CLASS : CC (Advanced)   **Binomial, Permutation & Combination, Probability**   **TEST-8**

M.M.: 72   **PART-A**   **Time: 60 Min.**

[SINGLE CORRECT CHOICE TYPE]

Q.1 to Q.5 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct.   **[5 × 3 = 15]**

Q.1   If the number of triplets (l, m, n) satisfying $1 \leq l \leq m \leq n \leq 20$ is $^{P}C_{q}$, where $l, m, n, p, q \in N$, then least value of $(p + q)$ is

(A) 9   (B) 12   (C) 25   (D) 27

Q.2   Coefficient of x^5 in $(1 + x^3)^{15}(1 + x^2)^{10}$ will be

(A) 100   (B) 150   (C) 200   (D) 250

Q.3   If one term is randomly selected from the expansion of the terms $\left(2^{\frac{1}{2}} + 3^{\frac{1}{3}} \right)^{30}$, then odds in favour of the term being irrational will be

(A) 5   (B) $\dfrac{1}{5}$ (C) $\dfrac{6}{25}$ (D) $\dfrac{25}{6}$

Q.4   A bag contains $(2n + 1)$ coins. It is known that n of these coin have a head on both sides, whereas the remaining $(n + 1)$ coins are fair. A coin is selected at random from the bag and tossed once. If the probability the toss results in a head is $\dfrac{31}{42}$, then n is equal to

(A) 10   (B) 11   (C) 12   (D) 13

Q.5   Let $S = \{1, 2, 3, 4, \ldots, 10\}$ and $f(x) = \dfrac{4\left(\sqrt{x+1} - \sqrt{3-x}\right)}{x-1}$.

If an element is selected from the set S then the probability that it lies in the range of $f(x)$ is

(A) $\dfrac{1}{5}$ (B) $\dfrac{3}{10}$ (C) $\dfrac{2}{5}$ (D) $\dfrac{1}{2}$

[PARAGRAPH TYPE]

Q.6 to Q.11 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct. **[6 × 3 = 18]**

Paragraph for question nos. 6 to 8

Consider, $f(x) = \left| \sin\left((2r_1 - 1)\frac{\pi}{6} \right)x \right| + \left| \cos\left(\frac{r_2\,\pi}{6} \right)x \right|.$

Let a fair dice is thrown twice and r_1, r_2 are the numbers obtained on the dice its first and second throw respectively.

Q.6 The probability that $f(1)$ is an integer, is

(A) $\dfrac{5}{9}$ (B) $\dfrac{4}{9}$ (C) $\dfrac{1}{3}$ (D) $\dfrac{2}{3}$

Q.7 The probability that $f(2)$ is irrational, is

(A) $\dfrac{7}{18}$ (B) $\dfrac{5}{9}$ (C) $\dfrac{2}{3}$ (D) $\dfrac{8}{9}$

Q.8 The probability that $f(x)$ has a period either 1 or 2 is

(A) $\dfrac{1}{9}$ (B) $\dfrac{5}{36}$ (C) $\dfrac{1}{6}$ (D) $\dfrac{1}{4}$

Paragraph for question nos. 9 to 11

A tosses 2 fair coins and B tosses 3 fair coins. The game is won by the person who throws greater number of heads. In case of tie the game is continued under identical rules until some one wins.

Q.9 Probability of the event that A wins the particular game is equal to

(A) $\dfrac{3}{16}$ (B) $\dfrac{1}{8}$ (C) $\dfrac{1}{7}$ (D) $\dfrac{1}{6}$

Q.10 Probability of A and B ties in a particular game is equal to

(A) $\dfrac{5}{16}$ (B) $\dfrac{7}{16}$ (C) $\dfrac{9}{16}$ (D) $\dfrac{3}{16}$

Q.11 Probability of A wins the game is equal to

(A) $\dfrac{1}{11}$ (B) $\dfrac{2}{11}$ (C) $\dfrac{3}{11}$ (D) $\dfrac{5}{11}$

[MULTIPLE CORRECT CHOICE TYPE]

Q.12 to Q.14 has four choices (A), (B), (C), (D) out of which **ONE OR MORE** may be correct. **[3 × 4 = 12]**

Q.12   In a knock out tournament of 8 players $P_i(i = 1, 2, 3, ... 8)$. Let A_k be the event that P_k reaches to the final. (In a match of P_i and P_j, P_i wins if $i > j$)

