Probability

Single Correct Answer Type

1. The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is

1)37/256 2)28/256 3)128/256 4)219/256

Key. 2

Sol. np=6

npq=2

$\Rightarrow q=\frac{1}{2}, p=\frac{1}{2}, n=8$

2. For a binomial distribution $\bar{x}=4,\ \sigma=\sqrt{3}$. Then P(X=r)=

1) ${}^{16}C_r\left(1/4\right)^r\left(3/4\right)^{16-r}$ 2) ${}^{12}C_r\left(1/4\right)^r\left(3/4\right)^{12-r}$

3) ${}^{16}C_r\left(3/4\right)^r\left(1/4\right)^{16-r}$ 4) ${}^{12}C_r\left(3/4\right)^r\left(1/4\right)^{12-r}$

Key. 1

Sol. np=4

npq=3

$\Rightarrow q=\frac{3}{4}, p=\frac{1}{4}, n=16$

3. If X is Poisson variate with $P\left(X=0\right)=P\left(X=1\right), then\ P\left(X=2\right)=$

1)e/2 2)e/6 3)1/(6e) 4)1/(2e)

Key. 4

Sol. $P\left(x=r\right)=\frac{e^{-\lambda}\lambda^r}{r!}$

4. In a Poisson distribution the variance is m. The sum of the terms in odd places in this distribution is

1) e^{-m} 2) $e^{-m}\cosh m$ 3) $e^{-m}\sinh m$ 4) $e^{-m}\coth m$

Key. 2

Sol. Conceptual

5. Two natural numbers *a* and *b* are selected at random. The probability that a^2+b^2 is divisible by 7 is

(a) 3/8 (b) 1/7 (c) 3/49 (d) 1/49

Key. D

Sol. $a_1 b$ are is of then form

$a_1 b \in \{7m, 7m+1, 7m+2, 7m+3, 7m+4, 7m+6\}$

$a_1^2 b^2 \in \{7m_1, 7m_1+1, 7m_1+4, 7m_1+2, 7m_1+2, 7m_1+4, 7m_1+1\}$

$\therefore a^2, b^2$ must be of the form 7m.

Probability = $\frac{1}{49}$

6. If a and b are chosen randomly from the set consisting of numbers 1, 2, 3, 4, 5, 6 with replacement. Then probability that $\underset{x\to 0}{Lt}\left(\frac{a^x+b^x}{2}\right)^{2/x}=6$ is

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{9}$ (d) $\frac{2}{9}$

Key. C

Sol. $\underset{x\to 0}{Lt}\left(\frac{a^x+b^x}{2}\right)^{2/x}=6$

$$=e^{\underset{x\to 0}{Lt}\left(\frac{a^x-1}{x}\right)+\left(\frac{b^x-1}{x}\right)}=6$$

$$=e^{\log a+\log b}=6$$

ab = 6

$(a,b)=(1,6),(6,1),(2,3),(3,2)$

Required probability = $\frac{4}{6\times 6}=\frac{1}{9}$

7. Thirty-two players ranked 1 to 32 are playing in a knockout tournament. Assume that in every match between any two players, the better-ranked player wins, the probability that ranked 1 and ranked 2 players are winner and runner up respectively, is

(A) 16/31 (B) 1/2 (C) 17/31 (D) None of these

Key. A

Sol. For ranked 1 and 2 players to be winners and runners up res., they should not be paired with each other in any rounded. Therefore, the required probability $\frac{30}{31}\times\frac{14}{15}\times\frac{6}{7}\times\frac{2}{3}=\frac{16}{31}$

8. A coin is tossed 7 times. Then the probability that at least 4 consecutive heads appear is

(A) 3/16 (B) 5/32 (C) 5/16 (D) 1/8

Key. B

Sol. Let H denote the head,
T the tail.
* Any of the head or tail

P(H) = $\frac{1}{2}$, P(T) = $\frac{1}{2}$ P(*) = 1

HHHH*** $=\left(\frac{1}{2}\right)^4\times 1=\frac{1}{16}$

THHHH** $= \left(\frac{1}{2}\right)^5 \times 1 = \frac{1}{32}$

THHHH $= \left(\frac{1}{2}\right)^5 \times 1 = \frac{1}{32}$

**THHHH = $\left(\frac{1}{2}\right)^5 \times 1 = \frac{1}{32}$

$\frac{5}{32}$

9. A fair coin is tossed 5 times then probability that two heads do not occur consecutively (No two heads come together)

1. $\frac{1}{16}$ 2. $\frac{15}{32}$ 3. $\frac{13}{32}$ 4. $\frac{7}{16}$

Key. 3

Sol. $p\left(\frac{E}{no\,heads}\right)+p\left(\frac{E}{1(head)}\right)+p\left(\frac{E}{2-heads}\right)+p\left(\frac{E}{3-heads}\right)$

Where $E \rightarrow$gtg n two consecutive heads.

$=\frac{1}{32}+\frac{5}{32}+\frac{6}{32}+\frac{1}{32}=\frac{14}{32}=\frac{7}{16}$

10. A man throws a die until he gets a number bigger than 3. The probability that he gets 5 in the last throw

1. $\frac{1}{3}$ 2. $\frac{1}{4}$ 3. $\frac{1}{6}$ 4. $\frac{1}{36}$

Key. 1

Sol. P(gtg a number bigger than 3) $=\frac{1}{2}$

P(gtg 5 in throw)=$\frac{1}{6}$

$E \rightarrow$gtg 5 in last throw when he gets a number bigger than 3

$P(E)=\frac{1}{6}+\frac{1}{2}.\frac{1}{6}+\frac{1}{2}.\frac{1}{2}.\frac{1}{6}+....\infty$

$=\frac{1}{6}\times\frac{1}{1-\frac{1}{2}}=\frac{1}{3}$

11. A bag contains 4-balls two balls are drawn from the bag and are found to be white then probability that all balls in the bag are white

1. $\frac{1}{5}$ 2. $\frac{2}{5}$ 3. $\frac{3}{5}$ 4. $\frac{4}{5}$

Key. 3

Sol. $P(E)=\dfrac{\frac{1}{3}\cdot\frac{4_{C_2}}{4_{C_2}}}{\frac{1}{3}\left\{\frac{2_{C_2}}{4_{C_2}}+\frac{3_{C_2}}{4_{C_2}}+\frac{4_{C_2}}{4_{C_2}}\right\}}$

$=\dfrac{1}{\frac{1+3+6}{6}}=\frac{6}{10}=\frac{3}{5}$

12. A randomly selected year is containing 53 Mondays then probability that it is a leap year

1. $\frac{2}{5}$ 2. $\frac{3}{5}$ 3. $\frac{4}{5}$ 4. $\frac{1}{5}$

Key. 1

Sol. Selected year may non leap year with a probability $\frac{3}{4}$

Selected year may leap year with a probability $\frac{1}{4}$

$E \rightarrow$Even that randomly selected year contains 53 Mondays

$P(E)=\frac{3}{4}\times\frac{1}{7}+\frac{1}{4}\times\frac{2}{7}=\frac{5}{28}$

$P\left(\frac{leapyear}{E}\right)=\dfrac{\frac{2}{28}}{\frac{5}{28}}=\frac{2}{5}$

13. When 5-boys and 5-girls sit around a table the probability that no two girls come together

1. $\frac{1}{120}$ 2. $\frac{1}{126}$ 3. $\frac{3}{47}$ 4. $\frac{4}{7}$

Key. 2

Sol. $E \rightarrow$first boys can be arranged in $\lfloor 4$ ways, then there are 5-gaps between boys in 5-gaps, 5-girls can be arranged in $\lfloor 5$ ways

$P(E)=\dfrac{\lfloor 5 \lfloor 4}{\lfloor 9}=\dfrac{5\times4\times3\times2}{5\times6\times7\times8\times9}=\frac{1}{126}$

14. There are m-stations on a railway line. A train has to stop at 3 intermediate stations then probability that no two stopping stations are adjacent

1. $\dfrac{1}{m\,c_3}$ 2. $\dfrac{3}{mc_3}$ 3. $\dfrac{m-2_{C_3}}{mc_3}$ 4. $\dfrac{mc_2}{m\,c_3}$

Key. 3

Sol. Let 3-stopping stations be S_1, S_2, S_3 then are m-3 stations remaining. Between these m-3 stations there are m-2 places select any 3 for S_1, S_2, S_3, then there are no two stopping stations are adjacent

$$P(E) = \frac{m-2_{C_3}}{m_{C_3}}$$

15. The probability that randomly selected positive integer is relatively prime to 6

1. $\frac{1}{2}$ 2. $\frac{1}{3}$ 3. $\frac{1}{6}$ 4. $\frac{5}{6}$

Key. 2

Sol. Among every 6-consecutive integers one divisible by 6 and other integers leaves remainders 1,2,3,4,5 when divided by 6

The numbers which leave the remainder 1 and 5 are relatively prime to 6

Required probability $\frac{2}{6} = \frac{1}{3}$

16. A and B are events such that P(A)=0.3 $P(A \cup B) = 0.8$. If A and B are independent then $P(B)$ =

1. $\frac{1}{7}$ 2. $\frac{3}{7}$ 3. $\frac{5}{7}$ 4. $\frac{6}{7}$

Key. 3

Sol. $P(A \cap B) = P(A).P(B)$

$P(A \cup B) = P(A) + P(B) - P(A)P(B)$

$0.8 = 0.3 + P(B)(1-0.3)$

$0.5 = P(B)(0.7) \Rightarrow P(B) = \frac{5}{7}$

17. In a 3×3 matrix the entries a_{ij} are randomly selected from the digits {0,1,2 __9} with replacement. The probability that the numbers of the form xyz where x,y,z are the elements in each row will be divisible by 11 is $\dfrac{7^{K_1}.13^{K_2}}{10^9}$ then $K_1 + K_2 =$ ____

Key. 6

Sol. The number of multiples of 11 from 000 to 999 is 91.

The required probability $=(\frac{91}{1000})^3=\frac{7^3\times 13^3}{10^9}$

18. Triangles are formed with vertices of a regular polygon of 20 sides. The probability that no side of the polygon is a side of the triangle is $\frac{\lambda}{57}$ Then $\frac{\lambda}{40}$ is ____

Key. 1

Sol. The total number of triangles $=20_{C_3}=1140$ there are 20 triangles with two sides of polygon there are 20×16 triangles with are side of polygon $\therefore$ required probability $=\frac{1140-20-320}{1140}=\frac{800}{1140}=\frac{40}{57}$

19. The probability that $\sin^{-1}(\sin x)+\cos^{-1}(\cos y)$ is an integer $x, y\in\{1,2,3,4\}$ is

A. $\frac{1}{16}$ B. $\frac{3}{16}$ C. $\frac{15}{16}$ D. $\frac{1}{17}$

Key. B

Sol. $Sin^{-1}(\sin x)+\cos^{-1}(\cos y)$ to be integer $x\in[-\frac{\pi}{2}\frac{\pi}{2}]$ and

$y\in[0\,\pi]\Rightarrow x=1 and\ y=1,2,3.$ required probability $=\frac{3}{16}$.

20. Seven coupons are selected at random one at a time with replacement from 15 coupons numbered 1 to 15. The probability that the largest number appearing on a selected coupon is 9, is

A) $\left(\frac{9}{16}\right)^6$ B) $\left(\frac{8}{15}\right)^7$ C) $\left(\frac{3}{5}\right)^7$ D) None of these

Key. D

Sol. Each coupon can be selected in 15 ways. The total number of ways of choosing 7 copouns is 15^7. If largest number is 9, then the selected numbers have to be from 1 to 9 excluding those consisting of only 1 to 8.

Probability desired is $\frac{9^7-8^7}{15^7}$

$$=\left(\frac{3}{5}\right)^7-\left(\frac{8}{15}\right)^7$$

21. If the letters of the word MATHEMATICS are arranged arbitrarily, the probability that C comes before E, E before H,H before I and I before S is

A) $\frac{1}{75}$ B) $\frac{1}{24}$ C) $\frac{1}{120}$ D) $\frac{1}{720}$

Key. C

Sol. The total numbers of arrangements is $\frac{11!}{2!2!2!}=\frac{11!}{8}$

The number of arrangements in which C, E, H, I, S appear in that order

$=\binom{11}{5}\frac{6!}{2!2!2!}=\frac{1}{8.5!}$

Probability $=\frac{11!}{8.5!}\div\frac{11!}{8!}=\frac{1}{5!}=\frac{1}{120}$

22. A signal which can be green or red with probability $\left(\frac{4}{5}\right)$ and $\left(\frac{1}{5}\right)$ respectively is received by the station A and Transmitted to B. The probability each station receive signal correctly $=\left(\frac{3}{4}\right)$. If signal in received in B is green. The probability original signal was green.

(A) $\frac{3}{5}$ (B) $\frac{6}{7}$ (C) $\frac{20}{23}$ (D) $\frac{9}{20}$

Key. C

Sol. G = Original signal green. A = A receive correct signal, B = B receive signal correct. E is signal received by B is green.

$P\left(\frac{G}{E}\right)=\frac{P(G\cap E)}{P(E)}$

$P(E)=P(GAB)+P(G\bar{A}\bar{B})+P(\bar{G}A\bar{B})+P(\bar{G}\bar{A}B)$

$=\frac{4}{5}.\frac{3}{4}.\frac{3}{4}+\frac{4}{5}.\frac{1}{4}.\frac{1}{4}+\frac{1}{5}.\frac{3}{4}.\frac{1}{4}+\frac{1}{5}.\frac{1}{4}.\frac{3}{4}=\frac{46}{80}$

$P(G\cap E)=P(GAB)+P(G\bar{A}\bar{B})=\frac{40}{80}$

23. Three fair and unbiased dice and rolled at a time. The probability that the numbers shown are totally different.

(A) $\frac{5}{9}$ (B) $\frac{27}{216}$ (C) $\frac{4}{9}$ (D) $\frac{2}{3}$

Key. A

Sol. $n(s)=6^3$

$n(E)=$ Available 6 different numbers or 3 places in $^6P_3$ Hence $P(E)\frac{5}{9}$

24. A bag contains 7 black and 4 white balls two balls are drawn at a time from the bag. The probability at least one white ball is selected is

(A) $\frac{7}{11}$ (B) $\frac{5}{11}$ (C) $\frac{28}{55}$ (D) $\frac{34}{55}$

Key. D

Sol. $1-\left[\frac{7_{C_2}}{11_{C_2}}\right]$

25. There are ten pairs of shoes in a cup board out or which 4 are picked up at random one after the other. The probability that there is at least one pair is

(A) $\frac{4}{11}$ (B) $\frac{3}{11}$ (C) $\frac{33}{107}$ (D) $\frac{99}{323}$

Key. D

Sol. Out of 20 shoes 4 be taken in 20_{P_4}.

Ways of getting no. pair = 20 x 18 x 16 x 14

Probability of no.pair $=\frac{224}{323}$

at least one pair = $1-\frac{224}{323}=\frac{99}{323}$

26. Out of 10 persons sitting at a round table. Three persons are selected at random one after the other. The chance that no two of the selected are together.

(A) $\frac{1}{2}$ (B) $\frac{2}{5}$ (C) $\frac{5}{9}$ (D) $\frac{5}{12}$

Key. D

Sol. Out of 10 persons 3 can be selected in 10_{P_3} ways = 10.9.8 = 720. First person in 10 ways.

Other 2 in 7_{C_2} ways in which 2 are together in 6 ways. Hence

$$P(E)=\frac{10\times\left(7_{C_2}-6\right)\times 2}{720}=\frac{5}{12}.$$

27. A bag contains n white and n red balls. Pairs of balls are drawn with out replacement until the bag is empty. The probability that each pair consists of one white and one red ball is

(A) $\frac{\lfloor n.\lfloor n}{\lfloor 2n}.2^{n-1}$ (B) $\frac{(\lfloor n).2^n}{\lfloor 2n}$

(C) $\frac{(\lfloor n)^2.2^n}{\lfloor 2n}$ (D) $\frac{(\lfloor n).2^{n-1}}{\lfloor 2n}$

Key. C

Sol. $n(s)=2n_{C_2}.2n-2_{C_2}......2_{C_2}$

$n(E)=n^2(n-1)^2(n-2)^2...1^2=(\lfloor n)^2$

28. Let w be complex cube root of unity with $w\neq 1$. A fair die is thrown 3 times. If r_1, r_2, r_3 be the numbers obtained on the die the probability that $w^{r_1}+w^{r_2}+w^{r_3}=0$ is

(A) $\frac{1}{18}$ (B) $\frac{1}{9}$ (C) $\frac{2}{9}$ (D) $\frac{1}{36}$

Key. C

Sol. r_1, r_2, r_3 must be of the four 3n, 3n+1, 3n+2 $P(E) = \frac{\lfloor 3 . {}^2C_1 . {}^2C_1 . {}^2C_1}{6^3} = \frac{2}{9}$

29. Four persons are selected at random out of 3 men, 2 women and 4 children. The probability that there are exactly 2 children in the selection is

A) 11/21 B) 9/21 C) 10/21 D) 8/21

Key. C

Sol. Req. = 2 childrens and 2 others $= \frac{{}^4C_2 \times {}^5C_2}{{}^9C_4} = \frac{10}{21}$

30. 10 different books and 2 different pens are given to 3 boys so that each gets equal number of things. The probability that the same boy does not receive both the pens is

A) $\frac{5}{11}$ B) $\frac{7}{11}$ C) $\frac{10}{11}$ D) $\frac{6}{11}$

Key. C

Sol. n (S) = ${}^{12}C_4 \times {}^8C_4 \times {}^4C_4 \times 3!$

n (E) = the number of ways in which one boy gets both the pens

= ${}^{10}C_2 \times {}^8C_4 \times {}^4C_4 \times (3!)$

$$\therefore P(E) = 1 - \frac{{}^{10}C_2 \times {}^8C_4 \times {}^4C_4 \times (3!)}{{}^{12}C_4 \times {}^8C_4 \times {}^4C_4 3!} = 1 - \frac{1}{11} = \frac{10}{11}$$

31. From a bag containing 10 distinct balls, 6 balls are drawn simultaneously and replaced. Then 4 balls are drawn. The probability that exactly 3 balls are common to the drawings is

A) 8/21 B) 6/19 C) 5/24 D) 9/22

Key. A

Sol. Let S be the sample space of the composite experiment of drawing 6 in the first draw and then four in second draw then $n(S) = {}^{10}C_6 \times {}^{10}C_4$

$\therefore$ Required Probability $= \frac{{}^{10}C_6 \times {}^6C_3 \times {}^4C_1}{{}^{10}C_6 \times {}^{10}C_4} = \frac{80 \times 24}{10 \times 9 \times 8 \times 7} = \frac{8}{21}$

32. If the letters of the word MATHEMATICS are arranged arbitrarily, the probability that C comes before E, E before H,H before I and I before S is

A) $\frac{1}{75}$ B) $\frac{1}{24}$ C) $\frac{1}{120}$ D) $\frac{1}{720}$

Key. C

Sol. The total numbers of arrangements is $\frac{11!}{2!2!2!} = \frac{11!}{8}$

The number of arrangements in which C, E, H, I, S appear in that order

$= \binom{11}{5} \frac{6!}{2!2!2!} = \frac{1}{8.5!}$

Probability $= \frac{11!}{8.5!} \div \frac{11!}{8!} = \frac{1}{5!} = \frac{1}{120}$

33. There are 12 pairs of shoes in a box. Then the possible number of ways of picking 7 shoes so that there are exactly two pairs of shoes are

A) 63360 B) 63300 C) 63260 D) 63060

Key. A

Sol. Total number of ways of picking up 7 shoes with 2 pairs is ${}^{12}C_2 \times {}^{10}C_3 \times 2^3$

34. A pair of unbiased dice are rolled together till a sum of either 5 or 7 is obtained. Probability that 5 comes before 7 is

A) $\frac{1}{5}$ B) $\frac{2}{5}$ C) $\frac{3}{5}$ D) $\frac{4}{5}$

Key. B

Sol. A-event that sum 5 occurs, B-sum 7 occurs

$P(A) = \frac{1}{9}$, $P(B) = \frac{1}{6}$,probability that neither a sum 5 or 7 occur $P = \frac{13}{18}$

$P = (A\,occurs\,before\,B) = \frac{1}{9} + \left(\frac{13}{18}\right)\left(\frac{1}{9}\right) + \ldots\ldots\infty = \frac{2}{5}$

35. Team A plays with 5 other teams exactly once. Assuming that for each match the probabilities of a win, draw and loss are equal, then

A) the probability that A wins and loses equal number of matches is $\frac{34}{81}$

B) the probability that A wins and loses equal number of matches is $\frac{17}{81}$

C) the probability that A wins more number of matches than it loses is $\frac{17}{81}$

D) the probability that A loses more number of matches than it wins is $\frac{16}{81}$

Key. B

Sol. Probability of equal number of W and L is

$(0)W,(0)L+(1)W,(1)L+(2)W,(2)L$

$=\left(\frac{1}{3}\right)^5+{}^5C_1.{}^4C_1\left(\frac{1}{3}\right)^5+{}^5C_2.{}^3C_2\left(\frac{1}{3}\right)^5=\frac{17}{81}$

36. A box contains 24 identical balls of which 12 are white and 12 black. The balls are drawn at random from the box one at a time with replacement. The probability that a white ball is drawn for the 4th time on the 7th draw is

A) $\frac{5}{64}$ B) $\frac{27}{32}$ C) $\frac{5}{32}$ D) $\frac{1}{2}$

Key. C

Sol. In any trail, P(getting white ball) $=\frac{1}{2}$

P(getting black ball) $=\frac{1}{2}$

Now, required event will occur if in the first six trails 3 white balls are drawn in any one of the

3 trails from six. The remaining 3 trails must be kept reserved for black balls. This can happen in ${}^6C_3\times{}^3C_3=20$ ways.

So, required probability $=20\times\left(\frac{1}{2}\right)^3\times\left(\frac{1}{2}\right)^3\times\frac{1}{2}=\frac{5}{32}$

37. Four identical dice are rolled once. Probability that atleast 3 different numbers appear on them is

A) $\frac{13}{42}$ B) $\frac{17}{42}$ C) $\frac{23}{42}$ D) $\frac{25}{42}$

Key. D

Sol. '$aaaa$' can appear in ${}^6C_1$ ways

'$aaab$' can appear in $2\left({}^{6}C_{2}\right)=30$

'$aabb$' can appear in ${}^{6}C_{2}=15$

'$aabc$' can appear in $3\left({}^{6}C_{3}\right)=60$

'$abcd$' can appear in ${}^{6}C_{4}=15$

Probability = $\dfrac{60+15}{6+30+15+60+15}=\dfrac{25}{42}$

38. If $x,y,z\in R$ and $x+y+z=5,\ xy+yz+zx=3$, probability for x to be positive only is

A) $\frac{3}{16}$ B) $\frac{5}{16}$ C) $\frac{13}{16}$ D) $\frac{15}{16}$

Key. C

Sol. Range of x is $\left[-1,\frac{13}{3}\right]$

Since probability of x to be

Positive is $\dfrac{\int_{0}^{\frac{13}{3}}dx}{\int_{-1}^{\frac{13}{3}}dx}=\dfrac{\frac{13}{3}}{\frac{13}{3}+1}=\dfrac{13}{16}$

39. If F is the set of all onto functions from a set of vowels to set having 3 elements and $f\in F$ is chosen randomly ,then the probability that $f^{-1}(x)$ is a singleton is

A) $\frac{7}{15}$ B) $\frac{8}{15}$ C) $\frac{9}{15}$ D) $\frac{10}{15}$

Key. A

Sol. No.of onto functions from A having 5 elements to set B having 3 elements is 150 we shall now count onto function which satisfy $f^{-1}(x)$ is singleton. We can choose a singleton in ${}^{5}C_{1}$ ways . The remain 4 elements can be mapped to remaining 2 elements in $2^4-2=14$ ways

$\therefore$ desired prob $=\dfrac{5(14)}{150}=\dfrac{7}{15}$

40. Probability that a random chosen three digit number has exactly 3 factors is

A) $\frac{2}{225}$ B) $\frac{7}{900}$ C) $\frac{1}{300}$ D) $\frac{4}{900}$

Key. B

Sol. A number has exactly 3 factors if the number is square of a prime number ,

squares of 11,13,17,19,23,29,31 are 3 digit numbers, required probability $=\frac{7}{900}$

41. If a, b are chosen randomly from the numbers present on a unbiased die with replacement. Probability that $\lim_{x\to 0}\left(\frac{a^x+b^x}{2}\right)^{\frac{2}{x}}=6$ is

A) $\frac{1}{3}$ B) $\frac{1}{4}$ C) $\frac{1}{9}$ D) $\frac{2}{9}$

Key. C

Sol. $\lim_{x\to 0}\left(\frac{a^x+b^x}{2}\right)^{\frac{2}{x}}=ab=6$

Total number of possible ways in which a, b can take values = 36

Possible ways are $\{(1,6),(6,1),(3,2),(2,3)\}$

Prob $=\frac{1}{9}$

42. Six persons stand at random in a queue for buying cinema tickets. Individually three of them have only a fifty rupee note each while each of the other three have a hundred rupee note only. The booking clerk has an empty cash box, probability that six persons get tickets without waiting for change is ----, (cost of one ticket is Rs.50/- and each person gets one ticket only)

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{5}$

Key. C

Sol. Here random experiment is arranging 6 persons in a line $n(S)=6!=720$

Let $'a'$ denote person having Rs.50/-, $'b'$ denote person having Rs.100/- note each since all the six person, should get ticket first place should be occupied by a person having Rs.50/- and sixth place should be occupied by person having Rs.100/- possible cases are

$\boxed{a}\ \boxed{aa}\ \boxed{bb}\ \boxed{b}$

$\boxed{a}\ \boxed{ab}\ \boxed{ab}\ \boxed{b}$

$\boxed{a}\ \boxed{ab}\ \boxed{ba}\ \boxed{b}$

$\boxed{a}\ \boxed{ba}\ \boxed{ab}\ \boxed{b}$

$\boxed{a}\ \boxed{ba}\ \boxed{ba}\ \boxed{b}$

'a's can arrange among themselves and

'b's can arrange among themselves in

$n(E) = 3!3! + 3!3! + 3!3! + 3!3! + 3!3! = 180$

Probability $= \dfrac{180}{720} = \dfrac{1}{4}$

43. A number is chosen at random from the set of all 4-digit numbers each of which contains not more than 2 different digits ,probability that it does not contain the digit zero is

A) $\dfrac{7}{64}$ B) $\dfrac{37}{64}$ C) $\dfrac{47}{64}$ D) $\dfrac{57}{64}$

Key. D

Sol. If $a \neq 0$ the numbers with O & a are

$aooo,\ aooa,\ aoao,\ aaoo,\ aaao,\ aaoa\ \&\ aoaa$. These are $9 \times 7 = 63\ (a \neq b,\ ab \neq 0)$

Now, $aaab,\ aaba,\ abaa,\ baaa$ are $4 \times 9 \times 8 = 288\ \&\ aabb,\ abab,\ abba$

are $3 \times 9 \times 8 = 216\ \&\ aaaa$ are 9

$\therefore$ prob $= \dfrac{288+216+9}{63+288+216+9} = \dfrac{513}{576} = \dfrac{57}{64}$

44. There are 3 bags .Bag 1 contain 2 red and $a^2 - 4a + 8$ black balls, bag 2 contains 1 red and $a^2 - 4a + 9$ black balls and bag 3 contains 3 red and $a^2 - 4a + 7$ black balls .A ball is draw at random from at random chosen bag. Then maximum value of probability that it is a red ball is

A) $\dfrac{1}{3}$ B) $\dfrac{1}{2}$ C) $\dfrac{2}{9}$ D) $\dfrac{4}{9}$

Key. A

Sol. Req. prob $= \dfrac{1}{3}\left(\dfrac{6}{a^2 - 4a + 10}\right)$

$(P(A))_{\max} = \dfrac{1}{3} \times \dfrac{6}{6} = \dfrac{1}{3}$

45. Let p, q be chosen one by one from the set $\left\{1,\sqrt{2},\sqrt{3},2,e,\pi\right\}$ with replacement. Now a circle is drawn taking (p, q) as its centre then the probability that atmost two rational points exist on circle is (Rational points are those points whose both co-ordinates are rational)

A) $\frac{3}{4}$ B) $\frac{5}{6}$ C) $\frac{7}{8}$ D) $\frac{8}{9}$

Key. D

Sol. Suppose there exist three rational points or more on the circle

$x^2+y^2+2gx+2fy+c=0$

If $(x_1,y_1)\,(x_2,y_2)\,(x_3,y_3)$ be those three points $S_{11}=0,\ S_{22}=0,\ S_{33}=0$

on solving we get g, f, c as rational

Possible values of p are 1,2

q are 1,2

(p, q) be chosen as 4 ways

(p, q) can be chosen without restriction in $6\times6=36$

Prob $=1-\frac{4}{36}=1-\frac{1}{9}=\frac{8}{9}$

46. A, B are two independent events such that $P(A)>\frac{1}{2}$ $P(B)>\frac{1}{2}$. If $P(A\cap\bar{B})=\frac{3}{25}$ and $P(\bar{A}\cap B)=\frac{8}{25}$ then $P(A\cap B)=$

A) 3/4 B) 2/3 C) 12/25 D) 18/25

Key. C

Sol. Let $P(A)=x$ and $P(B)=y$. $x>\frac{1}{2}, y>\frac{1}{2}$

$P(A-B)=x-xy=\frac{3}{25}$ and $P(B-A)=y-xy=\frac{8}{25}$

47. Two persons A and B are throwing 3 dice taking turns. If A throws 8 then the probability that B throws a higher number is

A) 5/27 B) 9/17 C) 8/27 D) 20/27

Key. D

Sol. Let E be the event that B throws a number more than 8. Then $P(E)=1-P(\overline{E})$

$\left|\overline{E}\right|$ = Number of positive integral solutions of $x+y+z\le8$

$\therefore\left|\overline{E}\right|={}^8c_3=56$ and $\left|S\right|=216$ $\quad\therefore P(\overline{E})=\frac{56}{216}=\frac{7}{27}$

48. Three groups of children contain respectively 3 girls and 1 boy; 2 girls and 2 boys; 1 girl and 3 boys. One child is selected at random from each group. The probability that the three selected consists of 1 girl and 2 boys is

A) 5/9 B) 13/32 C) 12/19 D) 25/64

Key. B

Sol. $\frac{3}{4}\times\frac{2}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{2}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{2}{4}\times\frac{1}{4}=\frac{26}{64}$

49. From a bag containing 10 distinct balls, 6 balls are drawn simultaneously and replaced. Then 4 balls are drawn. The probability that exactly 3 balls are common to the drawings is

A) 8/21 B) 6/19 C) 5/24 D) 9/22

Key. A

Sol. Let S be the sample space of the composite experiment of drawing 6 in the first draw and then four in second draw then $|S|={}^{10}C_6\times{}^{10}C_4$

$\therefore$ Required Probability = $\frac{{}^{10}C_6\times{}^{6}C_3\times{}^{4}C_1}{{}^{10}C_6\times{}^{6}C_4}=\frac{80\times 24}{10\times 9\times 8\times 7}=\frac{8}{21}$

50. Two persons each make a single throw with a pair of dice. The probability that their scores are equal is

A) 65/648 B) 69/648 C) 73/648 D) 91/648

Key. C

Sol. Required Probability = $=\frac{1^2+2^2+3^2+4^2+5^2+6^2+5^2+4^2+3^2+2^2+1^2}{36^2}$

51. A bag contains 4 red and 3 blue balls. Two drawings of two balls are made. The probability that the first drawing gives 2 red balls and the second drawing gives two blue balls if the balls are not returned to the bag after the first draw is

A) 2/49 B) 3/35 C) 3/10 D) 1/4

Key. B

Sol. $\frac{4c_2}{7c_2}\times\frac{3c_2}{5c_2}=\frac{4\times 3}{7\times 6}\times\frac{3}{10}=\frac{3}{35}$

52. A team has probability 2/3 of winning a game whenever it plays. If the team plays 4 games then the probability that it wins more than half of the games is

A) 17/25 B) 15/19 C) 16/27 D) 13/20

Key. C

Sol. Let p be the probability that the team wins a game. Let $q=1-p$ .Then the random variable "number of wins" follows the binomial distribution $P(X=K)={}^4C_k q^{4-k}p^k, k=0,1,2,3,4$.

Required probability = ${}^4C_3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3 + {}^4C_4\left(\frac{2}{3}\right)^4 = \frac{16}{27}$.

53. Four Identical oranges and six distinct apples (each a different variety) are distributed randomly into five distinct boxes. The probability that each box gets a total of two objects is

A) $\frac{813}{109375}$ B) $\frac{162}{21875}$ C) $\frac{323}{43750}$ D) $\frac{151}{21875}$

Key. B

Sol. The total number of ways to put four identical oranges and six distinct apples into five distinct boxes is

$\left({}^{5+4-1}C_4\right).5^6 = 70\times 5^6$

To satisfy the criteria that each box contains two object we make three cases

(1) Two oranges in each of the two boxes and no oranges in the other three boxes

Number of ways $= {}^5C_2 \times \frac{\lfloor 6}{(2!)^3} = 900$

(2) Two oranges in one box, one orange in each of the two other boxes =

$5\times\left({}^4C_2\right)\times\frac{\lfloor 6}{(2!)^2} = 5400$

(3) One orange in each of the four boxes $5.\frac{\lfloor 6}{2!} = 1800$

The total number of ways $= 900 + 5400 + 1800 = 8100$

Probability $= \frac{8100}{70\times 5^6} = \frac{162}{21875}$

54. A bag contains two red balls and two green balls. A person randomly pulls out a ball, replacing it with a red ball regardless of the colour.What is the probability that all the balls are red after three such replacement ?

A) 3/8 B) 7/16 C) 5/32 D) 9/32

Key. D

Sol. In order that all balls are red after 3 replacements, two of the three balls selected must have green.

There could be three cases.

I : Red, Green, Red.

II : Green, red, Green

III : Green,Green,Red.

The probabilities is

In case I = $\frac{1}{2}.\frac{1}{2}.\frac{1}{4} = \frac{1}{16}$

In case II = $\frac{1}{2}.\frac{3}{4}.\frac{1}{4} = \frac{3}{32}$

In case III = $\frac{1}{2}.\frac{1}{4}.1 = \frac{1}{8}$

The required probability = $\frac{1}{16}+\frac{3}{32}+\frac{1}{8}=\frac{9}{32}$

55. Each face of a cubical die is numbered with a distinct number from among the first six odd numbers; such that the sum of the two numbers on any pair of opposite face is 12, if ten such dies are thrown simultaneously, then find the probability that the sum of the numbers that turn up is exactly 53

A) $\frac{53}{6^{10}}$ B) $\frac{153}{6^{10}}$ C) $\frac{3}{6^{10}}$ D) 0

Key. D

Sol. With 10 dice, the number on each face being odd, we can never get an odd number as their sum

56. Three fair coins are tossed simultaneously .Let E be the event of getting three heads or three tails,F be the event of at least two heads and G be the event of at most two heads then which of the following is true.

A) $P(E\cap F)=P(E).P(F)$ B) $P(E\cap G)=P(E).P(G)$

C) $P(F\cap G)=P(F).P(G)$ D) None.

Key. A

Sol. $P(E)=\frac{2}{8}=\frac{1}{4}, P(F)=\frac{4}{8}=\frac{1}{2}, P(G)=\frac{7}{8},$

$P(E\cap F)=\frac{1}{8}, P(F\cap G)=\frac{3}{8}, P(E\cap G)=\frac{1}{8}$

57. If E and F are two independent events such that $P(E\cap F)=\frac{1}{6}, P(\overline{E}\cap\overline{F})=\frac{1}{3}$ and $P((E)-P(F))(1-P(F))>0$, Then

A) $P(E)=\frac{1}{2}, P(F)=\frac{1}{3}$ B) $P(E)=\frac{1}{3}, P(F)=\frac{1}{2}$

C) $P(E)=\frac{1}{4}, P(F)=\frac{2}{3}$ D) $P(E)=\frac{2}{3}, P(F)=\frac{1}{4}$

Key. A

Sol. $P(E)+P(F)=\frac{5}{6}$

$P(E)-P(F)=\frac{1}{6}$

58. Consider all the 3digit numbers abc (where $a\neq 0$) if a number is selected at random then the probability that the number is such that $a+b+c=6$ is

A) $\frac{2}{15}$ B) $\frac{7}{75}$ C) $\frac{7}{600}$ D) $\frac{7}{300}$

Key. D

Sol. Since $a+b+c=6$, the possible digit selections are

$(1,2,3),(1,1,4),(2,2,2),(0,1,5),(0,2,4),(0,3,3),(0,0,6)$

The required number of ways $6+3+1+4+4+2+1=21$

Required probability = $\dfrac{21}{9\times10\times10}=\dfrac{7}{300}$

59. The Probability that in a family of 5 members, exactly two members have birthday on Sunday is

A) $\dfrac{12\times5^3}{7^5}$ B) $\dfrac{10\times6^3}{5^7}$ C) $\dfrac{12\times6^2}{5^7}$ D) $\dfrac{10\times6^3}{7^5}$

Key. D

Sol. Required Probability = $\dfrac{{}^5C_2\times6\times6\times6}{7\times7\times7\times7\times7}$

60. If three numbers are chosen randomly from the set $\{1,3,3^2,.......,3^n\}$ without replacement then the probability that they form an increasing geometric progression is

A) $\dfrac{3}{2n}$ if n is odd B) $\dfrac{3}{2n}$ if n is even

C) $\dfrac{3n}{n^2-1}$ if n is even D) $\dfrac{3n}{2(n^2-1)}$ if n is odd

Key. A

Sol. Number of triplets $\left(3^r,3^{r+1},3^{r+2}\right)\left(0\le r\le n\right)$ is n-1

Number of triplets $\left(3^r,3^{r+2},3^{r+4}\right)\left(0\le r\le n\right)$ is n-3

Number of triplets $\left(3^r,3^{r+\frac{n-1}{2}},3^{r+n-1}\right)(n \text{ odd})$ is 2

And Number of triplets $\left(3^r,3^{r+\frac{n}{2}},3^{r+n}\right)(n \text{ even})$ is 1.

$\therefore$ If n is odd, the number of favorable out comes $=(n-1)+(n-3)+...+4+2=\dfrac{n^2-1}{4}$

And if n is even, the number of favorable out comes

$=(n-1)+(n-3)+...+3+1=\dfrac{n}{2}\times\dfrac{n}{2}=\dfrac{n^2}{4}$

Probability = $\dfrac{=(n^2-1)/4}{{}^{(n+1)}C_3}=\dfrac{3}{2n}$ if n is odd

$=\dfrac{n^2/4}{{}^{(n+1)}C_3}=\dfrac{3n}{2(n^2-1)}$ if n is even.

61. A fair coin is tossed until one of the two sides occurs twice in a row, The Probability that the number of tosses required is even is

A) 1/3 B) 2/3 C) 1/4 D) 3/4

Key. B

Sol. $A=\{HH, HTHH, HTHTHH, ...\}$

And $B=\{TT, THTT, THTHTT, ...\}$ P(A) = $\frac{1}{2^2}+\frac{1}{2^4}+.....=\frac{1}{3}$

$P(B)=\frac{1}{3}$

Required Probability $=\frac{1}{3}+\frac{1}{3}$

62. A determinant is chosen at random from the set of all determinants of order 2 with elements 1 and 0 only. The probability that the value of the determinant is positive is

A) $\frac{1}{8}$ B) $\frac{3}{16}$ C) $\frac{5}{16}$ D) $\frac{1}{16}$

Key. B

Sol. no. of determinants formed $=n(s)=2^4=16$

The determinants $\begin{vmatrix}1 & 0\\0 & 1\end{vmatrix}\begin{vmatrix}1 & 0\\1 & 1\end{vmatrix}\begin{vmatrix}1 & 1\\0 & 1\end{vmatrix}$ are positive $\Rightarrow n(E)=3$

$\therefore P(E)=\frac{3}{16}$

63. Two numbers x and y are chosen at random from the set $\{1,2,3,----3n\}$. The probability that x^2-y^2 is divisible by 3 is

A) $\frac{5n-3}{3(3n-1)}$ B) $\frac{2n}{3(3n-1)}$

C) $\frac{5n}{3(3n-1)}$ D) $\frac{n}{3n-1}$

Key. A

Sol. $G_1 = 1$ 4 7 ---- 3n–1

$G_2 = 2$ 5 8 ---- 32–2

$G_3 = 3$ 6 9 ---- 3n

We have $n(s)={}^{3n}C_2$ for n(E) both numbers can be selected from same group or one number from G_2 and one from G_3

$\therefore n(E)={}^{3n}C_2+nc_1\times nc_1$

$\therefore P(E)=\frac{3.{}^nC_2+n^2}{{}^{3n}C_2}$

64. In shuffling a pack of cards 3 are accidentally dropped. The chance that the missing cards are of different suits is.

A) $\frac{169}{425}$ B) $\frac{169}{1700}$

C) $\frac{1}{4}$ D) $\frac{169}{2550}$

Key. A

Sol. $n(S) = {}^{52}C_3$ $n(E) = {}^{4}C_3 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1$

65. Two small squares on a chess board are selected at random. The probability that they have a common side is.

A) $\frac{1}{36}$ B) $\frac{1}{9}$ C) $\frac{1}{3}$ D) $\frac{1}{18}$

Key. D

Sol. $n(S) = {}^{64}C_2$

n(E) = selecting two consecutive squares from a row or column

$= 7\times 8 + 7\times 8 = 112$

66. In a convex hexagon two diagnols are drawn at random. The probability that the diagnols intersect at an interior point of the hexagon is

A) $\frac{5}{12}$ B) $\frac{7}{12}$ C) $\frac{2}{5}$ D) $\frac{1}{3}$

Key. A

Sol. $n(S) = {}^{9}C_2$ $n(E) = {}^{6}C_4$

67. If 6 articles are distributed at random among 6 persons the probability that at least one person does not get any article is

A) $\frac{319}{324}$ B) $\frac{317}{324}$ C) $\frac{313}{324}$ D) $\frac{79}{162}$

Key. A

Sol. $n(S) = 6^6$ $n(E) = 6^6 - 6!$

68. A car is parked by an owner amongst 25 cars in a row not at either end. In his return he finds that exactly 15 places are still occupied. The probability that both the neighbouring places are empty is

A) $\frac{91}{276}$ B) $\frac{15}{184}$ C) $\frac{15}{92}$ D) none

Key. C

Sol. It is given that 15 places are occupied. 14 other cars are parked no. of ways of selecting 14 places from 24 in ${}^{24}C_{14}$ ways. Excluding the neighbouring places there are 22 places in where 14 cars can be parked in ${}^{22}C_{14}$ ways.

$$\therefore P(E)=\frac{{}^{22}C_{14}}{{}^{24}C_{14}}$$

69. Suppose $f(x)=x^3+ax^2+bx+c$, where a, b, c are chosen respectively by throwing a die three times. The probability that f(x) is an increasing function is.

A) $\frac{4}{9}$ B) $\frac{3}{8}$ C) $\frac{2}{5}$ D) $\frac{8}{17}$

Key. A

Sol. $f'(x)=3x^2+2ax+b$ f(x) is increasing $f'(x)\ge 0\forall x$ *and for* $f'(x)=0$ should not form an interval

$\therefore a^2-3b\le 0$

This is true for exactly 16 ordered pairs (a,b) $1\le a,b\le 6$ (1,1) (1,2) (1,3) (1,4) (1,5) (1,6) (2,2) (2,3) (2,4) (2,5) (2,6) (3,3) (3,4) (3,5) (3,6) (4,6)

$$\therefore P(E)=\frac{16}{36}=\frac{4}{9}$$

70. A and B throw alternatively with a pair of dice .A wins if he throws a sum 6 before B throws 7 and B wins if he throws a 7 before A throws sum 6.If A starts the game ,his chance of winning is

a) $\frac{30}{61}$ b) $\frac{31}{61}$ c) $\frac{15}{61}$ d) $\frac{60}{61}$

Key. A

Sol. A's chance of winning in a throw = $\frac{5}{36}$, B's chance of winning in a throw = $\frac{1}{6}$

A's chance of losing in a throw = $\frac{31}{36}$, B's chance of losing in a throw = $\frac{5}{6}$

A can winning the game = $\frac{5}{36}+\frac{31}{36}\times\frac{5}{6}\times\frac{5}{36}+\frac{31}{36}\times\frac{5}{6}\times\frac{31}{36}\times\frac{5}{6}\times\frac{5}{36}+----$

$$=\frac{5}{36}\left[1+\frac{155}{216}+\left(\frac{155}{216}\right)^2+----\right]=\frac{30}{61}$$

71. From a group of n persons arranged in a circle, 3 persons are selected at random .If probability that no two adjacent persons are selected is $\frac{2}{7}$ then n=

a) 6 b) 7 c) 8 d) 9

Key. C

Sol. P(no two adjacent persons are selected) = $\frac{n(n-4)(n-5)/6}{n(n-1)(n-2)/6} = \frac{(n-4)(n-5)}{(n-1)(n-2)} = \frac{2}{7}$

$\Rightarrow 5n^2 - 57n + 136 = 0 \Rightarrow n = 8$

72. A box contains 5 pairs of shoes. If 6 shoes are selected at random, the probability that exactly one pair of shoes is obtained is

a) $\frac{4}{21}$ b) $\frac{2}{21}$ c) $\frac{8}{21}$ d) $\frac{16}{21}$

Key. C

Sol. From 10 shoes, 6 can be selected in 10_{C_6} ways and if there is to be one pair among them if can be selected in 5_{C_1} ways. Other 4 shoes can be selected in $4_{C_4}(2^4)$ =16 ways .

Hence probability = $\frac{5\times16}{10_{C_6}} = \frac{8}{21}$

73. An unbiased coin is tossed 12 times .The probability that at least 7 consecutive heads show up is

a) $\frac{1}{128}$ b) $\frac{1}{64}$ c) $\frac{9}{256}$ d) $\frac{7}{256}$

Key. D

Sol. The sequence of consecutive heads may starts with 1st toss or 2nd toss or 3rd toss ---- or at 6th toss. In any case ,if it starts with r th throw , the first (r-2) throws may be head or tail but (r-1)st throw must be tail, after which again a head or tail can show up:

$\underbrace{\text{HHH------H}}_{\text{7 times}}$ x x x x x or $\underbrace{\text{x x x -------- x}}_{\text{(r-2)}}$ $\underset{\text{(r-1)st}}{\text{T}}$ $\underbrace{\text{H HH-------H}}$ x x------

$\therefore$ Probability = $\frac{1}{2^7} + \underbrace{\frac{1}{2}\cdot\frac{1}{2^7} + \frac{1}{2}\cdot\frac{1}{2^7} + \text{---------} + \frac{1}{2}\cdot\frac{1}{2^7}}_{\text{5 times}} = \frac{1}{2^7}\left[1+\frac{5}{2}\right] = \frac{7}{2^8}$

74. Six fair dice are thrown independently .The probability that there are exactly 2 different pairs (A pair is an ordered combination like 2, 2, 1, 3, 5, 6) is

a) $\frac{5}{72}$ b) $\frac{25}{72}$ c) $\frac{125}{144}$ d) $\frac{5}{36}$

Key. B

Sol. Total no. of outcomes $= 6^6$ .number of ways of choosing 4 other different numbers is 6_{C_2} and choosing 2 out of remaining 4 can be lone in 4_{C_2} ways. Also number of ways of arranging 6 numbers of which 2 are alike and 2 are alike is $\frac{6}{2!2!}$.

$\therefore$ Required probability= $\dfrac{6_{C_2} \times 4_{C_2} \times \dfrac{6}{2!2!}}{6^6} = \dfrac{25}{72}$.

75. Two integers x and y are chosen from the set {0, 1, 2, 3, -----, 2n}, with replacement, the probability that $|x-y| \le n (n \in N)$ is

a) $\dfrac{3n^2+3n+1}{(2n+1)^2}$ b) $\dfrac{3n^2+3n}{(2n+1)^2}$ c) $\dfrac{3n^2+1}{(2n+1)^2}$ d) $\dfrac{n^2+n+1}{(2n+1)^2}$

Key. A

Sol. x and y can be any one of (2n+1) numbers given . $|x-y| \le n \Rightarrow x-n \le y \le x+n$

Hence number of possibilities of y, for x=0,1,2,3,----n-1,n,n+1----2n are n+1,n+2,n+3n------2n,2,+1,2n,2,-1,-----n+1 respectively.

$$\therefore \text{Probability} = \frac{2\left(\overline{n+1}+\overline{n+2}+------+\overline{2n}\right)+2n+1}{(2n+1)^2} = \frac{3n^2+3n+1}{(2n+1)^2}$$

76. A die is rolled three times, the probability of getting large number than the previous number is

A) 1/54 B) 5/54 C) 5/108 D) 13/108

Key. B

Sol. If the 2nd number is $i(i>1)$ the no.of favourable ways = $(i-1)\times(6-i)$

$n(E) =$ total no.of favourable ways = $\sum_{i=1}^{6}(i-1)\times(6-i) = 1\times4+2\times3+3\times2+4\times1 = 20$

Required probability = $\dfrac{20}{216} = \dfrac{5}{54}$

77. 10 apples are distributed at random among 6 persons. The probability that at least one of them will receive none is

A) $\dfrac{6}{143}$ B) $\dfrac{{}^{14}C_4}{{}^{15}C_5}$ C) $\dfrac{137}{143}$ D) $\dfrac{143}{137}$

Key. C

Sol. The required probability = 1 – probability of each receiving at least one = $1-\dfrac{n(E)}{n(S)}$.

Now, the number of integral solutions of $x_1+x_2+x_3+x_4+x_5+x_6 = 10$

Such that $x_1 \ge 1, x_2 \ge 1, ..., x_6 \ge 1$ gives $n(E)$ and the number of integral solutions of $x_1+x_2+x_3+x_4+x_5+x_6 = 10$ such that $x_1 \ge 0, x_2 \ge 0, ..., x_6 \ge 0$ gives $n(S)$

$\therefore$ The required probability = $1-\frac{^{10-1}C_{6-1}}{^{10+6-1}C_{6-1}}=1-\frac{^{9}C_5}{^{15}C_5}=\frac{137}{143}$.

78. A, B are two independent events such that $P(A)>\frac{1}{2}$ $P(B)>\frac{1}{2}$. If $P(A\cap\bar{B})=\frac{3}{25}$ and $P(\bar{A}\cap B)=\frac{8}{25}$ then $P(A\cap B)=$

A) 3/4 B) 2/3 C) 12/25 D) 18/25

Key. C

Sol. Let $P(A)=x$ and $P(B)=y$. $x>1/2, y>1/2$

$P(A-B)=x-xy=3/25$ and $P(B-A)=y-xy=8/25$

79. Two persons A and B are throwing 3 dice taking turns. If A throws 8 then the probability that B throws a higher number is

A) 5/27 B) 9/17 C) 8/27 D) 7/27

Key. D

Sol. Let E be the event that B throws a number more than 8. Then $P(E)=1-P(\bar{E})$

$\left|\bar{E}\right|$ = Number of positive integral solutions of $x+y+z\le 8$

$\therefore\left|\bar{E}\right|={}^{8}c_3=56$ and $\left|S\right|=216$ $\therefore P(\bar{E})=\frac{56}{216}=\frac{7}{27}$

80. Three groups of children contain respectively 3 girls and 1 boy; 2 girls and 2 boys; 1 girl and 3 boys. One child is selected at random from each group. The probability that the three selected consists of 1 girl and 2 boys is

A) 5/9 B) 13/32 C) 12/19 D) 25/64

Key. B

Sol. $\frac{3}{4}\times\frac{2}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{2}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{2}{4}\times\frac{1}{4}=\frac{26}{64}$

81. A certain type of missile hits its target with probability 0.3. The number of missiles that should be fired so that there is atleast an 80% probability of hitting a target is

A) 3 B) 4 C) 5 D) 6

Key. C

Sol. Let n be the required number.

$\therefore$ The probability that 'n' missiles miss the target is $(0.7)^n$. We require $1-(0.7)^n>0.8$

i.e., $(0.7)^n<0.2$. The least value of 'n' satisfying this inequality is 5.

82. A team has probability 2/3 of winning a game whenever it plays. If the team plays 4 games then the probability that it wins more than half of the games is

A) 17/25 B) 15/19 C) 16/27 D) 13/20

Key. C

Sol. Let p be the probability that the team wins a game. Let $q=1-p$ .Then the random variable "number of wins" follows the binomial distribution $P\left(X=K\right)={}^4C_k q^{4-k}p^k, k=0,1,2,3,4$.

Required probability = ${}^4C_3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3+{}^4C_4\left(\frac{2}{3}\right)^4=\frac{16}{27}$.

83. From a bag containing 10 distinct balls, 6 balls are drawn simultaneously and replaced. Then 4 balls are drawn. The probability that exactly 3 balls are common to the drawings is

A) 8/21 B) 6/19 C) 5/24 D) 9/22

Key. A

Sol. Let S be the sample space of the composite experiment of drawing 6 in the first draw and then four in second draw then $|S|={}^{10}C_6\times{}^{10}C_4$

$\therefore$ Required Probability = $\frac{{}^{10}C_6\times{}^6C_3\times{}^4C_1}{{}^{10}C_6\times{}^6C_4}=\frac{80\times 24}{10\times 9\times 8\times 7}=\frac{8}{21}$

84. Two persons each make a single throw with a pair of dice. The probability that their scores are equal is

A) 65/648 B) 69/648 C) 73/648 D) 91/648

Key. C

Sol. Required Probability = $=\frac{1^2+2^2+3^2+4^2+5^2+6^2+5^2+4^2+3^2+2^2+1^2}{36^2}$

85. $'A'$ is a 3×3 matrix with entries from the set $\{-1,0,1\}$. The probability that $'A'$ is neither symmetric nor skew symmetric is

A) $\frac{3^9-3^6-3^3+1}{3^9}$ B) $\frac{3^9-3^6-3^3}{3^9}$ C) $\frac{3^9-1}{3^{10}}$ D) $\frac{3^9-3^3+1}{3^9}$

Key. A

Sol. Conceptual

86. The probability that in a family of 5 members, exactly two members have birthday on Sunday is

(A) $\frac{12\times 5^3}{7^5}$ (B) $\frac{10\times 6^3}{5^7}$

(C) $\frac{12\times 6^2}{5^7}$ (D) $\frac{10\times 6^3}{7^5}$

Key: D

Hint: Required probability $=\frac{{}^5C_2\times 6\times 6\times 6}{7\times 7\times 7\times 7\times 7}=\frac{10\times 6^3}{7^5}$

87. If a is an integer lying in the closed interval [- 5 ,30] , then the probability that the graph of $y = x^2 + 2(a+4)x - 5a + 64$ is strictly above the x-axis is

a) 2/9 b) 1/6 c) 1/2 d) 5/9

Key: C

Hint $(a+4)^2 + 5a - 64 < 0 \Rightarrow -16 < a < 3$

$\therefore probability = \frac{18}{36} = 2/9$

88. If three numbers are chosen randomly from the set $\{1, 3, 3^2, ..., 3^n\}$ without replacement, then the probability that they form an increasing geometric progression is

a) $\frac{3}{2n}$ $if\ n is\ odd$ b) $\frac{3}{2n}$ $if\ n is\ even$

c) $\frac{3n}{2(n^2-1)}$ $if\ n is\ even$ d) $\frac{3n}{2(n^2-1)}$ $if\ n is\ odd$

Key: A,C

Hint: Number of triplets $(3^r, 3^{r+1}, 3^{r+2})(0 \le r \le n) is\, n-1$

Number of triplets $(3^r, 3^{r+2}, 3^{r+4})(0 \le r \le n) is\, n-3$

Number of triplets $\left(3^r, 3^{r+\frac{n-1}{2}}, 3^{r+n-1}\right)(n\ odd) is\, 2$

and no of triplets $\left(3^r, 3^{r+\frac{n}{2}}, 3^{r+n}\right)(n\ even) is 1$

$\therefore$ If n is odd, the number of favourable outcomes

$= (n-1) + (n-3) + ... + 4 + 2 = \frac{n^2-1}{4}$

and if n is even, the number of favourable outcomes

$= (n-1) + (n-3) + ... + 3 + 1 = \frac{n}{2} \times \frac{n}{2} = n^2/4$

$\therefore$ Prob = $\frac{(n^2-1)/4}{(n+1)C_3} = 3/2n$ if n is odd

$= \frac{n^2/4}{(n+1)C_3} = \frac{3n}{2(n^2-1)}$ if n is even

89. The probabilities of A, B, C solving a problem independently are respectively $\frac{1}{4},\frac{1}{5},\frac{1}{6}$. If 21 such problems are given to A, B, C then the probability that atleast 11 problems can be solved by them is

a) ${}^{21}C_{11}\left(\frac{1}{2}\right)^{11}$ b) $\frac{1}{2}$ c) $\left(\frac{1}{2}\right)^{11}$ d) ${}^{21}C_{11}\frac{2^{11}}{3^{21}}$

Key: B

Hint: No. of trials n = 21

Success is solving the problem

$$\therefore p = P(A\cup B\cup C) = 1-P\left(\overline{A}\right)P\left(\overline{B}\right)P\left(\overline{C}\right)$$

$$=1-\frac{3}{4}\times\frac{4}{5}\times\frac{5}{6}$$

$$=\frac{1}{2}$$

$$q=\frac{1}{2}$$

Find $P\left(X\geq 11\right)$

90. Four identical oranges and six distinct apples (each a different variety) are distributed randomly into five distinct boxes. The probability that each box gets a total of two objects is

(A) $\frac{813}{109375}$ (B) $\frac{162}{21875}$ (C) $\frac{323}{43750}$ (D) $\frac{151}{21875}$

Key: B

Hint; The total number of ways to put four identical oranges and six distinct apples into five distinct boxes is

$$\left({}^{5+4-1}C_4\right).\,5^6 = 70\times 5^6$$

to satisfy the criteria that each box contains two objects we make three cases(based on number of oranges to go into a box)

1. two oranges in each of the two boxes and no oranges in the other three boxes. number of ways $= {}^5C_2 \times \frac{\lfloor 6}{\lfloor 2\lfloor 2\lfloor 2}$ = 900

2. two oranges in one box, one orange in each of the two other boxes $(5)\times\left({}^4C_2\right)\times\frac{\lfloor 6}{\lfloor 2\lfloor 2\lfloor 1\lfloor 1}$ $=5.6.180$

$= 5400$

3. one orange in each of the four boxes

$$=5\,.\frac{\lfloor 6}{\lfloor 2\lfloor 1\lfloor 1\lfloor 1} = 5\times 360 = 1800$$

the total number of ways $=900+5400+1800=8100$

probability $=\frac{8100}{70\times 5^6}=\frac{162}{21875}$

91. A bag contains two red balls and two green balls. A person randomly pulls out a ball, replacing it with a red ball regardless of the colour. What is the probability that all the balls are red after three such replacements?

