Parabola

Single Correct Answer Type

1. A straight line through A(6, 8) meets the curve $2x^2 + y^2 = 2$ at B and C. P is a point on BC such that AB, AP, AC are in H.P, then the minimum distance of the origin from the locus of 'P' is

A) $\frac{1}{\sqrt{52}}$ B) $\frac{5}{\sqrt{52}}$ C) $\frac{10}{\sqrt{52}}$ D) $\frac{15}{\sqrt{52}}$

Key. A

Sol. $(6 + r\cos\theta, 8 + r\sin\theta)$ lies on $2x^2 + y^2 = 2$

$\Rightarrow (2\cos^2\theta + \sin^2\theta)r^2 + 2(12\cos\theta + 8\sin\theta)r + 134 = 0$

AB, AP, AC are in H.P $\Rightarrow \frac{2}{r} = \frac{AB + AC}{AB.AC} \Rightarrow \frac{1}{r} = -\frac{(6\cos\theta + 4\sin\theta)}{67} \Rightarrow 6x + 4y - 1 = 0$

Minimum distance from $'O' = \frac{1}{\sqrt{52}}$

2. Let A (0, 2), B and C are points on parabola $y^2 = x + 4$ and such that $\underline{|CBA} = \frac{\Pi}{2}$, then the range of ordinate of C is

A) $(-\infty, 0) \cup (4, \infty)$ B) $(-\infty, 0] \cup [4, \infty)$

C) $[0, 4]$ D) $(-\infty, 0) \cup [4, \infty)$

Key. B

Sol. $A(0, 2)$, $B = (t_1^2 - 4, t_1)$ $C = (t^2 - 4, t)$

$\frac{2 - t_1}{4 - t_1^2} . \frac{t_1 - t}{t_1^2 - t^2} = -1 \Rightarrow \frac{1}{2 + t_1} . \frac{1}{t + t_1} = -1 \Rightarrow t_1^2 + (2 + t)t_1 + (2t + 1) = 0$

For real t_1, $\Rightarrow (2 + t)^2 - 4(2t + 1) = 0 \Rightarrow t^2 - 4t \geq 0 \Rightarrow t \in (-\alpha, 0] \cup [4, \alpha)$

3. If $2p^2 - 3q^2 + 4pq - p = 0$ and a variable line $px + qy = 1$ always touches a parabola whose axis is parallel to X-axis, then equation of the parabola is

A) $(y - 4)^2 = 24(x - 2)$ B) $(y - 3)^2 = 12(x - 1)$

C) $(y - 4)^2 = 12(x - 2)$ D) $(y - 2)^2 = 24(x - 4)$

Key. C

Sol. The parabola be $(y - a)^2 = 4b(x - c)$

Equation of tangent is $\left(y-a\right)=-\frac{p}{q}\left(x-c\right)-\frac{bq}{p}$

Comparing with $px+qy=1$, we get $cp^2-bq^2+apq-p=0$

$\therefore \frac{c}{2}=\frac{b}{3}=\frac{a}{4}=1 \Rightarrow$ the equation is $\left(y-4\right)^2=12\left(x-2\right)$

4. Consider the parabola $x^2+4y=0$. Let $p=\left(a,b\right)$ be any fixed point inside the parabola and let 'S' be the focus of the parabola. Then the minimum value at $SQ+PQ$ as point Q moves on the parabola is

A) $|1-a|$ B) $|ab|+1$ C) $\sqrt{a^2+b^2}$ D) $1-b$

Key. D

Sol. Let foot of perpendicular from Q to the directrix be N

$\Rightarrow SQ+PQ=QN+PQ$ is minimum it $P,Q\,\&\,N$ are collinear

So minimum value of $SQ+PQ=PN=1-b$

5. The locus point of intersection of tangents to the parabola $y^2=4ax$, the angle between them being always $45°$ is

A) $x^2-y^2+6ax-a^2=0$ B) $x^2-y^2-6ax+a^2=0$

C) $x^2-y^2+6ax+a^2=0$ D) $x^2-y^2-6ax-a^2=0$

Key. C

Sol. Equation of tangent is $y=mx+\frac{a}{m}$

$\Rightarrow m^2x-my+a=0 \Rightarrow m_1+m_2=\frac{y}{x}, m_1m_2=\frac{a}{x}$

$\tan 45°=\left|\frac{m_1-m_2}{1+m_1m_2}\right| \Rightarrow \left(\frac{y}{x}\right)^2-4\left(\frac{a}{x}\right)=\left(1+\frac{a}{x}\right)^2$

$\Rightarrow x^2-y^2+6ax+a^2=0$

6. The coordinates of the point on the parabola $y=x^2+7x+2$, which is nearest to the straight line $y=3x-3$ are

1) $\left(-2,-8\right)$ 2) $\left(1,10\right)$ 3) $\left(2,20\right)$ 4) $\left(-1,-4\right)$

Key. 1

Sol. Hint: Any point on the parabola is $\left(x,x^2+7x+2\right)$

Its distance from the line $y=3x-3$ is given by

$$P=\left|\frac{3x-\left(x^2+7x+2\right)-3}{\sqrt{9+1}}\right|=\left|\frac{x^2+4x+5}{\sqrt{10}}\right|=\frac{x^2+4x+5}{\sqrt{10}}\ \left(as\,x^2+4x+5>0\,\forall x\in R\right)$$

$\frac{dp}{dx}=0\Rightarrow x=-2$ the required point = $\left(-2,-8\right)$

7. The point P on the parabola $y^2=4ax$ for which $\left|PR-PQ\right|$ is maximum, where $R=\left(-a,0\right),Q=\left(0,a\right)$. is

1) $\left(a,2a\right)$ 2) $\left(a,-2a\right)$ 3) $\left(4a,4a\right)$ 4) $\left(4a,-4a\right)$

Key. 1

Sol. We know that any side of the triangle is more than the difference of the remaining two sides so that $\left|PR-PQ\right|\le RQ$

The required point P will be the point of intersection of the line RQ with parabola which is $\left(a,2a\right)$ as PQ is a tangent to the parabola

8. The number of point(s) $\left(x,y\right)$(where x and y both are perfect squares of integers) on the parabola $y^2=px$, p being a prime number, is

1) zero 2) one 3) two 4) infinite

Key. 2

Sol. If x is a perfect square ,then px will be a perfect square only if p is a perfect square, which is not possible as p is a prime number. Hence y cannot be a perfect square . So number of such points will be only one $\left(0,0\right)$

9. The locus of point of intersection of any tangent to the parabola $y^2=4a\left(x-2\right)$ with a line perpendicular to it and passing through the focus, is

1) $x=2$ 2) $y=0$ 3) $x=a$ 4) $x=a+2$

Key. 1

Sol. It is well known property of a parabola that a tangent and normal to it from focus intersect at tangent at vertex

10. If the parabola $y=\left(a-b\right)x^2+\left(b-c\right)x+\left(c-a\right)$ touches the $x-axis$ then the line $ax+by+c=0$

1) Always passes through a fixed point 2) represents the family of parallel lines

3) always perpendicular to x-axis 4) always has negative slope

Key. 1

Sol. Solving equation of parabola with x-axis (y=0)

We get $\left(a-b\right)x^2+\left(b-c\right)x+\left(c-a\right)=0,$ which should have two equal values of x, as x-axis touches the parabola $\Rightarrow\left(b-c\right)^2-4\left(a-b\right)\left(c-a\right)=0$

$\Rightarrow\left(b+c-2a\right)^2=0\ \Rightarrow-2a+b+c=0\Rightarrow ax+by+c=0$ always passes through $\left(-2,1\right)$

11. If one end of the diameter of a circle is $(3,4)$ which touches the $x-axis$ then the locus of other end of the diameter of the circle is

1) Circle 2) parabola 3) ellipse 4) hyperbola

Key. 2

Sol. Let other end of diameter (h,k)

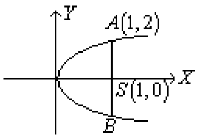
Hence centre is $\sqrt{\left(\frac{3+h}{2}-3\right)^2+\left(\frac{k+4}{2}-4\right)^2}$ gives the equation of parabola

12. The point $(1,2)$ is one extremity of focal chord of parabola $y^2=4x$. The length of this focal chord is

1) 2 2) 4 3) 6 4) none of these

Key. 2

Sol.



The parabola $y^2=4x,$ here $a=1$ and focus is $(1,0)$

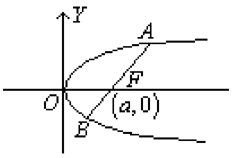
The focal chord is ASB. This is clearly latus rectum of parabola, its value = 4

13. If AFB is a focal chord of the parabola $y^2=4ax\, and\; AF=4, FB=5$ then the latus-rectum of the parabola is equal to

1) $\frac{80}{9}$ 2) $\frac{9}{80}$ 3) 9 4) 80

Key. 1

Sol.



FA = 4 , FB = 5

We know that $\frac{1}{a}=\frac{1}{AF}+\frac{1}{FB}$

$\Rightarrow a=\frac{20}{9}\Rightarrow 4a=\frac{80}{9},$

14. If at $x=1, y=2x$ tangent to the parabola $y=ax^2+bx+c,$ then respective values of a,b,c possible are

1) $\frac{1}{2},1,\frac{1}{2}$ 2) $1,\frac{1}{2},\frac{1}{2}$ 3) $\frac{1}{2},\frac{1}{2},1$ 4) $\frac{-1}{2},1,\frac{3}{2}$

Key. 1

Sol. for x =1 , $y=a+b+c$

Tangent at $(1,a+b+c) is \frac{1}{2}(y+a+b+c)=ax+\frac{b}{2}(x+1)+c$

Comparing with $y = 2x, c = a, b = 2(1-a)$

Which are true for choice (1) only

15. The number of focal chords of length 4/7 in the parabola $7y^2 = 8x$ is

1) one 2) zero 3) two 4) infinite

Key. 2

Sol. since length of latus –rectum = $\frac{8}{7}$

Latus-rectum is the smallest focal chord

Hence focal chord of length $\frac{4}{7}$ does not exist.

16. The length of the chord of the parabola $x^2 = 4y$ passing through the vertex and having slope $\cot\alpha$ is

(1) 4 $\cos\alpha . \cos ec^2\alpha$ (2) $4\tan\alpha\sec\alpha$ (3) $4\sin\alpha.\sec^2\alpha$ (4) none of these

Key. 1

Sol. Let $A =$vertex, AP = chord of $x^2 = 4y$ such that slope of AP is $\cot\alpha$

Let $P = \left(2t, t^2\right)$

Slope of $AP = \frac{1}{2} \Rightarrow \cot\alpha = \frac{1}{2} \Rightarrow t = 2\cot\alpha$

Now, $AP = \sqrt{4t^2 + t^4} = t\sqrt{4 + t^2}$

$= 4\cot\alpha\cos ec\alpha$

$= 4\cos\alpha.\cos ec^2\alpha$

17. Slope of tangent to $x^2 = 4y$ from $(-1, -1)$ can be

1) $\frac{-1 \pm \sqrt{5}}{2}$ 2) $\frac{-3 - \sqrt{5}}{2}$ 3) $\frac{1 - \sqrt{5}}{2}$ 4) $\frac{1 + \sqrt{5}}{2}$

Key. 1

Sol. $y^1 = \frac{x}{2} = m$

$\Rightarrow x = 2m \Rightarrow y = m^2$

So equation of tangent is $y - m^2 = m(x - 2m)$ which passes through $(-1, -1)$

$\Rightarrow -1 - m^2 = m(-1 - 2m)$

$\Rightarrow m^2 + m - 1 = 0 \quad \Rightarrow m = \frac{-1 \pm \sqrt{5}}{2}$

18. If line $y = 2x + \frac{1}{4}$ is tangent to $y^2 = 4ax,$ then a is equal to

1) $\frac{1}{2}$ 2) 1 3) 2 4) None of these

Key. 1

Sol. $c=\frac{a}{m}\Rightarrow a=2\left(\frac{1}{4}\right)=\frac{1}{2}$

19. The Cartesian equation of the curve whose parametric equations are $x=t^2+2t+3$ and $y=t+1$ is

1) $y=(x-1)^2+2(y-1)+3$ 2) $x=(y-1)^2+2(y-1)+5$

3) $x=y^2+2$ 4) none of these

Key. 3

Sol. $x=t^2+2t+3=(t+1)^2+2=y^2+2$

20. If the line $y-\sqrt{3}x+3=0$ cuts the parabola $y^2=x+2$ at A and B, then PA. PB is equal to (where $P\equiv(\sqrt{3},0)$)

1) $\frac{4(\sqrt{3}+2)}{3}$ 2) $\frac{4(2-\sqrt{3})}{3}$ 3) $\frac{4\sqrt{3}}{2}$ 4) $\frac{2(\sqrt{3}+2)}{3}$

Key. 1

Sol. $y-\sqrt{3}x+3=0$ can be rewritten as

$$\frac{y-0}{\frac{\sqrt{3}}{2}}=\frac{x-\sqrt{3}}{\frac{1}{2}}=r \quad (1)$$

Solving (1) with the parabola $y^2=x+2$

$$\frac{3r^2}{4}-\frac{r}{2}-\sqrt{3}-2=0\Rightarrow PA.PB=r_1r_2=\frac{4(\sqrt{3}+2)}{3}$$

21. The equation of the line of the shortest distance between the parabola $y^2=4x$ and the circle $x^2+y^2-4x-2y+4=0$ is.

1) $x+y=3$ 2) $x-y=3$ 3) $2x+y=5$ 4) none of these

Key. 1

Sol. Line of shortest distance is normal for both parabola and circle

Centre of circle is $(2,1)$

Equation of normal to circle is $y-1=m(x-2)\Rightarrow y=mx+(1-2m)$ (1)

Equation of normal for a parabola is $y=mx-2am-am^3$ (3)

Comparing (1) and (2)

$am^3=-1\Rightarrow m^3=-1\Rightarrow m=-1$ $(a=1)$

Equation is $y-1=-x+2\Rightarrow x+y=3$

22. If $x+k=0$ is equation of directrix to parabola $y^2=8(x+1)$ then $k=$

1) 1 2) 2 3) 3 4) 4

Key. 3

Sol. Focus is $(1,0)$ third vertex is $(-1,0)$. Hence directrix is $x+3=0$

23. If t is the parameter for one end of a focal chord of the parabola $y^2=4ax$, then its length is

1) $a\left(t+\frac{1}{t}\right)^2$ 2) $a\left(t-\frac{1}{t}\right)^2$ 3) $a\left(t+\frac{1}{t}\right)$ 4) $a\left(t-\frac{1}{t}\right)$

Key. 1

Sol. Conceptual

24. The ends of the latus rectum of the conic $x^2+10x-16y+25=0$ are

(1) $(3,-4),(13,4)$ (2) $(-3,-4),(13,-4)$ (3) $(3,4),(-13,4)$ (4) $(5,-8),(-5,8)$

Key. 3

Sol. $(x+5)^2=16y$ comparing it with $x^2=4ay$,

25. If the lines $(y-b)=m_1(x+a)$ and $(y-b)=m_2(x+a)$ are the tangents of $y^2=4ax$ then

1) $m_1+m_2=0$ 2) $m_1m_2=1$ 3) $m_1m_2=-1$ 4) $m_1+m_2=1$

Key. 3

Sol. $y=mx+\frac{a}{m}$

$\Rightarrow m^2x-3y+a=0, m_1.m_2=-1$

26. The equation of a parabola is $y^2=4x.Let\, P(1,3)$ and $Q(1,1)$ are two points in the xy plane. Then, for the parabola

1) P and Q are exterior points
2) P is an interior point while Q is an exterior point
3) P and Q are interior points
4) P is an exterior point while Q is an interior point

Key. 4

Sol. Here, $S\equiv y^2-4x=0$

$S(1,3)=3^2-4.1>0$

$\Rightarrow P(1,3)$ is an exterior point $S(1,1)=1^1-4.1<0$

$\Rightarrow Q(1,1)$ is an interior point

27. If the focus of a parabola is $(-2,1)$ and the directrix has the equation $x+y=3$, then the vertex is:

1) $(0,3)$ 2) $\left(-1,\frac{1}{2}\right)$ 3) $(-1,2)$ 4) $(2,-1)$

Key. 3

Sol. The vertex is the middle point of the perpendicular dropped from the focus to the directrix.

28. The length of the latus-rectum of the parabola $169\left\{(x-1)^2+(y-3)^2\right\}=(5x-12y+17)^2$ is

1) $\frac{12}{13}$ 2) $\frac{14}{13}$ 3) $\frac{28}{13}$ 4) $\frac{31}{13}$

Key. 3

Sol. $(x-1)^2+(y-3)^2=\left(\frac{5x-12y+17}{13}\right)^2$

Length of latus rectum $=4a$

Perpendicular distance from $(1,3)$ to the line $5x-12y+17=0$ is

$$2a=\frac{|5\times1-12\times3+17|}{\sqrt{169}}=\frac{14}{13}$$

29. The co-ordinates of a point on the parabola $y^2=8x$ whose focal distance is 4 is

1) $(2,4)$ 2) $(4,2)$ 3) $(2,-6)$ 4) $(4,-2)$

Key. 1

Sol. $a+x=4\Rightarrow2+x=4\Rightarrow x=2,y=4$

30. Co-ordinate of the focus of the parabola $x^2-4x-8y-4=0$ are

1) $(0,2)$ 2) $(2,1)$ 3) $\left(-3,\frac{-71}{10}\right)$ 4) $(2,-1)$

Key. 2

Sol. $(x-2)^2=8(y+1)$

Focus $x-2=0,y+1=2\Rightarrow x=2,y=1$

Focus (2,1)

31. If focal distance of a point on the parabola $y=x^2-4$ is $\frac{25}{4}$ and points are of the form $\left(\pm\sqrt{a},b\right)$ Then $a+b$ is equal to

1) 8 2) 4 3) 2 4) 0

Key. 1

Sol. $y+4=x^2$

$x^2=4.\frac{1}{4}(y+4)$

Focal distance $=\frac{25}{4}$

Distance from directrix $\left(y=\frac{-15}{4}\right)$

Ordinate of points on the parabola whose focal distance is $\frac{25}{4}$

$=\frac{-17}{4}+\frac{25}{4}=2$ points are $\left(\pm\sqrt{6},2\right)$ $\Rightarrow a+b=8$

32. Length of side of an equilateral triangle inscribed in a parabola $y^2-2x-2y-3=0$ whose one angular point is vertex of the parabola is

1) $2\sqrt{3}$ 2) $4\sqrt{3}$ 3) $-\sqrt{3}$ 4) $\sqrt{3}$

Key. 2

Sol. Length of side $=8\sqrt{3}a=8\sqrt{3}\frac{1}{2}=4\sqrt{3}$

33. Length of latus rectum of the parabola whose parametric equations are

$x=t^2+t+1,y=t^2-t+1$ where $t\in R$, is equal to

1) 4 2) +1 3) $\sqrt{2}$ 4) 3

Key. 3

Sol. $x+y=2\left(t^2+1\right)$ & $x-y=2t$

$$\therefore\left(x+y-2\right)=2\left(\frac{x-y}{2}\right)^2 \Rightarrow\left(\frac{x-y}{\sqrt{2}}\right)^2=\sqrt{2}\left(\frac{x+y-2}{\sqrt{2}}\right)$$

Length of latusrectum = $\sqrt{2}$

34. In the parabola, $y^2-2y+8x-23=0$, the length of double ordinate at a distance of 4 units from its vertex is

1) $4\sqrt{2}$ 2) $8\sqrt{2}$ 3) 6 4) 4

Key. 2

Sol. Length of double ordinate = $8\sqrt{2}$

35. If any point $P\left(x,y\right)$ satisfies the relation

$\left(5x-1\right)^2+\left(5y-2\right)^2=\lambda\left(3x-4y-1\right)^2$, represents parabola, then

1) $\lambda=1$ 2) $\lambda<1$ 3) $\lambda>1$ 4) $\lambda>2$

Key. 1

Sol. Conceptual

36. The locus of the vertex of the family of parabolas $y=\frac{a^3x^2}{3}+\frac{a^2x}{2}-2a$

(a is parameter) is

(A) $xy=\frac{105}{64}$ (B) $xy=\frac{3}{4}$ (C) $xy=\frac{35}{16}$ (D) $xy=\frac{64}{105}$

Key. A

Sol. $y=\frac{a^3x^2}{3}+\frac{a^2x}{2}-2a$

$$y=\frac{2a^3}{6}\left(x^2+\frac{3}{2a}x-\frac{12a}{2a^3}\right)$$

$$y=\frac{2a^3}{6}\left(x^2+2.\frac{3}{4a}x+\frac{9}{16a^2}-\frac{9}{16a^2}-\frac{12a}{2a^3}\right)$$

$$y=\frac{2a^3}{6}\left(\left(x+\frac{3}{4a}\right)^2-\frac{1059}{16a^3}\right)$$

$$\left(y+\frac{1059}{48}\right)=\frac{2a^3}{6}\left(x+\frac{3}{4a}\right)^2$$

$$x=\frac{-1059}{48}$$

$$y=\frac{-3}{49}$$

$$xy = \frac{1059}{48} \times \frac{3}{49} = \frac{105}{64}$$

37. Tangents are drawn from the point (-1, 2) to the parabola $y^2 = 4x$. The length of the intercept made by the line x = 2 on these tangents is
(A) 6 (B) $6\sqrt{2}$ (C) $2\sqrt{6}$ (D) none

Key. B

Sol. Equation of pair of tangent is

$$SS_1 = T^2$$
$$\Rightarrow (y^2 - 4x)(8) = 4(y - x + 1)^2$$
$$\Rightarrow y^2 - 2y(1-x) - (x^2 + 6x + 1) = 0$$
$$Put \quad x = 2$$
$$\Rightarrow y^2 + 2y - 17 = 0$$
$$\Rightarrow |y_1 - y_2| = 6\sqrt{2}$$

38. The given circle $x^2 + y^2 + 2px = 0$, $p \in R$ touches the parabola $y^2 = 4x$ externally, then
(A) p < 0 (B) p > 0 (C) 0 < p < 1 (D) p < - 1

Key. B

Sol. Centre of the circle is (- p, 0), If it touches the parabola, then according to figure only one case is possible.