(A) $P(A_4) = \dfrac{1}{35}$ (B) $P(A_5) = \dfrac{4}{35}$ (C) $P(A_6) = \dfrac{10}{35}$ (D) All of above

Q.13   Number of ways in which 32 identical blankets can be distributed among 4 persons so that each of them gets number of blankets which are divisible by 2 but not by 4, is (are)

(A) $^9C_3$

(B) $^{19}C_3$

(C) $^9C_6$

(D) $^4C_1 + 5 \cdot {}^4C_2 + 10 \cdot {}^4C_3 + 10 \cdot {}^4C_4$

Q.14   If different words are formed using all the letters from the word 'INDIANIDOL' in which 'a' denotes number of words which contains 'INDIA' but not 'INDIAN' and 'b' denotes number of words which contains 'INDIAN' but not 'IDOL' then

(A) a + b = 6! + 2   (B) a + b = 6! – 2   (C) a – b = 4.5! + 2   (D) a – b = 4.5! – 2

PART-B
[MATRIX TYPE]   [3 + 3 + 3 + 3 = 12]

Q.1 has **four** statements (A, B, C, D) given in **Column-I** and **four** statements (P, Q, R, S, T) given in **Column-II**. Any given statement in **Column-I** can have correct matching with one or more statement(s) given in **Column-II**.

Q.1 Consider four functions, $f(x) = \sqrt{5 - x^2}$, $g(x) = \text{sgn}(x^3 - 3x - 1)$

$$h(x) = \left[\sin^{-1}\left(\frac{2x}{1+x^2} \right) \right] \text{ and } k(x) = \left(e^{|x|-1} - 1 \right)\left| x^4 - x^2 \right|.$$

A function is selected at random, three events A, B and C are defined as

A : Selected function is continuous.
B : Selected function is derivable.
C : Selected function has 3 or more integers in its range.
Let P(E) denotes the probability of occurence of event E.
Note : [y] and sgn (y) denotes greatest integer function and signum function of y respectively.

Column-I		Column-II	
(A)	The value of $P\left(\dfrac{\overline{B}}{A}\right) + P\left(\dfrac{A}{B}\right)$ is equal to	(P)	$\dfrac{3}{4}$
(B)	The value of $P\left(\dfrac{B}{C}\right) + P\left(\dfrac{\overline{A}}{C}\right)$ is equal to	(Q)	1
(C)	The value of $P(\overline{B} \cap C) + P\left(\dfrac{A}{C}\right)$ is equal to	(R)	$\dfrac{5}{4}$
(D)	The value of $P\left(\dfrac{C}{\overline{B}}\right) - P\left(\dfrac{(A \cap \overline{B})}{C}\right)$ is equal to	(S)	$\dfrac{3}{2}$
		(T)	$\dfrac{7}{4}$

PART-D
[INTEGER TYPE]

Q.1 to Q.3 are "Integer Type" questions. (The answer to each of the questions **are upto 4 digits**) **[3 × 5 = 15]**

Q.1 Let $a = \left(4^{1/401} - 1\right)$ and for each $n \geq 2$, let $b_n = {}^nC_1 + {}^nC_2 \cdot a + {}^nC_3 \cdot a^2 + \ldots\ldots + {}^nC_n \cdot a^{n-1}$. If the value of $(b_{2006} - b_{2005})$ is 4^k where $k \in N$. Find the value of k.

Q.2 Find the value of $\displaystyle\sum_{r=0}^{19} r(r+1) \frac{{}^{20}C_{r+1}}{{}^{20}C_r}$.

Q.3 Let there be 4 boys and 4 girls are standing in a row. If m is the number of ways in which all the girls are consecutive and n is the number of ways in which exactly 3 boys are consecutive. If p denotes the number of ways in which 5 gentlemen & 5 ladies can stand in a row such that ladies and gentlemen are alternate then find the value of $\dfrac{p}{m+n}$.

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ANSWER KEY
PART-A

Q.1 C	Q.2 B	Q.3 D	Q.4 A	Q.5 A
Q.6 C	Q.7 C	Q.8 A	Q.9 A	Q.10 A
Q.11 C	Q.12 ABCD	Q.13 ACD	Q.14 BC	

PART-B

Q.1   (A) S; (B) P; (C) R; (D) P

PART-D

Q.1   5   Q.2   1330   Q.3   2