(A) $\frac{3}{8}$ (B) $\frac{7}{16}$

C) $\frac{5}{32}$ D) $\frac{9}{32}$

Key: D

Hint In order that all balls are red after 3 replacements , two of the three balls selected must have been green. There could be three cases

I : red , green , red

ii : green , red , green

iii : green , green , red (since they are now all red)

the probability is

case (i) is $\frac{1}{2}.\frac{1}{2}.\frac{1}{4}=\frac{1}{16}$

case (ii) is $\frac{1}{2}.\frac{3}{4}.\frac{1}{4}=\frac{3}{32}$

case (iii) is $\frac{1}{2}.\frac{1}{4}.1=\frac{1}{8}$

the required probability $=\frac{1}{16}+\frac{3}{32}+\frac{1}{8}=\frac{9}{32}$

92. In a test student either guesses or copies or knows the answer to a multiple choice questions with four choices in which exactly one choice is correct. The probability that he makes a guess is $\frac{1}{3}$; The probability that he copies the answer is $\frac{1}{6}$.The Probability that his answer is correct given that he copied it is $\frac{1}{8}$.Find the probability that he knew the answer to the question given that he correctly answered it is

(A) $\frac{29}{35}$ (B) $\frac{24}{29}$ (C) $\frac{1}{7}$ (D) $\frac{1}{9}$

Key: B

Hint: Let ‘A’ be the event of guessing the correct answer.

‘B’ be the event of copying the correct answer.

‘C’ be the event of knowing the correct answer.

‘D’ be the event that his answer is correct

$$P(A)=\frac{1}{3}$$

$$\left.\begin{array}{l} P(B)=\dfrac{1}{6} \\ P(C)=\dfrac{1}{2} \\ P(\dfrac{D}{B})=\dfrac{1}{8} \\ P(\dfrac{D}{A})=\dfrac{1}{4} \\ P(\dfrac{D}{C})=1 \end{array}\right\} \qquad P\left(\frac{C}{D}\right)=\frac{P(C).P\left(\dfrac{D}{C}\right)}{P(D\cap A)+P(D\cap B)+P(D\cap C)}=\frac{24}{29}$$

93. Consider the system of equations $ax+by=0$ and $cx+dy=0$ where $a,b,c,d\in\{1,2\}$. The probability that the system of equations has a unique solution is

A) 3/8 B) 5/16

C) 9/16 D) 5/8

Key: D

Hint: 1) $ad=1, bc=4 \Rightarrow (a=1, d=1, b=2, c=2)$

2) $ad=1, bc=2 \Rightarrow (a=1, d=1, b=1, c=2), (a=1, d=1, b=2, c=1)$

3) $ad=2, bc=1 \Rightarrow (a=1, d=2, b=1, c=1), (a=2, d=1, b=1, c=1)$

4) $ad=2, bc=4 \Rightarrow (a=1, d=2, b=2, c=2), (a=2, d=1, b=2, c=2)$

5) $ad=4, bc=1, \Rightarrow (a=2, d=2, b=1, c=1)$

6) $ad=4, bc=2 \Rightarrow (a=2, d=2, b=1, c=2), (a=2, d=2, b=2, c=1)$

$\Rightarrow$ required probability = $\dfrac{10}{16}=\dfrac{5}{8}$

94. Let A, B, C be any three events in a sample space of a random experiment. Let the events $E_1=$ exactly one of A, B occurs $E_2=$ exactly one of B, C occurs, $E_3=$ exactly one of C, A occurs, $E_4=$ all of A, B, C occurs, $E_5=$ atleast one of A, B, C occurs. $P(E_1)=P(E_2)=P(E_3)=1/3,\ P(E_4)=\dfrac{1}{9}$ then $P(E_5)=$

A) $\dfrac{1}{9}$ B) $\dfrac{7}{9}$ C) $\dfrac{5}{18}$ D) $\dfrac{11}{18}$

Key: D

Hint: $E_1=(A\cap\bar{B})\cup(\bar{A}\cap B),\ E_2=(B\cap\bar{C})\cup(\bar{B}\cap C), E_3=(C\cap\bar{A})\cup(\bar{C}\cap A)$

$E_4=A\cap B\cap C\ \ E_5=A\cup B\cup C$.

95. The numbers *a,b* are chosen from the set {1,2,3,4,---,10} such that $a\le b$ with replacement. The probability that *a* divides *b* is

(A) $\dfrac{5}{11}$ (B) $\dfrac{29}{55}$ (C) $\dfrac{27}{55}$ (D) None

Key: C

Hint: Number of ways of choosing *a&b* s.t $a\le b=10+9+8+---+1=55$

" " " *a&b* s.t *a* divides *b* =10+5+3+2+2+5=27

$\therefore$ Required probability= $\frac{27}{55}$

96. An electric component manufactured by a company is tested for its defectiveness by a sophisticated device. Let 'A' denote the event " the device is defective '' and 'B' the event "the testing device reveals the component to be defective'' . Suppose $P(A)=\alpha$ and $P(B/A)=P(\bar{B}/\bar{A})=1-\alpha$. Where $0<\alpha<1$. If it is given that the testing device revels it to be defective , then the probability that the component is not defective is

A) $\frac{1}{4}$ B) $\frac{3}{4}$ C) 0.7 D) 0.5

Key: D

Hint:
$$P\left(\frac{\bar{A}}{B}\right)=\frac{P(\bar{A}).P(B/\bar{A})}{P(A).P(B/A)+P(\bar{A}).P(B/\bar{A})}$$
$$=\frac{(1-\alpha)\alpha}{\alpha(1-\alpha)+(1-\alpha)\alpha}=\frac{1}{2}$$

97. P (B) =

a) $\frac{6^3}{7^3}$ b) $\frac{5^3}{7^3}$

c) $\frac{\left(2^6-2\right)^3}{2^{18}}$ d) $\frac{\left(2^6-1\right)^3}{2^{18}}$

Key: C

Hint: no. of ways of selecting atleast one but not al red balls for bag B_1 when considered them differently = ${}^6C_1+{}^6C_2+......+{}^6C_5=2^6-2$.

Similarly for black and white, no. of ways of selecting atleast one but not all for bag B_1 =

$2^6-2,\ Hence\, n(B)=\left(2^6-2\right)^3 \Rightarrow P(B)=\frac{\left(2^6-2\right)^3}{2^{18}}$

(Though no. of different ways of giving atleast one but not all Red (Identical balls) ball to bag B_1 is (4+1) = 5 i.e., no. of different ways of giving at least one ball of each colour in (4+1) (4+1) (4+1) = 5^3. but these are not equally likely so we can not use this.)

98. P (C/B) =

a) $\frac{{}^{12}C_6-1}{\left(2^6-1\right)^2}$ b) $\frac{\left({}^{12}C_6-2\right)}{\left(2^6-2\right)^2}$ c) $\frac{1}{5}$ d) $\frac{7}{25}$

Key: B

Hint: $P(C/B)=\frac{P(B\cap C)}{P(B)}=\frac{n(B\cap C)}{n(B)}$

$(B\cap C)$ event is same as selecting (1W, 1R), (2W, 2R), (3W, 3R) (4W, 4R) (5W, 5R) , and at leat one but not all black balls when considering all balls different

$$n(B\cap C)=\left({}^6C_1{}^6C_1+{}^6C_2{}^6C_2+......+{}^6C_5.{}^6C_5\right).\left({}^6C_1+{}^6C_2+......+{}^6C_5\right)$$

$$=\left({}^{12}C_6-2\right)\left(2^6-2\right)\Rightarrow P(C/B)=\frac{{}^{12}C_6-2}{\left(2^6-2\right)^2}$$

(Though no. of different ways of event $B\cap C$ is 5^2. when balls are identical. i.e., selecting (1W, 1R), (2W, 2R) .....(5W, 5R) → 5 ways and selecting atleast one but not all black balls → 5 ways.)

99. If E and F are two independent events, such that $P(E\cap F)=\frac{1}{6}, P\left(E^C\cap F^C\right)=\frac{1}{3}$ and $\left(P(E)-P(F)\right)\left(1-P(F)\right)>0$, then

(A) $P(E)=\frac{1}{2}$ (B) $P(E)=\frac{1}{4}$ (C) $P(F)=\frac{1}{3}$ (D) $P(F)=\frac{2}{3}$

Key: A, C

Hint: $P(E\cap F)=P(E).P(F)=\frac{1}{6}$ ......(i)

$P\left(E^c\cap F^c\right)=\left(1-P(E)\right)\left(1-P(F)\right)=\frac{1}{3}$

$\Rightarrow P(E)+P(F)=\frac{5}{6}$ ....(ii)

$\Rightarrow \left|P(E)-P(F)\right|=\frac{1}{6}$

As $\left(P(E)-P(F)\right)\left(1-P(F)\right)>0$

$\Rightarrow P(E)>P(F)\Rightarrow P(E)-P(F)=\frac{1}{6}......(iii)$

Solving (ii) and (iii) $\Rightarrow P(E)=\frac{1}{2}, P(F)=\frac{1}{3}$

100. Mr. A makes a bet with Mr. B that in a single throw with two dice he will throw a total of seven before B throws four. Each of them has a pair of dice and they throw simultaneously until one of them wins equal throws being disregarded. Probability that B wins, is

(A) $\frac{1}{3}$ (B) $\frac{4}{11}$ (C) $\frac{5}{16}$ (D) $\frac{6}{17}$

Key: A

Hint: We have $P(A)=P(7)=\frac{6}{36}\text{m}, P(B)=P(4)=\frac{3}{36}$

Since equal throws are disregarded,

Hence in each throw A is twice as likely to win as B.

Let $P(B)=p, P(A)=2p$

$3p = 1 \Rightarrow P = \frac{1}{3}$

101. Each face of a cubical die is numbered with a distinct number from among the first six odd numbers, such that the sum of the two numbers on any pair of opposite face is 12. if ten such dices are thrown simultaneously, then find probability that the sum of the numbers that turn up is exactly 53.

(A) $\frac{53}{6^{10}}$ (B) $\frac{153}{6^{10}}$

(C) $\frac{3}{6^{10}}$ (D) 0\

Key : D

Sol : With 10 dice, the number on each face being odd, we can never get an odd number, since the sum of 10 odd numbers can never be an odd. Hence required probability = 0.

102. Two cards are selected at randomly from a pack of ordinary playing cards. If there found to be of different colours (Red & Black), then conditional probability that both are face cards is

(A) $\frac{36}{325}$ (B) $\frac{18}{169}$

(C) $\frac{9}{169}$ (D) none of these

Key : C (final key)

Sol : Let A $\rightarrow$ they are face cards, B $\rightarrow$ they are of different colours

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{{}^{12}C_2 - 2 \times {}^{6}C_2}{13 \times 26} = \frac{18}{169}.$$

103. Sum of the series S= $\sum_{r=1}^{\infty} P_r$ is

a) 1 b) $\frac{1}{6}$

c) $\frac{1}{5}$ d) $\frac{2}{3}$

key : A (final key)

sol : $S = \frac{1}{5}\left[1 + \sum_{r=7}^{\infty}\left(\frac{1}{6}\right)^{r-6} - \sum_{r=2}^{\infty}\left(\frac{1}{6}\right)^{r-1}\right]$

$S = \frac{1}{5}$

104. In a knockout tournament 16 equally skilled players namely P_1, P_2, -------- P_{16} are participating. In each round players are divided in pairs at random and winner from each pair moves in the next round. If P_2 reaches the semifinal, then the probability that P_1 will win the tournament is.

a) $\frac{3}{64}$ b) $\frac{1}{16}$ c) $\frac{1}{20}$ d) $\frac{1}{15}$

Key: C

Hint: Let E_1 = P_1 win the tournament, E_2 = P_2 reaches the semifinal since all players are equally skilled and there are 4 persons in the semifinal $P(E_2) = \frac{^{15}C_3}{^{16}C_4} = \frac{4}{16} = \frac{1}{4}$

$E_1 \cap E_2$ = P_1 & P_2 both are in semifinal and P_1 wins in semifinal and final

$$P(E_1 \cap E_2) = \frac{^{16-2}C_2}{^{16}C_4} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{16.15} = \frac{1}{80}$$

$$P(E_1 / E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{1/80}{1/4} = \frac{1}{20}$$

105. $'A'$ is a 3×3 matrix with entries from the set $\{-1, 0, 1\}$. The probability that $'A'$ is neither symmetric nor skew symmetric is

A) $\frac{3^9 - 3^6 - 3^3 + 1}{3^9}$ B) $\frac{3^9 - 3^6 - 3^3}{3^9}$ C) $\frac{3^9 - 1}{3^{10}}$ D) $\frac{3^9 - 3^3 + 1}{3^9}$

Key. A

Sol. Total number of matrices that can be formed is 3^9.

Let $A = [a_{ij}]_{3\times3}$ where $a_{ij} \in \{-1, 0, 1\}$

If $'A'$ is symmetric then $a_{ij} = a_{ij} \quad \forall \ i, j$

If $'A'$ is skew symmetric then $a_{ij} = -a_{ij} \quad \forall \ i, j$

106. The digits of a nine – digit number are 1,2,3,4,5,6,7,8,9 written in random order, then the probability that the number is divisible by 11 is

a) $\frac{11}{126}$ b) $\frac{13}{126}$ c) $\frac{11}{104}$ d) $\frac{5}{63}$

Key. A

Sol. Total number of numbers having 9 digits = 9!

A number is divisible by 11 is the difference between the sum of the digits in odd places and the sum of the digits in even places is it self divisible by 11

As the sum digits is 45, the only possibility of the numbers being divisible by 11 is when the sum of the digits in odd places is 28 and the sum of the digits in even places in 17.

$\therefore$ Number of favorable cases = $11\times 5!\times 4!$

$\therefore$ Required probability $=\dfrac{11\times 5!\times 4!}{9!}=\dfrac{11}{126}$

107. That probability that a randomly chosen 3 digit in number has exactly 3 factors is

a) 2/225 b) 7/900 c) 7/300 d) 3/500

Key. B

Sol. A number has exactly 3 factors if the number is squares of a prime number. Squares of 11,13,17,19,23,29,31 are 3- digit numbers. Hence, the required probability is 7/900

108. If a is a positive integer and $a\in$[1,10], then the probability that the graph of the function $f\left(x\right)=x^2-2\left(4a-1\right)x+15a^2-2a-7$ is strictly above the x – axis is

a) $\dfrac{3}{10}$ b) $\dfrac{1}{10}$ c) $\dfrac{1}{5}$ d) $\dfrac{1}{4}$

Key. B

Sol. $f\left(x\right)=x^2-2\left(4a-1\right)x+15a^2-2a-7>0\ \forall x\in R$ if (i) coefficient of $x^2>0$ (ii) $\Delta<0$

$$\therefore D=4\left(4a-1\right)^2-4\left(15a^2-2a-7\right)<0\Rightarrow a^2-6a+8<0$$

$$\Rightarrow a\in\left(2,4\right)$$

$$\Rightarrow a=3\ only$$

$$Hence,\ probability=\frac{1}{10}$$

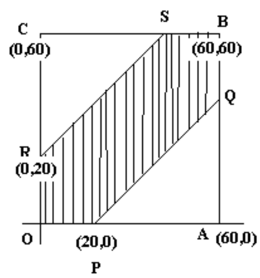
109. Two persons A and B agree to meet at a place between 10a.m to 11 a.m. The first one to arrive waits for 20 minutes and then leave. If the time of their arrival be independent and at random, what is the probability that A and B meet?

a) $\dfrac{1}{3}$ b) $\dfrac{4}{9}$ c) $\dfrac{5}{9}$ d) $\dfrac{2}{3}$

Key. C

Sol. Let A and B arrive at the place of their meeting x minutes and y minutes after 10 a.m.

Then they will meet if $\left|x-y\right|\le 20$



Then the area representing the favourable cases

= Area OPQBRS = 2000sq.units.

Total area = 3600sq. units

$\therefore$ Required probability = $\dfrac{5}{9}$

110. A man takes a step forward with probability 0.4 and one step backward with probability 0.6. Then the probability that at the end of eleven steps he is one step away from the starting point is

(a) ${}^{11}C_5 \times (0.48)^5$ (b) ${}^{11}C_6 \times (0.24)^5$

(c) ${}^{11}C_5 \times (0.12)^5$ (d) ${}^{11}C_6 \times (0.72)^6$

Key. B

Sol. It is possible if he moves (i) 6 steps forward 5 steps backward or (ii) 6 steps backward 5 steps forward

Required probability $= {}^{11}C_6 \left[(0.4)^6 (0.6)^5 + (0.4)^5 (0.6)^6\right] = {}^{11}C_6 \times (0.24)^5$

111. A drawer contains 6 black socks and r red socks $(r \geq 2)$. For the probability of drawing 2 red socks at random from the drawer is to be at least $\frac{1}{2}$, minimum number of socks in the drawer must be

a) 15 b) 16 c) 21 d) 22

Key. C

Sol. Given $\frac{{}^{r}C_2}{{}^{(r+6)}C_2} \geq \frac{1}{2} \Rightarrow (r-15)(r+2) \geq 0 \Rightarrow r \geq 15$

$\therefore$ minimum number of socks = 21

112. A fair coin is tossed 6 times. The probability of getting at least 4 consecutive heads is

a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) $\frac{1}{8}$ d) $\frac{1}{16}$

Key. C

Sol. P(at least 4 consecutive heads) = P (4 consecutive heads)

+ P(5 consecutive

$$= \left(2\left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^6\right) = \left(2\left(\frac{1}{2}\right)^6\right) + \left(\frac{1}{2}\right)^6 = 4\left(\frac{1}{2}\right)^6 + 2\left(\frac{1}{2}\right)^5 = \frac{1}{8}$$

113. A letter is known to have come from CHENNAI, JAIPUR, NAINITAL, MUMBAI. On the postmark only the two consecutive letters AI are legible. The probability that it came from MUMBAI is

(a) $\frac{39}{190}$ (b) $\frac{42}{149}$ (c) $\frac{39}{191}$ (d) $\frac{38}{149}$

Key. B

Sol. A_1 : Selecting a pair of consecutive letters from the word CHENNAI

A_2 : Selecting a pair of consecutive letters from the word JAIPUR

A_3 : Selecting a pair of consecutive letters from the word NAINITAL

A_4 :Selecting a pair of consecutive letters from the word MUMBAI

E: Selecting a pair of consecutive letters AI

Required probability $= P\left(\frac{A_1}{E}\right) = \dfrac{\frac{1}{5}}{\frac{1}{6}+\frac{1}{5}+\frac{1}{7}+\frac{1}{5}} = \frac{42}{149}$

114. X follows a binomial distribution with parameters n and p and Y follows a binomial distribution with parameters m and p. If X and Y are independent then $P\left(\frac{X=r}{X+Y=r+s}\right) =$------

(a) $\frac{{}^nC_r.{}^mC_s}{{}^{(m+n)}C_{(r+s)}}$
(b) $\frac{3\,{}^mC_{r+s}}{{}^{(m+n)}C_{(r+s)}}$
(c) $\frac{2\left({}^mC_r\right)\left({}^nC_s\right)}{{}^{(m+n)}C_{(r+s)}}$
(d) $\frac{\left({}^mC_r\right)\left({}^nC_r\right)}{{}^{(m+n)}C_{(r+s)}}$

Key. A

Sol. $P\left(\frac{X=r}{X+Y=r+s}\right) = \frac{P\left[(X=r)\cap(X+Y=r+s)\right]}{P(X+Y=r+s)} = \frac{P(X=r)P(Y=s)}{P(X+Y=r+s)}$

$\therefore P\left(\frac{X=r}{X+Y=r+s}\right) = \frac{\left({}^nC_r.q^{n-r}p^r\right)\left({}^mC_s.q^{m-s}p^s\right)}{{}^{(m+n)}C_{(r+s)}p^{r+s}.q^{m+n-r-s}} = \frac{{}^nC_r.{}^mC_s}{{}^{(m+n)}C_{(r+s)}}$

115. If the cube of a natural number ends with a prime digit then the probability of its fourth power ending not with a prime digit, is

(A) 3/10
(B) 9/10
(C) 4/9
(D) 3/4

Key. D

Sol. Total cases are with numbers ending with 3, 5, 7 or 8.
Favourable cases are with numbers ending with 3, 7 or 8.
So, the required probability = 3/4

116. Consider the following three words (written in capital letters): 'PRANAM', 'SALAAM' and 'HELLO'. One of the three words is chosen at random and a letter from it is drawn. The letter is found to be 'A' or 'L' then the probability that it has come from the word 'PRANAM', is

(A) 0
(B) 1/3
(C) 2/5
(D) 5/21

Key. D

Sol. Let Q $\rightarrow$ event that 'PRANAM' is selected. S $\rightarrow$ event that 'SALAAM' is selected
H $\rightarrow$ event that 'HELLO' is selected. E $\rightarrow$ event that the letter chosen is A or L.

P (Q/E) = $\dfrac{P(Q)P(E/Q)}{P(Q)P(E/Q)+P(S)P(E/S)+P(H)P(E/H)}$ = $\dfrac{\frac{1}{3}\times\frac{2}{6}}{\frac{1}{3}\times\frac{2}{6}+\frac{1}{3}\times\frac{4}{6}+\frac{1}{3}\times\frac{2}{5}}=\dfrac{5}{21}$

117. A fair coin is tossed 10 times and the outcomes are listed. Let H_i be the event that the i^{th} outcome is a head and A_m be the event that the list contains exactly m heads, then

(A) H_3 and A_4 are independent
(B) A_1 and A_9 are independent
(C) H_2 and A_5 are independent
(D) H_4 and H_8 are not independent

Key. C

Sol. $P(H_i) = \dfrac{1}{2}$, $P(A_m) = \dfrac{{}^{10}C_m}{2^{10}}$

$P(H_i \cap A_m) = \dfrac{{}^{9}C_{m-1}}{2^{10}}$

For H_i & A_m to be independent,

$\dfrac{{}^{9}C_{m-1}}{2^{10}} = \dfrac{1}{2}\times\dfrac{{}^{10}C_m}{2^{10}} \Rightarrow 1 = \dfrac{1}{2}\times\dfrac{10}{m} \Rightarrow m = 5.$

118. 64 players play in a knockout tournament. Assuming that all the players are of equal strength, the probability that P_1 loses to P_2 and P_2 becomes the eventual winner is

a) $\dfrac{1}{612}$ b) $\dfrac{1}{672}$ c) $\dfrac{1}{512}$ d) $\dfrac{1}{63.2^6}$

Key. B

Sol. $\dfrac{{}^{62}c_5}{{}^{63}c_6}\cdot\dfrac{1}{64}=\dfrac{1}{672}$

119. Team A plays with 5 other teams exactly once. Assuming that for each match the probabilities of a win, draw and loss are equal, then

a) The probability that A wins and loses equal number of matches is $\dfrac{34}{81}$

b) The probability that A wins and loses equal number of matches is $\dfrac{17}{81}$

c) The probability that A wins more number of matches than it loses is $\dfrac{17}{81}$

d) The probability that A loses more number of matches than it wins is $\dfrac{16}{81}$

Key. B

Sol.Prob.of equal no. of W and L = 0 wins, 0 losses + 1W, 1L + 2W,

2L = $\left(\dfrac{1}{3}\right)^5 + {}^5c_1.{}^4c_1.\left(\dfrac{1}{3}\right)^5 + {}^5c_2.{}^3c_2.\left(\dfrac{1}{3}\right)^5 = \dfrac{17}{81}$

120. The probability that the fourth powers of a number ends in 1 is

a) $\frac{2}{3}$ b) $\frac{2}{5}$ c) $\frac{1}{5}$ d) $\frac{1}{10}$

Key. B

Sol.The fourth power of a number ends with 1 if the last digit is 1, 3, 7, 9

$\therefore$ required probability = 4/10 = 2/5

121. One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife is

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{2}{5}$ d) $\frac{1}{5}$

Key. C

Sol. $\frac{{}^4c_1}{{}^4c_2 + {}^4c_1} = \frac{4}{10} = \frac{2}{5}$

122. Suppose $f(x) = x^3 + ax^2 + bx + c$ where a, b, c are chosen respectively by throwing a die three times, then the probability that f(x) is an increasing function is

a) $\frac{4}{9}$ b) $\frac{3}{8}$ c) $\frac{2}{5}$ d) $\frac{16}{34}$

Key. A

Sol. $f'(x) = 3x^2 + 2ax + b$

$f'(x) \geq 0, \forall x$ discriminiant, $(2a)^2 - 4 \text{ x} 3 \text{ x } (2a)^2 - 4\times 3\times b \leq 0 \Rightarrow a^2 - 3b \leq 0$

This is true for exactly 16 ordered pairs (a, b) namely (1, 1), (1, 2), (1, 3), (1,4), (1, 5) (1, 6), (2, 2) (2, 3), (2, 4, (2, 5), (2, 6), (3, 3) (3, 4), (3,5) , (3, 6) and (4, 6)

Thus, required probability = $\frac{16}{36} = \frac{4}{9}$

123. A bag initially contains one red ball and two blue balls. An experiment consisting of selecting a ball at random, noting its colour and replacing it together with an additional ball of the same colour. If three such trials are made, then

a) Probability that atleast one blue ball is drawn is 0.9

b) Probability that exactly one blue ball is drawn is 0.2

c) Probability that all the drawn balls are red given that all the drawn balls are of the same colour is 0.2

d) Probability that atleast one red ball is drawn is 0.6

Key. D

Sol. Prob. That atleast one blue ball is drawn

= 1- prob that all the balls drawn are red.

$= 1 - \frac{1}{3}.\frac{1}{2}.\frac{3}{5} = 1 - \frac{1}{10} = 0.9$

Prob. That exactly one blue ball is drawn

$= \frac{1}{3}.\frac{1}{2}.\frac{2}{5} + \frac{2}{3}.\frac{1}{4}.\frac{2}{5} = 0.2$

Prob. that all drawn balls are red given that all the drawn balls of the same colour

$$=\frac{\frac{1}{10}}{\frac{1}{10}+\frac{4}{10}}=\frac{1}{5}=0.2$$

Prob.that atleast one red ball is drawn $=1-\left(\frac{2}{3}.\frac{3}{4}.\frac{4}{5}\right)$ = 0.6

124. A has 3-shares in a lottery containing 3 prizes and 9-blanks, B has 2-Shares in a lottery containing 2 prizes and 6 – blanks then ratio of their success is

1. 952:715 2. 715:259 3. 265:233 4. 125 : 752

Key. 1

Sol. A has 3-shares $\Rightarrow$ $P\left(A\,gets\,success\right)=1-\frac{9_{C_3}}{12_{C_3}}=\frac{34}{55}$

$$P\left(B\,gets\,success\right)=1-\frac{6_{C_2}}{8_{C_2}}=1-\frac{15}{28}=\frac{13}{28}$$

$$P\left(A\right):P\left(B\right)=\frac{34}{55}:\frac{13}{28}=952:715$$

125. If a is an integer lying in [-5,30] then probability that graph of $y=x^2+2(a+4)x-5a+64$ is strictly above the x-axis

1. $\frac{1}{6}$ 2. $\frac{2}{9}$ 3. $\frac{3}{5}$ 4. $\frac{1}{5}$

Key. 2

Sol. n(s) =36

$y=x^2+2\left(a+4\right)x-5x+64$ lies above the X-axis is

If $4\left(a+4\right)^2-4\left(1\right)\left(-5a+64\right)<0$

$\Rightarrow -16<a<3$

$\Rightarrow a=-5,-4,-3,-2,-1,0,1,2$

n(E) =8

$\therefore P\left(E\right)=\frac{8}{36}=\frac{2}{9}$

126. There are 4-machines and it is known that exactly two of them are faulty they are tested one by one in a random order till both faulty machines are Identified. The probability that only two tests are needed

1. $\frac{1}{3}$ 2. $\frac{1}{6}$ 3. $\frac{1}{4}$ 4. $\frac{3}{4}$

Key. 2

Sol. $P(E)=\frac{2}{4}\times\frac{1}{3}=\frac{1}{6}$

127. Two numbers x and y are chosen such that $x\in[0,4], y\in[0,4]$ then probability that $y^2\le x$

1. $\frac{1}{3}$ 2. $\frac{2}{3}$ 3. $\frac{1}{4}$ 4. $\frac{3}{4}$

Key. 1

Sol. n(S)=16

n(E)= $\int_0^4 \sqrt{x}dx=\frac{16}{3}$

$P(E)=\frac{\frac{16}{3}}{16}=\frac{1}{3}$

128. A fair coin is tossed 5 times then probability that two heads do not occur consecutively (No two heads come together)

1. $\frac{1}{16}$ 2. $\frac{15}{32}$ 3. $\frac{13}{32}$ 4. $\frac{7}{16}$

Key. 3

Sol. $p\left(\frac{E}{no\,heads}\right)+p\left(\frac{E}{1(head)}\right)+p\left(\frac{E}{2-heads}\right)+p\left(\frac{E}{3-heads}\right)$

Where $E\rightarrow$gtg n two consecutive heads.

$=\frac{1}{32}+\frac{5}{32}+\frac{6}{32}+\frac{1}{32}=\frac{14}{32}=\frac{7}{16}$

129. A man throws a die until he gets a number bigger than 3. The probability that he gets 5 in the last throw

1. $\frac{1}{3}$ 2. $\frac{1}{4}$ 3. $\frac{1}{6}$ 4. $\frac{1}{36}$

Key. 1

Sol. P(gtg a number bigger than 3) = $\frac{1}{2}$

P(gtg 5 in throw)= $\frac{1}{6}$

$E\rightarrow$gtg 5 in last throw when he gets a number bigger than 3

$$P(E)=\frac{1}{6}+\frac{1}{2}.\frac{1}{6}+\frac{1}{2}.\frac{1}{2}.\frac{1}{6}+....\infty$$

$$=\frac{1}{6}\times\frac{1}{1-\frac{1}{2}}=\frac{1}{3}$$

130. A bag contains 4-balls two balls are drawn from the bag and are found to be white then probability that all balls in the bag are white

1. $\frac{1}{5}$ 2. $\frac{2}{5}$ 3. $\frac{3}{5}$ 4. $\frac{4}{5}$

Key. 3

Sol. $$P(E)=\frac{\frac{1}{3}.\frac{4_{C_2}}{4_{C_2}}}{\frac{1}{3}\left\{\frac{2_{C_2}}{4_{C_2}}+\frac{3_{C_2}}{4_{C_2}}+\frac{4_{C_2}}{4_{C_2}}\right\}}$$

$$=\frac{1}{\frac{1+3+6}{6}}=\frac{6}{10}=\frac{3}{5}$$

131. A randomly selected year is containing 53 Mondays then probability that it is a leap year

1. $\frac{2}{5}$ 2. $\frac{3}{5}$ 3. $\frac{4}{5}$ 4. $\frac{1}{5}$

Key. 1

Sol. Selected year may non leap year with a probability $\frac{3}{4}$

Selected year may leap year with a probability $\frac{1}{4}$

$E\rightarrow$Even that randomly selected year contains 53 Mondays

$$P(E)=\frac{3}{4}\times\frac{1}{7}+\frac{1}{4}\times\frac{2}{7}=\frac{5}{28}$$

$$P\left(\frac{leapyear}{E}\right)=\frac{\frac{2}{28}}{\frac{5}{28}}=\frac{2}{5}$$

132. When 5-boys and 5-girls sit around a table the probability that no two girls come together

1. $\frac{1}{120}$ 2. $\frac{1}{126}$ 3. $\frac{3}{47}$ 4. $\frac{4}{7}$

Key. 2

Sol. $E\rightarrow$first boys can be arranged in $\lfloor 4$ ways, then there are 5-gaps between boys in 5-gaps,

5-girls can be arranged in $\lfloor 5$ ways

$$P(E)=\frac{\lfloor 5 \lfloor 4}{\lfloor 9}=\frac{5\times4\times3\times2}{5\times6\times7\times8\times9}=\frac{1}{126}$$

133. There are m-stations on a railway line. A train has to stop at 3 intermediate stations then probability that no two stopping stations are adjacent

1. $\frac{1}{m\,c_3}$ 2. $\frac{3}{mc_3}$ 3. $\frac{m-2_{C_3}}{mc_3}$ 4. $\frac{mc_2}{m\,c_3}$

Key. 3

Sol. Let 3-stopping stations be S_1, S_2, S_3 then are m-3 stations remaining. Between these m-3 stations there are m-2 places select any 3 for S_1, S_2, S_3, then there are no two stopping stations are adjacent

$$P(E)=\frac{m-2_{C_3}}{m_{C_3}}$$

134. A has 3-shares in a lottery containing 3 prizes and 9-blanks, B has 2-Shares in a lottery containing 2 prizes and 6 – blanks then ratio of their success is

1. 952:715 2. 715:259 3. 265:233 4. 125 : 752

Key. 1

Sol. A has 3-shares $\Rightarrow$ $P(A\,gets\,success)=1-\frac{9_{C_3}}{12_{C_3}}=\frac{34}{55}$

$$P(B\,gets\,success)=1-\frac{6_{C_2}}{8_{C_2}}=1-\frac{15}{28}=\frac{13}{28}$$

$$P(A):P(B)=\frac{34}{55}:\frac{13}{28}=952:715$$

135. If a is an integer lying in [-5,30] then probability that graph of $y=x^2+2(a+4)x-5a+64$ is strictly above the x-axis

1. $\frac{1}{6}$ 2. $\frac{2}{9}$ 3. $\frac{3}{5}$ 4. $\frac{1}{5}$

Key. 2

Sol. n(s) =36

$y=x^2+2(a+4)x-5x+64$ lies above the X-axis is

If $4(a+4)^2-4(1)(-5a+64)<0$

$\Rightarrow -16<a<3$

$\Rightarrow a=-5,-4,-3,-2,-1,0,1,2$

n(E) =8

$\therefore P(E)=\frac{8}{36}=\frac{2}{9}$

136. There are 4-machines and it is known that exactly two of them are faulty they are tested one by one in a random order till both faulty machines are Identified. The probability that only two tests are needed

1. $\frac{1}{3}$ 2. $\frac{1}{6}$ 3. $\frac{1}{4}$ 4. $\frac{3}{4}$

Key. 2

Sol. $P(E)=\frac{2}{4}\times\frac{1}{3}=\frac{1}{6}$

137. Two numbers x and y are chosen such that $x\in[0,4], y\in[0,4]$ then probability that $y^2\le x$

1. $\frac{1}{3}$ 2. $\frac{2}{3}$ 3. $\frac{1}{4}$ 4. $\frac{3}{4}$

Key. 1

Sol. n(S)=16

n(E)= $\int_0^4 \sqrt{x}dx=\frac{16}{3}$

$$P(E)=\frac{\frac{16}{3}}{16}=\frac{1}{3}$$

138. The probability that randomly selected positive integer is relatively prime to 6

1. $\frac{1}{2}$ 2. $\frac{1}{3}$ 3. $\frac{1}{6}$ 4. $\frac{5}{6}$

Key. 2

Sol. Among every 6-consecutive integers one divisible by 6 and other integers leaves remainders 1,2,3,4,5 when divided by 6

The numbers which leave the remainder 1 and 5 are relatively prime to 6

Required probability $\frac{2}{6}=\frac{1}{3}$

139. A and B are events such that P(A)=0.3 $P(A\cup B)=0.8$. If A and B are independent then $P(B)$ =

1. $\frac{1}{7}$ 2. $\frac{3}{7}$ 3. $\frac{5}{7}$ 4. $\frac{6}{7}$

Key. 3

Sol. $P(A\cap B)=P(A).P(B)$

$$P(A\cup B)=P(A)+P(B)-P(A)P(B)$$

$0.8 = 0.3 + P(B)(1-0.3)$

$0.5 = P(B)(0.7) \Rightarrow P(B) = \frac{5}{7}$

140. Thirty-two players ranked 1 to 32 are playing in a knockout tournament. Assume that in every match between any two players, the better-ranked player wins, the probability that ranked 1 and ranked 2 players are winner and runner up respectively, is

(A) 16/31 (B) 1/2 (C) 17/31 (D) None of these

Key. A

Sol. For ranked 1 and 2 players to be winners and runners up res., they should not be paired with each other in any rounded. Therefore, the required probability $\frac{30}{31} \times \frac{14}{15} \times \frac{6}{7} \times \frac{2}{3} = \frac{16}{31}$

141. A coin is tossed 7 times. Then the probability that at least 4 consecutive heads appear is

(A) 3/16 (B) 5/32 (C) 5/16 (D) 1/8

Key. B

Sol. Let H denote the head,

T the tail.

* Any of the head or tail

P(H) = $\frac{1}{2}$, P(T) = $\frac{1}{2}$ P(*) = 1

HHHH*** = $\left(\frac{1}{2}\right)^4 \times 1 = \frac{1}{16}$

THHHH** = $\left(\frac{1}{2}\right)^5 \times 1 = \frac{1}{32}$

THHHH = $\left(\frac{1}{2}\right)^5 \times 1 = \frac{1}{32}$

**THHHH = $\left(\frac{1}{2}\right)^5 \times 1 = \frac{1}{32}$

$\frac{5}{32}$

142. Two natural numbers *a* and *b* are selected at random. The probability that $a^2 + b^2$ is divisible by 7 is

(a) 3/8 (b) 1/7 (c) 3/49 (d) 1/49

Key. D

Sol. $a_1 b$ are is of then form

$a_1 b \in \{7m, 7m+1, 7m+2, 7m+3, 7m+4, 7m+6\}$

$a_1^2 b^2 \in \{7m_1, 7m_1+1, 7m_1+4, 7m_1+2, 7m_1+2, 7m_1+4, 7m_1+1\}$

$\therefore a^2, b^2$ must be of the form 7m.

Probability = $\frac{1}{49}$

143. If a and b are chosen randomly from the set consisting of numbers 1, 2, 3, 4, 5, 6 with replacement. Then probability that $\underset{x\to 0}{Lt}\left(\frac{a^x+b^x}{2}\right)^{2/x}=6$ is

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{9}$ (d) $\frac{2}{9}$

Key. C

Sol. $\underset{x\to 0}{Lt}\left(\frac{a^x+b^x}{2}\right)^{2/x}=6$

$=e^{\underset{x\to 0}{Lt}\left(\frac{a^x-1}{x}\right)+\left(\frac{b^x-1}{x}\right)}=6$

$=e^{\log a+\log b}=6$

ab = 6

$(a,b)=(1,6),(6,1),(2,3),(3,2)$

Required probability = $\frac{4}{6\times 6}=\frac{1}{9}$

144. An urn contains five balls. Two balls are drawn and found to be white. Probability that all balls are white, is

(A) $\frac{1}{3}$ (B) $\frac{2}{9}$

(C) $\frac{1}{2}$ (D) $\frac{3}{4}$

Key. C

Sol. Event A_1 = urn contains 5 white balls

Event A_2 = urn contains 4 white balls

Event A_3 = urn contains 3 white balls

Event A_4 = urn contains 2 white balls

Event A = Drawing two white balls when two balls are drawn from five balls

$$P\left(\frac{A_1}{A}\right)=\frac{P(A_1)P\left(\frac{A}{A_1}\right)}{P(A_1)P\left(\frac{A}{A_1}\right)+P(A_2)P\left(\frac{A}{A_2}\right)+P(A_3)P\left(\frac{A}{A_3}\right)+P(A_4)P\left(\frac{A}{A_4}\right)}$$

$$P(A_1)=P(A_2)=P(A_3)=P(A_4)=\frac{1}{4}$$

$$P\left(\frac{A}{A_1}\right)=1,\ P\left(\frac{A}{A_2}\right)=\frac{{}^4C_2}{{}^5C_2},\ \ P\left(\frac{A}{A_3}\right)=\frac{{}^3C_2}{{}^5C_2}$$

$$P\left(\frac{A}{A_4}\right)=\frac{{}^2C_2}{{}^5C_2}$$

145. Three numbers a, b, c are choosen randomly from the set of natural numbers. The probability that '$a^2+b^2+c^{2}$' is divisible by 7 is

(A) 1/3 (B) 1/4

(C) 1/5 (D) 1/7

Key. D

Sol. Numbers are of the form: 7k, 7k + 1, 7k + 2, 7k + 3, 7k + 4, 7k + 5, 7k + 6 their squares: 7k, 7k + 1, 7k + 4, 7k + 2, 7k + 2, 7k + 1, 7k + 1.

So, for $a^2 + b^2 + c^2$ to be a multiple of 7, either all the three squares should be of the form 7k or they belong to the catagories 7k + 1, 7k + 2, 7k + 4 separately.

So, required prob., = $\left(\frac{1}{7}\right)^3 + 3!\left(\frac{2}{7}\right)\left(\frac{2}{7}\right)\left(\frac{2}{7}\right) = \frac{1}{7}$

146. If two events A and B are such that $P(\bar{A}) = 0.3$, $P(B) = 0.4$, $P(A \cap \bar{B}) = 0.5$, then the value of $P(B/(A \cup \bar{B}))$ is

(A) 1/2 (B) 1/4

(C) 3/4 (D) 4/5

Key. B

Sol. $P(B)\ (A \cup \bar{B})) = \frac{P(B \cap (A \cup \bar{B})}{P(A \cup \bar{B})} = \frac{P(A) - P(A \cap \bar{B})}{P(A) + P(\bar{B}) - P(A \cap \bar{B})}$

$= \frac{0.7 - 0.5}{0.7 + 0.6 - 0.5} = \frac{0.2}{0.8} = \frac{1}{4}$

147. If the third power of a natural no. ends with a prime digit then the probability of its fourth power ending not with a prime digit, is

(A) 3/10 (B) 9/10

(C) 4/9 (D) 3/4

Key. D

Sol. Total cases are when numbers ending with 3, 5, 7, or 8

Favourable cases are when numbers are ending with 3, 7, or 8

So, the required probability = 3/4

148. A fair coin is tossed 10 times and the outcomes are listed. Let H_i be the event that the ith out–come is a head and A_m be the event that the list contains exactly m heads, then

(A) H_3 and A_4 are independent (B) A_1 and A_9 are independent

(C) H_2 and A_5 are independent (D H_4 and H_8 are not independent

Key. C

Sol. $P(H_i) = \frac{1}{2}$, $P(A_m) = \frac{{}^{10}C_m}{2^{10}}$

$P(H_i \cap A_m) = \frac{{}^{9}C_{m-1}}{2^{10}}$

For H_i and A_m to be independent

$\frac{{}^{9}C_{m-1}}{2^{10}} = \frac{1}{2} \times \frac{{}^{10}C_m}{2^{10}} \Rightarrow 1 = \frac{1}{2} \times \frac{10}{m} \Rightarrow m = 5$

149. A fair coin is tossed 9 times. Heads are coming 7 times. The probability that among these heads atleast 6 are occurring consecutively, is

(A) 1/8 (B) 1/5

(C) 1/4 (D) 1/3

Key. C

Sol. Out of 7 heads exactly six consecutive heads occure in 6 ways and all seven heads consecutively can occur in 3 ways so the required probability = $\frac{6+3}{{}^{9}C_7} = \frac{9}{36} = \frac{1}{4}$.

150. One ticket is selected at random from 100 tickets numbered 00, 01, 02, ……. , 99. Suppose *X* and *Y* are the sum and product of the digits found on the ticket, then $P(X=7/Y=0)$ is given by

A) 2/3 B) 2/19 C) 1/50 D) None of these

Key. B

Sol. We have $(X=7)=\{07,16,25,34,43,52,61,70\}$

And $(Y=0)=\{00,01,02,...,09,10,20,30,.....,90\}$

Thus, $(X=7)\cap(Y=0)=\{07,70\}$

$\therefore P(X=7/Y=0)=\dfrac{P\{(X=7)\cap(Y=0)\}}{P(Y=0)}=\dfrac{2}{19}$.

151. If the mean and variance of a binomial variate *X* are 7/3 and 14/9 respectively, then the probability that *X* takes value 6 or 7 is equal to

A) $\dfrac{1}{729}$ B) $\dfrac{5}{729}$ C) $\dfrac{7}{729}$ D) $\dfrac{13}{729}$

Key. B

Sol. We have $np=\dfrac{7}{3}, npq=\dfrac{14}{9}$, therefore

$\dfrac{7}{3}q=\dfrac{14}{9}\Rightarrow q=\dfrac{2}{3}\Rightarrow p=\dfrac{1}{3}$.

Thus, $n(1/3)=7/3\Rightarrow n=7$

Now, $P(X=6\,or\,7)=P(X=6)+P(X=7)$

$={}^7C_6p^6q^1+{}^7C_7p^7q^0$

$=7\left(\dfrac{1}{3}\right)^6\left(\dfrac{2}{3}\right)+\left(\dfrac{1}{3}\right)^7=\dfrac{5}{729}$.

152. If *A* and *B* are events of the same random experiment with $P(A)=0.2, P(B)=0.5$, then maximum value of $P(\overline{A}\cap B)$ is

A) 1/4 B) 1/2 C) 1/8 D) 1/16

Key. B

Sol. $P(\overline{A}\cap B)\le P(B)$ if $P(A)+P(B)\le 1$

$\therefore$ maximum value = 0.5

153. A pair of fair dice is rolled together till a sum of either 5 or 7 is obtained. The probability that 5 comes before 7 is

A) 0.2 B) 0.3 C) 0.4 D) 0.5

Key. C

Sol. Let A denote the event that a sum of 5 occurs, B the event that a sum of 7 occurs and C the event that neither a sum of 5 nor a sum of 7 occurs we have

$P(A)=\dfrac{4}{36}=\dfrac{1}{9}, P(B)=\dfrac{6}{36}=\dfrac{1}{6}$ and $P(C)=\dfrac{26}{36}=\dfrac{13}{18}$.

Thus, P (A occurs before B)

$=P(A)+P(C)P(A)+P(C)P(C)P(A)+.......$

$$= \frac{P(A)}{1-P(C)} = \frac{\frac{1}{9}}{1-\frac{13}{18}} = \frac{2}{5}.$$

154. Sixteen players $P_1, P_2, ..., P_{16}$ play in a tournament. They are divided into eight pairs at random. From each pair a winner is decided on the basis of a game played between the two players of the pair. Assuming that all the players are of equal strength, the probability that exactly one of the two players P_1 and P_2 is among the eight winners is

A) 4/15 B) 7/15 C) 8/15 D) 17/30

Key. C

Sol. Let $E_1(E_2)$ denote the event that P_1 and P_2 are paired (not paired) together and let A denote the event that one of two players P_1 and P_2 is among the winners.

Since, P_1 can be paired with any of the remaining 15 players.

We have, $P(E_1) = \frac{1}{15}$

And $P(E_2) = 1 - P(E_1) = 1 - \frac{1}{15} = \frac{14}{15}$

In case E_1 occurs, it is certain that one of P_1 and P_2 will be among the winners. In case E_2 occurs, the probability that exactly one of P_1 and P_2 is among the winners is

$$P\left\{\left(P_1 \cap \overline{P_2}\right) \cup \left(\overline{P_1} \cap P_2\right)\right\} = P\left(P_1 \cap \overline{P_2}\right) + P\left(\overline{P_1} \cap P_2\right)$$

$$= \left(\frac{1}{2}\right)\left(1-\frac{1}{2}\right) + \left(1-\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

i.e, $P(A/E_1) = 1$ and $P(A/E_2) = \frac{1}{2}$

By the total probability Rule,

$$P(A) = P(E_1).P\left(\frac{A}{E_1}\right) + P(E_2).P\left(\frac{A}{E_2}\right)$$

$$= \frac{1}{15}(1) + \frac{14}{15}\left(\frac{1}{2}\right) = \frac{8}{15}.$$

155. If four cards are drawn at random from a pack of 52 playing cards, then the probability of getting at least two kings is

(a) $\frac{6961}{52_{C_4}}$ (b) $\frac{11}{52_{C_4}}$

(c) $\frac{6768}{52_{C_4}}$ (d) $\frac{24}{52_{C_4}}$

Key. A

Sol. Probability at least two king $= \frac{4_{C_2} 48_{C_2} + 4_{C_3} 48_{C_1} + 4_{C_4}}{52_{C_4}} = \frac{6961}{52_{C_4}}$

156. Two 8 faced dice (numbered from 1 to 8) are tossed. The probability that the product of two counts is a square number, is

(A) 1/8 (B) 7/32

(C) 3/16 (D) 3/8

Key. C

Sol. n(s) = 8 × 8 = 64

Square values that product can take are 1, 4, 9, 16, 25, 36, 49, 64

4 : (1, 4), (2, 2), (4, 1)

16 : (2, 8) (4, 4), (8, 2)

For other values, there is only one way of getting the product

$\therefore$ n(E) = 2 × 3 + 6 × 1 = 12

$\therefore$ P(E) = $\frac{12}{64} = \frac{3}{16}$.

157. A die is thrown a fixed number of times. If probability of getting even number 3 times is same as the probability of getting even number four times, then probability of getting even number exactly once is

(A) 1/4 (B) 3/128

(C) 5/64 (D) 7/128

Key. D

Sol. $^nC_3\left(\frac{1}{2}\right)^n = {}^nC_4\left(\frac{1}{2}\right)^n$

Where n = number of times die is thrown

$\Rightarrow {}^nC_3 = {}^nC_4 \Rightarrow n = 7$

$\therefore$ Required prob. = $^7C_1\left(\frac{1}{2}\right)^7 = \frac{7}{128}$

158. The sum of two natural numbers is 30 .The probability that their product is less than 150 is

a) $\frac{1}{5}$ b) $\frac{3}{29}$ c) $\frac{1}{6}$ d) $\frac{6}{29}$

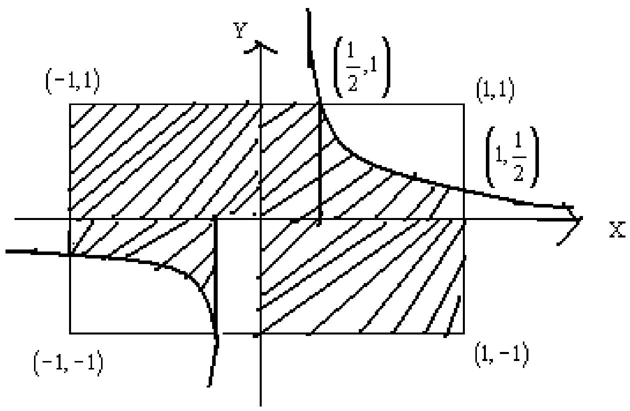
Key. D

Sol. Let the numbers be *x,y* .Given *x+y*=30, $x, y \in N \Rightarrow x = 1, 2, 3, ...29$

$$P(xy < 150) = P\left((30 - x)x < 150\right)$$

$$= P(x > 15 + \sqrt{75}\,) = P(x = 24, 25, 26, 27, 28, 29)$$

$$= \frac{5}{29}$$



159. A bag contains 7 white balls and 3 black balls , all being distinct . Balls are drawn one by one without replacement till all black balls are drawn . The probability that the procedure of drawing these balls comes to an end at the 4^{th} draw is

a) $\frac{1}{40}$ b) $\frac{1}{20}$ c) $\frac{1}{10}$ d) $\frac{1}{80}$

Key. A

Sol. The procedure comes to an end at 4 th draw if in the first 3 draws , 2 black balls drawn and in the 4 th drawn remaining black ball is drawn

$\therefore$ Required probability = $\dfrac{({}^{3}C_{2})({}^{7}C_{1})}{{}^{10}C_{3}}.\dfrac{1}{7}=\dfrac{1}{40}$

160. If a and b are selected at random from the range of y (a, b are distinct positive integers). Then the probability of selecting distinct ordered pairs (a, b) of prime numbers from the range of y, where $y=\dfrac{147}{x+\dfrac{1}{x}+5}\quad \forall\ x>0$

a) $\dfrac{3}{32}$ b) $\dfrac{2}{15}$ c) $\dfrac{5}{32}$ d) $\dfrac{2}{21}$

Key. B

Sol. $\left(x+\dfrac{1}{x}+5\right)\ge 2+5=7 \quad y\le\dfrac{147}{7}=21\Rightarrow 0<y\le 21$ Required prob

$=\dfrac{{}^{8}C_{2}.2!}{{}^{21}C_{2}.2!}=\dfrac{2}{15}$

161. 64 players play in a knockout tournament. Assuming that all the players are of equal strength, the probability that P_1 loses to P_2 and P_2 becomes the eventual winner is

a) $\dfrac{1}{612}$ b) $\dfrac{1}{672}$ c) $\dfrac{1}{512}$ d) $\dfrac{1}{63.2^{6}}$

Key. B

Sol. $\dfrac{{}^{62}C_{5}}{{}^{63}C_{6}}.\dfrac{1}{64}=\dfrac{1}{672}$

162. Team A plays with 5 other teams exactly once. Assuming that for each match the probabilities of a win, draw and loss are equal, then

a) The probability that A wins and loses equal number of matches is $\dfrac{34}{81}$

b) The probability that A wins and loses equal number of matches is $\dfrac{17}{81}$

c) The probability that A wins more number of matches than it loses is $\dfrac{17}{81}$

d) The probability that A loses more number of matches than it wins is $\dfrac{16}{81}$

Key. B

Sol. Prob.of equal no. of W and L = 0 wins, 0 losses + 1W, 1L + 2W,

2L = $\left(\dfrac{1}{3}\right)^{5}+{}^{5}C_{1}.{}^{4}C_{1}.\left(\dfrac{1}{3}\right)^{5}+{}^{5}C_{2}.{}^{3}C_{2}.\left(\dfrac{1}{3}\right)^{5}=\dfrac{17}{81}$

163. The probability that the fourth powers of a number ends in 1 is

a) $\dfrac{2}{3}$ b) $\dfrac{2}{5}$ c) $\dfrac{1}{5}$ d) $\dfrac{1}{10}$

Key. B

Sol. The fourth power of a number ends with 1 if the last digit is 1, 3, 7, 9

$\therefore$ required probability = 4/10 = 2/5

164. One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife is

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{2}{5}$ d) $\frac{1}{5}$

Key. C

Sol. $\frac{{}^4c_1}{{}^4c_2 + {}^4c_1} = \frac{4}{10} = \frac{2}{5}$

165. A bag contains 4 red and 3 blue balls. Two drawings of two balls are made. The probability that the first drawing gives 2 red balls and the second drawing gives two blue balls if the balls are not returned to the bag after the first draw is

A) 2/49 B) 3/35 C) 3/10 D) ¼

Key. B

Sol. $\frac{4c_2}{7c_2} \times \frac{3c_2}{5c_2} = \frac{4\times 3}{7\times 6} \times \frac{3}{10} = \frac{3}{35}$

166. A, B are two independent events such that $P(A) > \frac{1}{2}$ $P(B) > \frac{1}{2}$. If $P(A \cap \bar{B}) = \frac{3}{25}$ and $P(\bar{A} \cap B) = \frac{8}{25}$ then $P(A \cap B) =$

A) 3/4 B) 2/3 C) 12/25 D) 18/25

Key. C

Sol. Let $P(A) = x$ and $P(B) = y$. $x > 1/2, y > 1/2$

$P(A-B) = x - xy = 3/25$ and $P(B-A) = y - xy = 8/25$

167. Three groups of children contain respectively 3 girls and 1 boy; 2 girls and 2 boys; 1 girl and 3 boys. One child is selected at random from each group. The probability that the three selected consists of 1 girl and 2 boys is

A) 5/9 B) 13/32 C) 12/19 D) 25/64

Key. B

Sol. $\frac{3}{4}\times\frac{2}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{2}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{2}{4}\times\frac{1}{4}=\frac{26}{64}$

168. A certain type of missile hits its target with probability 0.3. The number of missiles that should be fired so that there is atleast an 80% probability of hitting a target is

A) 3 B) 4 C) 5 D) 6

Key. C

Sol. Let n be the required number.

$\therefore$ The probability that 'n' missiles miss the target is $(0.7)^n$. We require $1-(0.7)^n > 0.8$

i.e., $(0.7)^n < 0.2$. The least value of 'n' satisfying this inequality is 5.

169. A team has probability 2/3 of winning a game whenever it plays. If the team plays 4 games then the probability that it wins more than half of the games is

A) 17/25 B) 15/19 C) 16/27 D) 13/20

Key. C

Sol. Let p be the probability that the team wins a game. Let $q = 1 - p$ .Then the random variable "number of wins" follows the binomial distribution $P(X = K) = {}^4C_k q^{4-k} p^k, k = 0,1,2,3,4$.

Required probability = ${}^4C_3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3 + {}^4C_4\left(\frac{2}{3}\right)^4 = \frac{16}{27}$.

170. From a bag containing 10 distinct balls, 6 balls are drawn simultaneously and replaced. Then 4 balls are drawn. The probability that exactly 3 balls are common to the drawings is

A) 8/21 B) 6/19 C) 5/24 D) 9/22

Key. A

Sol. Let S be the sample space of the composite experiment of drawing 6 in the first draw and then four in second draw then $|S| = {}^{10}C_6 \times {}^{10}C_4$

$\therefore$ Required Probability = $\frac{{}^{10}C_6 \times {}^6C_3 \times {}^4C_1}{{}^{10}C_6 \times {}^6C_4} = \frac{80 \times 24}{10 \times 9 \times 8 \times 7} = \frac{8}{21}$

171. Two persons each make a single throw with a pair of dice. The probability that their scores are equal is

A) 65/648 B) 69/648 C) 73/648 D) 91/648

Key. C

Sol. Required Probability = $= \frac{1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 5^2 + 4^2 + 3^2 + 2^2 + 1^2}{36^2}$

172. Two numbers x and y are chosen without replacement from the set of {0, 1, 2, ….., 10}. The probability that $|x - y| \le 5$ is

(A) $\frac{2}{11}$ (B) $\frac{3}{11}$

(C) $\frac{15}{22}$ (D) $\frac{8}{11}$

Key. D

Sol. $|x - y| \ge 6$ ordered pair (x, y) are

(0, 6), (0, 7) ….. (0, 10

(1, 7) (1, 8) ….. (1, 10

(2, 8) (2, 9) (2, 10)

(3, 9) (3, 10)

(4, 10) = 15 ways

required probability $1 - \frac{15}{{}^{11}C_2} = 1 - \frac{3}{11} = \frac{8}{11}$

173. Four identical dice are rolled once. The probability that atleast three different numbers appears on them is

A) $\frac{15}{22}$ B) $\frac{25}{42}$ C) $\frac{32}{45}$ D) $\frac{17}{52}$

Key. B

Sol. All identical digits - ${}^{6}C_{1} = 6$

Only two different digits - $3 \times {}^{6}C_{2} = 45$

Three distinct digits - $3 \times {}^{6}C_{3} = 60$

Four different digits - ${}^{6}C_{4} = 15$

Total possible outcomes = 126

Favourable outcomes = 75.