Hence p > 0

39. The triangle PQR of area A is inscribed in the parabola $y^2 = 4ax$ such that P lies at the vertex of the parabola and base QR is a focal chord. The numerical difference of the ordinates of the points Q & R is
(A) $\frac{A}{2a}$ (B) $\frac{A}{a}$ (C) $\frac{2A}{a}$ (D) $\frac{4A}{a}$

Key. C

Sol. QR is a focal chord

$$\Rightarrow R(at^2, 2at) \,\&\, Q(\frac{a}{t^2}, -\frac{2a}{t})$$
$$\Rightarrow d = \left|2at + \frac{2a}{t}\right| = 2a\left|t + \frac{1}{t}\right|$$
$$Now \quad A = \frac{1}{2}\begin{vmatrix} at^2 & 2at & 1 \\ \frac{a}{t^2} & -\frac{2a}{t} & 1 \\ 0 & 0 & 1 \end{vmatrix} = a^2\left|t + \frac{1}{t}\right|$$
$$\Rightarrow 2a\left|t + \frac{1}{t}\right| = \frac{2A}{a}$$

40. Through the vertex O of the parabola $y^2 = 4ax$ two chords OP & OQ are drawn and the circles on OP & OQ as diameter intersect in R. If

$\theta_1, \theta_2 \& \phi$ are the inclinations of the tangents at P & Q on the parabola and the line through O, R respectively, then the value of $\cot\theta_1 + \cot\theta_2$ is

(A) – 2 tan ϕ (B) – 2 tan (π - ϕ) (C) 0 (D) 2 cot ϕ

Key. A

Sol. $Let \quad P(t_1) \& Q(t_2)$

$\Rightarrow$ Slope of tangent at P$(\frac{1}{t_1})$ & at Q$(\frac{1}{t_2})$ $\Rightarrow \cot\theta_1 = t_1$ *and* $\cot\theta_2 = t_2$

Slope of PQ = $\frac{2}{t_1 + t_2} = \tan\phi$

$\Rightarrow \tan\phi = -\frac{1}{2}(\cot\theta_1 + \cot\theta_2)$ $\Rightarrow \cot\theta_1 + \cot\theta_2 = -2\tan\phi$

41. AB and AC are tangents to the parabola $y^2 = 4ax$. $p_1, p_2 \& p_3$ are perpendiculars from A, B & C respectively on any tangent to the curve (otherthan the tangents at B&C), then $p_1, p_2 \& p_3$ are in

(A) A.P. (B) G.P. (C) H.P (D) none

Key. B

Sol. Let any tangent is tangent at vertex x = 0 and

$Let \quad B(t_1) \& C(t_2)$

$\Rightarrow A(at_1t_2, a(t_1 + t_2))$

$\Rightarrow p_1 = at_1^2; p_2 = at_2^2 \ \& \ p_3 = at_1t_2$

$\Rightarrow p_1, p_2 \ \& \ p$ are in G.P.

42. A tangent to the parabola $x^2 + 4ay = 0$ at the point T cuts the parabola $x^2 = 4by$ at A & B. Then locus of the mid point of AB is

(A) $(b+2a)x^2 = 4b^2y$ (B) $(b+2a)x^2 = 4a^2y$

(C) $(a+2b)y^2 = 4b^2x$ (D) $(a+2b)x^2 = 4b^2y$

Key. D

Sol. Let mid point of AB is M(h, k)

Then equation of AB is $hx - 2b(y+k) = h^2 - 4bk$

$Let\ T(2at, -at^2)$

$\Rightarrow$ *Equation of* tangent(AB) = $x(2at) = -2a(y - at^2)$

Compare these two equations, we get $\frac{h}{2at} = \frac{-2b}{2a} = \frac{h^2 - 2bk}{2a^2t^2}$

By eliminating t and Locus (h, k), we get $(a+2b)x^2 = 4b^2y$

43. A parabola $y = ax^2 + bx + c$ crosses the x-axis at A(p, 0) & B(q, 0) both to the right of origin. A circle also passes through these two points. The length of a tangent from the origin to the circle is

(A) $\sqrt{\frac{bc}{a}}$ (B) ac^2 (C) b/a (D) $\sqrt{\frac{c}{a}}$

Key. D

Sol. Use power of point for the point O figure

$$\Rightarrow OT^2 = OA.OB = pq = \frac{c}{a}$$

$$\Rightarrow OT = \sqrt{\frac{c}{a}}$$

44. The equation of the normal to the parabola $y^2 = 8x$ at the point t is

1. $y - x = t + 2t^2$ 2. $y + tx = 4t + 2t^3$ 3. $x + ty = t + 2t^2$ 4. $y - x = 2t - 3t^3$

Key. 2

Sol. Equation of the normal at 't' is $y + tx = 2(2)t + (2)t^3 \Rightarrow y + tx = 4t + 2t^3$

45. The slope of the normal at $(at^2, 2at)$ of the parabola $y^2 = 4ax$ is

1. $\frac{1}{t}$ 2. t 3. $-t$ 4. $-\frac{1}{t}$

Key. 3

Sol. Slope of the normal at 't' is $-t$.

46. If the normal at the point 't' on a parabola $y^2 = 4ax$ meet it again at t_1, then $t_1 =$

1. t 2. $-t - 1/t$ 3. $-t - 2/t$ 4. None

Key. 3

Sol. Equation of the normal at t is $tx + y = 2at + at^3 \rightarrow (1)$

Equation of the chord passing through t and t_1 is $y(t + t_1) = 2x + 2att_1 \rightarrow (2)$

Comparing (1) and (2) we get $\frac{t}{-2} = \frac{1}{t + t_1} \Rightarrow t + t_1 = -\frac{2}{t} \Rightarrow t_1 = -\frac{2}{t} - t$.

47. If the normal at t_1 on the parabola $y^2 = 4ax$ meet it again at t_2 on the curve, then $t_1(t_1 + t_2) + 2 =$

1. 0 2. 1 3. t_1 4. t_2

Key. 1

Sol. Equation of normal at t_1 is $t_1 x + y = 2at_1 + at_1^3$

It passes through $t_2 \Rightarrow at_1 t_2^2 + 2at_2 = 2at_1 + at_1^3$

$$\Rightarrow t_1(t_2^2 - t_1^2) = 2(t_1 - t_2) \Rightarrow t_1(t_1 + t_2) = -2 \Rightarrow t_1(t_1 + t_2) + 2 = 0$$

48. If the normal at $(1,2)$ on the parabola $y^2=4x$ meets the parabola again at the point $(t^2,2t)$, then the value of t is

1. 1 2. 3 3. -3 4. -1

Key. 3

Sol. $Let(1,2)=(t_1^2,2t_1)\Rightarrow t_1=1$

$t=-t_1-\frac{2}{t_1}=-1-\frac{2}{1}=-3$

49. If the normal to parabola $y^2=4x$ at $P(1,2)$ meets the parabola again in Q, then Q=

1. $(-6,9)$ 2. $(9,-6)$ 3. $(-9,-6)$ 4. $(-6,-9)$

Key. 2

Sol. $P=(1,2)=(t^2,2t)\Rightarrow t=1$

$Q=(t_1^2,2t_1)\Rightarrow t_1=-t-2/t=-1-2=-3\Rightarrow Q=(9,-6)$.

50. If the normals at the points t_1 and t_2 on $y^2=4ax$ intersect at the point t_3 on the parabola, then $t_1t_2=$

1. 1 2. 2 3. t_3 4. $2t_3$

Key. 2

Sol. Let the normals at t_1 and t_2 meet at t_3 on the parabola.

The equation of the normal at t_1 is $y+xt_1=2at_1+at_1^3\rightarrow(1)$

Equation of the chord joining t_1 and t_3 is $y(t_1+t_3)=2x+2at_1t_3\rightarrow(2)$

(1) and (2) represent the same line.

$\therefore\quad \frac{t_1+t_3}{1}=\frac{-2}{t_1}\Rightarrow t_3=-t_1-\frac{2}{t_1}$. Similarly $t_3=-t_2-\frac{2}{t_2}$

$\therefore\quad -t_1-\frac{2}{t_1}=-t_2-\frac{2}{t_2}\Rightarrow t_1-t_2=\frac{2}{t_2}-\frac{2}{t_1}\Rightarrow t_1-t_2=\frac{2(t_1-t_2)}{t_1t_2}\Rightarrow t_1t_2=2$

51. The number of normals thWSat can be drawn to the parabola $y^2=4x$ form the point $(1,0)$ is

1. 0 2. 1 3. 2 4. 3

Key. 2

Sol. $(1,0)$ lies on the axis between the vertex and focus $\Rightarrow$ number of normals =1.

52. The number of normals that can be drawn through $(-1,4)$ to the parabola $y^2-4x+6y=0$ are

1. 4 2. 3 3. 2 4. 1

Key. 4

Sol. $Let\ S \equiv y^2 - 4x + 6y$. $S_{(-1,4)} = 4^2 - 4(-1) + 6(4) = 16 + 4 + 24 = 44 > 0$

$\therefore$ $(-1, 4)$ lies out side the parabola and hence one normal can be drawn from $(-1, 4)$ to the parabola.

53. If the tangents and normals at the extremities of a focal chord of a parabola intersect at (x_1, y_1) and (x_2, y_2) respectively, then

1. $x_1 = x_2$ 2. $x_1 = y_2$ 3. $y_1 = y_2$ 4. $x_2 = y_1$

Key. 3

Sol. $Let\ A(t_1)\ B(t_2)$ the extremiues of a focal chard of $y^2 = 4ax$

$\therefore\ t_1t_2 = -1$

$(x_1, y_1) = [at_1t_2, a(t_1 + t_2)]; (x_2, y_2) = [a(t_1^2 + t_2^2 + t_1t_2 + 2), at_1t_2(t_1 + t_2)]$

$y_2 = -at_1t_2(t_1 + t_2) = -a(-1)(t_1 + t_2) = a(t_1 + t_2) = y_1$

54. The normals at three points P, Q, R of the parabola $y^2 = 4ax$ meet in (h, k). The centroid of triangle PQR lies on

1. $x = 0$ 2. $y = 0$ 3. $x = -a$ 4. $y = a$

Key. 2

Sol. $Let\ P(t_1), Q(t_2)\ \&\ R(t_3)$

Equation of a normal to $y^2 = 4ax$ is $y + tx = 2at + at^3$

This passes through $(h, k) \Rightarrow k + th = 2at + at^3 \Rightarrow at^3 + (2a - h)t - k = 0$

t_1, t_2, t_3 are the roots of this equation $t_1 + t_2 + t_3 = 0$

Centroid of ΔPQR is $G\left[\frac{a}{3}(t_1^2 + t_2^2 + t_3^2), \frac{2a}{3}(t_1 + t_2 + t_3)\right]$

$t_1 + t_2 + t_3 = 0 \Rightarrow \frac{2a}{3}(t_1 + t_2 + t_3) = 0 \Rightarrow G$ lies on $y = 0$.

55. The ordinate of the centroid of the triangle formed by conormal points on the parabola $y^2 = 4ax$ is

1. 4 2. 0 3. 2 4. 1

Key. 2

Sol. $Let\, t_1, t_2\ \&\, t_3$ be the conormal points drawn from (x_1, y_1) to $y^2 = 4ax$

Equation of the normal at point 't' to $y^2 = 4ax$ is $y + tx = 2at + at^3$

This passes through $(x_1, y_1) \Rightarrow y_1 + tx_1 = 2at + at^3 \Rightarrow at^3 + (2a - x_1)t - y_1 = 0$

t_1, t_2, t_3 are the roots of the equation. $\therefore$ $t_1 + t_2 + t_3 = 0$

The ordinate of the centroid of the triangle formed by the points $t_1, t_2 \,\&\, t_3$ is $\frac{2a}{3}(t_1 + t_2 + t_3) = 0$

56. The normals at two points P and Q of a parabola $y^2 = 4ax$ meet at (x_1, y_1) on the parabola. Then PQ^2 =

1. $(x_1 + 4a)(x_1 + 8a)$ 2. $(x_1 + 4a)(x_1 - 8a)$ 3. $(x_1 - 4a)(x_1 + 8a)$ 4. $(x_1 - 4a)(x_1 - 8a)$

Key. 2

Sol. $Let\, P = (at_1^2, 2at_1), Q = (at_2^2, 2at_2)$

Since the normals at P and Q meet on the parabola, $t_1 t_2 = 2$.

Point of intersection of the normals $(x_1, y_1) = \left(a\left[t_1^2 + t_2^2 + t_1 t_2 + 2\right], -at_1 t_2\left[t_1 + t_2\right]\right)$

$\Rightarrow$ $x_1 = a(t_1^2 + t_2^2 + t_1 t_2 + 2) = a(t_1^2 + t_2^2 + 4) \Rightarrow a(t_1^2 + t_2^2) = x_1 - 4a$

$PQ^2 = (at_1^2 - at_2^2)^2 + (2at_1 - 2at_2)^2 = a^2(t_1 - t_2)^2[(t_1 + t_2)^2 + 4]$
$= a(t_1^2 + t_2^2 - 4)a(t_1^2 + t_2^2 + 8) = (x_1 - 8a)(x_1 + 4a)$

57. If a normal subtends a right angle at the vertex of the parabola $y^2 = 4ax$, then its length is

1. $\sqrt{5}a$ 2. $3\sqrt{5}a$ 3. $6\sqrt{3}a$ 4. $7\sqrt{5}a$

Key. 3

Sol. $Leta(at_1^2, 2at_1), B(at_2^2, 2at_2)$.

The normal at A cuts the curve again at B. $\therefore$ $t_1 + t_2 = -\frac{2}{t_1}$........(1)

Again AB subtends a right angle at the vertex $0(0,0)$ of the parabola.

Slope $OA = \frac{2at_1}{at_1^2} = \frac{2}{t_1}$, slope of $OB = \frac{2}{t_2}$

$OA \perp OB \Rightarrow \frac{2}{t_1} . \frac{2}{t_2} = -t_1 t_2 = -4$......(2)

Slope of AB is $\frac{2a(t_2 - t_1)}{a(t_2^2 - t_1^2)} = \frac{2}{t_1 + t_2} = -t_1$. [By (1)]

Again from (1) and (2) on putting for t_2, we get $t_1 = \frac{4}{t_1} = -\frac{2}{t_1}$. $\quad \therefore \quad t_1^2 = 2$ or

$t_1 \pm \sqrt{2}$

$$t_2 = \frac{-4}{t_1} = \frac{-4}{(\pm\sqrt{2})} = \pm 2\sqrt{2}. \quad \therefore \quad A = (2a, \pm 2a\sqrt{2}), B = (8a, \pm 4\sqrt{a})$$

$$AB = \sqrt{(2a-8a)^2 + (2a\sqrt{2} + 4\sqrt{2}a)^2} = \sqrt{36a^2 + 72a^2} = \sqrt{108a^2} = 6\sqrt{3}a.$$

58. Three normals with slopes m_1, m_2, m_3 are drawn from any point P not on the axis of the parabola $y^2 = 4x$. If $m_1 m_2 = a$, results in locus of P being a part of parabola, the value of 'a' equals

1. 2 2. -2 3. 4 4. -4

Key. 1

Sol. Equation of normal to $y^2 = 4x$ is $y = mx - 2m - m^3$ ...(i)

It passes through (α, β) $\quad \therefore \quad m_1 m_2 m_3 \beta = m\alpha - 2, -m^3$

$\Rightarrow m^3 + (2-\alpha)\, m + \beta = 0$ $\quad \ldots.(ii)$

(Let m_1, m_2, m_3 are roots)

$\therefore \quad m_1 m_2 m_3 = -\beta \quad$ (as $m_1 m_2 = a$) $\Rightarrow m_3 = -\frac{\beta}{a}$

Now $-\frac{\beta^3}{a^3} - (2-\alpha) \times \frac{\beta}{a} + \beta = 0$

$\Rightarrow \beta^3 + (2-\alpha)a^2\beta - \beta a^3 = 0$

$\Rightarrow$ locus of P is $y^3 + (2-x)ya^2 - ya^3 = 0$

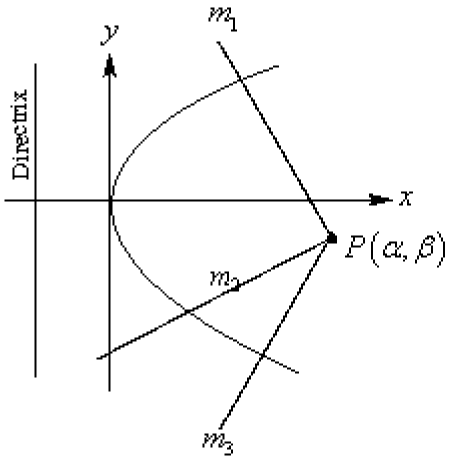
As P is not the axis of parabola

$\Rightarrow \quad y^2 = a^2 x - 2a^2 + a^3$ as it is the part of $y^2 = 4x$

$\therefore \quad a^2 = 4$ or $-2a^2 + a^3 = 0$, $a = \pm 2$ or $a^2(a-2) = 0$

$a = \pm 2$ or $a = 0$, $a = 2$

$\Rightarrow a = 2$ is the required value of a



59. The length of the normal chord drawn at one end of the latus rectum of $y^2 = 4ax$ is

1. $2\sqrt{2}a$ 2. $4\sqrt{2}a$ 3. $8\sqrt{2}a$ 4. $10\sqrt{2}a$

Key. 2

Sol. One end of the latus rectum $= (a, 2a)$

Equation of the normal at $(a, 2a)$ is $2a(x-a) + 2a(y-2a) = 0 \Rightarrow x + y - 3a = 0$

Solving; $y^2 = 4ax, x + y - 3a = 0$ we get the ends of normal chord are $(a, 2a), (9a, -6a)$.

Length of the chord $= \sqrt{(9a-a)^2 + (-6a-2a)^2} = \sqrt{64a^2 + 64a^2} = 8\sqrt{2}a.$

60. If the line $y = 2x + k$ is normal to the parabola $y^2 = 4x,$ then value of k equals

1. -2 2. -12 3. -3 4. -1/3

Key. 1

Sol. Conceptual

61. The normal chord at a point 't' on the parabola $y^2 = 4ax$ subtends a right angle at the vertex. Then $t^2 =$

1. 4 2. 2 3. 1 4. 3

Key. 2

Sol. Equation of the normal at point 't' is $y + tx = 2at + at^3 \Rightarrow \frac{y + tx}{2at + at^3} = 1$

Homoginising $y^2 = 4ax\left(\frac{y+tx}{2at+at^3}\right) \Rightarrow (2at + at^3)y^2 - 4ax(y + tx) = 0$

These lines re $\perp 1r \Rightarrow$ $2at + at^3 - 4at = 0 \Rightarrow at(t^2 - 2) = 0 \Rightarrow t^2 = 2$

62. A is a point on the parabola $y^2 = 4ax.$ The normal at A cuts the parabola again at B. If AB subtends a right angle at the vertex of the parabola, then slope of AB is

1. $\sqrt{2}$ 2. 2 3. $\sqrt{3}$ 4. 3

Key. 1

Sol. $Let\ A(at_1^2, 2at_1)$ and $B(at_2^2, 2at_2)$.

The normal at A cuts the curve again at B. $\therefore\ t_1 + t_2 = -2/t_1$....(1)

Again AB subtends a right angle at the vertex $O(0,0)$ of the parabola.

Slope of $OA = \dfrac{2at_1}{at_1^2} = \dfrac{2}{t_1}$, Slope of $OB = \dfrac{2}{t_2}$

$$OA \perp OB \Rightarrow \frac{2}{t_1}.\frac{2}{t_2} = -1 \Rightarrow t_1 t_2 = -4....(2)$$

Slope of AB is $\dfrac{2a(t_2 - t_1)}{a(t_2^2 - t_1^2)} = \dfrac{2}{t_1 + t_2} = -t_1$ by (1)

Again from (1) and (2) on putting for t_2 we get $t_1 - \dfrac{4}{t_1} = \dfrac{2}{t_1}$. $\therefore$ $t_1^2 = 2 \Rightarrow t_1 = \pm\sqrt{2}$.

$\therefore$ Slope $= \pm\sqrt{2}$.

