d) option is correct

25. The equation of a circle is $S_1 \equiv x^2+y^2=1$. The orthogonal tangents to S_1 meet at another circle S_2 and the orthogonal tangents to S_2 meet at the third circle S_3. Then

A) Radius of S_2 and S_3 are in the ratio $1:\sqrt{2}$

B) Radius of S_2 and S_3 are in the ratio $1:2$

C) The circles S_1, S_2 and S_3 are concentric

D) None of the above

Key. A,C

Sol. Orthogonal tangents to a circle meet at the director circle

$\therefore S_2 \equiv x^2+y^2=2.1 \Rightarrow S_2 \equiv x^2+y^2=2$

Also, $S_3 \equiv x^2+y^2=4$

Ratio of radius of S_2 and $S_3 = \sqrt{2} : 2 = 1 : \sqrt{2}$

Also, the three circles are concentric

26. Let k_1, k_2 be two integers such that $(n - a)! = (n - b)!$, $2a + b + 1 = k_1 n + k_2$ $\forall$ n where $a < b \le n$ and $a, b, n \in N$. Let P and Q be two points on the curve $y = \log_{1/2}(x + k_2/2) + \log_2\left(\sqrt{4x^2 + 4k_2x + k_1 + k_2}\right)$. Point P also lies on the circle $x^2 + y^2 = k_1^3 - 2k_2$, however Q lies inside the circle such that its abscissa is an integer then

(A)The values of k_1 and k_2 are respectively 2 and –1

(B) maximum value of $\overrightarrow{OP}.\overrightarrow{OQ}$ is 7

(C) minimum value of $\left|\overrightarrow{PQ}\right|$ is 1

(D) minimum value of $\overrightarrow{OP}.\overrightarrow{OQ}$ is 3

Key. A,B,C

Sol. Clearly $a = n-1$, $b = n \Rightarrow 2a + b + 1 = 2n-1 = k_1n + k_2$

$\Rightarrow k_1 = 2,\ k_2 = -1$

So, $y = \log_{1/2}(x - 1/2) + \log_2 \sqrt{4x^2 - 4x + 1} \Rightarrow y = 1$

the equation of circle is $x^2 + y^2 = 10$

So, $P = (3, 1)$ whereas $Q \equiv (1, 1)$ or $(2, 1)$

$\Rightarrow \overrightarrow{OP}.\overrightarrow{OQ} = 3 + 1 = 4$ or $6 + 1 = 7$, $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = -\hat{i}$ or $-2\hat{i}$

27. If a chord of the circle $x^2 + y^2 - 4x - 2y - c = 0$ is trisected at the points $\left(\frac{1}{3}, \frac{1}{3}\right)$ and $\left(\frac{8}{3}, \frac{8}{3}\right)$ then

A) length of the chord is $7\sqrt{2}$

B) $c = 20$

C) radius of the circle is 25

D) Midpoint of the chord is (1, 1)

Key. A,B,C

Sol. Conceptual

28. A line L_1 intersects *x* and *y* axes at P and Q respectively. Another line L_2 perpendicular to L_1 cuts the x and y axes at R and S respectively. The locus of the point of intersection of the lines PS and QR is a circle passing through

A) origin B) P C) Q D) R

Key. A,B,C

Sol. S is orthocentre of ΔPQR.

29. If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ cuts each of the circle $x^2 + y^2 - 4 = 0, x^2 + y^2 - 6x - 8y + 10 = 0$ and $x^2 + y^2 + 2x - 4y - 2 = 0$ at the extremities of a diameter then

A) $c = -4$ B) $g + f = c - 1$ C) $g^2 + f^2 - c = 17$ D) $gf = 6$

Key. A,B,C,D

Sol. Common chord is the diameter of the circles cut by $x^2 + y^2 + 2gx + 2fy + c = 0$

30. The equation of a circle C_1 is $x^2 + y^2 = 4$. The locus of point of intersection of perpendicular tangents to the circle is the curve C_2 and the locus of midpoints of the chords of the circle subtending a right angle at the origin is the curve C_3. Then

A) C_2 and C_3 are circles with same centre

B) area enclosed by C_3 is 2π

C) The angle between the tangents to C_3 from any point on C_1 is $\pi/3$.

D) Only one line touches C_1, C_2, C_3

Key. A,B

Sol. C_2 is $x^2 + y^2 = 8$ and C_3 is $x^2 + y^2 = 2$

31. A circle touches the line $x + y - 2 = 0$ at $(1, 1)$ and cuts the circle $x^2 + y^2 + 4x + 5y - 6 = 0$ at P and Q. Then

a) PQ can never be parallel to the given line $x + y - 2 = 0$

b) PQ can never be perpendicular to the given line $x + y - 2 = 0$

c) PQ always passes through (6, -4)

d) PQ always passes through (-6, 4)

Key. A,B,C

Sol. $(x-1)^2 + (y-1)^2 + \lambda(x + y - 2) = 0$

$$\Rightarrow x^2 + y^2 + (\lambda - 2)x + (\lambda - 2)y + 2 - 2\lambda = 0 \longrightarrow (1)$$

$$x^2 + y^2 + 4x + 5y - 6 = 0 \longrightarrow (2)$$

Eq. of common chord PQ is $s - s' = 0$.

$$\Rightarrow (\lambda - 6)x + (\lambda - 7)y + 8 - 2\lambda = 0 \longrightarrow (3).$$

(a) $PQ \parallel x + y - 2 = 0$

$$\Rightarrow \frac{6-\lambda}{\lambda - 7} = -1 \Rightarrow 6 = 7$$ which is impossible

(b) $PQ \perp x + y - 2 = 0$

$$\Rightarrow \frac{6-\lambda}{\lambda - 7} = 1 \Rightarrow \lambda = \frac{13}{2}$$ which is possible

But when $\lambda = \frac{13}{2}$, we can see that the circles (1) and (2) are not intersecting each other and their radical axis is perpendicular to the given line $x + y - 2 = 0$.

(c) and (d)

Eq. (3) can be written as

$$-6x - 7y + 8 + \lambda(x + y - 2) = 0$$

which is in the form $L_1 + \lambda L_2 = 0$

Solving L_1 and L_2, we get $(6, -4)$.

32. If $al^2 - bm^2 + 2dl + 1 = 0$ where a, b, d are fixed real numbers such that $a + b = d^2$ then the line $lx + my + 1 = 0$ touches a fixed circle. Then the fixed circle

a) which cuts the x-axis orthogonally

b) with radius equal to b

c) On which the length of the tangent from the origin is $\sqrt{d^2 - b}$

d) with centre (d, 0)

Key. A,C,D

Sol. $al^2 + 2dl + 1 = bm^2$

$$bl^2 + al^2 + 2dl + 1 = bm^2 + bl^2$$

$$d^2l^2 + 2dl + 1 = b\left(l^2 + m^2\right)$$

$$\Rightarrow \left|\frac{dl+1}{\sqrt{l^2+m^2}}\right| = \sqrt{b}$$

$\Rightarrow$ fixed circle with centre (d,0)

radius $\sqrt{b}$

$$\therefore \left(x-d\right)^2 + y^2 = b$$

33. Point M moved along the circle $\left(x-4\right)^2 + \left(y-8\right)^2 = 20$. Then it broke away from it and moving along a tangent to the circle cuts the x-axis at the point (-2, 0) the co-ordinate of the point on the circle at which the moving point broke away can be

a) $\left(\frac{-3}{5}, \frac{46}{5}\right)$ b) $\left(\frac{-2}{5}, \frac{44}{5}\right)$ c) (6, 4) d) (3, 5)

Key. B,C

Sol. $x^2 + y^2 - 8x - 16y + 60 = 0$ -------- (1)

equation of chord of contact from (-2,0) is $3x + 4y - 34 = 0$ ------ (2)

intersection (1) & (2) is

$$x^2 + \left(\frac{34-3x}{4}\right)^2 - 8x - 16\left(\frac{34-3x}{4}\right) + 60 = 0$$

$\Rightarrow 5x^2 - 28x - 12 = 0 \qquad \Rightarrow x = 6, -2/5$

$\therefore$ (6,4) & $\left(\frac{-2}{5}, \frac{44}{5}\right)$

Circles

Assertion Reasoning Type

1. STATEMENT 1: If the angle between the tangents drawn from a variable point to the circles $x^2 + y^2 = 1$ is equal to the angle between the tangents drawn form same point to the circle $x^2+y^2+10x-8y+37=0$. Then the locus of the point is $3x^2+3y^2-10x+8y+41=0$

STATEMENT 2: If the angle between tangents drawn from point

$P(x_1, y_1)$to the circle $x^2+y^2+2gx+2fy+c=0$ is 2θ then

$$\tan\theta=\sqrt{\frac{g^2+f^2-c}{x_1^2+y_1^2+2gx_1+2fy_1+c}}$$

Key. D

Sol. Conceptual

2. STATEMENT-1 : Suppose ABCD is a cyclic quadrilateral inscribed in a circle of radius one unit with then ABCD is a square

STATEMENT-2 : A cyclic quadrilateral is a square if its diagonals are the diameters of the circle

Key: A

Hint: If A and B are the same side of O, C and D are on the opposite sides of O then A, B, C, D can not be concyclic. Hence (d) is the correct answer.

3. STATEMENT-1

Number of circles passing through the points $(1,2),\left(3,\frac{1}{2}\right),\left(\frac{1}{3},\frac{5}{2}\right)$ is one.

STATEMENT–2

Through three non-collinear points in a plane only one circle can be drawn.

Key: D

Hint: The points $(1,2),\left(3,\frac{1}{2}\right),\left(\frac{1}{3},\frac{5}{2}\right)$ are collinear and no circle can be drawn from 3 collinear points

Also through 3 non-collinear points a unique circle can be drawn.

4. STATEMENT- 1 : If line $3x - 4y + 1 = 0$ touches the circle $x^2 + y^2 - 2x + 2y + \lambda = 0$ at P, then P is $\left(\frac{1}{25},\frac{7}{25}\right)$

STATEMENT 2 : Line joining center to point of contact of a circle is perpendicular to the tangent.

Key: A

Hint: Conceptual

5. Statement-1: Equation of a circle through the origin and belonging to the coaxial system, of which the limiting points are (1,1) and (3, 3) is $2x^2 + 2y^2 – 3x – 3y = 0$.

Statement-2: Equation of a circle passing through the points (1, 1) and (3, 3) is $x^2 + y^2 – 2x – 6y + 6 = 0$.

Key: b

Hint: Two members of the system of circles in statement-1 are the circles with centres at the limiting points and radius equal to zero i.e.

$(x-1)^2 + (y-1)^2 = 0$ and $(x-3)^2 + (y-3)^2 = 0$

or $x^2 + y^2 - 2x - 2y + 2 = 0$ and $x^2 + y^2 - 6x - 6y + 18 = 0$

Equation of the coaxial system is

$x^2 + y^2 - 6x - 6y + 18 + \lambda(x^2 + y^2 - 2x - 2y + 2) = 0$

which passes through the origin if $\lambda = -9$ and the equation of the required circle is

$2x^2 + 2y^2 - 3x - 3y = 0$. So that statement-1 is true.

Statement-2 is also true as the circle in it passes through (1, 1) and (3, 3) but does not lead to statement-1.

6. Statement – 1 : The minimum height of tangent drawn from point P on $3x + 4y - 20 = 0$ to the circle $x^2 + y^2 = 1$ is $\sqrt{15}$.

Statement – 2 : If tangent from point P touches the circle at T and secant passing through P intersects it at Q and R, then $PT^2 = PQ.PR$.

Key ; b

Sol : The minimum length of the tangent will be from the foot of the perpendicular from centre to the line

7. (L-1)Statement-1 : Limiting points of a family of co-axial system of circles are (1, 1) and (3, 3). The member of this family passing through the origin is $2x^2 + 2y^2 - 3x - 3y = 0$.

Statement-2 : Limiting points of a family of coaxial circles are the centres of the circle with zero radius.

Key : a

Sol : Equality member $x^2 + y^2 - 24 - 2y + 2, x^2 + y^2 - 64 - 6y + 18$ RA $x + y - 4 = 0$

$\therefore \tau = 1/2$ Required 0^x is $2x^2 - 2y^2 - 3x - 3y = 0$ and equation is correct.

8. (L-1I)Statement-1 : A circle can be inscribed in a quadrilateral whose sides are $3x - 4y = 0$, $3x - 4y = 5$, $3x + 4y = 0$ and $3x + 4y = 7$.

Statement-2 : A circle can be inscribed in a parallelogram if and only if it is a rhombus.

A) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1.

B) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1.

C) Statement-1 is true, Statement-2 is false.

D) Statement-1 is false, Statement-2 is true.

Key : c

Sol : Conceptional I is true II is false

9. (L-I1)Statement-1 : The circle of smallest radius passing through two given points A and B must be of radius $\frac{1}{2}AB$.

Statement-2 : A straight line is shortest distance between two points.

Key : D

10. (L-1I)Statement-1 : If three parallel chords of a circle have lengths 2, 3, 4 and subtend angle x, y, x + y at the centre (where x + y < 180°), then $\cos x = \frac{17}{32}$

Statement-2 : In a circle of radius r, if chord of length l subtends an angle θ at the centre, then $\cos\theta = 1 - \frac{l^2}{2r^2}$. Also $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta\ \forall\alpha,\beta\in R$

A) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1.

B) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1.

C) Statement-1 is true, Statement-2 is false.

D) Statement-1 is false, Statement-2 is true.

Key : a

Sol : $\sin\frac{\theta}{2} = \frac{l}{2r} \Rightarrow \cos\theta = 1 - 2\frac{l^2}{4r^2} = 1 - \frac{l^2}{2r^2}$

$\therefore \cos x = 1 - \frac{4}{2r^2}, \cos y = 1 - \frac{9}{2r^2}, \cos(x+y) = 1 - \frac{16}{2r^2}$

Solving $r^2 = \frac{64}{15} \Rightarrow \cos x = \frac{17}{32} \Rightarrow$ Statement-1 is true

Statement-2 is also true and is the correct explanation of 1

11. STATEMENT – I : If the circles $x^2+y^2+2gx+2fy=0$ and $x^2+y^2+2g^1x+2f^1y=0$ touch each other then $f^1g = fg^1$.

STATEMENT – II: Two circles touch each other if the line joining their centres is parallel to all possible common tangents.

Key. C

Sol. When two circles touch centres and point of contact are collinear.

12. STATEMENT – I: If C_1 and C_2 be two circles with C_2 lying inside C_1. A circle C lying inside C_1 touches C_1 internally and C_2 externally. The locus of centre of C is an ellipse.

STATEMENT – II: Let A, B be fixed points. The locus of a point P which moves such that $PA+PB=K$ ($K>AB$) is an ellipse.

Key. A

Sol. Let r, r_1, r_2 be radii and P, A, B, be the centres of circle C, C_1, C_2

$PA = r_1 - r, PB = r_2 + r$

$\Rightarrow PA + PB = r_1 + r_2 = constant$

13. STATEMENT- I: Let C be any circle with centre $(0, \sqrt{2})$. There can be atmost 2 points with rational coordinates on C.

STATEMENT – II: On any circle there will be even number points with rational coordinates.

Key. C

Sol. No three points on a circle can be collinear.

14. STATEMENT – I: Tangents drawn from the point (17, 7) to the circle $x^2 + y^2 = 169$ are perpendicular.

STATEMENT – II: The locus of point of intersection of perpendicular tangents to the circle $x^2 + y^2 = 169$ is $x^2 + y^2 = 338$.

Key. A

Sol. Conceptual

15. Consider the lines L_1: 2x + 3y + p – 3 = 0, L_2: 2x + 3y + p + 3 = 0, where 'p' is a real number, and C : $x^2 + y^2 + 6x - 10y + 30 = 0$

Statement I: If the line L_1 is a chord of the circle C, then the line L_2 is not always a diameter of circle.