174. A is a 3×3 matrix with entries from the set $\{-1, 0, 1\}$. The probability that A is neither symmetric nor skew symmetric is

A) $\dfrac{3^9 - 3^6 - 3^3 + 1}{3^9}$ B) $\dfrac{3^9 - 3^6 - 3^3}{3^9}$ C) $\dfrac{3^9 - 1}{3^{10}}$ D) $\dfrac{3^9 - 3^3 + 1}{3^9}$

Key. A

Sol. Total number of matrices that can be formed is 3^9.

Let $A = \left[a_{ij}\right]_{3\times3}$ where $a_{ij} \in \{-1, 0, 1\}$

If A is symmetric then $a_{ij} = a_{ji} \forall i, j$

If A is skew-symmetric then $a_{ij} = -a_{ji} \forall i, j$

175. If the cube of a natural number ends with a prime digit then the probability of its fourth power ending not with a prime digit, is

(A) 3/10 (B) 9/10

(C) 4/9 (D) 3/4

Key. D

Sol. Total cases are with numbers ending with 3, 5, 7 or 8.

Favourable cases are with numbers ending with 3, 7 or 8.

So, the required probability = 3/4

176. Consider the following three words (written in capital letters): 'PRANAM', 'SALAAM' and 'HELLO'. One of the three words is chosen at random and a letter from it is drawn. The letter is found to be 'A' or 'L' then the probability that it has come from the word 'PRANAM', is

(A) 0 (B) 1/3

(C) 2/5 (D) 5/21

Key. D

Sol. Let Q $\rightarrow$ event that 'PRANAM' is selected. S $\rightarrow$ event that 'SALAAM' is selected

H $\rightarrow$ event that 'HELLO' is selected. E $\rightarrow$ event that the letter chosen is A or L.

$$P(Q/E) = \frac{P(Q)P(E/Q)}{P(Q)P(E/Q) + P(S)P(E/S) + P(H)P(E/H)} = \frac{\frac{1}{3}\times\frac{2}{6}}{\frac{1}{3}\times\frac{2}{6} + \frac{1}{3}\times\frac{4}{6} + \frac{1}{3}\times\frac{2}{5}} = \frac{5}{21}$$

Probability

Multiple Correct Answer Type

1. The probability that a 50 year-old man will be alive at 60 is 0.83 and the probability that a 45 year-old woman will be alive at 55 is 0.87. Then

(A) The probability that both will be alive for the next 10 years is 0.7221

(B) At least one of them will alive for the next 10 years is 0.9779

(C) At least one of them will alive for the next 10 years is 0.8230

(D) The probability that both will be alive for the next 10 years is 0.6320

Key. A,B

Sol. The probability that both will be alive for 10 years, hence i.e., the prob. That the man and his wife Both be alive 10 years hence $0.83\times0.37=0.7221$.

The prob, that at least one of then will be alive is 1-P(That none of them remains alive 10 years)

$=1-(1-083)(1-0.87)$

$= 1-0.17\times0.13$

= 0.9779

2. Let us define the events A and B as

A : An year chosen at random contains 29 February.

B : An year chosen at random has 52 Fridays.

If P(E) denotes the probability of happening of event E then

(A) $P(\bar{B})=2/7$ (B) P(B) = 23/28

(C) P(A| $\bar{B}$) = 2/5 (D) P(A | B) = 5/23

Key. B,C,D

Sol. P(A) = 1/4, P(B/A) = 5/7, P(B/ $\bar{A}$) = 6/7

P(B) = P(A) P(B/A) + P $(\bar{A})$ P(B/ $\bar{A}$) = 1/4 × 5/7 + 3/4 × 6/7 = 23/28

P(A/B) = $\dfrac{P(A\cap B)}{P(B)}=\dfrac{1/4\times5/7}{1/4\times5/7+3/4\times6/7}=\dfrac{5}{23}$

P(A/ $\bar{B}$) = $\dfrac{P(A\cap\bar{B})}{P(\bar{B})}=\dfrac{\frac{1}{4}\times\frac{2}{7}}{1-\frac{23}{28}}=\dfrac{2}{5}$

3. Six balls of different colours are to be placed is 3 boxes of different sizes. Each box can hold all the six balls. Number of ways of placing the balls in the boxes so that no box remains empty, is

(A) $3^6-3.2^6+3$ (B) $6^3-6.5^3+6_{C_2}4^3$

(C) 540 (D) $\left(6_{C_4}+\dfrac{6!}{3!2!}+\dfrac{6!}{(2!)^3\,3!}\right)3!$

Key. A,C,D

Sol. Conceptual

4. There are n faculty members in a university . The faculty assembly consists of r members. Out of r assembly members k of them are selected for senate. The number of ways of selecting assembly members and senate is x. Then all possible values of x are.

(A) $n_{C_3}.n_{C_k}$ (B) $n_{C_r} + n_{C_k}$

(C) $n_{C_r} r_{C_k}$ (D) $n_{C_k} n - k_{C_{r-k}}$

Key. C,D

Sol. Conceptual

5. Triangles are formed by joining vertices of a octagon then number of triangle

(A) In which exactly one side common with the side of octagon is 32

(B) In which atmost one side common with the side of polygon is 48

(C) At least one side common with the side polygon 50

(D) Total number of triangle 56

Key. A,B,D

Sol. Total number of triangle = ${}^8C_3$ = 56

Number of triangle having exactly one side common with the polygon = 8×4 = 32

Number of triangle having exactly two side common with the polygon = 8

Number of triangle having no side common with the polygon = 16

6. If A and B are two invertible matrices of the same order, then adj(AB) is equal to

(A) $adj(B)\ adj(A)$ (B) $|B|\,|A|\ B^{-1}A^{-1}$

(C) $|B|\,|A|.A^{-1}B^{-1}$ (D) $|A|\,|B|(AB)^{-1}$

Key. A,B,D

Sol. Conceptual

7. If X = 144, then

a) no. of divisors (including 1 and X) of X = 15

b) sum of divisors (including 1 and X) of X = 403

c) product of divisors (including 1 and X) of X = 12^{15}

d) sum of reciprocals of divisors (including 1 and X) of X = $\frac{403}{144}$

Key. A,B,C,D

Sol. $144 = 2^4 . 3^2$

a) no. of divisors (4+1). (2+1) = 15

b) Sum of divisors $(1+2+2^2+2^3+2^4)(1+3+3^2) = 403$

c) Product of divisors $(144)^{\frac{15}{2}} = (12)^{15}$

d) Sum of reciprocals of divisors = $\frac{\text{sum of divisions}}{144} = \frac{403}{144}$

8. A, B are two events of a random experiment such that $P(\bar{A}) = 0.3, P(B) = 0.4$ and $P(A \cap \bar{B}) = 0.5$. Then

A) $P(A \cup B) = 0.9$ B) $P(B \cap \bar{A}) = 0.2$

C) $P(\bar{A} \cup \bar{B}) = 0.8$ D) $P\left(B/_{A \cup \bar{B}}\right) = 0.25$

Key. A,B,C,D

Sol. $P(A) = 0.7$; $P(B) = 0.4$. $P(A - B) = P(A) - P(AB)$

$\Rightarrow P(AB) = 0.2$, $\Rightarrow P(A + B) = 0.9 \Rightarrow P(B - A) = 0.2$, $\Rightarrow P(\bar{A} \cup \bar{B}) = 1 - P(AB) = 0.8$

$$\Rightarrow P(B / A \cup \bar{B}) = \frac{P(A \cap B)}{P(A \cup \bar{B})} = \frac{1}{4}$$

9. In a gambling between Mr. A and Mr. B a machine continues tossing a fair coin until the two consecutive throws either HT or TT are obtained for the first time. If it is HT, Mr, A wins and if it is TT, Mr, B wins. Which of the following is (are) true?

(A) probability of winning Mr.A is $\frac{3}{4}$ (B) Probability of Mr.B winning is $\frac{1}{4}$

(C) Given first toss is head probability of Mr. A winnings is 1

(D) Given first toss is tail, probability of Mr.A winning is $\frac{1}{2}$

Key: A,B,C,D

Hint: If T comes in first toss then Mr. B can win in only one case that is TT.

$\Rightarrow$ probability of Mr. B winning = $\frac{1}{4}$

$\Rightarrow$ Probability of Mr.A winning = $\frac{3}{4}$

Given first toss is head, Mr. A can win is successive tosses are T,HT, HHT, …..

Probability $= \dfrac{\frac{1}{2}}{1 - \frac{1}{2}} = 1$

Given first toss is head, Mr.A can win is successive tosses are HT, HHT, HHHT,…..

Probability = $= \dfrac{\frac{1}{4}}{1 - \frac{1}{2}} = \frac{1}{2}$

10. Contents of the two urns is as given in this table. A fair die is tossed. If the face 1,2,4 or 5 comes, a marble is drawn from the urn A other wise a marble is chosen from the urn B.

Urn	Red Marbles	White marbles	Blue marbles

A	5	3	8
B	3	5	0

Let E_1 : Denote the event that a red marble is chosen

E_2 : Denote the event that a white marble is chosen

E_3 : Denote the event that a blue is chosen

Then

(A) The event E_1, E_2 and E_3 are equiprobable

(B) $P(E_1), P(E_2), P(E_3)$ are in A.P

(C) If the marble drawn is red, the probability that it came from the urn A is $\frac{1}{2}$

(D) If the marble drawn is white, the probability that the face 5 appeared on the die is $\frac{3}{32}$

Key: A,B,D

Hint:

Urn	Red Marbles	White marbles	Blue marbles
A	5	3	8
B	3	5	0

$$P(E_1) = P(R) = \left(\frac{2}{3}\right)\left(\frac{5}{16}\right) + \left(\frac{1}{3}\right)\left(\frac{3}{8}\right) = \frac{10}{48} + \frac{6}{48} = \frac{1}{3}$$

$$P(E_2) = P(W) = \left(\frac{2}{3}\right)\left(\frac{3}{16}\right) + \left(\frac{1}{3}\right)\left(\frac{5}{8}\right) = \frac{6}{48} + \frac{10}{48} = \frac{1}{3}$$

$$P(E_3) = P(B) = \left(\frac{2}{3}\right)\left(\frac{8}{16}\right) = \frac{1}{3}$$

(C) Let A : event that urn A is chose

$$P(A/R) = \frac{P(A \cap R)}{P(R)} = \frac{\left(\frac{2}{3}\right)\left(\frac{5}{16}\right)}{\frac{1}{3}} = \left(\frac{10}{48}\right)(3) = \frac{5}{8} \Rightarrow (C)\text{ is incorrect}$$

(D) $$P(A/W) = \frac{P(A \cap W)}{P(W)} = \frac{\left(\frac{2}{3}\right)\left(\frac{3}{16}\right)}{\frac{1}{3}} = \left(\frac{6}{48}\right)(3) = \frac{3}{8}$$

$$P(\text{face five}/W) = \left(\frac{3}{8}\right)\left(\frac{1}{4}\right) = \frac{3}{32} \Rightarrow (D)\text{ is correct]}$$

11. Letters of the word SUDESH can be arranged in

a) 120 ways when two vowels are together

b) 180 ways when two vowels occupy in alphabetical order

c) 24 ways when vowels and consonants occupy their respective places

Key. A,B,C,D

Sol. $(a)\ (2!)\dfrac{5!}{2!}$

$(b)\dfrac{6!}{2!}$

$(c)\dfrac{4!}{2!}(2!)$

$(d)360-120$

12. Ram and Shyam select two numbers from the set 1 to n. If the probability that Shyam selects a number which is less than the number selected by Ram is $\dfrac{63}{128}$ then

a) n is even b) n is perfect square c) n is a cube d) none of these

Key. A,B,C

Sol. $\dfrac{\frac{1}{2}\left(n^2-n\right)}{n^2}=\dfrac{63}{128}$

$1-\dfrac{1}{n}=1-\dfrac{1}{64}$

13. There are n different gift coupons, each of which can occupy $N\ (N>n)$ different envelopes, with the same probability $\dfrac{1}{N}$

P_1: The probability that there will be one gift coupon in each of n definite envelopes of N given envelopes

P_2: The probability that there will be one gift coupon is each of n arbitary envelopes out of N given envelopes. Then

a) $P_1=P_2$ b) $P_1=\dfrac{n!}{N^n}$ c) $P_2=\dfrac{N!}{N^n(N-n)!}$ d) $P_1=\dfrac{N^n}{n!}$

Key. B,C

Sol. $P_1=\dfrac{n!}{N^n}$ and $P_2=\dfrac{{}^{N}c_n.n!}{N^n}$

14. If $|z_1|=2, |z_2|=3, |z_3|=4$ and $|2z_1+3z_2+4z_3|=4$ then absolute value of $8z_2z_3+27z_3z_1+64z_1z_2$ equals

a) 24 b)48 c) 72 d)96

Key. D

Sol. Conceptual

15. A random variable x takes values 0, 1, 2, 3, . . . with probability proportions to then $(x+1)\left(\frac{1}{5}\right)^x$

a) $p(x=0)=\frac{16}{25}$ b) $p(x\le 1)=\frac{112}{125}$ c) $p(x\ge 1)=\frac{9}{25}$ d) none of these

Key. A,C

Sol. We have, $P(X=x)\propto(x+1)\left(\frac{1}{5}\right)^x$ since, $\sum_{x=0}^{\infty}P(X=x)=1\Rightarrow$

$$k\left\{1+2\left(\frac{1}{5}\right)+3\left(\frac{1}{5}\right)^2+4\left(\frac{1}{5}\right)^3+\ldots\infty\right\}=1$$

$$k=\frac{16}{25}$$

a) $P(X=0)=k(0+1)\left(\frac{1}{5}\right)^0=k=\frac{16}{25}$

b) $P(X\le 1)=P(X=0)+P(X+1)$

c) $P(X\ge 1)=1-P(X=0)=1-k$

16. If $|f''(x)|\le 1$, $\forall x\in R$ and $f(0)=0=f'(0)$, then which of the following can't be true ?

A) $f\left(-\frac{1}{2}\right)=\frac{1}{5}$ B) $f(2)=-5$

C) $f(-2)=5$ D) $f\left(\frac{1}{2}\right)=-\frac{1}{5}$

Key. A,B,C,D

Sol. $-1\le f''(x)\le 1$, On integrating it twice with limits 0 to x, we get

$$|f(x)|\le\frac{x^2}{2}\Rightarrow\left|f\left(\pm\frac{1}{2}\right)\right|\le\frac{1}{8}\text{ and }|f(\pm 2)|\le 2$$

17. The sum of all three digited numbers that can be formed from the digits 1 to 9 and when the middle digit is perfect square is

A) 1,34,055 (When repetitions are allowed)

B) 1,70,555 (When repetitions are allowed)

C) 8,73,74 (When repetitions are not allowed)

D) 93,387 (When repetitions are not allowed)

Key. A,D

Sol. When repetitions are not allowed

$${}^{7}p_1(101)\left(\sum 9-1\right)+{}^{8}p_2\times 10+{}^{7}p_1(101)\left(\sum 9-4\right)+{}^{8}p_2\times 40\ +$$

$${}^{7}p_1(101)\left(\sum 9-9\right)+{}^{8}p_2\times 90 = 93,387$$

18. If A and B are two independent events such that $P(A'\cap B) = 2/15$ and $P(A\cap B') = 1/6$, then P(B) can be

A) 1/5 B) 1/6 C) 4/5 D) 5/6

Key. B,C

Sol. Since A and B are independent,

$\frac{2}{15} = P(A'\cap B) = P(A')\,P(B) = [1 - P(A)]\,P(B)$ ...(1)

and $\frac{1}{16} = P(A\cap B') = P(A)\,P(B') = P(A)\,[1 - P(B)]$ ... (2)

Subtracting (2) from (1), we get

$P(A) - P(B) = 1/30$ or $P(A) = P(B) + 1/30$

Put this value in (2), we get

$[P(B) + 1/30]\,[1 - P(B)] = 1/6$

$\Rightarrow 30[P(B)]^2 - 29\,P(B) + 4 = 0$

$\Rightarrow P(B) = 1/6, 4/5$

19. Which of the following are true Let $(x + 1)(x + 2)(x + 3) .... (x + n - 1)(x + n) = A_0 + A_1x + A_2x^2 + ... A_nx^n$ then

A) $A_0 + A_1 + A_2 + A_3 + ... + A_n$ is $= (n + 1)!$

B) $A_0 + 2A_1 + 3A_2 + ... (n + 1) A_n$ is $(n + 1)!\left(1 + \frac{1}{2} + ... + \frac{1}{n+1}\right)$

C) $A_1 + 2A_2 + 3A_3 + ... + nA_n$ is $(n + 1)!\left[\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+...\frac{n}{n+1}\right]$

D) $nA_0 + (n - 1) A_1 + (n - 2)A_2 + ... + A_{n-1}$ is $(n + 1)!\,(1/2 + 1/3 + ... + 1/(n + 1))$

Key. A,B,C,D

Sol. (a) Put x = 1,, We get $A_0 + A_1 + ... + A_n = (n + 1)!$

(b) Multiply by x on both sides and differenciate w.r.t. x and then put x = 1, we get

$A_0 + 2A_1 + ... (n + 1) A_n = (n + 1)! + \frac{(n+1)!}{2} + ...\frac{(n+1)!}{(n+1)!} = (n + 1)!\left(1 + \frac{1}{2} + ... + \frac{1}{n+1}\right)$

(c) Replace x by $\frac{1}{x}$, diff. and then put x =1 and we get $nA_0 + (n-1)A_1 + \ldots + A_{n-1} = (n+1)!$ $\left[\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\ldots\frac{n}{n+1}\right]$

(d) Taking log on both the sides and differentiate w.r.t. x and then put x = 1 we get $A_1 + 2A_2 + \ldots\ldots nA_n = (n+1)!\,(1/2 + 1/3 + \ldots + 1/(n+1))$

20. Which of the following are true

The total number of ways of selecting 5 letters from the letters of the word "INDEPENDENT" from the letters of the word with the condition.

A) There are 2 different 3 like letters are there is 20

B) There are 2 alike and 3 different letters are there is 30

C) There are two alike of one kind, two alike of another kind and one different letter are there is 12

D) there are 3 like of one kind, 2 like of another kind of letters are there is 6

Key. A,B,C,D

Sol. a) ${}^2C_1 . {}^5C_2 = 20$ b) ${}^3C_1 . {}^5C_2 = 30$ c) ${}^3C_2 . {}^4C_1 = 12$ d) ${}^3C_1 . {}^2C_1 = 6$

21. If X = 144, then

a) no. of divisors (including 1 and X) of X = 15

b) sum of divisors (including 1 and X) of X = 403

c) product of divisors (including 1 and X) of X = 12^{15}

d) sum of reciprocals of divisors (including 1 and X) of X = $\frac{403}{144}$

Key. A,B,C,D

Sol. $144 = 2^4 . 3^2$

a) no. of divisors (4+1). (2+1) = 15

b) Sum of divisors $(1+2+2^2+2^3+2^4)(1+3+3^2) = 403$

c) Product of divisors $(144)^{\frac{15}{2}} = (12)^{15}$

d) Sum of reciprocals of divisors = $\frac{\text{sum of divisions}}{144} = \frac{403}{144}$

22. $\sum_{K=0}^{n} {}^nc_k\left({}^{n+1}c_{K+1} + {}^{n+1}c_{K+2} + ----+ {}^{n+1}c_{n+1}\right)$ is equal to

a) ${}^{2n+1}c_0 + {}^{2n+1}c_1 + ----+ {}^{2n+1}c_n$ b) 4^n

c) ${}^{2n+1}c_0 + {}^{2n+1}c_1 + ----+ {}^{2n+1}c_{2n+1}$ d) 2^{2n+1}

Key. A,B

Sol. Consider the product of the expansions $(1+x)^n.\left(1+\frac{1}{x}\right)^{n+1} = \frac{(1+x)^{2n+1}}{x^{n+1}}$. The given expression is sum of the coeffts of negative powers of x in this product.

$\therefore$ It is equal to ${}^{2n+1}c_0 + ----+ {}^{2n+1}c_n = 2^{2n}$

23. The value of $c_0^2 + 3.c_1^2 + 5.c_2^2 + ----$ to (n+1) terms where $c_r = {}^nc_r$, is

a) ${}^{2n-1}c_{n-1}$ b) $(2n+1).{}^{2n-1}c_n$

c) $2(n+1).{}^{2n-1}c_n$ d) ${}^{2n-1}c_n + (2n+1)\,{}^{2n-1}c_{n-1}$

A) Both A and R are true and R is the correct explanation of A

B) Both A and R are true but R is not the correct explanation of A

C) A is true, R is false D) A is false, R is true

Key. C,D

Sol. Let $S = c_0^2 + 3.c_1^2 + 5.c_2^2 + ---+(2n+1).c_n^2$

$\Rightarrow S = (2n+1).c_n^2 + ------------+c_0^2$

Adding $2S = (2n+2).(c_0^2 + c_1^2 ----+c_n^2)$

$\Rightarrow S = (n+1).{}^{2n}c_n$

$\Rightarrow S = (n+1).\frac{2n}{n}\,{}^{2n-1}c_{n-1} = 2(n+1)\,{}^{2n-1}c_n$

$\therefore$ c is correct and a, b are not correct.

d is also correct, because ${}^{2n-1}c_n + (2n+1).{}^{2n-1}c_{n-1} = 2(n+1).{}^{2n-1}c_n$

24. If $(1+2x+3x^2)^{10} = a_0 + a_1x + a_2x^2 + ----+a_{20}x^{20}$,then

a) $a_1 = 20$ b) $a_2 = 210$

c) $a_{19} = 20.3^9$ d) $a_{20} = 2^2.3^7.7$

Key. A,B,C

Sol. $\left(1+2x+3x^2\right)^{10} = a_0 + a_1x + a_2x^2 + ------+a_{20}x^{20}$. Differentiate *w. r. t x*.

$10\left(1+2x+3x^2\right)^9(2+6x) = a_1 + 2a_2x + 3a_3x^2 + ------+20.a_{20}x^{19}$

Put *x* = 0 : we get $a_1 = 20$

Again differentiate *w. r. t x*

$10\left(1+2x+3x^2\right)^9.6 + 90(1+2x+3x^2)^8(2+6x)^2 = 2.a_2 + 6.a_3x + ----+20.19.a_{20}x^{18}$

Put *x* = 0; we get $2.a_2 = 60+360 \Rightarrow a_2 = 210$

Replace *x* by $\frac{1}{x}$ in the original expansion

$(1+\frac{2}{x}+\frac{3}{x^2})^{10} = \frac{(x^2+2x+3)^{10}}{x^{20}} = a_0 + \frac{a_1}{x} + \frac{a_2}{x^2} + ---+\frac{a_{20}}{x^{20}}$

$\Rightarrow (x^2+2x+3)^{10} = a_0.x^{20} + a_1x^{19} + ---+a_{19}x + a_{20}$

Put *x* = 0, we get $a_{20} = 3^{10}$.

Differentiate *w. r. t. x*

$$10(x^2+2x+3)^9(2x+2) = 20.a_0.x^{19} + - - - + a_{19}$$

Put *x* = 0, we get $a_{19} = 20.3^9$

25. Number of permutations of the word AUROBIND in which vowels appear in an alphabetical order is

A) ${}^8P_4$ B) ${}^8C_4$ C) ${}^8C_4 \times 4!$ D) ${}^8C_5 \times 5!$

Key. A,C

Sol. Required no. of permutation $= \frac{8!}{4!} = {}^8P_4 = {}^8C_4 \times 4!$

26. Triangles are formed by joining vertices of a octagon then the number of triangles

A) In which exactly one side common with the side of octagon is 32

B) In which atmost one side common with the side of polygon is 48

C) At least one side common with the side polygon 50

D) Without any restriction the number of triangles 56

Key. A,B,D

Sol. Total number of triangles $= {}^8C_3 = 56$

Number of triangles having exactly one side common with the polygon $= 8\left({}^4C_1\right) = 32$

Number of triangles having exactly two sides common with the polygon $= 8$

Number of triangles having no side common with the polygon $= 16$

27. A forecast is to be made of the results of five cricket matches, each of which can be a win or a draw or a loss for Indian team.

Let $p =$ number of forecasts with exactly 1 error

$q =$ number of forecasts with exactly 3 errors and

$r =$ number of forecasts with all five errors

then the correct statement(s) is/are

A) $8p = 5r$ B) $2q = 5r$ C) $8p = q$ D) $2(p+r) > q$

Key. B,C,D

Sol. Total number of possible forecast= 3^5

$p = {}^5C_4 \times 2 = 2.{}^5C_4$

$q = 2\times2\times2\times{}^5C_3$

$r = 2\times2\times2\times2\times2 = 2^5$

$\Rightarrow 8p = q$

$\Rightarrow 2q = 5r$

$2(p+r) > q$

28. Thirteen persons are sitting in a row. Number of ways in which four persons can be selected so that no two of them are consecutive is also equal to

A) number of ways in which the letters of the word MRINAL can be arranged if vowels are never separated.

B) number of numbers lying between 100 and 1000 using only the digits 1,2,3,4,5,6, 7 without repetition.

C) the number of ways in which 4 alike cadburies chocolate can be distributed in to 10 children each child getting atmost one.

D) number of triangles that can be formed by joining 12 points in plane of which 5 are collinear.

Key. B,C,D

Sol. $1\times2\times3\times4\times5$

$x_1+x_2+x_3+x_4+x_5 = 9$

$x_1 \ge 0; x_5 \ge 0$

$x_2, x_3, x_4 \ge 1$

$x_1 + 1 = t_1$

$t_1+x_2+x_3+x_4+t_5 = 11$

$= {}^{10}C_4 = \frac{10.9.8.7}{1.2.3.4} = 210$

(1) Required ways $= 6!-4!\,{}^5P_2 = 240$

(2) $\rightarrow {}^7P_3 = 7.6.5 = 210$

(3) 4 students to be selected among 10 students.

No. of ways = ${}^{10}C_4$

(4) ${}^{12}C_3 - {}^5C_3 = 210$

29. The sum of all three digited numbers that can be formed from the digits 1 to 9 and when the middle digit is perfect square is

A) 1,34,055 (When repetitions are allowed)

B) 1,70,555 (When repetitions are allowed)

C) 8,73,74 (When repetitions are not allowed)

D) 93,387 (When repetitions are not allowed)

Key. A,D

Sol. When repetitions are not allowed

$$^{7}P_1(101)\left(\sum 9-1\right)+{}^{8}P_2\times 10+{}^{7}P_1(101)\left(\sum 9-4\right)+{}^{8}P_2\times 40$$
$$+{}^{7}P_1(101)\left(\sum 9-9\right)+{}^{8}P_2\times 90=93,387$$

30. If M and N are any two events, the probability that the exactly one of them occurs is

A) $P(M)+P(N)-2P(M\cap N)$

B) $P(M)+P(N)-P(M\cap N)$

C) $P(M^C)+P(N^C)-2P(M^C\cap N^C)$

D) $P(M\cap N^C)+P(M^C\cap N)$.

Key. A,C,D

Sol. The required probability

= prob. that M occurs and N does not occur or N occurs and M does not occur.

$$=P\left(M\cap N^C\right)+P\left(M^C\cap N\right)[This\ is\ (d)]$$
$$=P(M)-P(M\cap N)+P(N)-P(M\cap N)$$

$$=P(M)+P(N)-2P(M\cap N)\qquad [This\ is\ (a)].$$
$$=1-P\left(M^C\right)+1-P\left(N^C\right)-2\left[1-P(M\cap N)^C\right]$$

$$=2P\left(M^C\cup N^C\right)-P\left(M^C\right)-P\left(N^C\right)$$
$$[By\ De-Morgan\ law]$$
$$=2\left[P\left(M^C\right)+P\left(N^C\right)-P(M^C\cup N^C\right]$$
$$-P\left(M^C\right)-P\left(N^C\right))$$
$$=P\left(M^C\right)+P\left(N^C\right)-P\left(M^C\cap N^C\right)\ [This\ is(c)]$$

31. If A and B are two events then

A) $P(A\cap B)\geq P(A)+P(B)-1$

B) $P(A\cap B)\geq P(A)+P(B)$

C) $P(A\cap B)=P(A)+P(B)-P(A\cup B)$

D) $P(A\cap B)=P(A).P(B)$

Key. A,C

Sol. $P(A\cup B)\leq 1\Rightarrow P(A)+P(B)-P(A\cap B)\leq 1$

$\Rightarrow P(A\cup B)-1\leq P(A\cap B)\Rightarrow P(A\cap B)\geq P(A)+P(B)-1$

So is true

Clearly $P(A\cap B)=P(A)+P(B)-P(A\cap B)$

So 3 also true

32. $P(A\cap B)=P(A\cup B)$ if the relation between P(A) and P(B) is _______

A) $P(A)=P(B)$ B) $P(A)=P(A\cap B)$ C) $P(A)=P(A\cup B)$ D) $P(B)=P(A\cap B)$

Key. A,B,C,D

Sol. We have, $P(A\cap B)=P(A\cup B)$

$\Leftrightarrow P(A\cap B)=P(A)+P(B)-P(A\cap B)$

$\Leftrightarrow P(A)+P(B)=2P(A\cap B)$

We know that

$P(A\cap B)\le P(A), P(A\cap B)\le P(B)$

$2P(A\cap B)<P(A)+P(A\cap B)$

$\Rightarrow 2P(A\cap B)<P(A)+P(B)$ $[\because P(A\cap B)\le P(B)]$

This contradicts (i). therefore $P(A\cap B)$=P(A). similarly

$P(A\cap B)$=P(B).

Thus, P(A)=P(B)=$P(A\cap B)$=$P(A\cup B)$

33. A, B are two events of a random experiment such that $P(\bar{A})=0.3, P(B)=0.4$ and $P(A\cap\bar{B})=0.5$. Then

A) $P(A\cup B)=0.9$ B) $P(B\cap\bar{A})=0.2$ C) $P(\bar{A}\cup\bar{B})=0.8$ D) $P\left(B/_{A\cup\bar{B}}\right)=0.25$

Key. A,B,C,D

Sol. $P(A)=0.7$; $P(B)=0.4$. $P(A-B)=P(A)-P(AB)$

$\Rightarrow P(AB)=0.2$, $\Rightarrow P(A+B)=0.9\Rightarrow P(B-A)=0.2, \Rightarrow P(\vec{A}\cup\vec{B})=1-P(AB)=0.8$

$\Rightarrow P(B/A\cup\vec{B})=\dfrac{P(A\cap B)}{P(A\cup\vec{B})}=\dfrac{1}{4}$

34. If A and B are two events such that $P(A\cup B)\ge\frac{3}{4}$ and $\frac{1}{8}\le P(A\cap B)\le\frac{3}{8}$ then

A) $P(A)+P(B)\le\frac{11}{8}$ B) $P(A).P(B)\le\frac{3}{8}$ C) $P(A)+P(B)\ge\frac{7}{8}$ D) $P(A).P(B)\le 1$

Key. A,C

Sol. $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\therefore 1\geq P(A)+P(B)-P(A\cap B)\geq \frac{3}{4}$

As the minimum value of $P(A\cap B)=\frac{1}{8},$ we get

$P(A)+P(B)-\frac{1}{8}\geq\frac{3}{4}\Rightarrow P(A)+P(B)\geq\frac{1}{8}+\frac{3}{4}=\frac{7}{8}$

As the maximum value of $P(A\cap B)=\frac{3}{8},$ we get

$1\geq P(A)+P(B)-\frac{3}{8}\Rightarrow P(A)+P(B)\leq 1+\frac{3}{8}=\frac{11}{8}$

35. If $X=144$, then

A) No.of divisors (including 1 and X) of X is 15

B) Sum of divisors (including 1 and X) of X is 403

C) Product of divisors (including 1 and X) of X is 12^{15}

D) Sum of reciprocals of divisors (including 1 and X) of X is $\frac{403}{144}$

Key. A,B,C,D

Sol. $144=2^4.3^2$

(A) No.of divisors $(4+1).(2+1)=15$

(B) Sum of divisors $(1+2+2^2+2^3+2^4)(1+3+3^2)=403$

(C) Product of divisors $(144)^{\frac{15}{2}}=(12)^{15}$

(D) Sum of reciprocals of divisors $=\frac{sum\ of\ divisors}{144}=\frac{403}{144}$

36. Letters of the word SUDESH can be arranged in

A) 120 ways when two vowels are together

B) 180 ways when two vowels occupy in alphabetical order

C) 24 ways when vowels and consonants occupy their respective places

D) 240 ways when vowels do not occur together

Key. A,B,C,D

Sol. a) $(2!)\frac{5!}{2!}$

b) $\frac{6!}{2!}\times\frac{1}{2}$

c) $\frac{4!}{2!}(2!)$

d) $360-120$

37. If x is the number of 5 digit numbers, sum of whose digits is even and y is the number of 5 digit numbers, sum of whose digits is odd, then

A) $x=y$ B) $x+y=90000$ C) $x=45000$ D) $x<y$

Key. A,B,C

Sol. $x=y$ since sum of digits is either even or odd, $x+y=$ total 5 digit no. $=9\times10\times10\times10\times10$

38. The total number of positive integers with distinct digits (in decimal system) must be

A) Infinite

B) Less than $\sum_{i=1}^{10}10^i$

C) Equal to $\sum_{i=1}^{10}10^i$

D) Equal to $9+9\times9+9\times9\times8+9\times9\times8\times7+.....+9\times9\times8!$

Key. B,D

Sol. A positive integer having more than 10 digits cannot have all distinct digits.

$\Rightarrow$The number of such numbers is finite.

Number of numbers having distinct digits.

$\Rightarrow 9+9\times9+9\times9\times8+......+9\times9\times8!<10+10^2+10^3+....+10^{10}$

39. A bag initially contains one red ball and two blue balls. An experiment consists of selecting a ball at random, noting its colour and replacing it together with an additional ball of the same colour. If three such trials are made, then

A) Probability that atleast one blue ball is drawn is 0.9

B) Probability that exactly one blue ball is drawn is 0.2

C) Probability that all the drawn balls are red given that all the drawn balls are of the same colour is 0.2

D) Probability that atleast one red ball is drawn is 0.6

Key. A,B,C,D

Sol. Probability that at least one blue ball is drawn

$=1-$ Probability that all the balls drawn are red.

$$=1-\frac{1}{3}.\frac{1}{2}.\frac{3}{5}=1-\frac{1}{10}=\frac{9}{10}=0.9$$

Probability that exactly one blue ball is drawn

$$=\frac{1}{3}.\frac{1}{2}.\frac{2}{5}+\frac{2}{3}.\frac{1}{4}.\frac{2}{5}+\frac{1.2.2}{3.4.5}=0.2$$

Probability that all drawn balls are red given that all the drawn balls of the same colour

$$=\frac{\frac{1}{10}}{\frac{1}{10}+\frac{4}{10}}=\frac{1}{5}=0.2$$

Probability that at least one red ball is drawn $=1-\left(\frac{2}{3}.\frac{3}{4}.\frac{4}{5}\right)=0.6$

40. Cards are drawn one by one in a pack of well shuffled cards with out replacement until all the cards are drawn then

A) Chance of getting a spade at the 13[th] trail is $\frac{1}{13}$

B) Chance of getting a spade at the 13[th] trail is $\frac{1}{4}$

C) Chance of getting last card as a spade is $\frac{1}{4}$

D) Chance of getting king at the 5[th] trail and queen at the 10[th] trail is $\frac{4}{663}$

Key. B,C,D

Sol. Let us make a sample space of 52 dimensions (i.e.,) each point of sample space is an ordered point of the form $\{x_1, x_2, \ldots, x_{52}\}$ .Total number of sample points are 52! Number of sample point on which x_{13} is spade is ${}^{13}C_1 \times 51!$

$\therefore$ Probability of getting spade on $x_{13}=\frac{{}^{13}C_1 \times 51!}{52!}=\frac{1}{4}$

Similarly number of sample points on x_{52} is spade $={}^{12}C_1 \times 51!$

$\therefore$ Probability of getting spade on last trail $= \frac{1}{4}$

Number of sample point having king at x_5 and queen x_{10} will be ${}^4C_1 \times {}^4C_1 \times 50!$

Required Probability $= \frac{16 \times 50!}{52!} = \frac{4}{663}$

41. A consignment of 15 record players contains 4 defectives. The record players are selected at random, one by one and examined. The one examined are not put back. Then

A) Probability of getting exactly 3 defectives in the examination of 8 record players is $\frac{{}^4C_3 \times {}^{11}C_5}{{}^{15}C_8}$

B) Probability that 9^{th} one examined is the last defective is $\frac{8}{195}$

C) Probability that 9^{th} examined record player is defective given that there were 3 defectives in the first 8 players examined is $\frac{1}{7}$

D) Probability 9^{th} one examined is last defective is $\frac{8}{197}$

Key. A,B,C

Sol. Let A be the event of getting exactly 3 defectives in the examination of 8 record players and B be the event 9^{th} record player is defective

$$P(A \cap B) = P(A) P\left(\frac{B}{A}\right)$$

$$P(A) = \frac{{}^4C_3 \times {}^{11}C_5}{{}^{15}C_8}, P\left(\frac{B}{A}\right) = \frac{1}{7}$$

Probability of 9^{th} one examined is the last defective $= \frac{{}^4C_3 \times {}^{11}C_5}{{}^{15}C_8} \times \frac{1}{7} = \frac{8}{195}$

42. If A and B are independent events such that $0 < P(A) < 1,\ 0 < P(B) < 1,$ then

A) A, B mutually exclusive

B) $A \text{ and } \overline{B}$ are independent

C) $\overline{A}, \overline{B}$ are independent

D) $P\left(\frac{A}{B}\right) + P\left(\frac{\overline{A}}{B}\right) = 1$

Key. B,C,D

Sol. Conceptual

43. Which of the following statements are true?

A) The probability that birthday of twelve people will fall in 12 Calendar months $= \frac{12!}{(12)^{12}}$

B) The probability that birthday of six people will fall in exactly two calendar months is $= \frac{{}^{12}C_2\left(2^6-2\right)}{(12)^6}$

C) The probability that birthday of six people will fall is exactly two Calendar months is $= \frac{{}^{12}C_3\left(2^7-2\right)}{(12)^7}$

D) The probability that birthdays of n $(n \le 365)$ people are different $\frac{{}^{365}P_n}{(365)^n}$

Key. A,B,D

Sol. A) Required Probability $= \frac{12}{12} \times \frac{11}{12} \times \frac{10}{12} \times \ldots\ldots \times \frac{1}{12} = \frac{12!}{(12)^{12}}$

B) Two months can be selected in ${}^{12}C_2$ ways. For each selection every person has two choices in 2^6 ways but it includes two cases in which all persons were born in the same month

Total number of favorable cases $= {}^{12}C_2\left(2^6-2\right)$

Required Probability $= \frac{{}^{12}C_2\left(2^6-2\right)}{(12)^6}$

d) Required Probability $= \frac{365}{365} \times \frac{364}{365} \times \ldots\ldots \times \frac{365-(n-1)}{365} = \frac{{}^{365}P_n}{(365)^n}$

44. A five digit number with distinct digits is formed by using the digits 0, 1, 2, 3, 4, 5. The probability that the number is divisible by 3 is

A) $\frac{{}^6P_5}{6^5}$ B) $\frac{9}{25}$ C) $\frac{\underline{|5}+4\underline{|4}}{5\underline{|5}}$ D) $\frac{{}^6C_5}{6^5}$

Key. B,C

Sol. Let S be the sample space .Then $|S| = 5 \times {}^5P_4$. If 0 is present then the number of 5 digit number divisible by 3 is $4\underline{|4} = 96$. If 0 is absent then the number of 5 digit number divisible by 3 is 120.

Required Probability $= \frac{216}{600}$

45. The probability that exactly one of the independent events A and B occurs is equal to

A) $P(A)+P(B)-2P(A\cap B)$ B) $P(A)+P(B)-P(A\cap B)$

C) $P(\bar{A})+P(\bar{B})-2P(\bar{A}\cap\bar{B})$ D) $P(A)+P(B)-3P(A\cap B)$

Key. A,C

Sol. The probability of exactly one of A and B to occur

$= P(A\bar{B})+P(\bar{A}B)$

$= P(A).P(\bar{B})+P(\bar{A}).P(B) = P(A).\{1-P(B)\}+\{1-P(A)\}P(B)$

$= P(A)+P(B)-2P(A).P(B) = P(A)+P(B)-2P(A\cap B)$

$= 1-P(\bar{A})+1-P(\bar{B})-2\{1-P(\bar{A})\}\{1-P(\bar{B})\}$

$= P(\bar{A})+P(\bar{B})-2P(\bar{A}).P(\bar{B})$

46.

A random variable X takes values $0,1,2,3,\ldots\ldots$ with probability proportional to $(x+1)\left(\frac{1}{5}\right)^x$. Then

A) $P(X=0)=\frac{16}{25}$ B) $P(X\geq 1)=\frac{9}{25}$

C) $P(X\geq 1)=\frac{7}{25}$ D) $E(X)=\frac{25}{32}$, (where $E(X)$ is mean of X)

Key. A,B

Sol. Let $P(X=x)=\alpha(x+1)\left(\frac{1}{5}\right)^x, \geq 0$

We have $\Rightarrow \alpha\left[1+2\left(\frac{1}{5}\right)+3\left(\frac{1}{5}\right)^2+\ldots\ldots\right]=1$

$\Rightarrow \alpha\frac{1}{(1-1/5)^2}=1 \Rightarrow \frac{25\alpha}{16} \Rightarrow \alpha=\frac{16}{25}$

Now $P(X=0)=\alpha(0+1)\left(\frac{1}{5}\right)^0=\alpha=\frac{16}{25}$

$\Rightarrow P(X\geq 1)=1-P(X=0)=1-\frac{16}{25}=\frac{9}{25}$

Also, $E(X)=\alpha\sum_{x=0}^{\infty} xP(X=x)=\alpha\sum_{x=0}^{\infty} x(x+1)\left(\frac{1}{5}\right)^x$

$= \alpha\left[(1)(2)\left(\frac{1}{5}\right)+(2)(3)\left(\frac{1}{5}\right)^2+(3)(4)\left(\frac{1}{5}\right)^3+\ldots\ldots\right]$ ....(1)

$$\frac{1}{5}E(X)=\alpha\left[(1)(2)\left(\frac{1}{5}\right)^2+(2)(3)\left(\frac{1}{5}\right)^3+.....\right]$$ ....(2)

$(1)-(2)$, we get

$$\Rightarrow \frac{4}{5}E(X)=\alpha\left[2\left(\frac{1}{5}\right)+2\left[2\left(\frac{1}{5}\right)^2+3\left(\frac{1}{5}\right)^3+4\left(\frac{1}{5}\right)^4+....\right]\right]$$

$$\Rightarrow E(X)=\frac{5}{4}\times\frac{16}{25}\left(\frac{2}{5}\times\frac{25}{16}\right)=\frac{1}{2}$$

47. If E_1, E_2 are two events such that $P(E_1)=\frac{1}{4}, P(E_2/E_1)=1/2$ and $P(E_1/E_2)=\frac{1}{4}$ then

A) E_1 and E_2 are independent

B) E_1 and E_2 are exhaustive

C) E_2 is twice as likely to occur as E_1

D) Probabilities of the events $E_1\cap E_2$, E_1, E_2 are in G.P

Key. A,C,D

Sol. $P\left(\frac{E_2}{E_1}\right)=\frac{P(E_1\cap E_2)}{P(E_1)}$

$\therefore P(E_1\cap E_2)=1/8$

$P(E_1/E_2)=\frac{P(E_1\cap E_2)}{P(E_2)}$

$\frac{1}{4}=\frac{1}{8}/P(E_2)\Rightarrow P(E_2)=\frac{1}{2}$

48. If A and B are two events. The probability that at most one of A, B occurs is

A) $1-P(A\cap B)$

B) $P(\overline{A})+P(\overline{B})-P(\overline{A}\cap\overline{B})$

C) $P(\overline{A})+P(\overline{B})+P(A\cup B)-1$

D) $P(A\cap\overline{B})+P(\overline{A}\cap B)+P(\overline{A}\cap\overline{B})$

Key. A,B,C,D

Sol. $P\left(\overline{A}\cup\overline{B}\right)=P\left(\overline{A\cap B}\right)=1-P(A\cap B)$

but $P\left(\overline{A}\cup\overline{B}\right)=P\left(\overline{A}\right)+P\left(\overline{B}\right)-P\left(\overline{A}\cap\overline{B}\right)$

$P\left(\overline{A}\right)+P\left(\overline{B}\right)-\{1-P(A\cup B)\}$

49. If $P(A)=\frac{3}{5}$ and $P(B)=\frac{2}{3}$ then

A) $P(A\cup B)\ge\frac{2}{3}$

B) $\frac{4}{15}\le P(A\cap B)\le\frac{3}{5}$

C) $P\left(A\cap\overline{B}\right)\le\frac{1}{3}$

D) $P(A\cup B)\ge\frac{3}{5}$

Key. A,B,C,D

Sol. $P(A\cup B)\ge P(A)P(A\cup B)\ge P(B)$

$P(A\cup B)\le 1\Rightarrow\frac{3}{5}+\frac{2}{3}-1\le P(A\cap B)\Rightarrow P(A\cap B)\ge\frac{4}{15}$

But $P(A\cap B)\le P(A),P(A\cap B)\le P(B)$

$P\left(A\cap\overline{B}\right)=P(A)-P(A\cap B)\le\frac{3}{5}-\frac{4}{15}\le\frac{1}{3}$

50. Let an ordinary coin be tossed 15 times. If P_r denotes the probability of getting r tails and P_k is maximum when K =

A) 7

B) 6

C) 8

D) 3

Key. A,C

Sol. $P_K=\frac{^{15}C_K}{2^{15}}$

It is max where $K=\frac{15-1}{2}or\frac{15+1}{2}$

51. If A and B are two mutually exclusive events then

A) $P(A)\le P\left(\overline{B}\right)$

B) $P(A)>P(B)$

C) $P(B)\le P\left(\overline{A}\right)$

D) $P(A)<P(B)$

Key. A,C

Sol. $P(A\cup B)\le 1\ \Rightarrow P(A)+P(B)\le 1\quad\Rightarrow P(A)\le P\left(\overline{B}\right)\ and\ P(B)\le P\left(\overline{A}\right)$

52. When a coin is flipped 'n' times and the probability that the first head comes after exactly m (n>m+1) tails is $\frac{1}{2^6}$ then

a) n=8,m=5 b) n=7,m=5 c) n=8, m=6 d) n=5,m=3

Key. A,B

Sol. There are 2^n out comes in all. The sequence of filps begins with m successive tails followed by a head followed head or tail.

$\underbrace{T\,T\,T\,T\,T\,T\,T\,T\,T\text{-------------}T}_{m \text{ tails}}\ H\ \underbrace{x\,x\,x\text{-----}x}_{n-(m+1) \text{ tails or heads:}}$

$\therefore$ Probability = $\frac{2^{n-(m+1)}}{2^n} = \frac{1}{2^{m+1}} = \frac{1}{2^6} \Rightarrow m = 5, n \in N$

53. If L_1 and L_2 are two parallel lines, m, n are number of points on them respectively. If the number of triangles that could be formed using these as vertices is 70 then

a) m=5, n=4 b) m=5, n=5 c) m=4, n=5 d) m=4,n=4

Key. A,C

Sol. Number of triangles formed is $= \left(^{m}C_2\right)n + \left(^{n}C_2\right)m = \frac{mn(m+n-2)}{2} = 70$

$$\Rightarrow mn(m+n-2) = 140$$

54. The slope of a common tangent to the parabola $y^2 = 4ax$ and hyperbola $x^2 - y^2 = a^2$ is

a) $\sqrt{\frac{\sqrt{5}+1}{2}}$ b) $\sqrt{\frac{\sqrt{5}-1}{2}}$ c) $-\sqrt{\frac{\sqrt{5}+1}{2}}$ d) $-\sqrt{\frac{\sqrt{5}-1}{2}}$

Key. A,C

Sol. $y = mx + \frac{a}{m}$ is a tangent to $x^2 - y^2 = a^2 \Rightarrow x^2(1-m^2) - 2ax - a^2(\frac{1}{m^2}+1) = 0$

For tangent, Discriminant=0 $\Rightarrow m^4 - m^2 - 1 = 0 \Rightarrow m = \pm\sqrt{\frac{\sqrt{5}+1}{2}}$

55. Let E, F be two independent events .The probability that both E and F happen is $\frac{1}{12}$ and probability that neither E nor F happens is $\frac{1}{2}$,then

a) 3P(E)=4P(F)=1 b) $P(E \cup F) = \frac{1}{2}$ c) 4P(E)=3P(F)=1 d) P(E)=P(F)

Key. A,B,C

Sol. Let P(E)=x , P(F)=y .Given xy=$\frac{1}{12}$, $(1-x)(1-y) = \frac{1}{2} \Rightarrow x + y = \frac{7}{12}$

$$\Rightarrow x = \frac{1}{3}, y = \frac{1}{4} \quad \text{or} \Rightarrow x = \frac{1}{4}, y = \frac{1}{3}$$

56. Suppose $A_1, A_2, A_3, ...A_{30}$ are thirty sets each with five elements and $B_1, B_2, B_3, ...B_n$ are *n* sets each with three elements such that $\bigcup_{i=1}^{30} A_i = \bigcup_{i=1}^{n} B_i = S$. If each elements of S belongs to exactly ten of the A_i's and exactly 9 of the B_i's then the value of *n* is

A) 15 B) 135 C) 45 D) $^{10}C_2$

Key. C,D

Sol. $\sum_{i=1}^{30} n(A_i) = 5 \times 30 = 150$

Suppose S has *m* elements

$150 = 10m \Rightarrow m = 15$

$\sum_{i=1}^{n} n(B_i) = 3n = 9m \Rightarrow n = 3m = 45 = ^{10}C_2$

57. Three six faced fair dice are thrown together. The probability that the sum of the numbers appearing on the dice is $K(3 \le K \le 8)$ is

A) $\frac{(K-1)(K-2)}{432}$ B) $\frac{K(K-2)}{432}$ C) $^{K-1}C_2 \times \frac{1}{216}$ D) $\frac{K^2}{432}$

Key. A,C

Sol. Coefficient of x^K in $(x^1 + x^2 + ...x^6)^3 = ^{K-1}C_2 = \frac{(K-1)(K-2)}{2}$

$\therefore$ Required probability = $\frac{^{K-1}C_2}{216}$

58. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and let $\vec{r}$ be a variable vector such that $\vec{r}.\hat{i}, \vec{r}.\hat{j}$ and $\vec{r}.\hat{k}$ are positive integers. If $\vec{r}.\vec{a} \le 12$ then the number of values of $\vec{r}$ is

A) $^{12}C_9 - 1$ B) $^{12}C_3$ C) $^{12}C_9$ D) $^{12}C_5$

Key. B,C

Sol. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then from the question x, y, z are positive integers.

Also $\vec{r}.\vec{a} \le 12 \Rightarrow x + y + z \le 12$

$\therefore$ the number of values of $\vec{r}$ = the number of positive integral solutions of $(x + y + z \le 12)$

$= \sum_{n=3}^{12} {^{n-1}C_2} = {^2C_2} + {^3C_2} + ... + {^{11}C_2} = {^3C_0} + {^3C_1} + {^4C_2} + ... + {^{11}C_9} \quad (\because {^nC_r} = {^nC_{n-r}})$

$= {^4C_1} + {^4C_2} + ... + {^{11}C_9} = ... = {^{12}C_9}$.

59. If $P = n(n^2 - 1^2)(n^2 - 2^2)(n^2 - 3^2)....(n^2 - r^2), \ n > r, n \in N$, then *P* is divisible by

A) $(2r+2)!$ B) $(2r-1)!$ C) $(2r+1)!$ D) $(2r-2)!$

Key. B,C

Sol. $P = n(n+1)(n-1)(n+2)(n-2)....(n+r)(n-r)$

$= (n-r)(n-\overline{r-1}).....(n-1)n(n+1)(n+2)....(n+r)$

= Product of $(2r+1)$ consecutive integers

$\therefore$ *P* is divisible by $(2r+1)!$ and so by $(2r-1)!$ also.

60. Let f(n) denote the number of ways in which n letters go into n envelops so that no letter is in the correct envelope, (where n>5), then $f(n) - nf(n-1) =$

a) $f(n-2)-(n-2)f(n-3)$ b) $f(n-1)-(n-1)f(n-2)$

c) $(n-3)f(n-4)-f(n-3)$ d) $(n-4)f(n-5)-f(n-4)$

Key. A,C

Sol. we know that $f(n) = (n-1)\{f(n-1)+f(n-2)\}$

61. The number of interior points that can be formed when diagonals of convex polygon of n-vertices, intersect if no three diagonals pass through the same interior point, is

a) ${}^nC_4$ b) ${}^nC_2$ c) ${}^nC_{n-4}$ d) ${}^nC_{n-2}$

Key. A,C

Sol. Each quadrilateral gives one point of intersection

62. The number of isosceles triangles with integer sides if no side exceeds 2008 is

a) $(1004)^2$ if equal sides do not exceed 1004

b) $2(1004)^2$ if equal sides exceed 1004

c) $3(1004)^2$ if equal sides have any length ≤ 2008

d) $(2008)^2$ if equal sides have any length ≤ 2008

Key. A,B,C

Sol. If the sides are a, a, b then the triangle forms only when $2a > b$ .so for any $a \varepsilon N$, b can change from 1 to 2a -1 when $a \le 1004$ then number of triangles = 1+3+5+..+(2(1004)-1)= $(1004)^2$ and if $1005 \le a \le 2008,$ b cam take any value from 1to 2008. but a has 1004 possibilities hence number of triangles = $1004 \times 2008 = 2(1004)^2$

$\therefore$ Total number of isosceles triangles $= 3(1004)^2$

63. Which of the following is/are true

a) $\overset{6}{5} - {}^5C_1.\overset{6}{4} + {}^5C_2.\overset{6}{3} - {}^5C_3.\overset{6}{2} + {}^5C_4.\overset{6}{1} = {}^6C_2.\underline{|5}$ b) $\overset{6}{5} - {}^6C_1.\overset{5}{5} + {}^6C_2.\overset{5}{4} - {}^6C_3.\overset{5}{3} + {}^6C_4.\overset{5}{2} - {}^6C_1.\overset{5}{1} = 0$

c) $\overset{6}{6} - {}^6C_1.\overset{6}{5} + {}^6C_2.\overset{6}{4} - {}^6C_3.\overset{6}{3} + {}^6C_4.\overset{6}{2} - {}^6C_5.\overset{6}{1} = 720$ d) $\overset{5}{6} - {}^6C_1.\overset{5}{5} + {}^6C_2.\overset{5}{4} - {}^6C_3.\overset{5}{3} + {}^6C_4.\overset{5}{2} - {}^6C_5.\overset{5}{1} = {}^5C_2.\underline{|6}$

Key. A,C

Sol. 1) Number of on to functions from a set containing 6 elements to a set containing 5 elements $={}^{6}C_{2}.\lfloor\underline{5}$

3) Number of on to functions from a set containing 6 elements to a set containing 6 elements $=\lfloor\underline{6}=720$

64. In a certain test a_i students gave wrong answers to at least i questions $(i=1,2,3....k)$. No student gave more than k wrong answers, then

a) Number of students who gave wrong answer to exactly i questions $=a_i-a_{i-1}$

b) Number of students who gave wrong answers to exactly i questions $=a_i-a_{i+1}$

c) The total no of wrong answers must be $a_1+2a_2+3a_3+....+ka_k$

d) Total no. of wrong answers must be $a_1+a_2+....+a_k$.

Key. B,D

Sol. Conceptual

65. A box contains 4 white balls, 5 black balls and 6 red balls. In how many ways can four balls be drawn from the box if at least one ball of each colour is to be drawn (If balls of same colour are different)

a) ${}^{4}C_{1}{}^{5}C_{1}{}^{6}C_{1}{}^{12}C_{1}$ b) 1440

c) 720 d) $\frac{1}{2}\,{}^{4}C_{1}{}^{5}C_{1}{}^{6}C_{1}{}^{12}C_{1}$

Key. C,D

Sol. Conceptual

66. In how many ways can the letters of the word INTERMEDIATE be arranged so that the order of the vowels as they occur in the given word do not change

a) ${}^{12}C_{6}\frac{6!}{2!}$ b) $\frac{12!}{3!2!}$ c) ${}^{12}C_{6}\frac{(6!)^2}{3!2!}$ d) $\frac{12!}{6!2!}$

Key. A,D

Sol. Conceptual

67. The no. of words formed with or without meaning, each of 3 vowels and 2 consonants from the letters of the word INVOLUTE is written in the form of $2^a.3^b.5^c.7^d$ then

a) $a=6$ b) $b=2$ c) $c=1$ d) d=0

Key. A,B,C,D

Sol. Number of ways selecting 3 vowels and 2 consonants and arranging them is ${}^{4}C_{3}.{}^{4}C_{2}.5!=2^6.3^2.5^1$

68. A five digit number with distinct digits is formed by using the digits 0, 1, 2, 3, 4, 5. The probability that the number is divisible by 3 is

A) $\frac{{}^{6}P_{5}}{6^5}$ B) $\frac{9}{25}$ C) $\frac{\lfloor\underline{5}+4\lfloor\underline{4}}{5\lfloor\underline{5}}$ D) $\frac{{}^{6}C_{5}}{6^5}$

Key. B,C

Sol. Let S be the sample space .Then $|S|=5.5_{P_4}$. If 0 is present then the number of 5 digit number divisible by 3 is $4\lfloor\underline{4}=96$. If 0 is absent then the number of 5 digit number divisible by 3 is 120.

Required Probability = $\frac{216}{600}$

69. A, B are two events such that $P(A\cup B)=\frac{5}{6}, P(A)=\frac{3}{4}$ and $P(\bar{B})=\frac{2}{3}$, then

A) A, B are independent B) $P(A\cap B)=\frac{1}{4}$

C) $P(A/B)=\frac{3}{4}$ D) $P(B/A)=\frac{1}{3}$

Key. A,B,C,D

Sol. $P(B)=\frac{1}{3}$ $P(AB)=\frac{3}{4}+\frac{1}{3}-\frac{5}{6}=\frac{1}{4}$

$P(A/B)=\frac{3}{4},\ P(B/A)=\frac{1}{3}$

70. Numbers are formed using all the digits 1, 2, 2, 2, 3, 3, 5. One number is picked up at random from the numbers so formed. The probability that the number

A) is divisible by 9 is 0 B) is divisible by 9 is 1

C) is odd is 4/7 D) is even is 3/7

Key. B,C,D

Sol. Conceptual

71. The probability that a 50 year-old man will be alive at 60 is 0.83 and the probability that a 45 year-old woman will be alive at 55 is 0.87. Then

(A) The probability that both will be alive for the next 10 years is 0.7221

(B) At least one of them will alive for the next 10 years is 0.9779

(C) At least one of them will alive for the next 10 years is 0.8230

(D) The probability that both will be alive for the next 10 years is 0.6320

Key. A,B

Sol. The probability that both will be alive for 10 years, hence i.e., the prob. That the man and his wife Both be alive 10 years hence $0.83\times 0.37=0.7221$.