63. If the normal at P meets the axis of the parabola $y^2 = 4ax$ in G and S is the focus, then SG =

1. SP 2. $2SP$ 3. $\dfrac{1}{2}SP$ 4. None

Key. 1

Sol. Equation of the normal at $P(at^2, 2at)$ is $tx + y = 2at + at^3$

Since it meets the axis, $y = 0 \Rightarrow tx = 2at + at^3 \Rightarrow x = 2a + at^2$

$\therefore\ G = (2a + at^2, 0)$, Focus $S = (a, 0)$

$$SG = \sqrt{(2a + at^2 - a)^2 + (0-0)^2} = \sqrt{(a + at^2)^2} = a + at^2 = a(1 + t^2)$$

$$SP = \sqrt{(at^2 - a)^2 + (2at - 0)^2} = \sqrt{(at^2 - a)^2 + 4a^2t^2} = \sqrt{(at^2 + a)^2} = at^2 + a = a(t^2 + 1)$$

$\therefore\ SG = SP$

64. The normal of a parabola $y^2 = 4ax$ at (x_1, y_1) subtends right angle at the

1. Focus 2. Vertex 3. End of latus rectum 4. None of these

Key. 1

Sol. Conceptual

65. The normal at P cuts the axis of the parabola $y^2 = 4ax$ in G and S is the focus of the parabola. If ΔSPG is equilateral then each side is of length

1. a 2. $2a$ 3. $3a$ 4. $4a$

Key. 4

Sol. $Let\ P(at^2, 2at)$

Equation of the normal at $P(t)$ is $y+tx=2at+at^3$

Equation to $y-$axis is $x=0$. Solving $G(2a+at^2,0)$

Focus $s(a,0)$

ΔSPG is equilateral $\Rightarrow$ $PG=GS$ $\Rightarrow$ $\sqrt{4a^2+4a^2t^2}=\sqrt{a^2(1+t^2)^2}$

$\Rightarrow$ $4a^2(1+t^2)=a^2(1+t^2)^2 \Rightarrow 4=1+t^2 \Rightarrow t^2=3$

Length of the side $=SG=a(1+t^2)=a(1+3)=4a$

66. If the normals at two points on the parabola $y^2=4ax$ intersect on the parabola, then the product of the abscissa is

1. $4a^2$
2. $-4a^2$
3. $2a$
4. $4a^4$

Key. 1

Sol. *Let* $P(at_1^2,2at_1);Q(at_2^2,2at_2)$

Normals at $P\,\&\,Q$ on the parabola intersect on the parabola $\Rightarrow t_1t_2=2$

$at_1^2\times at_2^2=a^2(t_1t_2)^2=a^2(2)^2=4a^2$

67. If the normals at two points on the parabola intersects on the curve, then the product of the ordinates of the points is

1. $8a$
2. $8a^2$
3. $8a^3$
4. $8a^4$

Key. 2

Sol. Let the normals at $P(t_1)$ and $Q(t_2)$ intersect on the parabola at $R(t_3)$.

Equation of any noemal is $tx+y=2at+at^3$

Since it passes through Q we get $t.at_3^2+2at_3=2at+at^3$

$\Rightarrow$ $at^3+(2a-at_3^2)t-2at_3=0,$ which is a cubic equation in t and hence its roots are t_1,t_2,t_3.

Product of the roots $=t_1t_2t_3 = \dfrac{-(-2at_3)}{a}=2t_3 \Rightarrow t_1t_2=2$

Product of the absisson of P and Q $=at_1^2.at_2^2=a^2(t_1t_2)^2=a^2(2)^2=4a^2$.

Product of the ordinates of P and Q $=2at_1.2at_2 4a^2.t_1t_2=4a^2(2)=8a^2$

68. The equation of the locus of the point of intersection of two normals to the parabola $y^2 = 4ax$ which are perpendicular to each other is

1. $y^2 = a(x-3a)$ 2. $y^2 = a(x+3a)$ 3. $y^2 = a(x+2a)$ 4. $y^2 = a(x-2a)$

Key. 1

Sol. *Let* $P(x_1, y_1)$ be the point of intersection of the two perpendicular normals at $A(t_1), B(t_2)$ on the parabola $y^2 = 4ax$.

Let t_3 be the foot of the third normal through P.

Equation of a normal at t to the parabola is $y + xt = 2at + at^3$

If this normal passes through P then $y_1 + x_1 t = 2at + at^3 \Rightarrow at^3 + (2a - x_1)t - y_1 = 0 \rightarrow (1)$

Now t_1, t_2, t_3 are the roots of (1). $\therefore$ $t_1 t_2 t_3 = y_1 / a$

Slope of the normal at t_1 is $-t_1$

Slope of the normal at t_2 is $-t_2$.

Normals at t_1 and t_2 are perpendicular $\Rightarrow$ $(-t_1)\ (-t_2) = -1 \Rightarrow t_1 t_2 = -1 \Rightarrow t_1 t_2 t_3 = -t_3$

$$\Rightarrow \frac{y_1}{a} = -t_3 \Rightarrow t_3 = -\frac{y_1}{a}$$

t_3 is a root of (1) $\Rightarrow a(-\frac{y_1}{a})^3 + (2a - x_1)(-\frac{y_1}{a}) - y_1 = 0 \Rightarrow -\frac{y_1^3}{a^2} - \frac{(2a - x_1)y_1}{a} - y_1 = 0$

$\Rightarrow$ $y_1^2 + a(2a - x_1) + a^2 = y_1^2 = a(x_1 - 3a)$.

$\therefore$ The locus of P is $y^2 = a(x - 3a)$

69. The three normals from a point to the parabola $y^2 = 4ax$ cut the axes in points, whose distances from the vertex are in A.P., then the locus of the point is

1. $27ay^2 = 2(x-2a)^3$ 2. $27ay^3 = 2(x-2a)^2$ 3. $9ay^2 = 2(x-2a)^3$ 4. $9ay^3 = 2(x-2a)^2$

Key. 1

Sol. *Let* $P(x_1, y_1)$ be any point.

Equation of any normal is $y = mx - 2am - am^3$

If is passes through P then $y_1 = mx_1 - 2am - am^3$

$\Rightarrow am^3 + (2a - x_1)m_1 + y_1 = 0$, which is cubic in m.

Let m_1, m_2, m_3 be its roots. Then $m_1 + m_2 + m_3 = 0, m_1m_2 + m_2m_3 + m_3m_1 = \dfrac{2a - x_1}{a}$

Normal meets the axis $(y = 0)$, where $0 = mx - 2am - am^3 \Rightarrow x = 2a + am^2$

$\therefore$ Distances of points from the vertex are $2a + am_1^2, 2a + am_2^2, 2a + am_3^2$

If these are in A.P., then $2(2a + am_2^2) = (2a + am_1^2) + (2a + am_3^2) \Rightarrow 2m_2^2 = m_1^2 + m_3^2$

$\Rightarrow 3m_2^2 = m_1^2 + m_2^2 = (m_1 + m_2 + m_3)^2 - 2(m_1m_2 + m_2m_3 + m_3m_1) = -2(2a - x_1)/a$

$\therefore m_2^2 = 2(x_1 - 2a)/3a$

But $y_1 = m_2(x_1 - 2a - am_2^2) \Rightarrow y_1^2 = m_2^2(x_1 - 2a - am_2^2)^2 = 2(x_1 - 2a)^3/27a$ Locus of P is $27ay^2 = 2(x - 2a)^3$

70. If the normals from any point to the parabola $x^2 = 4y$ cuts the line $y = 2$ in points whose abscissae are in A.P., then the slopes of the tangents at the 3 conormal points are in

1. AP 2. GP 3. HP 4. None

Key. 1

Sol. A point on $x^2 = 4y$ is $(2t, t_2)$ and required point be $P(x_1, y_1)$

Equation of normal at $(2t, t^2)$ is $x + ty = 2t + t^3$.......(1)

Given line equation is $y = 2$........(2)

Solving (1) & (3) $x + t(2) = 2t + t^3 \Rightarrow x = t^3$

This passes through $P(x_1, y_1) \Rightarrow t^3 = x_1$.............(3)

Let $(2t, t_1^2)(2t_2, t_2^2), (2t_3, t_3^2)$ be the co-normal points form P.

$2t_1, 2t_2, 2t_3$ in A.P. $\Rightarrow 4t_2 = 2(t_1 + t_3) \Rightarrow t_2 = \dfrac{t_1 + t_3}{2}$

$\therefore$ slopes of the tangents $t_1, t_2 \,\&\, t_3$ are in A.P.

71. The line $lx+my+n=0$ is normal to the parabola $y^2=4ax$ if

1. $al(l^2+2m^2)+m^2n=0$
2. $al(l^2+2m^2)=m^2n$
3. $al(2l^2+m^2)+m^2n=0$
4. $al(2l^2+m^2)=2m^2n$

Key. 1
Sol. Conceptual

72. The feet of the normals to $y^2=4ax$ from the point $(6a,0)$ are

1. $(0,0)$
2. $(4a,4a)$
3. $(4a,-4a)$
4. $(0,0),(4a,4a),(4a,-4a)$

Key. 4
Sol. Equation of any normal to the parabola $y^2=4ax$ is $y=mx-2am-am^3$

If passes through $(6a,0)$ then $0=6am-2am-am^3 \Rightarrow am^3-4am=0 \Rightarrow am(m^2-4)=0$

$\Rightarrow m=0,\pm 2$.

$\therefore$ Feet of the normals $=(am^2,-2am)=(0,0),(4a,-4a),(4a,4a)$.

73. The condition that parabola $y^2=4ax \,\&\, y^2=4c(x-b)$ have a common normal other than x-axis is $(a\neq b\neq c)$

1. $\dfrac{a}{a-c}<2$
2. $\dfrac{b}{a-c}>2$
3. $\dfrac{b}{a-c}<1$
4. $\dfrac{b}{a-c}>1$

Key. 2
Sol. Conceptual

74. Locus of poles of chords of the parabola $y^2=4ax$ which subtends 45^0 at the vertex is $\left(x+4a\right)^2=\lambda\left(y^2-4ax\right)$ then λ =________

1. 1
2. 2
3. 3
4. 4

Key. 4
Sol. Parabola is $y^2=4ax \rightarrow$ 1

Polar of a pole $\left(x_1y_1\right)=yy_1-2ax=2ax_1 \rightarrow$ 2

Making eq 1 homogeneous w.r.t 2

$$y^2-4ax\left(\frac{yy_1-2ax}{2ax_1}\right)=0$$

$$x_1y^2-2xyy_1+4ax^2=0$$

Angle between these pair of lines is 45^0

$$\therefore \tan 45^0 = \frac{2\sqrt{y_1^2 - 4ax_1}}{(x_1 + 4a)}$$

Locus of $(x_1 y_1)$ is

$$\Rightarrow (x + 4a)^2 = 4(y^2 - 4ax)$$

$$\Rightarrow \lambda = 4$$

75. Length of the latus rectum of the parabola $\sqrt{x} + \sqrt{y} = \sqrt{a}$

1. $a\sqrt{2}$ 2. $\frac{a}{\sqrt{2}}$ 3. a 4. 2a

Key. 1

Sol. $\sqrt{x} = \sqrt{a} - \sqrt{y}$

$$x = a + y - 2\sqrt{ay}$$

$$(x - y - a)^2 = 4ay$$

$$x^2 + (y + a)^2 - 2x(a + y) = 4ay$$

$$x^2 + y^2 - 2xy + 2ay + a^2 - 2ax = 4ay$$

$$x^2 + y^2 - 2xy = 2ax + 2ay - a^2$$

$$(x - y)^2 = 2a\left(x + y - \frac{a}{2}\right)$$

Axis is x-y=0

$$\left(\frac{x - y}{\sqrt{2}}\right)^2 = \frac{2a}{2}\left(\frac{x + y - \frac{a}{2}}{\sqrt{2}}\right) \times \sqrt{2}$$

$$\left(\frac{x - y}{\sqrt{2}}\right)^2 = a\sqrt{2}\left(\frac{x + y - \frac{a}{2}}{\sqrt{2}}\right)$$

$\therefore$ lengthy $L.R = a\sqrt{2}$

76. Equation of common tangent to $x^2 = 32y$ and $y^2 = 32x$

1. $x + y = 8$ 2. $x + y + 8 = 0$ 3. $x - y = 8$ 4. $x - y + 8 = 0$

Key. 2

Sol. Common tangets $y^2 = 4ax$ and $x^2 = 4ay$ is $xa^{\frac{1}{3}} + yb^{\frac{1}{3}} + a^{\frac{2}{3}}b^{\frac{2}{3}} = 0$

Here a=8, b=8

77. The angle subtended at the focus by the normal chord of the point $(\lambda,\lambda),\ \lambda \neq 0$ on the parabola $y^2 = 4ax$ is

A) $\frac{\pi}{4}$ B) $\frac{\pi}{3}$ C) $\frac{\pi}{2}$ D) $\frac{\pi}{6}$

Key. C

Sol. Putting (λ,λ) in $y^2 = 4ax$, gives $\lambda = 4a$

Slope of normal at $(4a,4a)$ is $-{}^{n}C_2$

Equation of normal at $(4a,4a)$ is $y-4a = -2(x-4a) \Rightarrow y+2x-12a = 0$

The coordinates of intersection points of the above normal,

$y+2\sum_{k=2}^{n}(k-1)-12a = 0 \Rightarrow y^2+2ay-24a^2 = 0$

$y = 4a-6a$ and $x = 4a, 9a,$

Then slope of SA, $m_1 = \frac{n(n-1)}{2} = {}^{n}C_2$

And slope of SB, $m_2 = \frac{6a}{8a} = \frac{-3}{4}$ $m_1 m_2 = -1$

78. A circle with its centre at the focus of the parabola $y^2 = 4ax$ and touching its directrix intersects the parabola at points A, B. Then length AB is equal to

A) $4a$ B) $2a$ C) a D) $7a$

Key. A

Sol. Centre of circle $(a,0)$ and radius $2a$

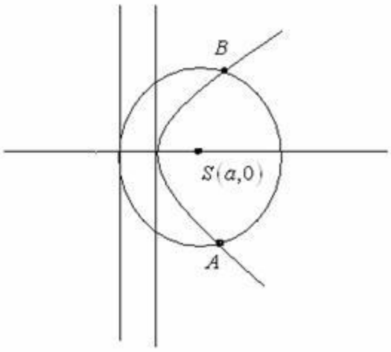
Equation of circle $(x-a)^2+y^2 = 4a^2$

$x^2+y^2-2ax-3a^2 = 0$ and $y^2 = 4ax$ solving $x^2+4ax-2ax-3a^2 = 0$

$x^2+2ax-3a^2 = 0$

$x = -3a, a$ and $y = \pm 2a$

$\therefore$ Length of $AB = 4a$



79. Tangents are drawn to $y^2 = 4ax$ from a variable point P moving on $x + a = 0,$ then the locus of foot of perpendicular drawn from P on the chord of contact of P is

A) $y = 0$ B) $(x-a)^2 + y^2 = a^2$ C) $(x-a)^2 + y^2 = 0$ D) $y(x-a) = 0$

Key. C

Sol. Portion of tangent intercepted between parabola and directrix subtends a right angle at the focus.

80. Three normals are drawn to the curve $y^2 = x$ from a point (c,0).Out of three one is always on x- axis. If two other normals are perpendicular to each other ,then the value of c

a) 3/4 b) 1/2 c) 3/2 d) 2

Key. A

Sol. Normal at $(at^2, 2at)$ is $y + tx = 2at + at^3$ $\left(a = \frac{1}{4}\right)$

If this passes through (c, 0)

We have $ct = 2at + at^3 = \frac{t}{2} + \frac{t^3}{4}$

$\Rightarrow t = 0 \text{ or } t^2 = 4c - 2$

If t = 0, the point at which the normal is drawn is (0, 0) if $t \neq 0$, then the two values of t represents slope of normals through (c, 0)

If these normals are perpendicular

then $(-t_1)(-t_2) = -1 \Rightarrow t_1t_2 = -1 \Rightarrow \left(\sqrt{4c-2}\right)\left(-\sqrt{4c-2}\right) = -1$

C = $\frac{3}{4}$

81. If area of Triangle formed by tangents fom the point (x_1,y_1) to the parabola $y^2 = 4ax$ and their chord of contact is

a) $\frac{\left(y_1^{\,2}-4ax_1\right)^{3/2}}{2a^2}$ b) $\frac{\left(y_1^{\,2}-4ax_1\right)^{3/3}}{a^2}$ c) $\frac{\left(y_1^2-4ax_1\right)^{3/2}}{2a}$ d) none of these

Key. C

Sol. Let $A\left(x_1, y_1\right)$ be any point outside the parabola and $B\left(\alpha, \beta\right), C\left(\alpha^1, \beta^1\right)$ be the points of contact of tangents from point A eq of chord BC, $YY_1 = 2a(x+x_1)$

Lengths of $\perp$ from A to BC

= $\frac{2a\left(x_1+x\right)-y_1y}{\sqrt{y^2+4a^2}}$ = $\frac{y_1^2-4ax}{\sqrt{y_1^2+4a^2}}$

Area of $\Delta\, ABC = \frac{1}{2}AL\times BC$

We get $\frac{\left(y_1^2-4ax_1\right)^{3/2}}{2a}$

82. Let 'P' be (1, 0) and Q be any point on the parabola $y^2 = 8x$. The locus of mid point of PQ must be

a) $y^2-4x+2=0$ b) $y^2+4x+2=0$

c) $x^2-4y+2=0$ d) $x^2+4y+2=0$

Key. A

Sol. Let Q be $\left(at^2, 2at\right)$, (for a =2) Q be $\left(2t^2,4t\right)$

Then locus will be eliminant of

x = $\frac{1+2t^2}{2}$, y = $\frac{0+4t}{2}$

We easily get $y^2-4x+2=0$

$\Rightarrow$ (a) is correct

83. Coordinates of the focus of the parabola $\sqrt{\frac{x}{a}}+\sqrt{\frac{y}{b}}=1$ is

A. $\left(\frac{ab}{a+b},\frac{ab}{a+b}\right)$ B. $\left(\frac{ab^2}{a^2+b^2},\frac{a^2b}{a^2+b^2}\right)$

C. $\left(\frac{a^2b}{a+b},\frac{ab^2}{a+b}\right)$ D. $\left(a,b\right)$

Key. B

Sol. $\sqrt{\frac{x}{a}}+\sqrt{\frac{y}{b}}=1$

For this parabola x is a tangent at P(a, 0)

Y-axis a tangent Q(0,b)

$\therefore$ O(0,0) is point if inter section perpendicular tagents

$\therefore$ directrix passing through this point

Clearly $\underline{|OSP} = 90^0$

Hence circle on OP as diameter passing though S

i.e., $x^2 + y^2 - ax = 0$ passing through S.

Ily, $\underline{|OSQ} = 90^0$ $\therefore x^2 + y^2 - bx = 0$ passing through S.

Point of intersecting above circle is focus.

$$x^2 + y^2 - ax = 0$$

$$x^2 + y^2 - bx = 0$$

$$ax - by = 0$$

$$y = \frac{ax}{b} \qquad \Rightarrow x^2 + \frac{a^2x^2}{b^2} = ax$$

$$x\left(\frac{b^2 + a^2}{b^2}\right) = a$$

$$x = \frac{ab^2}{a^2 + b^2}$$

Ily, $y = \dfrac{a^2b}{a^2 + b^2}$

Focus $S = \left(\dfrac{ab^2}{a^2 + b^2}, \dfrac{a^2b}{a^2 + b^2}\right)$.

84. The Length of Latusrectum of the parabola $x = t^2 + t + 1$, $y = t^2 + 2t + 3$ is

A. $\frac{1}{2}$ B. $\frac{1}{\sqrt{2}}$

C. $\frac{1}{2\sqrt{2}}$ D. $\frac{1}{8}$

Key. C

Sol. $\left.\begin{array}{l} x = t^2 + t + 1 \Rightarrow t^2 + t + 1 - x = 0 \\ y = t^2 + 2t + 3 \Rightarrow t^2 + 2t + 3 - u = 0 \end{array}\right\} eliminate\ t$

$$\begin{array}{cccc} 1 & 1-x & 1 & 1 \\ 2 & 3-y & 1 & 1 \end{array}$$

$$\frac{t^2}{3-y-2+2x} = \frac{t}{1-x-3+y} = \frac{1}{1}$$

$$\left.\begin{array}{l} t = -x + y - 2 \\ t = \dfrac{1-y+2x}{-x+y-2} \end{array}\right\} (x-y+2)^2 = (2x-y+1)$$

$$(x-y)^2 + 4(x-y) + 4 = (2x-y+1)$$

$$(x-y)^2 = -2x + 3y - 3$$

$$\therefore (x-y+\lambda)^2 = -2x + 3y - 3 + 2\lambda(x-y) + \lambda^2$$

$$(x-y+\lambda)^2 = x(2\lambda-2) + y(-2\lambda+3) + \lambda^2 - 3$$

$$\left.\begin{array}{l} \therefore\ slope\ of\ \ x-y+1=0\ \ is\ 1 \\ slope\ line\ on\ RHS\ is\ \ \dfrac{2-2\lambda}{3-2\lambda} \end{array}\right\} \frac{2-2\lambda}{3-2\lambda} = -1$$

$$2 - 2\lambda = -3 + 2\lambda$$

$$4\lambda = 5 \Rightarrow \lambda = \frac{5}{4}$$

ε of parabola is $\left(x-y+\frac{5}{4}\right)^2 = \frac{x}{2}+\frac{y}{2}+\frac{25}{16}-3$

$$\left(x-y+\frac{5}{4}\right)^2 = \frac{1}{2}\left(x+y-\frac{23}{16}\right)$$

$$\left(\frac{x-y+\frac{5}{4}}{\frac{1}{\sqrt{2}}}\right)^2 = \frac{1}{2\sqrt{2}}\left(\frac{x+y-\frac{23}{16}}{\sqrt{2}}\right) \qquad \text{LR} = \frac{1}{2\sqrt{2}}$$

85. For different values of k and l the two parabolas $y^2 = 16(x-k)$, $x^2 = 16(y-l)$ always touch each other then locus of point of contact is

A. $x^2+y^2=64$ B. $xy=8$

C. $y^2=8x$ D. $xy=64$

Key. D

Sol. $y^2 = 16(x-k)$ $x^2 = 16(y-l)$

$$2y\frac{dy}{dx} = 16 \qquad 2x = 16\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{8}{y} = m_1 \qquad \frac{dy}{dx} = \frac{x}{8} = m_2$$

Since two circle touch each other $m_1 = m_2 \Rightarrow \frac{8}{y} = \frac{x}{8} \Rightarrow xy = 64$

86. TP and TQ are any two tangents of a parabola $y^2 = 4ax$ and T is the point of intersection of two tangents. If the tangent at a third point on the parabola meets the above two tangents at P^1 and Q^1. Then $\frac{TP^1}{TP}+\frac{TQ^1}{TQ}$