Statement II: If the line L_1 is a diameter of the circle C, then the line L_2 is not a chord of circle C.

Key. D

Sol. Conceptual

16. Consider the circle, $S : x^2 + y^2 - 10x = 0$ and a point P(7, 11)

Statement – 1: Point P lies outside the circle S.

Statement – 2: If power of a point P with respect to a given circle is positive then the point P lies outside the circle.

Key: As

Hint:. Conceptual question

17. Let $C_1 = x^2 + y^2 = r_1^2, C_2 = x^2 + y^2 = r_2^2 \left(r_1 < r_2\right)$ be two circles. Let A be the fixed point $\left(r_1, 0\right)$ on C_1 and B be a variable point on C_2. Let the line BA meet the circle C_2 again at E. Then,

a) The maximum value of BE is $2r_2$

b) The minimum value of BE is $2\sqrt{r_2^2 - r_1^2}$

c) If O is origin, then, the best possible lower bound for $OA^2 + OB^2 + BE^2$ is, $5r_2^2 - 3r_1^2$

d) If O is origin, then, the best possible upper bound for $OA^2 + OB^2 + BE^2$ is, $r_1^2 + 5r_2^2$

Key. A,B,C,D

Sol. $\left(BE\right)_{\max} =$ diameter of circle $C_2 = 2r_2$

$\left(BE\right)_{\min} = 2\sqrt{r_2^2 - r_1^2}$

$\left(OA^2+OB^2+BE^2\right)_{\min}$ is , $r_1^2+r_2^2+4r_2^2-4r_1^2=5r_2^2-3r_1^2$

$\left(OA^2+OB^2+BE^2\right)_{\max}$ is , $r_1^2+r_2^2+4r_2^2=r_1^2+5r_2^2$

18. STATEMENT–1

The minimum distance of $4x^2 + y^2 + 4x - 4y + 5 = 0$ from the lines $-4x + 3y = 3$ is 1

because

STATEMENT–2

$4x^2 + y^2 + 4x - 4y + 5 = 0$ represents a point.

Key. A

Sol. The given curve represents the point $\left(-\frac{1}{2},2\right)$

$\therefore$ minimum distance = 1.

19. Let $(1+ax)^n = 1+8x+24x^2+.....$ and there exists a line through the point (a, n) in the cartesian plane

Statement - 1: If the line through (a, n) cuts the circle $x^2+y^2=4$ in A and B then PA.PB = 16

Statement - 2: The point (a, n) lies outside the circle

Key. B

Sol. From (i) $na = 8$

$$\frac{n(n-1)}{2}a^2 = 24$$

$$\therefore a^2 = \frac{64}{n^2} \Rightarrow a = 2 \text{ and } n = 4$$

Now from 92)

$$PA.PB = \left(\sqrt{S_1}\right)^2 = 2^2+4^2-4 = 16$$

20. STATEMENT – I : If the circles $x^2+y^2+2gx+2fy=0$ and $x^2+y^2+2g^1x+2f^1y=0$ touch each other then $f^1g = fg^1$.

STATEMENT – II: Two circles touch each other if the line joining their centres is parallel to all possible common tangents.

Key. C

Sol. When two circles touch centres and point of contact are collinear.

21. STATEMENT – I: If C_1 and C_2 be two circles with C_2 lying inside C_1. A circle C lying inside C_1 touches C_1 internally and C_2 externally. The locus of centre of C is an ellipse.

STATEMENT – II: Let A, B be fixed points. The locus of a point P which moves such that $PA+PB=K$ ($K>AB$) is an ellipse.

Key. A

Sol. Let r, r_1, r_2 be radii and P, A, B, be the centres of circle C, C_1, C_2

$PA = r_1 - r, PB = r_2 + r$

$\Rightarrow PA + PB = r_1 + r_2 = constant$

22. STATEMENT- I: Let C be any circle with centre $(0, \sqrt{2})$. There can be atmost 2 points with rational coordinates on C.

STATEMENT – II: On any circle there will be even number points with rational coordinates.

Key. C

Sol. No three points on a circle can be collinear.

23. STATEMENT – I: Tangents drawn from the point (17, 7) to the circle $x^2 + y^2 = 169$ are perpendicular.

STATEMENT – II: The locus of point of intersection of perpendicular tangents to the circle $x^2 + y^2 = 169$ is $x^2 + y^2 = 338$.

Key. A

Sol. Conceptual

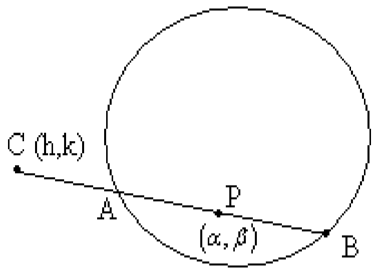
24. **STATEMENT-1 :** Through a fixed point (h, k) secants are drawn to the circle $x^2 + y^2 = r^2$ then the locus of the mid points of the secants intercepted by the circle $x^2 + y^2 = hx + ky$

STATEMENT-2 : The equation of the chord whose middle point at (x_1, y_1) is

$xx_1 + yy_1 = x_1^2 + y_1^2$

Key. A

Sol. equation of AB is $x\alpha + y\beta = \alpha^2 + \beta^2$ it, passes through (h,k)



$\therefore h\alpha + k\beta = \alpha^2 + \beta^2$

$\therefore$ locus of (α, β) is $x^2 + y^2 = hx + ky$

Circles

Comprehension Type

Paragraph – 1

Tangents PA and PB are drawn to the circle $(x-4)^2+(y-5)^2=4$ from the point P on the curve $y=\sin x,$ where A and B lie on the circle. Consider the function $y=f(x)$ represented by the locus of the center of the circumcircle of triangle PAB, then answer the following questions.

1. Range of y = f(x) is

   (A) [–2, 1] (B) [–1, 4] (C) [0, 2] (D)[2, 3]

Key. D

2. Fundamental period of y = f(x) is

   (A) 2π (B) 3π (C) π (D) Not defined

Key. C

3. Which of the following is true?

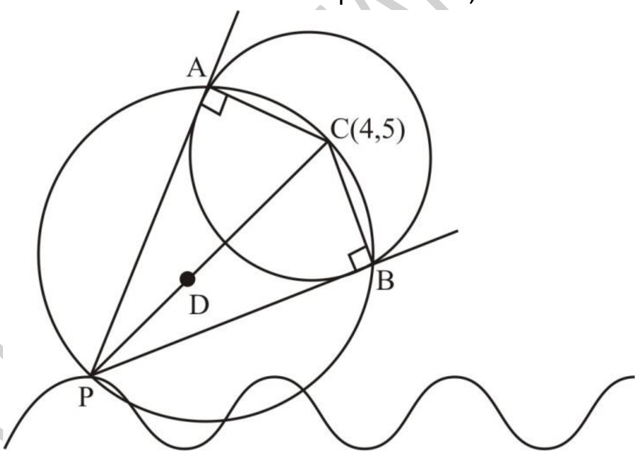
   (A) f(x) = 4 has real roots (B) f(x) = 1 has real roots

   (C) Range of $y=f^{-1}(x)$ is $\left[-\frac{\pi}{4}+2, \frac{\pi}{4}+2\right]$ (D) None of these

Key. C

Sol. 1 to 3

Centre of the given circle is C(4, 5). Points P, A, C, B are concyclic such that PC is diameter of the circle. Hence, centre D of the circumcircle of ΔABC is midpoint of PC, then we have



$$h=\frac{t+4}{2} \text{ and } k=\frac{\sin t+5}{2}$$

Eliminating t, we have

$$k=\frac{\sin(2h-4)+5}{2}$$

Or $\quad y=\frac{\sin(2x-4)+5}{2}$

$$\Rightarrow \quad f^{-1}(x)=\frac{\sin^{-1}(2x-5)+4}{2}$$

Thus range of $y=\frac{\sin(2x-4)+5}{2}$ is [2, 3] and period is π.

Also $f(x)=4 \Rightarrow \sin(2x-4)=3$ which has no real solutions.

But range of $y = \dfrac{\sin^{-1}(2x-5)+4}{2}$ is $\left[-\dfrac{\pi}{4}+2, \dfrac{\pi}{4}+2\right]$

Paragraph – 2

Given equation of two intersecting circle's $S_1 = 0$ & $S_2 = 0$

Equation of family of circles passing through the intersection point's of $S_1 = 0$ & $S_2 = 0$ is $S_1 + \lambda S_2 = 0$, (where $\lambda \neq -1$)

Equation of common chord is $S_1 - S_2 = 0$

Equation of chord of contact for circle $x^2 + y^2 + 2gx + 2fy + c = 0$ with respect to external point (x_1, y_1) is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

4. The equation of the circle described on the common chord of the circles $x^2 + y^2 + 2x = 0$ and $x^2 + y^2 + 2y = 0$ as diameter is

(A) $x^2 + y^2 + x - y = 0$
(B) $x^2 + y^2 - x - y = 0$
(C) $x^2 + y^2 - x + y = 0$
(D) $x^2 + y^2 + x + y = 0$

Key. D

5. Let P be any moving point on the circle $x^2 + y^2 - 2x - 1 = 0$. AB be the chord of contact of this point with respect to the circle $x^2 + y^2 - 2x = 0$. The locus of the circumcentre of the triangle PAB (C being centre of the circles) is

(A) $2x^2 + 2y^2 - 4x + 1 = 0$
(B) $x^2 + y^2 - 4x + 2 = 0$
(C) $x^2 + y^2 - 4x + 1 = 0$
(D) $2x^2 + 2y^2 - 4x + 3 = 0$

Key. A

6. The common chord of the circle $x^2 + y^2 + 6x + 8y - 7 = 0$ and a circle passing through the origin, and touching the line y = x, always passes through the point

(A) $\left(-\frac{1}{2}, \frac{1}{2}\right)$
(B) (1, 1)
(C) $\left(\frac{1}{2}, \frac{1}{2}\right)$
(D) none of these

Key. C

Sol. 31. Equation of common chord is $2x - 2y = 0$

Equation of family of circle is

$$x^2 + y^2 + 2x + \lambda(2x - 2y) = 0$$

Centre of circle is $(-\lambda - 1, +\lambda)$

Centre lies on y = x

$$\lambda = -\lambda - 1$$

$$2\lambda = -1$$

$$\lambda = -\frac{1}{2}$$

Equation is $x^2 + y^2 + x + y = 0$

32. Let P be $(1+\sqrt{2}\cos\theta, \sqrt{2}\sin\theta)$ and c is (1, 0).

Circumcentre of triangle ABC is mid point of PC.

$$2h = 1 + \sqrt{2}\cos\theta + 1$$

$2k = \sqrt{2}\sin\theta$

$(2(h-1)^2 + (2k)^2 = 2$

Locus of P(h, k) is $2x^2 + 2y^2 - 4x + 1 = 0$

33. Let the second circle $x^2 + y^2 + 2gx + 2fy = 0$

Hence, $x^2 + y^2 + 2gx + 2fy = 0$ lies equal roots f + g= 0

Equation of common chord is

$2(g-3)x + 2(-g-4)y + 7 = 0$

$(-6x - 8y + 7) + g(2x - 2y) = 0$

Passes through the intersection point of

$-6x - 8y + 7 = 0$ and $2x - 2y = 0$

$\Rightarrow \quad \left(\frac{1}{2}, \frac{1}{2}\right)$

$\cos 60° = \frac{\sqrt{(h+1)^2 + (k-1)^2}}{2}$

Paragraph – 3

Given P,Q are two points an the curve $y = \log_{\frac{1}{2}}(x - 0.5) + \log_2\sqrt{4x^2 - 4x + 1}$ P is also lies on the circle $x^2 + y^2 = 10$ .If Q lies inside the given circle such that its abscissae is integer then

7. $\overrightarrow{OP}.\overrightarrow{OQ}$ equal to

(a) 4 (b) 8 (c) 2 (d) 10

8. $Max\left\{\left|\overrightarrow{PQ}\right|\right\}$ Equal to

(a) 1 (b) 2 (c) 4 (d) 5

9. $Min\left\{\left|\overrightarrow{PQ}\right|\right\} =$

(a) 1 (b) 3 (c) 4 (d) 6

Sol. 7-9. (A) (B) (A) $y = \log_{\frac{1}{2}}(x - 0.5) + \log_2\sqrt{4x^2 - 4x + 1}$

$= -\log(2x - 1) + \log_2 2 + \log(2x - 1)$

$\Rightarrow$ y = 1 p(x , y) lies on the circle $x^2 + y^2 = 10$

$\Rightarrow$ x = ± 3 , neglecting x = -3

Q lies inside the circle abscissa is a integer

Q(1 , 1) , Q(2 , 1)

Paragraph – 4

The line x+2y+ a = 0 intersects the circle $x^2 + y^2 - 4 = 0$ at two distance points A and B. Another line 12x-6y-41=0 intersect the circle $x^2 + y^2 - 4x - 2y + 1 = 0$ at two distinct points C and D.

10. The value of a so that the line x+2y+a = 0 intersect the circle $x^2 + y^2 - 4 = 0$ at two distance points A and B is

a) $-2\sqrt{5} < a < 2\sqrt{5}$ b) $0 < a < 2\sqrt{5}$

c) $-\sqrt{5} < a < \sqrt{5}$ d) $0 < a < 2\sqrt{5}$

11. The value of 'a' for which the four points A,B,C and D are concyclic is
a) 1 b) 3 c) 4 d) 2

12. The equation of circle passing through the points A,B,C and D is
a) $5x^2+5y^2-8x-16y-36=0$ b) $5x^2+5y^2+8x-16y-36=0$
c) $5x^2+5y^2+8x+16y-36=0$ d) $5x^2+5y^2-8x-16y+36=0$

Sol. 10. (A) Lines x+2y+a=0 will intersect the circle $x^2+y^2=4$ is

$$\left|\frac{0+0+a}{\sqrt{1+4}}\right|<2 \Rightarrow -2\sqrt{5}<a<2\sqrt{5}$$

11. (D) Let lines x+2y+a=0 and 12x+6y-41=0 intersect at p, then PA.PB= PT^2 and $PC.PD=PT^{1^2}$ where T and T^1 are the points on the respective circles. A,B,C and D are concyclics.

$$PA.PB=PC.PD \Rightarrow PT^2=PT^{1^2}$$

Hence point P will lie on the radical axis of both the circles. Now equation of radical axis is 4x+2y-5=0

Since, radical axis and the lines x+2y+a=0 and 12x-6y-41=0 are concurrent at P, whe have

$$\begin{vmatrix} 4 & 2 & -5 \\ 1 & 2 & a \\ 12 & -6 & -41 \end{vmatrix}=0 \Rightarrow a=2$$

12. (A) Equation of the circle passing through point of intersection of circle $x^2+y^2-4=0$ and $x+2y+2=0$ is $x^2+y^2-4+\mathrm{D}(x+2y+2)=0 \to 1$

Common chord of circle represented by equation 1 and circle, is

$x^2+y^2-4x-2y+1=0$ is $(\lambda+4)x+2(\lambda+1)y+2\lambda-5=0 \to 2$

Since, equation 2 and $12x-6y-41=0$ and represents the same line, we get

$$5x^2+5y^2-8x-16y-36=0$$

Paragraph – 5

Consider the circles $S_1: x^2+y^2-6x+5=0,\ S_2: x^2+y^2-12x+35=0$ and a variable circle $S: x^2+y^2+2gx+2fy+c=0$

13. Number of common tangents to S_1 and S_2 is
a) 1 b) 2 c) 3 d) 4

14. Length of a transverse common tangent to S_1 and S_2 is
a) $11\sqrt{2}$ b) $\sqrt{35}$ c) $2\sqrt{11}$ d) 6

15. If the variable circle S = 0 with centre 'C' moves in such a way that it is always touching externally the circles $S_1 = 0$ and $S_2 = 0$ then the locus of the centre 'C' of the variable circle is.
a) a hyperbola b) an ellipse c) a parabola d) a circle

Sol. 13. (D) $C_1=(0,3)\ \ r_1=2$

$C_2=(6,0)\ \ r_2=1$

$C_1C_2>r_1+r_2$

14. (D) $\sqrt{d^2-(r_1+r_2)^2}=6$

15. (A) $\sqrt{g^2+(t+3)^2}=2+\sqrt{g^2+t^2-c}$

$\sqrt{(g+6)^2+t^2}=1+\sqrt{g^2+t^2-c}$

Paragraph – 6

ABC is a right angled triangle with $\angle B=90^0$ and $P=\frac{AB^2}{AC^2}+\frac{AB^3}{AC^3}+....\infty$. Let $x^2+y^2=1$ and $y^2-8x+16=0$ be the equations of a circle and parabola respectively.