The prob, that at least one of then will be alive is 1-P(That none of them remains alive 10 years)

$=1-(1-083)(1-0.87)$

$= 1-0.17\times 0.13$

= 0.9779

72. Ram and Shyam select two numbers from the set 1 to n. If the probability that Shyam selects a number which is less than the number selected by Ram is $\frac{63}{128}$ then

a)n is even b)n is perfect square c) n is a cube d) none of these

Key. A,B,C

Sol. Given $\frac{1}{2}\left(\frac{n^2-m}{n^2}\right)=\frac{63}{128}\Rightarrow n=64$

73. A bag initially contains one red ball and two blue balls. An experiment consisting of selecting a ball at random, noting its colour and replacing it together with an additional ball of the same colour. If three such trials are made, then

a) Probability that atleast one blue ball is drawn is 0.9

b) Probability that exactly one blue ball is drawn is 0.2

c) Probability that all the drawn balls are red given that all the drawn balls are of the same colour is 0.2

d) Probability that atleast one red ball is drawn is 0.6

Key. A,B,C,D

Sol. Prob. That atleast one blue ball is drawn

= 1- prob that all the balls drawn are red.

= $1-\frac{1}{3}.\frac{1}{2}.\frac{3}{5}=1-\frac{1}{10}=0.9$

Prob. That exactly one blue ball is drawn

= $\frac{1}{3}.\frac{1}{2}.\frac{2}{5}+\frac{2}{3}.\frac{1}{4}.\frac{2}{5}=0.2$

Prob. that all drawn balls are red given that all the drawn balls of the same colour $=\frac{\frac{1}{10}}{\frac{1}{10}+\frac{4}{10}}=\frac{1}{5}=0.2$

Prob.that atleast one red ball is drawn $=1-\left(\frac{2}{3}.\frac{3}{4}.\frac{4}{5}\right)$ = 0.6

74. There are n different gift coupons, each of which can occupy N $(N>n)$ different envelopes, with the same probability $\frac{1}{N}$

P_1 : The probability that there will be one gift coupon in each of n definite envelopes of N given envelopes

P_2 : The probability that there will be one gift coupon is each of n arbitary envelopes out of N given envelopes. Then

a) $P_1=P_2$ b) $P_1=\frac{n!}{N^n}$ c) $P_2=\frac{N!}{N^n(N-n)!}$ d) $P_1=\frac{N^n}{n!}$

Key. B,C

Sol. $P_1=\frac{n!}{N^n}$ and $P_2=\frac{{}^Nc_n.n!}{N^n}$

75. A five digit number with distinct digits is formed by using the digits 0, 1, 2, 3, 4, 5. The probability that the number is divisible by 3 is

A) $\frac{{}^6P_5}{6^5}$ B) $\frac{9}{25}$ C) $\frac{\lfloor 5+4\lfloor 4}{5\lfloor 5}$ D) $\frac{{}^6C_5}{6^5}$

Key. B,C

Sol. Let S be the sample space .Then $|S| = 5.5_{P_4}$. If 0 is present then the number of 5 digit number divisible by 3 is $4\underline{|4} = 96$. If 0 is absent then the number of 5 digit number divisible by 3 is 120.

Required Probability = $\frac{216}{600}$

76. A, B are two events such that $P(A \cup B) = \frac{5}{6}, P(A) = \frac{3}{4}$ and $P(\bar{B}) = \frac{2}{3}$, then

A) A, B are independent

B) $P(A \cap B) = \frac{1}{4}$

C) $P(A/B) = \frac{3}{4}$

D) $P(B/A) = \frac{1}{3}$

Key. A,B,C,D

Sol. $P(B) = \frac{1}{3}$ $P(AB) = \frac{3}{4} + \frac{1}{3} - \frac{5}{6} = \frac{1}{4}$

$P(A/B) = \frac{3}{4},\ P(B/A) = \frac{1}{3}$

77. Numbers are formed using all the digits 1, 2, 2, 2, 3, 3, 5. One number is picked up at random from the numbers so formed. The probability that the number

A) is divisible by 9 is 0

B) is divisible by 9 is 1

C) is odd is 4/7

D) is even is 3/7

Key. B,C,D

Sol. Conceptual

78. A, B are two events of a random experiment such that $P(\bar{A}) = 0.3, P(B) = 0.4$ and $P(A \cap \bar{B}) = 0.5$. Then

A) $P(A \cup B) = 0.9$ B) $P(B \cap \bar{A}) = 0.2$ C) $P(\bar{A} \cup \bar{B}) = 0.8$ D) $P\left(B/_{A \cup \bar{B}}\right) = 0.25$

Key. A,B,C,D

Sol. $P(A) = 0.7$; $P(B) = 0.4$. $P(A - B) = P(A) - P(AB)$

$\Rightarrow P(AB) = 0.2$, $\Rightarrow P(A + B) = 0.9 \Rightarrow P(B - A) = 0.2$, $\Rightarrow P(\bar{A} \cup \bar{B}) = 1 - P(AB) = 0.8$

$\Rightarrow P(B / A \cup \bar{B}) = \frac{P(A \cap B)}{P(A \cup \bar{B})} = \frac{1}{4}$

79. If M and N are two events, the probability that exactly one of them occurs is

a) P (M) + P (N) – 2 P $(M \cap N)$

b) $P(M) + P(N) - 2P\left(\overline{M \cup N}\right)$

c) $P\left(\overline{M}\right) + P\left(\overline{N}\right) - 2P\left(\overline{M} \cap \overline{N}\right)$

d) $P\left(M \cap \overline{N}\right) - P\left(\overline{M} \cap N\right)$

Key. A,C

Sol. a) $P\left(M \cap \overline{N}\right) + P\left(\overline{M} \cap N\right) = P(M) - P(M \cap N) + P(N) - P(M \cap N)$

` c) $P\left(M \cap \overline{N}\right) + P\left(\overline{M} \cap N\right) = P(M \cup N) - P(M \cap N)$

$= P(M \cup N) - P(M) - P(N) + P(M \cup N)$

$= 2P(M \cup N) - P(M) - P(N)$

$$= \left(1-P\left(M\right)\right)+\left(1-P\left(N\right)\right)-2\left(1-P\left(M\cup N\right)\right)$$
$$= P\left(\overline{M}\right)+P\left(\overline{N}\right)-2P\left(\overline{M\cup N}\right)$$

80. A box contains 11 tickets numbered from 1 to 11. Six tickets are drawn simultaneously at random, let E_1 be the event that the sum of the numbers on the tickets drawn is even, E_2 be the event that the sum of the number on the tickets drawn is odd which of following hold good.

a) E_1, E_2 are equally likely
b) E_1, E_2 are exhaustive
c) $P(E_2) > P(E_1)$
d) $P(E_1/E_2) = P(E_2/E_1)$

Key. B,C,D

Sol. $P\left(E_2\right)=\dfrac{118}{231}\quad P\left(E_1\right)=\dfrac{113}{231}$

$E_2 \Rightarrow$ 1 odd + 5 even or 3 odd + 3 even or 5 odd + one even

$P\left(E_1\cap E_2\right)=0\Rightarrow P\left(E_1/E_2\right)=P\left(E_2/E_1\right)=0$

81. If $A_1, A_2, \ldots, A_n$ are n independent events such that $P(A_i) = \dfrac{1}{i+1}$, i = 1, 2, ... n. The probability that none of $A_1, A_2, \ldots A_n$ occurs is

(A) $\dfrac{n}{n+1}$
(B) $\dfrac{1}{n+1}$
(C) less than $\dfrac{1}{n}$
(D) none of these

Key. B,C

Sol. $P(A' \cap A'_2 \ldots \cap A'_n)$

$= P(A'_1)\, P(A'_2) \ldots P(A'_n)$

[Q $A_1, A_2, \ldots, A_n$ are independent]

$= \left(1-\dfrac{1}{2}\right)\left(1-\dfrac{1}{3}\right)\ldots\left(1-\dfrac{1}{n+1}\right)$

$= \dfrac{1}{2}\times\dfrac{2}{3}\times\dfrac{3}{4}\times\ldots\times\dfrac{n-1}{n}\times\dfrac{n}{n+1}=\dfrac{1}{n+1}<\dfrac{1}{n}$

82. A random variable X takes values x = 0, 1, 2, 3, ... with probability proportional to $(x+1)\left(\dfrac{1}{5}\right)^x$. Then

(A) $P(X = 0) = 16/25$
(B) $P(X \geq 1) = 9/25$
(C) $P(X \geq 1) = 7/25$
(D) $E(X) = 25/32$

Key. A,B,D

Sol. Let $P(X = x) = \alpha\,(x + 1)\left(\dfrac{1}{5}\right)^x$, $x \geq 0$

We have

$\Rightarrow \alpha\left[1+2\left(\dfrac{1}{5}\right)+3\left(\dfrac{1}{5}\right)^2+\ldots\right] = 1$

$\Rightarrow \alpha\dfrac{1}{(1-1/5)^2}=1 \Rightarrow \dfrac{25\alpha}{16} \Rightarrow \alpha = \dfrac{16}{25}$

Now, P(X = 0) = α (0 + 1) $\left(\frac{1}{5}\right)^{\circ}$ = α = 16/25

$\Rightarrow$ P(X $\geq$ 1) = 1 - P(X = 0) = 1 - $\frac{16}{25} = \frac{7}{25}$

Also, E(X) = $\sum_{x=0}^{\infty} xP(X = x) = \sum_{x=0}^{\infty} x(x+1)\left(\frac{1}{5}\right)^x$

= (1) (2) $\left(\frac{1}{5}\right) + (2)(3)\left(\frac{1}{5}\right)^2$ + (3) (4) $\left(\frac{1}{5}\right)^3$ + ...

$\frac{1}{5}$ E(X) = (1) (2) $\left(\frac{1}{5}\right)^2$ + (2) (3) $\left(\frac{1}{5}\right)^3$ + ...

Subtracting, we get

$$\Rightarrow \frac{4}{5}E(x) = 2\left(\frac{1}{5}\right) + 2\left[2\left(\frac{1}{5}\right)^2 + 3\left(\frac{1}{5}\right)^3 + 4\left(\frac{1}{5}\right)^4 + \ldots\right]$$

$$= \frac{2/5}{(1-1)/5)^2} = \frac{2}{5} \times \frac{25}{16} = \frac{5}{8}$$

$\Rightarrow$ E(X) = 25/32

83. Let us define the events A and B as
A : An year chosen at random contains 29 February.
B : An year chosen at random has 52 Fridays.
If P(E) denotes the probability of happening of event E then

(A) $P(\bar{B}) = \frac{2}{7}$ (B) P(B) = $\frac{23}{28}$

(C) P(A/$\bar{B}$) = 2/5 (D) P(A/B) = 5/23

Key. B,C,D

Sol. P(A) = 1/4, P(B/A) = 5/7, P(B/$\bar{A}$) = 6/7

P(B) = P(A) P(B/A) + P($\bar{A}$) P(B/$\bar{A}$) = 1/4 × 5/7 + 3/4 × 6/7 = 23/28

P(A/B) = $\frac{P(A \cap B)}{P(B)} = \frac{1/4 \times 5/7}{1/4 \times 5/7 + 3/4 \times 6/7} = \frac{5}{23}$

P(A/$\bar{B}$) = $\frac{P(A \cap \bar{B})}{P(\bar{B})} = \frac{\frac{1}{4} \times \frac{2}{7}}{1 - \frac{23}{28}} = \frac{2}{5}$

84. A fair coin is tossed 10 times and the outcomes are listed. Let H_i be the event that the i^{th} outcome is a head and A_m be the event that the list contains exactly m heads, then

(A) H_3 and A_4 are independent (B) A_1 and A_9 are independent

(C) H_2 and A_5 are independent (D) H_4 and H_8 are not independent

Key. C

Sol. P(H_i) = $\frac{1}{2}$, P(A_m) = $\frac{{}^{10}C_m}{2^{10}}$

$P(H_i \cap A_m) = \dfrac{{}^9C_{m-1}}{2^{10}}$

For H_i & A_m to be independent,

$\dfrac{{}^9C_{m-1}}{2^{10}} = \dfrac{1}{2} \times \dfrac{{}^{10}C_m}{2^{10}} \Rightarrow 1 = \dfrac{1}{2} \times \dfrac{10}{m} \Rightarrow m = 5.$

85. Two numbers are chosen from {1, 2, 3, 4, 5, 6, 7, 8} one after another without replacement. Then the probability that
(A) The smaller value of two is less than 3 is 13/28
(B) The bigger value of two is more than 5 is 9/14
(C) Product of two number is even is 11/14
(D) none of these

Key. A,B,C

Sol. (A) $\dfrac{{}^8C_2 - {}^6C_2}{{}^8C_2} = \dfrac{13}{28}$

(B) $\dfrac{{}^8C_2 - {}^5C_2}{{}^8C_2} = \dfrac{9}{14}$

(C) $\dfrac{{}^8C_2 - {}^4C_2}{{}^8C_2} = \dfrac{11}{14}$

Probability

Assertion Reasoning Type

A) Both Statements are true and Statement-2 is the correct explanation of Statement-1

B) Both Statements are true but Statement-2 is not the correct explanation of Statement-1

C) Statement-1 is true, Statement-2 is false D) Staement-1 is false,Statement-2 is true

1. Statement – 1 : The number of selections of four letters taken from the word PARALLEL must be 15

Statement – 2 : Coefficient of x^4 in the expansion of $(1-x)^{-3}$ is 15 $(|x|<1)$

Key. D

Sol. $1'P,\ 2'A,\ 1R,\ 3'L,\ 1E$

4 diff : $5c_4 = 5$

3 alike of 1 kind & 1 diff = $1c_1.4c_1 = 4$

2 alike of 1 kind & 2 diff = $2c_1.4c_2 = 2.6 = 12$

2 alike of 1 kind & 2 diff of 2nd kind = $2c_2 = 1$

Total = 22

2. Let A^c denote the complement of an event A.

Statement – 1: If $P(A)=0.5, P(B)=0.7, P(C)=0.9,$ then $P(A^c \cap B \cap C)$ lies in the interval $[0.1, 0.5]$

Statement – 2: If $P(E_i)=C_i, i=1,2,....,n,$ then $P(E_1 \cap E_2 \cap E_3 ..... \cap E_n) \ge C_1 + C_2 + C_3 + ... + C_n - (n-1)$

Key. A

Sol. $P(A^C)=0.5, P(B)=0.7, P(C)=0.9$

$\Rightarrow P(A^C \cap B \cap C) \le \min\{0.5, 0.7, 0.9\}$

$P(A^C \cap B \cap C) \ge 0.5+0.7+0.9-2 = 0.1$

$\therefore P(A^C \cap B \cap C)$ *lies in* $[0.1,\ 0.5]$

3. Let $H_1, H_2, ..., H_n$ be mutually exclusive and exhaustive events with $P(H_i) > 0, i=1,2,....,n.$ Let E be any other event with $0 < P(E) < 1$

Statement 1: $P(H_i / E) > P(E / H_i).P(H_i)$ for i = 1,2,.....,n

Statement 2: $\sum_{i=1}^{n} P(H_i) = 1$

Key. D

Sol. Statement – 1 is not always true. For instance, if $P(H_i \cap E) = 0$ for some i, then

$$P(H_i / E) = \frac{P(H_i \cap E)}{P(E)} = 0 \text{ and } P(E / H_i).P(H_i) = \frac{P(E \cap H_i)}{P(H_i)} = 0$$

That is $P(H_i / E) = P(E / H_i).P(H_i) = 0$

However, if $0 < P(H_i \cap E) < 1,\ for\ i = 1, 2, ..., n$ then

$$P(H_i / E) = \frac{P(H_i \cap E)}{P(E)} = \frac{P(H_i \cap E)}{P(H_i)} \cdot \frac{P(H_i)}{P(E)} = \frac{P(E / H_i).P(H_i)}{P(E)} > P(E / H_i).P(H_i)$$

4. Consider the system of equations $ax + by = 0$, $cx + dy = 0$ where a, b, c, d $\in \{0,1\}$

Statement 1:- The probability that the system of equations has a unique solution is 3/8

Statement 2:- The probability that the system has a solutions is 1

Key. B

Sol. The system of equations has a solution if and only if $\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc \neq 0$

As a, b, c, d $\in \{0,1\}$ The system has a solution if $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}$

$\therefore n(E) = 6, n(S) = 16\ \&\ P(E) = \frac{3}{8}$

Since x = y = 0 satisfy the system of equations irrespective of the values of a, b, c, d

$\therefore$ The probability that the system has a solution is 1

5. n identical dies are rolled simultaneously.

STATEMENT – 1

The number of distinct throws is ${}^{n+5}C_5$.

because

STATEMENT – 2

$\sum_{r=1}^{6} {}^{6}C_r\ {}^{n-1}C_{r-1} = {}^{n+5}C_5$

Key. A

Sol. The number of distinct throws when exactly

r ($1 \le r \le 6$) numbers appear will be

${}^{6}C_r$ × (the number of ways of putting n identical things into r distinct boxes with no box empty)

= ${}^{6}C_r \times {}^{n-1}C_{r-1}$

The total number of distinct throws = $\sum_{r=1}^{6} {}^6C_r\ {}^{n-1}C_{r-1}$

$= \sum_{r=1}^{6} {}^6C_r\ {}^{n-1}C_{n-r} = {}^{n+5}C_n = {}^{n+5}C_5$

(A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for Statement – 1.

(B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is NOT a correct explanation for Statement – 1.

(C) Statement – 1 is True, Statement – 2 is False.

(D) Statement – 1 is False, Statement – 2 is True.

6. A card from a well shuffled pack of 52 cards is drawn. Let A be the event that the card is a diamond and B be the event that it is a queen.

STATEMENT – 1

A and B are independent events

because

STATEMENT – 2

A and B are not mutually exclusive events.

Key. B

Sol. I: P(A) = $\frac{13}{52} = \frac{1}{4}$, P(B) = $\frac{4}{52} = \frac{1}{13}$

$P(A \cap B) = \frac{1}{52} = P(A)\,P(B)$

II: $P(A \cap B) \neq 0$

7. In a bag there are n balls of either red or green colour. Let G_k be the event that it contains exactly k green balls and its probability is proportional to k^2. Now a ball is drawn at random. Let A be the event that the ball drawn is green.

STATEMENT – 1

$P(A) = \frac{3(n+1)}{2(n+2)}$

because

STATEMENT – 2

$\sum_{k=0}^{n} P(G_k) = 1$

Key. D

Sol. $P(G_k) \propto k^2 \Rightarrow P(G_k) = \lambda k^2$

$\sum_{k=0}^{n} P(G_k) = 1$ (as these are mutually exclusive and exhaustive events)

$\Rightarrow \lambda \sum_{k=0}^{n} k^2 = 1$

$\Rightarrow \lambda = \dfrac{6}{n(n+1)(2n+1)}$

P(A) = $\sum_{k=0}^{n} P(G_k)$ P(A/G$_k$) = $\sum_{k=0}^{n} \lambda k^2 . \dfrac{k}{n} = \dfrac{\lambda}{n} . \dfrac{n^2(n+1)^2}{4} = \dfrac{3(n+1)}{2(2n+1)}$.

8. STATEMENT – 1

Total number of different functions from set A having 4 elements to a set B having 2 elements is 16.

because

STATEMENT – 2

The number of ways in which m different things can be distributed into n different parcels, blank lots being admissible, is m^n.

Key. C

Sol. Every element of set A has 2 options to go with an element of set B.

So, the total number of functions = 2^4 = 16.

II. The number of ways = n^m.

9. STATEMENT-I: If P is a natural number having number of divisors (including unity and P) equal to 105 then $\{\sqrt{P}\} = 0$ where $\{x\}$ stands for fractional part of x .

STATEMENT-II : $2^2.3^4.5^6$ is one of such numbers P.

Key. B

Sol. If $P = a^x.b^y.c^z$ ——, where *a,b,c* etc are prime factors, then we know that no. of divisors of

$P = (x+1).(y+1).(z+1)---$ etc = 105.

$\Rightarrow x+1, y+1, z+1, ---$ all must be odd

$\Rightarrow x, y, z, ---$ all must be even

$\Rightarrow P$ is a perfect square

$\therefore$ Statement-I is true.

Statement-II is also true, but it is not the correct explanation.

10. STATEMENT-I: ${}^{30}C_{15} - 1$ is divisible by 31

STATEMENT-II : If n is a prime, then ${}^{n}C_r$ is divisible by n for r = 1,2,3,----,n-1 and

Key. D

Sol. ${}^{n}C_r = \dfrac{n(n-1)----(n-r+1)}{1.2.3.---r}$

${}^{n}C_r$ is a positive integer and hence all the factors of the denominator must cancel out with some of the factors in the numerator. But n, being prime, remains without cancellation. Hence ${}^{n}C_r$ is divisible by *n* for *r* = 1,2,---,*n-1*.

Now ${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$ is well known.

$\Rightarrow {}^{n}C_r = {}^{n+1}C_r - {}^{n}C_{r-1}$

Using this formula again and again, we can show that

${}^{30}C_{15} = {}^{31}C_{15} - {}^{31}C_{14} + {}^{31}C_{13} --- + {}^{31}C_3 - {}^{31}C_2 + {}^{31}C_1 - {}^{30}C_0$

$\Rightarrow {}^{30}c_{15}+1={}^{31}c_{15}-{}^{31}c_{14}+---+{}^{31}c_1$

On the R.H.S each term is divisible by 31

$\therefore {}^{30}c_{15}+1$ is divisible by 31.

A. Statement – 1 is True, Statement – 2 is True ; Statement – 2 is a correct explanation for Statement – 1.
B. Statement – 1 is True, Statement – 2 is True ; Statement – 2 is NOT a correct explanation for Statement – 1.
C. Statement – 1 is True, Statement – 2 is False.
D. Statement – 1 is False, Statement – 2 is True.

11. STATEMENT-1: The term independent of x in the expansion of $\left(x+\frac{1}{x}+2\right)^m$ is $\frac{\angle 4m}{(\angle 2m)^2}$

STATEMENT-2: The coefficient of x^k in the expansion of $(1+x)^n$ is nC_k

Key. D

Sol. $T_{r+1}=10C_r(-k)^2 x^5-\frac{5r}{2}$

$\therefore 5-\frac{5r}{2}=0\Rightarrow r=2$

$\therefore 10C_2k^2=405$, $\therefore k^2=9$, $\therefore k=\pm 3$

12. STATEMENT -1: The number of ways of writing 1400 as a product of two positive integers is 12

STATEMENT-2: $1400=2^3\times 5^2\times 7$

Key. A

Sol. $\left(x+\frac{1}{x}+2\right)^m=\frac{(x+1)^{2m}}{x^m}$

$\therefore$ Term independent of $x=\frac{2mC_m x^m}{x^m}=(2m)C_m$

13. STATEMENT -1: The number of selections of four letters from the letters of word PARALLEL is 15.

STATEMENT-2: Coefficient of x^2 in the expansion of $(1+x)^6$ is 15

Key. A

Sol. The number of divisors of $1400=(3+1)(2+1)(1+1)=24$

$\therefore$ No. of ways of writing as product of two numbers $=\frac{24}{2}=12$

14. STATEMENT -1: If n is a positive integer less than 20, then $\angle n\ \angle(20-n)$ is minimum when n = 10

STATEMENT-2: $(2m)C_r$ is maximum when r = m.

Key. D

Sol. $20C_n = \dfrac{\angle 20}{\angle n \angle 20-n}$

$\angle n \angle 20-n$ is minimum $\Rightarrow 20C_n$ is maximum

$\therefore n=10$

15. Statement – 1 : For the given system of non-homogeneous linear equations of the form

A x = B, if $|A|=0$, then the system of equations have either no solution or infinite number of solutions

Because

Statement – 2 : For the given system of non-homogeneous linear equations of the form

A x = B, if $|A|=0$ & (Adj A) B = 0, then it will have no solution

Key. C

Sol. If $|A|=0 \& (Adj\ A)\ B \neq 0$, no solution

If $|A|=0 \& (Adj\ A)\ B = 0$, infinite solution

16. Statement – 1 : The number of selections of four letters taken from the word PARALLEL must be 15

Because

Statement – 2 : Coefficient of x^4 in the expansion of $(1-x)^{-3}$ is 15 $(|x|<1)$

Key. D

Sol. $1'p,\ 2'A,\ 1R,\ 3'L,\ 1E$

4 diff : $5c_4 = 5$

3 alike of 1 kind & 1 diff = $1c_1.4c_1 = 4$

2 alike of 1 kind & 2 diff = $2c_1.4c_2 = 2.6 = 12$

2 alike of 1 kind & 2 diff of 2nd kind = $2c_2 = 1$

Total = 22

17. Statement – 1 : If $f:\{1,2,3,4,5\} \to \{1,2,3,4,5\}$ then the number of onto functions such that $f(i) \neq i$ is 42

Statement – 2 : If n things are arranged in row, the number of ways in which they can be de- arranged so that no one of them occupies its original place is

$$n!\left(1-\frac{1}{1!}+\frac{1}{2!}+.......(-1)^n\frac{1}{n!}\right)$$

Key. D

Sol. Conceptual

18. Statement – 1 : Number of ways of distribution of 12 identical balls into 3 identical boxes is 19

Because

Statement – 2 : Number of ways of distribution of *n* identical objects among r persons, each one of whom can receive any number of objects is $n+r-1\ c_{r-1}$

Key. B

Sol. Total 12 identical in 3 distinct

$$12+3-1_{C_{3-1}}=91 \qquad ie.(x+y+z=91)$$

Case (i) When each box contains equal number

$$x=y=z=4=1way$$

Case (ii) When two boxes contains equal number

$$2x+z=12 \Rightarrow (x=6,z=0)(x=5,z=2),(x=3,z=6)$$

$$(x=2,z=8)(x=1,z=10),(x=0,z=12)$$

$3c_2.6=18$ ways $=\dfrac{18}{\left(\dfrac{3!}{2!}\right)}=6\,ways$

Case (iii) distinct number

Total $-(1+18)=72=\dfrac{72}{3!}=12$

$$\therefore Total=1+6+12=19$$

19. Let A and B are two candidates seeking admission in IIT. the probability that A is selected is 0.5 and the probability that both A and B are selected is at most 0.3 then consider the following statements

Statement I:. The probability of B getting selected is 0.9

Statement II: If E_1 and E_2 are the events of A and B selected respectively, then $P(E_1 \cap E_2) =$ $P(E_1).P(E_2)$.

Key. D

Sol. Given $P(E_1 \cap E_2) \le 0.3$

$\Rightarrow P(E_1).P(E_2) \le 0.3$

$\Rightarrow P(0.5)P(E_2) \le 0.3$

$\therefore \quad P(E_2) \le \frac{(0.3)}{(0.5)}$

$\Rightarrow \quad P(E_2) \le 0.6$

$P(E_2) \ne 0.9$

20. Statement I: : If two numbers are drawn from the set {1,2, 3, …..n} (n > 1), then the probability that the second drawn number is larger than first is $\frac{1}{2n}$.

Statement II: The sample space will have $\frac{n(n-1)}{2}$ elements out of which half of them will be favouring the event described above.

Key. D

Sol. Statement 1 is false and statement 2 is true.

$\text{prob} = \frac{1}{2}$

21. Statement I: If A,B,C are mutually independent events then $(A \cup B)$ and C are also independent

Because.

Statement II: If A, B, C are pair wise independent and if A is independent of $(B \cup C)$, then A, B and C are not mutually independent

Key. C

Sol. Statement 1 is true and statement 2 is false.

22. Statement I: Let α and β be two fixed non-zero complex numbers and z is a variable complex number. If the lines $\alpha\bar{z}+\bar{\alpha}z+1=0$ and $\beta\bar{z}+\bar{\beta}z-1=0$ are mutually perpendicular, then $\alpha\bar{\beta}+\bar{\alpha}\beta=0$

Statement II: Two lines passing through the points z_1, z_2 and z_3, z_4 in argand plane are mutually perpendicular if $\arg\frac{z_1-z_2}{z_3-z_4}=\pm\frac{\pi}{2}$

Key. B

Sol. Conceptual

23. Statement-1: Number of non negative Integral solutions of the equation $x_1+x_2+x_3=10$ is equal to 34.

Statement-2: Number of non negative integral solutions of the equation

$x_1+x_2+x_3+\dots\dots\dots\dots\dots+x_r=n$ is equal to ${}^{(n+r-1)}C_{r-1}$

Key. D

Sol. Total number of integral solution of $x_1+x_2+x_3=10$ is

${}^{(10+3-1)}C_2={}^{12}C_2=66$

Statement 2 is true and statement 1 is false.

24. Statement-1: The total number of different 3-digits number of type $N=abc$, where $a<b<c$ is 84.

Statement-2: O cannot appear at any position, so total numbers are ${}^9C_3$.

Key. A

Sol. 0 cannot be appear in first position. With the given condition in the question 0 can not appear in any position. Now three digit can be selected out of 9 remaining digits in ${}^9C_3$ ways. Cross pending to each we will get three digit number with condition $a<b<c$.

25. Statement1 : A polygon has 44 diagonals and number of sides are 11.

because

Statement2 : From n distinct object r object can be selected in ${}^nC_r$ ways.

Key. A

Sol. Let no of sides are n.

${}^nC_2$ n = 44

n = 8 or 11 n = 11.

26. Let y = x + 3, y = 2x + 3, y = 3x + 2 and y + x = 3 are four straight lines

Statement-I: The number of triangles formed is ${}^4C_3$

because

Statement-II: Number of distinct point of intersection between various lines will determine the number of possible triangle.

Key. D

Sol. Obviously A is correct answer.

27. Let A and B be two events such that $P(A)=\frac{3}{5};\ P(B)=\frac{2}{3}$ then

Statement I :

$\frac{4}{15}\le P(A\cap B)\le\frac{3}{5}$

Statement II :

$\frac{2}{5}\le P\left(\frac{A}{B}\right)\le\frac{9}{10}$

Key. A

Sol. $P(A\cap B)\ge P(A)+P(B)-1\ge\frac{4}{15}$

$P(A\cap B)\le P(A)\le P(B)$

$P(A\cap B)\le \frac{3}{5}$

$\frac{4}{15P(B)}\le \frac{P(A\cap B)}{P(B)}\le \frac{3}{5P(B)}$

28. When a fair die is rolled

Statement I : Probability on getting a composite number $=\frac{1}{3}$

Statement II : When a die is rolled. There are 3 out comes namely
(1) getting a composite number (2) getting a prime number (3) getting 1 (neither composite nor prime).
Hence probability of getting composite number $=\frac{1}{3}$.

Key. C

Sol. Conceptual.

29. Statement I : Let the odds against an event $=\frac{2}{3}$ then the probability of occurring the event $=\frac{3}{5}$

Statement II : For any two events $P(\bar{A}\cap\bar{B})=1-P(A\cup B)$.

Key. B

Sol. $P(\bar{E}):P(E)=2:3$

$\Rightarrow P(E)=\frac{3}{2+3}=\frac{3}{5}$

30. Statement I : Two non negative intigers are chosen at random the probability that the sum of their squares is divisible by 5 is $\frac{9}{25}$

Statement II : At unit place if 0 is there in any number then it is divisible by only with 5.

Key. C

Sol. Let $x=5a+p,\ y=5b+q$

$n(E)$ is so that p^2+q^2 is divisible by 5

$x^2+y^2=5N+p^2+q^2$, were $0\le p,q\le 4$

$n(S)=25$ and $n(E)$ are $(0,0)(1,2),(2,1),(3,4),(4,3),(3,1),(1,3),(4,2),(2,4)=9$

31. STATEMENT-I: $n^n-{}^nc_1(n-1)^n+{}^nc_2(n-2)^n-{}^nc_3(n-3)^n+.........+(-1)^{n-1}\,{}^nc_{n-1}=n!$

STATEMENT-II: If A and B have the same number of elements then No. of onto functions from A to B = No.of one – one functions from A to B.

Key. A

Sol. No. of onto functions from a set of n elements to a set of r elements.

$= r^n - {}^r c_1(r-1)^n + {}^r c_2(r-2)^n + {}^r c_3(r-3)^n + ----- + (-1)^{r-1}\,{}^r c_{r-1}$

No. of one-one functions from a set of n elements to another set of n elements = n!

$\therefore$ Ans = A.

32. STATEMENT-I: If P is a natural number having number of divisors (including unity and P) equal to 105 then $\{\sqrt{P}\} = 0$ where $\{x\}$ stands for fractional part of x.

STATEMENT-II : $2^2.3^4.5^6$ is one of such numbers P.

Key. B

Sol. If $P = a^x.b^y.c^z$ ——, where *a,b,c* etc are prime factors, then we know that no. of divisors of $P = (x+1).(y+1).(z+1) ---$ etc = 105.

$\Rightarrow x+1, y+1, z+1, ---$ all must be odd

$\Rightarrow x, y, z, ---$ all must be even

$\Rightarrow P$ is a perfect square

$\therefore$ Statement-I is true.

Statement-II is also true, but it is not the correct explanation.

33. STATEMENT-I: If $(2x+3y)^{12} = T_1 + T_2 + ---- + T_{13}$ where *T_1, T_2,-----T_{13}* are terms of binomial expansion when $x = \frac{1}{3}$ and $y = \frac{1}{2}$,then $\sum_{r=1}^{12} \operatorname{sgn}(T_r - T_{r+1}) = -4$.

STATEMENT-II : $T_1 < T_2 < ---- < T_8 < T_9 = T_{10} > T_{11} > T_{12} > T_{13}$

Key. D

Sol. $\left(\frac{2}{3}+\frac{3}{2}\right)^{12} = \left(\frac{2}{3}\right)^{12}\left(1+\frac{9}{4}\right)^{12}$

$$\frac{(n+1)|x|}{|x|+1} = \frac{13\times\frac{9}{4}}{\frac{9}{4}+1} = 9$$

$\therefore$ $T_9 = T_{10}$ are greatest terms and $T_1<T_2<$ ------ $< T_8<T_9 = T_{10}>T_{11}>T_{12}>T_{13}$

$$\sum_{r=1}^{12} \operatorname{sgn}(T_r - T_{r+1}) = (-1)\times 8 + 0 + 1\times 3 = -5$$

34. STATEMENT-I: ${}^{30}c_{15} - 1$ is divisible by 31

STATEMENT-II : If n is a prime, then ${}^n c_r$ is divisible by n for r = 1,2,3,----,n-1 and

Key. D

Sol. $${}^n c_r = \frac{n(n-1)----(n-r+1)}{1.2.3.---r}$$

${}^n c_r$ is a positive integer and hence all the factors of the denominator must cancel out with some of the factors in the numerator. But n, being prime, remains without cancellation. Hence ${}^n c_r$ is divisible by *n* for *r* = 1,2,---,*n-1*.

Now ${}^n c_r + {}^n c_{r-1} = {}^{n+1}c_r$ is well known.

$\Rightarrow {}^n c_r = {}^{n+1}c_r - {}^n c_{r-1}$

Using this formula again and again, we can show that

$$^{30}c_{15} = {}^{31}c_{15} - {}^{31}c_{14} + {}^{31}c_{13} ---+ {}^{31}c_3 - {}^{31}c_2 + {}^{31}c_1 - {}^{30}c_0$$

$$\Rightarrow {}^{30}c_{15} + 1 = {}^{31}c_{15} - {}^{31}c_{14} + --- + {}^{31}c_1$$

On the R.H.S each term is divisible by 31

$\therefore {}^{30}c_{15} + 1$ is divisible by 31.

35. STATEMENT-1: The term independent of x in the expansion of $\left(x+\frac{1}{x}+2\right)^m$ is $\frac{\angle 4m}{(\angle 2m)^2}$

STATEMENT-2: The coefficient of x^k in the expansion of $(1+x)^n$ is nC_k

Key. D

Sol. $T_{r+1} = 10C_r(-k)^2 x^5 - \frac{5r}{2}$

$\therefore 5 - \frac{5r}{2} = 0 \Rightarrow r = 2$

$\therefore 10C_2 k^2 = 405$, $\therefore k^2 = 9$, $\therefore k = \pm 3$

36. STATEMENT -1: The number of ways of writing 1400 as a product of two positive integers is 12

STATEMENT-2: $1400 = 2^3 \times 5^2 \times 7$

Key. A

Sol. $\left(x+\frac{1}{x}+2\right)^m = \frac{(x+1)^{2m}}{x^m}$

$\therefore$ Term independent of $x = \frac{2mC_m x^m}{x^m} = (2m)C_m$

37. STATEMENT -1: The number of selections of four letters from the letters of word PARALLEL is 15.

STATEMENT-2: Coefficient of x^2 in the expansion of $(1+x)^6$ is 15

Key. A

Sol. The number of divisors of $1400 = (3+1)(2+1)(1+1) = 24$

$\therefore$ No. of ways of writing as product of two numbers $= \frac{24}{2} = 12$

38. STATEMENT -1: If n is a positive integer less than 20, then $\angle n \angle(20-n)$ is minimum when n = 10

STATEMENT-2: $(2m)C_r$ is maximum when r = m.

Key. D

Sol. $20C_n = \frac{\angle 20}{\angle n \angle 20 - n}$

$\angle n \angle 20 - n$ is minimum $\Rightarrow 20C_n$ is maximum

$\therefore n = 10$

39. Statement 1: If P is a natural number having number of divisors (including unity and P) equal to 105 then $\{\sqrt{P}\}=0$ where $\{x\}$ stands for fractional part of x

Statement 2: $2^2.3^4.5^6$ is one of such numbers P.

Key. B

Sol. If $P=a^x.b^y.c^z.....$ where a,b,c etc are prime factors, then we know that no.of divisors of $P=(x+1)(y+1)(z+1).....$ etc = 105.

$\Rightarrow x+1, y+1, z+1, ...$ all must be odd

$\Rightarrow x, y, z, ...$ all must be even

$\Rightarrow P$ is a perfect square

40. Statement 1: ${}^{30}C_{15}-1$ is divisible by 31

Statement 2: If n is a prime, then ${}^nC_r$ is divisible by n for $r=1,2,3,...n-1$

Key. D

Sol. $${}^nC_r=\frac{n(n-1)...(n-r+1)}{1.2.3......r}$$

${}^nC_r$ is a positive integer and hence all the factors of the denominator must cancel out with some of the factors in the numerator. But n, being prime, remains without cancellation. Hence ${}^nC_r$ is divisible by n for $r=1,2,....n-1$

Now ${}^nC_r+{}^nC_{r-1}={}^{n+1}C_r$ is well known.

$\Rightarrow {}^nC_r={}^{n+1}C_r-{}^nC_{r-1}$

Using this formula again and again, we can show that

${}^{30}C_{15}={}^{31}C_{15}-{}^{31}C_{14}+{}^{31}C_{13}......+{}^{31}C_3-{}^{31}C_2+{}^{31}C_1-{}^{30}C_0$

$\Rightarrow {}^{30}C_{15}+1={}^{31}C_{15}-{}^{31}C_{14}+......+{}^{31}C_1$

On the R.H.S each term is divisible by 31

$\therefore {}^{30}C_{15}+1$ is divisible by 31.

41. Statement 1: The number of selections of four letters taken from the word PARALLEL must be 15

Statement 2: Coefficient of x^4 in the expansion of $(1-x)^{-3}$ is 15 $(|x|<1)$

Key. D

Sol. $1'P.2'A.1R.3'L.1E$

4 diff: ${}^5C_4=5$

3 alike of 1 kind & 1 diff $={}^1C_1.{}^4C_1=4$

3 alike of 1 kind & 2 diff $= {}^2C_1 . {}^4C_2 = 2.6 = 12$

2 alike of 1 kind & 2 diff of 2nd kind $= {}^2C_2 = 1$

Total = 22

42. Statement 1: If $f:\{1,2,3,4,5\} \to \{1,2,3,4,5\}$ then the number of onto functions such that $f(i) \neq i$ is 42

Statement 2: If n things are arranged in row, the number of ways in which they can be de-arranged so that no one of them occupies its original place is $n!\left(1-\frac{1}{1!}+\frac{1}{2!}+.....(-1)^n\frac{1}{n!}\right)$

Key. D

Sol. Conceptual

43. Statement 1: Number of ways of distribution of 12 identical balls into 3 identical boxes is 19

Statement 2: Number of ways of distribution of n identical objects among r persons, each one of whom can receive any number of objects is $n+r-1C_{r-1}$

Key. B

Sol. Total 12 identical in 3 distinct

${}^{12+3-1}C_{3-1} = {}^{14}C_2 = 91 \ i.e., (x+y+z=91)$

Case (1): When each box contains equal number $x=y=z=4=1 way$

Case (2): When two boxes contains equal number

$2x+z=12 \Rightarrow (x=6,z=0),(x=5,z=2),(x=3,z=6)$

$(x=2,z=8),(x=1,z=10),(x=0,z=12)$

${}^3C_2 .6 = 18 \ ways = \frac{18}{\left(\frac{3!}{2!}\right)} = 6 \ ways$

$= 91-(1+18) = 72 \Rightarrow \frac{72}{3!} = 12$

Case (3): Distinct numbers

$\therefore Total = 1+6+12 = 19$

44. Statement 1: The number of ways of writing 1400 as a product of two positive integers is 12.

Statement 2: 1400 is divisible by exactly three prime numbers.

Key. B

Sol. Since $1400 = 2^3 .5^2 .7^1$

$\Rightarrow$Number of factors $= (3+1)(2+1)(1+1) = 24$

$\Rightarrow$Number of ways of expressing 1400 as a product of two numbers $= \frac{1}{2}\times 24 = 12$. But this does not follow from R which is obviously true.

45. Statement-1: Number of ways in which two persons A and B select objects from two different groups each having 20 different objects such that B always selects more objects than A(including the case when A selects no object) is $\frac{2^{40} - {}^{40}C_{20}}{2}$

Statement-2: $\sum\sum_{0\le i<j\le n} {}^nC_i\,{}^nC_j = \frac{2^{2n} - {}^{2n}C_n}{2}$

Key. B

Sol. Conceptual

46. Statement-1: Number of ways in which 10 identical toys can be distributed among three students if each receives atleast two toys is ${}^9C_2$

Statement-2: Number of positive integral solutions of $x + y + z + w = 7$ is ${}^6C_3$.

Key. D

Sol. Conceptual

47. Statement – 1: If $x, y, z \in R$ and $3x+4y+5z = 10\sqrt{2}$ then the least value of $x^2+y^2+z^2$ is 4.

Statement – 2: If $a_i, b_i \in R$, $i = 1, 2, 3$ then $(a_1b_1+a_2b_2+a_3b_3)^2 \le (a_1^2+a_2^2+a_3^2)(b_1^2+b_2^2+b_3^2)$

Key. A

Sol. Conceptual

48. Statement – 1: If $x > 0$ then the least value of $\frac{x^4+x^2+4}{x}$ is 6.

Statement – 2: If A, G respectively are the A.M and G.M of '*n*' positive numbers then $A \ge G$.

Key. A

Sol. Conceptual

49. Statement – 1: If A, B are two mutually exclusive events with non-zero probabilities then A, B are dependent.

Statement – 2: If A, B are mutually exclusive events then $P(A \cup B) = P(A) + P(B)$.

Key. B

Sol. Conceptual

50. Statement – 1: If the perimeter of a triangle is constant then the area of the triangle is maximum when the triangle is equilateral.

Statement – 2: If the circumradius of a triangle is constant then the area of the triangle is maximum when the triangle is equilateral.

Key. B

Sol. Conceptual

51. Statement – 1: If A and B are two mutually exclusive events then $P(A) \le P(B^1)$

Statement – 2: If A and B be any two events then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ and Probability of any event is always less than or equal to one.

Key. A

Sol. If A and B are two mutually exclusive events then $P(A \cap B) = 0$

$\therefore P(A \cup B) = P(A) + P(B) \le 1$

$P(A) \le 1 - P(B)$

$P(A) \le P(B^1)$

52. Statement – 1: If the Probability of A failing in an examination is 0.2 and that for B is 0.3, The Probability that either A or B fails is 0.44.

Statement – 2: If A and B are independent then A' and B' need not be independent.

Key. C

Sol. $P(A \cup B) = 1 - P(A')P(B') = 0.44 \Rightarrow$ statement-1 is true,

Statement-2 us false.

53. Statement – 1: $P(A/B) + P(A'/B) = 1$

Statement – 2: Least value of $P(A) + P(B) = 1 + P(A \cap B)$

Key. C

Sol. $P(A \cap B) + P(\overline{A} \cap B) = P(B) \Rightarrow$ Statement-1 is true

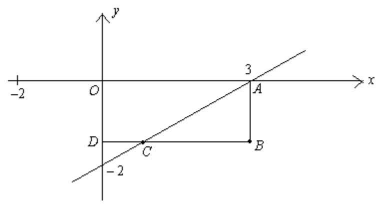
Maxi. Value of $P(A) + P(B) = 1 + P(A \cap B) \Rightarrow$ Statement-2 is false.

54. Statement – 1: On the real line R, points a and b are selected at random such that $-2 \le b \le 0$ and $0 \le a \le 3$ then the probability that the distance between a and b is greater than 3 is $\frac{1}{3}$.

Statement – 2: Uncountable sample space is said to be discrete.

Key. C

Sol. The Sample space S consists of the ordered pairs (a,b) and so forms the rectangular region on the other hand, the set A of points (a,b) for which $|a-b| > 3$ .consists of those points of S which lie below the line $x - y = 3$,which is given the following diagram.



$$\text{Required Probability} = \frac{\text{area of } \Delta ABC}{\text{area of } \square OABD} = \frac{1}{3}$$

STATEMENT -1 is true.

Finite or countable infinite probability space is said to be discrete and an uncountable space is said to be no discrete.

STATEMENT-2 is false.

An urn contains 4 white and 9 black balls. Red balls are drawn with replacement. Let P_r be the probability that no two white balls appear in succession. Answer the following questions.

55. *Assertion:* The probability that 10 is the second smallest integer in a subset of 4 different numbers chosen from 1 through 20 is $\frac{27}{323}$.

REASON: For 10 to be second smallest number there must be one number smaller than 10 and two other larger numbers greater than 10

Key. A

Sol. Conceptual

56. *Assertion:* If A,B are two events such that $P(A) = \frac{3}{5}, P(B) = \frac{2}{3}$ then $P(A/B) \in \left[\frac{2}{5}, \frac{9}{10}\right]$

Reason: $P(A \cap B) \geq P(A) + P(B) - 1, P(A \cap B) \leq P(A) or P(B)$

Key. A

Sol. Conceptual

57. *Assertion:* There can not exist a Binomial variate whose μ and σ are given by 5, 2 respectively

Reason: If n, p are parameters of Binomial variate then np=5, npq=4

Key. D

Sol. Conceptual

58. *Assertion:* An unbiased die is tossed till a number greater 4 appears, then probability that even number of tosses is needed is $\frac{1}{2}$

Reason: Probability of getting a 5 or 6 when a die is tossed is $\frac{1}{3}$

Key. D

Sol. Conceptual

59. Statement I : The probability of solving a new problem by 3 students are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. The probability that the problem will be solved by them is $\frac{1}{24}$.

Statement II : If *A*, *B* and *C* are three independent events then the probability of atleast one of them happening = $1-P(\bar{A})P(\bar{B})P(\bar{C})$.

Key. D

Sol. Conceptual

60. Let *A* and *B* are two independent events.

Statement I : If $P(A)=0.3$ and $P(A\cup\bar{B})=0.8$ then $P(B)$ is $\frac{2}{7}$.

Statement II : $P(\bar{E})=1-P(E)$ where *E* is any event.

Key. A

Sol. Conceptual

61. Statement I : If A and B be mutually exclusive events in a sample space such that $P(A)=0.3, P(B)=0.6$ then $P(\bar{A}\cap\bar{B})=0.28$

Statement II : If A & B are mutually exclusive events then $P(A\cap B)=0$

Key. D

Sol. Conceptual

62. Statement I : No.of terms in the expansion of $(x_1+x_2+...+x_{11})^6={}^{16}C_6$

Statement II : No.of ways of distributing *n* identical things among *r* persons when each person get zero or more things = ${}^{n+r-1}C_n$

Key. A

Sol. Conceptual

63. A fair coin is tossed 3 times, consider the events

P: First toss is head; Q: Second toss is head.

R: Exactly two consecutive heads or exactly two consecutive tails

Statement – 1: P, Q, R are independent events.

Statement – 2: P, Q, R are pair wise independent.

Key: B

Hint: Conceptual Question

64. A bag contains four tickets having numbers 112, 121, 211, 222 written on them. Denote by $A_i\left(i=1,2,3\right)$ the event that the i[th] digit from left of the number on a randomly drawn ticket is 1.

Statement- 1 : A_1, A_2, A_3 are independent events

Statement-2 : $P\left(A_iA_j\right)=P\left(A_i\right)P(A_j);1\le i<j\le 3$.

Key: D

Hint: CONCEPTUAL

65. STATEMENT – 1 If A and B are two mutually exclusive events then P(A) ≤ P(B′).

STATEMENT – 2 If A and B be any two events then P(A ∪ B) = P(A) + P(B) – P(A ∩ B) and probability of any event is always less than equal to one.

Key: A

Hint: If A and B are two mutually exclusive events then P(A $\cap$ B) = 0

$\therefore$ P(A $\cup$ B) = P(A) + P(B) $\le$ 1

P(A) $\le$ 1 – P(B)

P(A) $\le$ P(B′)

$\therefore$ statement – I is true, statement – II is the correct explanation of statement – I.

66. Statement- I: Out of 5 tickets consecutively numbered, three are drawn at random. The chance that the numbers on them are in AP is $\frac{2}{5}$.

Statement – II: Out of $(2n+1)$ tickets consecutively numbered, if three are drawn at random, the chance that the numbers on them are in AP is $\frac{(4n-2)}{(4n^2-1)}$.

Key. C

Sol. Total ways = ${}^{2n+1}C_3 = \frac{(2n+1)2n(2n-1)}{1.2.3}$

$$= \frac{n(4n^2-1)}{3}.$$

Let the three numbers a, b, c are drawn where $a < b < c$ and given a, b, c are n AP.

$\therefore$ $b = \frac{a+c}{2}$ $or\ 2b = a+c$ ....(i)

It is clear that form Eq. (i) a and C are both odd or both even.

Out of $(2n+1)$ tickets consecutively numbers either $(n+1)$ of them will be odd and n of them will be even..

$\therefore$ Favourable ways = ${}^{n+1}C_2 + {}^{n}C_2$

$$= \frac{(n+1)n}{1.2} + \frac{n(n-1)}{1.2} = n^2.$$

$\therefore$ Required probability $= \frac{n^2}{\frac{n(4n^2-1)}{3}} = \frac{3n}{4n^2-1}$.

67. Statement – I: A number is chosen at random from the numbers 1, 2, 3, …., 6n + 3. Let *A* and *B* be defined as follows:

A : number is divisible by 2

B : number is divisible by 3

Then, *A* and *B* are independent.

Statement – II: If events *A* and *B* are independent, then $P(A \cap B) = P(A).P(B)$.

Key. D

Sol. We have

$$A = \{2,4,6,8,....,6n,6n+2\}$$

$$B = \{3,6,9,12,....,6n,6n+3\}$$

And $A \cap B = \{6,12,18,...,6n\}$

Here, $n(A) = 3n+1, n(B) = 2n+1$ and $n(A \cap B) = n$

$\therefore \quad P(A)=\frac{n(A)}{6n+3}=\frac{3n+1}{6n+3}, P(B)=\frac{n(B)}{6n+3}=\frac{1}{3}$

And $\quad P(A\cap B)=\frac{n(A\cap B)}{6n+3}=\frac{n}{(6n+3)}$.

Since, $\quad P(A\cap B)\neq P(A)P(B)$

$\therefore$ A and B are not independent.

68. Let A and B are two events such that P(A $\cup$B)^C = $\frac{1}{6}$, P(A $\cap$B) = $\frac{1}{4}$, P (A^C) = $\frac{1}{4}$.

STATEMENT–1
Events A and B are independent events.
because
STATEMENT–2
Events A and B are equally likely.

Key. C

Sol. P(A^c) = 1/4 $\Rightarrow$ P(A) = 3/4
P(A$\cup$B)^C = 1/6
$\Rightarrow$ 1 – P (A$\cup$ B) = 1/6
$\Rightarrow$ P(A $\cup$ B) = 5/6
$\Rightarrow$ P(A) + P(B) – P(A $\cap$B) = 5/6
$\Rightarrow$ $\frac{3}{4}$ + P(B) – $\frac{1}{4}$ = 5/6
$\Rightarrow$ P(B) = 1/3.
Clearly A and B are independent events but not equally likely.

69. Let A and B be two independent events of a random experiment.
STATEMENT–1
P(A $\cap$ B) = P(A). P(B)
because
STATEMENT–2
Probability of occurrence of A is independent of occurrence or non–occurrence of B.

Key. A

Sol. Statement –II is true as this is the definition of the independent events.
Statement – I is also true, as if events are independent, then P$\left(\frac{A}{B}\right)$ = P(A)
$\Rightarrow \frac{P(A\cap B)}{P(B)}$ = P(A) $\Rightarrow$ P(A $\cap$ B) = P(A). P(B).
Obviously statement – II is a correct reasoning of statement – I
Hence (A) is the correct answer.

70. Statement – 1 : If two numbers are drawn from the set {1,2, 3, …..n} (n > 1), then the probability that the second drawn number is larger than first is $\frac{1}{2n}$

Statement – 2 : The sample space will have $\frac{n(n-1)}{2}$ elements out of which half of them will be favouring the event described above.

Key. D

Sol. Statement 1 is false and statement 2 is true.

$$\text{prob} = \frac{1}{2}$$

71. Statement – 1 : If A,B,C are mutually independent events then $(A \cup B)$ and C are also independent

Because

Statement – 2 : If A, B, C are pair wise independent and if A is independent of $(B \cup C)$, then A, B and C are not mutually independent.

Key. C

Sol. Statement 1 is true and statement 2 is false.

72. Statement - 1: If 12 coins are thrown simultaneously, then probability of appearing exactly five heads is equal to probability of appearing exactly 7 heads.

Statement - 2: ${}^nC_r = {}^nC_s \Rightarrow \text{either } r = s$ or $r + s = n$ and $P(H) = P(T)$ in a single trial.

Key. A

Sol. Conceptual

Probability

Comprehension Type

Paragraph – 1

A lot contains 10 defective and 10 non-defective bulbs. 2 bulbs are drawn at random, One at time with replacement. We define the events A, B and C is follows:

A = {The first bulb is defective}

B= {The second bulb is non-defective}

C= {Both bulbs are either defective or non-defective}

1. P(A) will be equal to

(A) $\frac{1}{4}$ (B) $\frac{3}{4}$ (C) $\frac{1}{2}$ (D) $\frac{1}{3}$

Key. C

2. P(B).P(C) will be equal to

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{16}$ (D) $\frac{1}{8}$

Key. A

3. $P(A \cap B \cap C)$ will be equal to

(A) 0 (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{1}{8}$

Key. A

Sol. Conceptual

Paragraph – 2

Let S be the set of first 18 natural Numbers. Then attempt the following.

4 The probability of choosing {x, y}$\subseteq$S such that $x^3 + y^3$ is divisible by 3.

A. 1/3 **B.** 1/6 **C.** 1/5 **D.** 1/4

Key. A

Sol.

1	4	7	10	13	16
2	5	8	11	14	17
3	6	9	12	15	18

$$probability = \frac{6_{c_2} + (6_{c_1})^2}{18_{c_2}} = \frac{1}{3}$$

5. The probability of choosing {x, y, z} $\subseteq$S such that x, y, z are in A.P is

A. 1/17 **B.** 2/17 **C.** 5/34 **D.** 3/34

Key. D

Sol.

1	3	5	7	9	11	13	15
2	4	6	8	10	12	14	16

$$probability = \frac{2(9_{c_2})}{18_{c_3}} = \frac{3}{34}$$

6. The probability of choosing { x, y, z} $\subseteq$ S such that no two of the numbers x, y, z are consecutive is

A. 35/51 **B.** 2/17 **C.** 4/17 **D.** 6/17

Key. A

Sol. $$probability = \frac{(18-3+1)_{c_3}}{18_{c_3}} = \frac{35}{31}$$

Paragraph – 3

All the 52 cards of a well shuffled pack of playing cards are distributed equally or unequally among 4 players named P_1, P_2, P_3 & P_4.

For i = 1, 2, 3, 4 let

α_i = number of ace(s) given to P_i

β_i = number of black card(s) given to P_i

γ_i = number of red card(s) given to P_i

δ_i = number of diamond(s) given to P_i

7. The probability that $\delta_i \geq 1 \ \forall$ i = 1, 2, 3, 4 is

(A) $^{13}C_4\ 4!/4^{13}$

(B) $[2^{24} + 3\times2^{12} - 3^{13} - 1]/2^{24}$

(C) $[13^4 - {}^4C_1\ 12^4 + {}^4C_2\ 11^4 - {}^4C_3\ 10^4 + {}^4C_4\ 9^4]/13^4$

(D) $1 - 4(3/4)^{13} - 6(1/2)^{13} - (1/4)^{12}$

Key. B

Sol. Probability of giving atleast one diamond to every player is $\frac{\left[4^{13} - {}^4C_1 3^{13} + {}^4C_2 2^{13} - {}^4C_1 1^{13}\right]\times 4^{39}}{4^{52}}$.

8. If $\beta_i + \gamma_i = 13 \ \forall$ i = 1, 2, 3, 4 then the probability that $\alpha_i = 1 \ \forall$ i = 1, 2, 3, 4 is

(A) $\frac{5^4\times7^2}{13^4}$ (B) $\frac{13^3}{17\times7^2\times5^2}$

(C) $\frac{3^4\times13^2}{17^4\times2^7}$ (D) $\frac{7^3\times3^2}{13^4}$

Key. B

Sol. They get equal number of cards. The probability of each getting an ace

$$= \frac{4!\times\frac{48!}{(12!)^4}}{\frac{52!}{(13!)^4}} = \frac{13^3}{17\times7^2\times5^2}$$

9. If $\beta_i + \gamma_i = 13\ \forall\ i = 1, 2, 3, 4$ then the probability that $|\beta_i - \gamma_i| = 1\forall\ i = 1, 2, 3, 4$ is

(A) $\left[\dfrac{26!}{(6!)^2\,(7!)^2}\right]^2 \div \left[\dfrac{52!}{(13!)^4}\right]$ (B) $\left[\dfrac{(26!)\,(4!)}{(6!)^2\,(7!)^2\,(2!)^2}\right]^2 \div \left[\dfrac{52!}{(13!)^4}\right]$

(C) $4!\left[\dfrac{26!}{(6!)^2\,(7!)^2\,(2!)^2}\right]^2 \div \left[\dfrac{52!}{(13!)^4}\right]$ (D) $4!\left[\dfrac{26!}{(6!)^2\,(7!)^2\,(2!)}\right]^2 \div \left[\dfrac{52!}{(13!)^4}\right]$

Key. D

Sol. Two players get 7 red and 6 black cards each while other two get 6 red and 7 black cards each.