A. (-1) B. $\frac{1}{2}$ C. $-\frac{1}{2}$ D. 2

Key. A

Sol. $T=(at_1t_2,\ a(t_1+t_2))$

$$P^1=\left(at_1t_3\quad a(t_1+t_3)\right)$$

$$Q^1=\left(at_2t_3\quad a(t_2+t_3)\right)$$

$$TP^1:TP=\lambda:1$$

$$\lambda=\frac{at_1t_3-at_1t_2}{at_1t_2-at_1^2}$$

$$=\frac{t_3-t_2}{t_2-t_1}$$

$$\therefore\frac{TP^1}{TP}=\frac{t_3-t_2}{t_2-t_1}$$

lly, Let $TQ^1:TQ=\mu:1$

$$\frac{TQ^1}{TQ}=\frac{at_2t_3-at_1t_2}{at_1t_2-at_2^2}=\frac{t_3-t_1}{t_1-t_2}$$

$$\therefore\frac{TP^1}{TP}+\frac{TQ^1}{TQ}=\frac{t_3-t_2}{t_2-t_1}+\frac{t_3-t_1}{t_1-t_2}=\frac{t_1-t_2}{t_2-t_1}=-1$$

87. The locus of the Orthocentre of the triangle formed by three tangents of the parabola $(4x-3)^2=-64(2y+1)$ is

A) $y=\frac{-5}{2}$ B) $y=1$ C) $x=\frac{7}{4}$ D) $y=\frac{3}{2}$

Key. D

Sol. The locus is directrix of the parabola

88. A pair of tangents with inclinations α,β are drawn from an external point P to the parabola $y^2=16x$. If the point P varies in such a way that $\tan^2\alpha+\tan^2\beta=4$ then the locus of P is a conic whose eccentricity is

A) $\frac{\sqrt{5}}{2}$ B) $\sqrt{5}$ C) 1 D) $\frac{\sqrt{3}}{2}$

Key. B

Sol. Let $m_1=\tan\alpha, m_2=\tan\beta$, Let $P=(h,k)$

m_1, m_2 are the roots of $K = mh + \frac{4}{m} \Rightarrow hm^2 - Km + 4 = 0$

$m_1 + m_2 = \frac{K}{h}; \quad m_1 m_2 = \frac{4}{h}$

$m_1^2 + m_2^2 = \frac{K^2}{h^2} - \frac{8}{h} = 4$

Locus of P is $y^2 - 8x = 4x^2 \Rightarrow y^2 = 4(x+1)^2 - 4 \Rightarrow \frac{(x+1)^2}{1} - \frac{y^2}{4} = 1$

89. The length of the latusrectum of a parabola is $4a$. A pair of perpendicular tangents are drawn to the parabola to meet the axis of the parabola at the points A, B. If S is the focus of the parabola then $\frac{1}{|SA|} + \frac{1}{|SB|} =$

A) $2/a$ B) $4/a$ C) $1/a$ D) $2a$

Key. C

Sol. Let $y^2 = 4ax$ be the parabola

$y = mx + \frac{a}{m}$ and $y = \left(-\frac{1}{m}\right)x - am$ are perpendicular tangents

$S = (a, 0), A = \left(-\frac{a}{m^2}, 0\right), B = (-am^2, 0)$

$|SA| = a\left(1 + \frac{1}{m^2}\right) = \frac{a(1+m^2)}{m^2}$

$|SB| = a(1+m^2)$

90. Length of the focal chord of the parabola $(y+3)^2 = -8(x-1)$ which lies at a distance 2 units from the vertex of the parabola is

A) 8 B) $6\sqrt{2}$ C) 9 D) $5\sqrt{3}$

Key. A

Sol. Lengths are invariant under change of axes

consider $y^2 = 8x$. Consider focal chord at $(2t^2, 4t)$

Focus = (2, 0). Equation of focal chord at t is $y = \frac{2t}{t^2 - 1} 9x - 2) \Rightarrow 2tx + (1-t^2)y - 4t = 0$

$$\frac{4|t|^2}{\sqrt{4t^2 + (1-t^2)^2}} = 2 \Rightarrow (|t| - 1)^2 = 0$$

Length of focal chord at 't'= $2\left(t + \frac{1}{t}\right)^2 = \frac{2(t^2+1)^2}{t^2} = 8$

91. The slope of normal to the parabola $y=\frac{x^2}{4}-2$ drawn through the point $(10,-1)$

A) -2 B) $-\sqrt{3}$ C) $-1/2$ D) $-5/3$

Key. C

Sol. $x^2=4(y+2)$ is the given parabola

Any normal is $x=m(y+2)-2m-m^3$. If $(10,-1)$ lies on this line then

$10=+m-2m-m^3 \Rightarrow m^3+m+10=0 \Rightarrow m=-2$

Slope of normal = $1/m$.

92. m_1, m_2, m_3 are the slope of normals $(m_1<m_2<m_3)$ drawn through the point $(9,-6)$ to the parabola $y^2=4x$. $A=[a_{ij}]$ is a square matrix of order 3 such that $a_{ij}=1$ if $i \neq j$ and $a_{ij}=m_i$ if $i=j$. Then detA =

A) 6 B) –4 C) –9 D) 8

Key. D

Sol. $y=mx-2m-m^3.(9,-6)$ lies on this

$\therefore -6=9m-2m-m^3 \Rightarrow m^3-7m-6=0$

Roots are $-1,-2,3 \therefore$ $|A|=\begin{vmatrix} -2 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 3 \end{vmatrix}=(-2)(-4)-(3-1)+2=8$

93. If parabola of latusrectum ‘u’ touches a fixed equal parabola, the axes of two curves being parallel, then the locus of the vertex of the moving curve is

(a) A parabola of latusrectum ‘2u’

(b) A parabola of latusrectum ‘u’

(c) An ellipse whose major axis is ‘2u’

(d) An ellipse whose minor axis is ‘2u’

Key. A

Sol. Let (α,β) be the vertex of the moving parabola and its equation is

$(y-\beta)^2=-4a(x-\alpha)$ ---------------- (1)

Let the equation of fixed parabola be $y^2=4ax$ ---------- (2) (Here 4a = u)

From (1) & (2) $(y-\beta)^2=-4a\left(\frac{y^2}{4a}-\alpha\right)$

$\Rightarrow 2y^2-2\beta y+\beta^2-4a\alpha=0$

The above is a quadratic equation in y having same roots

$\Rightarrow \Delta=0$ $\Rightarrow \beta^2=8a\alpha$

Hence locus is $y^2=8ax$ i.e., $y^2=2ux$

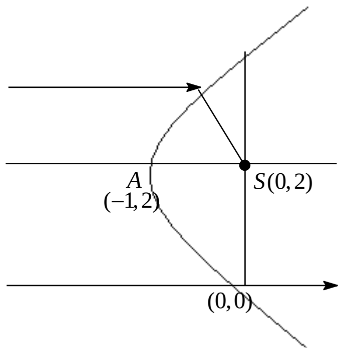
94. A ray of light moving parallel to the x-axis gets reflected form a parabolic mirror whose equation is $(y-2)^2 = 4(x+1)$. After reflection, the ray must pass through the point ……

(a) (0, 2) (b) (2, 0) (c) (0, -2) (d) (-1, 2)

Key. A

Sol. The equation of the axis of the parabola y – 2 = 0

Which is parallel to the x-axis so, a ray parallel to x-axis of parabola. W.K.T any ray parallel to the axis of a parabola passes through this focus after reflection. Here (0, 2) is the focus.



95. If the normal to the parabola $y^2 = 4ax$ at $(at^2, 2at)$ cuts the parabola again at $(aT^2, 2aT)$ then

(a) $-2 \le T \le 2$ (b) $T \in (-\infty, -8) \cup (8, \infty)$

(c) $T^2 < 8$ (d) $T^2 \ge 8$

Key. D

Sol. $T = -t - \dfrac{2}{t}$

$|T| = \left|t + \dfrac{2}{t}\right| \ge 2\sqrt{2}$

$T^2 \ge 8$

96. Let α is the angle which a tangent to $y^2 = 4ax$ makes with its axis, the distance between the tangent and a parallel normal will be

(a) $a\sin^2\alpha\cos^2\alpha$ (b) $a\,\mathrm{cosec}\,\alpha \cdot \sec^2\alpha$ (c) $a\tan^2\alpha$ (d) $a\cos^2\alpha \cdot \mathrm{cosec}^5\alpha$

Key. B

Sol. Equation of Tangent is $yt = x + at^2$

$\therefore Tan\alpha = \dfrac{1}{t}; t = \cot\alpha$

Equation of parallel normal is $yt = x + K$

$a \cdot 1^3 + 2a.1.(-t)^2 + (-t)^2 \cdot K = 0$

$K = \dfrac{-(a + 2at^2)}{t^2}$

$$\text{Distance } = \frac{at^2 + \frac{a+2at^2}{t^2}}{\sqrt{1+t^2}} = \frac{at^4 + 2at^2 + a}{t^2\sqrt{1+t^2}} = \frac{a(t^2+1)^{3/2}}{t^2}$$

97. If the normal at a point P on $y^2 = 4ax(a > 0)$ meet it again at Q in such a way that PQ is of minimum length. If 'O' is vertex then ΔOPQ is

(a) a right angled triangle (b) an obtuse angled triangle

(c) an equilateral triangle (d) right angled isosceles triangle

Key. A

Sol. $PQ = 6a\sqrt{3}; OP = 2a\sqrt{3};\ OQ = 4a\sqrt{6}$

98. Coordinates of the focus of the parabola $\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} = 1$ is

A. $\left(\frac{ab}{a+b}, \frac{ab}{a+b}\right)$ B. $\left(\frac{ab^2}{a^2+b^2}, \frac{a^2b}{a^2+b^2}\right)$

C. $\left(\frac{a^2b}{a+b}, \frac{ab^2}{a+b}\right)$ D. (a,b)

Key. B

Sol. $\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} = 1$

For this parabola x is a tangent at P(a, 0)

Y-axis a tangent Q(0,b)

$\therefore$ O(0,0) is point if inter section perpendicular tagents

$\therefore$ directrix passing through this point

Clearly $\underline{|OSP} = 90^0$

Hence circle on OP as diameter passing though S

i.e., $x^2 + y^2 - ax = 0$ passing through S.

Illy, $\underline{|OSQ} = 90^0$ $\therefore x^2 + y^2 - bx = 0$ passing through S.

Point of intersecting above circle is focus.

$x^2 + y^2 - ax = 0$

$x^2 + y^2 - bx = 0$

$ax - by = 0$

$y = \frac{ax}{b} \qquad \Rightarrow x^2 + \frac{a^2x^2}{b^2} = ax$

$x\left(\frac{b^2 + a^2}{b^2}\right) = a$

$x = \frac{ab^2}{a^2 + b^2}$

lly, $y = \frac{a^2b}{a^2 + b^2}$

Focus $S = \left(\frac{ab^2}{a^2 + b^2}, \frac{a^2b}{a^2 + b^2}\right)$.

99. The Length of Latusrectum of the parabola $x = t^2 + t + 1$, $y = t^2 + 2t + 3$ is

A. $\frac{1}{2}$ B. $\frac{1}{\sqrt{2}}$

C. $\frac{1}{2\sqrt{2}}$ D. $\frac{1}{8}$

Key. C

Sol. $\left.\begin{array}{l} x = t^2 + t + 1 \Rightarrow t^2 + t + 1 - x = 0 \\ y = t^2 + 2t + 3 \Rightarrow t^2 + 2t + 3 - u = 0 \end{array}\right\} eliminate\ t$

1 $\quad 1 - x \quad$ 1 $\quad$ 1

2 $\quad 3 - y \quad$ 1 $\quad$ 1

$$\frac{t^2}{3-y-2+2x}=\frac{t}{1-x-3+y}=\frac{1}{1}$$

$$\left.\begin{array}{l} t=-x+y-2 \\ t=\dfrac{1-y+2x}{-x+y-2} \end{array}\right\}(x-y+2)^2=(2x-y+1)$$

$$(x-y)^2+4(x-y)+4=(2x-y+1)$$

$$(x-y)^2=-2x+3y-3$$

$$\therefore (x-y+\lambda)^2=-2x+3y-3+2\lambda(x-y)+\lambda^2$$

$$(x-y+\lambda)^2=x(2\lambda-2)+y(-2\lambda+3)+\lambda^2-3$$

$$\left.\begin{array}{l} \therefore\ \textit{slope of } x-y+1=0 \textit{ is } 1 \\ \textit{slope line on RHS is } \quad \dfrac{2-2\lambda}{3-2\lambda} \end{array}\right\}\frac{2-2\lambda}{3-2\lambda}=-1$$

$$2-2\lambda=-3+2\lambda$$

$$4\lambda=5 \Rightarrow \lambda=\frac{5}{4}$$

ε of parabola is $\left(x-y+\dfrac{5}{4}\right)^2=\dfrac{x}{2}+\dfrac{y}{2}+\dfrac{25}{16}-3$

$$\left(x-y+\frac{5}{4}\right)^2=\frac{1}{2}\left(x+y-\frac{23}{16}\right)$$

$$\left(\frac{x-y+\frac{5}{4}}{\frac{1}{\sqrt{2}}}\right)^2=\frac{1}{2\sqrt{2}}\left(\frac{x+y-\frac{23}{16}}{\sqrt{2}}\right) \qquad \text{LR} = \frac{1}{2\sqrt{2}}$$

100. For different values of k and l the two parabolas $y^2=16(x-k)$, $x^2=16(y-l)$ always touch each other then locus of point of contact is

A. $x^2+y^2=64$ B. $xy=8$

C. $y^2 = 8x$ D. $xy = 64$

Key. D

Sol. $y^2 = 16(x-k)$ $x^2 = 16(y-l)$

$$2y\frac{dy}{dx} = 16 \qquad 2x = 16\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{8}{y} = m_1 \qquad \frac{dy}{dx} = \frac{x}{8} = m_2$$

Since two circle touch each other $m_1 = m_2 \Rightarrow \frac{8}{y} = \frac{x}{8} \Rightarrow xy = 64$

101. TP and TQ are any two tangents of a parabola $y^2 = 4ax$ and T is the point of intersection of two tangents. If the tangent at a third point on the parabola meets the above two tangents at P^1 and Q^1. Then $\frac{TP^1}{TP} + \frac{TQ^1}{TQ}$

A. (-1) B. $\frac{1}{2}$ C. $-\frac{1}{2}$ D. 2

Key. A

Sol. $T = (at_1t_2,\ a(t_1+t_2))$

$$P^1 = (at_1t_3 \quad a(t_1+t_3))$$

$$Q^1 = (at_2t_3 \quad a(t_2+t_3))$$

$$TP^1 : TP = \lambda : 1$$

$$\lambda = \frac{at_1t_3 - at_1t_2}{at_1t_2 - at_1^2}$$

$$= \frac{t_3 - t_2}{t_2 - t_1}$$

$$\therefore \frac{TP^1}{TP} = \frac{t_3 - t_2}{t_2 - t_1}$$

lly, Let $TQ^1 : TQ = \mu : 1$

$$\frac{TQ^1}{TQ} = \frac{at_2t_3 - at_1t_2}{at_1t_2 - at_2^2} = \frac{t_3 - t_1}{t_1 - t_2}$$

$$\therefore \frac{TP^1}{TP} + \frac{TQ^1}{TQ} = \frac{t_3 - t_2}{t_2 - t_1} + \frac{t_3 - t_1}{t_1 - t_2} = \frac{t_1 - t_2}{t_2 - t_1} = -1$$

102. A normal, whose inclination is 30^0, to a parabola cuts it again at an angle of

a) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ b) $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$ c) $\tan^{-1}(2\sqrt{3})$ d) $\tan^{-1}\left(\frac{1}{2\sqrt{3}}\right)$

Key. D

Sol. The normal at $P(at_1^{\,2}, 2at_1)$ is $y + xt_1 = 2at_1 + at_1^{\,3}$ with slope say $\tan\alpha = -t_1 = \frac{1}{\sqrt{3}}$. If it meets curve at $Q(at_2^{\,2}, 2at_2)$ then $t_2 = -t_1 - \frac{2}{t_1} = \frac{7}{\sqrt{3}}$. Then angle θ between parabola (tangent at Q) and normal at P is given by $\tan\theta = \frac{-t_1 - \frac{1}{t_2}}{1 - \frac{t_1}{t_2}} = \frac{1}{2\sqrt{3}}$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{1}{2\sqrt{3}}\right)$$

103. The locus of vertices of family of parabolas, $y = ax^2 + 2a^2x + 1$ is $(a \neq 0)$ a curve passing through

a) (1,0) b) (1,1) c) (0,1) d) (0,0)

Key. C

Sol.

$$y = ax^2 + 2a^2x + 1 \Rightarrow \frac{y - (1 - a^3)}{a} = (x + a)^2$$

$$\therefore Vertex = (\alpha, \beta) = (-a, 1 - a^3)$$

$$\Rightarrow \beta = 1 + \alpha^3$$

$$\Rightarrow curve\ \ is\ \ y = 1 + x^3$$

104. Equation of circle of minimum radius which touches both the parabolas $y = x^2 + 2x + 4$ and $x = y^2 + 2y + 4$ is

a) $2x^2 + 2y^2 - 11x - 11y - 13 = 0$

b) $4x^2 + 4y^2 - 11x - 11y - 13 = 0$

c) $3x^2 + 3y^2 - 11x - 11y - 13 = 0$

d) $x^2+y^2-11x-11y-13=0$

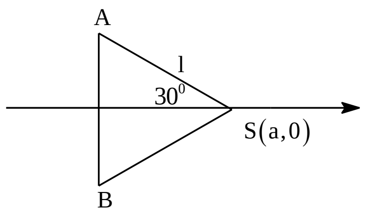
Key. B

Sol. Circle will be touching both parabolas. Circles centre will be on the common normal

105. An equilateral triangle SAB is inscribed in the parabola $y^2=4ax$ having it's focus at 'S'. If the chord AB lies to the left of S, then the length of the side of this triangle is :

a) $3a\left(2-\sqrt{3}\right)$
b) $4a\left(2-\sqrt{3}\right)$
c) $2a\left(2-\sqrt{3}\right)$
d) $8a\left(2-\sqrt{3}\right)$

Key. B



Sol.

$A\left(a-l\cos 30^0, l\sin 30^0\right)$

Point 'A' lies on $y^2=4ax$

$\Rightarrow$ a quadratic in 'l'

106. Let the line lx + my = 1 cuts the parabola $y^2=4ax$ in the points A & B. Normals at A & B meet at a point C. Normal from C other than these two meet the parabola at a point D, then D =

a) $(a,2a)$
b) $\left(\frac{4am}{l^2},\frac{4a}{l}\right)$
c) $\left(\frac{2am^2}{l^2},\frac{2a}{l}\right)$
d) $\left(\frac{4am^2}{l^2},\frac{4am}{l}\right)$

Key. D

Sol. Conceptual

107. The normals to the parabola $y^2=4ax$ at points Q and R meet the parabola again at P. If T is the intersection point of the tangents to the parabola at Q and R, then the locus of the centroid of ΔTQR, is

a) $y^2=3a(x+2a)$
b) $y^2=a(2x+3a)$
c) $y^2=a(3x+2a)$
d) $y^2=2a(2x+3a)$

Key. C

Sol. Let $Q=\left(at_1^2, 2at_1\right)$

$R=\left(at_2^2, 2at_2\right)$

Normals at Q & R meet on parabola

Also $T=\left(at_1t_2, a\left(t_1+t_2\right)\right)$

Let (α, β) be centroid of ΔQRT

Then $3\alpha=a\left(t_1^2+t_2^2+t_1t_2\right) \& \beta=a\left(t_1+t_2\right)$

Eliminate $\left(t_1+t_2\right)$

108. The normal at a point P of a parabola $y^2=4ax$ meets its axis in G and tangent at its vertex in H. If A is the vertex of the parabola and if the rectangle AGQH is completed, then equation to the locus of vertex Q is

a) $y^2(y-2a)=ax^2$ b) $y^2(y+2a)=ax^2$

c) $x^2(x-2a)=ay^2$ d) $x^2(x+2a)=ay^2$

Key. C

Sol. $A=(a,0), H=\left(0, 2at+at^3\right), G=\left(2at+at^2, 0\right), Q=(h,k)$

$(h,k)=\left(2a+at^2, 2at+at^3\right)$

eliminating 't', $x^3=2ax^2+ay^2$

109. If the focus of the parabola $(y-\beta)^2=4(x-\alpha)$ always lies between the lines $x+y=1$ and $x+y=3$, then,

a) $3<\alpha+\beta<4$ b) $0<\alpha+\beta<3$

c) $0<\alpha+\beta<2$ d) $-2<\alpha+\beta<2$

Key. C

Sol. origin & focus line on off side of $x+y=1 \Rightarrow \alpha+\beta>0$

origin & focus line on same side of $x+y=3 \Rightarrow \alpha+\beta<2.$

110. Consider the two parabolas $y^2=4a(x-\alpha) \& x^2=4a(y-\beta)$, where 'a' is the given constant and α, β are variables. If α and β vary in such a way that these parabolas touch each other, then equation to the locus of point of contact

a) circle b) Parabola

c) Ellipse d) Rectangular hyperbola

Key. D

Sol. Let POC be (h,k). Then, tangent at (h,k) to both parabolas represents same line.