16. If P = shortest distance between circle and parabola, then $\frac{AB}{AC}$ is equal to

(A) $\left(\frac{\sqrt{5}-1}{2}\right)$ (B) $\frac{-\sqrt{5}-1}{2}$

(C) $\frac{\sqrt{5}+2}{2}$ (D) Both (A) & (B) are correct

17. If exp (P ln 2) = distance between the center of the circle and the focus of parabola. Then $\frac{AB}{AC}$ can be equal to

(A) $\frac{-1+\sqrt{17}}{2}$ (B) $\frac{-1-\sqrt{17}}{2}$

(C) $\frac{2-\sqrt{17}}{2}$ (D) $\sqrt{3}-1$

18. D, E are the two points on the circle and the parabola such that, the distance between them is minimum. Let T is mid point of DE. Then area of ΔTFG (where $F\,\&\,G$ are the points of intersection of circle and Y-axis)

(A) $\frac{3}{2}$ sq.unit (B) 2 sq.unit

(C) $\frac{9}{2}$ sq.unit (D) 3 sq.units

KEY : A-D-A

HINT

16. p = 1

sin θ = $\frac{\sqrt{5}-1}{2}$

$\therefore \frac{AB}{AC}=\frac{\sqrt{5}-1}{2}$

Paragraph – 7

An equation of the family of circles passing through a given pair of points (x_1, y_1) and (x_2, y_2) can be taken as $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)+k\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}=0$, k being a real parameter. If a member of this family satisfies some other condition then that enables us to determine k and hence the member.

19. The number of values of $\lambda \in R$ for which there exists exactly one circle passing through the points $(2,-3)$ and $(\lambda, 2\lambda-1)$ and touching the line $16x-2y+27=0$, is

(A) 0 (B) 1 (C) 2 (D) Infinitely many

20. There exist exactly two circles that pass through $(3,-5)$ and $(5,-3)$ and touch the line $2x+2y+13=0$. Let the ratio of radii of the two circles be m/n with $m(>0)$ and $n(>0)$ having no common factors except 1. Then $(m+n)$ equals

(A) 2 (B) 3 (C) 5 (D) 7

21. Consider the set S of all possible circles that pass through $(4,-3)$ and $(-3,4)$ and touch the line $x+y-5\sqrt{2}=0$. Then which of the following statements is correct?

(A) S consists of exactly one circle, having radius 5
(B) S consists of exactly one circle, having radius 10
(C) S consists of exactly two circles, the ratio of radii being 1 : 2
(D) S consists of exactly two circles, the ratio of radii being 2 : 3

KEY : C-A-A

HINT

19. There will exist exactly one circle if the line passing through A(2, -3) and $B(\lambda, 2\lambda-1)$ is parallel to the given line $16x-2y+27=0$

Also, if the point $B(\lambda, 2\lambda-1)$ lies on the line $16x-2y+27=0$, then we will have exactly one circle.

Thus two values of λ are possible.

20. The line joining (3, -5) and (5, -3) has slope 1 and thus it is perpendicular to $2x+2y+13=0$. Hence the two circles will have same radii.

21. The equation to circle with A(4, -3) and B(-3, 4) as diameter is

$(x-4)(x+3)+(y+3)(y-4)=0$

$\Rightarrow x^2+y^2-x-y-24=0$

The equation to line AB is x + y – 1 = 0

The system of circle is $x^2+y^2-x-y-24+\lambda(x+y-1)=0$

i.e., $x^2+y^2-(1+\lambda)x-(1+\lambda)y-24+\lambda=0$

we apply the condition that the circle touches the line $x+y-5\sqrt{2}=0$ to determine λ. We get λ = - 1 .

The circle then is $x^2+y^2=25$.

Paragraph – 8

P(a, 5a) and Q(4a, a) are two points. Two circles are drawn through these points touching the axis of y.

22. Centre of these circles are at

(a) (a, a), (2a, 3a)

(b) $\left(\frac{205a}{18}, \frac{29a}{3}\right), \left(\frac{5a}{2}, 3a\right)$

(c) $\left(3a, \frac{29a}{3}\right), \left(\frac{205a}{9}, \frac{29a}{18}\right)$

(d) none of these

Key: B

Hint:

23. Angle of intersection of these circles is

(a) $\tan^{-1}(4/3)$

(b) $\tan^{-1}(40/9)$

(c) $\tan^{-1}(84/187)$

(d) $\pi/4$

Key: B

24. If C_1, C_2 are the centres of these circles then area of ΔOC_1C_2, where O is the origin, is

(a) a^2

(b) $5a^2$

(c) $10a^2$

(d) $20a^2$

key: B

Hint: Equation of any circle through the given points is

$(x - a)(x - 4a) + (y - 5a)(y - a) + \lambda(4x + 3y - 19a) = 0$, for some $\lambda \in R$.

As it touches the y-axis, $\left(-3a + \frac{3\lambda}{2}\right)^2 = 9a^2 - 19\lambda a$

Solving $\lambda = 0, \frac{-40a}{9}$.

The required circles are

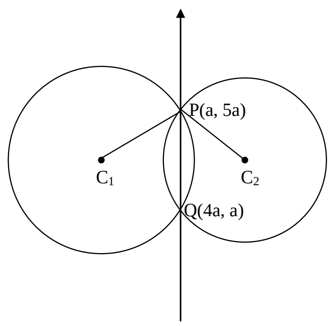
$x^2 + y^2 - 5ax - 6ay + 9a^2$

$x^2 + y^2 - 5ax - 6ay + 9a^2 - \frac{40a}{9}(4x + 3y - 19a) = 0$

Hence centre are $\left(\frac{5a}{2}, 3a\right)$ and $\left(\frac{205a}{18}, \frac{29a}{3}\right)$.,

The centres of the given circles are

$C_1\left(\frac{205a}{18}, \frac{29a}{3}\right)$ and $C_2\left(\frac{5a}{2}, 3a\right)$



Now the angle of intersection θ of these two circles is the angle between the radius vectors at the common point P to the two circles

i.e. $\angle C_1PC_2 = \theta$

Slope of $C_1P = \frac{\frac{29}{3}a - 5a}{\frac{205}{18}a - a} = \frac{84}{187}$

and slope of C_2P = $\dfrac{5a-3a}{a-\dfrac{5a}{2}} = -\dfrac{4}{3}$

So that tan θ = $\dfrac{\dfrac{84}{187}+\dfrac{4}{3}}{1-\dfrac{84}{187}\times\dfrac{4}{3}} = \dfrac{252+748}{561-336} = \dfrac{40}{9}$ $\Rightarrow$ θ = $\tan^{-1}\left(\dfrac{40}{9}\right)$

Area of ΔOC_1C_2 = $\dfrac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ \dfrac{205}{18} & \dfrac{29}{3} & 1 \\ \dfrac{5}{2} & 3a & 1 \end{vmatrix} a^2 = 5a^2$

Paragraph – 9

P is a variable point on the line L=0.Tangents are drawn to the circle $x^2+y^2=4$ from P to touch it at Q and R. The parallelogram PQSR is completed.

25. If $L \equiv 2x+y=6$, then the locus of circum centre of ΔPQR is

a) 2x-y=4 b) 2x+y=3 c) x-2y=4 d) x+2y=3

26. If P =(2,3) then the centre of circum circle of Δ QRS is

a) $\left(\dfrac{2}{13},\dfrac{7}{26}\right)$ b) $\left(\dfrac{2}{13},\dfrac{3}{26}\right)$ c) $\left(\dfrac{3}{13},\dfrac{9}{26}\right)$ d) $\left(\dfrac{3}{13},\dfrac{2}{13}\right)$

27. If P=(3,4) then the coordinates of S are

a) $\left(\dfrac{-46}{25},\dfrac{-63}{25}\right)$ b) $\left(\dfrac{-51}{25},\dfrac{-68}{25}\right)$

c) $\left(\dfrac{-46}{25},\dfrac{-68}{25}\right)$ d) $\left(\dfrac{-68}{25},\dfrac{-51}{25}\right)$

Key: B-C-B

Hint 25. Circumcentre of Δ PQR is the midpoint of P and centre of circle $x^2+y^2=4$

26. Find the image of cicrumcentre of Δ PQR w.r.t. chord of contact of P w.r.t. to circle

27. Find the image of P w.r.t chord of contact of P w.r.t to circle

Paragraph – 10

Two perpendicular tangents are drawn from a point P to a circle C_1. A circle C_2 is drawn touching the circle C_1 and also the perpendicular tangents from P. If r_1 and r_2 are the radii of circles $C_1 \text{ and } C_2$ respectively.

28. Let $r_1 = 3 + 2\sqrt{2}$ then r_2 is

a) $\frac{1}{2}$ b) 1 c) 2 d) 4

29. If A, B are points of contact of tangents to the circle C_1, then area enclosed between the tangents PA, PB circle $C_1 \text{ and } C_2$ is

a) less than area of circle C_2

b) greater than area of circle C_2

c) equal to area of circle C_2

d) will be greater than area of circle C_2 if $r_1 > 4$ and less than area of circle C_2 if $2 < r_1 < 4$

30. If $r_1 = 3 + 2\sqrt{2}$, then the length of common tangents to the circle C_1 and C_2 intercepted between the perpendicular tangents from P is

a) $\frac{1}{2}\left(\sqrt{2}+1\right)$ b) $\left(\sqrt{2}-1\right)$

c) $2\left(\sqrt{2}+1\right)$ d) $4\left(2\sqrt{2}-1\right)$

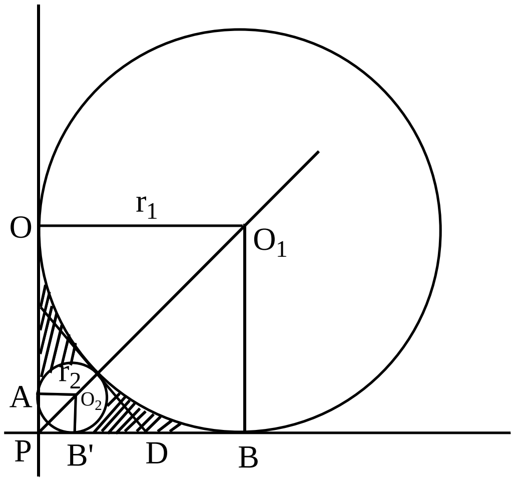
Key : b-b-c

Sol : In $\Delta PA'O_2$ and ΔPAO_1

$$\frac{AA'}{A'P} = \frac{O_1O_2}{O_2P}$$

$$\Rightarrow \frac{r_1 - r_2}{r_2} = \frac{r_1 + r_2}{\sqrt{2}r_2}$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{\sqrt{2}+1}{\sqrt{2}-1} = 3 + 2\sqrt{2}$$



area of shaded region = $r_1^2 - \frac{\pi}{4}r_1^2 - \pi r_2^2 = A_1$

and area of circle $C_1 = \pi r_2^2 = A_2$

$\Rightarrow$ Let $A = A_1 - A_2 = r_1^2 - \frac{\pi}{4}r_1^2 - 2\pi r_2^2$

$= r_2^2\left(\left(\frac{r_1}{r_2}\right)^2\left(1-\frac{\pi}{4}\right) - 2\pi\right) > 0$. Hence $A_1 > A_2$ always

Let CD be the common tangent

$\Rightarrow PE = \left(\sqrt{2}+1\right)r_2 = CE \Rightarrow 2\left(\sqrt{2}+1\right)r_2,$ where $r_2 = 1$

$\Rightarrow$ length CD = $2\left(\sqrt{2}+1\right)$

Paragraph – 11

P is a variable point on the line $2x + y = 4$ and the tangents from P to the circle $x^2 + y^2 = 1$ touch it at A,B. The chord AB always passes through the fixed point Q.

Then answer the following questions.

31. The locus of circumcentre S of ΔPAB is

A) $4(x^2 + y^2) = 2x + y$ B) $2x + y = 2$ C) $x^2 + y^2 = 21$ D) None

Key. B

32. The coordinates of Q are

A) $\left(-\frac{1}{2}, 3\right)$ B) $\left(\frac{1}{2}, \frac{1}{4}\right)$ C) $\left(\frac{1}{2}, -2\right)$ D) $\left(\frac{3}{4}, \frac{1}{2}\right)$

Key. B

33. The midpoint of AB always lies on

A) $x-2y=0$ B) $x^2+y^2=6x-y$ C) $4(x^2+y^2)=2x+y$ D) $2x+y=2$

Key. C

Sol. 31. Circumecentre of ΔPAB is midpoint of OP where O is centre of $x^2+y^2=1$

32. Let $P=(a,4-2a)$. Chord of contact of P w.r.t circle $x^2+y^2=1$ is

$(4y-1)+a(x-2y)=0$ (1)

33. Let (x_1,y_1) be the midpoint of chord AB. So equation of AB is $S_1=S_{11}$ (2)

and (1) & (2) represent same line

$\therefore$ eliminate 'a'

Paragraph – 12

C_1, C_2 are circles of unit radius with centres at $P(0,0)$ and $Q(1,0)$ respectively. C_3 is a circle of unit radius which passes through P and Q and having its centre 'R' above x-axis.

Then answer the following questions:

34. The length of a common tangent to the circles C_2 and C_3 is

A) 2 B) $\sqrt{3}/2$ C) 1 D) 5

Key. C

35. The equation of a common tangent to C_1 and C_3 which does not intersect C_2 is

A) $\sqrt{3}x-y+2=0$ B) $\sqrt{3}x-y-2=0$ C) $x+\sqrt{3}y-2=0$ D) None

Key. A

36. The length of the common chord of the circles on $\overline{PQ}$ and $\overline{PR}$ as diameters is

A) $1/2$ B) $\sqrt{3}/2$ C) 2 D) 1

Key. B

Sol. 34. $RP=RQ=1 \Rightarrow$ R is point of intersection of C_1 & C_2 in first quadrant . C_2, C_3 are circles of equal radius $\Rightarrow$ length of common target =QR

35. ΔPQR is equilateral $\Rightarrow \angle RPQ=60^0$

36. Common chord is the altitude through P in ΔPQR

Paragraph – 13

Let $A(0,0)$ and $B(4,0)$ be given points. The locus of a point 'P' which moves such that $PA=KPB$ ($K>0$ and not equal to 1) is the circle 'S'. Let the line AB intersect S at the points D, E.

Answer the following questions

37. If a circle with centre (3, 2) touches the line AB at D then, E =

A) $(2,0)$ B) $(-3,0)$ C) $(6,0)$ D) $(8,0)$

Key. C

Sol. S a circle with $\overrightarrow{DE}$ as diameter where D,E divide $\overrightarrow{AB}$ in the ratio $K:1$ internally and externally.

38. If a circle 'C' passing through (1, 1) bisects the circumference of S then the radius of 'C' is

A) $\sqrt{\frac{65}{2}}$ B) $\sqrt{189}$ C) $\sqrt{\frac{173}{2}}$ D) $\sqrt{\frac{105}{2}}$

Key. A

39. If θ is an angle between the pair of tangents from (2, 0) to S then $\tan\theta =$

A) $\frac{3}{4}$ B) $\frac{24}{25}$ C) $\frac{7}{25}$ D) $\frac{24}{7}$

Key. D

Sol. Conceptual

Paragraph – 14

A point $P(x, y)$ in a plane is called lattice point if $x, y \in Z$ and a rational point if $x, y \in Q.$ Every lattice point is then a rational point .