So, the required probability = $\dfrac{{}^4C_2 \times \left({}^{26}C_7\,{}^{19}C_7\ {}^{12}C_6\ {}^6C_6\right)\times\left({}^{26}C_6\,{}^{20}C_6\,{}^{14}C_7\,{}^7C_7\right)}{{}^{52}C_{13}\ {}^{39}C_{13}\ {}^{26}C_{13}\ {}^{13}C_{13}}$.

Paragraph – 4

Considering the rectangular hyperbola xy = 15!. The number of points (α, β) lying on it, where

10. $\alpha, \beta \in I$, is

(A) 2016 (B) 4032

(C) 4033 (D) 8064

Key. D

11. $\alpha, \beta \in I^+$ and HCF $(\alpha, \beta) = 1$, is

(A) 64 (B) 785

(C) 4032 (D) 94185

Key. A

12. $\alpha, \beta \in I^+$ and α divides β, is

(A) 96 (B) 511

(C) 1344 (D) 4032

Key. A

Sol. **10.** $xy = 15! = 2^{11}3^6\ 5^3\ 7^2\ 11^1\ 13^1$

Number of +ve integral solutions = no. of ways of fixing x = the number of factors of 15!

= (1 + 11) (1 + 6) (1 + 3) (1 + 2) (1 + 1) (1 + 1) = 4032

Total number of integral solutions (positive or negative) = 2 × 4032 = 8064

11. HCF $(\alpha, \beta) = 1$. α and β will not have common factor other than 1 so, identical prime numbers should not be separated. e.g. 2^{11} will completely go with either α or β.

So the number of solutions = 2 × 2 ×2 × 2 × 2 × 2 = 64.

12. The largest number whose perfect square can be made with 15! is $2^5\ 3^3\ 5^1\ 7^1$

So that number of ways of selecting x will be

(1 + 5) (1 + 3) (1 + 1) (1 + 1) = 96.

Paragraph – 5

10-digit numbers are formed by using all the digits 0,1,2,3,4,5,6,7,8 and 9 such that they are divisible by 11111.

13. The digit in the ten's place, in the smallest of such numbers, is

a) 9 b) 8 c) 7 d) 6

Key. D

14. The digit in the unit's place, in the greatest of such numbers, is

a) 4 b) 3 c) 2 d) 1

Key. A

15. The total number of such numbers is

a) 6543 b) 5634 c) 3456 d) 4365

Key. C

Sol. Let a b c d e f g h i j be one of such numbers where a b c d e f g h i j is some permutation of the digits 0,1,2,3,4,5,6,7,8,9 where $a \neq 0$.

Sum of digits of the number = 0+1+2+3+4+5+6+7+8+9 = 45, which is divisible by 9 and hence the number is divisible by 9. But it is divisible by 11,111 also and 11,111 is not divisible by 3 or 9. Therefore, the number is divisible by $11{,}111 \times 9 = 99{,}999$.

And a b c d e f g h i j = a b c d e $\times$ 10^5 + f g h i j

= a b c d e $\times$ (99,999+1) + f g h i j

= a b c d e $\times$ 99,999 + a b c d e + f g h i j is divisible by 99,999.

$\Rightarrow$ a b c d e + f g h i j is divisible by 99,999.

But a b c d e < 99,999

And f g h i j < 99,999

$\Rightarrow$ a b c d e + f g h i j < 2 $\times$ 99,999

$\therefore$ a b c d e + f g h i j = 99,999

$\Rightarrow$ e + j = d + i = c + h = b + g = a + f = 9

13.

a	b	c	d	e	f	g	h	i	j
1	0	2	3	4	8	9	7	6	5

For smallest number a must be 1 (since a can not be 0) and hence f = 8.

Then, b = 0 $\Rightarrow$ g = 9

Then, c = 2 $\Rightarrow$ h = 7

Then, d = 3 $\Rightarrow$ i = 6

Then, e = 4 $\Rightarrow$ j = 5

` $\therefore$ The smallest of such numbers is 1023489765 and the digit in the ten's place is 6.

14.

a	b	c	d	e	f	g	h	i	j
9	8	7	6	5	0	1	2	3	4

For greatest number a = 9 $\Rightarrow$ f = 0

Then, b = 8 $\Rightarrow$ g = 1

Then, c = 7 $\Rightarrow$ h = 2

Then, d = 6 $\Rightarrow$ i = 3

Then, e = 5 $\Rightarrow$ j = 4

$\therefore$ The greatest of such numbers is 9876501234 and the digit in the units place is 4.

15.

a	b	c	d	e	f	g	h	i	j
9	8	6	4	2	1	1	1	1	1

The blank ‘a’ can be filled in 9 ways (except 0).
Then blank f can be filled in only one way (by 9-a).
Now, blank ‘b’ can be filled by any of the remaining 8 digits.
Then blank ‘g’ can be filled in only one way (by 9-b)
Now, blank ‘c’ can be filled by any of the remaining 6 digits.
Then blank ‘h’ can be filled in only one way (by 9-c).
Now, blank ‘d’ can be filled by any of the remaining 4 digits.
Then blank ‘i’ can be filled in only one way (by 9-d).
Now, blank ‘e’ can be filled by any of the remaining 2 digits.
Then blank ‘j’ can be filled in only one way (by 9-e).
$\therefore$ The total number of such numbers = $9\times1\times8\times1\times6\times1\times4\times1\times2\times1$
= 3456.

Paragraph – 6

D_1, D_2,-----,D_{1000} are 1000 doors and P_1, P_2,-------P_{1000} are 1000 persons. Initially all the doors are closed. P_1 opens all the doors. Then, P_2 closes D_2, D_4, D_6---- D_{998}, D_{1000}. Then P_3 changes the status of

D_3, D_6, D_9, D_{12},-----etc.(doors having numbers which are multiples of 3). Changing the status of a door means closing it if it is open and opening it if it is closed. Then P_4 changes the status of D_4, D_8, D_{12}, D_{16},-----etc (doors having numbers which are multiples of 4). And so on until lastly P_{1000} changes the status of D_{1000}.

16. Finally, how many doors are open?
a) 30 b) 31 c) 32 d) 33

Key. B

17. What is the greatest number of consecutive doors that are closed finally?
a) 56 b) 58 c) 60 d) 62

Key. C

18. The door having the greatest number that is finally open is
a) D_{960} b) D_{961} c) D_{962} d) D_{963}

Key. B

Sol. 16. Consider any door, for example, D_{72}. It is operated by
$P_1, P_2, P_3, P_4, P_6, P_8, P_9, P_{12}, P_{18}, P_{24}, P_{36}, P_{72}$, (Remember that D_m is operated by P_n if m is a multiple of n)

Here 1,2,3,4,6,8,9,12,18,24,36,72 are all the factors of 72. Initially all the doors are closed. Therefore, if odd numbers of persons operate it, it will be finally open. Otherwise it will be closed finally.

$\therefore$ D_m will be finally open, if m has an odd number of factors. And, we know that m has an odd number of factors if and only if m is a perfect square.

$\therefore$ $1^2, 2^2, 3^2, 4^2$,------, 31^2 are the numbers of the doors that are open finally.

$\therefore$ No. of doors finally open = 31.

17. $D_1, D_4, D_9, D_{16}, D_{25}$,-----,$D_{900}, D_{961}$ are the 31doors that are open finally.

$\therefore$ $D_{901}, D_{902}, D_{903}$, -----, D_{960} are the 60 consecutive doors that are closed and 60 is clearly greatest.

18. Ans: D_{961}

Paragraph – 7

If n is a positive integer, then $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$ where

$C_r = nC_r = \dfrac{\angle n}{\angle r \angle (n-r)}$. Answer the questions 15, 16 and 17.

19. The value of $nC_0 - nC_2 + nC_4 - nC_6 + ...$ is

a) $2^{\frac{n}{2}} \cos\dfrac{n\pi}{2}$ b) $2^{\frac{n}{2}} \sin\dfrac{n\pi}{2}$

c) $2^{\frac{n}{2}} \cos\dfrac{n\pi}{4}$ d) $2^{\frac{n}{2}} \sin\dfrac{n\pi}{4}$

Key. C

20. $(nC_0 - nC_2 + nC_4 - nC_6 + ...)^2 + (nC_1 - nC_3 + nC_5 - nC_7 + ...)^2$ is

a) 2^{2n} b) 2^n c) 2^{n^2} d) $2^{\frac{n+1}{2}}$

Key. B

21. $nC_0 + nC_4 + nC_8 + ...$ is equal to

a) $2^{\frac{n}{2}} \cos\dfrac{n\pi}{8}$ b) $2^{\frac{n}{2}} \sin\dfrac{n\pi}{8}$ c) $2^n + 2^{\frac{n}{2}} \cos\dfrac{n\pi}{4}$ d) $2^{n-2} + 2^{\frac{n-2}{2}} \cos\dfrac{n\pi}{4}$

Key. D

Sol. 19. put x = i in the expansion and equate real and imaginary parts.

20. In problem 15, when we put x = i. Then $(nC_0 - nC_2 + nC_4 - nC_6) + i(nC_1 - nC_3 + nC_5 ...)$

$= 2^{\frac{n}{2}}\left[\cos\dfrac{n\pi}{4} + i\sin\dfrac{n\pi}{4}\right]$

Take modulus both sides and square it.

21. Use $C_0 + C_2 + C_4 + C_6 + ... = 2^{n-1}$

Paragraph – 8

ABCDEF is regular hexagon. Answer the questions 18, 19 and 20.

22. The number of triangles that can be made by using the vertices is :

a) 10 b) 15 c) 20 d) 30

Key. C

23. The number of equilateral triangles whose vertices are the vertices of the hexagon, but sides are not the sides of the hexagon is :

a) 6 b) 4 c) 3 d) 2

Key. D

24. The number of diagonals of the hexagons is :

a) 15 b) 12 c) 24 d) 6

Key. A

Sol. Direct procedure for 18, 19, 20.
22, 23, and 24 are routine type.

Paragraph – 9

There are 'n' intermediate stations on a railway line from one terminus to another. In how many ways can the train stop at 3 of these intermediate stations, if

25. All the three stations are consecutive

a) (n + 2) b) (n + 1) c) (n – 1) d) (n – 2)

Key. D

26. Atleast two of the stations are consecutive

a) (n + 2) (n - 1) b) (n – 2) (n – 1) c) $(n – 2)^2$ d) None

Key. C

27. No two of these stations are consecutive

a) n_{c_3} b) $(n-2)_{c_3}$ c) $\frac{(n-2)(n-3)}{6}$ d) none

Key. B

Sol. 25. $(s_1, s_2, s_3),(s_2, s_3, s_4)......(s_{n-2}, s_{n-1}, s_n) = (n-2)$

26. (n – 2) ways (n – 1) ways - (n – 2) = $(n – 2)^2$

27. $n_{c_3} - (n-2)^2 = (n-2)_{c_3}$

Paragraph – 10

A is a set containing 'n' elements. A subset 'P' of 'A' is chosen at random. The set A is reconstructed by replacing the elements of 'P'. A subset Q is again chosen at random. Then the number of ways of selecting P & Q so that

28. P = Q

a) 3^n b) 2^n c) $n.3^{n-1}$ d) $3n$

Key. B

29. $P \cap Q$ contains just one element

a) 3^n b) 2^n c) $n.3^{n-1}$ d) $3n$

Key. C

30. $P \cup Q$ contains just one element

a) 3^n b) 2^n c) $n.3^{n-1}$ d) $3n$

Key. D

Sol. 28. If P contains r elements

Then number of ways of selecting P is nc_5

$$Q\ P = Q \quad \sum_{r=0}^{n} nc_r = 2^n$$

29. P can be nc_r ways

$Q / P \cap Q$ contains just one element

$$rc_1.\left(n - rc_0 + n - rc_1 + ...n - rc_{n-r}\right)$$

$$\Rightarrow nc_r\left[rc_1.\left\{n - rc_0 + n - rc_1 + ...n - rc_{n-1}\right\}\right]$$

$$\frac{n}{r}.n - 1c_{r-1}.\ r.2^{n-r}$$

$$\Rightarrow \sum_{r=1}^{n} n.n - 1c_{r-1}.2^{n-r} = n.3^{n-1}$$

30. $nc_1 + nc_0.nc_1 + nc_1.nc_0 = 3n$

Paragraph – 11

If A is a square matrix of order n, we can form the matrix $A - \lambda I$, where λ is a scalar and I is the unit matrix of order n. The determinant of this matrix equated to zero (i.e.,$\left|A - \lambda I\right| = 0$) is called as characteristic equation of A. On expanding the determinant, the characteristic equation can be written as a polynomial equation of degree n in λ of the form.

$\left(-1\right)^n \lambda^n + k_1\lambda^{n-1} + k_2\lambda^{n-2} + ...k_n = 0$. The roots of this equation are called the characteristic roots (or) Eigen values of A. The sum of the Eigen values of matrix A is equal to trace of A. Every square matrix ‘A’ satisfies its own characteristic equation.

(i.e., $(-1)^n A^n + k_1A^{n-1} + k_2A^{n-2} + ...k_nI = 0$) on multiplying the above equation by A^{-1} we can easily obtain the value of A^{-1}. This is the other way of finding A^{-1}.

31. The Eigen values of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

a) 3, -3, 5 b) 3, -3, -5 c) -3, -3, 5 d) -2, 4, -3

Key. C

32. If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ then A^{-1} =

a) $\frac{1}{4}\left[A^2 + 6A - 9I\right]$ b) $\frac{1}{4}\left[A^2 + 6A + 9I\right]$

c) $\frac{-1}{4}\left[A^2 - 6A + 9I\right]$ d) $\frac{1}{4}\left[A^2 - 6A + 9I\right]$

Key. D

33. If $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$, find the matrix represented by

$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$

a) $\begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}$ b) $\begin{bmatrix} 8 & 5 & 5 \\ 0 & 0 & 3 \\ 5 & 8 & 5 \end{bmatrix}$ c) $\begin{bmatrix} 5 & 8 & 5 \\ 3 & 0 & 0 \\ 5 & 5 & 8 \end{bmatrix}$ d) $\begin{bmatrix} 8 & -5 & 5 \\ 0 & 0 & -3 \\ 5 & -8 & 5 \end{bmatrix}$

Key. A

Sol. 31. $\lambda^3 + \lambda^2 - 21\lambda - 45 = 0$

$\lambda = -3, -3, 5$

32. $\lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0$

$\Rightarrow A^2 - 6A + 9I - 4A^{-1} = 0$

$\Rightarrow A^{-1} = \frac{1}{4}\left(A^2 - 6A + 9I\right)$

33. $A^3 - 5A^2 + 7A - 3I = 0$

$$= A^5\left(A^3 - 5A^2 + 7A - 3I\right) + \left(A^3 - 5A^2 + 7A - 3I\right)$$

$$+A^2 + A + I = A^2 + A + I$$

Paragraph – 12

A box contains n coins. Let $P(E_i)$ be the probability that exactly i out of n coins are biased.

If $P(E_i)$ is directly proportional to $i(i+1);\ 1 \le i \le n$

34. Proportionality constant K is equal to

a) $\frac{3}{n(n^2+1)}$ b) $\frac{1}{(n^2+1)(n+2)}$

c) $\frac{3}{n(n+1)(n+2)}$ d) $\frac{1}{(n+1)(n+2)(n+3)}$

Key. C

35. If P be the probability that a coin selected at random is biased then $\underset{n\to\infty}{\mathrm{Lt}}\ P$ is

a) $\frac{1}{4}$ b) $\frac{3}{4}$ c) $\frac{3}{5}$ d) $\frac{7}{8}$

Key. B

36. If a coin selected at random is found to be biased then the probability that it is the only biased coin in the box is

a) $\frac{1}{(n+1)(n+2)(n+3)(n+4)}$ b) $\frac{12}{n(n+1)(n+2)(3n+1)}$

c) $\frac{24}{n(n+1)(n+2)(2n+1)}$ d) $\frac{24}{n(n+1)(n+2)(3n+1)}$

Key. D

Sol.34. $P(E_i) = Ki(i+1)$

$$\because P(E_1) + P(E_2) + ... + P(E_n) = 1$$

$$K\sum_{i=1}^{n} i(i+1) = 1$$

$$K\left[\frac{1}{6}n(n+1)(2n+1) + \frac{n(n+1)}{2}\right] = 1 \qquad \therefore K = \frac{3}{n(n+1)(n+2)}$$

35. $P(E) = \sum_{i=1}^{n} P(E_i).P(E/E_i)$

$$= Ki(i+1).\frac{i}{n} = \frac{k}{n}\sum_{i=1}^{n}\left(i^3 + i^2\right)$$

$$=\frac{K}{n}\left[\left(\frac{n(n+1)}{2}\right)^2+\frac{1}{6}n(n+1)(2n+1)\right]$$

$$=\frac{(3n+1)(n+2)}{4n(n+2)}$$

$$\therefore \underset{n\to\infty}{Lt}\ p = \frac{3}{4}$$

36. $P(E_1/E)=\frac{P(E_1)P(E/E_1)}{P(E)}=\frac{K\times 2\times\frac{1}{n}}{\frac{(3n+1)(n+2)}{4n(n+2)}}=\frac{24}{n(n+1)(n+2)(3n+1)}$

Paragraph – 13

Let z_1,z_2,z_3 be complex numbers associated with the vertices A,B,C of a triangle ABC which is circumscribed by a circle $|z|=1$. Perpendicular from A is drawn which meet BC at D and circle $|z|=1$ at E.If P be image of E about BC and F be image of E about origin (O). Then answer the following question.

37. If $\frac{z_1-z}{\overline{z}_1-\overline{z}}+\frac{z_3-z_2}{\overline{z}_3-\overline{z}_2}=0\, and\, \frac{z_2-z}{z_1-z_3}+\frac{\overline{z}_2-\overline{z}}{\overline{z}_1-\overline{z}_3}=0$ then z is complex number associated with

a) E b) 0 c)P d) F

Key. C

38. The complex number of P is equal to

a) $\frac{z_1 - z_2+z_3}{3}$ b) $\frac{2}{3}(z_1+z_2+z_3)$ c) $z_1+z_3+z_3$ d) none of the these

Key. C

39. The complex no. associated with E is

a) $-\frac{z_2z_3}{z_1}$ b) $\frac{z_1z_2}{z_3}$ c) $\frac{z_2z_3}{z_1}$ d) none

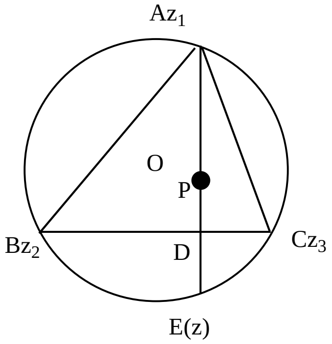
Key. A

Sol.37. Clearly P is orthocenter let Z= T

And BT $\perp$ BC

$\Rightarrow$ T is orthocenter of ΔABC is T=P

38. G= $\frac{2s+p}{3}$



P = $z_1 + z_2 + z_3$

39. Let E= z

$$AD \perp BC \Rightarrow \arg\left(\frac{z - z_1}{z_2 - z_3}\right) = \pm\frac{\pi}{2}$$

$$\frac{z - z_1}{z_2 - z_3} = \frac{\bar{z}-\bar{z}_1}{\bar{z}_2 - \bar{z}_3}$$

Paragraph – 14

A JEE aspirant estimates that she will be successful with an 80% chance if she studies 10 hours per day, with a 60% chance if she studies 7 hours per day and with a 40% chance if she studies 4 hours per day. She further believes that she will study10 hours, 7 hours and 4 hours per day with probabilities 0.1, 0.2 and 0.7 respectively

40. The probability that she will be successful is

a) 0.28 b) 0.38 c) 0.48 d) 0.58

Key. C

41. Given that she is successful, the probability she studied for 4 hours is

a) $\frac{6}{12}$ b) $\frac{7}{12}$ c) $\frac{8}{12}$ d) $\frac{9}{12}$

Key. B

42. Given that she does not achieve success the probability she studied for 4 hours is

a) $\frac{18}{26}$ b) $\frac{19}{26}$ c) $\frac{20}{26}$ d) $\frac{21}{26}$

Key. D

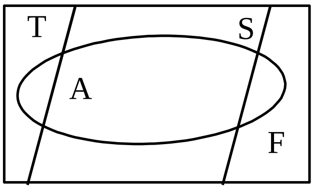
Sol. 40.41.42

A : She get a success

T : She studies 10 hrs : P (T) = 0.1

S : She studies 7 hrs : P (S) = 0.2

F : She studies 4 hrs : P (F) = 0.7



P(A/T)= 0.8; P(A/S)= 0.6; P(A/F)= 0.4

Now, P(A)= $P(A\cap T)+P(A\cap S)+P(A\cap F)$

= P(T) .P(A/T)+ P(S).P(A/S)+ P(F).P(A/F)

= (0.1)(0.8)+(0.2) (0.6) + (0.7)(0.4)

= 0.08+0.12 + 0.28 = 0.48

P(F/A) = $\frac{P(F\cap A)}{P(A)}=\frac{(0.7)(0.4)}{0.48}=\frac{0.28}{0.48}=\frac{7}{12}$

$P(F/\bar{A})=\frac{P(F\cap \bar{A})}{P(\bar{A})}=\frac{P(F)-P(F\cap A)}{0.52}$

$=\frac{(0.7)-0.28}{0.52}=\frac{0.42}{0.52}=\frac{21}{26}$

Paragraph – 15

Five balls are to be places in three boxes. Each can hold all the five balls. In how many different ways can be place the balls so that no box remains empty, if

43. Balls and boxes are all different

A) 50 B) 100 C) 125 D) 150

Key. D

44. Balls are identical but boxes are different

A) 2 B) 6 C) 4 D) 8

Key. B

45. Balls are different but boxes are identical

A) 25 B) 15 C) 10 D) 35

Key. A

Sol.

43. Number of ways $\left(3^5-3.\,2^5+3\right)=150$

44. $x_1+x_2+x_3=5$

$x_1, x_2, x_3\geq 1$

$t_1+t_2+t_3=2$

$t_1,t_2,t_3 \geq 0$

Number of ways $={}^4C_2=6$

45. Possibilities are $(3,1,1),\ (2,2,1)$

Number of ways $=\dfrac{{}^5C_2.\,{}^3C_2.\,{}^1C_1}{2!}+\dfrac{{}^5C_3.\,{}^2C_1.\,{}^1C_1}{2!}=(10+15)=25$

Paragraph – 16

Let $x_1\,x_2\,x_3\,x_4\,x_5\,x_6$ be a six digit number find the number of such numbers

46 $x_1<x_2<x_3<x_4<x_5<x_6$ is

A) ${}^9C_3$ B) ${}^{10}C_3$ C) ${}^{10}C_6$ D) ${}^9C_5$

Key. A

47. $x_1<x_2<x_3=x_4<x_5<x_6$ is

A) ${}^9C_3$ B) ${}^9C_4$ C) ${}^{10}C_3$ D) ${}^{10}C_4$

Key. B

48. $x_1<x_2<x_3\leq x_4<x_5<x_6$ is

A) ${}^9C_4$ B) ${}^9C_3$ C) ${}^{10}C_4$ D) ${}^{10}C_3$

Key. C

Sol.

46. $x_1<x_2<x_3<x_4<x_5<x_6 \rightarrow {}^9C_6$ ways

47. For $x_1<x_2<x_3=x_4<x_5<x_6={}^9C_4$ ways

48. For

$x_1<x_2<x_3 \leq x_4<x_5<x_6={}^9C_6+{}^9C_4+={}^9C_3+{}^9C_4={}^{10}C_4\left[\because {}^nC_{r-1}+{}^nC_r={}^{n+1}C_r\right]$

Paragraph – 17

A ten digit number is formed using the digits 0 to 9 every digit being used only once. In the same number.

49. The probability that the number is divisible by 4 is

(A) $\frac{20}{81}$ (B) $\frac{2}{27}$ (C) $\frac{14}{81}$ (D) $\frac{25}{27}$

Key. A

50. The probability that the number consists zero at unit or tenth place and is divisible by Four is

(A) $\frac{20}{81}$ (B) $\frac{2}{27}$ (C) $\frac{14}{81}$ (D) $\frac{25}{27}$

Key. B

51. The probability that the number contains zero at tens place and is divisible by 4 is

(A) $\frac{20}{81}$ (B) $\frac{4}{81}$ (C) $\frac{6}{81}$ (D) $\frac{2}{81}$

Key. D

Sol. $n(s) = 9\lfloor 9$

Divisible by $4 \Rightarrow$ last 2 places number is divisible by 4.

04, 08, 12, 16, 20, 24, 28, 32, 36, 40, 48, 52, 56, 60, 64, 68, 72, 76, 80, 84, 92, 96 = 22

and with these numbers $= 22 \times \lfloor 8$ in which starts with zero $= 16 \times \lfloor 7$

$$n(E) = \lfloor 7(160)$$

$$P(E) = \frac{20}{81}$$

having 0 in unit or ten place

$$n(E) = 6.\lfloor 8 \text{ and } P(E) = \frac{2}{27}$$

having zero in ten place

$$n(E) = 2\lfloor 8 \text{ and } P(E) = \frac{2}{81}$$

Paragraph – 18

There are 9 balls and 3 boxes, which can hold all the nine balls. A man wants to keep all these balls in the boxes and carry. So that no bag is empty.

52. If the balls are of the same colour and identical and boxes are also identical. The no. of ways he can arrange the balls.

(A) 12 (B) 7 (C) 28 (D) 55

Key. B

53. If the boxes are of different colour and the balls are identical and of same colour. The no.of ways in which he can arrange balls.

(A) 28 (B) 55 (C) 24 (D) 36

Key. A

54. If the balls are of different colour and the boxes are also different colour and are not identical. The no. of ways in which he can carry balls is

(A) 531441 (B) 729 (C) 6050 (D) 18150

Key. D

Sol. We can arrange is bags as (117), (123), (135) (144) (225) (234) (333) = 7

If boxes are different it will be $3.\frac{\lfloor 3}{\lfloor 2}+3.\lfloor 3+\frac{\lfloor 3}{\lfloor 3}=28$

If balls and boxes are different it is $3^9-{}^3C_1 2^9+{}^3C_2$.

Paragraph – 19

10-digit numbers are formed by using all the digits 0,1,2,3,4,5,6,7,8 and 9 such that they are divisible by 11111.

55. The digit in the ten's place, in the smallest of such numbers, is

a) 9 b) 8 c) 7 d) 6

Key. D

56. The digit in the unit's place, in the greatest of such numbers, is

a) 4 b) 3 c) 2 d) 1

Key. A

57. The total number of such numbers is

a) 6543 b) 5634 c) 3456 d) 4365

Key. C

Sol. Let a b c d e f g h i j be one of such numbers where a b c d e f g h i j is some permutation of the digits 0,1,2,3,4,5,6,7,8,9 where $a \neq 0$.

Sum of digits of the number = 0+1+2+3+4+5+6+7+8+9 = 45, which is divisible by 9 and hence the number is divisible by 9. But it is divisible by 11,111 also and 11,111 is not divisible by 3 or 9. Therefore, the number is divisible by $11{,}111\times 9=99{,}999$.

And a b c d e f g h i j = a b c d e $\times$ 10^5 + f g h i j

= a b c d e $\times$ (99,999+1) + f g h i j

= a b c d e $\times$ 99,999 + a b c d e + f g h i j is divisible by 99,999.

$\Rightarrow$ a b c d e + f g h i j is divisible by 99,999.

But a b c d e < 99,999

And f g h i j < 99,999

$\Rightarrow$ a b c d e + f g h i j < 2 $\times$ 99,999

$\therefore$ a b c d e + f g h i j = 99,999

$\Rightarrow$ e + j = d + i = c + h = b + g = a + f = 9

55.

a	b	c	d	e	f	g	h	i	j
1	0	2	3	4	8	9	7	6	5

For smallest number a must be 1 (since a can not be 0) and hence f = 8.

Then, b = 0 $\Rightarrow$ g = 9

Then, c = 2 $\Rightarrow$ h = 7

Then, d = 3 $\Rightarrow$ i = 6

Then, e = 4 $\Rightarrow$ j = 5

`

$\therefore$ The smallest of such numbers is 1023489765 and the digit in the ten's place is 6.

56.

a	b	c	d	e	f	g	h	i	j
9	8	7	6	5	0	1	2	3	4

For greatest number a = 9 $\Rightarrow$ f = 0

Then, b = 8 $\Rightarrow$ g = 1

Then, c = 7 $\Rightarrow$ h = 2

Then, d = 6 $\Rightarrow$ i = 3

Then, e = 5 $\Rightarrow$ j = 4

$\therefore$ The greatest of such numbers is 9876501234 and the digit in the units place is 4.

57.

a	b	c	d	e	f	g	h	i	j
9	8	6	4	2	1	1	1	1	1

The blank 'a' can be filled in 9 ways (except 0).

Then blank f can be filled in only one way (by 9-a).

Now, blank 'b' can be filled by any of the remaining 8 digits.

Then blank 'g' can be filled in only one way (by 9-b)

Now, blank 'c' can be filled by any of the remaining 6 digits.

Then blank 'h' can be filled in only one way (by 9-c).

Now, blank 'd' can be filled by any of the remaining 4 digits.

Then blank 'i' can be filled in only one way (by 9-d).

Now, blank 'e' can be filled by any of the remaining 2 digits.

Then blank 'j' can be filled in only one way (by 9-e).

$\therefore$ The total number of such numbers = $9 \times 1 \times 8 \times 1 \times 6 \times 1 \times 4 \times 1 \times 2 \times 1$

= 3456.

Paragraph – 20

D_1, D_2,-----,D_{1000} are 1000 doors and P_1, P_2,-------P_{1000} are 1000 persons. Initially all the doors are closed. P_1 opens all the doors. Then, P_2 closes D_2, D_4, D_6---- D_{998}, D_{1000}. Then P_3 changes the status of

D_3, D_6, D_9, D_{12},-----etc.(doors having numbers which are multiples of 3). Changing the status of a door means closing it if it is open and opening it if it is closed. Then P_4 changes the status of D_4, D_8, D_{12}, D_{16},-----etc (doors having numbers which are multiples of 4). And so on until lastly P_{1000} changes the status of D_{1000}.

58. Finally, how many doors are open?

a) 30 b) 31 c) 32 d) 33

Key. B

59. What is the greatest number of consecutive doors that are closed finally?

a) 56 b) 58 c) 60 d) 62

Key. C

60. The door having the greatest number that is finally open is

a) D_{960} b) D_{961} c) D_{962} d) D_{963}

Key. B

Sol. 58. Consider any door, for example, D_{72}. It is operated by $P_1,P_2,P_3,P_4,P_6,P_8,P_9,P_{12},P_{18},P_{24},P_{36},P_{72}$, (Remember that D_m is operated by P_n if m is a multiple of n)

Here 1,2,3,4,6,8,9,12,18,24,36,72 are all the factors of 72. Initially all the doors are closed. Therefore, if odd numbers of persons operate it, it will be finally open. Otherwise it will be closed finally.

$\therefore$ D_m will be finally open, if m has an odd number of factors. And, we know that m has an odd number of factors if and only if m is a perfect square.

$\therefore$ $1^2,2^2,3^2,4^2$,------, 31^2 are the numbers of the doors that are open finally.

$\therefore$ No. of doors finally open = 31.

59. $D_1,D_4,D_9,D_{16},D_{25}$,-----,$D_{900},D_{961}$ are the 31doors that are open finally.

$\therefore$ D_{901},D_{902},D_{903}, -----,D_{960} are the 60 consecutive doors that are closed and 60 is clearly greatest.

60. Ans: D_{961}

Paragraph – 21

Consider the expansion of $(a + b + c +d)^{10}$

61. The number of terms in the expansion is

a) ${}^{14}c_4$ b) ${}^{14}c_3$ c) ${}^{13}c_4$ d) ${}^{13}c_3$

Key. D

62. The greatest coefficient in the expansion is

a) 25200 b) 14400 c) 22500 d) 32400

Key. A

63. The number of terms in the expansion having the greatest coefficient is

a) 2 b) 4 c) 6 d) 8

Key. C

Sol. 61. No.of terms in the expansion of $(x_1+x_2+---+x_r)^n = {}^{n+r-1}c_n$

$\therefore$ No.of terms in $(a+b+c+d)^{10} = {}^{10+4-1}c_{10} = {}^{13}c_{10} = {}^{13}c_3$

62. Greatest coefficient = $\dfrac{10!}{2!2!3!3!} = 25200$

Greatest coefficient occurs in 6 terms. They are $a^2b^2c^3d^3$, $a^2b^3c^2d^3$, $a^2b^3c^3d^2$, $a^3b^2c^2d^3$, $a^3b^2c^3d^2$ and $a^3b^3c^2d^2$.

Paragraph – 22

If n is a positive integer, then $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$ where

$C_r = nC_r = \frac{\angle n}{\angle r \angle (n-r)}$. Answer the questions 15, 16 and 17.

64. The value of $nC_0 - nC_2 + nC_4 - nC_6 + ...$ is

a) $2^{\frac{n}{2}} \cos\frac{n\pi}{2}$ b) $2^{\frac{n}{2}} \sin\frac{n\pi}{2}$

c) $2^{\frac{n}{2}} \cos\frac{n\pi}{4}$ d) $2^{\frac{n}{2}} \sin\frac{n\pi}{4}$

Key. C

Sol. Conceptual

65. $(nC_0 - nC_2 + nC_4 - nC_6 + ...)^2 + (nC_1 - nC_3 + nC_5 - nC_7 + ...)^2$ is

a) 2^{2n} b) 2^n c) 2^{n^2} d) $2^{\frac{n+1}{2}}$

Key. B

Sol. put x = i in the expansion and equate real and imaginary parts.

66. $nC_0 + nC_4 + nC_8 + ...$ is equal to

a) $2^{\frac{n}{2}} \cos\frac{n\pi}{8}$ b) $2^{\frac{n}{2}} \sin\frac{n\pi}{8}$ c) $2^n + 2^{\frac{n}{2}} \cos\frac{n\pi}{4}$ d) $2^{n-2} + 2^{\frac{n-2}{2}} \cos\frac{n\pi}{4}$

Key. D

Sol. In problem 15, when we put x = i. Then $(nC_0 - nC_2 + nC_4 - nC_6) + i(nC_1 - nC_3 + nC_5 ...)$

$= 2^{\frac{n}{2}}\left[\cos\frac{n\pi}{4} + i\sin\frac{n\pi}{4}\right]$

Take modulus both sides and square it.

Paragraph – 23

ABCDEF is regular hexagon. Answer the questions 18, 19 and 20.

67. The number of triangles that can be made by using the vertices is :

a) 10 b) 15 c) 20 d) 30

Key. C

Sol. Conceptual

68. The number of equilateral triangles whose vertices are the vertices of the hexagon, but sides are not the sides of the hexagon is :

a) 6 b) 4 c) 3 d) 2

Key. D

Sol. Conceptual

69. The number of diagonals of the hexagons is :

a) 15 b) 12 c) 24 d) 6

Key. A

Sol. Conceptual

Paragraph – 24

We can find the exponent of a prime number $'p'$ in $n!$ by using the given formula

$E_p(n!) = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \ldots\ldots + \left[\frac{n}{p^s}\right]$ where $'s'$ is the largest natural number such that $p^s \le n < p^{s+1}$

70. $10! = 2^p.3^q.5^r.7^s$ then maximum among p, q, r, s is

A) p B) q C) r D) s

Key. A

Sol. Clearly 2 will come more times than any other

71. The exponent of 2 in $N = 20\times19\times18\times\ldots\ldots\times11.$ is

A) 10 B) 15 C) 20 D) 12

Key. A

Sol. $N = 20\times19\times18\times\ldots\ldots\times11 = \frac{20!}{10!}$

$\text{Exponent of 2 in } 20! = \left[\frac{20}{2}\right] + \left[\frac{20}{4}\right] + \left[\frac{20}{8}\right] + \left[\frac{20}{16}\right]$

$= 10+5+2+1 = 18$

$\text{Exponent of 2 in } 10! = \left[\frac{10}{2}\right] + \left[\frac{10}{4}\right] + \left[\frac{10}{8}\right]$

$= 5+2+1 = 8$

$\text{Exponent 2 in } \frac{20!}{10!} = 10$

72. The number of zeros at the end of 60!

A) 10 B) 12 C) 14 D) 16

Key. C

Sol. $$\text{Exponent of } 2 \text{ in } 60! = \left[\frac{60}{2}\right]+\left[\frac{60}{4}\right]+\left[\frac{60}{8}\right]+\left[\frac{60}{16}\right]+\left[\frac{60}{16}\right]+\left[\frac{60}{32}\right]$$

$$= 30+15+7+3+1=56$$

$$\text{Exponent of } 5 \text{ in } 60! = \left[\frac{60}{5}\right]+\left[\frac{60}{25}\right]=12+2=14$$

Hence number of zeros at the end of $60\,! = 14$

Paragraph – 25

There are three pots and four coins. All these coins are to be distributed into these pots where any pot can contain any number of coins.

73. The number of ways all these coins can be distributed if all coins are identical but all pots are different is

A) 15 B) 16 C) 17 D) 81

Key. A

Sol. Required number of ways = ${}^{4+3-1}C_{3-1}$

74. The number of ways all these coins can be distributed if all coins are different but all pots are identical is

A) 14 B) 21 C) 27 D) None of these

Key. A

Sol. Since pots are identical, these will be 4 cases (4,0,0), (3,1,0), (2,2,0) and (2,1,1) but since all coins are different hence selection of coins matters.

Therefore, the first case number of selections = ${}^4C_4=1$

For the second case number of selections

$${}^4C_3 \times {}^2C_1 = 4$$

For the third case number of selections

$$= \frac{{}^4C_2 \times {}^2C_2}{2!} = 3$$

For the fourth case number of selections

$$= \frac{{}^4C_2 \times {}^2C_1 \times {}^1C_1}{2!} = 6$$

Hence, the total number of distributions = 14

Paragraph – 26

A triangle is called an integer triangle if all the sides are integers. If a, b, c are sides of a triangle then we can assume $a \le b \le c$ (any other permutation will yield same triangle).

Since sum of two sides is greater than the third side therefore if c is fixed $a+b$ will vary from $c+1$ to $2c$. The number of such integer triangles can be found by finding integer solutions of $a+b=c+1, a+b=c+2, ..., a+b=2c.$

75. The number of integer isosceles or equilateral triangle none of whose sides exceed 4 must be

A) 9 B) 10 C) 11 D) 12

Key. D

76. The number of integer isosceles or equilateral triangles none of whose sides exceed 2c must be

A) c^2 B) $2c^2$ C) $3c^2$ D) $\frac{3c^2}{2}$

Key. C

77. If $'c'$ is fixed and odd, the number of integer isosceles or equilateral triangle whose sides are $a, b, c, a \le b \le c$ must be

A) $\frac{2c-1}{2}$ B) $\frac{2c+1}{2}$ C) $\frac{3c+1}{2}$ D) $\frac{3c-1}{2}$

Key. D

Sol.

75. If the equal side is unity and x be the value of the third side then $x<2$ must be satisfied (sum of two sides must be greater than the third side)

Now $x<2$ has only solution namely $x=1$.

If equal side is 2 and $'x'$ be the value of the third side then $x<4$ which has 3 solutions $x=1,2,3$.

If equal side is 3 then $x<6$ has 4 solutions $x=1,2,3,4$. (largest value of side is 4)

Finally if equal side is 4 then $x<8$ has again 4 solutions.

$\Rightarrow$Total triangles $=1+3+4+4=12$

76. The value of equal side can vary from 1 to $2c$

The number of triangles can be found by finding integer solutions of

$x<2, x<4, x<6, ...x<2c,$

$x<2c+2, ..., x<4c$

$\Rightarrow$Total number of solutions

$=1+3+5+......+(2c-1)+(2c)+(2c)+(2c)+.....+(2c)$

Inequation $x<2c$ each inequation will have $2c$ solutions only $=c^2+2c^2=3c^2$

77. If $c=3$ then triangle having 3 as largest sides and which are isosceles or equilateral are $3,3,1;\ 3,3,2;\ 3,3,3;\ 3,2,2$

$\Rightarrow$ Choices (a), (b), (c) are easily riled out.

Paragraph – 27

Given are six 0`s, five 1`s and four 2`s. consider all possible permutations of all these numbers. [A permutation can have its leading digit 0].

78. How many permutations have the first 0 preceding the first 1?

A) ${}^{15}C_4 \times {}^{10}C_5$ B) ${}^{15}C_5 \times {}^{10}C_4$ C) ${}^{15}C_6 \times {}^{10}C_5$ D) ${}^{15}C_4 \times {}^{9}C_4$

Key. A

79. In how many permutations does the first 0 preceed the first 1 and the first 1 preceed first 2.

A) ${}^{14}C_5 \times {}^{8}C_6$ B) ${}^{14}C_5 \times {}^{8}C_4$ C) ${}^{14}C_6 \times {}^{8}C_4$ D) ${}^{12}C_5 \times {}^{7}C_4$

Key. B

80. The no. of permutations in which all 2`s are together but no two of the zeroes are together is :

A) 42 B) 40 C) 84 D) 80

Key. A

Sol.

78. The number of ways of arranging 2`s is ${}^{15}C_4$.
Fill the first empty position left after arranging the 2`s with a 0(1 way) and pick the remaining five places the position the remaining five zeros $\rightarrow {}^{10}C_5$ ways.
$\therefore {}^{15}C_4 \times 1 \times {}^{10}C_5$

79. Put 0 in the first position, (1 way). Pick five other positions for the remaining 0`s ( ${}^{14}C_5$ ways), put a 1 in the first of the remaining positions ( 1 way), then arrange the remaining four 1`s (${}^{8}C_4$ ways)
$\therefore {}^{14}C_5 \times {}^{8}C_4$

80. Conceptual

Paragraph – 28

There are n intermediate stations on a railway line from one terminal to another. In how many ways can the train stop at 3 of these intermediate stations, if

81. All the three stations are consecutive

A) $(n+2)$ B) $(n+1)$ C) $(n-1)$ D) $(n-2)$

Key. D

82. Atleast two of the stations are consecutive

A) $(n+2)(n-1)$ B) $(n-2)(n-1)$ C) $(n-2)^2$ D) None of these

Key. C

83. No two of these stations are consecutive

A) ${}^nC_3$ B) ${}^{(n-2)}C_3$ C) $\dfrac{(n-2)(n-3)}{6}$ D) None of these

Key. B

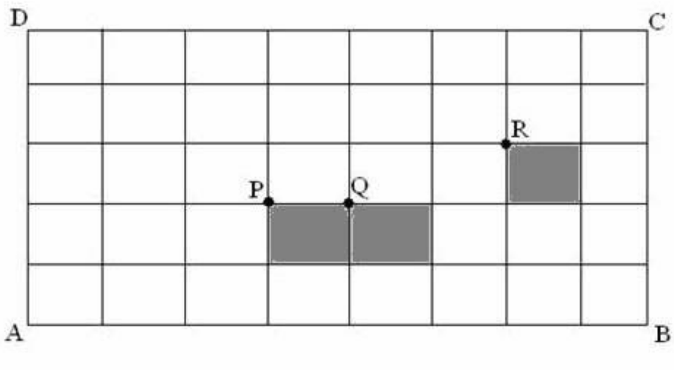
Sol. 81. $(S_1,S_2,S_3),(S_2,S_3,S_4)\ldots\ldots(S_{n-2},S_{n-1},S_n)=(n-2)$

82. $(n-2)_{\text{ways}}\ (n-1)_{\text{ways}}\ -(n-2)=(n-2)^2$

83. ${}^nC_3-(n-2)^2={}^{(n-2)}C_3$

Paragraph – 29

Consider the network of equally spaced parallel lines (6 horizontal and 9 vertical) shown in the figure. All small squares are of the same size. A shortest route from A to C is defined as a route consisting 8 horizontal steps and 5 vertical steps. Since any shortest route is a typical arrangement of 8H and 5V. The number of shortest routes $=\dfrac{13!}{5!8!}$. Answer the following questions:



84. The number of shortest routes through the junction P

A) 240 B) 216 C) 560 D) None of these

Key. C

85. The number of shortest routes which go following street PQ must be

A) 324 B) 350 C) 512 D) None of these

Key. B

86. The number of shortest routes which pass through junctions P and R

A) 144 B) 240 C) 216 D) None of these

Key. B

Sol. 84. Number of shortest routes through P

= (Number of shortest routes from A to P) (Number of shortest routes from P to C0

$$= \frac{5!}{3!2!} \times \frac{8!}{3!5!} = 560$$

85. Number of shortest routes $= \frac{5!}{3!2!} \times 1 \times \frac{7!}{3!4!} = 350$

86. $\frac{5!}{3!2!} \times \frac{4!}{3!1!} \times \frac{4!}{2!2!} = 240$

Paragraph – 30

10 digit numbers are formed by using all the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 such that they are divisible by 11111.

87. The digit in the tens place, in the smallest of such numbers, is

A) 9 B) 8 C) 7 D) 6

Key. D

88. The digit in the units place, in the greatest of such numbers, is

A) 4 B) 3 C) 2 D)1

Key. A

89. The total number of such numbers is

A) 6543 B) 5634 C) 3456 D) 4365

Key. C

Sol. 87. Let a b c d e f g h i j be one of such numbers where a b c d e f g h i j is some permutation of the digits 0,1,2,3,4,5,6,7,8,9 where $a \neq 0$.

Sum of digits of the number $= 0+1+2+3+4+5+6+7+8+9 = 45$, which is divisible by 9 and hence the number is divisible by 9. But it is divisible by 11, 111 also and 11, 111 is not divisible by 3 or 9. Therefore, the number is divisible by $11{,}111 \times 9 = 99{,}999$.

And $abcdefghij = abcde \times 10^5 + fghij$

$= abcde \times (99{,}999 + 1) + fghij$

$= abcde \times 99{,}999 + abcde + fghij$ is divisible by $99{,}999$.

$\Rightarrow abcde + fghij$ is divisible by $99{,}999$.

But $abcde < 99{,}999$

And $fghij < 99{,}999$

$\Rightarrow abcde + fghij < 2 \times 99{,}999$

$\therefore abcde + fghij = 99{,}999$

$\Rightarrow e + j = d + i = c + h = b + g = a + f = 9$

a	b	c	d	e	f	g	h	i	j
1	0	2	3	4	8	9	7	6	5

For smallest number a must be 1 (since a can not be 0) and hence $f = 8$.

Then, $b = 0 \Rightarrow g = 9$

Then, $c = 2 \Rightarrow h = 7$

Then, $d = 3 \Rightarrow i = 6$

Then, $e = 4 \Rightarrow j = 5$

$\therefore$ The smallest of such numbers is 1023489765 and the digit in the tens place is 6.

88.

a	b	c	d	e	f	g	h	i	j
9	8	7	6	5	0	1	2	3	4

For greatest number $a = 9 \Rightarrow f = 0$

Then, $b = 8 \Rightarrow g = 1$

Then, $c = 7 \Rightarrow h = 2$

Then, $d = 6 \Rightarrow i = 3$

Then, $e = 5 \Rightarrow j = 4$

$\therefore$ The greatest of such numbers is 9876501234 and the digit in the units place is 4.

89.

a	b	c	d	e	f	g	h	i	j
9	8	6	4	2	1	1	1	1	1

The blank a can be filled in 9 ways (except 0).

Then blank f can be filled in only one way (by $9-a$).

Now, blank b can be filled by any of the remaining 8 digits.

Then blank g can be filled by only one way (by $9-b$).

Now, blank c can be filled by any of the remaining 6 digits.

Then blank h can be filled in only one way (by $9-c$).

Now, blank d can be filled by any of the remaining 4 digits.

The blank i can be filled in only one way (by $9-d$).

Now, blank e can be filled by any of the remaining 2 digits.

Then blank j can be filled in only one way (by $9-e$).

$\therefore$ The total number of such numbers $=9\times1\times8\times1\times6\times1\times4\times1\times2\times1=3456$

Paragraph – 31

$D_1, D_2, \ldots\ldots, D_{1000}$ are 1000 doors and $P_1, P_2, \ldots. P_{1000}$ are 1000 persons. Initially all the doors are closed. P_1 opens all the doors. Then, P_2 closes $D_2, D_4, D_6, \ldots\ldots D_{998}, D_{1000}$. Then P_3 changes the status of $D_3, D_6, D_9, D_{12}, \ldots.$ etc. (doors having numbers which are multiples of 3). Changing the status of a door means closing it if it is open and opening it if it is closed. Then P_4 changes the status of $D_4, D_8, D_{12}, D_{16}, \ldots.$ etc (doors having numbers which are multiples of 4). And so on until lastly P_{1000} changes the status of D_{1000}.

90. Finally, how many doors are open?

A) 30 B) 31 C) 32 D) 33

Key. B

91. What is the greatest number of consecutive doors that are closed finally?

A) 56 B) 58 C) 60 D) 62

Key. C

92. The door having the greatest number that is finally open is

A) D_{960} B) D_{961} C) D_{962} D) D_{963}

Key. B

Sol. 90. Consider any door, for example, D_{72} it is operated by $P_1, P_2, P_3, P_4, P_6, P_8, P_9, P_{12}, P_{18}, P_{24}, P_{36}, P_{72},$ (Remaining that D_m is operated by P_n if m is a multiple of n)

Here 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72 are all the factors of 72. Initially all the doors are closed. Therefore, if odd numbers of persons operate it, it will be finally open. Otherwise it will be closed finally.

$\therefore$ D_m will be finally open, if m has an odd numbers of factors. And, we know that m has an odd number of factors if and only if m is a perfect square.

$\therefore$ $1^2, 2^2, 3^2, 4^2, \ldots\ldots 31^2$ are the numbers of the doors that are open finally.

$\therefore$ No.of doors finally open = 31.

91. $D_1, D_4, D_9, D_{16}, D_{25}, \ldots\ldots, D_{900}, D_{961}$ are the 31 doors that are open finally.

$\therefore$ $D_{901}, D_{902}, D_{903}, \ldots\ldots, D_{960}$ are the 60 consecutive doors that are closed and 60 is clearly greatest.

92. D_{961}

Paragraph – 32

A box contains n coins. Let $P(E_i)$ be the probability that exactly 'i' out of n coins are biased. If $P(E_i)$ is directly proportional to $i(i+1)$; $1 \le i \le n$

93. Proportionality constant K is equal to

A) $\frac{3}{n(n^2+1)}$ B) $\frac{1}{(n^2+1)(n+2)}$ C) $\frac{3}{n(n+1)(n+2)}$ D) $\frac{1}{(n+1)(n+2)(n+3)}$

Key. C

Sol. $P(E_i) = Ki(i+1)$

$$\because P(E_1)+P(E_2)+\ldots\ldots+P(E_n)=1 \Rightarrow K\sum_{i=1}^{n} i(i+1)=1 \Rightarrow$$

$$K\left[\frac{1}{6}n(n+1)(2n+1)+\frac{n(n+1)}{2}\right]=1$$

$$\therefore K=\frac{3}{n(n+1)(n+2)}$$

94. If P be the probability that a coin selected at random is biased then $\lim_{n\to\infty} P$ is

A) $\frac{1}{4}$ B) $\frac{3}{4}$ C) $\frac{3}{5}$ D) $\frac{7}{8}$

Key. B

Sol. $$P(E)=\sum_{i=1}^{n} P(E_i).P(E/E_i)=\sum_{i=1}^{n} Ki(i+1).\frac{i}{n}$$

$$= \frac{k}{n}\sum_{i=1}^{n}\left(i^3+i^2\right) = \frac{K}{n}\left[\left(\frac{n(n+1)}{2}\right)^2 + \frac{1}{6}n(n+1)(2n+1)\right] = \frac{(3n+1)(n+2)}{4n(n+2)}$$

$$\therefore \lim_{n\to\infty} P = \frac{3}{4}$$

95. If a coin selected at random is found to be biased then the probability that it is the only biased coin in the box is

A) $\frac{1}{(n+1)(n+2)(n+3)(n+4)}$ B) $\frac{12}{n(n+1)(n+2)(3n+1)}$

C) $\frac{24}{n(n+1)(n+2)(2n+1)}$ D) $\frac{24}{n(n+1)(n+2)(3n+1)}$

Key. D

Sol.
$$P(E_1/E) = \frac{P(E_1)P(E/E_1)}{P(E)} = \frac{K\times 2\times\frac{1}{n}}{\frac{(3n+1)(n+2)}{4n(n+2)}} = \frac{24}{n(n+1)(n+2)(3n+1)}$$

Paragraph – 33

The sides of a triangle a,b,c be positive integers and given $a\le b\le c$. If c is given, then

96. The number of triangles that can be formed when c is odd, is

A) $\frac{(c+1)^2}{4}$ B) $\frac{(3c-1)}{2}$ C) $\frac{c(c+2)}{4}$ D) $\frac{(3c-2)}{2}$

Key. A

Sol. Let $c = 2m+1$

$\therefore$ Number of triangles $= (2m+1)+(2m-1)+\ldots+1 = (m+1)^2 = \frac{(c+1)^2}{4}$

97. The number of triangles that can be formed when c is even, is

A) $\frac{(c+1)^2}{4}$ B) $\frac{(3c-1)}{2}$ C) $\frac{c(c+2)}{4}$ D) $\frac{(3c-2)}{2}$

Key. C

Sol. Let $c = 2m$

$\therefore$ Number of triangles $= (2m)+(2m-2)+.....+2 = m(m+1) = \frac{c(c+2)}{4}$

98. The number of isosceles or equilateral triangles than can be formed when c is odd, is

A) $\frac{(c+1)^2}{4}$ B) $\frac{(3c-1)}{2}$ C) $\frac{c(c+2)}{4}$ D) $\frac{(3c-2)}{2}$

Key. B

Sol. Let $c = 2m+1$

$\therefore$ Number of isosceles or equilateral triangles

$= (2m+1)+1+1+.....+1 = 3m+1 = \frac{3c-1}{2}$

Paragraph – 34

A slip of paper is given to a person A who marks it either with a plus sign or a minus sign. The probability of his writing a plus sign is 1/3. A passes the slip to B, who may either leave it alone or change the sign before passing it to C. Next C passes the slip to D after perhaps changing the sign. Finally D passes it to a referee after perhaps changing the sign. B, C, D each change the sign with probability 2/3.

99. The probability that the referee observes a plus sign on the slip if it is known that A wrote a plus sign is

A) 14/27 B) 16/27 C) 13/27 D) 17/27

Key. C

100. The probability that the referee observes a plus sign on the slip if it is known that A wrote a minus sign is

A) 16/27 B) 14/27 C) 13/27 D) 11/27

Key. B

101. If the referee observes a plus sign on the slip then the probability that A originally wrote a plus sign is

A) 13/41 B) 19/27 C) 17/25 D) 21/37

Key. A

Sol. **: (**58, 59, 60)

Let $E_1 =$ Event that A wrote a plus sign .

$E_2 =$ Event that A wrote a minus sign .

$E =$ Event that the referce observes a plus sign.

Given $P(E_1) = \frac{1}{3} \Rightarrow P(E_2) = \frac{2}{3}$

$P(E/E_1) =$ Probability that none of B,C,D change sign+Probability that exactly two of B,C,D Change sign.

$= \frac{1}{27} + 3\left(\frac{1}{3} \times \frac{2}{3} \times \frac{2}{3}\right) = \frac{13}{27}$

$P(E/E_2) =$ Probability that all of B,C,D change the sign+Probability that exactly one of them changes the sign.

$= \frac{8}{27} + 3 \times \left(\frac{2}{3} \times \frac{1}{3} \times \frac{1}{3}\right) = \frac{14}{27}$

$\therefore P(E_1/E) = \frac{13}{41}$

Using Baye's theorem.

Paragraph – 35

Let $x_1, x_2, x_3, ..., x_n \in R^+$ and A, G respectively be the AM and GM of these 'n' numbers. Then $A \geq G$ and the equality holds only when $x_1 = x_2 = ... = x_n$.

102. If $x, y \in R^+$ and $x^3 y^2 = 6$ then the least value of $4x + 3y$ is

A) $\sqrt[3]{24}$ B) 10 C) $\sqrt[3]{32}$ D) 12

Key. B

103. If $x > -1$ then the least value of $\frac{x^2 + 7x + 10}{x + 1}$ is

A) $\sqrt{12}$ B) 10 C) $\sqrt{18}$ D) 9

Key. D

104. If $x, y, z \in R^+$ and $x^2 + y^2 + z^2 = 1$ then the maximum value of $x^2 y^3 z^4$ is

A) $2^6.3^{-9/2}$ B) $2^4.3^{-7}$ C) $2^5.3^{-15/2}$ D) $2^4.3^{-8}$

Key. C

Sol. 102. Apply $A.M \geq G.M$ for the 5 numbers $\frac{4x}{3}, \frac{4x}{3}, \frac{4x}{3}, \frac{3y}{2}, \frac{3y}{2}$

103. Put $x+1=y, y>0$

then $\frac{(x+5)(x+2)}{x+1} = 5+y+\frac{4}{y} \geq 5+4$

104. Let $P = x^2y^3z^4$, then $P^2 = (x^2)^2(y^2)^3(z^2)^4$

Paragraph – 36

An urn contains 4 white and 9 black balls. r balls are drawn with replacement. Let P_r be the probability that no two white balls appear in succession. Answer the following questions.