111. The points on the axis of the parabola $3y^2+4y-6x+8=0$ from where 3 distinct normals can be drawn is given by

(A) $\left(a, \frac{4}{3}\right); a > \frac{19}{9}$ (B) $\left(a, -\frac{2}{3}\right); a > \frac{19}{9}$

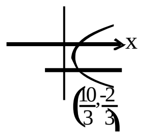
(C) $\left(a, -\frac{2}{3}\right); a > \frac{16}{9}$ (D) $\left(a, -\frac{2}{3}\right); a > \frac{7}{9}$

Key. B

Sol. $3y^2 + 4y = 6x - 8$

$\Rightarrow 3\left(y^2 + \frac{4}{3}y\right) = 6x - 8 \quad \Rightarrow \left(y + \frac{2}{3}\right)^2 = 2x - \frac{8}{3} + \frac{4}{9} \quad \Rightarrow \left(y + \frac{2}{3}\right)^2 = 2\left(x - \frac{10}{9}\right)$

Let any point on the axis $\left(a, -\frac{2}{3}\right)$



$y + \frac{2}{3} = m\left(x - \frac{10}{9}\right) - m - \frac{1}{2}m^3$

$\Rightarrow 0 = m\left[a - \frac{10}{9} - 1 - \frac{1}{2}m^2\right]$

$\Rightarrow a - \frac{19}{9} = \frac{1}{2}m^2 \Rightarrow m^2 = 2\left(a - \frac{19}{9}\right)$ $\quad \therefore a > \frac{19}{9}$

112. Tangents $\overline{PA}$ and $\overline{PB}$ are drawn to $y^2 = 4ax$. If $m_{\overline{PA}}$ & $m_{\overline{PB}}$ are the slopes of the tangents satisfying $\left(m_{\overline{PA}}\right)^2 + \left(m_{\overline{PB}}\right)^2 = 4$ then the locus of P is

(A) $y^2 = 2x(2x + a)$ (B) $y^2 = 2x(2x - a)$

(C) $y^2 = x(x - a)$ (D) None of these

Key. A

Sol. Let $P \equiv (h, k)$

$y = mx + \frac{a}{m}$

$k = mh + \frac{a}{m} \Rightarrow m^2h + a - mk = 0$ $\quad \Rightarrow m_{PA} + m_{PB} = \frac{k}{h}$

$m_{\overline{PA}}.m_{PB} = \frac{a}{h}$

$\frac{k^2}{h^2} - \frac{2a}{h} = 4$

$\Rightarrow k^2 - 2ah = 4h^2$ $\quad \therefore y^2 = 2ax + 4x^2 = 2x(2x + a)$

113. Minimum distance between $y^2 = 4x$ and $x^2 + y^2 - 12x + 31 = 0$.

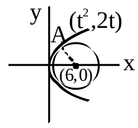
(A) $\sqrt{21}$ (B) $\sqrt{26} - \sqrt{5}$

(C) $\sqrt{20} - \sqrt{5}$ (D) $\sqrt{28} - \sqrt{5}$

Key. C

Sol. $y + tx = 2t + t^3$

$6t = 2t + t^3$



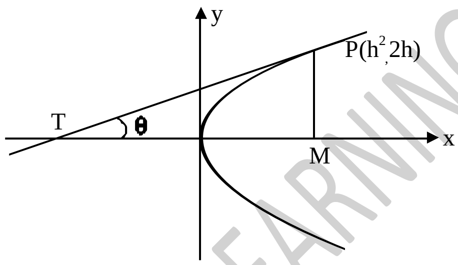
$\Rightarrow \quad t^2 + 2 - 6 = 0$

$t = \pm 2$

$\therefore \quad A \equiv (4,4)$

$\therefore$ Minimum distance $\sqrt{4+16} - \sqrt{5} = \sqrt{20} - \sqrt{5}$.

114. The triangle formed by the tangent to the parabola $y^2 = 4x$ at the point whose abscissa lies in the interval $[a^2, 4a^2]$, the ordinate and the $x-$ axis has the greatest area equal to



(A) $12a^3$ (B) $8a^3$

(C) $16a^3$ (D) None

Key. C

Sol. $P \equiv (h^2, 2h)$

$\tan\theta = \frac{1}{h}$

And $\Delta PTM = \frac{1}{2} \times 2h \times 2h\cot\theta = 2h^3$

$a^2 \le h^2 \le 4a^2$

$\therefore$ maximum area = $2(2a)^3 = 16a^3$

115. Minimum distance between $y^2 - 4x - 8y + 40 = 0$ and $x^2 - 8x - 4y + 40 = 0$

(A) 0 (B) $\sqrt{3}$

(C) $2\sqrt{2}$ (D) $\sqrt{2}$

Key. D

Sol. since two parabolas are symmetrical about $y = x$.

Solving $y = x$ & $y^2 - 4x - 8y + 40 = 0$

$\Rightarrow x^2 - 12x + 40 = 0$

has no real solution

$\therefore$ They don't intersect

Point on $(x-4)^2 = 4(y-6)$ is (6,7) and the corresponding point on $(y-4)^2 = 4(x-6)$ is (7, 6) minimum distance is $\sqrt{2}$.

116. Minimum distance between the parabolas $y^2 - 4x - 8y + 40 = 0$ and $x^2 - 8x - 4y + 40 = 0$ is

(A) 0 (B) $\sqrt{3}$

(C) $2\sqrt{2}$ (D) $\sqrt{2}$

Key. D

Sol. Since two parabolas are symmetrical about

$y = x$

Minimum distance is distance between tangents to the parabola parallel to y = x.

Differentiating $x^2 - 8x - 4y + 40 = 0$ w.r.t x, we get $2x - 8 - 4y' = 0$

$y' = \frac{x-4}{2} = 1$

x = 6 and y = 7

Corresponding point on $(y - 4)^2 = 4(x - 6)$

is (7, 6) so minimum distance = $\sqrt{2}$.

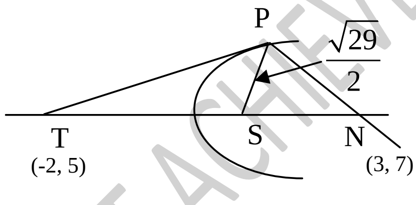
117. If (–2, 5) and (3, 7) are the points of intersection of the tangent and normal at a point on a parabola with the axis of the parabola, then the focal distance of that point is

(A) $\frac{\sqrt{29}}{2}$ (B) $\frac{5}{2}$

(C) $\sqrt{29}$ (D) $\frac{2}{5}$

Key. A

Sol.



118. The locus of the Orthocentre of the triangle formed by three tangents of the parabola $(4x-3)^2 = -64(2y+1)$ is

A) $y = \frac{-5}{2}$ B) $y = 1$ C) $x = \frac{7}{4}$ D) $y = \frac{3}{2}$

Key. D

Sol. The locus is directrix of the parabola

119. A pair of tangents with inclinations α, β are drawn from an external point P to the parabola $y^2 = 16x$. If the point P varies in such a way that $\tan^2\alpha + \tan^2\beta = 4$ then the locus of P is a conic whose eccentricity is

A) $\frac{\sqrt{5}}{2}$ B) $\sqrt{5}$ C) 1 D) $\frac{\sqrt{3}}{2}$

Key. B

Sol. Let $m_1 = \tan\alpha, m_2 = \tan\beta$, Let $P = (h,k)$

m_1, m_2 are the roots of $K = mh + \frac{4}{m} \Rightarrow hm^2 - Km + 4 = 0$

$m_1 + m_2 = \frac{K}{h}; \quad m_1 m_2 = \frac{4}{h}$

$m_1^2 + m_2^2 = \frac{K^2}{h^2} - \frac{8}{h} = 4$

Locus of P is $y^2 - 8x = 4x^2 \Rightarrow y^2 = 4(x+1)^2 - 4 \Rightarrow \frac{(x+1)^2}{1} - \frac{y^2}{4} = 1$

120. The length of the latusrectum of a parabola is $4a$. A pair of perpendicular tangents are drawn to the parabola to meet the axis of the parabola at the points A, B. If S is the focus of the parabola then $\frac{1}{|SA|} + \frac{1}{|SB|} =$

A) $2/a$ B) $4/a$ C) $1/a$ D) $2a$

Key. C

Sol. Let $y^2 = 4ax$ be the parabola

$y = mx + \frac{a}{m}$ and $y = \left(-\frac{1}{m}\right)x - am$ are perpendicular tangents

$S = (a,0), A = \left(-\frac{a}{m^2}, 0\right), B = (-am^2, 0)$

$|SA| = a\left(1 + \frac{1}{m^2}\right) = \frac{a(1+m^2)}{m^2}$

$|SB| = a(1+m^2)$

121. Length of the focal chord of the parabola $(y+3)^2 = -8(x-1)$ which lies at a distance 2 units from the vertex of the parabola is

A) 8 B) $6\sqrt{2}$ C) 9 D) $5\sqrt{3}$

Key. A

Sol. Lengths are invariant under change of axes

consider $y^2 = 8x$. Consider focal chord at $(2t^2, 4t)$

Focus = (2, 0). Equation of focal chord at t is $y = \frac{2t}{t^2-1} 9x - 2) \Rightarrow 2tx + (1-t^2)y - 4t = 0$

$$\frac{4|t|^2}{\sqrt{4t^2 + (1-t^2)^2}} = 2 \Rightarrow \left(|t| - 1\right)^2 = 0$$

Length of focal chord at 't'= $2\left(t+\frac{1}{t}\right)^2 = \frac{2(t^2+1)^2}{t^2} = 8$

122. The slope of normal to the parabola $y = \frac{x^2}{4} - 2$ drawn through the point $(10,-1)$

A) -2 B) $-\sqrt{3}$ C) $-1/2$ D) $-5/3$

Key. C

Sol. $x^2 = 4(y+2)$ is the given parabola

Any normal is $x = m(y+2) - 2m - m^3$. If $(10,-1)$ lies on this line then

$10 = +m - 2m - m^3 \Rightarrow m^3 + m + 10 = 0 \Rightarrow m = -2$

Slope of normal = $1/m$.

123. m_1, m_2, m_3 are the slope of normals $(m_1 < m_2 < m_3)$ drawn through the point $(9,-6)$ to the parabola $y^2 = 4x$. $A = [a_{ij}]$ is a square matrix of order 3 such that $a_{ij} = 1$ if $i \neq j$ and $a_{ij} = m_i$ if $i = j$. Then detA =

A) 6 B) –4 C) –9 D) 8

Key. D

Sol. $y = mx - 2m - m^3.(9,-6)$ lies on this

$\therefore -6 = 9m - 2m - m^3 \Rightarrow m^3 - 7m - 6 = 0$

Roots are $-1,-2,3$ $\therefore$ $|A| = \begin{vmatrix} -2 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 3 \end{vmatrix} = (-2)(-4) - (3-1) + 2 = 8$

124. PQ is any focal chord of the parabola $y^2 = 32x$. The length of PQ can never be less than

(A) 8 unit (B) 16 unit

(C) 32 unit (D) 48 unit

Key. C

Sol. Length of focal chord is $a\left(t+\frac{1}{t}\right)^2$, if $(at^2, 2at)$ is one extremity of the parabola $y^2 = 4ax$.

$\therefore\ t+\frac{1}{t} \geq 2$ (AM $\geq$GM)

$\Rightarrow a\left(t+\frac{1}{t}\right)^2 \geq 4a$

Here, 4a = 32

125. PN is the ordinate of any point P on $y^2 = 4x$. The normal at P to the curve meets the axis at G, then

(A) NG = 1 (B) NG = 2

(C) NG = 4 (D) NG = 6

Key. B

Sol. Let P be $(t^2, 2t)$, then the normal at P, is $y + tx = 2t + t^3$ which meets x-axis at G$(2 + t^2, 0)$. Now as N is $(t^2, 0)$.

$\therefore$ NG = 2

126. The coordinates of the focus of the parabola $y^2 = 4(x + y)$, are

(A) (-1, 1) (B) (0, 2)

(C) (2, 1) (D) (2, -1)

Key. B

SOL. $y^2 = 4x + 4y$

$\Rightarrow (y - 2)^2 = 4(x + 1)$

focus (0, 2)

127. The straight line y = mx + c touches the parabola $y^2 = 4a(x + a)$, if

(A) c = am - a/m (B) c = m - a/m

(C) c = am + a/m (D) c = m + am

Key. C

Sol. Putting y = mx + c in parabola $y^2 = 4a(x + a)$

$\Rightarrow (mx + c)^2 = 4a(x + a)$

$\Rightarrow m^2x^2 + 2(mc - 2a)x + (c^2 - 4a^2) = 0$

If roots are equal i.e., D = 0

$\Rightarrow 4(mc - 2a)^2 - 4m^2(c^2 - 4a^2) = 0$

$\Rightarrow -mc + a + am^2 = 0 \Rightarrow c = am + a/m$

Alternative

Equation of any tangent to the parabola y = m(x + a) = a/m

comparing with y = mx + c

c = am + a/m.

128. Three normals are drawn to the curve $y^2 = x$ from a point (c, 0). Out of three one is always on x-axis. If two other normals are perpendicular to each other, then the value of c is

(A) 3/4 (B) 1/2

(C) 3/2 (D) 2

Key. A

SOL. Normal at $(at^2, 2at)$ is $y + tx = 2at + at^3$ $\left(a = \frac{1}{4}\right)$

if this passes through (c, 0), we have

$$ct = 2at + at^3 = \frac{t}{2} + \frac{t^3}{4}$$

$\Rightarrow t[t^2 + 2 - 4c] = 0$

$\Rightarrow t = 0$ or $t^2 = 4c - 2$

if t = 0 the point at which the normal is drawn is (0, 0).

if $t \neq 0$ then the two values of t represents slope of normals through (c, 0).

if these normals are perpendicular then $(-t_1)(-t_2) = -1$

$\Rightarrow t_1t_2 = -1$

$\Rightarrow (\sqrt{4c-2})(-\sqrt{4c-2}) = -1$

$\Rightarrow \quad c = \frac{3}{4}$

129. Let $y^2 = 4ax$ be a parabola and PQ be a focal chord of parabola. Let T be the point of intersection of tangents at P and Q. Then

(A) area of circumcircle of ΔPQT is $\left(\frac{\pi(PQ)^2}{4}\right)$

(B) orthocenter of ΔPQT will lie on tangent at vertex

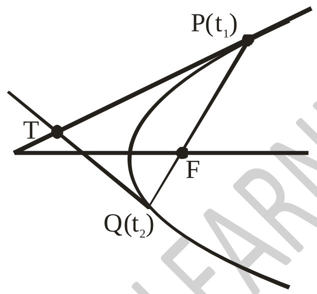
(C) incenter of ΔPQT will be vertex of parabola

(D) incentre of ΔPQT will lie on directrix of parabola

Key. A

Sol. Equation of tangent at P $\rightarrow ty = x + at^2$ …..(i)

Equation of tangent at Q $\rightarrow \frac{-1}{t}y = x + \frac{a}{t^2}$ …..(ii)



$\Rightarrow x = -a.$

$\therefore$ t lies on the directrix and thus ΔPTQ is right angled triangle. thus circle passing through P, Q and T must have P and Q are end points of diameter. thus area of required circle is $\frac{\pi(PQ)^2}{4}$

130. Axis of a parabola is y = x and vertex and focus are at a distance $\sqrt{2}$ and $2\sqrt{2}$ respectively from the origin. Then equation of the parabola is

(A) $(x – y)^2 = 8(x + y – 2)$ (B) $(x + y)^2 = 2 (x + y – 2)$

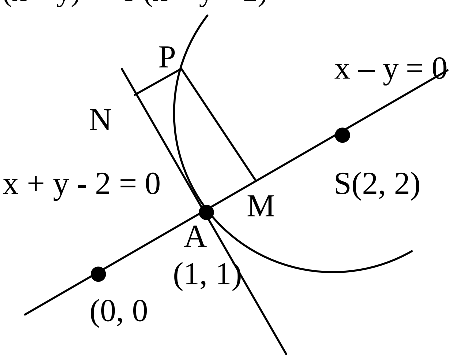
(C) $(x – y)^2 = 4 (x + y – 2)$ (D) $(x + y)^2 = 2(x – y + 2)$

Key. A

Sol. $PM^2 = 4a(PN)$

$$\frac{(x-y)^2}{2} = 4\sqrt{2}\,\frac{(x+y-2)}{\sqrt{2}}$$

$(x – y)^2 = 8 (x + y – 2)$



131. If m_1, m_2 are slopes of tangents drawn from (1, 4) to the parabola $y^2 = 4x$, then

(A) $m_1 + m_2 = 4$ (B) $|m_1 - m_2| = 2\sqrt{3}$

(C) $m_1.m_2 = -1$ (D) $m_1 = m_2$

Key. A

Sol. Any tangent of the parabola $y = mx + \frac{a}{m}$

$\Rightarrow 4 = m + \frac{1}{m}$ $\Rightarrow 4m = m^2 + 1$

$\Rightarrow m^2 - 4m + 1 = 0$

$\Rightarrow m_1 + m_2 = 4$ and $m_1m_2 = 1$

132. The locus of point of intersection of two tangents to the parabola $y^2 = 4x$ such that their chord of contact subtends a right angle at the vertex is

A) $x+4=0$ B) $y+4=0$ C) $x-4=0$ D) $y-4=0$

Key : A

Sol. Chord of contact of (t_1t_2, t_1+t_2) with respect to $y^2 = 4x$ is $(t_1+t_1)y = 2(x+t_1t_2)$

$\Rightarrow \frac{(t_1+t_2)y-2x}{2t_1t_2} = 1 =$ $y^2 = 4x.1 \Rightarrow t_1t_2 + 4 = 0 \Rightarrow t_1t_2 = -4$

$x = -4 \Rightarrow x+4=0$

133. If the line y = x + 2 does not intersect any member of family of parabolas $y^2 = ax$, ($a \in R^+$) at two distinct point, then maximum value of latus rectum of parabola is

(A) 4 (B) 8

(C) 16 (D) 32

KEY : B

HINT

$y^2 = ax$

$y = x + 2$

$(x + 2)^2 - ax = 0$

$x^2 + x(4 - a) + 4 = 0$

$D \le 0$

$a \le 8$

134. Equation of the circle of minimum radius which touches both the parabolas $y = x^2+2x+4$ and

$x = y^2+2y+4$ is

A) $2x^2+2y^2-11x-11y-13 = 0$ B) $4x^2+4y^2-11x-11y-13 = 0$

C) $3x^2+3y^2-11x-11y-13 = 0$ D) $x^2+y^2-11x-11y-13 = 0$

KEY : B

HINT : Given parabolas are symmetric about the line y = x so they have a common normal with slope -1 it meets the parabolas at $\left(\frac{-1}{2},\frac{13}{4}\right),\left(\frac{13}{4},\frac{-1}{2}\right)$ hence the req circles is x^2+y^2

$$-\frac{11}{4}x-\frac{11}{4}y-\frac{13}{4}=0$$

135. The slope of the line which belongs to family of these $(1 + \lambda)x + (\lambda - 1)y + 2(1 - \lambda) = 0$ and makes shortest intercept on $x^2 = 4y - 4$

(A) $\frac{1}{2}$ (B) 1 (C) 0 (D) 2

Key : C

Hint : Family of lines passes through focus hence latus rectum will makes shortest intercept.

136. If the tangents at two points (1, 2) and (3, 6) as a parabola intersect at the point (– 1, 1), then the slope of the directrix of the parabola is

(A) $\sqrt{2}$ (B)– 2

(C) – 1 (D)none of these

Key : C

Hint : If the tangents at P and Q intersect at T, then axis of parabola is parallel to TR, where R is the mid point of P and Q. So, slope of the axis is 1.

$\therefore$ slope of the directrix = – 1.

137. A variable chord PQ of the parabola $y = 4x^2$ substends a right angle at the vertex. Then the locus of points of intersection of the tangents at P and Q is

a) $4y+1=16x^2$ b) $y+4=0$ c) $4y+4=4x^2$ d) $4y+1=0$

Key: D

Hint: $Let\ P\left(t_1,4t_1^2\right),Q\left(t_2,4t_2^2\right)$

Slope of OP x slope of OQ = -1

$\Rightarrow 4t_1.4t_2=-1$

Eq of tangent at $\left(t_1,4t_1^2\right) is$

$y-4t_1^2=8t_1\left(x-t_1\right)\Rightarrow y+4t_1^2=8t_1x$

Eq of tangent at $\left(t_2,4t_2^2\right) is\ y+4t_2^2=8t_2x$

Let (x_1 , y_1) is the point of intersection

$eq(1)-eq(2)\Rightarrow x_1=\frac{t_1+t_2}{2}$

$y_1=8t_1\left(\frac{t_1+t_2}{2}\right)-4t_1^2=4t_1t_2=\frac{-1}{4}$

$\Rightarrow 4y_1+1=0$

138. Let A ≡ (9, 6), B(4, -4) be two points on parabola $y^2 = 4x$ and P(t^2, 2t), t∈[-2, 3] be a variable point on it such that area of ΔPAB is maximum, then point P will be

(A) (4, 4) (B) (3, $-2\sqrt{3}$)

(C) (4, 1) (D) $\left(\frac{1}{4}, 1\right)$

Key: D

Hint: Let P be (t^2, 2t) area of Δ PAB

$$\frac{1}{2}\begin{vmatrix} t^2 & 2t & 1 \\ 9 & 6 & 1 \\ 4 & -4 & 1 \end{vmatrix} = |5t^2 - 5t - 30|$$

it is maximum at t = 1/2.

139. Let (2, 3) be the focus of a parabola and x + y = 0 and x - y = 0 be its two tangents, then equation of its directrix will be

(A) 2x - 3y = 0 (B) 3x + 4y = 0

(C) x + y = 5 (D) 12x - 5y + 1 = 0

Key: A

Hint: Mirror image of focus in the tangent of parabola lie on its directrix.

140. The line x + y = 6 is a normal to the parabola $y^2 = 8x$ at the point

(a) $(18, -12)$ (b) (4, 2) (c) (2, 4) (d) (3, 3)

Key: c

Hint: Slope of the normal is given to be –1. We know that, foot of the normal is (am^2, –2am). Here a = 2, m = –1. Hence the required point is (2, 4).