Answer the following

40. The number of lattice points inside the circle $x^2 + y^2 = 16$ is

a) 16 b) 45 c) 28 d) 36

Key. B

41. A rational point on $x^2 + y^2 = 1$ is of the form

a) $\left(\frac{m-n}{m+n}, \frac{2\sqrt{mn}}{m+n}\right), m, n \in Z, m+n \neq 0$

b) $\left(\frac{m^2-n^2}{m^2+n^2}, \frac{2mn}{m^2+n^2}\right), m, n \in Q, m^2+n^2 \neq 0$

c) $\left(\frac{m}{m+n}, \frac{n}{m+n}\right), m, n \in Q, m+n \neq 0$ d) $\left(\frac{2\sqrt{mn}}{m+n}, \frac{m-n}{m+n}\right), m, n \in Z, m+n \neq 0$

Key. B

42. For a circle whose centre is not a rational point , maximum number of rational points on it is

a) 1 b) 2 c) 3 d) 4

Key. B

Sol. 40 to 42

For $x^2 + y^2 = 16$, a point (x,y) is internal if $-4 < x < 4, -4 < y < 4$ *and* $x^2 + y^2 - 16 < 0$

$$x = 0 \Rightarrow y = -3, -2, -1, 0, 1, 2, 3 \rightarrow 7$$
$$x = \pm 1 \Rightarrow y = -3, -2, -1, 0, 1, 2, 3 \rightarrow 14$$
$$x = \pm 2 \Rightarrow y = -3, -2, -1, 0, 1, 2, 3 \rightarrow 14$$
$$x = \pm 3 \Rightarrow y = -2, -1, 0, 1, 2 \rightarrow 10$$

Total=45

$$x=\frac{m^2-n^2}{m^2+n^2}, y=\frac{2mn}{m^2+n^2} \Rightarrow x^2+y^2=1.$$

$As\ m,n \in Q$, $\left(\frac{m^2-n^2}{m^2+n^2}, \frac{2mn}{m^2+n^2}\right)$ is a rational point, others are not.

Paragraph – 15

A circle C whose radius is 1 unit, touches the x-axis at point A. The centre Q of C lies in first quadrant. The tangent from origin O to the circle touches it at T and a point P lies on it such that ΔOAP is a right angled triangle at A and its perimeter is 8 units.

43. The length of QP is

A) $\frac{1}{2}$ B) $\frac{4}{3}$ C) $\frac{5}{3}$ D) $\frac{5}{2}$

Key. C

44. Equation of circle C is

A) $(x-2)^2+(y-1)^2=1$ B) $\left\{x-\left(2+\sqrt{3}\right)\right\}^2+(y-1)^2=1$

C) $\left(x-\sqrt{3}\right)^2+(y-1)^2=1$ D) none of these

Key. A

45. Equation of tangent OT is

A) $4x-3y=0$ B) $x-\sqrt{3}y=0$

C) $y-\sqrt{3}x=0$ D) $x+\sqrt{3}y=0$

Key. A

Sol. Solutions for 43-45

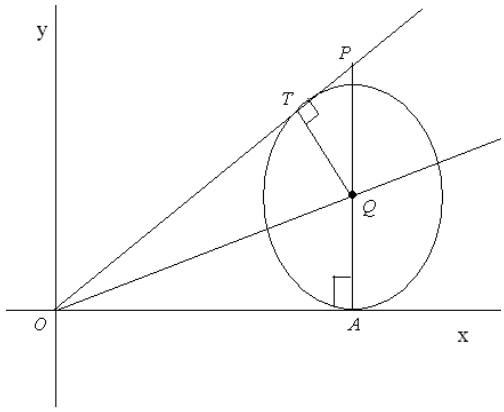
Given $QT=QA=1$

Let $PQ=x$, then $PT=\sqrt{x^2-1}$

Then ΔTQP and ΔAPO are similar triangles

Then $OT=OA=\frac{x+1}{\sqrt{x^2-1}}$

$$\Rightarrow 1+x+\frac{2(x+1)}{\sqrt{x^2-1}}+\sqrt{x^2-1}=8 \Rightarrow x=\frac{5}{3}$$



$AP=\frac{8}{3}, OP=\frac{10}{3}$; and OA=2

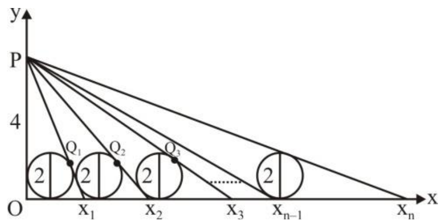
$\therefore Q=(2,1)$

Equation of the circle is $(x-2)^2+(y-1)^2=1$

Coordinates of P are $\left(2, \frac{8}{3}\right)$

$\therefore$ equation of OT is 4x-3y=0

Paragraph – 16



A circle of diameter 2 is drawn tangent to the x and y axes as shown ; another tangent is drawn to the circle from P, meeting the circle at point Q_1 and the x-axis at $x = x_1$. A new circle of diameter 2 is drawn tangent to the x-axis and to $\overleftrightarrow{PQ_1}$ as shown; another tangent is drawn to this circle from P, meeting this circle at point Q_2 and the x-axis at $x = x_2$. This process is continued as shown, then

46. The formula for x_n in terms of x_{n-1}, is

(A) $x_n = \dfrac{5x_{n-1} + 3\sqrt{(x_{n-1})^2 + 16}}{4}$ (B) $x_n = \dfrac{5x_{n-1} + 3\sqrt{x_{n-1}^2 + 4}}{4}$

(C) $x_n = \dfrac{5x_{n-1} + 3\sqrt{(x_{n-1})^2 - 16}}{4}$ (D) $x_n = 5x_{n-1} + 4$

Key. A

47. The values of x_2, x_3 and x_4 in the form of a/b, (where a and b are integers)

(A) $\frac{15}{2}, \frac{64}{5}, \frac{255}{8}$ (B) $\frac{15}{2}, \frac{63}{4}, \frac{255}{8}$

(C) $\frac{3}{2}, \frac{7}{4}, \frac{15}{8}$ (D) $\frac{15}{4}, \frac{63}{8}, \frac{255}{16}$

Key. B

48. The function expressing x_n explicitly as a function of n is

(A) $\dfrac{2^n - 1}{2^{2n}}$ (B) $\dfrac{2^n - 1}{2^{n-1}}$

(C) $\dfrac{2^{2n} - 1}{2^n}$ (D) $\dfrac{2^{2n} - 1}{2^{n-1}}$

Key. D

Sol. 42 to 44

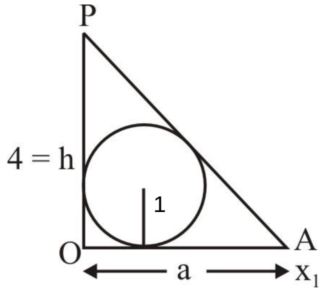
46. (A)

47. (B)

48. (D)

Δ = rs

$$\Rightarrow Sr = 1\left(\frac{a + 4 + \sqrt{a^2 + 16}}{2}\right) = \frac{1}{2}ah$$



$$= \frac{1}{2} \times a \times 4 = 2a$$

$$\Rightarrow \sqrt{a^2 + 16} = 3a - 4$$

$\Rightarrow$ a = 3

$\therefore$ x_1 = 3

In ΔPQR

$$\Delta s = \frac{1}{2} bh$$

Let x_{n-1} = u

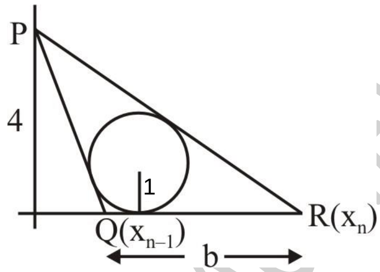
x_n = v

$$\Rightarrow \frac{(v-u) + \sqrt{u^2+16} + \sqrt{v^2+16}}{2} = \frac{(v-u)(4)}{2}$$

$$\Rightarrow \sqrt{u^2+16} = 3(v-u) - \sqrt{v^2+16}$$

$$\Rightarrow 3(v-u)\sqrt{v^2+16} = 5v^2 - 9uv + 8u^2$$

$$= (v-u)(5v-4u)$$



$$\Rightarrow 3\sqrt{v^2+16} = 5v - 4u$$

$$\Rightarrow 2v^2 - 5uv + \left(2u^2 - 18\right) = 0$$

$$\Rightarrow v = \frac{5u + 3\sqrt{u^2+16}}{4},$$ so

$$x_n = \frac{5x_{n-1} + 3\sqrt{(x_{n-1})^2 + 16}}{4}$$

$$x_2 = \frac{15 + 3\sqrt{25}}{4} = \frac{15}{2},\ x_3 = \frac{63}{4},\ x_4 = \frac{255}{8}$$

Noting, $x_3 = \dfrac{2^6 - 1}{2^2},\ x_2 = \dfrac{2^4 - 1}{2^1}$

$$x_n = \frac{2^{2n} - 1}{2^{n-1}}$$

Paragraph – 17

Let C be a curve defined by $y.e^{-\beta x^2} = e^{\alpha}$. The curve C passes through the point P(1, 1) and the slope of the tangent at P is (-2). Also S_1 and S_2 are the Circles $x^2+y^2-2\alpha x-2\beta y+\alpha^2+\beta^2-3=0, x^2+y^2-12x-22y+130=0$ respectively.

49. The value of $6\left(\left(\frac{\alpha}{2}\right)^2+\left(\frac{\beta}{2}\right)^2\right)$ is equal to

A) 2 B) 3 C) 8 D) 12

50. The length of the shortest line segment AB which is tangent to S_1 at A and to S_2 at B is

A) $9\sqrt{3}$ B) $10\sqrt{3}$ C) 11 D) 12

51. If f is a real valued derivable function satisfying $f\left(\frac{x}{y}\right)=\frac{f(x)}{f(y)}$ with $f'(1) = 2$. Then the value of the integral $\int_{\beta}^{\alpha} f(x)\,d(\ln x)$ is equal to

A) 0 B) $\frac{e^2+e^{-2}}{2}$

C) $\frac{e^{-2}-e^2}{2}$ D) $\frac{(e^2+1)(e^2-1)}{2e^2}$

Sol. 49. Ans. (b)

$y = e^{\alpha+\beta x^2}$, passes through (1, 1)

$\Rightarrow \quad 1 = e^{\alpha+\beta} \quad \Rightarrow \alpha + \beta = 0$ .... (1)

Also, $\left(\frac{dy}{dx}\right)_{(1,\,1)} = -2$

$\Rightarrow \quad e^{\alpha+\beta x^2}2\beta x = -2$

$\Rightarrow \quad e^{\alpha+\beta}.2\beta(1) = -2 \Rightarrow \beta = -1$ and $\alpha = 1$

$\Rightarrow \quad (\alpha, \beta) = (1, -1) \Rightarrow 6\left(\left(\frac{\alpha}{2}\right)^2+\left(\frac{\beta}{2}\right)^2\right) = 3$

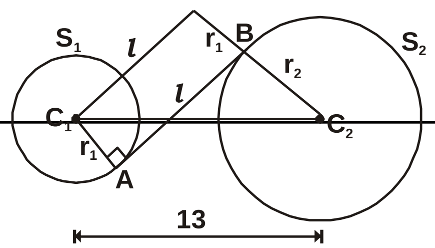
50. Ans. (c)

$S_1 : \quad (x-1)^2 + (y+1)^2 = \left(\sqrt{3}\right)^2$

$S_2 : (x-6)^2 + (y-11)^2 = \left(3\sqrt{2}\right)^2$

$AB^2 = l^2 = (C_1C_2)^2 - (r_1 + r_2)^2 = 169 - \left(4\sqrt{3}\right)^2 = 121$

$\Rightarrow \quad AB = 11$



{ In the figure, $r_1 = \sqrt{3}$, $r_2 = 3\sqrt{3}$

C_1 (1, −1), C_2 (6, 11) }

51. Ans. (d)

Simplifying f(x) = x^2

$$\int_{\beta}^{\alpha} f(x)d\left(\log x\right)dx = \int_{-1}^{1} x^2\, d\left(\log x\right)dx$$

$$= \int_{1/e}^{e} x^2 . \frac{1}{x} dx = \frac{e^2 - e^{-2}}{2} = \frac{e^4 - 1}{2e^2} = \frac{\left(e^2+1\right)\left(e^2-1\right)}{2e^2}$$

Paragraph – 18

Circle C whose radius is 1 unit, touches the x–axis at point A. The centre Q of C lies in first quadrant. The tangent from origin O to the circle touches it at T and a point P lies on it such that ΔOAP is right angle triangle at A and its perimeter is 8 units.

52. The length QP is

(A) $\frac{1}{2}$ (B) $\frac{4}{3}$

(C) $\frac{5}{3}$ (D) $\frac{1}{3}$

53. Equation of circle C is

(A) $\left(x-(2+\sqrt{3})\right)^2 + (y-1)^2 = 1$ (B) $\left(x-(\sqrt{3}+\sqrt{2})\right)^2 + (y-1)^2 = 1$

(C) $\left(x-\sqrt{3}\right)^2 + (y-1)^2 = 1$ (D) $\left(x+\sqrt{3}\right)^2 + (y-1)^2 = 1$

54. Equation of tangent OT is

(A) $x - \sqrt{3}y = 0$ (B) $x - \sqrt{2}y = 0$

(C) $y - \sqrt{3}x = 0$ (D) $x - y = 0$

Sol. 52. (c)

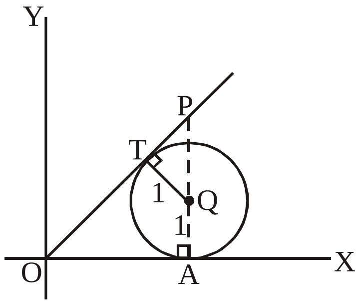
Let PQ = x, $PT = \sqrt{x^2 - 1}$

ΔTQP and ΔAPO are similar triangles then

$$OT = OA = \frac{x+1}{\sqrt{x^2-1}}$$

$$1 + x + \frac{2(x+1)}{\sqrt{x^2-1}} + \sqrt{x^2-1} = 8 \Rightarrow x = \frac{5}{3}$$

53. (a)



$AP = \frac{8}{3},\ OP = \frac{16}{3}$, Let $\angle AOP = 2\theta$

$\sin 2\theta = \frac{1}{2}$ from ΔOAQ $\tan\theta = \frac{1}{OA}$

$OA = \frac{1}{\tan\theta} \Rightarrow \sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta} = \frac{1}{2}$

$\tan\theta = 2-\sqrt{3}$

Hence $OA = 2+\sqrt{3}$

54. (a)

$\frac{x-0}{\cos 2\theta} = \frac{y-0}{\sin 2\theta} \Rightarrow x-\sqrt{3}y = 0$

Paragraph – 19

Consider the relation $4l^2 - 5m^2 + 6l + 1 = 0$, where $l, m \in R$, then the line $lx + my + 1 = 0$ touches a fixed circle whose

55. Centre and radius are

(A) (2, 0), 3 (B) (–3, 0), $\sqrt{3}$ (C) (3, 0), $\sqrt{5}$ (D) None of these

Key. C

56. Tangents PA and PB are drawn to the above fixed circle from the points P on the line x + y – 1 = 0. Then chord of contact AB passes though the fixed point

(A) $\left(\frac{1}{2}, -\frac{5}{2}\right)$ (B) $\left(\frac{1}{3}, \frac{4}{3}\right)$ (C) $\left(-\frac{1}{2}, \frac{3}{2}\right)$ (D) None of these

Key. A

Sol. 55-56.