105. The Value of P_4 must be

A) $\frac{81}{(169)^2}$ B) $\frac{81\times 273}{(169)^2}$ C) $1-\left(\frac{4}{13}\right)^4$ D) $\left(\frac{4}{13}\right)^4$

Key. B

106. The recursion relation for P_r must be

A) $P_r = P_{(r-2)}\frac{9}{13}\times\frac{4}{13} + P_{(r-1)}\times\frac{9}{13}$ B) $P_r = P_{(r-2)}\left(\frac{9}{13}\right)^2 + P_{(r-1)}\left(\frac{4}{13}\right)$

C) $P_r = P_{(r-2)}\left(\frac{4}{13}\right)^2 + P_{(r-1)}\left(\frac{9}{13}\right)$ D) $P_r = P_{(r-2)}\left(\frac{9}{13}\right)^3 + P_{(r-1)}\left(\frac{4}{13}\right)$

Key. A

107. P_r Must be equal to

A) $\frac{16\times 12^r - (-3)^r}{12\times 13^r}$ B) $\frac{16\times 12^r - (-3)^r}{9^r \times 13^r}$

C) $\frac{16\times 12^r - (-3)^r}{15\times 13^r}$ D) $\frac{16\times 12^r - (-3)^r}{9^r \times 15^r}$

Key. C

Sol. 105. At any trial chance of getting a white ball $=\frac{4}{13}$

Chance of getting a black ball $=\frac{9}{13}$

$P_4 = P$ (When 4 balls are drawn no 2 white balls appear in succession)

$= P(\text{This occurs with no white balls}) + P(\text{This occurs with one white ball}) +$

$P(\text{This occurs with two white balls})$

$= \left(\frac{9}{13}\right)^4 + 4\left(\frac{4}{13}\right)\left(\frac{9}{13}\right)^3 + 3\left(\frac{4}{13}\right)^2\left(\frac{9}{13}\right)^2 = \frac{81\times 273}{13^4}$

106. (1) No two white balls appear in succession up to (r-2) draws and then Black, White occur

(2) In the second way, no two white balls appear in succession up to (r-1) draws and then B occurs.

$$P_r = P_{r-2} \times \frac{9}{13} \times \frac{4}{13} + P_{r-1} \times \frac{9}{13}$$

107. For r =2

$$P_2 = \frac{4}{13} \times \frac{9}{13} \times 2 + \left(\frac{9}{13}\right)^2 = \frac{153}{169}$$

Choice (C) also becomes $\frac{153}{169}$.

If n distinct objects are distributed randomly into n distinct boxes, what is the probability that

Paragraph – 37

If n distinct objects are distributed randomly into n distinct boxes, what is the probability that

108. No box is empty

A) $\frac{\lfloor \underline{n-1}}{n^n}$ B) $\frac{\lfloor \underline{n-1}}{n^n \lfloor \underline{2}}$ C) $\frac{\lfloor \underline{n}}{n^n}$ D) $\frac{\lfloor \underline{n-1} \lfloor \underline{2}}{n^n}$

Key. C

109. Exactly one box empty

A) $\frac{{}^nC_2 \lfloor \underline{n}}{n^{n-1}}$ B) $\frac{{}^nC_2 \lfloor \underline{n}}{n^n}$ C) $\frac{n \lfloor \underline{n}}{n^n}$ D) $\frac{\lfloor \underline{n}}{n^n}$

Key. B

110. A Particular box get exactly r objects

A) $\frac{{}^nC_r (n-1)^{n-r-1}}{n^n}$ B) $\frac{{}^nC_r (n-1)^{n-r+1}}{n^n}$ C) $\frac{{}^nC_r (n-1)^{n-r}}{n^{n-1}}$ D) $\frac{{}^nC_r (n-1)^{n-r}}{n^n}$

Key. D

Sol. 108. Since no box is empty,

Number of favorable ways = $\lfloor \underline{n}$

109. Since exactly one box empty,

Number of favorable ways $= {}^nC_1 . {}^{n-1}C_1 . {}^nC_2 . \lfloor \underline{n-2}$

110. A Particular box get exactly r objects $= {}^nC_r (n-1)^{n-r}$.

Paragraph – 38

Consider the sets $A = \{1, 2, 3\}$ $B = \{1, 2, 3, 4, 5, 6\}$. A function is selected at random from the set of all functions from A to B:

111. The probability that the selected function is such that $f(i) < f(j)$ whenever $i < j$ is

A) $\frac{1}{27}$ B) $\frac{2}{27}$

C) $\frac{5}{54}$ D) $\frac{7}{54}$

Key. C

112. The probability of selecting a function satisfying $f(i) \le f(j)$ whenever $i < j$ is

A) $\frac{7}{27}$ B) $\frac{5}{27}$

C) $\frac{2}{27}$ D) $\frac{8}{27}$

Key. A

113. The probability of selecting a one-one function from B to B such that $f(i) \ne i, i = 1,2,3,4,5,6$ is

A) $\frac{25}{72}$ B) $\frac{29}{72}$

C) $\frac{55}{144}$ D) $\frac{35}{144}$

Key. C

Sol. 111. $n(S) = 6^3$, $n(E) = {}^6C_3$ $\Rightarrow P(E) = \frac{20}{216} = \frac{5}{54}$

112. $f(1) < f(2) < f(3) \to {}^6C_3$

$f(1) < f(2) = f(3) \to {}^6C_2$

$f(1) = f(2) < f(3) \to {}^6C_2$

$f(1) = f(2) = f(3) \to {}^6C_1$

$\therefore P(E) = \frac{56}{216} = \frac{7}{27}$

113. n(E) = no. of derangements $= 6!\left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} - \frac{1}{6!}\right]$

$P(E) = \frac{275}{720} = \frac{55}{144}$

Paragraph – 39

A is a set containing 10 elements. A subset P of A is chosen at random and the set A is reconstructed by replacing the elements of A. Another subset Q of A is now chosen at random.

114. The probability that $P \cap Q = \phi$ is

A) $\left(\frac{2}{3}\right)^{10}$ B) $\left(\frac{3}{4}\right)^{10}$

C) $\left(\frac{4}{5}\right)^{10}$ D) $\left(\frac{5}{6}\right)^{10}$

Key. B

115. The probability that $P \cap Q$ contains exactly 3 elements is

A) $\frac{5\times 3^8}{2^{17}}$ B) $\frac{15\times 3^9}{2^{17}}$

C) $\frac{3\times 5^8}{2^{17}}$ D) $\frac{7\times 3^8}{2^{17}}$

Key. A

116. The probability that $P \cup Q = A$ is

A) $\left(\frac{1}{3}\right)^{10}$ B) $\left(\frac{2}{3}\right)^{10}$

C) $\left(\frac{3}{4}\right)^{10}$ D) $\left(\frac{2}{5}\right)^{10}$

Key. C

Sol. 114. $n(E) = 3^{10}, n(S) = 4^{10}$

$$\therefore P(E) = \frac{3^{10}}{4^{10}} \quad \therefore For\ any\ element\ x, \begin{array}{l} x \in P, x \notin Q \\ x \notin P, x \in Q \\ x \notin P, x \notin Q \\ and\ x \in P,\ x \in Q \end{array}$$

115. $n(E) = {}^{10}C_3 \times 3^7 \qquad \therefore P(E) = \frac{{}^{10}C_3 \times 3^7}{4^{10}}$

116. $n(E) = 3^{10}, \qquad P(E) = \frac{3^{10}}{4^{10}} (\because 4th\ case\ is\ not\ possible)$

Paragraph – 40

A is a set containing 10 elements .A subset P of A is chosen at random and the set A is then reconstructed by replacing elements of P. A subset Q of A is again chosen at random, then (n(P)=number of elements in P)

117. The probability that n(P)=n(Q) is

a) $\frac{20_{C_{10}}}{2^{10}}$ b) $\frac{20_{C_{10}}}{2^{20}}$ c) $\frac{10_{C_5}}{2^{10}}$ d) $\frac{20_{C_{10}}}{2^{40}}$

Key. B

118. The probability that n(P)>n(Q) is

a) $\frac{1}{2}+\frac{20_{C_{10}}}{2^{21}}$ b) $\frac{1}{4}-\frac{20_{C_{10}}}{2^{21}}$ c) $\frac{1}{2}-\frac{20_{C_{10}}}{2^{21}}$ d) $\frac{1}{4}+\frac{20_{C_{10}}}{2^{21}}$

Key. C

119. The probability that $Q \subset P$ is

a) $\left(\frac{1}{4}\right)^{10}$ b) $\left(\frac{1}{2}\right)^{10}$ c) $\frac{1}{2}$ d) $\left(\frac{3}{4}\right)^{10}$

Key. D

Sol. (117-119)

n (P)=n(Q) $\Rightarrow$ P,Q have same number of elements, say r $(0 \le r \le 10)$

$\therefore$ There are 10_{C_r} ways each of forming P, Q, number of ways of forming P,Q is

$\sum_{r=0}^{10}(10_{C_r})(10_{C_r}) = \sum_{r=0}^{10}(10_{C_r})^2 = 20_{C_{10}}$

$\therefore P\left(n(P)=n(Q)\right)=\frac{20c_{10}}{4^{10}}$ (For any $x \in A \Longrightarrow x \in P, x \in Q$ or $x \in P$,$x \notin Q$ or $x \notin P$,$x \in Q$ or $x \notin P$,$x \notin Q$) For P(n(P)>n(Q)), number of ways of forming P ,Q ,both = $\sum_{r>s}\left(10_{C_r}\right)\left(10_{C_s}\right)$

$$\equiv \frac{\left(\sum_{r=0}^{10}10_{C_r}\right)^2-\sum_{r=0}^{10}\left(10_{C_r}\right)^2}{2}$$

$$= \frac{2^{20}-20c_{10}}{2}$$

$\therefore$ P(n(P)>n(Q))= $\frac{2^{20}-20c_{10}}{2(4^{10})} = \frac{1}{2} - \frac{20c_{10}}{2^{21}}$.

For $Q \subset P$, number of ways of choosing $x \in A$ such that $Q \subset P$ is 3 out of 4 choices,

hence $P(Q \subset P) \equiv \left(\frac{3}{4}\right)^{10}$

Paragraph – 41

Of three independent events A, B, C the chance that the only A occurs is a that only B occurs is b and only the third occurs is c.If probability that none of them occurs is x then

120. P (C)=

a) $\frac{x}{c+x}$ b) $\frac{c}{x+c}$ c) $\frac{1}{c+x}$ d) $\frac{xc}{x+c}$

Key. B

121. x is a root of the equation

a) $x^3 = (a+x)(b+x)(c+x)$ b) $(a+x)(b+x)(c+x) = x$

c) $(a+x)(b+x)(c+x) = 1$ d) $(a+x)(b+x)(c+x) = x^2$

Key. D

122. Probability that exactly two of the events A, B, C occurs is—

a) $\frac{x^2}{ab+bc+ca}$ b) $(ab+bc+ca)x$ c) $\frac{ab+bc+ca}{x^2}$ d) $\frac{ab+bc+ca}{x}$

Key. D

Sol. 120-122)

Given P(A $\cap B^1 \cap C^1$)=a, $P(A^1 \cap B \cap C^1) = b$, $P(A^1 \cap B^1 \cap C) = c$ and $P(A^1 \cap B^1 \cap C^1) = x$

$$\frac{P(A^1 \cap B^1 \cap C)}{P(A^1 \cap B^1 \cap C^1)} = \frac{c}{x}$$

$\frac{P(C)}{1-P(C)} = \frac{c}{x}$ => P(C)= $\frac{c}{c+x}$

Also P(A)= $\frac{a}{a+x}$, P(B)= $\frac{b}{b+x}$

$\therefore$ abc=P(A)P(B)P(C)(x^2)= $\frac{abc(x^2)}{(a+x)(b+x)(c+x)} \Rightarrow (a+x)(b+x)(c+x) = x^2$

P(exactly two of A,B,C occur) = $P(A \cap B) + P(B \cap C) + P(A \cap C) - 3P(A \cap B \cap C)$

$$= \frac{ab}{(a+x)(b+x)} + \frac{bc}{(b+x)(c+x)} + \frac{ac}{(a+x)(c+x)} - \frac{3abc}{(a+x)(b+x)(c+x)}$$

$$= \frac{x(ab+bc+ca)}{x^2} = \frac{ab+bc+ca}{x}$$

Paragraph – 42

By definition, every permutation of 'r' out of 'n' distinct objects can be treated as a work done in 2 stages .We first choose ' r' out of 'n' objects in n_{C_r} ways and then arrange chosen' r' objects in r! ways giving n_{C_r} $\times$ r!= n_{P_r} .

Use this idea in answering the following questions

123. Total number of ways of selecting 5 letters from the letters of the word INDEPENDENT is

a) 54 b) 44 c) 34 d) 24

Key. B

124. Number of ways of arranging 5 letters from the letters of INDEPENDENT is

a) 3610 b) 4160 c) 6310 d) 3160

Key. D

125. If a five lettered word is formed from the letters of the word INDEPENDENT, probability that it contains 3 alike letters is

a) $\frac{7}{89}$ b) $\frac{7}{69}$ c) $\frac{7}{59}$ d) $\frac{7}{79}$

Key. D

Sol. Conceptual

Paragraph – 43

If *n* positive integers taken at random & multiplied together, then the chance that the last digit of the product would be:

126. 1, 3, 5, 7 or 9 is

A) $\left(\frac{2}{5}\right)^n$ B) $\left(\frac{1}{2}\right)^n$ C) $\frac{2^n-1}{5^n}$ D) $\frac{5^n-4^n}{10^n}$

Key. B

127. 1, 3, 7 or 9 is

A) $\frac{2^n-1}{5^n}$ B) $\frac{5^n-4^n}{10^n}$ C) $\left(\frac{2}{5}\right)^n$ D) $\left(\frac{1}{2}\right)^n$

Key. C

128. 5 is

A) $\frac{5^n-4^n}{10^n}$ B) $\frac{2^n-1}{5^n}$ C) $\frac{10^n-8^n-5^n+4^n}{10^n}$ D) $\left(\frac{2}{5}\right)^n$

Key. A

Sol. Let *n* positive integers be $x_1, x_2, x_3....x_n$

Let $a = x_1.x_2.x_3....x_n$

Let S be the sample space since the last digit in each of the numbers $x_1, x_2, x_3....x_n$ can be any one of the digits 0, 1, 2, 3, ..., 9 (total 10)

$n(S) = (10)^n$

Let E_1 and E_2 be the events when the last digit in a is 1, 3, 5, 7 or 9 and 1, 3, 7 or 9 respectively.

$n(E_1) = 5^n, n(E_2) = 4^n$

126. $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{5^n}{10^n} = \left(\frac{1}{2}\right)^n$

127. $P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4^n}{10^n} = \left(\frac{2}{5}\right)^n$

128. Let E be the event that the last digit in *a* is 5

$n(E) = n(E_1) - n(E_2) = 5^n - 4^n$

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{5^n - 4^n}{10^n}$

Paragraph – 44

A sequence of ellipses $E_1, E_2, ..., E_n$ is constructed as follows: Ellipse E_n is drawn so as to touch ellipse E_{n-1} as the extremities of the major axis of E_{n-1} and to have its foci at the extremities of the minor axis of E_{n-1}

129. If E_n is independent of *n*, then the eccentricity of ellipse E_{n-2} is

A) $\left(\frac{3-\sqrt{5}}{2}\right)$ B) $\left(\frac{\sqrt{5}-1}{2}\right)$ C) $\frac{2-\sqrt{3}}{2}$ D) $\frac{\sqrt{3}-1}{2}$

Key. B

130. If eccentricity of ellipse E_n is e_n then the locus of (e_n^2, e_{n-1}^2) is

A) a parabola B) an ellipse C) a hyperbola D) a rectangular hyperbola

Key. D

131. If eccentricity E_n is independent of *n*, then the locus of mid point of chords of slope –1 of E_n is (if axis of E_n along Y-axis)

A) $\left(\sqrt{5}-1\right)x = 2y$ B) $\left(\sqrt{5}+1\right)x = 2y$

C) $\left(3-\sqrt{5}\right)x = 2y$ D) $\left(3+\sqrt{5}\right)x = 2y$

Key. B

Sol. 129. If $E_n : \frac{x^2}{a_n^2} + \frac{y^2}{b_n^2} = 1$ and eccentricity of E_n is e_n

If $a_n > b_n$ then, $b_n^2 = a_n^2(1-e_n^2)$ ..... (i)

According to question $b_n = b_{n-1}$ .... (ii)

And $a_{n-1} = a_n e_n$ ..... (iii)

For ellipse E_{n-1},

$a_{n-1}^2 = b_{n-1}^2\left(1-e_{n-1}^2\right)$ .... (iv)

From equations (i) and (ii)

$b_{n-1}^2 = a_n^2\left(1-e_n^2\right)$ ..... (v)

Substituting the value of a_{n-1} and b_{n-1}^2 from equation (iii) and (v) in (iv), then

$a_n^2 e_n^2 = a_n^2(1-e_n^2)(1-e_{n-1}^2)$ ...... (vi)

$\because E_n$ is independent of *n*,

$\therefore e_n = e_{n-1} = e$ (say)

$\Rightarrow e^2 = (1-e^2)^2 \Rightarrow e^4 - 3e^2 + 1 = 0$

$$\therefore e^2 = \frac{3\pm\sqrt{5}}{2} = \frac{6\pm2\sqrt{5}}{4} = \left(\frac{\sqrt{5}\pm1}{2}\right)^2$$

$$\therefore e = \frac{\sqrt{5}-1}{2} \quad (\because 0 < e < 1)$$

130. From equation (vi)

$e_n^2 = (1-e_n^2)(1-e_{n-1}^2)$

Locus of (e_n^2, e_{n-1}^2) is $x = (1-x)(1-y) \Rightarrow xy - 2x - y + 1 = 0$

Here $a = 0, b = 0, L = 1, f = -1/2, g = -1, h = 1/2$

$\therefore \Delta = 0 + 2\times\frac{-1}{2}\times -1\times\frac{1}{2} - 0 - 0 - 1\times\frac{1}{4} = \frac{1}{4} \neq 0$

$h^2 > ab$ and $a + b = 0$ $\Rightarrow$ rectangular hyperbola

131. Equation of chord whose mid-point (x_1, y_1) is

$T = S_1 \Rightarrow \frac{xx_1}{a_n^2} + \frac{yy_1}{b_n^2} - 1 = \frac{x_1^2}{a_n^2} + \frac{y_1^2}{b_n^2} - 1$

Slope = $\frac{-b_n^2 x_1}{a_n^2 y_1} = -1$ (given)….. (viii)

$\because$ Eccentricity of E_n is independent of n

$\therefore$ Axis of E_n along x-axis are along y-axis in each case

$e = \frac{\sqrt{5}-1}{2} \therefore e^2 = \frac{3-\sqrt{5}}{2}$

from equation (viii) $b_n^2 x_1 = a_n^2 y_1 = b_n^2(1-e_n^2)y_1$ or $x_1 = \left(1 - \frac{3-\sqrt{5}}{2}\right) y_1$

$\Rightarrow 2x_1 = (\sqrt{5}-1)y_1$ (or) $2x_1(\sqrt{5}+1) = 4y_1$ (or) $(\sqrt{5}+1)x_1 = 2y_1$

$\therefore$ Required locus is $(\sqrt{5}+1)x = 2y$

Paragraph – 45

Suppose a lot of *n* objects contains n_1 objects of one kind, n_2 objects of second kind, n_3 objects of third kind,….., n_K objects of Kth kind. Such that $n_1 + n_2 + n_3 + ...n_K = n$, then the number of possible arrangements or permutations of R objects out of this lot is the coefficient of x^r in the expansion of $r!\prod\left(\sum_{\lambda=0}^{n_1}\frac{x^\lambda}{\lambda!}\right)$

132. The number of permutations of the letters of the word INDIA taken three at a time must be

A) 27 B) 30 C) 33 D) 57

Key. C

133. If $n_1 = n_2 = n_3 = ... = n_K = 1$, then the number of permutations of *r* objects must be

A) ${}^nP_r$ B) ${}^nC_r$ C) ${}^KP_r$ D) ${}^KC_r$

Key. A

134. If $n_1 + n_2 + n_3 + ... + n_K = r$, then number of permutations must be

A) ${}^nC_r$ B) ${}^nP_r$ C) $(K+r)!$ D) $\frac{r!}{n_1!n_2!...n_K!}$

Key. D

Sol. 132. There are 5 letters A, D, I, I, N.

Number of permutations = coefficient of x^3 in

$$3!\left(1+\frac{x}{1!}+\frac{x^2}{2!}\right)\left(1+\frac{x}{1!}\right)^3 \quad (\because 1A, 1D, 2I's, 1N)$$

= coefficient of x^3 in $\dfrac{6(2+2x+x^2)(1+x)^3}{2}$

= coefficient of x^3 in $3\left[\{1+(1+x)^2\}(1+x)^3\right]$

= coefficient of x^3 in $3\{(1+x)^3+(1+x)^5\}$

$=3\left\{{}^3C_3+{}^5C_3\right\}=3(1+10)=33$

133. $\because n_1=n_2=n_3=......=n_K=1$

i.e., *n* distinct objects in a line taken *r* at a time is ${}^nP_r$

134. $n_1+n_2+n_3+....+n_K=r$

$\Rightarrow n=r$

$\therefore$ number of permutations = number of permutation of *r* objects in a line of which n_1 are of one kind, n_2 of second kind, n_3 of third kind,…. n_K of Kth kind

Paragraph – 46

$$A=\{a_1,a_2,.....a_n\},\ A\times A=\left\{\left(a_i,a_j\right);a_i,a_j\varepsilon A,1\le i,j\le n\right\}$$

$$A*A=\left\{\{a_i,a_j\}:a_i,a_j\varepsilon A,1\le i,j\le n\right\}$$

135. Number of functions defined form $A\times A\to A$

a) n^{n^2} b) $n^{(n-1)^2}$ c) $n^{(n+1)^2}$ d) n^{2n}

Key. A

136. Number of functions defined from $A*A\to A$

a) $n^{\frac{(n+1)^2}{2}}$ b) $n^{\frac{n^2}{2}}$ c) $n^{\frac{n(n+1)}{2}}$ d) $n^{\frac{n(n-1)}{2}}$

Key. C

137. Number of functions $f:A*A\to A\times A$ defined by $f\left(\{a_i,a_j\}\right)=\left(a_i,a_j\right)$

a) $2^{n(n-1)}$ b) $2^{n(n+1)}$ c) 2^{n^2} d) 2^n

Key. A

Sol. 135. Number of elements in $A\times A=n^2$

136. Number of elements in $A*A=\dfrac{n(n+1)}{2}$

137. if $i=j$ then $\left\{a_i,a_j\right\}$ can be maped in only one way.

if $i \neq j$ then $\{a_i \neq a_j\}$ can be maped in two way.

Paragraph – 47

For a finite set A, let $|A|$ denote the number of elements in the Set A. Also Let F denote the set of all functions $f:\{1,2,....,n\} \to \{1,2,...,k\},(n \geq 3, k \geq 2)$ satisfying $f(i) \neq f(i+1)$ for every i, $1 \leq j \leq n-1$

138. $|F| =$

a) $k^n(k-1)$ b) $k(k-1)^n$ c) $k^{n-1}(k-1)$ d) $k(k-1)^{n-1}$

Key. D

139. If $c(n,k)$ denote the number of functions in F satisfying $f(n) \neq f(1)$, then for $n \geq 4, C(n,k)$

a) $k(k-1)^{n-1} - c(n-1,k)$ b) $k(k-1)^n - c(n-1,k-1)$

c) $k^{n-1}(k-1)^n - c(n-1,k)$ d) $k^n(k-1) - c(n-1,k)$

Key. A

140. For $n \geq k, c(n,k)$, where $c(n,k)$ has the same meaning as in question no.37, equals.

a) $k^n + (-1)^n(k-1)$ b) $(k-1)^n + (-1)^{n-1}(k-1)$

c) $(k-1)^n + (-1)^n(k-1)$ d) $k^n + (-1)^{n-1}(k-1)$

Key. C

Sol. 138. The image of the element 1 can be chosen in k ways and for each of the remaining $(n-1)$ elements, the image can be defined in $(k-1)$ ways, since $f(i) \neq f(i+1)$

$\therefore$ Total number of mapping in $F = k(k-1)^{n-1}$

139. Out of the total number of mappings in F, the number of mapping which satisfy $f(n) = f(1)$ is same as the number of mappings which satisfy $f(n-1) \neq f(1)$ and this number is $C(n-1,k)$

$\therefore \; C(n,k) = |F| - C(n-1,k)$

140. $C(n,k) = k(k-1)^{n-1} - c(n-1,k)$

$= (k-1)^n + (k-1)^{n-1} - C(n-1,k)$

$C(n,k) - (k-1)^n = (-1)\left\{C(n-1,k) - (k-1)^{n-1}\right\}$

$= (-1)^{n-3}\left\{c(3,k) - (k-1)^3\right\}$

but $c(3,k)$ = number of mappings f in F for which $f(3) \neq f(1)$

$\therefore C(3,k) = k(k-1)(k-2)$

$\therefore C(n,k) - (k-1)^n = (-1)^{n-1}(k-1)\left\{k(k-2) - (k-1)^2\right\}$

$\left(-1\right)^n\left(k-1\right)$

$\therefore c\left(n,k\right)=\left(k-1\right)^n+\left(-1\right)^n\left(k-1\right)$

Paragraph – 48

Consider all permutations of the letters of the word MORADABAD

141. The no. of permutations which contain the word BAD is

a) $21\times 5!$ b) $7\times 5!$ c) $6\times 5!$ d) $2\times 5!$

Key. A

142. The no. of permutations with the letter D occurring in the first and the last places is :

a) $21\times 5!$ b) $7\times 5!$ c) $6\times 5!$ d) $2\times 5!$

Key. B

143. The no. of permutations with the letters M , A, O occurring only in odd positions is:

a) $21\times 5!$ b) $7\times 5!$ c) $6\times 5!$ d) $2\times 5!$

Key. D

Sol. Conceptual

Paragraph – 49

Given are six 0`s, five 1`s and four 2`s. consider all possible permutations of all these numbers. [A permutation can have its leading digit 0].

144. How many permutations have the first 0 preceeding the first 1?

a) ${}^{15}C_4\times{}^{10}C_5$ b) ${}^{15}C_5\times{}^{10}C_4$

c) ${}^{15}C_6\times{}^{10}C_5$ d) ${}^{15}C_5\times{}^{10}C_5$

Key. A

145. In how many permutations does the first 0 preceed the first 1 and the first 1 preceed first 2.

a) ${}^{14}C_5\times{}^{8}C_6$ b) ${}^{14}C_5\times{}^{8}C_4$

c) ${}^{14}C_6\times{}^{8}C_4$ d) ${}^{14}C_6\times{}^{8}C_6$

Key. B

146. The no. of permutations in which all 2`s are together but no two of the zeroes are together is

a) 42 b) 40 c) 84 d) 80

Key. A

Sol. 144. The no. of ways of arranging 2`s is ${}^{15}C_4$. Fill the first empty position left after arranging the 2`s with a 0(1 way) and pick the remaining five places the position the remaining five zeros $\rightarrow{}^{10}C_5$ ways.

$\therefore\ {}^{15}C_4\times 1\times{}^{10}C_5$

145. Put a) in the first position, (1 way). Pick five other positions for the remaining O`s ( ${}^{14}c_5$ ways), put a 1 in the first of the remaining positions ( 1 way), then arrange the remaining four 1`s (${}^{8}C_4$ ways)

$\therefore{}^{14}C_5\times{}^{8}C_4$

Paragraph – 50

Box A contains 8 items of which 3 are defective and box B contains 5 items of which two are defective. An item is drawn at random from each box.

147. The probability that atleast one item is defective is

A) 3/8 B) 5/8 C) 3/7 D) 5/9

Key. B

148. The probability that exactly one item is defective is

A) 17/32 B) 15/34 C) 19/40 D) 19/37

Key. C

149. If one item is defective and the other is non-defective then the probability that the defective item came from box A is

A) 15/31 B) 9/19 C) 8/15 D) 21/32

Key. B

Sol. (147) $1-\frac{5}{8}\times\frac{3}{5}=\frac{5}{8}$

(148) $\frac{3}{8}\times\frac{3}{5}+\frac{5}{8}\times\frac{2}{5}=\frac{19}{40}$

(149) E_1 – Event of drawing a defective item from A

E_2 – Event of drawing a defective item from B

E – Event of drawing one defective item and one non-defective item.

$$P\left(E_1/E\right)=\frac{P\left(E_1E\right)}{P\left(E\right)}=\frac{\frac{3}{8}\times\frac{3}{5}}{\frac{19}{40}}=\frac{9}{19}$$

Paragraph – 51

A slip of paper is given to a person A who marks it either with a plus sign or a minus sign. The probability of his writing a plus sign is 1/3. A passes the slip to B, who may either leave it alone or change the sign before passing it to C. Next C passes the slip to D after perhaps changing the sign. Finally D passes it to a referee after perhaps changing the sign. B, C, D each change the sign with probability 2/3.

150. The probability that the referee observes a plus sign on the slip if it is known that A wrote a plus sign is

A) 14/27 B) 16/27 C) 13/27 D) 17/27

Key. C

151. The probability that the referee observes a plus sign on the slip if it is known that A wrote a minus sign is

A) 16/27 B) 14/27 C) 13/27 D) 11/27

Key. B

152. If the referee observes a plus sign on the slip then the probability that A originally wrote a plus sign is

A) 13/41 B) 19/27 C) 17/25 D) 21/37

Key. A

Sol. (150, 151, 152)

Let $E_1 =$ Event that A wrote a plus sign.

$E_2 =$ Event that A wrote a minus sign.

$E =$ Event that the referce observes a plus sign.

Given $P\left(E_1\right)=\frac{1}{3} \Rightarrow P\left(E_2\right)=\frac{2}{3}$

$P\left(E/E_1\right)=$ Probability that none of B,C,D change sign+Probability that exactly two of B,C,D Change sign.

$=\frac{1}{27}+3\left(\frac{1}{3}\times\frac{2}{3}\times\frac{2}{3}\right)=\frac{13}{27}$

$P\left(E/E_2\right)=$ Probability that all of B,C,D change the sign+Probability that exactly one of them changes the sign.

$=\frac{8}{27}+3\times\left(\frac{2}{3}\times\frac{1}{3}\times\frac{1}{3}\right)=\frac{14}{27}$

$\therefore P(E_1/E)=\frac{13}{41}$

Using Baye's theorem.

Paragraph – 52

done his home work, then he is sure to identify the correct answer, otherwise , he chooses an answer at random.

Let E : denotes the event that a student does his home work with P (E) = p and

F : denotes the event that he answer the question correctly.

153. If p = 0.6 the value of P(E/F) equals

a) $\frac{8}{16}$ b) $\frac{10}{16}$ c) $\frac{12}{16}$ d) $\frac{6}{7}$

154. The relation $P\left(E/F\right)\geq P\left(E\right)$ holds good for

a) all values of p in [0,1] b) all values of p in (0,1) only

c) all values of p in [0,5,1] d) no value of p

155. Suppose that each question has n alternative answers of which only one is correct, and p is fixed but not equal to 0 or 1then P(E/F)

a) decreases as n increases for all $p \in (0,1)$

b) increases as n increases for all $p \in (0,1)$

c) remains constant for all $p \in (0,1)$

d) decreases if $p \in (0,0.5)$ and increases if $p \in (0.5,1)$ as 'n' increases

Sol. 153. Ans. (d)

154. Ans. (a)

155. Ans. (b)

$$P(E) = P$$

$$P(F) = P(E).P(F/E) + P(\overline{E}).P(F/\overline{E})$$

$$= P.1 + (1-P)\frac{1}{4} = \frac{3P+1}{4}$$

If P = 0.6 P(F) = 0.7

$$P(E/F) = \frac{6}{7}$$

$$P(E/F) = \frac{4P}{3P+1} \geq P$$

Paragraph – 53

Consider the independent event A,B,C corresponding to a random experiment. Suppose the event $A \times B \times C$ represents the event of occurrence of atleast one of A, B, C and the event $A.B.C$ represents the event of simultaneous occurrence of A, B,C. Also, the event $\overline{A}$ represents non-occurrence of A. It is given that $P(A) = a, P(A \times B \times C) = 1-b, P(A.B.C) = 1-c$ and $P(\overline{A}.\overline{B}.C) = x$. Now answer the following questions.

156. The probability of occurrence of event B is

a) $\frac{x}{x+b}$ b) $\frac{(1-c)(x+b)}{ax}$

c) $\frac{(1-a)^2 + ab}{1-a}$ d) $\frac{(1-b)(x-c)}{ax}$

157. The probability x satisfies the equation

a) $ax^2 + \{ab + (a-1)(a+c-1)\}x + b(a-1)(c-1) = 0$

b) $ax^2 + \{ac + (b-1)(b+c-1)\}x + c(b-1)(a-1) = 0$

c) $ax^2 + \{bc + (a-1)(a+b-1)\}x + a(b-1)(c-1) = 0$

d) $(1-b)x^2 + (ab+bc+ca)x + (a-1)(b-1)(c-1) = 0$

158. As $0 < x < 1$, so both the roots of the equation obtained in Q. No: 18 must be positive. Using this information select the appropriate answer

a) $c > \frac{ab+(1-a)^2}{1-a}$ b) $c < \frac{ab+(1-a)^2}{1-a}$ c) $c = \frac{ab+(1-a)^2}{1-a}$ d) None of these

Sol. 156. (B) $x = P(\bar{A}.\bar{B}.C) = P\{(\overline{A\times B}).C\} = P(C) - P\{(A\times B).C\}$

$= P(C) - P(A.C\times A.B) = P(C) - P(A.C) - P(B.C) + P(A.B.C)$

$= P(C) - P(A)P(C) - P(B)P(C) + P(A)P(B)P(C) = P(C)\{1-P(A)\}\{1-P(B)\}$

$\therefore X = (1-a)P(C)\{1-P(B)\} \Rightarrow P(C) - P(C)P(B) = \frac{x}{1-a}$ ....(a)

Also, $1-c = P(A.B.C) = P(A).P(B).P(C) \Rightarrow P(B)P(C) = \frac{1-c}{a}$ ....(b)

And

$P(\bar{A}.\bar{B}.\bar{C}) = b \Rightarrow \{1-P(A)\}\{1-P(B)\}\{1-P(C)\} = b \Rightarrow \{1-P(B)\}\{1-P(C)\} = \frac{b}{1-a}$ .....(c)

From (a) and (c), $\frac{P(C)}{1-P(C)}\,\frac{P(C)}{1-P(C)} = \frac{x}{b} \Rightarrow P(C) = \frac{x}{x+b}$ and from (b)

$P(B) = \frac{(1-c)(x+b)}{ax}$

157. (A) From (a) and (b) $P(C) = \frac{x}{1-a} + \frac{1-c}{a} = \frac{x}{x+b}$ from previous solution.

$\therefore [ax + (1-c)(1-a)](x+b) = a(1-a)x \Rightarrow ax^2 + \{ab + (1-c)(1-a) - a(1-a)\}x + b(1-c)(1-a) = 0$

or $ax^2 + \{ab + (a-1)(a+c-1)\}x + b(a-1)(c-1) = 0$

158. (A)As $a > 0, b(a-1)(c-1) > 0,$ so the above equation has positive roots if

$ab + (a-1)(a+c-1) < 0 \Rightarrow ab + (a-1)^2 + (a-1)c < 0 \Rightarrow c > \frac{ab+(1-a)^2}{1-a}$ $[\because 1-a > 0]$

Paragraph – 54

Consider all the 3 digit numbers abc (where $a \neq 0$). If a number is selected at random then

159. The probability that the number is such that $a < b < c$ is

(A) $\frac{2}{15}$ (B) $\frac{7}{75}$ (C) $\frac{7}{600}$ (D) $\frac{7}{300}$

160. The probability that the number is such that $a > b > c$ is

(A) $\frac{2}{15}$ (B) $\frac{7}{75}$ (C) $\frac{7}{600}$ (D) $\frac{7}{300}$

161. The probability that the number is such that $a + b + c = 6$ is

(A) $\frac{2}{15}$ (B) $\frac{7}{75}$ (C) $\frac{7}{600}$ (D) $\frac{7}{300}$

Key: 1)B 2)A 3)D

Hint: If $a<b<c$ then required probability $=\frac{{}^{9}C_3}{9\times10\times10}=\frac{7}{75}$

If $a>b>c$ then required probability $=\frac{{}^{10}C_3}{9\times10\times10}=\frac{2}{15}$

If $a+b+c=6$ then the possible digit selections are

$(1,2,3),(1,1,4),(2,2,2),(0,1,5),(0,2,4)(0,3,3),(0,0,6)$

The required number of ways $=6+3+1+4+4+2+1=21$

Required probability $=\frac{21}{9\times10\times10}=\frac{7}{300}$

Paragraph – 55

Three fair coins are tossed simultaneously. Let E be the event of getting three heads or three tails, F be the event of at least two heads & G be the event of atmost two heads then.

161. Which of the following is True

(A) $P(E\cap F)=P(E).P(F)$ (B) $P(E\cap G)=P(E).P(G)$

(C) $P(F\cap G)=P(F).P(G)$ (D) none.

162. Probability that at least one head occur is

(A) 5/8 (B) 3/8 (C) $\frac{7}{8}$ (D) none

163. $P(G)=$

(A) $\frac{3}{4}$ (B) $\frac{7}{8}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$

Key: 161)A 16 2)C 163)B

Hint: $P(E)=\frac{2}{8}=\frac{1}{4},P(F)=\frac{4}{8}=\frac{1}{2};P(G)=\frac{7}{8}$

$P(E\cap F)=\frac{1}{8};P(F\cap G)=\frac{3}{8};P(E\cap G)=\frac{1}{8}$

Paragraph – 56

Starting at *(0,0)* ,an object moves in *x-y* plane via a sequence of steps, each of length 1 unit. Each step is left, right, up or down ,all the four being equally likely. The probability that object reaches *(2,2)* in

164. Exactly 4 steps is

(A) $\frac{5}{128}$ (B) $\frac{3}{128}$ (C) $\frac{1}{128}$ (D) $\frac{1}{256}$

165. Exactly 6 steps is

(A) $\frac{6}{4^4}$ (B) $\frac{1}{4^6}$ (C) $\frac{6}{4^6}$ (D) $\frac{15}{4^4}$

166. Six or fewer steps is

(A) $\frac{1}{16}$ (B) $\frac{1}{32}$ (C) $\frac{3}{64}$ (D) $\frac{5}{64}$

Key: 164)A 165) C 166)A

Hint: Since the net movement must be two steps right (R) and two steps up (U) there must be atleast 4 steps to reach (2,2) in 6 or fewer steps .(2,2) can be reached in 4 steps if the sequence of steps is some permutations of R,R,U,U

$\therefore$ Probability of reaching (2,2) in 4 steps = $\frac{\left(\frac{4!}{2!2!}\right)}{4^4} = \frac{6}{4^4}$

A six step sequence moludes the steps R,R,U,U in same order as well as a pair of steps consisting of R,L or U,D in same order .R,R,U,U,R,L or R,R,U,U,U,D can be permuted in 2(60)ways of which 2×12 correspond to exactly 4 steps .

Hence probability for exactly six steps = $\frac{2(60-13)}{4^6} = \frac{9^6}{4^6}$

For Q.No 16 Answer is $\frac{6}{4^4} + \frac{6}{4^4} = \frac{3}{64}$

Paragraph – 57

A box contains n coins of which at least one is biased. Let E_k denote the event that exactly k out of the n coins are biased. Also let $P(E_k)$ be directly proportional to $k(k+1)$ for $1 \le k \le n$. Then

167. The proportionality constant is equal to

A) $\frac{3}{n(n^2+1)}$ B) $\frac{1}{(n^2+1)(n+2)}$ C) $\frac{3}{n(n+1)(n+2)}$ D) $\frac{1}{(n+1)(n+2)(n+3)}$

168. If p(n) denotes the probability that a coin selected out of the n coins at random is biased, then $\lim_{n\to\infty} p(n) =$

A) $\frac{1}{4}$ B) $\frac{3}{4}$ C) $\frac{1}{2}$ D) $\frac{7}{8}$

169. If a coin selected at random is found to be biased, then the probability that it is the only biased coin in the box is

A) $\frac{1}{(n+1)(n+2)(n+3)(n+4)}$ B) $\frac{12}{n(n+1)(n+2)(3n+1)}$

C) $\frac{24}{n(n+1)(n+2)(n+3)}$ D) $\frac{24}{n(n+1)(n+2)(3n+1)}$

Key: 167)C 168)B 169)D

Hint: 167. If $P(E_k)\,\alpha\, k(k+1)$, then $P(E_k) = \lambda k(k+1)$ for $1 \le k \le n$

And $\lambda\sum_{k=1}^{n} k(k+1) = 1$ gives $\lambda = \frac{3}{n(n+1)(n+2)}$

168. $$p(n)=P(E)=P\left(\bigcup_{k=1}^{n} E\cap E_k\right)$$

$$=\sum_{k=1}^{n}P(E\cap E_k)=\sum_{k=1}^{n}\lambda\ k(k+1).\frac{k}{n}$$

$$=\frac{3}{n^2(n+1)(n+2)}\left[\frac{n^2(n+1)^2}{4}+\frac{n(n+1)(2n+1)}{6}\right]\to\frac{3}{4}\ as\ n\to\infty$$

169. $$P(E_1/E)=\frac{P(E_1)P(E/E_1)}{\sum_{k=1}^{n}P(E_k)P(E/E_k)}=\frac{(2\lambda)\left(\frac{1}{n}\right)}{\sum_{k=1}^{n}\lambda\ k(k+1).\frac{k}{n}}$$

$$=\frac{2}{\sum_{k=1}^{n}k^2(k+1)}=\frac{24}{n(n+1)(n+2)(3n+1)}$$

Paragraph – 58

If n distinct objects are distributed randomly into n distinct boxes, what is the probability that

170. No box is empty

1) $\frac{\lfloor n-1}{n^n}$ 2) $\frac{\lfloor n-1}{\lfloor 2n^n}$ 3) $\frac{\lfloor n}{n^n}$ 4) $\frac{\lfloor 2\lfloor n-1}{n^n}$

Key: **3**

Hint: no box empty the no of favorable ways $=\lfloor n$

171. Exactly one box empty

1) $\frac{\lfloor n\ {}^nC_2}{n^{n-1}}$ 2) $\frac{\lfloor n\ {}^nC_2}{n^n}$ 3) $\frac{n\lfloor n}{n^n}$ 4) $\frac{\lfloor n}{n^n}$

Key: **2**

Hint: exactly one box empty, then no. of favorable ways $={}^nC_1.{}^{n-1}C_1{}^nC_2\lfloor n-2$

172. A particular box get exactly r objects

1) $\frac{{}^nC_r(n-1)^{n-r-1}}{n^n}$ 2) $\frac{{}^nC_r(n-1)^{n-r+1}}{n^n}$

3) $\frac{{}^nC_r(n-1)^{n-r}}{n^{n-1}}$ 4) $\frac{{}^nC_r(n-1)^{n-r}}{n^n}$

Key: **4**

Hint: a particular box get exactly r objects $={}^nC_r(n-1)^{n-r}$

Paragraph – 59

A box contains n coins. Let $P(E_i)$ be the probability that exactly i out of n coins are biased. If $P(E_i)$ is directly proportional to $i(i+1);\ 1 \le i \le n$.

173. Proportionality constant K is equal to

a) $\dfrac{3}{n(n^2+1)}$ b) $\dfrac{1}{(n^2+1)(n+2)}$

c) $\dfrac{3}{n(n+1)(n+2)}$ d) $\dfrac{1}{(n+1)(n+2)(n+3)}$

Key. C

Sol. $P(E_i) = Ki(i+1)$

$$\because P(E_1)+P(E_2)+......+P(E_n)=1 \Rightarrow K\sum_{i=1}^{n} i(i+1)=1 \Rightarrow$$

$$K\left[\frac{1}{6}n(n+1)(2n+1)+\frac{n(n+1)}{2}\right]=1 \qquad \therefore K=\frac{3}{n(n+1)(n+2)}$$

174. If P be the probability that a coin selected at random is biased then $\underset{n\to\infty}{\text{Lt}}\ P$ is ______

a) $\dfrac{1}{4}$ b) $\dfrac{3}{4}$ c) $\dfrac{3}{5}$ d) $\dfrac{7}{8}$

Key. B

Sol. $P(E)=\sum_{i=1}^{n} P(E_i).P(E/E_i)$

$$=Ki(i+1).\frac{i}{n}=\frac{k}{n}\sum_{i=1}^{n}\left(i^3+i^2\right)=\frac{K}{n}\left[\left(\frac{n(n+1)}{2}\right)^2+\frac{1}{6}n(n+1)(2n+1)\right]=\frac{(3n+1)(n+2)}{4n(n+2)}$$

$$\therefore \underset{n\to\infty}{\text{Lt}}\ P = \frac{3}{4}$$

175. If a coin selected at random is found to be biased then the probability that it is the only biased coin in the box is _______

a) $\dfrac{1}{(n+1)(n+2)(n+3)(n+4)}$ b) $\dfrac{12}{n(n+1)(n+2)(3n+1)}$

c) $\dfrac{24}{n(n+1)(n+2)(2n+1)}$ d) $\dfrac{24}{n(n+1)(n+2)(3n+1)}$

Key. D

Sol. $P\left(E_1/E\right)=\frac{P\left(E_1\right)P\left(E/E_1\right)}{P\left(E\right)}=\frac{K\times 2\times\frac{1}{n}}{\frac{(3n+1)(n+2)}{4n(n+2)}}=\frac{24}{n(n+1)(n+2)(3n+1)}$

Paragraph – 60

A positive integer n(>1) can be written as a product of power of distinct primes in one and only one way, except for the order of the factorization. That is $n=p_1^{\alpha_1}p_2^{\alpha_2}.....p_k^{\alpha_k}$ (α_i s are being positive integers) A positive divisor of n is of the form $p_1^{\beta_1}p_2^{\beta_2}.....p_k^{\beta_k}$ where β_i s are non – negative integers such that $0\le p_i\le\alpha_i$

176. The number of positive divisors of $n=4^5.3^7.5^{11}$ which are perfect squares is
(a) 144 (b) 75 (c) 480 (d) 288

Key. A

Sol. We haven $n=2^{10}.3^7.5^{11}$

To form the division that are perfect square, powers of 2 that are available = 6

Powers of 3 that are available = 4

Powers of 5 that are available = 6

The number of divisors that are perfect square = $6\times 4\times 6=144$

177. The product of positive divisors of $n=4^4\times 27^3$ is
(a) $2^{405}.3^{360}$ (b) $4^{180}.27^{135}$
(c) $2^{180}.3^{360}$ (d) $4^{135}.27^{180}$

Key. B

Sol. $n=4^4.27^3=2^8.3^9$

Number of divisors = $(8+1)(9+1)=90$

The product of all divisors = $\left(2^8.3^9\right)^{90/2}=\left(2^8.3^9\right)^{45}=2^{360}.3^{405}=4^{180}.27^{135}$

178. The product of those positive divisors of $n=2^2.3^3.5^5$, which are divisible by 5 is
(a) $4^{30}.27^{30}.25^{60}$ (b) $2^{60}.3^{90}.5^{180}$
(c) $4^{60}.27^{15}.25^{40}$ (d) $2^{90}.3^{60}.5^{180}$

Key. B

Sol. $n=2^2.3^3.5^5$

The number of divisors = 3.4.6 = 72

The product of all divisors = $\left(2^2.3^3.5^5\right)^{36}=2^{72}.3^{108}.5^{180}$

The product of divisors not divisible by $5 = \left(2^2.3^3\right)^{12/2} = \left(2^2.3^3\right)^6 = 2^{12}.3^{18}$

Thus the product of divisors divisible by 5 = $\dfrac{2^{72}.3^{108}.5^{180}}{2^{12}.3^{18}} = 2^{60}.3^{90}.5^{180}$

Paragraph – 61

In an objective paper, there are two sections of 10 questions each. For 'section I', each question has 5 options and only one option is correct and 'section II', each question has 4 options with multiple answers and marks for a question in this section is awarded only if he ticks all correct answers. Marks for each question in 'section I' is 1 and in 'section II' is 3. (There is no negative marking).

179. If a candidate attempts only two questions by guessing, one from 'section I' and one from 'section II', the probability that he scores in both questions is

(a) 74/75 (b) 1/25 (c) 1/15 (d) 1/75

Key. D

180. If a candidate in total attempts 4 questions all by guessing, then the probability of scoring 10 marks is

(a) $1/15(1/15)^3$ (b) $4/5(1/15)^3$ (c) $1/5(14/15)^3$ (d) None of these

Key. D

181. The probability of getting a score less than 40 by answering all the questions by guessing in this paper is

(a) $(1/75)^{10}$ (b) $1-\left(\dfrac{74}{75}\right)^{10}$ (c) $(74/75)^{10}$ (d) None of these

Key. B

Sol. 179. Let P_1 be the probability of being an answer correct from section I then $P_1 = \dfrac{1}{5}$

And $P_2 = \dfrac{1}{15}$

Required probability = $\dfrac{1}{5}\times\dfrac{1}{15} = \dfrac{1}{75}$

180. To get 10 marks we must choose 3 questions from section 2 and one question from section 1.

Required probability = $\dfrac{10C_3 \times 10C_1}{20C_4}\times\dfrac{1}{5}\times\left(\dfrac{1}{15}\right)^3$

181. To get marks he case to answer all questions correctly = $\left(\dfrac{1}{5}\right)^{10}\left(\dfrac{1}{15}\right)^{10}$

Required probability = $1-\left(\dfrac{1}{75}\right)^{10}$

Paragraph – 62

A lot contains 10 defective and 10 non-defective bulbs. 2 bulbs are drawn at random, One at time with replacement. We define the events A, B and C is follows:

A = {The first bulb is defective}

B= {The second bulb is non-defective}

C={Both bulbs are either defective or non-defective}

182. P(A) will be equal to

(A) $\frac{1}{4}$ (B) $\frac{3}{4}$ (C) $\frac{1}{2}$ (D) $\frac{1}{3}$

Key. C

183. P(B).P(C) will be equal to

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{16}$ (D) $\frac{1}{8}$

Key. A

184. $P(A \cap B \cap C)$ will be equal to

(A) 0 (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{1}{8}$

Key. A

Sol. Conceptual

Paragraph – 63

In an objective paper, there are two sections of 10 questions each. For ‘section I’, each question has 5 options and only one option is correct and ‘section II’, each question has 4 options with multiple answers and marks for a question in this section is awarded only if he ticks all correct answers. Marks for each question in ‘section I’ is 1 and in ‘section II’ is 3. (There is no negative marking).

185. If a candidate attempts only two questions by guessing, one from ‘section I’ and one from ‘section II’, the probability that he scores in both questions is

(a) 74/75 (b) 1/25 (c) 1/15 (d) 1/75

Key. D

186. If a candidate in total attempts 4 questions all by guessing, then the probability of scoring 10 marks is

(a) $1/15(1/15)^3$ (b) $4/5(1/15)^3$ (c) $1/5(14/15)^3$ (d) None of these

Key. D

187. The probability of getting a score less than 40 by answering all the questions by guessing in this paper is

(a) $(1/75)^{10}$ (b) $1-\left(\frac{74}{75}\right)^{10}$ (c) $(74/75)^{10}$ (d) None of these

Key. B

Sol. 185. Let P_1 be the probability of being an answer correct from section I then $P_1 = \frac{1}{5}$

And $P_2 = \frac{1}{15}$

Required probability = $\frac{1}{5} \times \frac{1}{15} = \frac{1}{75}$

186. To get 10 marks we must choose 3 questions from section 2 and one question from section 1.

Required probability = $\frac{10C_3 \times 10C_1}{20C_4} \times \frac{1}{5} \times \left(\frac{1}{15}\right)^3$

187. To get marks he case to answer all questions correctly = $\left(\frac{1}{5}\right)^{10} \left(\frac{1}{15}\right)^{10}$

Required probability = $1 - \left(\frac{1}{75}\right)^{10}$

Paragraph – 64

All the 52 cards of a well shuffled pack of playing cards are distributed equally or unequally among 4 players named P_1, P_2, P_3 & P_4.

For i = 1, 2, 3, 4, let

α_i = number of ace(s) given to P_i

β_i = number of black card(s) given to P_i

γ_i = number of red card(s) given to P_i

δ_i = number of diamond(s) given to P_i

188. The probability that $\delta_i \geq 1 \ \forall$ i = 1, 2, 3, 4, is

(A) ${}^{13}C_4 \times \frac{4!}{4^{13}}$

(B) $[2^{24} + 3 \times 2^{12} - 3^{13} - 1]/2^{24}$

(C) $[13^4 - {}^4C_1\ 12^4 + {}^4C_2\ 11^4 - {}^4C_3\ 10^4 + {}^4C_4\ 9^4]/13^4$

(D) $1 - 4(3/4)^{13} - 6\,(1/2)^{13} - (1/4)^{12}$

Key. B

Sol. Probability of giving atleast one diamond to every player is

$$\frac{\left[4^{13} - {}^4C_1 3^{13} + {}^4C_2 2^{13} - {}^4C_1 1^{13}\right] \times 4^{39}}{4^{52}}.$$

189. If $\beta_i + \gamma_i = 13 \ \forall$ i = 1, 2, 3, 4 then the probability that $\alpha_i = 1 \ \forall$ i = 1, 2, 3, 4, is

(A) $5^4\ 7^2/13^4$

(B) $\frac{13^3}{17 \times 7^2 \times 5^2}$

(C) $3^4\ 13^2/17^4\ 2^7$

(D) $\frac{7^3 \times 3^2}{13^4}$

Key. B

Sol. They get equal number of cards. The probability of each getting an ace

$$= \frac{4! \times \frac{48!}{(12!)^4}}{\frac{52!}{(13!)^4}} = \frac{13^3}{17 \times 7^2 \times 5^2}$$

190. If $\beta_i + \gamma_i = 13 \ \forall$ i = 1, 2, 3, 4 then the probability that $|\beta_i - \gamma_i| = 1 \forall$ i = 1, 2, 3, 4, is

(A) $\left[\frac{26!}{(6!)^2 (7!)^2}\right]^2 \div \left[\frac{52!}{(13!)^4}\right]$ (B) $\left[\frac{(26!)\,(4!)}{(6!)^2\,(7!)^2 (2!)^2}\right]^2 \div \left[\frac{52!}{(13!)^4}\right]$

(C) $4!\left[\frac{26!}{(6!)^2 (7!)^2 (2!)^2}\right]^2 \div \left[\frac{52!}{(13!)^4}\right]$ (D) $4!\left[\frac{26!}{(6!)^2 (7!)^2 (2!)}\right]^2 \div \left[\frac{52!}{(13!)^4}\right]$

Key. D

Sol. Two players get 7 red and 6 black cards each while other two get 6 red and 7 black cards each.

So, the required probability = $\frac{{}^4C_2 \times \left({}^{26}C_7\,{}^{19}C_7\;{}^{12}C_6\;{}^{6}C_6\right)\times\left({}^{26}C_6\,{}^{20}C_6\,{}^{14}C_7\,{}^{7}C_7\right)}{{}^{52}C_{13}\;\;{}^{39}C_{13}\;\;{}^{26}C_{13}\;\;{}^{13}C_{13}}$.

Paragraph – 65

A player throws a fair cubical die and scores the number appearing on the die. If he throws a 1, he gets a further compulsory throw. Let p_r denote the probability of getting a total score of exactly *r*.

191. If $2 \le r \le 6,\ p_r$ equals

A) $1-\left(\frac{1}{6}\right)^{r-1}$ B) $\frac{1}{5}\left[1-\left(\frac{1}{6}\right)^{r-1}\right]$ C) $5^r/6^r$ D) None of these

Key. B

192. If $r > 6,\ p_r$ equals

A) $1-\left(\frac{1}{6}\right)^{r-1}$ B) $\frac{1}{5}\left[1-\left(\frac{1}{6}\right)^{r-1}\right]$ C) $\frac{1}{5}\left[\left(\frac{1}{6}\right)^{r-6}-\left(\frac{1}{6}\right)^{r-1}\right]$ D) None of these

Key. C

193. Sum of the series $S=\sum_{r=1}^{\infty} p_r$ is

A) 1 B) 1/6 C) 1/5 D) 2/3

Key. A

Sol. 191. The player score 6 or less the following ways; the player scores r at the very first trial; he scores 1 at the first trial and $r-1\ \left(if\ r-1>1\right)$ at the second trial; 1 at each of the first two trials and $r-2\ \left(if\ r-2>1\right)$ at the third trial; and so on. This leads to the probability.

$$\frac{1}{6}+\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)+\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)+........+\left(\frac{1}{6}\right)^{r-2}\left(\frac{1}{6}\right)$$

$$=\frac{1}{5}\left[1-\left(\frac{1}{6}\right)^{r-1}\right] if\ 2 \le r \le 6.$$

192. We now enumerate the ways in which the player can score a value $r > 6$. He can score 1 at each of the first $r-2$ trials and 2 at the $(r-1)$ th trial; 1 at each of the first $r-3$ trials and 3 at the $(r-2)$ th trial; 1 at each of the first $r-4$ trials and 4 at the $(r-3)$ th trial; and so on. This gives the probability.

$$\left(\frac{1}{6}\right)^{r-2}\left(\frac{1}{6}\right)+\left(\frac{1}{6}\right)^{r-3}\left(\frac{1}{6}\right)+\ldots\ldots+\left(\frac{1}{6}\right)^{r-6}\left(\frac{1}{6}\right)$$

$$=\frac{1}{5}\left[\left(\frac{1}{6}\right)^{r-6}-\left(\frac{1}{6}\right)^{r-1}\right] \textit{for } r>6.$$

193. Answer is obviously 1.

Paragraph – 66

A JEE aspirant estimate that she will be successful with on80% chance if she studies 10 hours per day, with a 60% chance if she studies 7 hours per day and with a 40% chance if she studies 4 hours per day. She further believes that she will study 10 hours, 7 hours and 4 hours per day with probabilities 0.1, 0.2 and 0.7, respectively.

194. The chance she will be successful, is

(A) 0.28 (B) 0.38

(C) 0.48` (D) 0.58

Key. C

195. Given that she is successful, the chance she studied for 4 hours, is

(A) $\frac{6}{12}$ (B) $\frac{7}{12}$

(C) $\frac{8}{12}$ (D) $\frac{9}{12}$

Key. B

196. Given that she does not achieve success, the chance she studied for 4 hours, is

(A) $\frac{18}{26}$ (B) $\frac{19}{26}$

(C) $\frac{20}{26}$ (D) $\frac{21}{26}$

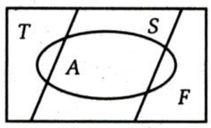
Key. D

Sol. 194-196. A : She get a success

T: She studies 10 h : P(T) = 0.1

S : She studies 7 H : P(S) = 0.2

F : She studies 4 h : P(F) = 0.7



P(A/T) = 0.8; P(A/S) = 0.6; P(A/F) = 0.4

$P(A) = P(A \cap T) + P(A \cap S) + P(A \cap F)$

$= P(T) . P(A/T) + P(S) . P(A/S) + P(F) . P(A/F)$

$= (0.1) (0.8) + (0.2) (0.6) + (0.7) (0.4)$

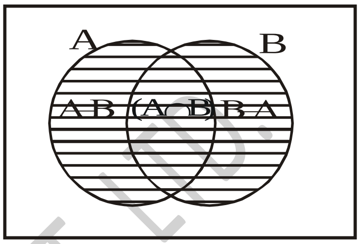
$= 0.08 + 0.12 + 0.28 = 0.48$

$$P(F/A)=\frac{P(F\cap A)}{P(A)}=\frac{(0.7)(0.4)}{0.48}=\frac{0.28}{0.48}=\frac{7}{12}$$

$$P\left(F/\bar{A}\right)=\frac{P\left(F\cap\bar{A}\right)}{P\left(\bar{A}\right)}=\frac{P\left(F\right)-P\left(F\cap A\right)}{0.52}$$

$$=\frac{\left(0.7\right)-0.28}{0.52}=\frac{0.42}{0.52}=\frac{21}{26}.$$

Paragraph – 67



1. Venn diagram
2. $P(A\cup B) = P(A) + P(B) - P(A\cap B)$
3. $P\left(\frac{A}{B}\right)$ means probability of occurrence of A given that B has occurred $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}$
4. $P(A\cap B) = P(A)P(B) \Leftrightarrow$ Events A & B are independent.
5. If events A & B are independent, then A and $\bar{B}$ are also independent, $\bar{A}$ and B are also independent. $\bar{A}$ and $\bar{B}$ are also independent.