141. The tangent and normal at the point P(4, 4) to the parabola, $y^2 = 4x$ intersect the x–axis at the points Q and R respectively. Then the cirucm centre of the ΔPQR is

(A) (2, 0) (B) (2, 1)

(C) (1, 0) (D) (1, 2)

Key : C

Sol : Eq. of tangent 2y = x + 4

∴ Q≡ (– 4, 0)

Eq. of normal is y – 4 = – 2 (x – 4)

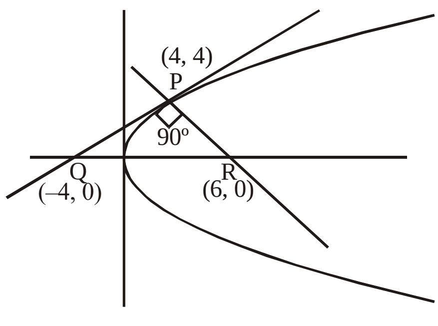
⇒ y + 2x = 12

Clearly QR is diameter of the required circle.

⇒ (x + 4) (x – 6) + y^2 = 0

⇒ $x^2 + y^2 - 2x - 24 = 0$

centre (1, 0)



142. The mirror image of the parabola $y^2 = 4x$ in the tangent to the parabola to the point (1,2) is

(A) $(x-1)^2 = 4(y+1)$ (B) $(x+1)^2 = 4(y+1)$

(C) $(x+1)^2 = 4(y-1)$ (D) $(x-1)^2 = 4(y-1)$

Key : C

Sol : Any point on the given parabola is $\left(t^2, 2t\right)$. The equation of the tangent at (1,2) is x-y +1 = 0.

The image (h,k) of the point $(t^2,2t)$ in x-y + 1 = 0 is

given by $\frac{h-t^2}{1} = \frac{k-2t}{-1} = \frac{-2\left(t^2-2t+1\right)}{1+1}$

$\therefore \quad h = t^2 - t^2 + 2t - 1 = 2t - 1$

and $\quad k = 2t + t^2 - 2t + 1 = t^2 + 1$

Eliminating t from h = 2t – 1 and k = t^2+1

we get, $(h+1)^2 = 4(k-1)$

The required equation of reflection is $\left(x+1\right)^2 = 4\left(y-1\right)$

143. $Min\{(x_1-x_2)^2 + (12+\sqrt{1-x_1^2}-\sqrt{4x_2})^2\} \forall x_1, x_2 \in R$ is

A. $4\sqrt{5}-1$ B. $4\sqrt{5}+1$ C. $\sqrt{5}+1$ D. $\sqrt{5}-1$

Key. A

Sol. Let $y_1 = 12+\sqrt{1-x_1^2}$ and $y_2 = \sqrt{4x_2}$

$(y_1-12)^2 = 1-x_1^2 \Rightarrow x_1^2 + (y_1-12)^2 = 1; y_2^2 = 4x_2$

Required answer is shortest distance between two curves $x^2+(y-12)^2=1$ and $y^2=4x$

144. The radius of largest circle which passes through focus of parabola $y^2 = 4(x+y)$ and also contained in it is

A. 4 B. 1 C. 3 D. 2

Key. A

Sol. Parabola is $y^2-4y=4x \Rightarrow (y-2)^2 = 4(x+1)$

Focus = (0,2)

Let radius of circle = r then centre = (r,2)

Circle is $(x-r)^2+(y-2)^2=r^2$

$\Rightarrow (x-r)^2+4(x+1)=r^2$ has equal roots $\Delta = 0 \Rightarrow r = 4$

145. Length of the latus rectum of the parabola $\sqrt{x}+\sqrt{y}=\sqrt{a}$

1. $a\sqrt{2}$ 2. $\frac{a}{\sqrt{2}}$ 3. a 4. 2a

Key. 1

Sol. $\sqrt{x} = \sqrt{a} - \sqrt{y}$

$x = a + y - 2\sqrt{ay}$

$(x - y - a)^2 = 4ay$

$x^2 + (y + a)^2 - 2x(a + y) = 4ay$

$x^2 + y^2 - 2xy + 2ay + a^2 - 2ax = 4ay$

$x^2 + y^2 - 2xy = 2ax + 2ay - a^2$

$(x - y)^2 = 2a\left(x + y - \frac{a}{2}\right)$

Axis is x-y=0

$$\left(\frac{x-y}{\sqrt{2}}\right)^2 = \frac{2a}{2}\left(\frac{x + y - \frac{a}{2}}{\sqrt{2}}\right) \times \sqrt{2}$$

$$\left(\frac{x-y}{\sqrt{2}}\right)^2 = a\sqrt{2}\left(\frac{x + y - \frac{a}{2}}{\sqrt{2}}\right)$$

$\therefore$ lengthy $L.R = a\sqrt{2}$

146. Equation of common tangent to $x^2 = 32y$ and $y^2 = 32x$

1. $x + y = 8$
2. $x + y + 8 = 0$
3. $x - y = 8$
4. $x - y + 8 = 0$

Key. 2

Sol. Common tangets $y^2 = 4ax$ and $x^2 = 4ay$ is $xa^{\frac{1}{3}} + yb^{\frac{1}{3}} + a^{\frac{2}{3}}b^{\frac{2}{3}} = 0$

Here a=8, b=8

147. Locus of poles of chords of the parabola $y^2 = 4ax$ which subtends 45^0 at the vertex is $(x + 4a)^2 = \lambda(y^2 - 4ax)$ then λ =________

1. 1
2. 2
3. 3
4. 4

Key. 4

Sol. Parabola is $y^2 = 4ax \rightarrow$ (1)

Polar of a pole $(x_1y_1) = yy_1 - 2ax = 2ax_1 \rightarrow$ (2)

Making eq (1) homogeneous w.r.t (2)

$$y^2 - 4ax\left(\frac{yy_1 - 2ax}{2ax_1}\right) = 0$$

$$x_1y^2 - 2xyy_1 + 4ax^2 = 0$$

Angle between these pair of lines is 45^0

$$\therefore \tan 45^0 = \frac{2\sqrt{y_1^2 - 4ax_1}}{(x_1 + 4a)}$$

Locus of (x_1y_1) is

$$\Rightarrow (x+4a)^2 = 4(y^2 - 4ax)$$

$$\Rightarrow \lambda = 4$$

148. The equation of the normal to the parabola $y^2 = 8x$ at the point t is

1. $y - x = t + 2t^2$ 2. $y + tx = 4t + 2t^3$ 3. $x + ty = t + 2t^2$ 4. $y - x = 2t - 3t^3$

Key. 2

Sol. Equation of the normal at 't' is $y + tx = 2(2)t + (2)t^3 \Rightarrow y + tx = 4t + 2t^3$

149. The slope of the normal at $(at^2, 2at)$ of the parabola $y^2 = 4ax$ is

1. $\frac{1}{t}$ 2. t 3. $-t$ 4. $-\frac{1}{t}$

Key. 3

Sol. Slope of the normal at 't' is $-t$.

150. If the normal at the point 't' on a parabola $y^2 = 4ax$ meet it again at t_1, then $t_1 =$

1. t 2. $-t - 1/t$ 3. $-t - 2/t$ 4. None

Key. 3

Sol. Equation of the normal at t is $tx + y = 2at + at^3 \rightarrow (1)$

Equation of the chord passing through t and t_1 is $y(t + t_1) = 2x + 2att_1 \rightarrow (2)$

Comparing (1) and (2) we get $\frac{t}{-2} = \frac{1}{t + t_1} \Rightarrow t + t_1 = -\frac{2}{t} \Rightarrow t_1 = -\frac{2}{t} - t$.

151. If the normal at t_1 on the parabola $y^2 = 4ax$ meet it again at t_2 on the curve, then $t_1(t_1 + t_2) + 2 =$

1. 0 2. 1 3. t_1 4. t_2

Key. 1

Sol. Equation of normal at t_1 is $t_1x + y = 2at_1 + at_1^3$

It passes through $t_2 \Rightarrow at_1t_2^2 + 2at_2 = 2at_1 + at_1^3$

$\Rightarrow t_1(t_2^2 - t_1^2) = 2(t_1 - t_2) \Rightarrow t_1(t_1 + t_2) = -2 \Rightarrow t_1(t_1 + t_2) + 2 = 0$

152. If the normal at $(1,2)$ on the parabola $y^2 = 4x$ meets the parabola again at the point $(t^2, 2t)$, then the value of t is

1. 1 2. 3 3. -3 4. -1

Key. 3

Sol. $Let(1,2) = (t_1^2, 2t_1) \Rightarrow t_1 = 1$

$t = -t_1 - \frac{2}{t_1} = -1 - \frac{2}{1} = -3$

153. If the normal to parabola $y^2 = 4x$ at $P(1,2)$ meets the parabola again in Q, then Q =

1. $(-6,9)$ 2. $(9,-6)$ 3. $(-9,-6)$ 4. $(-6,-9)$

Key. 2

Sol. $P = (1,2) = (t^2, 2t) \Rightarrow t = 1$

$Q = (t_1^2, 2t_1) \Rightarrow t_1 = -t - 2/t = -1 - 2 = -3 \Rightarrow Q = (9,-6)$.

154. If the normals at the points t_1 and t_2 on $y^2 = 4ax$ intersect at the point t_3 on the parabola, then $t_1t_2 =$

1. 1 2. 2 3. t_3 4. $2t_3$

Key. 2

Sol. Let the normals at t_1 and t_2 meet at t_3 on the parabola.

The equation of the normal at t_1 is $y + xt_1 = 2at_1 + at_1^3 \rightarrow (1)$

Equation of the chord joining t_1 and t_3 is $y(t_1 + t_3) = 2x + 2at_1t_3 \rightarrow (2)$

(1) and (2) represent the same line.

$\therefore \frac{t_1 + t_3}{1} = \frac{-2}{t_1} \Rightarrow t_3 = -t_1 - \frac{2}{t_1}$. Similarly $t_3 = -t_2 - \frac{2}{t_2}$

$\therefore -t_1 - \frac{2}{t_1} = -t_2 - \frac{2}{t_2} \Rightarrow t_1 - t_2 = \frac{2}{t_2} - \frac{2}{t_1} \Rightarrow t_1 - t_2 = \frac{2(t_1 - t_2)}{t_1t_2} \Rightarrow t_1t_2 = 2$

155. The number of normals thWSat can be drawn to the parabola $y^2 = 4x$ form the point $(1,0)$ is

1. 0 2. 1 3. 2 4. 3

Key. 2

Sol. $(1,0)$ lies on the axis between the vertex and focus $\Rightarrow$ number of normals =1.

156. The number of normals that can be drawn through $(-1,4)$ to the parabola $y^2 - 4x + 6y = 0$ are

1. 4 2. 3 3. 2 4. 1

Key. 4

Sol. $Let\ S \equiv y^2 - 4x + 6y$. $S_{(-1,4)} = 4^2 - 4(-1) + 6(4) = 16 + 4 + 24 = 44 > 0$

$\therefore$ $(-1,4)$ lies out side the parabola and hence one normal can be drawn from $(-1,4)$ to the parabola.

157. If the tangents and normals at the extremities of a focal chord of a parabola intersect at (x_1, y_1) and (x_2, y_2) respectively, then

1. $x_1 = x_2$ 2. $x_1 = y_2$ 3. $y_1 = y_2$ 4. $x_2 = y_1$

Key. 3

Sol. $Let\ A(t_1)\ B(t_2)$ the extremiues of a focal chard of $y^2 = 4ax$

$\therefore\ t_1 t_2 = -1$

$(x_1, y_1) = [at_1t_2, a(t_1 + t_2)]; (x_2, y_2) = [a(t_1^2 + t_2^2 + t_1t_2 + 2), at_1t_2(t_1 + t_2)]$

$y_2 = -at_1t_2(t_1 + t_2) = -a(-1)(t_1 + t_2) = a(t_1 + t_2) = y_1$

158. The normals at three points P, Q, R of the parabola $y^2 = 4ax$ meet in (h,k). The centroid of triangle PQR lies on

1. $x = 0$ 2. $y = 0$ 3. $x = -a$ 4. $y = a$

Key. 2

Sol. $Let\ P(t_1), Q(t_2) \,\&\, R(t_3)$

Equation of a normal to $y^2 = 4ax$ is $y + tx = 2at + at^3$

This passes through $(h,k) \Rightarrow k + th = 2at + at^3 \Rightarrow at^3 + (2a - h)t - k = 0$

t_1, t_2, t_3 are the roots of this equation $t_1 + t_2 + t_3 = 0$

Centroid of ΔPQR is $G\left[\frac{a}{3}(t_1^2+t_2^2+t_3^2), \frac{2a}{3}(t_1+t_2+t_3)\right]$

$t_1+t_2+t_3=0 \Rightarrow \frac{2a}{3}(t_1+t_2+t_3)=0 \Rightarrow G$ lies on $y=0$.

159. The ordinate of the centroid of the triangle formed by conormal points on the parabola $y^2=4ax$ is

1. 4 2. 0 3. 2 4. 1

Key. 2

Sol. $Let\, t_1, t_2 \,\&\, t_3$ be the conormal points drawn from (x_1, y_1) to $y^2=4ax$

Equation of the normal at point 't' to $y^2=4ax$ is $y+tx=2at+at^3$

This passes through $(x_1, y_1) \Rightarrow y_1+tx_1=2at+at^3 \Rightarrow at^3+(2a-x_1)t-y_1=0$

t_1, t_2, t_3 are the roots of the equation. $\therefore$ $t_1+t_2+t_3=0$

The ordinate of the centroid of the triangle formed by the points $t_1, t_2 \,\&\, t_3$ is $\frac{2a}{3}(t_1+t_2+t_3)=0$

160. The normals at two points P and Q of a parabola $y^2=4ax$ meet at (x_1, y_1) on the parabola. Then PQ^2=

1. $(x_1+4a)(x_1+8a)$ 2. $(x_1+4a)(x_1-8a)$ 3. $(x_1-4a)(x_1+8a)$ 4. $(x_1-4a)(x_1-8a)$

Key. 2

Sol. $Let\, P=(at_1^2, 2at_1), Q=(at_2^2, 2at_2)$

Since the normals at P and Q meet on the parabola, $t_1t_2=2$.

Point of intersection of the normals $(x_1, y_1)=\left(a\left[t_1^2+t_2^2+t_1t_2+2\right], -at_1t_2\left[t_1+t_2\right]\right)$

$\Rightarrow$ $x_1=a(t_1^2+t_2^2+t_1t_2+2)=a(t_1^2+t_2^2+4) \Rightarrow a(t_1^2+t_2^2)=x_1-4a$

$$PQ^2=(at_1^2-at_2^2)^2+(2at_1-2at_2)^2=a^2(t_1-t_2)^2[(t_1+t_2)^2+4]$$
$$=a(t_1^2+t_2^2-4)a(t_1^2+t_2^2+8)=(x_1-8a)(x_1+4a)$$

161. If a normal subtends a right angle at the vertex of the parabola $y^2=4ax$, then its length is

1. $\sqrt{5}a$ 2. $3\sqrt{5}a$ 3. $6\sqrt{3}a$ 4. $7\sqrt{5}a$

Key. 3

Sol. $Let\, a(at_1^2, 2at_1), B(at_2^2, 2at_2)$.

The normal at A cuts the curve again at B. $\therefore \quad t_1 + t_2 = -\frac{2}{t_1}$........(1)

Again AB subtends a right angle at the vertex $0(0,0)$ of the parabola.

Slope $OA = \frac{2at_1}{at_1^2} = \frac{2}{t_1}$, slope of $OB = \frac{2}{t_2}$

$OA \perp OB \Rightarrow \frac{2}{t_1}.\frac{2}{t_2} = -t_1 t_2 = -4$......(2)

Slope of AB is $\frac{2a(t_2 - t_1)}{a(t_2^2 - t_1^2)} = \frac{2}{t_1 + t_2} = -t_1$. [By (1)]

Again from (1) and (2) on putting for t_2, we get $t_1 = \frac{4}{t_1} = -\frac{2}{t_1}$. $\quad\therefore \quad t_1^2 = 2$ or $t_1 \pm \sqrt{2}$

$t_2 = \frac{-4}{t_1} = \frac{-4}{(\pm\sqrt{2})} = \pm 2\sqrt{2}. \quad \therefore \quad A = (2a, \pm 2a\sqrt{2}), B = (8a, \pm 4\sqrt{a})$

$AB = \sqrt{(2a - 8a)^2 + (2a\sqrt{2} + 4\sqrt{2}a)^2} = \sqrt{36a^2 + 72a^2} = \sqrt{108a^2} = 6\sqrt{3}a.$

162. Three normals with slopes m_1, m_2, m_3 are drawn from any point P not on the axis of the parabola $y^2 = 4x$. If $m_1 m_2 = a$, results in locus of P being a part of parabola, the value of 'a' equals

1. 2 2. -2 3. 4 4. -4

Key. 1

Sol. Equation of normal to $y^2 = 4x$ is $y = mx - 2m - m^3$ ...(i)

It passes through (α, β) $\quad \therefore \quad m_1 m_2 m_3 \beta = m\alpha - 2, -m^3$

$\Rightarrow \quad m^3 + (2 - \alpha)\, m + \beta = 0 \qquad \ldots.(ii)$

(Let m_1, m_2, m_3 are roots)

$\therefore \quad m_1 m_2 m_3 = -\beta \qquad$ (as $\quad m_1 m_2 = a$) $\quad \Rightarrow \quad m_3 = -\frac{\beta}{a}$

Now $-\frac{\beta^3}{a^3} - (2 - \alpha) \times \frac{\beta}{a} + \beta = 0$

$\Rightarrow \beta^3+(2-\alpha)a^2\beta-\beta a^3=0$

$\Rightarrow$ locus of P is $y^3+(2-x)ya^2-ya^3=0$

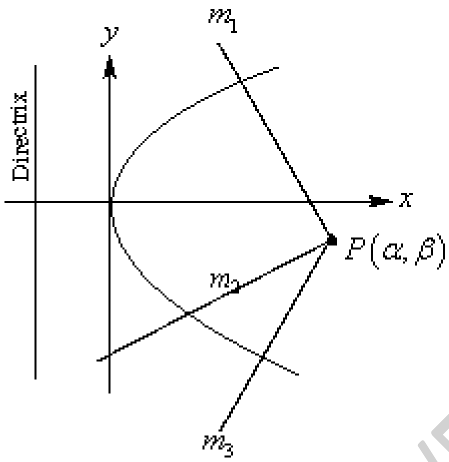
As P is not the axis of parabola

$\Rightarrow y^2=a^2x-2a^2+a^3$ as it is the part of $y^2=4x$

$\therefore a^2=4$ or $-2a^2+a^3=0$, $a=\pm 2$ or $a^2(a-2)=0$

$a=\pm 2$ or $a=0$, $a=2$

$\Rightarrow a=2$ is the required value of a



163. The length of the normal chord drawn at one end of the latus rectum of $y^2=4ax$ is

1. $2\sqrt{2}a$ 2. $4\sqrt{2}a$ 3. $8\sqrt{2}a$ 4. $10\sqrt{2}a$

Key. 3

Sol. One end of the latus rectum $=(a,2a)$

Equation of the normal at $(a,2a)$ is $2a(x-a)+2a(y-2a)=0 \Rightarrow x+y-3a=0$

Solving; $y^2=4ax, x+y-3a=0$ we get the ends of normal chord are $(a,2a),(9a,-6a)$.

Length of the chord $=\sqrt{(9a-a)^2+(-6a-2a)^2} = \sqrt{64a^2+64a^2}=8\sqrt{2}a.$

164. If the line $y=2x+k$ is normal to the parabola $y^2=4x,$ then value of k equals

1. -2 2. -12 3. -3 4. -1/3

Key. 2

Sol. Conceptual

165. The normal chord at a point 't' on the parabola $y^2 = 4ax$ subtends a right angle at the vertex. Then $t^2 =$

1. 4 2. 2 3. 1 4. 3

Key. 2

Sol. Equation of the normal at point 't' is $y + tx = 2at + at^3 \Rightarrow \frac{y + tx}{2at + at^3} = 1$

Homoginising $y^2 = 4ax\left(\frac{y + tx}{2at + at^3}\right) \Rightarrow (2at + at^3)y^2 - 4ax(y + tx) = 0$

These lines re $\perp 1r \Rightarrow$ $2at + at^3 - 4at = 0 \Rightarrow at(t^2 - 2) = 0 \Rightarrow t^2 = 2$

166. A is a point on the parabola $y^2 = 4ax.$ The normal at A cuts the parabola again at B. If AB subtends a right angle at the vertex of the parabola, then slope of AB is

1. $\sqrt{2}$ 2. 2 3. $\sqrt{3}$ 4. 3

Key. 1

Sol. $Let\ A(at_1^2, 2at_1)$ and $B(at_2^2, 2at_2)$.

The normal at A cuts the curve again at B. $\therefore\ t_1 + t_2 = -2/t_1 ....(1)$

Again AB subtends a right angle at the vertex $O(0,0)$ of the parabola.

Slope of $OA = \frac{2at_1}{at_1^2} = \frac{2}{t_1}$, Slope of $OB = \frac{2}{t_2}$

$OA \perp OB \Rightarrow \frac{2}{t_1}.\frac{2}{t_2} = -1 \Rightarrow t_1t_2 = -4....(2)$

Slope of AB is $\frac{2a(t_2 - t_1)}{a(t_2^2 - t_1^2)} = \frac{2}{t_1 + t_2} = -t_1$ by (1)

Again from (1) and (2) on putting for t_2 we get $t_1 - \frac{4}{t_1} = \frac{2}{t_1}.\ \therefore\ t_1^2 = 2 \Rightarrow t_1 = \pm\sqrt{2}$.

$\therefore$ Slope $= \pm\sqrt{2}$.