$9l^2 + 6l + 1 = 5l^2 + 5m^2$

$\left(\frac{3l+1}{\sqrt{l^2+m^2}}\right) = 5$

Hence the centre is (3, 0) and radius = $\sqrt{5}$.

Paragraph – 20

A circle 'C' of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D, E, F respectively. The line PQ is given by the equation $\sqrt{3}x + y - 6 = 0$ and the point D is $\left(3\sqrt{3}/2,\ 3/2\right)$. Further it is given that the origin and the centre of C are on the same side of PQ.

57. The equation of circle C is

A) $\left(x-2\sqrt{3}\right)^2 + \left(y-1\right)^2 = 1$ B) $\left(x-2\sqrt{3}\right)^2 + \left(y+1/2\right)^2 = 1$

C) $\left(x-\sqrt{3}\right)^2 + \left(y+1\right)^2 = 1$ D) $\left(x-\sqrt{3}\right)^2 + \left(y-1\right)^2 = 1$

Key. D

Sol. Centre of C lies on the line through D perpendicular to PQ. Thus centre of C lie on

$y-3/2=\left(1/\sqrt{3}\right)\left(x-3\sqrt{3}/2\right) \Rightarrow\ x=\sqrt{3}y$

Let the centre of the circle C be $\left(\sqrt{3}y_1,\ y_1\right)$ then $\left(\frac{3\sqrt{3}}{2}-\sqrt{3}y_1\right)^2+\left(\frac{3}{2}-y_1\right)^2=1$

$\Rightarrow 4\left(\frac{3}{2}-y_1\right)^2=1 \Rightarrow \frac{3}{2}-y_1=\pm\frac{1}{2} \Rightarrow\ y_1=1,\ 2$

Thus, centre of C can be $\left(\sqrt{3},1\right)$, or $\left(2\sqrt{3},\ 2\right)$. Since centre of the circle and origin lie on the same side of $\sqrt{3}x+y-6=0$

and $\sqrt{3}\left(0\right)+0-6<0$ and $\sqrt{3}\left(\sqrt{3}\right)+1-6<0$ we get centre of the circle C to be $I\left(\sqrt{3},\ 1\right)$ and Hence its equation is $\left(x-\sqrt{3}\right)^2+\left(y-1\right)^2=1$

58. Points E and F are given by

A) $\left(\sqrt{3}/2,\ 3/2\right),\left(\sqrt{3},\ 0\right)$

B) $\left(\sqrt{3}/2,\ 1/2\right),\left(\sqrt{3},\ 0\right)$

C) $\left(\sqrt{3}/2,\ 3/2\right),\left(\sqrt{3}/2,\ 1/2\right)$

D) $\left(\ 3/2,\ \sqrt{3}/2\right),\ \left(\sqrt{3}/2,\ 1/2\right)$

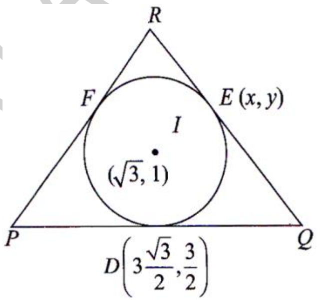
Key. A

Sol. Next, if m is the slope of QR $\pm\tan\left(\frac{\pi}{3}\right)=\frac{m-\left(-\sqrt{3}\right)}{1+m\left(-\sqrt{3}\right)} \Rightarrow \pm\sqrt{3}\left(1-m\sqrt{3}\right)=m+\sqrt{3}$ $\Rightarrow m=0$ or $\sqrt{3}$

Let the slope of QR be 0 and the coordinate of E be $\left(x,y\right)$, then since IE is perpendicular to

$QR\ \frac{x-\sqrt{3}}{y-1}=0 \Rightarrow x=\sqrt{3}$ and as $IE=1,\ \left(x-\sqrt{3}\right)^2+\left(y-1\right)^2=1 \Rightarrow y=0$ or 2 so the coordinates of E

are $\left(\sqrt{3},0\right)$ as $y=2$ is not given in the choices.



Similarly, let the slope of PR be $\sqrt{3}$. If the coordinates of F are $\left(p,q\right)$ $\frac{q-1}{p-\sqrt{3}}\times\sqrt{3}=-1$

$\Rightarrow\ p-\sqrt{3}=-\sqrt{3}\left(q-1\right)$ Also $\left(p-\sqrt{3}\right)^2+\left(q-1\right)^2=1 \Rightarrow\ 4\left(q-1\right)^2=1 \Rightarrow q-1=\pm\frac{1}{2} \Rightarrow\ q=\frac{3}{2}$ or $\frac{1}{2}$

if $q=\frac{3}{2},\ p=\sqrt{3}-\frac{\sqrt{3}}{2}=\frac{\sqrt{3}}{2}$ if $q=\frac{1}{2}, p=\sqrt{3}+\frac{\sqrt{3}}{2}=\frac{3\sqrt{3}}{2}$ So the coordinates of F are $\left(\sqrt{3}/2,\ 3/2\right)$

59. Equations of the sides QR, RP are

A) $y=\left(2/\sqrt{3}\right)x+1,\ y=\left(-2/\sqrt{3}\right)x-1$ B) $y=\left(1/\sqrt{3}\right)x,\ \ y=0$

C) $y=\left(\sqrt{3}/2\right)x+1,\ \ y=\left(-\sqrt{3}/2\right)x-1$ D) $y=\sqrt{3}x,\ y=0$

Key. D

Sol. Now equation of QR is $y-0=0\left(x-\sqrt{3}\right)\Rightarrow y=0$

Equation of RP is $y-3/2=\sqrt{3}\left(x-\sqrt{3}/2\right)\Rightarrow\ y=\sqrt{3}x$

Paragraph – 21

A system of circles is said to be coaxal when every pair of the circles has the same radical axis. It follows from this definition that

1. The centres of all circles of a coaxal system lie on one straight line, which is perpendicular to the common radical axis.

2. Circles passing through two fixed points form a coaxal system with line joining the points as common radical axis.

3. The equation to a coaxal system of which two members are $S_1=0$ and $S_2=0$ is $S_1+\lambda S_2=0,\ \lambda$ is parameter. If we choose the line of centres as x-axis and the common radical axis as y-axis, then the simplest form of equation of coaxal circles is $x^2+y^2+2gx+c=0...(1)$ where c is fixed and g is variable.

If $g=\pm\sqrt{c}, c>0$, then the radius g^2-c vanishes and the circles become point circles. The points $\left(\pm\sqrt{c},0\right)$ are called the limiting points of the system of coaxal circles given by (1).

60. The coordinates of the limiting points of the coaxal system to which the circles $x^2+y^2+4x+2y+5=0$ and $x^2+y^2+2x+4y+7=0$ belong are

A) $(0,-3),(0,3)$ B) $(0,3),(-2,-1)$

C) $(-2,\ -1),(0,\ -3)$ D) $(2,\ 1),(-2,\ -1)$

Key. C

Sol. The equation of the coaxal system is $x^2+y^2+4x+2y+5+\lambda\left(x^2+y^2+2x+4y+7\right)=0$ or

$$x^2+y^2+\frac{2(2+\lambda)}{1+\lambda}x+\frac{2(1+2\lambda)}{1+\lambda}y+\frac{5+7\lambda}{1+\lambda}=0$$

Equating radius to zero, we get

$$\frac{(2+\lambda)^2+(1+2\lambda)^2-(5+7\lambda)(1+\lambda)}{(1+\lambda)^2}=0$$

$\Rightarrow 2\lambda^2+4\lambda=0\Rightarrow\lambda=0$ or -2

The centre of above system is $\left(-\frac{2+\lambda}{1+\lambda},-\frac{1+2\lambda}{1+\lambda}\right)$ Substituting the values of $\lambda,$ we get the Coordinates of limiting points $(-2,\ -1)$ and $(0,\ -3)$

61. The equation to the circle which belongs to the coaxal system of which the limiting points are $(1,-1),(2,\ 0)$ and which passes through the origin is

A) $x^2+y^2-4x=0$ B) $x^2+y^2+4x=0$ C) $x^2+y^2-4y=0$ D) $x^2+y^2+4y=0$

Key. D

Sol. The point circles represented by the limiting points are $(x-1)^2+(y+1)^2=0$ and $(x-2)^2+y^2=0$ So, the equation of coaxal system is, $(x-1)^2+(y+1)^2+\lambda\{(x-2)^2+y^2\}=0....(1)$ it passes through $(0,\ 0)$, so, $\lambda=-\dfrac{1}{2}$ putting into (1) we get the equation to the desired circle as $x^2+y^2+4y=0$

62. If origin be a limiting point of a coaxal system one of whose member is $x^2+y^2-2\alpha x-2\beta y+c=0$, then the other limiting point is

A) $\left(\dfrac{c\alpha}{\alpha^2+\beta^2},-\dfrac{c\beta}{\alpha^2+\beta^2}\right)$ B) $\left(\dfrac{c\alpha}{\alpha^2+\beta^2},\dfrac{c\beta}{\alpha^2+\beta^2}\right)$

C) $\left(\dfrac{\alpha\beta}{\alpha^2+\beta^2},\dfrac{c\alpha}{\alpha^2+\beta^2}\right)$ D) $\left(-\dfrac{c\beta}{\alpha^2+\beta^2},\dfrac{c\alpha}{\alpha^2+\beta^2}\right)$

Key. B

Sol. The equation of the given coaxal system is $x^2+y^2-2\alpha x-2\beta y+c+\lambda\left(x^2+y^2\right)=0$ or

$$x^2+y^2-\frac{2\alpha}{1+\lambda}x-\frac{2\beta}{1+\lambda}y+\frac{c}{1+\lambda}=0$$

Its centre is $\left(\dfrac{\alpha}{1+\lambda},\dfrac{\beta}{1+\lambda}\right)$ and radius is $\dfrac{\sqrt{\alpha^2+\beta^2-c(1+\lambda)}}{|1+\lambda|}$ The radius vanishes if $1+\lambda=\dfrac{\alpha^2+\beta^2}{c}$

So, the other limiting point is $\left(\dfrac{c\alpha}{\alpha^2+\beta^2},\dfrac{c\beta}{\alpha^2+\beta^2}\right)$.

Paragraph – 22

Consider two circles S_1 and S_2 whose equation are respectively $x^2+y^2-2x=0$, $x^2+y^2+6x=0$. Let the direct common tangents of circles touches S_1 at A and B and S_2 at C and D. Further these direct common tangents meet at P and intersect a transverse common tangent at Q and R.

63. Centre of incircle of ΔPCD is

(A) (–2, 0) (B) (–1, 0) (C) (0, 0) (D) (1, 0)

Key. C

64. Orthocentre of ΔPQR is

(A) (–2, 0) (B) (–1, 0) (C) (0, 0) (D) (1, 0)

Key. D

65. Centre of circle through A, B, C, D and centre of circle touching the circles S_1 and S_2 internally is

(A) (–1, 0), (–2, 0) (B) (–1, 0), (0, 0)

(C) (1, 0), (–1, 0) (D) none of these

Key. A

Sol. 63–65. Point P is (3, 0)

ΔPQR is equilateral

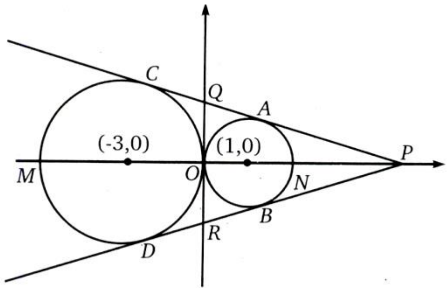
$\Rightarrow$ orthocenter = incentre = (1, 0)

ΔPCD is equilateral.

$\Rightarrow$ incentre = circumcentre = (0, 0)

Centre of circle touching S_1 and S_2 internally is mid point of M and N (–2, 0)

Now, PQ = $\sqrt{3}$, PC = $3\sqrt{3}$



So, equation of ABCD are $x-\frac{3}{2}=0$ and $x+\frac{3}{2}=0$ respectively.

Circles through CD and AB are respectively

$$x^2+y^2+6x+\lambda\left(x+\frac{3}{2}\right)=0$$

$$x^2+y^2-2x+\mu\left(x-\frac{3}{2}\right)=0$$

$\Rightarrow \quad 6+\lambda=\mu-2$ and $\frac{3}{2}\lambda=-\frac{3}{2}\mu \quad \Rightarrow \quad \lambda=-\mu$ and $\mu = 4$

$\therefore$ Equation of circle through A, B, C, D is

$x^2+ y^2+2x-6=0$

$\therefore$ centre of circle $(-1, 0)$

Paragraph – 23

Let ABCD is a rectangle with AB = a and BC = b. A circle is drawn passing through A and B and touching side CD. Another circle is drawn passing through B and C and touching side AD. Let r_1 and r_2 be the radii of these two circles respectively.

66. r_1 equals

(A) $\frac{4b^2-a^2}{8b}$ (B) $\frac{4b^2+a^2}{8b}$

(C) $\frac{4a^2+b^2}{8a}$ (D) $\frac{a^2-4b^2}{8a}$

Key. B

67. $\frac{r_1}{r_2}$ equals

(A) $\frac{a}{b}\left(\frac{4b^2+a^2}{4a^2+b^2}\right)$ (B) $\frac{b}{a}\left(\frac{4a^2+b^2}{4b^2+a^2}\right)$

(C) $\frac{a}{b}\left(\frac{4b^2-a^2}{4a^2-b^2}\right)$ (D) $\frac{b}{a}\left(\frac{a^2-4b^2}{4a^2-b^2}\right)$

Key. A

68. Minimum value of $(r_1 + r_2)$ equals

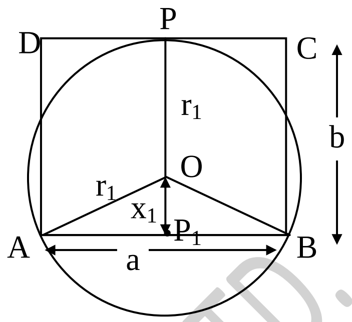
(A) $\frac{5}{8}$ (a – b) (B) $\frac{5}{8}$ (a + b)

(C) $\frac{3}{8}$ (a – b) (D) $\frac{3}{8}$ (a + b)

Key. B

Sol.

66–68. Let $r_1 = b - x_1 = OP = OA$



$\therefore AP_1 = a/2$

$r_1^2 = x_1^2 + (a/2)^2 = (b - x_1)^2$

$\Rightarrow x_1^2 + \dfrac{a^2}{4} = b^2 + x_1^2 - 2bx_1 \Rightarrow x_1 = \dfrac{4b^2 - a^2}{8b}$

$\therefore r_1 = b - x_1 = \dfrac{4b^2 + a^2}{8b}$

Similarly for the circle passing through B and C and touching side AD,

$r_2 = \dfrac{4a^2 + b^2}{8a}$.

Now, $r_1 + r_2 = \dfrac{4b^2 + a^2}{8b} + \dfrac{4a^2 + b^2}{8a}$

$= \dfrac{a^3 + b^3 + 4ab(a+b)}{8ab}$

$= \dfrac{(a+b)(a^2 + 3ab + b^2)}{8ab} = \dfrac{(a+b)}{8}.\dfrac{(a^2 - 2ab + b^2 + 5ab)}{ab} = \dfrac{(a+b)}{8}.\dfrac{[(a-b)^2 + 5ab]}{ab}$

$= \dfrac{(a+b)}{8}.\dfrac{[(a-b)^2 + 5ab]}{ab}$

But $(a - b)^2 \geq 0$

$\therefore r_1 + r_2 \geq \dfrac{(a+b)}{8}.\dfrac{5ab}{ab} \Rightarrow r_1 + r_2 \geq \dfrac{5(a+b)}{8}$

Paragraph – 24

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D, E, F respectively. The line PQ is given by the equation $\sqrt{3}x + y - 6 = 0$ and the point D is $\left(\dfrac{3\sqrt{3}}{2}, \dfrac{3}{2}\right)$. Further, it is given that the origin and the centre of C are on the same side of the line PQ.