197. If $P(A)=\frac{3}{4}, P(B)=\frac{2}{3}, P(\bar{A}\cap\bar{B})=\frac{1}{12}$, then $P\left(\frac{A}{B}\right)-P\left(\frac{A}{\bar{B}}\right)$

(A) 0 (B) $\frac{1}{3}$

(C) $\frac{1}{2}$ (D) $\frac{3}{4}$

Key. A

198. If $P(A)=\frac{1}{3}, P(B)=\frac{1}{2}$, then the range of $P(\bar{A}\cap\bar{B})$ is

(A) $\left[0,\frac{1}{3}\right]$ (B) $\left[0,\frac{1}{2}\right]$

(C) $\left[\frac{1}{3},\frac{1}{2}\right]$ (D) $\left[\frac{1}{3},\frac{2}{3}\right]$

Key. B

199. If $P(A)=\frac{1}{3}$, $P(B)=\frac{3}{4}$, then the range of $P(A\cap\bar{B})$ is

(A) $\left[0,\frac{1}{3}\right]$ (B) $\left[0,\frac{1}{4}\right]$

(C) $\left[\frac{1}{3},\frac{3}{4}\right]$ (D) $\left[\frac{1}{4},\frac{3}{4}\right]$

Key. B

Sol. 197. $P(A\cup B)=1-\frac{1}{12}=\frac{11}{12}$

$\Rightarrow \quad \frac{11}{12}=\frac{3}{4}+\frac{2}{3}-P(A\cap B)$

$\Rightarrow \qquad P(A \cap B) = \frac{3}{4} + \frac{2}{3} - \frac{11}{12} = \frac{9+8-11}{12} = \frac{1}{2}$

$\therefore \qquad P(A \cap B) = \frac{1}{2} = P(A)P(B)$

So A & B are independent

$\Rightarrow \qquad \left(P\left(\frac{A}{B}\right)\right) = P\left(\frac{A}{\bar{B}}\right)$

198. $P(A \cup B) \geq \max.\{P(A), P(B)\}$

$P(A \cup B) \geq \frac{1}{2} \Rightarrow 1 - P(\bar{A} \cap \bar{B}) \geq \frac{1}{2}$

$\Rightarrow \qquad P(\bar{A} \cap \bar{B}) \leq \frac{1}{2}$

obviously $P(\bar{A} \cap \bar{B}) \geq 0$

199. $\frac{3}{4} \leq P(A \cup B) \leq 1$

$\Rightarrow \frac{3}{4} \leq \frac{3}{4} + \frac{1}{3} - P(A \cap B) \leq 1$

$\Rightarrow -\frac{1}{3} \leq -P(A \cap B) \leq -\frac{1}{12}$

$\Rightarrow \frac{1}{12} \leq P(A \cap B) \leq \frac{1}{3}$

$\Rightarrow -\frac{1}{3} \leq -P(A \cap B) \leq -\frac{1}{12}$

$\Rightarrow P(A) - \frac{1}{3} \leq -P(A \cap B) \leq -\frac{1}{12}$

$\Rightarrow P(A) - \frac{1}{3} \leq P(A) - P(A \cap B) \leq P(A) - \frac{1}{12}$

$\Rightarrow \frac{1}{3} - \frac{1}{3} \leq P(A \cap \bar{B}) \leq \frac{1}{3} - \frac{1}{12}$

$\Rightarrow 0 \leq P(A \cap \bar{B}) \leq \frac{1}{4}$.

Paragraph – 68

Die A has 4 red and two white faces and die B has two red and 4 white faces. An unbiased coin is flipped to select the dice . If it falls head, die A is thrown and if it falls tail then die B is thrown Assuming that the same die is used for all throws once it is selected, answer the following.

200. Probability of getting a red face at first throw is

a) $\frac{1}{3}$ b) $\frac{1}{2}$ c) $\frac{1}{4}$ d) $\frac{1}{6}$

Key. B

201. If the first two throws result in red face then the probability of getting red face at the third throw is

a) $\frac{2}{5}$ b) $\frac{1}{5}$ c) $\frac{4}{5}$ d) $\frac{3}{5}$

Key. D

202. If the red face turns up at the first 4 throws then the probability that die A is thrown is

a) $\frac{8}{9}$ b) $\frac{4}{5}$ c) $\frac{16}{17}$ d) $\frac{32}{33}$

Key. C

Sol. 200. E_1 =event of die A thrown, E_2 = event of die B thrown

C=event of red face appearing in any throw.

$$P(E_1) = P(E_2) = \frac{1}{2}, P\left(\frac{C}{E_1}\right) = \frac{2}{3}, P\left(\frac{C}{E_2}\right) = \frac{1}{3}$$

$$P(C) = P(E_1)P\left(\frac{C}{E_1}\right) + P(E_2)P\left(\frac{C}{E_2}\right)$$

$$= \frac{1}{2}\left(\frac{2}{3}\right) + \frac{1}{2}\left(\frac{1}{3}\right) = \frac{1}{2}$$

201. D= event of red face appearing in 3 rd throw

E= event of red face appearing in first two throws

$$P\left(\frac{E}{E_1}\right) = \left(\frac{2}{3}\right)^2, P\left(\frac{D}{E \cap E_1}\right) = \frac{2}{3}$$

$$P\left(\frac{E}{E_2}\right) = \left(\frac{1}{3}\right)^2, P\left(\frac{D}{E \cap E_2}\right) = \frac{1}{3}$$

$$P\left(\frac{D}{E}\right) = \frac{P(E_1)P\left(\frac{E}{E_1}\right)P\left(\frac{D}{E \cap E_1}\right) + P(E_2)P\left(\frac{E}{E_2}\right)P\left(\frac{D}{E \cap E_2}\right)}{P(E_1)P\left(\frac{E}{E_1}\right) + P(E_2)P\left(\frac{E}{E_2}\right)}$$

$$= \frac{3}{5}$$

202. F= event of red face appearing in each of the first 4 throws

Then $P\left(\frac{F}{E_1}\right) = \left(\frac{2}{3}\right)^4, P\left(\frac{F}{E_2}\right) = \left(\frac{1}{3}\right)^4$

.: P (die A is thrown) = $\frac{P(E_1)P\left(\frac{F}{E_1}\right)}{P(E_1)P\left(\frac{F}{E_1}\right) + P(E_2)P\left(\frac{F}{E_2}\right)} = \frac{16}{17}$

Paragraph – 69

A box contains n coins. Let $P(E_i)$ be the probability that exactly i out of n coins are biased. If $P(E_i)$ is directly proportional to $i(i+1);\ 1 \le i \le n$.

203. Proportionality constant K is equal to

a) $\frac{3}{n(n^2+1)}$ b) $\frac{1}{(n^2+1)(n+2)}$ c) $\frac{3}{n(n+1)(n+2)}$ d) $\frac{1}{(n+1)(n+2)(n+3)}$

Key. C

204. If P be the probability that a coin selected at random is biased then $\underset{n\to\infty}{Lt}\ P$ is _______

a) $\frac{1}{4}$ b) $\frac{3}{4}$ c) $\frac{3}{5}$ d) $\frac{7}{8}$

Key. B

205. If a coin selected at random is found to be biased then the probability that it is the only biased coin in the box is _______

a) $\frac{1}{(n+1)(n+2)(n+3)(n+4)}$ b) $\frac{12}{n(n+1)(n+2)(3n+1)}$

c) $\frac{24}{n(n+1)(n+2)(2n+1)}$ d) $\frac{24}{n(n+1)(n+2)(3n+1)}$

Key. D

Sol. 203. $P(E_i) = Ki(i+1)$

$$\because P(E_1)+P(E_2)+\ldots\ldots+P(E_n)=1$$

$$K\sum_{i=1}^{n} i(i+1)=1$$

$$K\left[\frac{1}{6}n(n+1)(2n+1)+\frac{n(n+1)}{2}\right]=1 \qquad \therefore K=\frac{3}{n(n+1)(n+2)}$$

204. $P(E)=\sum_{i=1}^{n} P(E_i).P(E/E_i)$

$$=Ki(i+1).\frac{i}{n}=\frac{k}{n}\sum_{i=1}^{n}(i^3+i^2)$$

$$=\frac{K}{n}\left[\left(\frac{n(n+1)}{2}\right)^2+\frac{1}{6}n(n+1)(2n+1)\right]$$

$$=\frac{(3n+1)(n+2)}{4n(n+2)}$$

$$\therefore \underset{n\to\infty}{Lt}\ p = \frac{3}{4}$$

205. $P(E_1/E) = \frac{P(E_1)P(E/E_1)}{P(E)} = \frac{K \times 2 \times \frac{1}{n}}{\frac{(3n+1)(n+2)}{4n(n+2)}} = \frac{24}{n(n+1)(n+2)(3n+1)}$

Paragraph – 70

A player tosses a coin and scores one point for every head and two points for every tail that turns up. He plays on until his score reaches or passes *n*. If P_n denotes the probability of getting a score of exactly *n*.

Answer the following questions.

206. Value of P_n for $n \geq 1$

A) $\frac{1}{3}\left\{2+\frac{(-1)^n}{2^n}\right\}$ B) $\frac{1}{4}\left\{3+\frac{(-1)^n}{3^n}\right\}$ C) $\frac{1}{5}\left\{4+\frac{(-1)^n}{2^n}\right\}$ D) $\frac{1}{6}\left\{3+\frac{(-1)^n}{5^n}\right\}$

Key. A

207. Value of P_3 is

A) 1/2 B) 5/8 C) 1/3 D) 1/6

Key. B

208. Value of P_4 is

A) 1/5 B) 1/7 C) 11/16 D) 7/11

Key. C

Sol. **206 – 208**

The score of *n* can be achieved in the following mutually exchesive ways

(i) by throwing a head when the score is $(n-1)$

(ii) by throwing a tail when the score is $(n-2)$

$A_i =$ Getting score of exactly $i,\ i = 1, 2, ...$

$H =$ Getting head in a toss, $T =$ Getting tail in a toss

$$P(A_n) = P\{(A_{n-1} \cap H) \cup (A_{n-2} \cap T\} = P(A_{n-1})P(H) + P(A_{n-2})P(T)$$

$$P(A_n) = \frac{1}{2}(P_{n-1} + P_{n-2}) \Rightarrow P_n + \frac{1}{2}P_{n-1} = P_{n-1} + \frac{1}{2}P_{n-2} = P_{n-2} + \frac{1}{2}P_{n-3} = P_2 + \frac{1}{2}P_1$$

$$P_1 = P(H) = \frac{1}{2}, P_2 = P(T \cup HH) = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

$$P_n + \frac{1}{2}P_{n-1} = 1$$

$$P_n + \frac{1}{2}P_{n-1} = \frac{2}{3} + \frac{1}{2}\left(\frac{2}{3}\right) \Rightarrow P_n - \frac{2}{3} = -\frac{1}{2}\left(P_{n-1} - \frac{2}{3}\right) = -\frac{1}{2}\left\{-\frac{1}{2}\left(P_{n-1} - \frac{2}{3}\right)\right\}$$

$$= \left(-\frac{1}{2}\right)^2\left(P_{n-2} - \frac{2}{3}\right) = \frac{1}{3}\left(-\frac{1}{2}\right)^n \Rightarrow P_n = \frac{2}{3} + \frac{1}{3}\left(-\frac{1}{2}\right)^n = \frac{1}{3}\left\{2 + \frac{(-1)^n}{2^n}\right\} \text{ for } n \geq 1$$

Paragraph – 71

Box A contains 8 items of which 3 are defective and box B contains 5 items of which two are defective. An item is drawn at random from each box.

209. The probability that atleast one item is defective is

A) 3/8 B) 5/8 C) 3/7 D) 5/9

Key. B

210. The probability that exactly one item is defective is

A) 17/32 B) 15/34 C) 19/40 D) 19/37

Key. C

211. If one item is defective and the other is non-defective then the probability that the defective item came from box A is

A) 15/31 B) 9/19 C) 8/15 D) 21/32

Key. B

Sol. 209. $1-\frac{5}{8}\times\frac{3}{5}=\frac{5}{8}$

210. $\frac{3}{8}\times\frac{3}{5}+\frac{5}{8}\times\frac{2}{5}=\frac{19}{40}$

211. E_1 – Event of drawing a defective item from A

E_2 – Event of drawing a defective item from B

E – Event of drawing one defective item and one non-defective item.

$$P\left(E_1/E\right)=\frac{P\left(E_1E\right)}{P\left(E\right)}=\frac{\frac{3}{8}\times\frac{3}{5}}{\frac{19}{40}}=\frac{9}{19}$$

Paragraph – 72

A slip of paper is given to a person A who marks it either with a plus sign or a minus sign. The probability of his writing a plus sign is 1/3. A passes the slip to B, who may either leave it alone or change the sign before passing it to C. Next C passes the slip to D after perhaps changing the sign. Finally D passes it to a referee after perhaps changing the sign. B, C, D each change the sign with probability 2/3.

212. The probability that the referee observes a plus sign on the slip if it is known that A wrote a plus sign is

A) 14/27 B) 16/27 C) 13/27 D) 17/27

Key. C

213. The probability that the referee observes a plus sign on the slip if it is known that A wrote a minus sign is

A) 16/27 B) 14/27 C) 13/27 D) 11/27

Key. B

214. If the referee observes a plus sign on the slip then the probability that A originally wrote a plus sign is

A) 13/41 B) 19/27 C) 17/25 D) 21/37

Key. A

Sol. (212, 213, 214)

Let E_1 = Event that A wrote a plus sign.

$E_2 =$ Event that A wrote a minus sign .

$E =$ Event that the referce observes a plus sign.

Given $P\left(E_1\right) = \frac{1}{3} \Rightarrow P\left(E_2\right) = \frac{2}{3}$

$P\left(E / E_1\right) =$ Probability that none of B,C,D change sign+Probability that exactly two of B,C,D Change sign.

$= \frac{1}{27} + 3\left(\frac{1}{3} \times \frac{2}{3} \times \frac{2}{3}\right) = \frac{13}{27}$

$P\left(E / E_2\right) =$ Probability that all of B,C,D change the sign+Probability that exactly one of them changes the sign.

$= \frac{8}{27} + 3 \times \left(\frac{2}{3} \times \frac{1}{3} \times \frac{1}{3}\right) = \frac{14}{27}$

$\therefore P(E_1 / E) = \frac{13}{41}$

Using Baye's theorem.

Probability

Integer Answer Type

1. In a multiple-choice question, there are five alternative answers, of which one or more than one are correct. A candidate will get marks on the question if he ticks all the correct answers. So he decides to tick answers at random, if the least number of chances ,he should be allowed so that the probability of his getting marks on the question exceeds 1/8 is K, then K =

(the student always attempt the question)

Key. 4

Sol. The probability that he get marks $=\dfrac{1}{31}$

The probability that he get marks in second trial is $\dfrac{30}{31}\times\dfrac{1}{30}=\dfrac{1}{31}$

The probability that he get marks in third trial is $\dfrac{1}{31}$

Continuing this process the probability from *r* trial is $\dfrac{r}{31}>\dfrac{1}{8}$

$\Rightarrow r>\dfrac{31}{8}$

$r=4$

2. If n(X) = (K + 1), then the probability of selecting 2 subsets A and B of the set 'X' such that B = A^C is equal to $\dfrac{1}{2^{m-1}}$ where m - k is equal to

Key. 2

Sol. $n(X)=k+1$

No. of ways to construct $A=2^{k+1}$

No. of ways to construct $B=2^{k+1}$

$\therefore$ Total ways to construct A and B = $2^{k+1}\times 2^{k+1}$

Favourable ways to construct A = 2^{k+1}

Favourable ways to construct B such that B = A^C is = 1

$\therefore$ Favourable ways = $2^{k+1}\times 1$

Required Probability = $\dfrac{2^{k+1}}{\left(2^{k+1}\right)^2}=\dfrac{1}{2^{k+1}}$

$\Rightarrow m-1=k+1$

$\Rightarrow m-k=2$

3. A bag contains 10 different balls. Five balls are drawn simultaneously and then replaced and then seven balls are drawn. The probability that exactly three balls are common to the two drawn is *p*, then the value of 12*p* is

Key. 5

Sol. The no. of ways of drawing 7 balls = ${}^{10}C_7$

For each set of 7 balls of the second draw, 3 must be common to the set of 5 balls of the draw, i.e., 2 other balls can be drawn in ${}^{3}C_2$ ways thus, for each set of 7 balls of the second draw, there are ${}^{7}C_3 \times {}^{3}C_2$ ways of making the first draw so that there are 3 balls common.

Hence, the probability of having three balls in common $\frac{{}^{7}C_3 \times {}^{3}C_2}{{}^{10}C_7} = \frac{5}{12}$.

4. The number of ways of arranging the letters of the word NALGONDA, such that the letters of the word GOD occur in that order (G before and O and O before D), is P then $\frac{P}{420} =$

Key. 4

Sol. No. of ways $= \frac{\lfloor 8}{\lfloor 2 \lfloor 2 \lfloor 3} = 1680$

$\Rightarrow \quad \frac{P}{420} = 4$

5. In a group of people, if 4 are selected at a random, the probability that the any two of the four do not have same month of birth is p then $\frac{96p}{11}$ is equal to

Key. 5

Sol. Required probability $= \frac{{}^{12}C_4 \lfloor 4}{12^4} = \frac{55}{96}$

6. Two numbers are selected at random from set of the first 100 natural numbers. The probability that the product obtained is divisible by 3 is k then $\frac{150k}{83}$ is equal to

Key. 1

Sol. Required probability $= \frac{{}^{33}C_2 + {}^{33}C_1\, {}^{67}C_1}{{}^{100}C_2}$

$= \frac{83}{150}$

7. Functions are formed form A = {1, 2, 3,} to set B = {1, 2, 3, 4, 5} and one function is elected at random. If P the probability that function satisfying f(i) ≤ f(j) whenever i < j then value of 25 p is equal to

Key. 7

Sol. Total number of function = $5^3 = 125$

Number of function satisfying $f(i) \le f(j)$ if $i < j$

$= {}^5C_3 + {}^5C_2\,(1 + 1) + {}^5C_1 = 35$

Required probability $= \dfrac{35}{125} = \dfrac{7}{25}$

8. If the sides of triangle are decided by throwing a die thrice, the probability that the triangle is isosceles or equilateral is $\dfrac{1}{k}$ then k =

Key. 8

Sol. Let the sides be a, b, c

$a = b = 1, c = 1$

$a = b = 2, c = 1, 2, 3$

$a = b = 3, c = 1, 2, 3, 4, 5$

$a = b = 4, c = 1, 2, 3, 4, 5, 6$

$a = b = 5, c = 1, 2, 3, 4, 5, 6$

$a = b = 6, c = 1, 2, 3, 4, 5, 6$

The number of these triangles is $1 + 3 + 5 + 3 \times 6 = 27$

Probability $= \dfrac{27}{6^3} = \dfrac{1}{8}$

9. Four identical dice are rolled once the probability that all the members on them are primes is $\dfrac{L}{8L+2}$ then L =

Key. 5

Sol. The total number of outcomes:

$aaaa$ appear in $\binom{6}{1} = 6$ ways

$aaab$ appear in $2\binom{6}{2} = 30$ ways

$aabb$ appear in $\binom{6}{2} = 15$ ways

$aabc$ appear in $3\binom{6}{3} = 60$ ways

$abcd$ appear in $\binom{6}{4} = 15$ ways

Total $= 6 + 30 + 15 + 60 + 15 = 126$

The number of ways of primes appearing

$aaaa$ appear in $\binom{3}{1}=3$ ways

$aaab$ appear in $2\binom{3}{2}=6$ ways

$aabb$ appear in $\binom{3}{2}=3$ ways

$aabc$ appear in $3\binom{3}{3}=3$ ways

Total $=3+6+3+3=15$

Probability $=\frac{15}{126}=\frac{5}{42}$

10. If $\{x, y\}$ is a subset of the first 30 natural numbers, then the probability, that x^3+y^3 is divisible by 3, is $\frac{S}{9}$ then S =

Key. 3

Sol. x^3+y^3 is divisible by $3 \Rightarrow x+y$ is divisible by $3 \Rightarrow x, y$ are multiples of 3 or one leaves remainder 1 and the other 2 when divided by 3.

$3, 6, 9.....30$ are multiple of $3; 1, 4, 7, ..., 28$ leave remainder 1

$2, 5, 8, ..., 29$ leave remainder 2

Probability $=\frac{\binom{10}{1}\binom{10}{1}+\binom{10}{2}}{\binom{30}{2}}$

$=\frac{145}{15\times 29}=\frac{1}{3}$

11. If p, q are chosen randomly with replacement from the set $\{1, 2, 3, ...10\}$, the probability, that the roots of the equation $x^2+px+q=0$ are real, is $\frac{k^2+6}{50}$ then k =

Key. 5

Sol.

p	q
2	1
3	1, 2

4	1 to 4
5	1 to 6
6	1 to 9
7, 8, 9, 10	1 to 10

$p^2 \geq 4q \Rightarrow$

The total number of pairs $(p,q) is 1+2+4+6+9+40=62$

Probability $= \dfrac{62}{10.10} = \dfrac{31}{50}$

12. Total number of divisors of $3^5.5^7.7^9$ which are of the form $4\lambda+1,\ \lambda \geq 0$, is $(60)l$ then l is

Key. 2

Sol. Any positive integral power of 5 is of the form $4\lambda+1$. Even power of 3 and 7 are of the form $4\lambda+1$ and odd powers of 3 and 7 are of the form $4\lambda-1$. The required number $=8(3\times5+3\times5)$

13. If $f(x)=ax^3+bx^2+cx+d$, (a,b,c,d are rational) and roots of $f(x)=0$ are eccentricities of a parabola and a rectangular hyperbola then $a+b+c+d$ equals

Key. 0

Sol. Roots of $f(x)$ are $1, \sqrt{2}, -\sqrt{2}$

$$ax^3+bx^2+cx+d=(x-1)(x-\sqrt{2})(x+\sqrt{2})=(x-1)(x^2-2)=x^3-x^2-2x+2$$
$$a=1, b=-1, c=-2, d=2 \Rightarrow a+b+c+d=0$$

14. An unbiased coin is tossed 12 times .The probability that at least 7 consecutive heads show up is $\dfrac{K}{256}$ then K =

Key. 7

Sol. The sequence of consecutive heads may starts with 1st toss or 2nd toss or 3rd toss ---- or at 6th toss. In any case ,if it starts with r th throw , the first (r-2) throws may be head or tail but (r-1)st throw must be tail, after which again a head or tail can show up:

$\underbrace{\text{HHH------H}}_{\text{7 times}}$ x x x x x or $\underbrace{\text{x x x -------- x}}_{\text{(r-2)}}$ $\underset{\text{(r-1)st}}{\text{T}}$ $\underbrace{\text{H HH-------H}}$ x x------

$\therefore$ Probability = $\frac{1}{2^7}+\frac{1}{2}\cdot\frac{1}{2^7}+\frac{1}{2}\cdot\frac{1}{2^7}+\text{---------}+\frac{1}{2}\cdot\frac{1}{2^7}=\frac{1}{2^7}\left[1+\frac{5}{2}\right]=\frac{7}{2^8}$

5 times

15. An unbiased coin is tossed. If the result is a head, a pair of unbiased dice is rolled and the number obtained by adding the numbers on the two faces is noted. If the result is a tail, a card from a well shuffled pack of eleven cards numbered 2, 3, 3, 4, ... 12 is picked and the number on the card is noted. The probability that the noted number is either 7 or 8 is $\frac{P}{792}$ then the digit in tens place of P is

Key. 9

Sol. Let E_1 = the toss result in a head

E_2 = the toss result in a tail

A = noted number is 7 or 8

$\therefore$ $P(A) = P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)$

$= \frac{1}{2}\times\frac{11}{36}+\frac{1}{2}\times\frac{2}{11}=\frac{193}{792}$

$\therefore \frac{P}{792}=\frac{193}{792}$

$\therefore$ P = 193.

16. If the number of ways can the letters of the word INSURANCE be arranged, so that the vowels are never separated is $(1080)n$ then the value of 'n' is

Key. 8

Sol. The word INSURANCE has nine different letters, combining the vowels into one bracket as (IUAE) and treating them as one letter we have six letters viz.

(IUAE), N, S, R, N, C and these can be arranged among themselves in $\frac{6!}{2!}$ ways and four vowels within the bracket can be arranged themselves in $4!$ ways.

Required number of words $=\frac{6!}{2!}\times 4! = 8640$

17. In a class of 10 students, there are 3 girls. If the number of different ways that all the students be arranged in a row such that no two of the three girls are consecutive is $(564480)k$ then the value of 'k' is

Key. 3

Sol. Number of girls = 3, number of boys = 7. Since there is no restriction on boys, therefore first of all arrange the 7 boys in ${}^{7}P_{7} = 7!$ ways.

B B B B B B B

If the girls are arranged at the places (including the two ends) indicated by crosses, no two of three girls will be consecutive.

Now there are 8 places for 3 girls

3 girls can be arranged in ${}^{8}P_{3}$ ways

Required number $= {}^{8}P_{3}7! = \frac{8!}{5!} \times 7! = 336 \times 5040 = 3 \times (564480)$

18. If the number of 3 digit odd numbers divisible by 3, which can be formed using the digits 3, 4, 5, 6 when repetition of digits within the number is allowed is $2k+5$ then the value of 'k' is

Key. 3

Sol. Three digits odd numbers using only 3 and only 5 are 2.

Three digit odd numbers using 3, 4 and 5, are 4.

Three digit odd numbers using 4, 5 and 6 are 2.

Three digit odd numbers using two 6 and one 3 are 1.

Three digit odd numbers using two 3 and one 6 are 2.

So, total three digit numbers $= 11$

19. The number of ways of arranging the letters of the word NALGONDA, such that the letters of the word GOD occur in that order (G before and O and O before D), is P then $\frac{P}{420} =$

Key. 4

Sol. No. of ways $= \frac{\lfloor 8}{\lfloor 2 \lfloor 2 \lfloor 3} = 1680$

$\Rightarrow \quad \frac{P}{420} = 4$

20. $1, \alpha_1, \alpha_2, \alpha_3, \ldots\ldots, \alpha_k$ are divisions of number $N = 2^{n-1}(2^n - 1)$ where $2^n - 1$ is a prime number and $1 < \alpha_1 < \alpha_2 < \alpha_3 < \ldots\ldots < \alpha_k$ then value of $\left(1 + \frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \ldots\ldots + \frac{1}{\alpha_k}\right)$ is

Key. 2

Sol. Divisors of $N = 2^{n-1}\left(2^n - 1\right)$ are

$$1, 2, 2^2, \ldots\ldots, 2^{n-1}, 2^n - 1, 2\left(2^n - 1\right), 2^2\left(2^n - 1\right), \ldots\ldots, 2^{n-1}\left(2^n - 1\right)$$

$$1 + \frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \ldots\ldots + \frac{1}{\alpha_k} = 1 + \frac{1}{2} + \ldots\ldots + \frac{1}{2^{n-1}} + \frac{1}{2^n - 1} + \frac{1}{2\left(2^n - 1\right)} + \ldots\ldots + \frac{1}{2^{n-1}\left(2^n - 1\right)}$$

$$= \left(1 + \frac{1}{2} + \ldots. + \frac{1}{2^{n-1}}\right) + \frac{1}{2^n - 1}\left(1 + \frac{1}{2} + \ldots\ldots + \frac{1}{2^{n-1}}\right)$$

$$= \left(1 + \frac{1}{2} + \ldots. + \frac{1}{2^{n-1}}\right)\left(1 + \frac{1}{2^n - 1}\right)$$

$$= \frac{1.\left(1 - \left(\frac{1}{2}\right)^n\right)}{1 - \frac{1}{2}}\left(\frac{2^n}{2^n - 1}\right) = \frac{2^n}{2^{n-1}} = 2$$

21. If the number of ordered triplets (x, y, z) such that $L.C.M(x, y) = 3375,\ L.C.M(y, z) = 1125,\ L.C.M(z, x) = 3375$ is equal to $'k'$, then $k - 47$ is equal to

Key. 3

Sol. $3375 = 5^3.3^3,\ 1125 = 5^3.3^2$

Clearly, 3^3 is a factor of $'x'$ and 3^2 is factor of atleast one of $y\ \&\ z$. This can be done in 5 ways.

Also, 5^3 is a factor of atleast two of the numbers x, y, z which can be done in

$^3C_2 \times 4 - 2 = 10$

$\therefore\ k = 50$

22. If k be the number of 3 digit natural numbers, having sum of their digits atleast 10, then the value of $\frac{\mathrm{k} - 35}{100}$ is

Key. 7

Sol. We have to calculate number of solution of $a + b + c > 9, 1 \le a \le 9,\ 0 \le b, c \le 9$

$a + b + c + d = 9,\ d \ge 0$

Number of solutions is co-efficient of t^9 in

$\left(t + t^2 + \ldots. + t^9\right)\left(1 + t + \ldots\ldots + t^9\right)^2\left(1 + t + t^2 + \ldots\ldots t^9\right)$

$=$ coefficient of t^8 in $\left(1 - t^9\right)\left(1 - t^{10}\right)^2\left(1 - t\right)^{-4}$

$=$ coefficient of t^8 in $(1-t)^{-4} = {}^{8+4-1}C_{4-1} = {}^{11}C_3 = 165$

So, required number of natural numbers $900-165=735$

$\therefore \dfrac{k-35}{100}=7$

23. Eighteen guests have to be seated, half on each side of a long table. Four particular guests desire to sit on one particular side and three others on other side. If the number of ways in which the seating arrangements can be made is $\dfrac{11!\times(9!)^2}{(p!)(p+1)!}$.
Then the value of $'p'$ is

Key. 5

Sol. Out of 18 guests half i.e., 9 to be seated on side A and rest 9 on side B.
Now out of 18 guests, 4 particular guess desire to sit on one particular side say side A and other 3 on other side B. Out of rest $18-4-3=11$ guests
we can select 5 more for side A and rest 6 can be done in
${}^{11}C_5$ ways and 9 guests on each side of table can be seated in $9!\,9!$ ways.
Thus there are total ${}^{11}C_5\,9!\,9!$ arrangements.

24. If the number of arrangements of the letters of the word BANANA in which the two $N's$ do not appear adjacently is $5k$ then $'k'$ equals

Key. 8

Sol. Required number of ways $=\dfrac{\lfloor 6}{\lfloor 3\lfloor 2}-\dfrac{\lfloor 5}{\lfloor 3}=40$

25. $'m'$ men and $'n'$ women are to be seated in a row so that no two women sit together. If $m>n,$ then the number of ways in which they can be seated is $\dfrac{m!(m+1)!\times\lambda}{(m-n+1)!}$.
Then the value of $'\lambda'$ is

Key. 1

Sol. $'m'$ men can be seated in $m!$ ways creating $(m+1)$ for ladies to sit.
$'n'$ ladies out of $(m+1)$ places (as $n<m$) can be seated in ${}^{m+1}P_n$ ways
$\therefore$ Total ways $=m!\times{}^{m+1}P_n=m!\,\dfrac{(m+1)!}{(m+1-n)!}$

26. A seven digit number made up of all distinct digits $8,7,6,4,2,x,y$ is divisible by 3. The possible number of ordered pairs (x,y) is

Key. 8

Sol. We know that a number is divisible by 3. If sum of its digits is divisible by 3.

Hence we must have $8+7+6+4+2+(x+y)=3k$

$27+x+y=3k$

$\Rightarrow x+y$ is multiple of 3

Hence required (x,y) order pairs

$=(0,3),(0,9),(1,5),(3,0),(3,9),(5,1),(9,0),(9,3)$

27. The number of integral solutions of the equation $2x+2y+z=20$ where $x\ge 0, y\ge 0$ *and* $z\ge 0$ is $11k$ then $k=$

Key. 6

Sol. $2x+2y=20-z \Rightarrow x+y=10-\dfrac{z}{2}$

Coefficients of $P^{10-\frac{z}{2}}$ in $\left(1+P+P^2....\right)^2=11-\dfrac{z}{2}$

Patting $z=0,2,4,6,8,10,12,14,16,18,20$

The number of integral solutions $=66$

28. The number of numbers from 1 to 100, which are neither divisible by 3 not by 5 nor by 7 is n. Then n/9 is.

Key. 5

Sol. Conceptual

29. There are four balls of different colors and four boxes of colors, same as those of the balls. The number of ways in which the balls, one each in a box, could be placed such that a ball does not go to a box of its own color, is

Key. 9

Sol. $\therefore$ Number of ways of putting all the 4 balls into boxes of different colour.

$$=4!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right]=4!\left(\frac{1}{2}-\frac{1}{6}+\frac{1}{24}\right)=24\left(\frac{12-4+1}{24}\right)=9$$

30. The digit in tenth place of $1!+2!+3!+......+49!=$

Key. 1

Sol. $1!+2!+3!+4!=33$ Also $5!=120, 6!=720, 7!=5040, 8!=40320 \ \ and \ \ 9!=326880$

The digit in tenth place $1!+2!+3!+....+9!=1$

Also note that $n!$ is divisible by 100 for $n \geq 10$

so that the digit in tenth place $10!+11!+.....+49!$ is zero.

There fore The digit in tenth place of $1!+2!+3!+.....+49!$ is 1.

31. In a group of people, if 4 are selected at a random, the probability that any two of the four do not have same month of birth is p then $\frac{96\,P}{11}$ is equal to

Key. 5

Sol. Required probability $= \frac{{}^{12}C_4 \lfloor 4}{12^4} = \frac{55}{96}$

32. Two persons X and Y go to a hotel. There are two hotels having three rooms each and one hotel having four rooms each, The probability that X and Y are in the same hotel having four rooms is $\frac{k}{15}$ then k is ---

Key. 2

Sol. $P(E) = \frac{2.\,{}^4C_2}{10.9}$

33. Two numbers are selected at random from set of the first 100 natural numbers. The probability that the product obtained is divisible by 3 is k then $\frac{150\,k}{83}$ is equal to

Key. 1

Sol. Required probability $= \frac{{}^{33}C_2 + {}^{33}C_1\,{}^{67}C_1}{{}^{100}C_2}$ $= \frac{83}{150}$

34. If A and B throw a die each. The probability that A s throw is not greater than Bs throw is K/12, then K=

Key. 7

Sol. If A gets 1, then Bs chances are 1,2,.6 = 6

If A gets 2, then Bs chances are 2,3,... = 5

Similarly so on up to 6 = $\sum 6$

$\therefore P(E) = \frac{21}{36}$

35. Two squares are chosen at random on a chess board. If the chance that may have a contact at corner is $\frac{\mu}{144}$ then μ should be equal to

Key. 7

Sol. Total cases of choosing two squares on a chess board $= 64 \times 63$

Favourable cases $= 4 \times 1$ (corners) $+ 24 \times 2 + 36 \times 4$ $= 4 \times 49$

Required probability $= \frac{4 \times 49}{64 \times 63} = \frac{7}{144}$

$\Rightarrow \mu = 7$

36.

The probability that a teacher will give a surprise test in a class is $\frac{1}{5}$. If a student is absent twice, the probability that he will miss atleast one test will be $\frac{k}{25}$, then k must be

Key. 9

Sol. Required probability $= 1 - \frac{4}{5} \times \frac{4}{5} = \frac{9}{25}$

$\Rightarrow$ k = 9

37. A box contains 24 balls of which 12 are black and 12 are white. The balls are drawn at random from the box one at a time with replacement . The probability that a white ball is drawn for the 4^{th} time on the 7^{th} draw is K/32 , then K=

Key. 5

Sol. Required probability = Probability of drawing of 3 W & 3 B balls in first 6 draws and a white ball in 7^{th} draw.

$= {}^6C_3 \cdot \frac{1}{2^7} = \frac{5}{32}$

38. Three tangents are drawn at random to a given circle. The odds against the circle being inscribed in the triangle formed by them is K to 1 , then K =

Key. 2

Sol. $P(E)=\frac{2}{6}=\frac{1}{3}$

$P(\overline{E})=\frac{2}{3}$

$\therefore$ odds against = 2 : 1

39. Two non-negative integers are chosen at random. The probability that the sum of their squares is divisible by 5 is $\frac{K}{25}$ then $k=$

Key. 9

Sol. Let the non-negative integers be $x, y, x=5a+\alpha,\ y=5b+\beta$ where
$0\le\alpha\le4, 0\le\beta\le4$

$x^2+y^2=25\left(a^2+b^2\right)+10\left(a\,\alpha+b\,\beta\right)+\alpha^2+\beta^2$

$\alpha^2+\beta^2$ is divisible by 5

$\alpha,\beta\in\{(0,0)(1,2)(2,1)\ (1,3)(3,1)(2,4)(4,2)(3,4)(4,3)\}$

Probability $=\frac{9}{25}\quad\therefore\ k=9$

40. If the integers m and n are chosen at random from $\{1,2,3,\ldots\ldots,100\}$ then the probability that a number of the form 7^m+7^n is divisible by 5 is equal to $\frac{1}{k}$.

The numerical value of k is

Key. 4

Sol. Total ways of choosing m and n is $n(S)=100\times100$

Now, $7^1=7=5k+2,\ 7^2=49=5k+4,\ 7^3=343=5k+3,\ 7^4=2301=5k+1$

The same sequence will repeat for next four powers

$7^5=2k+2,\ 7^6=5k+4,\ 7^7=5k+3,\ 7^8=5k+1$

$\therefore\ 7^1,7^5,7^9,\ldots\ldots7^{97}$ are of the type $5k+2$

$7^2,7^6,7^{10},\ldots\ldots7^{98}$ are of the type $5k+4$

$7^3,7^7,7^{11},\ldots\ldots7^{99}$ are of the type $5k+3$

& $7^4,7^8,7^{12},\ldots\ldots7^{100}$ are of the type $5k+1$

There will be only four favourable combinations in which 7^m+7^n will be divisible by 5.

$7^m: 5k+2,\ 5k+4,\ 5k+3,\ 5k+1$

$7^n: 5k+3,\ 5k+1,\ 5k+2,\ 5k+4$

$\therefore$ Number of favourable cases is

$n(E)=25\times25+25\times25+25\times25+25\times25=4\times625$

$\therefore$ Required probability $= \frac{n(E)}{n(S)} = \frac{4\times 625}{100\times 100} = \frac{1}{4}$

41. A man parks his car among n cars standing in a row ,his car not being parked at an end, on his return he finds that exactly m of the n cars are still there, probability that both the cars parked on two sides of his car ,have left is

$\frac{(n-m)(n-m-1)}{(n-A)(n-B)}$ then $A+B$ is

Key. 3

Sol. Number of ways in which remaining $'m-1'$ cars can take their places

(excluding the car of man) $= {}^{n-1}C_{m-1}$

No.of ways in which remaining $(m-1)$ cars can take places keeping the two places on two sides of his car vacant $= {}^{n-3}C_{m-1}$

Prob $= \frac{{}^{n-3}C_{m-1}}{{}^{n-1}C_{m-1}} = \frac{(n-m)(n-m-1)}{(n-1)(n-2)}$

$\Rightarrow A = 1$

$B = 2$

$A+B = 3$

42. Five horses are in a race. Mr. A selects two of the horses at random and bets on them. If the probability that Mr. A selected the winning horse is $\frac{P}{5}$ then the value of P is

Key. 2

Sol. Out of 5 horses only one is the winning horse. The probability that Mr. A selected the loss horse $= \frac{4}{5}\times\frac{3}{4}$

$\therefore$ The probability that Mr. A selected the winning horse $= 1-\frac{4}{5}.\frac{3}{4} = \frac{2}{5}$

43. You are given a box with 20 cards in it. 10 cards of this have the letter I printed on them. The other ten have the letter T printed on them. If you pick up 3 cards at random and keep them in the same order, the probability of making the word IIT is $\frac{K}{38}$. The numerical value of K is

Key. 5

Sol. $\frac{{}^{10}C_1}{{}^{20}C_1}\times\frac{{}^{9}C_1}{{}^{19}C_1}\times\frac{{}^{10}C_1}{{}^{18}C_1} = \frac{10\times 9\times 10}{20\times 19\times 18} = \frac{5}{38}$

44. Two squares are chosen at random from small squares (one by one) drawn on a chess board and

the chance that two squares chosen have exactly one corner in common is $\frac{k}{144}$ then $k =$

Key. 7

Sol. Total number of ways to select 2 unit squares $= {}^{64}C_2$

No.of ways of selecting squares which have a corner in common $= 98$

$\therefore$ probability $= \frac{98}{{}^{64}C_2} = \frac{7}{144} \Rightarrow k = 7$

45. A die is rolled three times, the probability of getting a large number than the previous number is $\frac{k}{54}$ then the value of k is

Key. 5

Sol. Let the second number be x (where $1 < x < 6$)

Then first number can be chosen in $(x-1)$ ways and third in $(6-x)$ ways

$\therefore$ Favourable cases $= \sum_{x=2}^{5}(x-1)(6-x) = 20$

46. From a bag containing 10 distinct balls, 6 balls are drawn simultaneously and replaced. Then 4 balls are drawn. The probability that exactly 3 balls are common to the drawings is $\frac{m}{21}$. Then the numerical value of m is

Key. 8

Sol. Let S be the sample space of the composite experiment of drawing 6 in the first draw and then four in second draw then $|S| = {}^{10}C_6 \times {}^{10}C_4$

$\therefore$ Required probability $= \frac{{}^{10}C_6 \times {}^{6}C_3 \times {}^{4}C_1}{{}^{10}C_6 \times {}^{10}C_6}$

$= \frac{80 \times 24}{10 \times 9 \times 8 \times 7} = \frac{8}{21}$

47. If two natural numbers x, y are selected at randomly and probability that $x^2 + y^2$ is multiple of 5 is p, then 25 p is

Ans: 9

Hint: Total number of ways of end digits of x,y is 100

and favourable is $8 \times 4 + 2 \times 2 = 36$

So, p = $\frac{36}{100}=\frac{9}{25}$

CONDITIONAL PROBABILITY

48. Two cards are selected at randomly from a pack of ordinary playing cards. If there found to be of different colours (Red & Black), then conditional probability that both are face cards is

(A) $\frac{36}{325}$ (B) $\frac{18}{169}$

(C) $\frac{9}{169}$ (D) none of these

Key: B

Hint: Let A $\to$ they are face cards, B $\to$ they are of different colours

$$P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{{}^{12}C_2-2\times{}^{6}C_2}{13\times 26}=\frac{18}{169}$$

49. The probability of a bomb hitting a bridge is ½ and two direct hits are needed to destroy it. The leastnumber of bombs required so that the probability of the bridge being destroyed is greater than 0.9 is

a) 7 b) 9 c) 8 d) 10

KEY : A

HINT. $P(X\geq 2)\geq 0.9$ X follows B.D with parameter n,p= $\frac{1}{2}$

50. A special die is so constructed that the probabilities of throwing 1, 2, 3, 4, 5 and 6 are (1 – k)/6, (1 + 2k)/6, (1 – k)/6, (1 + k)/6, (1 – 2k)/6 and (1 + k)/6 respectively. If two such dice are thrown and the probability of getting a sum equal to 9 lies in $\left[\frac{1}{9},\frac{2}{9}\right]$. Then find the number of integral solutions of k.

Key. 1

Sol. Let E_1, E_2, E_3 E_4, E_5 and E_6 be the events of occurrence of 1, 2, 3, 4, 5 and 6 on the dice respectively, and let E be the event

$$\therefore P(E_1)=\frac{1-k}{6};\ P(E_2)=\frac{1+2k}{6};P(E_3)=\frac{1-k}{6}$$

$$P(E_4)=\frac{1+k}{6};\ P(E_5)=\frac{1-2k}{6};\ P(E_6)=\frac{1+k}{6}$$ and $\frac{1}{9}\leq P(E)\leq\frac{2}{9}$

Then, $E\equiv\{(3,6),(6,3),(4,5),(5,4)\}$

Hence, $P(E)=P(E_3E_6)+P(E_6E_3)+P(E_4E_5)+P(E_5E_4)$

$$=P(E_3)P(E_6)+P(E_6)P(E_3)+P(E_4)P(E_5)+P(E_5)P(E_4)$$

$$=2P(E_3)P(E_6)+2P(E_4)P(E_5)$$

{Since E_1, E_2, E_3, E_4, E_5 and E_6 are independent}

$=2\left(\frac{1-k}{6}\right)\left(\frac{1+k}{6}\right)+2\left(\frac{1+k}{6}\right)\left(\frac{1-2k}{6}\right)$

$=\frac{1}{18}\left[2-k-3k^2\right]$

Since, $\frac{1}{9}\le P(E)\le\frac{2}{9}$

$\therefore$ $-\frac{1}{3}\le k\le 0$

$\therefore$ Set of integral value of k = {0}

$\therefore$ Number of integral solution of k is 1

51. If number of numbers greater than 3000, which can be formed by using the digits 0, 1, 2, 3, 4, 5 without repetition, is n then $\frac{n}{230}$ is equal to

Key. 6

Sol. No. of 4 digit numbers = 3 × 5 × 4 × 3 = 180

No. of 5 digit numbers = 5 × 5 × 4 × 3 × 2 = 600

No. of 6 digit numbers = 5 × 5 × 4 × 3 × 2 = 600

$n=1380$

$\Rightarrow$ $\frac{n}{230}=6$

52. Nine hundred distinct n-digit numbers are to be formed using only the 3 digits 2, 5, 7. The smallest value of n for which this is possible is

Key. 7

Sol. $3^n\ge 900\Rightarrow n\ge 7$

53. Out of 5 apples, 10 mangoes and 15 oranges, the number of ways of distributing 15 fruits each to two persons, is n then $\frac{n}{22}$ is equal to

Key. 3

Sol. $x_1+x_2+x_3=15$

$0\le x_1\le 5,\ 0\le x_2\le 10,\ 0\le x_3\le 15$

n = co-efficient of $x^{15}(1-x^6)(1-x^{11})(1-x^{16})(1-x)^{-3}$

n = 66

$\frac{n}{22}=3$

54. A bag contains 10 different balls. Five balls are drawn simultaneously and then replaced and then seven balls are drawn. The probability that exactly three balls are common to the two drawn is *p*, then the value of 12*p* is

Key. 5

Sol. The no. of ways of drawing 7 balls = ${}^{10}C_7$

For each set of 7 balls of the second draw, 3 must be common to the set of 5 balls of the draw, i.e., 2 other balls can be drawn in ${}^{3}C_2$ ways thus, for each set of 7 balls of the second draw, there are ${}^{7}C_3 \times {}^{3}C_2$ ways of making the first draw so that there are 3 balls common.

Hence, the probability of having three balls in common $\dfrac{{}^{7}C_3 \times {}^{3}C_2}{{}^{10}C_7} = \dfrac{5}{12}$.

55. In a multiple-choice question, there are five alternative answers, of which one or more than one are correct. A candidate will get marks on the question if he ticks all the correct answers. So he decides to tick answers at random, if the least number of chances ,he should be allowed so that the probability of his getting marks on the question exceeds 1/8 is K, then K =

(the student always attempt the question)

Key. 4

Sol. The probability that he get marks $= \dfrac{1}{31}$

The probability that he get marks in second trial is $\dfrac{30}{31} \times \dfrac{1}{30} = \dfrac{1}{31}$

The probability that he get marks in third trial is $\dfrac{1}{31}$

Continuing this process the probability from *r* trial is $\dfrac{r}{31} > \dfrac{1}{8}$

$\Rightarrow r > \dfrac{31}{8}$

$r = 4$

56. 3 couples have to be seated around a circle. Let p be the probability that no couple is together then the value of 30 p is

Key. 8

Sol. $p = \dfrac{5! - {}^{3}C_1(4!).(2!) + {}^{3}C_2(3!).(2!).(2!) - {}^{3}C_3(2!).(2!).(2!).(2!)}{5!} = \dfrac{120 - 144 + 72 - 16}{120} = \dfrac{4}{15}$

So 30p = 8

57. A coin is tossed m + n times (m > n) , the probability of getting m consecutive heads is $\dfrac{n+k}{2^{m+2}}$ then k = ___

Key. 3

Sol. If m = 3, n = 2

Coin is tossed 5 times, then 3 consecutive heads can come is 5 cases

Probability = $\dfrac{5}{2^5} = \dfrac{2+3}{2^{3+2}}$

58. Three identical dies are rolled, the probability that they will get same number on them. If $\dfrac{K}{28}$ then *K* = ________

Key. 3

Sol. $n(S)=56$

$P(E)=\frac{31}{50}$

59. Two distinct numbers are chosen at random from set {1, 2, ……. 3n}. The probability that x^2-y^2 is divisible by 3 is $\frac{pn+q}{r(3n-1)}$ then $p+q+r=$

Key. 5

Sol. $n(s)={}^{3n}C_2$

Let $A_0=\{3,6,9..........3n\}$

$A_1=\{1,4,7..........3n-2\}$

$A_2=\{5,8,11.........3n-1\}$

(x^2-y^2) divisible by 3. If both x,y should come from A_0 or A_1 or A_2 or one is from A_1 and other from A_2

$n(E)=3{}^nC_2+{}^nC_1+{}^nC_1=\frac{n}{2}(5n-3)$

$P(E)=\frac{5n-3}{3(3n-1)}$

60. 3 numbers are chosen at random without replacement from $\{1,2,3.......14\}$ Let

Let A = { min of chosen number is 5}

B = { max of chosen no is 11 }

$P(A\cup B)=\frac{K+11}{91}$ then K = _____

Key. 8

Sol. $P(A)=\frac{{}^9C_2}{{}^{14}C_3}=\frac{9}{91},\ P(B)=\frac{{}^{10}C_2}{{}^{14}C_3}=\frac{45}{364}$

$P(A\cap B)=\frac{{}^5C_1}{{}^{14}C_3}=\frac{5}{364}$

$P(A\cup B)=\frac{19}{91}$

61. There are n lines in a pane, No two of which are parallel and No three of concurrent Let plane be divided in U_n parts then U_3=

Key. 7

Sol. $U_0=1, U_1=2$

The n[th] line will rise to n additional parts when U_{n-1} parts are already there $U_n=U_{n-1}+n$

62. Two natural numbers x, y are selected at random, probability that x^2+y^2 is divisible by 5 is $\frac{k}{25}$ then k = _______

Key. 9

Sol. $P(E)=\frac{9}{25}$

Sample space $S=\{0,1,2,3,4\}\times\{0,1,2,3,4\}$

$E=\{(0,0)(1,2)(2,1)(1,3)(3,1)(2,4)(4,2)(3,4)(4,3)\}$

63. In a group of people, if 4 are selected at a random, the probability that the any two of the four do not have same month of birth is p then $\frac{96p}{11}$ is equal to

Key. 5

Sol. Required probability $=\frac{{}^{12}C_4\lfloor 4}{12^4}=\frac{55}{96}$

64. Two numbers are selected at random from set of the first 100 natural numbers. The probability that the product obtained is divisible by 3 is k then $\frac{150k}{83}$ is equal to

Key. 1

Sol. Required probability $=\frac{{}^{33}C_2+{}^{33}C_1\,{}^{67}C_1}{{}^{100}C_2}$

$=\frac{83}{150}$

65. Functions are formed form A = {1, 2, 3,} to set B = {1, 2, 3, 4, 5} and one function is elected at random. If P the probability that function satisfying f(i) ≤ f(j) whenever i < j then value of 25 p is equal to

Key. 7

Sol. Total number of function = 5^3 = 125

Number of function satisfying f(i) ≤ f(j) if i < j

$= {}^5C_3 + {}^5C_2\,(1 + 1) + {}^5C_1 = 35$

Required probability $=\frac{35}{125}=\frac{7}{25}$

66. In a bag there are 15 balls of either red or green colour. Let G_k be the event that it contains exactly k green balls and its probability is proportional to k^2. Now a ball is drawn at random. Let P(A) be the probability that the ball drawn is green. If P(A) = p/q in lowest form then q - p is _____.

Key. 7

Sol. $P(G_k)\ \alpha\ k^2 \Rightarrow P(G_k)=\lambda k^2$

$\sum_{k=0}^{n} P(G_k)=1$ (as these are mutually exclusive and exhaustive events)

$\Rightarrow \lambda \sum_{k=0}^{n} k^2=1 \Rightarrow \lambda=\frac{6}{n(n+1)(2n+1)}$

$P(A)=\sum_{k=0}^{n} P(G_k)\ P(A/G_k)=\sum_{k=0}^{n}\lambda k^2.\frac{k}{n}=\frac{\lambda}{n}.\frac{n^2(n+1)^2}{4}=\frac{3(n+1)}{2(2n+1)}$.

Take n = 15.

67. The probability that a random chosen 3 digit number has exactly 3 factors is $\frac{p}{900}$ (where $p\in N$) then the value of p is _____.

Key. 7

Sol. A number has exactly 3 factors if the number is squares of a prime number. Squares of 11, 13, 17, 19, 23, 29, 31 are 3–digit numbers

$\therefore$ required probability = $\frac{7}{900}$

68. Die A has four red and two white faces whereas die B has two red and four white faces. A coin is flipped once. If it falls a head, the game continues by throwing die A, if it falls tail then die B is to be used. If the probability that die A used is $\frac{32}{33}$ when it is given that red turns up every time in first n throws, then the value of n is ______.

Key. 5

Sol. Let R be the event that a red face appears in each of the first n throws.

E_1 : Die A is used when head has already fallen

E_2 : Die B is used when tail has already fallen.

$\therefore$ P $(R/E_1) = \left(\frac{2}{3}\right)^n$ and $P\left(\frac{R}{E_2}\right) = \left(\frac{1}{3}\right)^n$

As per the given condition

$$\frac{P(E_1).P(R/E_1)}{P(E_1).P(R/E_1)+P(E_2).P(R/E_2)} = \frac{32}{33} \Rightarrow \frac{1/2.(2/3)^n}{\frac{1}{2}.\left(\frac{2}{3}\right)^n + \frac{1}{2}\left(\frac{1}{3}\right)^n} = \frac{32}{33} \Rightarrow \frac{2^n}{2^n+1} = \frac{32}{33}$$

$\Rightarrow$ n = 5.

Probability

Matrix-Match Type

1. *'n'* whole numbers are randomly chosen and multiplied.

	Column - I		Column – II
(a)	The probability that the last digit is 1, 3, 7 or 9 is	p.	$\frac{8^n - 4^n}{10^n}$
(b)	The probability that the last digit is 2, 4, 6, 8 is	q.	$\frac{5^n - 4^n}{10^n}$
(c)	The probability that the last digit is 5 is	r.	$\frac{4^n}{10^n}$
(d)	The probability that the last digit is zero is	s.	$\frac{10^n - 8^n - 5^n + 4^n}{10^n}$

Key. a $\rightarrow$ r, b $\rightarrow$ p, c $\rightarrow$ q, d $\rightarrow$ s

Sol. (a) The required event will occur if last digit in all chosen numbers is 1, 5, 7 or 9.

Required probability = $\left(\frac{4}{10}\right)^n$

(b) The required probability is equal to the probability that last digit is 2, 4, 6, 8

$$P(1,2,3,4,5,6,7,8,9) = P(1,3,7,9) = \frac{8^n - 4^n}{10^n}$$

(c) $P(1,3,5,7,9) = P(1,3,7,9) = \frac{5^n - 4^n}{10^n}$

(d) $P(0,5) - P(5) = (10^n - 8^n) - (5^n - 4^n) = \frac{10^n - 8^n - 5^n + 4^n}{10^n}$

2. Match the functions given in Column I with their domain in Column II A is a set containing n elements . A subset P of A is chosen at random . The set A is reconstructed by replacing the elements of the subset P. A subset Q of A is again chosen at random then the probability that

	Column I		Column II
(A)	$P \cap Q = \Phi$	(P)	$\frac{n(3^{n-1})}{4^n}$
(B)	$P \cap Q$ is a singleton	(Q)	$(3/4)^n$
(C)	$P \cap Q$ contain 2 elements	(R)	$\frac{{}^{2n}C_n}{4^n}$
(D)	$\lvert P\rvert = \lvert Q\rvert$ where $\lvert X\rvert$ = number of elements in X	(S)	$\frac{3^{n-2}(n-1)n}{2(4^n)}$

Key. A-Q, B-P, C-S, D-R

Sol. If $x_i \in A$ $then$ $x_i \in P, xi \in Q$

$x_i \notin P,\ xi \notin Q$

$x_i \notin p, x_i \in Q$

$x_i \in P, x_i \notin Q$

A. $P \cap Q = \phi \Rightarrow x_i \in P, x_i \notin Q$

$x_i \notin P, x_i \notin Q$

$x_i \notin P, x_i \in Q$

$n(E) = 3^n,\ n(S) = 4^n$

B. $P \cap Q\, is\, a\, \sin gle ton$

$x_i \in P, x_i \in Q$

$n(E) = {}^n C_1 (1) 3^{n-1}, n(s) = 4^n$

C. $P \cap Q$ contain 2 elements

$n(E) = {}^n c_2 (1)^2\, 3^{n-2}$

D. $n(P) = n(Q) \Rightarrow P(E) = \dfrac{{}^nC_0\,{}^nC_0 + {}^nC_1\,{}^nC_1 + ... + {}^nC_n.{}^nC_n}{2^n.2^n}$

3. There are 10 pairs of shoes in a cup board from which 4 shoes are taken at random.
If P(E) denotes the probability of the event E .
Match the following:

	Column I		Column II
(A)	P (getting no pair)	(P)	$\dfrac{99}{323}$
(B)	P (getting at least one pair)	(Q)	$\dfrac{96}{323}$
(C)	P (getting exactly two pairs)	(R)	$\dfrac{224}{323}$
(D)	P (getting exactly one pair)	(S)	$\dfrac{3}{323}$

Key. A-R, B-P, C-S, D-Q

Sol. A) P (no pair) = $\dfrac{20}{20}.\dfrac{18}{19}.\dfrac{16}{18}.\dfrac{14}{17} = \dfrac{224}{323}$

B) P (at least one pair) = $1 - \dfrac{224}{323} = \dfrac{99}{323}$

C) P (exactly two pairs) = $\dfrac{{}^{10}C_2}{{}^{20}C_4} = \dfrac{3}{323}$

D) P (exactly one pair) = 1 - $\left[\dfrac{224}{323} + \dfrac{3}{323}\right] = \dfrac{96}{323}$

4. Five unbiased cubical dies are rolled simultaneously. Let m and n be the smallest and the largest number appearing on the upper faces of the dies, then match the probabilities given in the column II corresponding to the events given in the column I:

	Column I		Column II
(A)	m = 3	(p)	$\left(\dfrac{2}{3}\right)^5$
(B)	n = 4	(q)	$\left(\dfrac{2}{3}\right)^5 + \left(\dfrac{1}{3}\right)^5 - \left(\dfrac{1}{2}\right)^4$
(C)	$2 \le m \le 4$	(r)	$\left(\dfrac{5}{6}\right)^5 - \left(\dfrac{1}{3}\right)^5$
(D)	m = 2 and n = 5	(s)	$\left(\dfrac{2}{3}\right)^5 - \left(\dfrac{1}{2}\right)^5$

Key. (A) – (s)
(B) – (s)
(C) – (r)
(D) – (q)

Sol. The number appearing on upper face of any dice can be 3, 4, 5 or 6 i.e. maximum 4 cases.