167. If the normal at P meets the axis of the parabola $y^2 = 4ax$ in G and S is the focus, then SG =

1. SP 2. $2SP$

3. $\frac{1}{2}SP$ 4. None

Key. 1

Sol. Equation of the normal at $P(at^2, 2at)$ is $tx + y = 2at + at^3$

Since it meets the axis, $y=0 \Rightarrow tx = 2at+at^3 \Rightarrow x = 2a+at^2$

$\therefore G=(2a+at^2,0)$, Focus $S=(a,0)$

$SG=\sqrt{(2a+at^2-a)^2+(0-0)^2} = \sqrt{(a+at^2)^2} = a+at^2 = a(1+t^2)$

$SP=\sqrt{(at^2-a)^2+(2at-0)^2} = \sqrt{(at^2-a)^2+4a^2t^2} = \sqrt{(at^2+a)^2} = at^2+a = a(t^2+1)$

$\therefore SG=SP$

168. The normal of a parabola $y^2=4ax$ at (x_1, y_1) subtends right angle at the

1. Focus 2. Vertex 3. End of latus rectum 4. None of these

Key. 1

Sol. Conceptual

169. The normal at P cuts the axis of the parabola $y^2=4ax$ in G and S is the focus of the parabola. If ΔSPG is equilateral then each side is of length

1. a 2. $2a$ 3. $3a$ 4. $4a$

Key. 4

Sol. $Let\ P(at^2, 2at)$

Equation of the normal at $P(t)$ is $y+tx=2at+at^3$

Equation to $y-$axis is $x=0$. Solving $G(2a+at^2,0)$

Focus $s(a,0)$

ΔSPG is equilateral $\Rightarrow PG=GS \Rightarrow \sqrt{4a^2+4a^2t^2} = \sqrt{a^2(1+t^2)^2}$

$\Rightarrow 4a^2(1+t^2) = a^2(1+t^2)^2 \Rightarrow 4 = 1+t^2 \Rightarrow t^2=3$

Length of the side $= SG = a(1+t^2) = a(1+3) = 4a$

170. If the normals at two points on the parabola $y^2=4ax$ intersect on the parabola, then the product of the abscissa is

1. $4a^2$ 2. $-4a^2$ 3. $2a$ 4. $4a^4$

Key. 1

Sol. $Let\ P(at_1^2, 2at_1); Q(at_2^2, 2at_2)$

Normals at $P\ \&\ Q$ on the parabola intersect on the parabola $\Rightarrow t_1t_2 = 2$

$at_1^2 \times at_2^2 = a^2(t_1t_2)^2 = a^2(2)^2 = 4a^2$

171. If the normals at two points on the parabola intersects on the curve, then the product of the ordinates of the points is

1. $8a$ 2. $8a^2$ 3. $8a^3$ 4. $8a^4$

Key. 2

Sol. Let the normals at $P(t_1)$ and $Q(t_2)$ intersect on the parabola at $R(t_3)$.

Equation of any noemal is $tx + y = 2at + at^3$

Since it passes through Q we get $t.at_3^2 + 2at_3 = 2at + at^3$

$\Rightarrow at^3 + (2a - at_3^2)t - 2at_3 = 0,$ which is a cubic equation in t and hence its roots are t_1, t_2, t_3.

Product of the roots $= t_1t_2t_3 = \dfrac{-(-2at_3)}{a} = 2t_3 \Rightarrow t_1t_2 = 2$

Product of the absisson of P and Q $= at_1^2.at_2^2 = a^2(t_1t_2)^2 = a^2(2)^2 = 4a^2$.

Product of the ordinates of P and Q $= 2at_1.2at_2 4a^2.t_1t_2 = 4a^2(2) = 8a^2$

172. The equation of the locus of the point of intersection of two normals to the parabola $y^2 = 4ax$ which are perpendicular to each other is

1. $y^2 = a(x-3a)$ 2. $y^2 = a(x+3a)$ 3. $y^2 = a(x+2a)$ 4. $y^2 = a(x-2a)$

Key. 1

Sol. *Let* $P(x_1, y_1)$ be the point of intersection of the two perpendicular normals at $A(t_1), B(t_2)$ on the parabola $y^2 = 4ax$.

Let t_3 be the foot of the third normal through P.

Equation of a normal at t to the parabola is $y + xt = 2at + at^3$

If this normal passes through P then $y_1 + x_1t = 2at + at^3 \Rightarrow at^3 + (2a - x_1)t - y_1 = 0 \rightarrow (1)$

Now t_1, t_2, t_3 are the roots of (1). $\therefore$ $t_1t_2t_3 = y_1 / a$

Slope of the normal at t_1 is $-t_1$

Slope of the normal at t_2 is $-t_2$.

Normals at t_1 and t_2 are perpendicular $\Rightarrow$ $(-t_1)$ $(-t_2) = -1 \Rightarrow t_1t_2 = -1 \Rightarrow t_1t_2t_3 = -t_3$

$\Rightarrow \dfrac{y_1}{a} = -t_3 \Rightarrow t_3 = -\dfrac{y_1}{a}$

t_3 is a root of (1) $\Rightarrow a(-\dfrac{y_1}{a})^3 + (2a - x_1)(-\dfrac{y_1}{a}) - y_1 = 0 \Rightarrow -\dfrac{y_1^3}{a^2} - \dfrac{(2a - x_1)y_1}{a} - y_1 = 0$

$\Rightarrow y_1^2 + a(2a - x_1) + a^2 = y_1^2 = a(x_1 - 3a)$.

$\therefore$ The locus of P is $y^2 = a(x - 3a)$

173. The three normals from a point to the parabola $y^2 = 4ax$ cut the axes in points, whose distances from the vertex are in A.P., then the locus of the point is

1. $27ay^2 = 2(x-2a)^3$ 2. $27ay^3 = 2(x-2a)^2$ 3. $9ay^2 = 2(x-2a)^3$ 4. $9ay^3 = 2(x-2a)^2$

Key. 1

Sol. *Let* $P(x_1, y_1)$ be any point.

Equation of any normal is $y = mx - 2am - am^3$

If is passes through P then $y_1 = mx_1 - 2am - am^3$

$\Rightarrow am^3 + (2a - x_1)m_1 + y_1 = 0$, which is cubic in m.

Let m_1, m_2, m_3 be its roots. Then $m_1 + m_2 + m_3 = 0, m_1m_2 + m_2m_3 + m_3m_1 = \dfrac{2a - x_1}{a}$

Normal meets the axis $(y = 0)$, where $0 = mx - 2am - am^3 \Rightarrow x = 2a + am^2$

$\therefore$ Distances of points from the vertex are $2a + am_1^2, 2a + am_2^2, 2a + am_3^2$

If these are in A.P., then $2(2a + am_2^2) = (2a + am_1^2) + (2a + am_3^2) \Rightarrow 2m_2^2 = m_1^2 + m_3^2$

$\Rightarrow 3m_2^2 = m_1^2 + m_2^2 = (m_1 + m_2 + m_3)^2 - 2(m_1m_2 + m_2m_3 + m_3m_1) = -2(2a - x_1)/a$

$\therefore m_2^2 = 2(x_1 - 2a)/3a$

But $y_1 = m_2(x_1 - 2a - am_2^2) \Rightarrow y_1^2 = m_2^2(x_1 - 2a - am_2^2)^2 = 2(x_1 - 2a)^3/27a$ Locus of P is $27ay^2 = 2(x - 2a)^3$

174. If the normals from any point to the parabola $x^2 = 4y$ cuts the line $y = 2$ in points whose abscissae are in A.P., then the slopes of the tangents at the 3 conormal points are in

1. AP 2. GP 3. HP 4. None

Key. 1

Sol. A point on $x^2 = 4y$ is $(2t, t_2)$ and required point be $P(x_1, y_1)$

Equation of normal at $(2t, t^2)$ is $x + ty = 2t + t^3$.......(1)

Given line equation is $y = 2$........(2)

Solving (1) & (3) $x + t(2) = 2t + t^3 \Rightarrow x = t^3$

This passes through $P(x_1, y_1) \Rightarrow t^3 = x_1$.............(3)

Let $(2t, t_1^2)(2t_2, t_2^2), (2t_3, t_3^2)$ be the co-normal points form P.

$2t_1, 2t_2, 2t_3$ in A.P. $\Rightarrow 4t_2 = 2(t_1 + t_3) \Rightarrow t_2 = \frac{t_1 + t_3}{2}$

$\therefore$ slopes of the tangents $t_1, t_2 \,\&\, t_3$ are in A.P.

175. The line $lx + my + n = 0$ is normal to the parabola $y^2 = 4ax$ if

1. $al(l^2 + 2m^2) + m^2 n = 0$
2. $al(l^2 + 2m^2) = m^2 n$
3. $al(2l^2 + m^2) + m^2 n = 0$
4. $al(2l^2 + m^2) = 2m^2 n$

Key. 1
Sol. Conceptual

176. The feet of the normals to $y^2 = 4ax$ from the point $(6a, 0)$ are

1. $(0,0)$
2. $(4a, 4a)$
3. $(4a, -4a)$
4. $(0,0), (4a, 4a), (4a, -4a)$

Key. 4
Sol. Equation of any normal to the parabola $y^2 = 4ax$ is $y = mx - 2am - am^3$

If passes through $(6a, 0)$ then $0 = 6am - 2am - am^3 \Rightarrow am^3 - 4am = 0 \Rightarrow am(m^2 - 4) = 0$

$\Rightarrow m = 0, \pm 2$.

$\therefore$ Feet of the normals $= (am^2, -2am) = (0,0), (4a, -4a), (4a, 4a)$.

177. The condition that parabola $y^2 = 4ax \,\&\, y^2 = 4c(x - b)$ have a common normal other than x-axis is $(a \neq b \neq c)$

1. $\dfrac{a}{a-c}<2$ 2. $\dfrac{b}{a-c}>2$ 3. $\dfrac{b}{a-c}<1$ 4. $\dfrac{b}{a-c}>1$

Key. 2
Sol. Conceptual

178. Tangents are drawn from the point (-1, 2) to the parabola $y^2=4x$. The length of the intercept made by the line x = 2 on these tangents is
(A) 6 (B) $6\sqrt{2}$ (C) $2\sqrt{6}$ (D) none

Key. B
Sol. Equation of pair of tangent is

$$SS_1=T^2$$
$$\Rightarrow (y^2-4x)(8)=4(y-x+1)^2$$
$$\Rightarrow y^2-2y(1-x)-(x^2+6x+1)=0$$
$$Put \quad x=2$$
$$\Rightarrow y^2+2y-17=0$$
$$\Rightarrow |y_1-y_2|=6\sqrt{2}$$

179. The given circle $x^2+y^2+2px=0$, $p\in R$ touches the parabola $y^2=4x$ externally, then
(A) p < 0 (B) p > 0 (C) 0 < p < 1 (D) p < - 1

Key. B
Sol. Centre of the circle is (- p, 0), If it touches the parabola, then according to figure only one case is possible.

Hence p > 0

180. The triangle PQR of area A is inscribed in the parabola $y^2=4ax$ such that P lies at the vertex of the parabola and base QR is a focal chord. The numerical difference of the ordinates of the points Q & R is
(A) $\dfrac{A}{2a}$ (B) $\dfrac{A}{a}$ (C) $\dfrac{2A}{a}$ (D) $\dfrac{4A}{a}$

Key. C
Sol. QR is a focal chord

$$\Rightarrow R(at^2,2at)\,\&\,Q\left(\frac{a}{t^2},-\frac{2a}{t}\right)$$
$$\Rightarrow d=\left|2at+\frac{2a}{t}\right|=2a\left|t+\frac{1}{t}\right|$$
$$Now \quad A=\frac{1}{2}\begin{vmatrix} at^2 & 2at & 1 \\ \frac{a}{t^2} & -\frac{2a}{t} & 1 \\ 0 & 0 & 1 \end{vmatrix}=a^2\left|t+\frac{1}{t}\right|$$
$$\Rightarrow 2a\left|t+\frac{1}{t}\right|=\frac{2A}{a}$$

181. Through the vertex O of the parabola $y^2 = 4ax$ two chords OP & OQ are drawn and the circles on OP & OQ as diameter intersect in R. If $\theta_1, \theta_2 \& \phi$ are the inclinations of the tangents at P & Q on the parabola and the line through O, R respectively, then the value of $\cot\theta_1 + \cot\theta_2$ is

(A) – 2 tan ϕ (B) – 2 tan (π - ϕ) (C) 0 (D) 2 cot ϕ

Key. A

Sol. $Let \quad P(t_1) \& Q(t_2)$

$\Rightarrow$ Slope of tangent at P($\frac{1}{t_1}$) & at Q($\frac{1}{t_2}$) $\quad \Rightarrow \cot\theta_1 = t_1$ *and* $\cot\theta_2 = t_2$

Slope of PQ = $\frac{2}{t_1 + t_2} = \tan\phi$

$\Rightarrow \tan\phi = -\frac{1}{2}(\cot\theta_1 + \cot\theta_2) \quad \Rightarrow \cot\theta_1 + \cot\theta_2 = -2\tan\phi$

182. AB and AC are tangents to the parabola $y^2 = 4ax$. $p_1, p_2 \& p_3$ are perpendiculars from A, B & C respectively on any tangent to the curve (otherthan the tangents at B&C), then $p_1, p_2 \& p_3$ are in

(A) A.P. (B) G.P. (C) H.P (D) none

Key. B

Sol. Let any tangent is tangent at vertex $x = 0$ and

$Let \quad B(t_1) \& C(t_2)$

$\Rightarrow A(at_1t_2, a(t_1 + t_2))$

$\Rightarrow p_1 = at_1^2; p_2 = at_2^2 \& p_3 = at_1t_2$

$\Rightarrow p_1, p_2 \& p$ are in G.P.

183. A tangent to the parabola $x^2 + 4ay = 0$ at the point T cuts the parabola $x^2 = 4by$ at A & B. Then locus of the mid point of AB is

(A) $(b+2a)x^2 = 4b^2y$ (B) $(b+2a)x^2 = 4a^2y$

(C) $(a+2b)y^2 = 4b^2x$ (D) $(a+2b)x^2 = 4b^2y$

Key. D

Sol. Let mid point of AB is M(h, k)

Then equation of AB is $\quad hx - 2b(y+k) = h^2 - 4bk$

$Let\ T(2at, -at^2)$

$\Rightarrow$ *Equation of* tangent(AB) = $x(2at) = -2a(y - at^2)$

Compare these two equations, we get $\frac{h}{2at} = \frac{-2b}{2a} = \frac{h^2 - 2bk}{2a^2t^2}$

By eliminating t and Locus (h, k), we get $(a+2b)x^2 = 4b^2y$

184. A parabola $y = ax^2 + bx + c$ crosses the x-axis at A(p, 0) & B(q, 0) both to the right of origin. A circle also passes through these two points. The length of a tangent from the origin to the circle is

(A) $\sqrt{\frac{bc}{a}}$ (B) ac^2 (C) b/a (D) $\sqrt{\frac{c}{a}}$

Key. D

Sol. Use power of point for the point O figure

$$\Rightarrow OT^2 = OA.OB = pq = \frac{c}{a}$$

$$\Rightarrow OT = \sqrt{\frac{c}{a}}$$

185. The locus of the vertex of the family of parabolas $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$ (a is parameter) is

(A) $xy = \frac{105}{64}$ (B) $xy = \frac{3}{4}$ (C) $xy = \frac{35}{16}$ (D) $xy = \frac{64}{105}$

Key. A

Sol. $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$

$$y = \frac{2a^3}{6}\left(x^2 + \frac{3}{2a}x - \frac{12a}{2a^3}\right)$$

$$y = \frac{2a^3}{6}\left(x^2 + 2.\frac{3}{4a}x + \frac{9}{16a^2} - \frac{9}{16a^2} - \frac{12a}{2a^3}\right)$$

$$y = \frac{2a^3}{6}\left(\left(x + \frac{3}{4a}\right)^2 - \frac{1059}{16a^3}\right)$$

$$\left(y + \frac{1059}{48}\right) = \frac{2a^3}{6}\left(x + \frac{3}{4a}\right)^2$$

$$x = \frac{-1059}{48}$$

$$y = \frac{-3}{49}$$

$$xy = \frac{1059}{48} \times \frac{3}{49} = \frac{105}{64}$$

186. Equation of a common tangent to the curves $y^2 = 8x$ and $xy = -1$ is

(a) $3y = 9x + 2$ (b) $y = 2x + 1$ (c) $2y = x + 8$ (d) $y = x + 2$

Key. D

Sol. $y^2 = 8k, xy = -1$

Let $P\left(t, \frac{-1}{t}\right)$ be any point on xy = - 1

Equation of the tangent to $xy = -1$ at $P\left(t, \frac{-1}{t}\right)$ is

$$\frac{xy_1 + yx_1}{2} = -1$$

$$\frac{-x}{t} + yt = -2$$

$$y = \frac{x}{t^2} + \left(\frac{-2}{t}\right)................(1)$$

If (1) is tangent to the parabola $y^2 = 8x$ then

$$\frac{-2}{t} = \frac{2}{1/t^2} \Rightarrow t^3 = -1$$

$t = -1$

$\therefore$ Common tangent is y = x+2

187. The locus of the mid-point of the line segment joining the focus to a moving point on the parabola $y^2 = 4ax$ is another parabola with directrix

1. $x = -a$ 2. $x = -a/2$ 3. $x = 0$ 4. $x = a/2$

Key. 3

Sol. The focus of the parabola $y^2 = 4ax$ is $S(a, 0)$,Let $P(at^2, 2at)$ be any point on the parabola then coordinates of the mid-point of SP are given by

$$x = \frac{a(t^2+1)}{2},\ y = \frac{2at+0}{2}$$

Eliminating 't' we get the locus of the mid-point

As $y^2 = 2ax - a^2$ or $y^2 = 2a(x - a/2)$ (1)

Which is a parabola of the form $Y^2 = 4AX$ (2)

Where $Y = y, X = x - a/2$ and $A = a/2$

Equation of the directrix of (2) is $X = -A$

So equation the directrix of (1) is $x - a/2 = -a/2$

$\Rightarrow x = 0$

188. The tangent at the point $P(x_1, y_1)$ to the parabola $y^2 = 4ax$ meets the parabola $y^2 = 4a(x+b)$ at Q and R, then the coordinates of the mid-point of QR are

1. $(x_1 - a, y_1 + b)$ 2. (x_1, y_1) 3. $(x_1 + b, y_1 + a)$ 4. $(x_1 - b, y_1 - b)$

Key. 2

Sol. Equation of the tangent at $P(x_1, y_1)$ to the parabola $y^2 = 4ax$ is

$yy_1 = 2a(x + x_1)$ Or $2ax - y_1y + 2ax_1 = 0$ (i)

If $M(h,k)$ is the mid-point of QR , then equation of QR a chord of the parabola $y^2 = 4a(x+b)$ in terms of its mid-point is $ky - 2a(x+h) - 4ab = k^2 - 4a(h+b)$

(using $T = S'$) or $2ax - ky + k^2 - 2ah = 0$ (ii)

Since (i) and (ii) represent the same line, we have

$$\frac{2a}{2a} = \frac{y_1}{k} = \frac{2ax_1}{k^2 - 2ah} \Rightarrow k = y_1 \text{ and } k^2 - 2ah = 2ax_1$$

$$\Rightarrow \quad y_1^2 - 2ah = 2ax_1 \Rightarrow 4ax_1 - 2ax_1 = 2ah$$

(as $P(x_1, y_1)$lies on the parabola $y^2 = 4ax$)

$\Rightarrow h = x_1$ so that $h = x_1$ $k = y_1$ and the midpoint of QR is (x_1, y_1)

189. Equation of the common tangent touching the circle $(x-3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$ above the x-axis is

1. $\sqrt{3}y = 3x+1$ 2. $\sqrt{3}y = -(x+3)$ 3. $\sqrt{3}y = x+3$ 4. $\sqrt{3}y = -(3x+1)$

Key. 3

Sol. Equation of a tangent to the parabola $y^2 = 4x$ is $y = mx + 1/m$. it will touch the circle $(x-3) + y^2 = 9$ whose centre is $(3,0)$ and radius is 3 if $\left|\frac{0 + m(3) + (1/m)}{\sqrt{1+m^2}}\right| = 3$

Or if $(3m + 1/m)^2 = 9(1+m^2)$

Or if $9m^2 + 6 + 1/m^2 = 9 + 9m^2$

Or if $m^2 = 1/3, i.e. m = \pm 1/\sqrt{3}$

As the tangent is above the x-axis, we take $m = 1/\sqrt{3}$ and thus the required equation is $\sqrt{3}y = x + 3.$

190. If the normal chord at a point 't' on the parabola $y^2 = 4ax$ subtends a right angle at the vertex, then the value of t is

1. 4 2. $\sqrt{3}$ 3. $\sqrt{2}$ 4. 1

Key. 3

Sol. Equation of the normal at 't' to the parabola $y^2 = 4ax$ is $y = -tx + 2at + at^3$

The joint equation of the lines joining the vertex (origin)to the points of intersection of the parabola and the line (i) is $y^2 = 4ax\left[\frac{y + tx}{2at + at^3}\right]$

$\Rightarrow \quad (2t+t^3)y^2 = 4x(y+tx)$

$\Rightarrow \quad 4tx^2 - (2t+t^3)y^2 + 4xy = 0$

Since these lines are at right angles co efficient of x^2 + coefficient of $y^2 = 0$

$\Rightarrow \quad 4t - 2t - t^3 = 0 \quad \Rightarrow \quad t^2 = 2$

For $t = 0$, the normal line is $y = 0$, *i.e.* axis of the parabola which passes through the vertex $(0,0)$.

191. If the focus of a parabola divides a focal chord of the parabola in segments of length 3 and 2, then the length of the latus rectum of the parabola is

1. 3/2 2. 6/5 3. 12/5 4. 24/5

Key. 4

Sol. Let $y^2 = 4ax$ be the equation of the parabola, then the focus is $S(a,0)$. Let $P(at_1^2, 2at_1)$ and $Q(at_2^2, 2at_2)$ be vertices of a focal chord of the parabola, then $t_1t_2 = -1.$ Let $SP = 3 \;\; SQ = 2$

$$SP = \sqrt{a^2\left(1-t_1^2\right) + 4a^2t_1^2} = a\left(1+t_1^2\right) = 3 \qquad \text{(i)}$$

And $$SQ = a\left(1 + \frac{1}{t_1^2}\right) = 2 \qquad \text{(ii)}$$

From (i) and (ii) we get $t_1^2 = 3/2$ and $a = 6/5$

Hence the length of the latus rectum =24/5.