69. The equation of the circle C is

(A) $(x - 2\sqrt{3})^2 + (y - 1)^2 = 1$ (B) $(x - 2\sqrt{3})^2 + \left(y + \dfrac{1}{2}\right)^2 = 1$

(C) $(x - \sqrt{3})^2 + (y + 1)^2 = 1$ (D) $(x - \sqrt{3})^2 + (y - 1)^2 = 1$

Key. D

70. Points E and F are given by

(A) $\left(\dfrac{\sqrt{3}}{2}, \dfrac{3}{2}\right), (\sqrt{3}, 0)$ (B) $\left(\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right), (\sqrt{3}, 0)$

(C) $\left(\dfrac{\sqrt{3}}{2}, \dfrac{3}{2}\right), \left(\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right)$ (D) $\left(\dfrac{3}{2}, \dfrac{\sqrt{3}}{2}\right), \left(\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right)$

Key. A

71. Equations of the sides QR, RP are

(A) $y = \dfrac{2}{\sqrt{3}}x + 1, \; y = -\dfrac{2}{\sqrt{3}}x - 1$ (B) $y = \dfrac{x}{\sqrt{3}}, \; y = 0$

(C) $y=\frac{\sqrt{3}}{2}x+1,\ y=-\frac{\sqrt{3}}{2}x-1$ (D) $y=\sqrt{3}x,\ y=0$

Key. D

Sol. 69. PQ makes 120º with x-axis $(\because m=-\sqrt{3})$. So, PQ is parallel to the x-axis

$\therefore$ IE is parallel to y-axis passing through $(\sqrt{3},1)$.

$\therefore$ the equation of EI (and so EP) is x = $\sqrt{3}$

Solving x = $\sqrt{3}$ with $\sqrt{3}$ x + y = 6, we get P = ($\sqrt{3}$, 3).

I divides PE in the ratio 2 : 1. So, E = ($\sqrt{3}$, 0)

$\therefore$ QR has the equation y = 0. Solving with $\sqrt{3}$ x + y = 6, Q = (2 $\sqrt{3}$, 0).

I divides FQ in the ratio 1 : 2.

So, $F=\left(\frac{\sqrt{3}}{2},\frac{3}{2}\right)$

70. Slope PR is tan 60º ($\because$ QR || x-axis, its equation being y = 0) and it passes through F.

$\therefore$ The equation of PR is $y-\frac{3}{2}=\tan 60°.\left(x-\frac{\sqrt{3}}{2}\right)$ or $y=\sqrt{3}x$.

Paragraph – 25

A point $P(x,y)$ in a plane is called lattice point if $x,y\in Z$ and a rational point if $x,y\in Q.$ Every lattice point is then a rational point .

Answer the following

72. The number of lattice points inside the circle $x^2+y^2=16$ is

a) 16 b) 45 c) 28 d) 36

Key. B

73. A rational point on $x^2+y^2=1$ is of the form

a) $\left(\frac{m-n}{m+n},\frac{2\sqrt{mn}}{m+n}\right),m,n\in Z,m+n\neq 0$

b) $\left(\frac{m^2-n^2}{m^2+n^2},\frac{2mn}{m^2+n^2}\right),m,n\in Q,m^2+n^2\neq 0$

c) $\left(\frac{m}{m+n},\frac{n}{m+n}\right),m,n\in Q,m+n\neq 0$

d) $\left(\frac{2\sqrt{mn}}{m+n},\frac{m-n}{m+n}\right),m,n\in Z,m+n\neq 0$

Key. B

74. For a circle whose centre is not a rational point , maximum number of rational points on it is

a) 1 b) 2 c) 3 d) 4

Key. B

Sol. 72 to 74

For $x^2+y^2=16$, a point (x,y) is internal if $-4<x<4,-4<y<4$ *and* $x^2+y^2-16<0$

$x=0\Rightarrow y=-3,-2,-1,0,1,2,3\rightarrow 7$

$x=\pm 1 \Rightarrow y=-3,-2,-1,0,1,2,3 \to 14$

$x=\pm 2 \Rightarrow y=-3,-2,-1,0,1,2,3 \to 14$

$x=\pm 3 \Rightarrow y=-2,-1,0,1,2 \to 10$

Total=45

$x=\dfrac{m^2-n^2}{m^2+n^2}, y=\dfrac{2mn}{m^2+n^2} \Rightarrow x^2+y^2=1.$

$As\ m,n \in Q$, $\left(\dfrac{m^2-n^2}{m^2+n^2}, \dfrac{2mn}{m^2+n^2}\right)$ is a rational point, others are not.

Paragraph – 26

Circle C whose radius is 1 unit, touches the x–axis at point A. The centre Q of C lies in first quadrant. The tangent from origin O to the circle touches it at T and a point P lies on it such that ΔOAP is right angle triangle at A and its perimeter is 8 units.

75. The length QP is

(A) $\frac{1}{2}$ (B) $\frac{4}{3}$

(C) $\frac{5}{3}$ (D) $\frac{1}{3}$

Key. C

76. Equation of circle C is

(A) $\left(x-(2+\sqrt{3})\right)^2+(y-1)^2=1$ (B) $\left(x-(\sqrt{3}+\sqrt{2})\right)^2+(y-1)^2=1$

(C) $\left(x-\sqrt{3}\right)^2+(y-1)^2=1$ (D) $\left(x+\sqrt{3}\right)^2+(y-1)^2=1$

Key. A

77. Equation of tangent OT is

(A) $x-\sqrt{3}y=0$ (B) $x-\sqrt{2}y=0$

(C) $y-\sqrt{3}x=0$ (D) $x-y=0$

Key. A

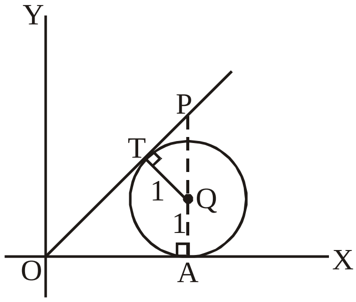
Sol. 75. Let PQ = x, $PT=\sqrt{x^2-1}$

ΔTQP and ΔAPO are similar triangles then

$$OT=OA=\frac{x+1}{\sqrt{x^2-1}}$$

$$1+x+\frac{2(x+1)}{\sqrt{x^2-1}}+\sqrt{x^2-1}=8 \Rightarrow x=\frac{5}{3}$$

76. $AP = \frac{8}{3}, OP = \frac{16}{3}$, Let $\angle AOP = 2\theta$



$\sin 2\theta = \frac{1}{2}$ from ΔOAQ $\tan\theta = \frac{1}{OA}$

$$OA = \frac{1}{\tan\theta} \Rightarrow \sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta} = \frac{1}{2}$$

$$\tan\theta = 2-\sqrt{3}$$

Hence $OA = 2+\sqrt{3}$

77. $\frac{x-0}{\cos 2\theta} = \frac{y-0}{\sin 2\theta} \Rightarrow x-\sqrt{3}y = 0$

Paragraph – 27

A circle C_1 of radius r units rolls outside the circle

$C_2: x^2+y^2+2rx=0$ touching it externally .The line of centers has an inclination 60^0.Then

78. The point of contact of C_1 & C_2 is

a) $\left(-r, r\sqrt{3}\right)$ b) $\left(-r, -r\sqrt{3}\right)$ c) $\left(\frac{-r}{2}, \frac{-r\sqrt{3}}{2}\right)$ d) $\left(\frac{-r}{2}, \frac{r\sqrt{3}}{2}\right)$

Key. D

79. The equation of direct common tangents are

a) $\sqrt{3}x-y+r\left(2\pm\sqrt{3}\right)=0$ b) $\sqrt{3}x-y+r\left(\sqrt{3}\pm 2\right)=0$

c) $\sqrt{3}x-y+2r\left(2\pm\sqrt{3}\right)=0$ d) $\sqrt{3}x-y+2r\left(\sqrt{3}\pm 2\right)=0$

Key. B

80. The transverse common tangent is

a) $x+\sqrt{3}y+r=0$ b) $x+\sqrt{3}y+2r=0$

c) $x+\sqrt{3}y-r=0$ d) $x+\sqrt{3}y-2r=0$

Key. C

Sol. 78,79,80

The point of contact is at a distance of r units from (-r,0) on a line of inclination 60^0

$\therefore$ Point of contact $=\left(-r+r\cos 60^0, 0+r\sin 60^0\right)$

$$=\left(-\frac{r}{2}, \frac{r\sqrt{3}}{2}\right)$$

C_1 centre = $\left(-r+2r\cos 60^0, 0+2r\sin 60^0\right) = \left(0, r\sqrt{3}\right)$

DCT’s are parallel to line of centres.

Equation of DTCs are of the form : $x\sqrt{3}-y+K=0$ to find K use CP=r.

Transverse common tangent is $\perp r$ to line of centers and passes through point of contact.

Paragraph – 28

P is a variable point on the line $2x+y=4$ and the tangents from P to the circle $x^2+y^2=1$ touch it at A,B. The chord AB always passes through the fixed point Q.
Then answer the following questions.

81. The locus of circumcentre S of ΔPAB is

A) $4(x^2+y^2)=2x+y$ B) $2x+y=2$ C) $x^2+y^2=21$ D) None

Key. B

82. The coordinates of Q are

A) $\left(-\frac{1}{2},3\right)$ B) $\left(\frac{1}{2},\frac{1}{4}\right)$ C) $\left(\frac{1}{2},-2\right)$ D) $\left(\frac{3}{4},\frac{1}{2}\right)$

Key. B

83. The midpoint of AB always lies on

A) $x-2y=0$ B) $x^2+y^2=6x-y$ C) $4(x^2+y^2)=2x+y$ D) $2x+y=2$

Key. C

Sol. 81. Circumecentre of ΔPAB is midpoint of OP where O is centre of $x^2+y^2=1$

82. Let $P=(a,4-2a)$. Chord of contact of P w.r.t circle $x^2+y^2=1$ is

$(4y-1)+a(x-2y)=0$ $\quad(1)$

83. Let (x_1,y_1) be the midpoint of chord AB. So equation of AB is $S_1=S_{11}$ $\quad(2)$

and (1) & (2) represent same line

$\therefore$ eliminate 'a'

Paragraph – 29

C_1,C_2 are circles of unit radius with centres at $P(0,0)$ and $Q(1,0)$ respectively. C_3 is a circle of unit radius which passes through P and Q and having its centre 'R' above x-axis.
Then answer the following questions:

84. The length of a common tangent to the circles C_2 and C_3 is

A) 2 B) $\sqrt{3}/2$ C) 1 D) 5

Key. C

85. The equation of a common tangent to C_1 and C_3 which does not intersect C_2 is

A) $\sqrt{3}x-y+2=0$ B) $\sqrt{3}x-y-2=0$ C) $x+\sqrt{3}y-2=0$ D) None

Key. A

86. The length of the common chord of the circles on $\overline{PQ}$ and $\overline{PR}$ as diameters is

A) $1/2$ B) $\sqrt{3}/2$ C) 2 D) 1

Key. B

Sol. 84. $RP=RQ=1\Rightarrow$ R is point of intersection of $C_1\,\&\,C_2$ in first quadrant . C_2,C_3 are circles of equal radius $\Rightarrow$ length of common target =QR

85. ΔPQR is equilateral $\Rightarrow$ $\angle RPQ=60^0$

86. Common chord is the altitude through P in ΔPQR

Paragraph – 30

Let $A(0,0)$ and $B(4,0)$ be given points. The locus of a point 'P' which moves such that $PA = KPB$ ($K > 0$ and not equal to 1) is the circle 'S'. Let the line AB intersect S at the points D, E.
Answer the following questions

87. If a circle with centre (3, 2) touches the line AB at D then, E =

A) $(2,0)$ B) $(-3,0)$ C) $(6,0)$ D) $(8,0)$

Key. C

88. If a circle 'C' passing through (1, 1) bisects the circumference of S then the radius of 'C' is

A) $\sqrt{\frac{65}{2}}$ B) $\sqrt{189}$ C) $\sqrt{\frac{173}{2}}$ D) $\sqrt{\frac{105}{2}}$

Key. A

89. If θ is an angle between the pair of tangents from (2, 0) to S then $\tan\theta =$

A) $\frac{3}{4}$ B) $\frac{24}{25}$ C) $\frac{7}{25}$ D) $\frac{24}{7}$

Key. D

Sol. 87. S a circle with $\overrightarrow{DE}$ as diameter where D,E divide $\overrightarrow{AB}$ in the ratio $K:1$ internally and externally.

Paragraph – 31

The line $x+2y+a=0$ intersects the circle $x^2+y^2=4$ at two distinct points A and B another line $12x-6y-41=0$ intersects the circle $x^2+y^2-4x-2y+1=0$ at two distinct points C and D.

90. The number of integral values of "a" are given by

a) 9 b) 7 c) 8 d) 6

Key. A

91. The value of "a" for which the points A, B, C, D are concyclic is

a) 1 b) 3 c) 4 d) 2

Key. D

92. The equation of circle passing through the points A, B, C, D is

a) $5x^2+5y^2-8x-16y-36=0$ b) $5x^2+5y^2+8x-16y-36=0$

c) $5x^2+5y^2+8x+16y-36=0$ d) $5x^2+5y^2-8x-16y+36=0$

Key. A

Sol. 90. x = - 2y – a

$$\therefore 4y^2+a^2+4ay+y^2-4=0$$

$$5y^2+4ay+\left(a^2-4\right)=0$$

since the line intersects at two district points, D > O

$$\Rightarrow a^2<20 \Rightarrow -2\sqrt{5}<a<2\sqrt{5}$$

$\therefore$ number of interval values of 'a' are 9

91. equation of circle, passing through the points A and B is $x^2+y^2-4+\lambda\left(x+2y+a\right)=0$ -----(1) equation of circle, passing through the points C and D is

$\left(x^2+y^2-4x-2y+1\right)+\mu\left(12x-6y-41\right)=0$ ------- (2)

since (1) + (2) represent the same circle, compare the coefficients of x,y and constant terms.

$$\lambda=-4+12\mu$$

$$2\lambda=-2-6\mu$$

$$a\lambda-4=1-41\mu$$

$\Rightarrow \lambda=\frac{-8}{5}$, $\mu=\frac{1}{5}$, a = 2

92. Put $\lambda=\frac{-8}{5}$ in eq.no-(1)

We get $5x^2+5y^2-8x-16y-36=0$

Circles

Integer Answer Type

1. Let $S_1 \equiv x^2 + y^2 - 4x - 8y + 4 = 0$ and S_2 its image in the line y = x. The radius of the circle touching y = x at (1, 1) and orthogonal to S_2 is $\frac{3}{\sqrt{\lambda}}$, then $\lambda^2 + 2 =$

Key. 6

Sol. Centre of circle $S_1 = (2, 4)$

Centre of circle $S_2 = (4, 2)$

Radius of circle S_1 = radius of circle $S_2 = 4$

$\therefore$ equation of circle S_2

$(x-4)^2 + (y-2)^2 = 16$

$\Rightarrow x^2 + y^2 - 8x - 4y + 4 = 0$ . . . (i)

Equation of circle touching y = x at (1, 1) can be taken as

$(x-1)^2 + (y-1)^2 + \lambda(x-y) = 0$

or, $x^2 + y^2 + x(\lambda - 2) + y(-\lambda - 2) + 2 = 0$ . . . (ii)

As this is orthogonal to S_2

$\Rightarrow 2\left(\frac{\lambda-2}{2}\right)\cdot(-4) + 2\left(\frac{-\lambda-2}{2}\right)\cdot(-2) = 4 + 2$

$\Rightarrow -4\lambda + 8 + 2\lambda + 4 = 6$

$\therefore$ required equation of circle is

$x^2 + y^2 + x - 5y + 2 = 0$.