P(m = 3) = P(m ≥ 3) - P(m ≥ 4) = $\dfrac{4^5 - 3^5}{6^5} = \left(\dfrac{2}{3}\right)^5 - \left(\dfrac{1}{2}\right)^5$

(B) The number appearing on upper face of any dice can be 1, 2, 3, or 4 i.e. maximum 4 cases.

P(n = 4) = P(n ≤ 4) - P (n ≤ 3) = $\dfrac{4^5 - 3^5}{6^5} = \left(\dfrac{2}{3}\right)^5 - \left(\dfrac{1}{2}\right)^5$

(C) P(2 ≤ m ≤ 4) = P(m ≥ 2) - P(m ≥ 5)

= $\dfrac{5^5 - 2^5}{6^5} = \left(\dfrac{5}{6}\right)^5 - \left(\dfrac{1}{3}\right)^5$

(D) P(m = 2, n = 5) = P(2, 3, 4 or 5) - P(2, 3 or 4) - P (3, 4 or 5) + P(3 or 4)

= $\dfrac{4^5 - 2\times 3^5 + 2^5}{6^5} = \left(\dfrac{2}{3}\right)^5 - \left(\dfrac{1}{2}\right)^4 + \left(\dfrac{1}{3}\right)^5$

5. In a single throw with 3 dice, find the chances of throwing

	Column I		Column II

(A)	One-two-three	(P)	$\frac{5}{8}$
(B)	Sum of eleven	(Q)	$\frac{1}{2}$
(C)	Less than eleven	(R)	$\frac{1}{4}$
(D)	More than ten	(S)	$\frac{1}{8}$
		(T)	$\frac{1}{36}$

Key. A – T, B – S;C – Q, D – Q

Sol. (B) $x_1 + x_2 + x_3 = 11$ $1 \le x_i \le 6$, i = 1, 2, 3

no. of solution = 27

$\Rightarrow$ required probability $= \frac{27}{216} = \frac{1}{8}$

(C) $x_1 + x_2 + x_3 \le 10$

$\Rightarrow$ $x_1 + x_2 + x_3 + t = 10$

$1 \le x_i \le 6$, i = 1, 2, 3 $t \ge 0$

No. of solution = 108.

Required probability = $\frac{1}{2}$

6. Find the number of integers between 1 and 1000, both inclusive,

	Column I		Column II
(A)	Which are divisible by either of 10, 15 and 25,	(P)	54
(B)	Which are divisible by neither 10 nor 15 nor 25	(Q)	48
(C)	Which are divisible by at least two of 10, 15 or 25	(R)	146
(D)	Which are divisible by exactly two of 10, 15 or 25	(S)	352
		(T)	854

Key. A – R, B – T; C – P, D – Q

Sol. between 18000

numbers divisible by 10 = n(A) = 100

numbers divisible by 15 = n(B) = 66

numbers divisible by 25 = n(C) = 40

$n(A\cap B) = 33$, $n(A\cap C) = 20$ $n(B\cap C) = 13$ $n(A\cap B\cap C) = 6$

(A) $n(A\cup B\cup C) = 146$

(B) $n(\bar{A} \cap \bar{B} \cap \bar{C}) = 1000 - n(A \cup B \cup C) = 854$

(C) n(A∩B) + n(B∩C) + n(A∩C) – 2n(A∩B∩C) = 54

(D) n(A∩B) + n(B∩C) + n(A∩C) – 3(A∩B∩C) = 48

7. List – I | List – II

A) In the expansion of $\left(2^{x}+4^{-x}\right)^{n}$ if the ratio of second term to the third is $\frac{1}{7}$ and the sum of the coefficients of second and third terms is 36, then *x* value is : P) $\frac{1}{3}$

B) Let *n* be a positive integer. If the coefficients of 2nd, 3rd and 4th terms in the expansion of $(1+x)^{n}$ are in AP, then The value of *n* is : Q) 7

C) A basket contains 4 oranges, 5 apples and 6 mangoes. All of them are fruits. The number of ways that a person can select at least one fruit from the basket is : R) $\frac{-1}{3}$

D) If $31C_{3r}=31C_{r+3}$, then r equals : S) 209

Key. A → R; B → Q; C → S; D → Q

Sol. Conceptual

8. List – I | List - II

A) If $(2n+1)P_{n-1}:(2n-1)P_{n}=3:5$, then *n* is equal to P) 0

B) The position of the term independent of x in the expansion of $\left(x^{2}-\frac{1}{3x}\right)^{9}$ is Q) 4

C) The coefficient of x^{50} in the expansion of $(1+x)^{41}\left(1-x+x^{2}\right)^{40}$ is : R) 6

D) If $nC_{2}=nC_{5}$, then *n* value is : S) 7

Key. A → Q; B → S; C → P; D → S

Sol. Conceptual

9. List – I | List - II

A) The number of arrangements of the letters of the word 'BANANA' in which the two N's do not appear adjacently is : P) 40

B) the number of words that can be formed by using all the letters of the word IITJEE is : Q) 180

C) The number of divisions of the number $2^{2}.3^{2}.5.7^{9}$ is R) 5

D) If the coefficients of $(r-1)$ th term and $(2r+3)$ th in the expansion of $(1+x)^{15}$ are equal, then the value of r is : S) 6

Key. A $\rightarrow$ P; B $\rightarrow$ Q; C $\rightarrow$ Q; D $\rightarrow$ R

Sol. Conceptual

10. Match the following :

Column I		Column II	
(A)	The maximum number of points at which 5 straight lines intersect is	(p)	120
(B)	The number of distinct positive divisors of $2^4\ 3^5\ 5^3$ is	(q)	2^n-1
(C)	How many triangles can be drawn through 5 given points on a circle	(r)	${}^5C_2$
(D)	The value of $\sum_{r=1}^{n}\frac{{}^nP_r}{r!}$	(s)	${}^5C_3$

Key.

Sol. (A) Two straight lines intersect at only one point. For selecting two out of 5 straight lines is ${}^5C_2$. So maximum number of point of intersection is ${}^5C_2$.

(B) The number of distinct positive divisors of $2^4\ 3^5\ 5^3$ = (4 + 1) (5 + 1) (3 + 1) = 120

(C) Total number of triangles formed = ${}^5C_3$

(D) = ${}^nC_1 + {}^nC_2 + \ldots\ldots + {}^nC_n = 2^n - 1$

11. Match the Following: Let m and n be two positive integers such that $m > n$. The number of ways of

Column I		Column II	
(A)	Distributing m distinct books among n children	(p)	0
(B)	Arranging n distinct books at m places	(q)	${}^{m-1}C_{m-n}(m!)$
(C)	Selecting m persons out of n persons so that two particular persons are not selected	(r)	n^m
(D)	Distributing m distinct books among n children so that every child get at least one book	(s)	${}^mC_n(n!)$

Key.

Sol. (A) Each book can be given in n ways. Since, there are m books, the number of ways is n^m.

(B) We can choose n places out of m in ${}^mC_n$ ways and then can arrange n books at these places in $n!$ ways. Thus, the required number of ways in $\left({}^mC_n\right)(n!)$.

(C) When two particular persons are excluded the number of persons become $n-2$.

Since $m-n > n-2$, it is impossible to choose m persons out of $n-2$.

12. Match the following:

Column I		Column II	
(A)	Number of triangle that can be made using the vertices of a polygon of 10 sides as their vertices and having exactly one side common with the polygon is	(p)	75
(B)	Number of triangles that can be made using the vertices of a polygon of 10 sides as their vertices and having exactly 2 sides common with the polygon is	(q)	110
(C)	Number of quadrilaterals that can be made using the vertices of a polygon of 10 sides as their vertices and having exactly 2 sides common with the polygon	(r)	60
(D)	Number of quadrilaterals that can be made using the vertices of a polygon of 10 sides as their vertices had having 3 sides common with the polygon is	(s)	10

Key.

Sol. (A) No. of such triangles = 10 ${}^6C_1$ = 60

(B) No. of such triangles = 10

(C) Number of such quadrilaterals = 10 ${}^5C_1$ + = 75.

(D) Number of such quadrilaterals = 10 (when four consecutive points are taken)

13. Match the following :

COLUMN – I	COLUMN – II

A)	7 identical white balls and 3 identical black balls are placed in a row at random.The probability that no two black balls are adjacent is	P)	$\frac{5}{11}$
B)	4 gentlemen and 4 ladies take seats at random round a table. The probability that they are sitting alternately is	Q)	$\frac{1}{16}$
C)	10 different books and 2 different pens are given to 3 boys, so that each gets equal number of things. The probability that the same boy does not receive both the pens, is	R)	$\frac{7}{15}$
D)	A fair coin is tossed repeatedly. The probability of getting a result in the fifth toss different from those obtained in the first four tosses is	S)	$\frac{1}{35}$
		T)	$\frac{3}{16}$

Key. A-R, B-S, C-P, D-Q

Sol. (a) $n(S)=\frac{10!}{(7!)(3!)}$

$n(E)={}^{8}C_{3}=\frac{8!}{(3!)(5!)}$, because there are 8 places for 3 black balls.

$$\therefore P(E)=\frac{\frac{8!}{(3!)(5!)}}{\frac{10!}{(7!)(3!)}}=\frac{(8!)(7!)}{(10!)(5!)}=\frac{7.6}{10.9}=\frac{7}{15}$$

b) $n(S)=7!,\ n(E)=(3!)\times(4!)$

(Θ after making 4 gentlemen sit in 3! ways, 4 ladies can sit in 4! ways in between the gentlemen)

$$P(E)=\frac{(3!)\times(4!)}{7!}=\frac{6}{7\times 6\times 5}=\frac{1}{35}$$

c) $n(S)={}^{12}C_4\times{}^{8}C_4\times{}^{4}C_4$

$n(E)=n(S)-$ the number of ways in which one boy gets both the pens.

$=n(S)-{}^{10}C_2\times{}^{8}C_4\times{}^{4}C_4\times(3!)$

$$\therefore P(E)=1-\frac{{}^{10}C_2\times{}^{8}C_4\times{}^{4}C_4\times(3!)}{{}^{12}C_4\times{}^{8}C_4\times{}^{4}C_4}=\frac{5}{11}$$

d) Required probability= $P(E\ E\ E\ E\ \overline{E})+P(\overline{E}\ \overline{E}\ \overline{E}\ \overline{E}\ E)$

= $\{P(E)\}^4.P(\overline{E})+\{P(\overline{E})\}^4.P(E)=\frac{1}{16}$

14. MATCH THE FOLLOWING.

Column-I	Column-II

A)	If the letters of the word SUCCESS are arranged in all possible ways as in the dictionary then rank of the word SUCCESS is	P)	13
B)	A factor 'P' of 10000000099 lies between 9000 and 10,000 then sum of the digits of 'P' is	Q)	19
C)	Number of zeroes at the end of $83!$ is	R)	271
D)	There are 4 identical yellow strips3 identical red strips and 2 identical Pink strips, the number of flags with three strips in order can be formed	S)	331
		T)	20

Key. A-S,B-Q,C-Q,D-T

Sol. b) 10000000099 = x^5+x-1 where x= 100

$$x^5+x-1 = \left(x^2-x+1\right)\left(x^3+x^2-1\right)$$

Since x^2-x+1 is a factor $\Rightarrow$ 9901 is a factor

d) No. of required flags= $coeff\ of$

$$x^3\ in\ \lfloor 3\left[1+x+\frac{x^2}{\lfloor 2}+\frac{x^3}{\lfloor 3}+\frac{x^4}{\lfloor 4}\right]\left[1+x+\frac{x^2}{\lfloor 2}+\frac{x^3}{\lfloor 3}\right]\left[1+x+\frac{x^2}{\lfloor 2}\right]$$

= 20

15. A man and a women appear in an inter view for two vacancies. Probability man selected $=\frac{1}{4}$ and that of women selected $=\frac{1}{3}$. Then the probability

Column – I		Column – II	
(A)	Both will be selected	(p)	$\frac{1}{12}$
(B)	Only one of them selected	(q)	$\frac{5}{12}$
(C)	None is selected	(r)	$\frac{1}{2}$
(D)	At least one get selection	(s)	$\frac{1}{3}$

Key. A –P; B – Q; C – R; D – R

Sol. Conceptual

16. Let A and B be two independent events such that $P(A)=\frac{1}{3}$ and $P(B)=\frac{1}{4}$. Now match the following.

	Column – I		Column – II
(A)	$P(A\cup B)=$	(p)	$\frac{1}{12}$
(B)	$P\left(\frac{A}{A\cup B}\right)$	(q)	$\frac{1}{2}$
(C)	$P\left(\frac{B}{\bar{A}\cap\bar{B}}\right)$	(r)	$\frac{2}{3}$
(D)	$P\left(\frac{\bar{A}}{\bar{B}}\right)$	(s)	0

Key. A – Q; B – R; C – S; D – R

Sol. Conceptual

17. A bag has 6 red, 4 white, 8 blue balls. If 3 balls are drawn at random then the probability.

	Column – I		Column – II
(A)	All 3 balls are blue	(P)	0.466
(B)	Balls drawn are of different colour	(Q)	0.08
(C)	Balls drawn are of the same colour	(R)	0.24
(D)	No write Ball is drawn	(S)	0.068

Key. A –S; B – R; C – Q; D – P

Sol. Conceptual

18. List – I List – II

A) In the expansion of $\left(2^{x}+4^{-x}\right)^{n}$ if the ratio of second term to the third is $\frac{1}{7}$ and the sum of the coefficients of second and third terms is 36, then x value is : P) $\frac{1}{3}$

B) Let n be a positive integer. If the coefficients of 2^{nd}, 3^{rd} and 4^{th} terms in the expansion of $(1+x)^{n}$ are in AP, then

The value of *n* is : Q) 7

C) A basket contains 4 oranges, 5 apples and 6 mangoes. All of them are fruits. The number of ways that a person can select at least one fruit from the basket is : R) $\frac{-1}{3}$

D) If $31C_{3r} = 31C_{r+3}$, then r equals : S) 209

Key. A $\to$ R; B $\to$ Q; C $\to$ S; D $\to$ Q

Sol. Conceptual

19. List – I / List - II

A) If $(2n+1)P_{n-1} : (2n-1)P_n = 3:5$, then *n* is equal to P) 0

B) The position of the term independent of x in the expansion of $\left(x^2 - \frac{1}{3x}\right)^9$ is Q) 4

C) The coefficient of x^{50} in the expansion of $(1+x)^{41}(1-x+x^2)^{40}$ is : R) 6

D) If $nC_2 = nC_5$, then *n* value is : S) 7

Key. A $\to$ Q; B $\to$ S ; C $\to$ P ;D $\to$ S

Sol. Conceptual

20. List – I / List - II

A) The number of arrangements of the letters of the word 'BANANA' in which the two N's do not appear adjacently is : P) 40

B) the number of words that can be formed by using all the letters of the word IITJEE is : Q) 180

C) The number of divisions of the number $2^2.3^2.5.7^9$ is R) 5

D) If the coefficients of $(r-1)$ th term and $(2r+3)$th in the expansion of $(1+x)^{15}$ are equal, then the value of *r* is : S) 6

Key. A $\to$ P; B $\to$ Q; C $\to$ Q; D $\to$ R

Sol. Conceptual

21. Match the following:

Column -I		Column -II	
(A)	Number of integral solutions of $x+y+z=1, x \geq -4, y \geq -4, z \geq -4$ is	(p)	132

(B)	Greatest term in the expansions of $\frac{4}{3\sqrt{2}}\left(1+\frac{1}{\sqrt{2}}\right)^{12}$ is	(q)	99
(C)	If $a_1, a_2, a_3, \ldots\ldots, a_{100}$ are in H.P then value of $\sum_{i=1}^{99} \frac{a_i a_{i+1}}{a_1 a_{100}}$ is	(r)	105
(D)	If 8 points out of 11 points are in same straight line then the number of triangle formed is	(s)	109

Key. (A) $\rightarrow$(r); (B) $\rightarrow$(p); (C) $\rightarrow$(r); (D) $\rightarrow$(s)

Sol. A) $x+4=t_1$

$y+4=t_2$

$z+4=t_3$

$t_1+t_2+t_3=13$

No. of solutions ${}^{13+3-1}C_{3-1}$

$= {}^{15}C_2 = 105$

$$\frac{(n+1)|x|}{|x|+1} = \frac{13.\frac{1}{\sqrt{2}}}{\frac{\sqrt{2}+1}{\sqrt{2}}} = 13(0.44) = 5.382$$

B)

$|T_6|$ is greatest

$\frac{4}{3\sqrt{2}}\,{}^{12}C_5\left(\frac{1}{3\sqrt{2}}\right)=132$

$$C = \frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots\ldots \frac{1}{a_{100}} \rightarrow A.P$$

C)

D) No. of triangles ${}^{11}C_3 - {}^{8}C_3$

22. Match the following

Column I		Column II	
(A)	The number of permutations of the letters of the word HINDUSTAN such that neither the pattern HIN nor DUS nor TAN appears, are	(p)	169194
(B)	Taking all the letters of the word MATHEMATICS how many words can be formed in which either M or T are together?	(q)	$\frac{9.9!}{2!}$

(C)	The number of ways in which we can choose 2 distinct integers from 1 to 100 such that difference between them is at most 10 is	(r)	${}^{100}C_2 - {}^{90}C_2$
(D)	The total number of eight-digit numbers, the sum of whose digits is odd, is	(s)	45×10^6

Key. (A) $\rightarrow$(p); (B) $\rightarrow$(q); (C) $\rightarrow$(r); (D) $\rightarrow$(s)

Sol. (A) Total number of permutations $=\frac{9!}{2!}$

Number of those containing HIN $=7!$

Number of those containing DUS $=\frac{7!}{2!}$

Number of those containing TAN $=7!$

Number of those containing HIN and DUS $=5!$

Number of those containing HIN and TAN $=5!$

Number of those containing TAN and DUS $=5!$

Number of those containing HIN, DUS and TAN $=3!$

Required number $=\frac{9!}{2!}-\left(7!+7!+\frac{7!}{2}\right)+3\times5!-3!=169194$

(B) M 2, T 2, A 2, H 1, E 1, I 1, C 1, S 1

(Number of words in which both M are together) + (Number of words in which both T are together).

–(Number of words in which both T and both M are together) = required number of words

Required number of words $=\frac{10!}{2!2!}+\frac{10!}{2!2!}-\frac{9!}{2!}=\frac{5.9!+5.9!-9!}{2!}=\frac{9.9!}{2!}$

(C) Let the chosen integers be x_1 and x_2

Let there be 'a' integer before x_1, 'b' integer between x_1 and x_2 and 'c' integer after x_2

$\therefore a+b+c=98$. Where $a\geq 0, b\geq 10, c\geq 0$

Now if we consider the choices where difference is at least 11, then the number of solutions is ${}^{88+3-1}C_{3-1}={}^{90}C_2$

$\therefore$ Number of ways in which b is less than 10 is ${}^{100}C_2-{}^{90}C_2$

(D) The numbers will vary from 10000000 to 99999999. If sum of digits of a particular number is even, then the sum of digits of its next consecutive number will be odd.

As sum of digits of first number is odd and sum of digits of last number is even.

So number of numbers with sum of digits as odd

$=\frac{\text{total number of 8 − digit numbers}}{2}=\frac{90000000}{2}=45\times10^6$

23. Two dice are thrown. Let A be the event that sum of the points on the two dice is odd and B be the event that atleast one 3 is there, then match the following

Column I		Column II	
(A)	$P(A\cup B)$	(p)	$\frac{12}{36}$
(B)	$P(A\cap B)$	(q)	$\frac{6}{36}$
(C)	$P(A\cap \overline{B})$	(r)	$\frac{23}{36}$
(D)	$P(B)$	(s)	$\frac{11}{36}$

Key. (A) $\rightarrow$(r); (B) $\rightarrow$(q); (C) $\rightarrow$(p); (D) $\rightarrow$(s)

Sol. (ii) $P(A)=\frac{18}{36}$

$$P(B)=1-\left(\frac{5}{6}\right)^2=\frac{11}{36}$$

$$P(A\cap B)=\frac{6}{36}$$

$$P(A\cup B)=\frac{18}{36}+\frac{11}{36}-\frac{6}{36}=\frac{23}{36}$$

24. Match the following

Column I		Column II	
(a)	Out of four faulty machines, exactly two are faulty, they are tested one by one in a random order till both faulty machines are identified then the probability that only two test are needed.	(p)	$1/2$
(b)	A die with 6 faces marked 1,1,4,3,3,3, is tossed twice. Find the probability of getting sum 4.	(q)	$1/4$
(c)	$(a_1,b_1c_1),(a_2,b_2c_2)$ and (a_3,b_3c_3) are direction ratios of three perpendicular lines and direction ratio of line equally inclined to them is given by $k(a_1+a_2+a_3),k(b_1+b_2+b_3),k(c_1+c_2+c_3)$ .Then k is given by	(r)	$1/3$
(d)	If three points are lying in a plane what is the probability that a triangle will be formed by joining them.	(s)	$1/6$

Key. (a) (r); (b) (r); (c) (p,q,r,s); (d) (p)

Sol. (A) P(two faulty identified) + p(two correct identified)

$$2\left(\frac{{}^{2}C_{2}}{{}^{4}C_{2}}\right)=2\left(\frac{1.2}{4.3}\right)=2\left(\frac{2}{4}.\frac{1}{3}\right)=\frac{1}{3}$$

(B) $n(s)=6\times6=36$

$n(E)=12$ p= $\frac{12}{36}=\frac{1}{3}$

(C) k may be any real constant

(D) $n(E)=1, n(S)=2$

25. Match the following:

Column -I		Column -II	
(A)	The number of arrangements of the letters of the word BANANA in which the two Ns do not appear adjacently is	(p)	40
(B)	The number of words that can be formed by using all the letters of the word IITJEE is	(q)	180
(C)	The number of divisors of the number $2^2.3^2.5.7^9$ is	(r)	209
(D)	A basket contains 4 oranges, 5 apples and 6 mangoes. All of them are fruits. The number of ways that a person can select atleast one fruit from the basket is	(s)	6

Key. (A) $\rightarrow$(p); (B) $\rightarrow$(q); (C) $\rightarrow$(q); (D) $\rightarrow$(r)

Sol. Conceptual

26. Match the following:

Column -I		Column -II	
(A)	The total number of selections containing one or more fruits which can be made from 3 bananas, 4 apples and 2 oranges is	(p)	Greater than 50
(B)	If 7 points out of 12 distinct points are collinear and no three of remaining points are collinear then the number of triangles formed is	(q)	Greater than 100
(C)	The number of ways of selecting 10 balls from unlimited number of Red, Black, White and Green balls is	(r)	Greater than 150
(D)	The total number of divisors of 38808 is	(s)	Greater than 200

Key. (A) $\rightarrow$(p); (B) $\rightarrow$(p, q, r); (C) $\rightarrow$(p, q, r, s) ; (D) $\rightarrow$(p)

Sol. Conceptual

27. Match the following:

	Column -I		Column -II
(A)	The number of ways in which 12 Red balls, 12 Black balls, 12 White balls can be given to 2 children so that each gets 18 is (Assume that balls of same colour are identical)	(p)	125
(B)	The number of ways of forming two teams from 5 boys and 5 girls so that each team has 5 children and in each team there are children of different genders is	(q)	127
(C)	Six bundles of books are to be kept in 6 distinct boxes one in each box. If two of the boxes are too small for three of the bundles, the number of ways of keeping the bundles in the boxes is	(r)	135
(D)	A bag contain 30 balls of 5 different colours, the number of balls of each colour being same. The balls are numbered from 1 to 6 in each colour. The number of ways of drawing two balls from the bag such that the balls are of the same colour or of the same number is	(s)	144

Key. (A) $\rightarrow$(q); (B) $\rightarrow$(p); (C) $\rightarrow$(s); (D) $\rightarrow$(r)

Sol. Conceptual

28. Match the following:

In a tournament, there are sixteen players $S_1, S_2, S_3, \ldots\ldots, S_{16}$ and divided into eight pairs at random. From each game a winner is decided on the basis of a game played between the two players of the pair. Assuming that all the players are of equal strength

	Column –I		Column -II
(A)	The Probability that S_1 is one of the winners	(p)	$\frac{8}{15}$
(B)	The Probability that exactly one of S_1 or S_2 is a winner	(q)	$\frac{1}{2}$
(C)	The probability that Both S_1 and S_2 are among eight winners	(r)	$\frac{7}{30}$

(D)	The probability that none of S_1 and S_2 is among eight winners	(s)	$\frac{4}{8}$
		(t)	$\frac{23}{30}$

Key. (A) $\rightarrow$(q, s); (B) $\rightarrow$(p); (C) $\rightarrow$(r); (D) $\rightarrow$(r)

Sol. give one victory to S_1 then, remaining 7 wins are to be given to any of 15 players.

$\Rightarrow P(S_1$ is the winner) $= \frac{^{15}C_7}{^{16}C_8} = \frac{1}{2}$

Now P(S_1 or S_2 (not both)are winner) $= \frac{2\times^{14}C_7}{^{16}C_8} = \frac{8}{15}$

Finally P(both S_1 or S_2 are winners) $= \frac{^{14}C_6}{^{16}C_8} = \frac{7}{30}$

29. Match the following

Four digit numbers without repetition are formed using the digits 1, 2, 3, 4, 5, 6, 7, 8. One of these numbers so formed is picked up at random. The probability that the selected number is

Column I		Column II	
(A)	divisible by 2 is	(p)	$\frac{5}{8}$
(B)	divisible by 4 is	(q)	$\frac{1}{8}$
(C)	divisible by 5 is	(r)	$\frac{2}{3}$
(D)	neither divisible by 5 nor by 4 is	(s)	$\frac{1}{2}$
		(t)	$\frac{1}{4}$

Key. (A) $\rightarrow$(s); (B) $\rightarrow$(t); (C) $\rightarrow$(q); (D) $\rightarrow$(p)

Sol. Conceptual

30. Match the following: One word is chosen at random from all the words that can be formed (with or without meaning) by arranging all the letters of the word INTERMEDIATE. Then the probability that in the word

Column -I		Column -II	
(A)	vowels occupy the position of vowels in the given word	(p)	$\frac{1}{462}$

(B)	all the vowels retain their respective positions in the given word is	(q)	$\frac{1}{55440}$
(C)	the vowels are together and the consonants are together is	(r)	$\frac{1}{12}$
(D)	the first letter is M is	(s)	$\frac{1}{984}$
		(t)	$\frac{1}{924}$

Key. (A) $\to$(t); (B) $\to$(q); (C) $\to$(p); (D) $\to$(r)

Sol. Conceptual

31.

A is a set containing n elements. A subset P of A is chosen at random. The set A is reconstructed by replacing the elements of the subset P. A subset Q of A is again chosen at random. The probability that where $|X|$ = number of elements in X

Column -I		Column -II	
(A)	$P \cap Q = \phi$	(p)	$\frac{n\left(3^{n-1}\right)}{4^n}$
(B)	$P \cap Q$ is a singleton	(q)	$\left(\frac{3}{4}\right)^n$
(C)	$P \cap Q$ contains 2 elements	(r)	$\frac{{}^{2n}C_n}{4^n}$
(D)	$\lvert P\rvert = \lvert Q\rvert$	(s)	$\frac{3^{n-2}\left(n(n-1)\right)}{2\left(4^n\right)}$

Key. (A) $\to$(q); (B) $\to$(p); (C) $\to$(s); (D) $\to$(r)

Sol.
$$P(P \cap Q = \phi) = \left(\frac{3}{4}\right)^n$$

$$P(P \cap Q \text{ has one element}) = \frac{n \times 3^{n-1}}{4^n}$$

$$P(P \cap Q \text{ has two elements}) = {}^nC_2 \frac{3^{n-2}}{4^n}$$

$$P(\lvert P\rvert = \lvert Q\rvert) = \frac{\left({}^nC_0\right)^2 + \left({}^nC_1\right)^2 + \ldots + \left({}^nC_n\right)^2}{4^n} = \frac{{}^{2n}C_n}{4^n}$$

32. One word is chosen at random from all the words that can be formed (with or without meaning) by arranging all the letters of the word INTERMEDIATE. Then the probability that in the word

<u>Column – I</u>
<u>Column – II</u>

a) Relative positions of vowels and consonants in the given word remain unaltered is p) 1/462

b) Each vowel retains its original position in the given word is q) 1/55440

c) The vowels are together and the consonants are together is r) 1/12

d) The first letter is M is s) 1/924

Key. a) s; b) q; c) p; d) r

Sol. Conceptual

33. Four digit numbers without repetition are formed using the digits 1, 2, 3, 4, 5, 6, 7, 8. One of these numbers so formed is picked up at random. The probability that the selected number is

<u>Column – I</u>	<u>Column – II</u>
a) divisible by 2 is	p) 5/8
b) divisible by 4 is	q) 1/8
c) divisible by 5 is	r) 1/4
d) neither divisible by 5 nor by 4 is	s) 1/2

Key. a) s; b) r; c) q; d) p

Sol. Conceptual

34. A straight line with negative slope passes through the point (8, 1) and cuts the coordinate axes at *A, B*. *O* is the origin

<u>Column – I</u>	<u>Column – II</u>
a) The minimum area of ΔAOB is	p) 26
b) The minimum length of AB is	q) $5\sqrt{5}$
c) The minimum value of $OA+OB$ is	r) $9+4\sqrt{2}$
d) The minimum perimeter of ΔAOB is	s) 16

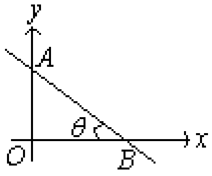
Key. a) s; b) q; c) r; d) p

Sol. $OA+OB=9+8\tan\theta+\cot\theta, 0<\theta<\frac{\pi}{2}$

$AB=8\sec\theta+\operatorname{cosec}\theta$

Area of $\Delta AOB=8+\frac{1}{2}(\cot\theta+64\tan\theta)$

Perimeter of $\Delta AOB=9+8(\tan\theta+\sec\theta)+(\cot\theta+\operatorname{cosec}\theta)$



35. 'n' whole numbers are randomly chosen and multiplied, then probability that

Column-I	Column-II
a) The last digit is 1,3,7 or 9	p) $\frac{8^n-4^n}{10^n}$
b) The last digit is 2,4,6,8	q) $\frac{5^n-4^n}{10^n}$
c) The last digit is 5	r) $\frac{4^n}{10^n}$
d) The last digit is zero	s) $\frac{10^n-8^n-5^n+4^n}{10^n}$

Key. a) r; b) p; c) q; d) s

Sol. (a)The required event will occur if last digit in all the chosen numbers is 1,3,7 or 9

$\therefore$ Required Probability $=\left(\frac{4}{10}\right)^n$

(b) Required Probability = P (that the last digit is 1,2,3,4,6,7,8,9) – P(that last digit is 1,3,7,9)

$=\frac{8^n-4^n}{10^n}$

(c) P(1,3,5,7,9)-P(1,3,7,9) = $=\frac{5^n-4^n}{10^n}$

(d) P (0,5) – P(5) = $=\frac{\left(10^n-8^n\right)-\left(5^n-4^n\right)}{10^n}$

36. Let A and B be events $P(A)=\frac{1}{3}, P(B)=\frac{1}{4}$ and $P(A\cup B)=\frac{1}{2}$.

Column-I	Column-II
a) $P\left(\frac{B^C}{A^C}\right)$	p) $\frac{1}{4}$
b) $P\left(\frac{B}{A^C}\right)$	q) $\frac{3}{4}$
c) $P\left(A^C/B^C\right)$	r) $\frac{1}{3}$
d) $P(B/A)$	s) $\frac{2}{3}$

Key. a) q; b) p; c) s; d) p

Sol. a) $P\left(\frac{B^C}{A^C}\right)=\frac{P\left(\overline{AB}\right)}{P\left(\overline{A}\right)}=\frac{3}{4}$

b) $P\left(\frac{B}{A^C}\right)=\frac{P\left(B\overline{A}\right)}{P\left(\overline{A}\right)}=\frac{1}{4}$

c) $P\left(\frac{A^C}{B^C}\right)=\frac{P\left(\overline{AB}\right)}{P\left(\overline{B}\right)}=\frac{2}{3}$

d) $P\left(\frac{B}{A}\right)=\frac{P(AB)}{P(A)}=\frac{1}{4}$

37. Column-I Column-II

a) A box contain 6 balls of unknown colours.3 balls are drawn and they are found to the white. Then the probability that all the 6 balls are white is — p) 31/42

b) If a 4 digit number is formed at random using 0,1,2,3,4,5 Then the probability that it is divisible by 6 is — q) 4/7

c) A purse contains 4 one rupee coins and 6 ten paise coins 6 ten paise coins. If 5 coins are selected at random, then the Probability that the sum on the exceed Rs. 2.25 is — r) 1/3

d) Four digit numbers are formed with 1,2,3,4,5,6 with repetition Then the probability that they are divisible by 3 is — s) 13/75

Key. a) q; b) s; c) p; d) r

Sol. (a) Required Probability $=\frac{{}^6C_3}{{}^3C_3+{}^4C_3+{}^5C_3+{}^6C_3}=\frac{4}{7}$

(b) Using $1,2,4,5=3!\times 2$

Using $0,3,4,5=3!+(3!-2!)=10$

Using $0,2,3,4=3!+2(3!-2!)=14$

Using $0,1,3,5=3! \qquad =6$

Using $0,1,2,3=3!+(3!-2!)=10$

$n(E)=52$

$$P(E)=\frac{52}{{}^6P_4-{}^5P_3}=\frac{13}{75}$$

(c)

Rupee (4)	Paise (6)
2	3
3	2
4	1

Required Probability $=\frac{{}^4C_3.{}^6C_2+{}^4C_2.{}^6C_3+{}^4C_4.{}^6C_1}{{}^{10}C_5}$

(d) $n(E)=6\times6\times6\times2$

$n(s)=6\times6\times6\times6$

$P(E)=\frac{1}{3}$

38.

	Column I		Column II
(A)	If 3 identical dice are rolled once the probability that the 3 numbers on them are different	(p)	$\frac{1}{6}$
(B)	The probability of selecting a divisor of the form 4n + 2, n = 0,1,2……. of the integer 720 if a divisor is selected at random is	(q)	$\frac{5}{14}$
(C)	If 3 numbers are selected from the set of first 10 natural numbers the probability that they form an A.P. is	(r)	$\frac{7}{15}$
(D)	The probability of selecting 3 men out of 10 men sitting in a row so that no two of them are from adjacent seats is	(s)	$\frac{1}{5}$
		(t)	$\frac{2}{5}$

Key. A- q; B- s; C- p; D- r

Sol. Conceptual

39. Let A and B be two events such that P(A) = 0.4 and $P(A\cup B)=0.7$ then

	Column I		Column II
(A)	If A and B are disjoint then P(B) =	(p)	0.5
(B)	If A is a subset of B then P(B) =	(q)	0.7
(C)	$P\left(\overline{A}\cap\overline{B}\right)$ =	(r)	0.3
(D)	If A and B are disjoint then $P\left(\overline{A}\cap B\right)$=	(s)	0.2
		(t)	0.1

Key. A- r; B- q; C- r; D- r

Sol. Conceptual

40. There are 2 Indian couples, 2 American couples and one unmarried person

Column-I	Column-II
a) The total number of ways in which they can sit in a row such that an Indian wife and American wife are always on either side of the unmarried person, is	p) 22680
b) The total numbers of ways in which they can sit in row such that an unmarried person always occupy the middle position is	q) 5760
c) The total number of ways in which they can sit round a circular table such that an Indian wife and an American wife are always on either side of the unmarried person, is	r) 40320
d) If all the nine persons are to be interviewed one by one then the total number of ways of arranging their interviews such that no wife gives interview before her husband, is	s) 24320

Key. a) r; b) r; c) q; d) p

Sol. a) one Indian wife and one American wife can be selected in ${}^{2}C_{1}\times{}^{2}C_{1}$ ways and keeping an unmarried person in between these two wives the total number of linear arrangements are ${}^{2}C_{1}\times{}^{2}C_{1}\times\underline{|7}\times\underline{|2}=40320$

b) Required number of ways $\underline{|8}=40320$

c) Required number of ways $\underline{|(7-1)}\times\underline{|2}\times{}^{2}C_{1}\times{}^{2}C_{1}=5760$

d) Number of ways in which interviews can be arranged $=9\times{}^{8}C_{2}\times{}^{6}C_{2}\times{}^{4}C_{2}\times{}^{2}C_{2}=22680$

41. Column-I | Column-II

a) The number of positive unequal integral solutions of the equation $x+y+z+t=20$ — p) 504

b) The number of zeros at the end of $\underline{|100}$ is — q) 36

c) Number of congruent triangles that can be formed using the vertices of a regular polygon of 72 vertices such that the number of vertices of the polygon between any two consecutive vertices of triangle must be same, is — r) 24

d) The number of ways in which the letters of the word " SUNDAY" be arranged so that they neither begin with s nor end with Y, is — s) 552

Key. a) s ; b) r; c) r; d) p

Sol. a)We can assume that $x<y<z<t$ without loss of genterality. Now put $x_1=x, x_2=y-x, x_3=z-y$ and $x_4=t-z$, Then $x_1,x_2,x_3,x_4\geq 1$ and the given equation becomes $4x_1+3x_2+2x_3+x_4=20$.The number of positive integer solutions of this equation = 552

b) $\underline{|100}=2^{97}\times 3^{b}\times 5^{24}\times 7^{d}\times\ldots.$

c) $\dfrac{72}{3}=24$

d) $6!-2(5!)+4!=504$

42. Match the following

Column-I | Column-II

a) The number of ways of answering one or more of n different questions is — p) $\dfrac{{}^{n}P_{r}}{2r}$

b) The number of ways of answering one or more of n different questions when each question has an alternative is — q) 2^{n}

c) The number of circular permutations of n different things taken r at a time is r) $\frac{^nP_r}{r}$

d) The number of circular permutations of n different things taken r at a time, given that an anticlockwise and a clockwise arrangement in the same order are considered to be equivalent is s) $3^n - 1$

t) $2^n - 1$

Key. a-t, b-s, c-r, d-p

Sol. Conceptual

43. Match the following

Given a convex octagon. The no. of triangles that can be formed having

Column-I	Column-II
a) one side common with the octagon	p) 16
b) two sides common with the octagon	q) 7
c) no side common with the octagon	r) 32
d) the number of diagonals of the octagon	s) 20
	t) 8

Key. a-r, b-t, c-p, d-s

Sol. Conceptual

Match the following

44. One word is chosen at random from all the words that can be formed (with or without meaning) by arranging all the letters of the word INTERMEDIATE. Then the probability that in the word

Column – I	Column – II
a) vowels occupy the position of vowels in the given word	p) 1/462
b) all the vowels retain their respective positions in the given word is	q) 1/55440
c) the vowels are together and the consonants are together is	r) 1/12
d) the first letter is M is	s) 1/984
	t) 1/924

Key. a) t; b) q; c) p; d) r

Sol. Conceptual

45. Four digit numbers without repetition are formed using the digits 1, 2, 3, 4, 5, 6, 7, 8. One of these numbers so formed is picked up at random. The probability that the selected number is

Column – I	Column – II
a) divisible by 2 is	p) 5/8
b) divisible by 4 is	q) 1/8
c) divisible by 5 is	r) 2/3
d) neither divisible by 5 nor by 4 is	s) 1/2
	t) 1/4

Key. a) s; b) t; c) q; d) p

Sol. Conceptual

46. Match the following:

Column -I   Column -II

(A)	An urn contains five balls, two balls are drawn and are found to be white. If probability of all the balls in urn are white is $'k'$, then	(p)	$\frac{2}{5} < k < \frac{3}{4}$
(B)	Out of 15 consecutive integers three are selected at random, then the probability of the sum is divisible by 3 is $'k'$, then	(q)	$\frac{1}{6} < k < \frac{1}{3}$
(C)	If 3 cards are placed at random and independently in 4 boxes lying in a straight line. Then the probability of the cards going into 3 adjacent boxes so that each box contain one card, is $'k'$, then	(r)	$\frac{2}{3} < k < 1$
(D)	A box contains 4 balls which are either red or black, 2 balls are drawn and found to be red if these are replaced, then the probability that next draw will result in a red ball is $'k'$, then	(s)	$\frac{1}{7} < k < \frac{2}{3}$
		(t)	$\frac{1}{6} < k < \frac{1}{2}$

Key. A $\rightarrow$ p, s; B $\rightarrow$ s, t; C $\rightarrow$ q, s, t; D $\rightarrow$ r

Sol. A) Let $A_i\ (i = 1,2,3,4)$ be the event that urn contains $2,3,4$ or $5W$ balls and B the event that two white balls is drawn we have to find $P\left(\frac{A_4}{B}\right)$

Applying, Bayes theorem

$$P\left(\frac{A_4}{B}\right) = \frac{\frac{1}{4}\times 1}{\frac{1}{4}\times\frac{1}{10}+\frac{1}{4}\times\frac{3}{10}+\frac{1}{4}\times\frac{6}{10}+\frac{1}{4}\times 1} = \frac{1}{2}$$

B) $n, n+3, \ldots\ldots m+12$

$n+1, \ldots\ldots$

$n+2, \ldots\ldots, n+14$

Required probability $= \frac{3\times {}^5C_3 + 5^3}{{}^{15}C_3} = \frac{31}{91}$

C) Required probability $= \frac{3!\,(2)}{4\times 4\times 4} = \frac{3}{16}$

D) For the balls in box there are three possibility

i) All the 4 balls are red

ii) 3 of the 4 balls are red

iii) 2 of the 4 balls are red

Let these represented by E_1, E_2 and E_3 respectively.

Assuming that E_1, E_2 and E_3 are given to be equally likely

$P(E_1)=P(E_2)=P(E_3)=\frac{1}{3}$

Let A be the event two balls drawn are red

$P\left(\frac{A}{E_1}\right)=\frac{{}^4C_2}{{}^4C_2}=1,\ P\left(\frac{A}{E_2}\right)=\frac{{}^3C_2}{{}^4C_2}=\frac{1}{2}$ and $P\left(\frac{A}{E_3}\right)=\frac{{}^2C_2}{{}^4C_2}=\frac{1}{6}$

$P(A)=\sum_{i=1}^{3}P(E_i).P\left(\frac{A}{E_i}\right)=\frac{1}{3}\times 1+\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times\frac{1}{6}=\frac{5}{9}$

Thus, $P\left(\frac{E_1}{A}\right)=\frac{\frac{1}{3}\times 1}{\frac{5}{9}}=\frac{3}{5},\ P\left(\frac{E_2}{A}\right)=\frac{\frac{1}{3}\times\frac{1}{2}}{\frac{5}{9}}=\frac{3}{10}$ and $P\left(\frac{E_3}{A}\right)=\frac{\frac{1}{3}\times\frac{1}{6}}{\frac{5}{9}}=\frac{1}{10}$

Now, denote the events $\frac{E_1}{A},\frac{E_2}{A},\frac{E_3}{A}$ by F_1,F_2,F_3 respectively, then

we have $P(F_1)=\frac{3}{5},\ P(F_2)=\frac{3}{10},\ P(F_3)=\frac{1}{10}$

Let B is the event that next draw is red ball.

$P\left(\frac{B}{F_1}\right)=1,\ P\left(\frac{B}{F_2}\right)=\frac{3}{4},\ P\left(\frac{B}{F_3}\right)=\frac{2}{4}$

$P(B)=\frac{3}{5}\times 1+\frac{3}{10}\times\frac{3}{4}+\frac{1}{10}\times\frac{2}{4}=\frac{7}{8}$

47. We are given M urns, numbered 1 to M and n balls (n<M) and P(A) denote the probability that each of the urns numbered 1 to n, will contain exactly one ball.

	Column I		Column II
(A)	If the balls are different and any number of balls can go to any urn then P(A)=___	(p)	$\frac{1}{{}^MC_n}$
(B)	If the balls are identical and any number of balls can go to any urn then P(A)=___	(q)	$\frac{1}{{}^{(M+n-1)}C_{M-1}}$
(C)	If the balls are identical but at most one ball can be put in any box, then P(A)=___	(r)	$\frac{n!}{{}^MC_n}$
(D)	If the balls are different and at most one ball can be put in any box, then P(A)=___	(s)	$\frac{n!}{M^n}$

Key: $(a\to s, b\to q, c\to p, d\to p)$

Hint: a) $n(s)=m^n\ \ n(A)=n!\Rightarrow P(A)=\frac{n!}{M^n}$

b) $n(s) = {}^{(M+n+1)}C_{M-1}$ n(A)=1 $\Rightarrow P(A) = \frac{1}{{}^{M+n+1}C_{M-1}}$

c) $n(s) = {}^{M}C_{n}$ $n(A) = 1 \Rightarrow P(A) = \frac{1}{{}^{M}C_{n}}$

d) $n(s) = {}^{M}C_{n}\ n!$ $n(A) = n! \Rightarrow P(A) = \frac{1}{{}^{M}C_{n}}$

48. Match the following

Column-I		Column-II	
(A)	There are two balls in an urn. Each ball is equally likely to be black or white. A white ball is now put in the urn. What is the chance of drawing a white ball now	(p)	$\frac{1}{2}$
(B)	From a group of 10 persons consisting of 5 lawyers, 3 doctors and 2 engineers, four persons are selected at random. The probability that the selection contains at least one person of each category is	(q)	$\frac{1}{10}$
(C)	If number of ways of distributing 5 identical books among 3 persons is n then $\frac{n}{35} - \frac{1}{2} =$	(r)	$\frac{2}{3}$
(D)	Eccentricity of the conic represented by complex equation $\lvert z-3\rvert + \lvert z+3\rvert = 9$ is	(s)	$\frac{8}{13}$

Key: $A \to r; B \to p; C \to q; D \to r$

Hint: (A) There can be three possibility in the urn

(i) W, W corresponding probability $= \frac{1}{4}$

(ii) B, W corresponding probability $= \frac{1}{2}$

(iii) B, B corresponding probability $= \frac{1}{4}$

$$P(W) = \frac{1}{4} \times 1 + \frac{1}{2} \times \frac{2}{3} + \frac{1}{4} \times \frac{1}{3} = \frac{2}{3}$$

(B) $n(S) = {}^{10}C_4 = 210$

$n(E) = 105$

(C) Let ith person gets x_i books

$\Rightarrow x_1 + x_2 + x_3 = 5$

No. of ways $= {}^{5+3-1}C_{3-1} = {}^{7}C_2 = 21$

$\Rightarrow \frac{21}{35} - \frac{1}{2} = \frac{1}{10}$

(D) $PS + PS' = 9 = 2a$

$SS' = 6 = 2ac$

$\Rightarrow e = \dfrac{6}{9} = \dfrac{2}{3}$

49. 'n' whole numbers are randomly chosen and multiplied, then probability that

	Column I		Column II
(A)	The last digit is 1, 3, 7 or 9	(p)	$\dfrac{8^n - 4^n}{10^n}$
(B)	The last digit 2, 4, 6, 8	(q)	$\dfrac{5^n - 4^n}{10^n}$
(C)	The last digit is 5	(r)	$\dfrac{4^n}{10^n}$
(D)	The last digit is zero	(s)	$\dfrac{10^n - 8^n - 5^n + 4^n}{10^n}$

Key: (A-r), (B-p), (C-q), (D-s)

Hint (A) (a – r) The required event will occur if last digit in all the chosen numbers is 1, 3, 7 or 9.

$\therefore$ Req. probability = $\left(\dfrac{4}{10}\right)^n$.

(B) required probability = P (that the last digit is (2, 4, 6, 8) = P(That the last digit is 1, 2, 3, 4, 6, 7, 8, 9) –P (that the last digit is 1, 3, 7, 9) = $\dfrac{8^n - 4^n}{10^n}$.

(C) P(1, 3, 5, 7, 9) – P(1, 3, 7, 9)

= $\dfrac{5^n - 4^n}{10^n}$

(D) required prob = P(0, 5) – P(5)

= $\dfrac{(10^n - 8^n) - (5^n - 4^n)}{10^n}$

= $\dfrac{10^n - 8^n - 5^n + 4^n}{10^n}$.

50. *Match the following.*

	Column I		Column II
(A)	If the total number of points of intersection of the diagonals inside the convex polygon of n sides is 210 then n =	(P)	18
(B)	A convex polygon have equal number of sides and diagonals then the number of sides is	(Q)	16
(C)	A regular convex polygon with an even number of sides is inscribed in a circle. If	(R)	10

	126 diagonals of the polygon do not passes through the centre of the circle in which it is inscribed, then the number of sides of the polygon is		
(D)	If n lines (no two lines are parallel and no three lines are concurrent) drawn in a plane divide the plane into 172 regions then n =	(S)	5

Key. A-R, B-S, C – P, D - P

Sol. (A) $^{n}C_4 = 210 = ^{10}C_4 \Rightarrow n = 10$

(B) $n = \frac{n(n-3)}{2} \Rightarrow n = 5$

(C) If polygon has n sides (even) then $\frac{n}{2}$ of its diagonals will passes through the centre of the circle in which it inscribed

$\therefore {}^{n}C_2 - n - \frac{n}{2} = 126$

(D) $1 + \sum n = 172 \Rightarrow \sum n = 171 \Rightarrow n = 18$

51. *'n'* whole numbers are randomly chosen and multiplied.

	Column - I		Column – II
(a)	The probability that the last digit is 1, 3, 7 or 9 is	p.	$\frac{8^n - 4^n}{10^n}$
(b)	The probability that the last digit is 2, 4, 6, 8 is	q.	$\frac{5^n - 4^n}{10^n}$
(c)	The probability that the last digit is 5 is	r.	$\frac{4^n}{10^n}$
(d)	The probability that the last digit is zero is	s.	$\frac{10^n - 8^n - 5^n + 4^n}{10^n}$

Key. a$\rightarrow$r, b$\rightarrow$p, c$\rightarrow$q, d$\rightarrow$s

Sol. (a) The required event will occur if last digit in all chosen numbers is 1, 5, 7 or 9.

Required probability = $\left(\frac{4}{10}\right)^n$

(b) The required probability is equal to the probability that last digit is 2, 4, 6, 8

$P(1,2,3,4,5,6,7,8,9) = P(1,3,7,9) = \frac{8^n - 4^n}{10^n}$

(c) $P(1,3,5,7,9) = P(1,3,7,9) = \frac{5^n - 4^n}{10^n}$

(d) $P(0,5)-P(5)=\left(10^n-8^n\right)-\left(5^n-4^n\right)=\dfrac{10^n-8^n-5^n+4^n}{10^n}$

52. A box contains 10 chits numbered 1 to 10. Chits are drawn one by one with replacement. If m and n are the least and greatest numbers drawn, then match list I with the corresponding probability in list II

	Column I		Column II
(A)	$m \ge 4; n \le 8$	(P)	$\frac{29}{125}$
(B)	$m \le 4 ; n \ge 7$	(Q)	$\frac{1}{16}$
(C)	$m \ge 5 ; n \ge 8$	(R)	$\frac{243}{2000}$
(D)	$m \le 4; n \le 7$	(S)	$\frac{224}{625}$

Key. A – Q; B – S; C – R; D – P

Sol. A)$m \ge 4, n \le 8$; probability: $\left(\frac{5}{10}\right)^4 = \frac{1}{16}$

B)$m \le 4, n \ge 7$; probability: $\left(\frac{8}{10}\right)^4 - 2\left(\frac{4}{10}\right)^4 = \frac{224}{625}$

C)$m \ge 5, n \ge 8$; probability: $\left(\frac{6}{10}\right)^4 - \left(\frac{3}{10}\right)^4 = \frac{243}{2000}$

D)$m \le 4, n \le 7$; probability: $\left(\frac{7}{10}\right)^4 - \left(\frac{3}{10}\right)^4 = \frac{29}{125}$

Match the following

53. One word is chosen at random from all the words that can be formed (with or without meaning) by arranging all the letters of the word INTERMEDIATE. Then the probability that in the word

<u>**Column – I**</u>	<u>**Column – II**</u>
a) vowels occupy the position of vowels in the given word	p) 1/462
b) all the vowels retain their respective positions in the given word is	q) 1/55440
c) the vowels are together and the consonants are together is	r) 1/12

d) the first letter is M is | s) 1/984

t) 1/924

Key. a) t; b) q; c) p; d) r

Sol. Conceptual

54. Three distinct numbers a, b, c are chosen at random from the numbers 1, 2, …. 100. The probability that

A) a, b, c are in A.P p) $\frac{53}{161700}$

B) a, b, c are in G.P q) $\frac{1}{66}$

C) $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in G.P r) $\frac{1}{22}$

D) $a+b+c$ is divisible by 2 s) $\frac{1}{2}$

Key. A$\to$ q, B$\to$ p, C$\to$p, D$\to$s

Sol. $n(s)={}^{100}C_3$

a) 2b = a + c = even

$\Rightarrow a,c$ are both even or odd

$n(E)={}^{50}C_2+{}^{50}C_2=50\times 49$

b) Taking r = 2, 3, …….10,

a, b, c can be in GP in 53 way

c) $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in GP = a, b, c are in GP

d) P(a + b + c is even)

= 1 – P (a + b + c odd)

= $1-\left(\frac{{}^{50}C_3+{}^{50}C_1-{}^{50}C_2}{{}^{100}C_3}\right)$

55. In a single throw with 3 dice, find the chances of throwing

	Column I		Column II
(A)	One-two-three	(P)	$\frac{5}{8}$
(B)	Sum of eleven	(Q)	$\frac{1}{2}$
(C)	Less than eleven	(R)	$\frac{1}{4}$
(D)	More than ten	(S)	$\frac{1}{8}$
		(T)	$\frac{1}{36}$

Key. A – T, B – S, C – Q, D – Q

Sol. (B) $x_1 + x_2 + x_3 = 11$ $1 \le x_i \le 6$, i = 1, 2, 3

no. of solution = 27

$\Rightarrow$ required probability $= \dfrac{27}{216} = \dfrac{1}{8}$

(C) $x_1 + x_2 + x_3 \le 10$

$\Rightarrow$ $x_1 + x_2 + x_3 + t = 10$

$1 \le x_i \le 6$, i = 1, 2, 3 $t \ge 0$

No. of solution = 108.

Required probability = $\dfrac{1}{2}$

56. Find the number of integers between 1 and 1000, both inclusive,

	Column I		Column II
(A)	Which are divisible by either of 10, 15 and 25,	(P)	54
(B)	Which are divisible by neither 10 nor 15 nor 25	(Q)	48
(C)	Which are divisible by at least two of 10, 15 or 25	(R)	146
(D)	Which are divisible by exactly two of 10, 15 or 25	(S)	352
		(T)	854

Key. A – R, B – T, C – P, D – Q

Sol. between 18000

numbers divisible by 10 = n(A) = 100

numbers divisible by 15 = n(B) = 66

numbers divisible by 25 = n(C) = 40

$n(A\cap B) = 33$, $n(A\cap C) = 20$ $n(B\cap C) = 13$ $n(A\cap B\cap C) = 6$

(A) $n(A\cup B\cup C) = 146$

(B) $n(\bar{A}\cap\bar{B}\cap\bar{C}) = 1000 - n(A\cup B\cup C) = 854$

(C) $n(A\cap B) + n(B\cap C) + n(A\cap C) - 2n(A\cap B\cap C) = 54$

(D) $n(A\cap B) + n(B\cap C) + n(A\cap C) - 3(A\cap B\cap C) = 48$

57. Five unbiased cubical dies are rolled simultaneously. Let m and n be the smallest and the largest number appearing on the upper faces of the dice, then match the probabilities given in the column II corresponding to the events given in the column I:

	Column I		Column II
(A)	m = 3	(p)	$\left(\frac{2}{3}\right)^5$
(B)	n = 4	(q)	$\left(\frac{2}{3}\right)^5 + \left(\frac{1}{3}\right)^5 - \left(\frac{1}{2}\right)^4$
(C)	$2 \le m \le 4$	(r)	$\left(\frac{5}{6}\right)^5 - \left(\frac{1}{3}\right)^5$
(D)	m = 2 and n = 5	(s)	$\left(\frac{2}{3}\right)^5 - \left(\frac{1}{2}\right)^5$

Key. (A–s), (B–s), (C–r), (D–q)

Sol. The number appearing on upper face of any dice can be 3, 4, 5 or 6 i.e. maximum 4 cases.

P(m = 3) = P(m ≥ 3) - P(m ≥ 4) = $\frac{4^5 - 3^5}{6^5} = \left(\frac{2}{3}\right)^5 - \left(\frac{1}{2}\right)^5$

(B) The number appearing on upper face of any dice can be 1, 2, 3, or 4 i.e. maximum 4 cases.

P(n = 4) = P(n ≤ 4) - P (n ≤ 3) = $\frac{4^5 - 3^5}{6^5} = \left(\frac{2}{3}\right)^5 - \left(\frac{1}{2}\right)^5$

(C) P(2 ≤ m ≤ 4) = P(m ≥ 2) - P(m ≥ 5)

= $\frac{5^5 - 2^5}{6^5} = \left(\frac{5}{6}\right)^5 - \left(\frac{1}{3}\right)^5$

(D) P(m = 2, n = 5) = P(2, 3, 4 or 5) - P(2, 3 or 4) - P (3, 4 or 5) + P(3 or 4)

= $\frac{4^5 - 2\times 3^5 + 2^5}{6^5} = \left(\frac{2}{3}\right)^5 - \left(\frac{1}{2}\right)^4 + \left(\frac{1}{3}\right)^5$