Radius = $\sqrt{\frac{1}{4}+\frac{25}{4}-2} = \sqrt{\frac{26-8}{4}} = \sqrt{\frac{18}{4}} = \frac{3}{2}\sqrt{2}$.

2. The centre of each of a set of circles, each of radius 3, lie on the circle $x^2 + y^2 = 25$. The locus of any point in the set is a ring whose area is $\lambda\pi$, then $\frac{\lambda}{10} =$

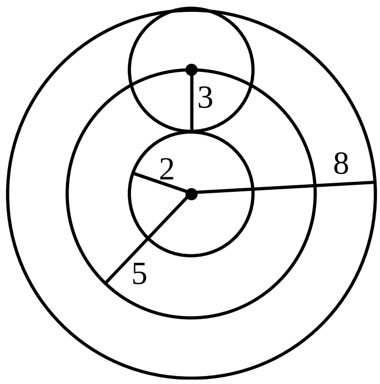
Key. 6

Sol.

From figure it is clear that point will lie between two concentric circles

$x^2 + y^2 = 4$ and $x^2 + y^2 = 64$

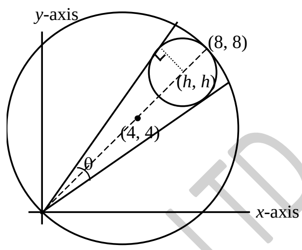
$\therefore$ Required locus $4 \le x^2 + y^2 \le 64$



3. If the radius of the circle touching the pair of lines $7x^2 - 18xy + 7y^2 = 0$ and the circle $x^2 + y^2 - 8x - 8y = 0$, and contained in the given circle is equal to k, then k^2 is equal to

Key. 8

Sol.



$\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right| = \frac{4\sqrt{2}}{7}$

$\tan\frac{\theta}{2} = \frac{1}{2\sqrt{2}}$ on solving

$\sin\frac{\theta}{2} = \frac{1}{3} = \frac{\sqrt{2}(8-h)}{\sqrt{2}\,h}$

Hence equation of circle is $(x-6)^2 + (y-6)^2 = 8$.

4. For the circle $x^2 + y^2 = r^2$ the value of r for which the area enclosed by the tangents from the point P(6, 8) to the circle and the chord of contact is maximum is ________

Key. 5

Sol. $f(r) = \Delta = \frac{r.s_{11}^{3/2}}{s_{11} + r^2} = \frac{r\left(100 - r^2\right)^{3/2}}{100}$

$f'(r) = 0 \Rightarrow -3r^2 + 100 - r^2 = 0 \Rightarrow r = 5$

5. Circles are drawn through (1, 1) touching the circle $x^2 + y^2 - 6x + 2y + 1 = 0$. If r_1 *and* r_2 are the radii of smallest and largest circles then the value of $(r_2 + r_1)^2 - (r_2 - r_1)^2$ equals ____

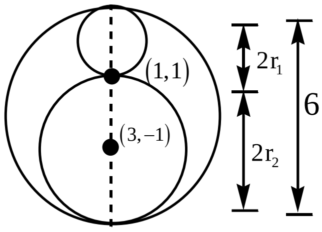
Key. 1

Sol. $2r_1 + 2r_2 = 6 \Rightarrow r_1 + r_2 = 3 \longrightarrow (1)$

Also $2r_2 - 2\sqrt{3} = 3 \Rightarrow r_2 = \frac{3 + 2\sqrt{2}}{2}$

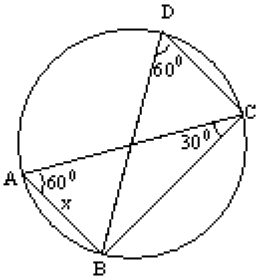
$\therefore r_1 = \frac{3 - 2\sqrt{2}}{2}$

$G.E = 4r_1r_2 = 4 \times \frac{1}{4} = 1$



6. Line segment AC and BD are diameters of circle of radius one. If $\angle BDC = 60^0$, the length of line segment AB is

Key. 1



Sol.

$\angle A = 60^0 = \angle D$

AC = 2(given)

$\angle ABC = 90^0$

$\Rightarrow x = 1$

7. If $m(x-2)+\sqrt{1-m^2}.y=3$, is tangent to a circle for all $m \in [-1,1]$ then the radius of the circle.

Key. 3

Sol. $(x-2)\cos\theta + y\sin\theta = 3$ is tangent to the circle $(x-2)^2 + y^2 = 3^2$

8. If the portion of the line $ax+by-1=0$ intercepted between the lines $ax+y+1=0$ and $x+by=0$ subtends a right angle at the origin, then the value of $4a+b^2+(b+1)^2$

Key. 1

Sol. Homogenise $(ax+by-1)(x+by)=0$ using $ax+y+1=0$

9. The tangents drawn from the origin to the circle $x^2+y^2-2rx-2hy+h^2=0$ are perpendicular then sum of all possible values of $\frac{h}{r}$ is ___

Key. 0

Sol. Combined equation of the tangents drawn from (0, 0)to the circle is

$\left(x^2+y^2-2rx-2hy+h^2\right)h^2=\left(-rx-hy+h^2\right)^2$ here coefficient of

$x^2+\text{coffecient of } y^2=0 \Rightarrow \left(h^2-r^2\right)+\left(h^2-r^2\right)=0$

$\Rightarrow \frac{h}{r}=\pm 1$

10. The number of points on $y=\tan^{-1}x,\ \forall\ x\in(0,\pi),$ whose image in $y=x$ is the centre of the circle with radius $\frac{\pi}{2\sqrt{2}}$ units and which is at a minimum distance of $\frac{\pi}{2\sqrt{2}}$ units from the circle.

Key. 8

Sol. Let (h,k) be the point on the curve $y=\tan^{-1}x$.

Image of (h,k) in $y=x$ is (k,h) which is the centre of a circle of radius $\frac{\pi}{2\sqrt{2}}$

Given $P.M=\frac{\pi}{2\sqrt{2}}$ (shortest distance)

And $C.M=\frac{\pi}{2\sqrt{2}}$ (radius of circle)

Now, $CP=\sqrt{(h-k)^2+(k-h)^2}=\frac{\pi}{2}$

$\Rightarrow \sqrt{2}\,|h-k|=\frac{\pi}{\sqrt{2}} \Rightarrow |h-k|=\frac{\pi}{2}$

$\Rightarrow h-k=\pm\frac{\pi}{2} \Rightarrow k=h\pm\frac{\pi}{2}$

Since, (h,k) lies on $y=\tan^{-1}x$

$\Rightarrow k=h-\frac{\pi}{2}$

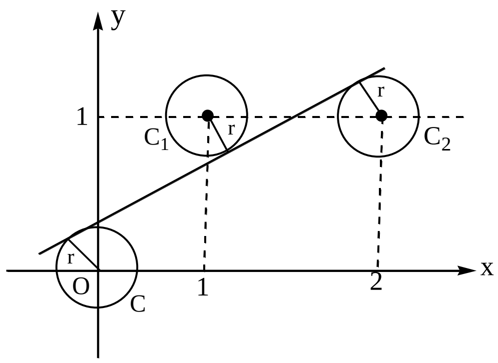
Now, $h=\frac{x}{2}=\tan^{-1}h$

Since, $0<h<\pi \Rightarrow \frac{-\pi}{2}<h-\frac{\pi}{2}<\frac{\pi}{2}$

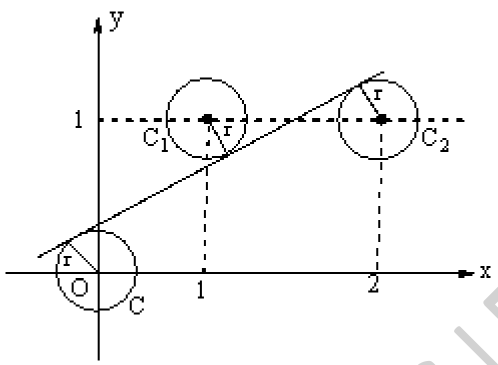
$h=\tan\left(h-\frac{\pi}{2}\right)=-\cot h \Rightarrow -h=\cot h$

11. As shown in figure three circles which have the same radius r, have centres at (0, 0), (1, 1), (2,

1). If they have a common tangent line, as shown, then the value of $10\sqrt{5}$ r is.



Key. 5



Sol.

Equation of line joining origin and centre of circle $C_2 \equiv (2,1)$ is, $y = \dfrac{x}{2}$

$\Rightarrow x - 2y = 0$

Let equation of common tangent is x – 2y + c = 0…………..(1)

$\therefore$ perpendicular distance from (0, 0) on this line

= perpendicular distance from (1, 1)

$\Rightarrow \left|\dfrac{c}{\sqrt{5}}\right| = \left|\dfrac{c-1}{\sqrt{5}}\right|$

$\Rightarrow c = 1 - c \Rightarrow c = \dfrac{1}{2}$

Equation of common tangent is

$x - 2y + \dfrac{1}{2} = 0$ or $2x - 4y + 1 = 0$ ………..(2)

Perpendicular from (2, 1) on the line (2)

$r = \left|\dfrac{4-4+1}{\sqrt{20}}\right| = \dfrac{1}{2\sqrt{5}} = \dfrac{\sqrt{5}}{10}$

12. The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let 'P' be the midpoint of the line segment joining the centres of C_1 and C_2

and C be a circle touching circles C_1 and C_2 externally. If a common tangents to C_1 and C passing through $'P'$ is also a common tangent to C_2 and C. Then the radius of the circle C is

Key. 8

Sol. $(r+1)^2 = \alpha^2 + 9$

$r^2 + 8 = \alpha^2$

$\Rightarrow r^2 + 2r + 1 = r^2 8 + 9 \Rightarrow 2r = 16$

$\therefore\ r = 8$

13. The tangents drawn from the origin to the circle $x^2 + y^2 - 2rx - 2hy + h^2 = 0$ are perpendicular then sum of all possible values of $\frac{h}{r}$ is ___

Ans: 0

Hint. Combined equation of the tangents drawn from (0, 0)to the circle is

$\left(x^2 + y^2 - 2rx - 2hy + h^2\right)h^2 = \left(-rx - hy + h^2\right)^2$ here coefficient of

$x^2 + \text{coffecient of } y^2 = 0 \Rightarrow \left(h^2 - r^2\right) + \left(h^2 - r^2\right) = 0$

$\Rightarrow \frac{h}{r} = \pm 1$

14. If the curves $\frac{x^2}{4} + y^2 = 1$ and $\frac{x^2}{a^2} + y^2 = 1$ for suitable value of a cut on four concyclic points, then find the radius of the smallest circle passing through these 4 points

Key: 1

Hint: $\left(\frac{x^2}{4} + y^2 - 1\right) + \lambda\left(\frac{x^2}{a^2} + y^2 - 1\right) = 0$

$x^2\left(\frac{a^2 + 4\lambda}{4a^2(1+\lambda)}\right) + y^2 = 1$

Clearly radius is 1 unit

15. The neighbouring sides AB and BC of a square ABCD of side $\left(2+\sqrt{2}\right)$ units are tangents to a circle. The vertex D of the square lies on the circumference of the circle. The radius of the circle is ……….

Key : 4

Sol : Let y = mx +c be the tangnt to the given ellipse $y = 2x \pm \sqrt{a^2 + 4^e + l^2}$ Which passes the $(-2,0)$ $4a^2 + l^2 = 16$ let s = al $s^2 = a^2 l^2 = a^2\left(16 - 4a^2\right)$

$s_{\max} at - a\sqrt{l}\ then\, l = \sqrt{8}$

Max. value of al = 4

16. D, E, F are mid points of sides BC, CA, AB of ΔABC and the circum circles of ΔDEF, ΔABC touch each other then $\left[\sum \cos^2 A\right] =$ ________ (where [.] denotes N.G.I.F)

Key. 1

Sol. $(OS)^2 = (2SN)^2$

$$= 4R^2\left[\sum \cos^2 A - \frac{3}{4}\right]$$

17. The radius of the circles which pass through the point $(2,3)$ and cut off equal chords of length 6 units along the lines $y - x - 1 = 0$ and $y + x - 5 = 0$ is $'r'$ then $[r]$ is (where [.] denotes greatest integer function)

Key. 4

Sol. The given two lines pass through the point $(2,3)$ and are inclined at 45^0 and 135^0 to the x–axis the other ends of chords can easily be calculated as

$\left(2+3\sqrt{2},\ 3+3\sqrt{2}\right)$ and $\left(2-3\sqrt{2},\ 3-3\sqrt{2}\right)$

There is symmetry about the line $x = 2$ and therefore the centres of circles lie on $x = 2$

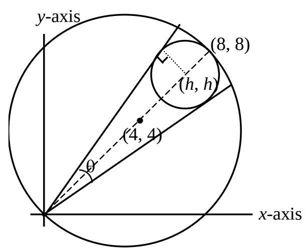
As the chords subtend right angles at the centre.

$\therefore\ 2r^2 = 6^2 \Rightarrow r = 3\sqrt{2}$

18. If the radius of the circle touching the pair of lines $7x^2 - 18xy + 7y^2 = 0$ and the circle $x^2 + y^2 - 8x - 8y = 0$, and contained in the given circle is equal to k, then k^2 is equal to

Key. 8

Sol.



$\tan\theta = \left|\dfrac{2\sqrt{h^2 - ab}}{a+b}\right| = \dfrac{4\sqrt{2}}{7}$

$\tan\dfrac{\theta}{2} = \dfrac{1}{2\sqrt{2}}$ on solving

$\sin\dfrac{\theta}{2} = \dfrac{1}{3} = \dfrac{\sqrt{2}(8-h)}{\sqrt{2}\,h}$

Hence equation of circle is $(x - 6)^2 + (y - 6)^2 = 8$.

19. The number of integral values of α for which the point $(\alpha - 1, \alpha + 1)$ lies in the larger segment of the circle $x^2 + y^2 - x - y - 6 = 0$ made by the chord whose equation is $x + y - 2 = 0$ is

Key. 1

Sol. S(x, y) = $x^2 + y^2 - x - y - 6 = 0$ (1)

has centre at $C \equiv \left(\frac{1}{2}, \frac{1}{2}\right)$

According to the required conditions, the given point P(α - 1, α + 1) must lie inside the given circle.

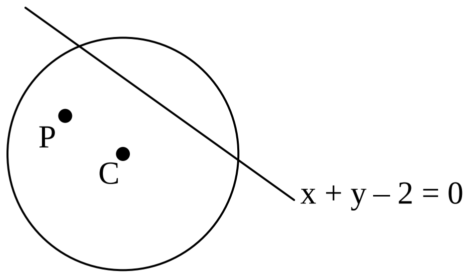
i.e. S(α - 1, α + 1) < 0

$\Rightarrow \quad (\alpha-1)^2 + (\alpha+1)^2 - (\alpha-1) - (\alpha+1) - 6 < 0$

$\Rightarrow \quad \alpha^2 - \alpha - 2 < 0$, i.e., $(\alpha-2)(\alpha+1) < 0$

$\Rightarrow \quad -1 < \alpha < 2$ (2)

Also P and C must lie on the same side of the line (see figure)



L (x, y) $\equiv x + y - 2 = 0$ (3)

i.e. L(1/2, ½) and $L(\alpha-1, \alpha+1)$ must have the same sign.

Since $L\left(\frac{1}{2}, \frac{1}{2}\right) = \frac{1}{2} + \frac{1}{2} - 2 < 0$

$L(\alpha-1, \alpha+1) = (\alpha-1) + (\alpha+1) - 2 < 0$, i.e., $\alpha < 1$ (4)

Inequalities (2) and (4) together give the permissible values of α as $-1 < \alpha < 1$.

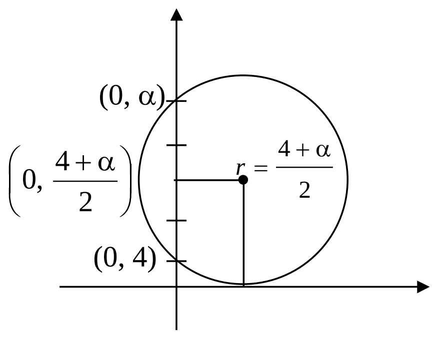
20. Radius of the smallest circle that can be drawn to pass through the point (0, 4) and touching the *x*–axis is

Key. 2

Sol.

$r = \frac{4+\alpha}{2}, \ \alpha \geq 0$

when α = 0, smallest radius = 2.



21. Let $M(-1, 2)$ and $N(1, 4)$ be two points in a plane rectangular coordinate system XOY. P is a moving point on the x-axis. When ∠MPN takes its maximum value, the x-coordinate of point P is

Key. 1

Sol. The centre of a circle passing through points M and N lies on the perpendicular bisector y = 3 – x of MN. Denote the centre by C(a, 3 – a), the equation of the circle is

$(x - a)^2 + (y - 3 + a)^2 = 2(1 + a^2)$

Since for a chord with a fixed length the angle at the circumference subtended by the corresponding arc will become larger as the radius of the circle becomes smaller. When $\angle$MPN reaches its maximum value the circle through the three points M, N and P will be tangent to the x-axis at P, which means

$2(1 + a^2) = (a - 3)^2 \Rightarrow a = 1$ or $a = -7$

Thus the point of contact are P(1, 0) or $P'(-7,0)$ respectively.

But the radius of circle through the points M, N and P' is larger than that of circle through points M, N and P.

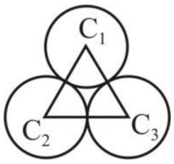
Therefore, $\angle MPN > \angle MP'N$. Thus P = (1, 0)

$\therefore$ x-coordinate of P = 1.

22. *r* be radius of incircle of triangle formed by joining centres of $(x-a)^2+(y-b)^2=9$, $(x-a)^2+(y-b-7)^2=16$ and circle touching above two circles and having radius 5 units. Find r^2.

Key. 5

Sol. All three circles touch each other externally



$C_1C_2 = 7$

$C_2C_3 = 9$

$C_3C_1 = 8$

$$s=\frac{7+8+9}{2}=12$$

$$\Delta=\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{12\times5\times3\times4}$$

$$r=\frac{\Delta}{s}=\sqrt{5}$$

Circles

Matrix-Match Type

1. Match the following

	Column I		Column II
(A)	The number of common tangents that can be drawn to the two circles $C_1 : x^2 + y^2 - 4x - 6y - 3 = 0$ $C_2 : x^2 + y^2 + 2x + 2y + 1 = 0$ is	(p)	2
(B)	The common chord of $x^2 + y^2 - 4x - 4y = 0$ and $x^2 + y^2 = 16$ subtends at the origin an angle equal to $\frac{\pi}{k}$, then k = ?	(q)	0
(C)	Shortest distance from the point (2, –7) to the circle $x^2 + y^2 - 14x - 10y - 151 = 0$ is	(r)	3
(D)	If real number x and y satisfy $(x + 5)^2 + (y - 12)^2 = (14)^2$ then the minimum value of $\sqrt{x^2 + y^2}$ is	(s)	1

Key. a – r ; b – p ; c – p ; d – s

Sol. (A) $C_1 = (2,3)\ r_1 = \sqrt{4+3+3} = 4$

$C_2 = (-1,-1)\ r_2 = \sqrt{1+1-1} = 1$

$C_1C_2 = 5$

$r_1 + r_2 = 5$

$C_1C_2 = r_1 + r_2$

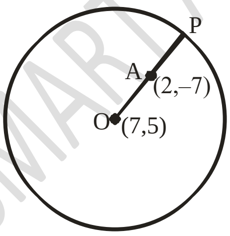
$\therefore$ No. of common tangent is 3.

(B) Common chord is $S_1 = S_2$

$x + y = 4$

x + y = 4 at origin subtend an angle of $\frac{\pi}{2}$

(C)



C = (7, 5)

$r = \sqrt{49+25+151} = 15$

$AP = OP - OA \quad OA = \sqrt{(7-2)^5 + (5+7)^2}$

AP = 15 – 13

= 2

(D) Let $x + 5 = 14\cos\theta$

$y - 12 = 14\sin\theta$

$x^2 + y^2 = 365 + 28(12\sin\theta - 5\cos\theta)$

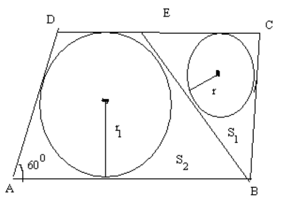
$\left(\sqrt{x^2+y^2}\right)_{\min} = \sqrt{365 - 28\times 13} = 1$

2. In the parallelogram ABCD with angle $A = 60^0$, the bisector of angle B is drawn which cuts the side CD at point E. A circle S_1 of radius 'r' is inscribed in the ΔECB. Another circle 'S_2' is inscribed in the trapezoid ABED.

	Column I		Column II
(A)	The value of radius of S_2 is	(p)	$2\sqrt{3}r$
(B)	The value of distance between the centres of S_1 and S_2 is	(q)	$\frac{\sqrt{3}}{2}r$
(C)	The value of the length of internal common tangent of $S_1 \& S_2$ is	(r)	$\sqrt{7}\ r$
(D)	The value of the length CE is	(s)	$\frac{3}{2}r$
		(t)	r

Key: A$\to$ s, B$\to$ r C$\to$ q, D$\to$ p

Hint:



3. Match the column:

Column – I | Column - II

a) If the circle $x^2 + y^2 + 2x + c = 0$ and $x^2 + y^2 + 2y + c = 0$ touch each other, then the set of 'c' values is contained in or equal to — p) the set satisfying c = 1

b) If the circle $x^2 + y^2 + 2x + 3y + c = 0$ and $x^2 + y^2 - x + 2y + c = 0$ intersect orthogonally, then the set of 'c' values is contained in or equal to — q) the set satisfying $|c| < 2$

c) If the circle $x^2+y^2=9$ contains the r) the set satisfying $c=\frac{1}{2}$

circle $x^2+y^2-2x+1-c^2=0$, then

the set of 'c' values is contained in or equal to

d) If the circle $x^2+y^2=9$ is contained in the circle s) the set satisfying $|c|>8$

$x^2+y^2-6x-8y+25-c^2=0$, then

the set of 'c' values is contained in or equal to

t) the set satisfying $2<|c|<8$

Key. $(A)-q,r;(B)-p,q$
$(C)-q\ ;(D)-s$

Sol. a) $c=1/2$

b) $c=1$

c) $C_1(0,0)\quad C_2(1,0)$

$r_1=3\quad r_2=|c|$

$C_1C_2<r_1-r_2\ \Rightarrow |c|<2$

d) $C_1(0,0)\ \ r_1=3$

$C_2(3,4)\ \ r_2=|c|$

$|c|>8$

4. List – I List – II

a) If $2a,b,c$ are in A.P then the lines $ax+by=c$ are concurrent at p) $(-1,1)$

b) The ortho centre of the triangle form by the lines q) $(1,-1)$

$x+y=0, x-y+4=0,\ x+2y+1=0$

c) $x-y+8=0,\ \ x=0,\ \ y=0$ forms ΔOAB then the centre of the circle r) $(2,-2)$

passing through middle points of sides of ΔOAB

d) The in – centre of the triangle formed by $x=0, y=0$, s) $(-2,2)$

$3x-4y-12=0$

Key. $a-s;b-s;$
$c-s;d-q$

Sol. Circle passing through midpoints is nine point circle

5. List – I List – II

a) The line $x+2y=5$ touches the circle $x^2+y^2-4x-8y+15=0$ at p) $\left(\frac{1}{2},\sqrt{2}\right)$

b) A, B are two points on the circle $x^2+y^2-4x-8y-1=0$, $O(0,0)$ and $\overline{OA},\overline{OB}$ are tangents to the circle then the circumcentre of the triangle ΔOAB q) $(1,2)$

c) Transverse common tangents of the circles $x^2+y^2-2x+4y+4=0$, $x^2+y^2+4x-2y+1=0$ meet at r) $\left(\frac{1}{2},-\sqrt{2}\right)$

d) Circle passes through $(0,0),(1,0)$ touching the circle $x^2+y^2=9$ then the centre of the circle s) $(0,-1)$

Key. $a-q; b-q;$
$c-s; d-p,r$

Sol. (x_1,y_1) lie on circle $x^2+y^2=4$, (x_2,y_2) lie on circle, $(x-1)^2+y^2=16$

6. Match the following:

Column-I		Column-II	
a)	Locus of centre of circles touching $x^2+y^2-4x-4y=0$ internally and $x^2+y^2-6x-6y+17=0$ externally is,	p)	Straight line
b)	The locus of the point $(3h-2,3k)$ where (h,k) lies on the circle $x^2+y^2-2x-4y-4=0$ is	q)	Circle
c)	Locus of centres of the circles touching the two circles $x^2+y^2+2x=0$ and $x^2+y^2-6x+5=0$ externally is	r)	Ellipse
d)	emities of a diagonal of a rectangle are $(0,0)$ and $(4,4)$. The locus of the extremities of the diagonal is	s)	Part of Hyperbola

Key. A-r, C-s, B-a, D-q

Sol. $a\to r, b\to q, c\to s, d\to q$

a) $sp+s^1p=2a$

b) $\alpha=32-2, \beta=3k \Rightarrow \frac{\alpha+2}{3}=h, \frac{\beta}{3}=k$

c) $|sp-s^1p|=2a$

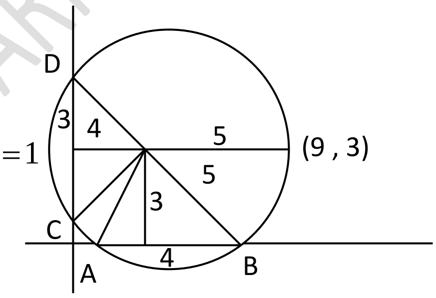
d) Locus is a circle with the given diagonal as diameter

7.

	Column I		Column II
A	A circle cut off an intercept of 8 unit on the x – axis and k units on y-axis. If tangent at (9, 3) is parallel to y-axis then k equal to	p	12
B	If $y = 2[2x-1]-1 = 3[2x-2]+1$ then the values of $[y+5x]$ can be, [ ] denotes the G.I. F	q	8
C	The number of solution of equation $\cos x\sqrt{16\sin^2 x} = 1$ in $(-\pi, \pi)$ is	r	7
d	If one root of the equation $(a-6)x^2 - (a+6)x + 10 = 0$ is smaller than 1 and the other root greater than 2 then the value of a can be	s	6
		t	4

Key: A$\to$S; B$\to$Q,R,S;C$\to$T; D$\to$P,Q,R

Hint: a) see figure



CD = 6

$$(b)\ y = 2[2x]-3 = 3[2x]-5 \Rightarrow [2x] = 2 \Rightarrow y = 1$$

$$2 \le 2x < 3 \Rightarrow 5 \le 5x \le 7.5 \Rightarrow 6 \le 5x + y \le 8.5$$

$$[5x+y] = 6, 7, or\, 8.$$

$$(c)\ 4|\sin x|\cos x = 1$$

solutions are possible if $\cos x > 0$

$$\Rightarrow \frac{-\pi}{2} \le x \le \frac{\pi}{2} \Rightarrow -\pi \le 2x \le \pi$$

$$if\ \ x \in \left[-\frac{\pi}{2}, 0\right)$$

$$\sin 2x = \frac{-1}{2} \Rightarrow 2x = \frac{-\pi}{6}, \frac{-5\pi}{6} \Rightarrow x = \frac{-\pi}{12}, \frac{-5\pi}{12}$$

$$if\ x \in \left(0, \frac{\pi}{2}\right)$$

$$\sin 2x = \frac{1}{2} \Rightarrow 2x = \frac{\pi}{6}, \frac{5\pi}{6} \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}$$

$$(d)(a-6)\, f(1) < 0 \Rightarrow a > 6$$

$$(a-6)\, f(2) < 0 \Rightarrow 6 < a < 13$$

8. Match the following

COLUMN_I	COLUMN_II
A) The radical axis of two circles	P) subtends a right angle at a point of intersection
B) The common tangent to two intersecting circles of equal radii	Q) is perpendicular to the line joining the centres
C) The common chord of two intersecting Circles	R) is parallel to the line joining the centres
D) The line joining the centres of two circles intersecting orthogonally	S) is bisected by the line joining the centres.

Key. A-Q;B-R;C-QS;D-P

Sol. Let the equations of the circles be $x^2+y^2+2g_1x+2f_1y+c_1=0$ and
$x^2+y^2+2g_2x+2f_2y+c_2=0$

(A) Equation of the radical axis is $2(g_1-g_2)x+2(f_1-f_2)y+c_1-c_2=0$

Slope of the radical axis $=-\dfrac{g_1-g_2}{f_1-f_2}$

Slope of the line joining the centres $=\dfrac{f_1-f_2}{g_1-g_2}$

So the radical axis is perpendicular to the line joining the centre

(B) Common tangent to the intersecting circles of equal radii is at the same distance from the centres of the two circles and hence is parallel to the line joining the centres

(C) Since the line joining the centres of the circles to the mid-point of the common chord is perpendicular to the chord, it bisects the chord.

(D) The line joining the centre of one to a point of intersection is tangent to the other circle. SO by definition of orthogonality, they are perpendicular

9. Match the following

COLUMN_I	COLUMN_II
A) If a circle passes through $A(1,0)\,B(0,-1)$ and $C\left(\frac{1}{\sqrt{3}},\sqrt{\frac{2}{3}}\right)$ such that the tangent at B makes an angle θ with line AB then $\tan\theta$ equals	P) -4
B) From a point $(h,0)$ common tangents are drawn to the circles $x^2+y^2=1$ and the $(x-2)^2+y^2=4.$ The value of h can be	Q) -2
C) If the common chord of the circle $x^2+y^2=8$ and $(x-a)^2+y^2=8$ subtends right angle at the	R) 1

origin then a can be

D) If the tangents drawn from S) 2

$(4,k)$ to the circle $x^2+y^2=10$

are at right angles then k can be T) 4

Key. A-R; B-Q; C-P,T; D-Q,S

Sol. (A) Origin is the Circumcentre $\Rightarrow$ circle is $x^2+y^2=1 \Rightarrow \theta=\dfrac{\pi}{4}$

(B) A tangent to $x^2+y^2=1$ is $y=mx\pm\sqrt{1+m^2}$ It touches $(x-2)^2\, y^2=4$ if

$$\left|\frac{2m\pm\sqrt{1+m^2}}{\sqrt{1+m^2}}\right|=2$$

$\Rightarrow m=\pm\dfrac{1}{\sqrt{3}}$ The common tangents are $y=\dfrac{1}{\sqrt{3}}x+\dfrac{2}{\sqrt{3}}$ and $y=\dfrac{1}{\sqrt{3}}x-\dfrac{2}{\sqrt{3}}$ which intersect at $(-2,\ 0)$

(C) Common chord of the given circles is $(x^2+y^2-8)-\left[(x-a)^2+y^2-8\right]=0$

$\Rightarrow 2x-a=0$

$\Rightarrow \dfrac{2x}{a}=1$

Homogenising $x^2+y^2-8=0 \Rightarrow x^2+y^2-8\left(\dfrac{2x}{a}\right)^2=0$ It represents perpendicular lines

$\Rightarrow 1-\dfrac{32}{a^2}+1=0 \Rightarrow a^2=16 \Rightarrow a=\pm 4$

(D) $(4,k)$ must lie on the director circle of the given circle. which is $x^2+y^2=20.$ Thus

$16+k^2=20 \Rightarrow k=\pm 2$