192. The common tangents to the circle $x^2 + y^2 = a^2/2$ and the parabola $y^2 = 4ax$ intersect at the focus of the parabola

1. $x^2 = 4ay$ 2. $x^2 = -4ay$ 3. $y^2 = -4ax$ 4. $y^2 = 4a(x+a)$

Key. 3

Sol. Equation of a tangent to the parabola $y^2 = 4ax$ is $y = mx + a/m$. If it touches the circle $x^2 + y^2 = a^2/2$

$$\frac{a}{m} = \left(\frac{a}{\sqrt{2}}\right)\sqrt{1+m^2} \;\; \Rightarrow \;\; 2 = m^2(1+m^2)$$

$\Rightarrow \quad m^4 + m^2 - 2 = 0 \;\; \Rightarrow \;\; (m^2-1)(m^2+2) = 0$

$\Rightarrow \quad m^2 = 1 \;\Rightarrow\; m = \pm 1$

Hence the common tangents are $y = x + a$ and $y = -x - a$ which intersect at the point $(-a, 0)$ which is the focus of the parabola $y^2 = -4ax.$

193. If $a \neq 0$ and the line $2bx+3cy+4d=0$ passes through the point of intersection of the parabolas $y^2=4ax$ and $x^2=4ay$, then

1. $d^2+(2b-3c)^2=0$ 2. $d^2+(3b+2c)^2=0$ 3. $d^2+(2b+3c)^2=0$ 4. $d^2+(3b-2c)^2=0$

Key. 3

Sol. The pints of intersection of the two parabolas are $(0,0)$ and $(4a,4a)$. If the given line passes through these two points then $d=0$ and $2b+3c=0$ (As $a \neq 0$)so that $d^2(2b+3c)^2=0$.

194. If PQ is a focal chord of the parabola $y^2=4ax$ with focus at S, then $\dfrac{2SP.SQ}{SP+SQ}=$

1. a 2. $2a$ 3. $4a$ 4. a^2

Key. 2

Sol. Let the coordinates of P be $(at_1^2, 2at_1)$ and of Q be $(at_2^2, 2at_2)$.Since PQ is a focal chord,

$$t_1t_2=-1$$

Focus is $S(a,0) \Rightarrow SP=\sqrt{a^2(1-t_1^2)^2+4a^2t_1^2}=a(1+t_1^2)$

And $SQ=a(1+1/t_1^1)=\dfrac{a(1+t_1^2)}{t_1^2}$

So that $\dfrac{2SP.SQ}{SP+SQ}=\dfrac{2a^2(1+t_1^2)^2}{t_1^2a\left[\left(1+t_1^2\right)+\left(1+\dfrac{1}{t_1^2}\right)\right]}=2a$

195. If the tangents at the extremities of a chord PQ of a parabola intersect at T, then the distances of the focus of the parabola from the points P,T,Q are in

1. A.P 2. G.P 3. H.P 4. None of these

Key. 2

Sol. Let the equation of the parabola be $y^2=4ax$ and $P(at_1^2, 2at_1)$, $Q(at_2^2, 2at_2)$ be the extremities of the chord PQ. The coordinates of T, the point of intersection of the tangents at P and Q are $(at_1t_2, a(t_1+t_2))$

Now $SP=a\left(1+t_1^2\right)$

$SQ=a\left(1+t_2^2\right)$

And $ST^2=(at_1t_2-a)^2+[a(t_1+t_2)-0]^2$

$=a^2\left(t_1^2+t_2^2+t_1^2t_2^2+1\right)$

$$= a^2(1+t_1^2)(1+t_2^2) = SP.SQ$$

So that SP, ST, SQ are in $G.P.$

196. If perpendiculars are drawn on any tangent to a parabola $y^2 = 4ax$ from the points $(a \pm k, 0)$ on the axis. The difference of their squares is

1. 4 2. $4a$ 3. $4k$ 4. $4ak$

Key. 4

Sol. Any tangent is $y = mx + a/m.$ Required difference is

$$\left[\frac{m(a+k)+a/m}{\sqrt{1+m^2}}\right]^2 - \left[\frac{m(a-k)+a/m}{\sqrt{1+m^2}}\right]^2$$

$$= \frac{1}{1+m^2} \times 4(ma + a/m)mk = 4ak.$$

197. Which of the following parametric equations does not represent a parabola

1. $x = t^2 + 2t + 1, y = 2t + 2$ 2. $x = a(t^2 - 2t + 1), y = 2at - 2a$
3. $x = 3\sin^2 t, y = 6\sin t$ 4. $x = a\sin t, y = 2a\cos t$

Key. 4

Sol. $x = aT^2, y = 2aT$ Represents a parabola.

In (a) $a = 1, T = t+1$, in (b) $a = a, T = (t-1)$

In (c) $a = 3, T + \sin t$ But in (d) if $2aT = 2a\cos t$

$\Rightarrow T = \cos t$ Which does not satisfy $x = aT^2$.

198. $y = -2x + 12a$ is a normal to the parabola $y^2 = 4ax$ at the point whose distance from the directrix of the parabola is

1. $4a$ 2. $5a$ 3. $4\sqrt{2}a$ 4. $8a$

Key. 2

Sol. $y = -2x + 12a$ is a normal at the point $(a(-2)^2, -2a(-2))$ $i,e.,$ $(4a, 4a)$ whose distance from $x = -a$ is $5a$.

199. If the area of the triangle inscribed in the parabola $y^2 = 4ax$ with one vertex at the vertex of the parabola and other two vertices at the extremities of a focal chord is $5a^2/2$,then the length of the focal chord is

1. $3a$ 2. $5a$ 3. $25a/4$ 4. None of these

Key. 3

Sol. Let the vertices be O $(0,0)$, $A(at^2, 2at), B\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$ then

$$\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ at^2 & 2at & 1 \\ \frac{a}{t^2} & \frac{-2a}{t} & 1 \end{vmatrix} = \frac{5a^2}{2} \Rightarrow 2t^2 - 5t + 2 = 0$$

$\Rightarrow$ $t = 2$ or $1/2$ so the vertices of a focal chord are $(4a, 4a)$ and $(a/4, -a)$ (Taking $t = 2$)and length of this focal chord is 25 $a/4$.

200. If the tangents at the extremities of a focal chord of the parabola $x^2 = 4ay$ meet the tangent at the vertex at points whose abcissae are x_1 and x_2 then $x_1 x_2 =$

1. a^2 2. $a^2 - 1$ 3. $a^2 + 1$ 4. $-a^2$

Key. 4

Sol. One extremity of the focal chord be $(2at, at^2)$. Equation of the tangent is $tx = y + at^2$ which meets the tangent at the vertex, $y = 0$ at $x = at$ so $x_1 = at$ and $x_2 = a\left(-\frac{1}{t}\right)$ thus $x_1 x_2 = -a^2$.

201. Area of a trapezium whose vertices lie on the parabola $y^2 = 4x$ and its diagonals pass through $(1,0)$ and having length $\frac{25}{4}$ units each is

(A) $\frac{75}{4} sq.units$ (B) $\frac{625}{16} sq.units$ (C) $\frac{25}{4} sq.units$ (D) $\frac{25}{8} sq.units$

Key. 1

Sol. Focus of parabola is $(1,0) \Rightarrow$ diagonals are focal chords

$$AS = 1 + t^2 = CE \qquad \frac{1}{C} + \frac{1}{\frac{25}{4} - c} = 1 \qquad C = \frac{5}{4}, 5$$

For $C = \frac{5}{4}$ $\quad t = \pm\frac{1}{2}$

$C = 5 \quad t = \pm 2$

$$\Rightarrow A = \left(\frac{1}{4}, 1\right) \quad B = (4,4) \quad C = (4,-4) \quad D = \left(\frac{1}{4}, -1\right)$$

$AD = 2 \,\&\, BC = 8$ distance between $AD \,\&\, BC = \frac{15}{4}$

Area of trapezium $= \frac{75}{4} sq.units$

202. Maximum number of common normals of $y^2 = 4ax \,\&\, x^2 = 4by$ may be equal to

(A) 2 (B) 4 (C) 5 (D) 3

Key. 3

Sol. Equation of normal to $y^2 = 4ax$ is $y = mx - 2am - am^3$ & for $x^2 = 4by$ is

$y = mx + 2b + \frac{b}{m^2}$

We get $2b + \frac{3}{m^2} + 4m + am^3 = 0$

$am^5 + 2am^3 + 2bm^2 + b = 0$

Max 5 normals

203. If the normal to the parabola $y^2 = 4ax$ at a point t_1 cuts the parabola again at t_2, then

(A) $2 \le t_2^2 \le 8$ (B) $t_2^2 \le 2$ (C) $t_2^2 \ge 8$ (D) $t_2^2 \le 1$

Key. 3

Sol. As $t_2 = -t_1 - \frac{2}{t_1}$ $\quad t_1 \in R \Rightarrow t_2^2 \ge 8$

204. The normal at a point P of a parabola $y^2 = 4ax$ meets its axis in G and tangent at its vertex in H. If A is the vertex of the parabola and if the rectangle AGQH is completed, then equation to the locus of vertex Q is

a) $y^2(y - 2a) = ax^2$ b) $y^2(y + 2a) = ax^2$

c) $x^2(x - 2a) = ay^2$ d) $x^2(x + 2a) = ay^2$

Key. C

Sol. $A = (a, 0), H = (0, 2at + at^3), G = (2at + at^2, 0), Q = (h, k)$

$(h, k) = (2a + at^2, 2at + at^3)$

eliminating 't', $x^3 = 2ax^2 + ay^2$

205. If the focus of the parabola $(y - \beta)^2 = 4(x - \alpha)$ always lies between the lines $x + y = 1$ and $x + y = 3$, then,

a) $3 < \alpha + \beta < 4$ b) $0 < \alpha + \beta < 3$

c) $0 < \alpha + \beta < 2$ d) $-2 < \alpha + \beta < 2$

Key. C

Sol. origin & focus line on off side of $x + y = 1 \Rightarrow \alpha + \beta > 0$

origin & focus line on same side of $x + y = 3 \Rightarrow \alpha + \beta < 2$.

206. Consider the two parabolas $y^2 = 4a(x - \alpha) \,\&\, x^2 = 4a(y - \beta)$, where 'a' is the given constant and α, β are variables. If α and β vary in such a way that these parabolas touch each other, then equation to the locus of point of contact

a) circle b) Parabola

c) Ellipse d) Rectangular hyperbola

Key. D

Sol. Let POC be (h,k). Then, tangent at (h,k) to both parabolas represents same line.

207. A parabola y = ax^2 + bx + c crosses x-axis at (α, 0) and (β, 0) both right of origin. A circle passes through these two points. The length of tangent from origin to the circle is

(a) $\sqrt{\frac{bc}{a}}$ (b) ac^2

(c) $\frac{b}{a}$ (d) $\sqrt{\frac{c}{a}}$

Key. D

SOL. ROOTS OF $AX^2 + BX + C = 0$ ARE α, β

$$\alpha+\beta=-\frac{b}{a},\ \alpha\beta=\frac{c}{a}$$

EQUATION OF CIRCLE THROUGH (α, 0) AND (β, 0)

$$S \equiv (X-\alpha)(X-\beta)+Y^2+\lambda Y=0$$

LENGTH OF TANGENT FROM ORIGIN IS

$$=\sqrt{\alpha\beta}=\sqrt{\frac{c}{a}}$$

208. Equation of the line passing through (α, β), cutting the parabola $y^2 = 4ax$ at two distinct points A and B such that AB subtends right angle at the origin is

(A) $\beta x+(4a-\alpha)y-4a\beta=0$ (B) $2\beta x+(\alpha-4a)y-2a\beta=0$

(C) $\beta x+(\alpha-4a)y-2a\beta=0$ (D) none of these

Key. A

Sol. Any line through (α,β)

$$y-\beta=m(x-\alpha) \quad ..(i)$$

Solving equation (i) with equation of the parabola.

$$\Rightarrow\ 2at-\beta=m(at^2-\alpha)$$

$$\Rightarrow\ amt^2-2at+\beta-m\alpha=0$$

$$\Rightarrow\ t_1t_2=\frac{\beta-m\alpha}{am}=-4$$

$$\Rightarrow\ m=\left(\frac{\beta}{\alpha-4a}\right)$$

Hence required equation

$$y-\beta=\frac{\beta}{\alpha-4a}(x-\alpha)$$

$$\Rightarrow\ y(\alpha-4a)-\alpha\beta+4a\beta=\beta x-\alpha\beta$$

$$\Rightarrow\ \beta x+(4a-\alpha)y-4a\beta=0$$

209. Let 3x – y – 8 = 0 be the equation of tangent to a parabola at the point (7, 13). If the focus of the parabola is at (–1, –1). Its directrix is

(A) x – 8y + 19 = 0 (B) 8x + y + 19 = 0

(C) 8x – y + 19 = 0 (D) x + 8y + 19 = 0

Key. D

Sol. Foot of perpendicular from focus upon tangent is say (α, β). So $\frac{\alpha+1}{3}=\frac{\beta+1}{-1}=\frac{-(-3+1-8)}{3^2+(-1)^2}=1$

$\Rightarrow (\alpha, \beta) \equiv (2, -2)$.

Images of (7, 13) and (–1, –1) w.r.t. (2, –2) will lie on respectively the axis and the directrix of the parabola. The two points are respectively (–3, –17) and (5, –3). Slope of axis = $\frac{-1+17}{-1+3}$ = 8. So equation of directrix: $y + 3 = -\frac{1}{8}(x-5)$

i.e., $x + 8y + 19 = 0$.

210. A parabola having focus at (2,3) touches both the axes then the equation of its directrix is

a) $2x+3y = 0$ b) $3x+2y = 0$ c) $2x-3y= 0$ d) $3x-2y = 0$

Key. B

Sol. The foot of the perpendicular from focus (2,3) to the axes are (2,0),(0,3) lie on the tangent at the vertex hence it's slopes $\frac{-3}{2}$. $\therefore$ Equation of directory is $3x+2y = 0$

211. Equation of the circle of minimum radius which touches both the parabolas $y = x^2+2x+4$ and $x = y^2+2y +4$ is

a) $2x^2+2y^2-11x-11y-13 = 0$ b) $4x^2+4y^2-11x-11y-13= 0$

c) $3x^2+3y^2 -11x-11y-13 = 0$ d) $x^2+y^2-11x-11y-13 = 0$

Key. B

Sol. Given parabolas are symmetric about the line y = x so they have a common normal with slope -1 it meets the parabolas at $\left(\frac{-1}{2},\frac{13}{4}\right),\left(\frac{13}{4},\frac{-1}{2}\right)$ hence the req circles is $x^2+y^2 -\frac{11}{4}x-\frac{11}{4}y-\frac{13}{4}=0$

212. If $a_1x + by + c = 0$
$a_2x + by + c = 0$ are two tangents to $y^2 = 8a(x - 2a)$, then

(A) $\left(\frac{a_1}{b}\right)+\frac{a_2}{b}=0$ (B) $1+\frac{a_1}{b}+\frac{a_2}{b}=0$

(C) $a_1a_2 + b^2 = 0$ (D) $a_1a_2 - b^2 = 0$

Key. C

Sol. The tangents are drawn from $\left(0, -\frac{c}{b}\right)$ on. Y-axis which is directrix of the given parabola.

$$\Rightarrow \left(-\frac{a_1}{b}\right)\left(-\frac{a_2}{b}\right)=-1 \Rightarrow a_1a_2+b^2=0$$

213. A normal, whose inclination is 30^0, to a parabola cuts it again at an angle of

a) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ b) $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$ c) $\tan^{-1}(2\sqrt{3})$ d) $\tan^{-1}\left(\frac{1}{2\sqrt{3}}\right)$

Key. D

Sol. The normal at $P(at_1^2, 2at_1)$ is $y + xt_1 = 2at_1 + at_1^3$ with slope say $\tan\alpha = -t_1 = \frac{1}{\sqrt{3}}$. If it meets curve at $Q(at_2^2, 2at_2)$ then $t_2 = -t_1 - \frac{2}{t_1} = \frac{7}{\sqrt{3}}$. Then angle θ between parabola (tangent at Q) and normal at P is given by $\tan\theta = \frac{-t_1 - \frac{1}{t_2}}{1 - \frac{t_1}{t_2}} = \frac{1}{2\sqrt{3}}$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{1}{2\sqrt{3}}\right)$$

214. The locus of vertices of family of parabolas, $y = ax^2 + 2a^2x + 1$ is $(a \neq 0)$ a curve passing through

a) (1,0) b) (1,1) c) (0,1) d) (0,0)

Key. C

Sol.
$$y = ax^2 + 2a^2x + 1 \Rightarrow \frac{y-(1-a^3)}{a} = (x+a)^2$$
$$\therefore Vertex = (\alpha, \beta) = (-a, 1-a^3)$$
$$\Rightarrow \beta = 1 + \alpha^3$$
$$\Rightarrow curve\ \ is\ \ y = 1 + x^3$$

215. The locus of the Orthocentre of the triangle formed by three tangents of the parabola $(4x-3)^2 = -64(2y+1)$ is

A) $y = \frac{-5}{2}$ B) $y = 1$ C) $x = \frac{7}{4}$ D) $y = \frac{3}{2}$

Key. D

Sol. The locus is directrix of the parabola

216. A pair of tangents with inclinations α, β are drawn from an external point P to the parabola $y^2 = 16x$. If the point P varies in such a way that $\tan^2\alpha + \tan^2\beta = 4$ then the locus of P is a conic whose eccentricity is

A) $\frac{\sqrt{5}}{2}$ B) $\sqrt{5}$ C) 1 D) $\frac{\sqrt{3}}{2}$

Key. B

Sol. Let $m_1 = \tan\alpha, m_2 = \tan\beta$, Let $P = (h, k)$

m_1, m_2 are the roots of $K = mh + \frac{4}{m} \Rightarrow hm^2 - Km + 4 = 0$

$$m_1 + m_2 = \frac{K}{h}; \quad m_1 m_2 = \frac{4}{h}$$

$$m_1^2 + m_2^2 = \frac{K^2}{h^2} - \frac{8}{h} = 4$$

Locus of *P* is $y^2 - 8x = 4x^2 \Rightarrow y^2 = 4(x+1)^2 - 4 \Rightarrow \frac{(x+1)^2}{1} - \frac{y^2}{4} = 1$

217. The length of the latusrectum of a parabola is $4a$. A pair of perpendicular tangents are drawn to the parabola to meet the axis of the parabola at the points A, B. If S is the focus of the parabola then $\frac{1}{|SA|}+\frac{1}{|SB|}=$

A) $2/a$ B) $4/a$ C) $1/a$ D) $2a$

Key. C

Sol. Let $y^2=4ax$ be the parabola

$y=mx+\frac{a}{m}$ and $y=\left(-\frac{1}{m}\right)x-am$ are perpendicular tangents

$S=(a,0), A=\left(-\frac{a}{m^2},0\right), B=(-am^2,0)$

$|SA|=a\left(1+\frac{1}{m^2}\right)=\frac{a(1+m^2)}{m^2}$

$|SB|=a(1+m^2)$

218. Length of the focal chord of the parabola $(y+3)^2=-8(x-1)$ which lies at a distance 2 units from the vertex of the parabola is

A) 8 B) $6\sqrt{2}$ C) 9 D) $5\sqrt{3}$

Key. A

Sol. Lengths are invariant under change of axes

consider $y^2=8x$. Consider focal chord at $(2t^2,4t)$

Focus = (2, 0). Equation of focal chord at *t* is $y=\frac{2t}{t^2-1}9x-2)\Rightarrow 2tx+(1-t^2)y-4t=0$

$$\frac{4|t|^2}{\sqrt{4t^2+(1-t^2)^2}}=2\Rightarrow(|t|-1)^2=0$$

Length of focal chord at '*t*'= $2\left(t+\frac{1}{t}\right)^2=\frac{2(t^2+1)^2}{t^2}=8$

219. The slope of normal to the parabola $y=\frac{x^2}{4}-2$ drawn through the point $(10,-1)$

A) -2 B) $-\sqrt{3}$ C) $-1/2$ D) $-5/3$

Key. C

Sol. $x^2=4(y+2)$ is the given parabola

Any normal is $x=m(y+2)-2m-m^3$. If $(10,-1)$ lies on this line then

$10=+m-2m-m^3\Rightarrow m^3+m+10=0\Rightarrow m=-2$

Slope of normal = $1/m$.

220. m_1,m_2,m_3 are the slope of normals $(m_1<m_2<m_3)$ drawn through the point $(9,-6)$ to the parabola $y^2=4x$. $A=[a_{ij}]$ is a square matrix of order 3 such that $a_{ij}=1$ if $i\neq j$ and $a_{ij}=m_i$ if $i=j$. Then det*A* =

A) 6 B) –4 C) –9 D) 8

Key. D

Sol. $y=mx-2m-m^3.(9,-6)$ lies on this

$\therefore -6 = 9m - 2m - m^3 \Rightarrow m^3 - 7m - 6 = 0$

Roots are $-1, -2, 3 \therefore \ |A| = \begin{vmatrix} -2 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 3 \end{vmatrix} = (-2)(-4) - (3-1) + 2 = 8$

221. A line L passing through the focus of the parabola $y^2 = 4(x-1)$ intersects the parabola in two distinct points. If '*m*' be the slope of the line L, then

A) $m \in (-1, 1)$ B) $m \in (-\infty, -1) \cup (1, \infty)$

C) $m \in R$ D) $m \in R - \{0\}$

Key. D

Sol. Focus (2, 0)

$y - 0 = m(x-2) \Rightarrow \dfrac{y}{m} + 2 = x \Rightarrow y^2 - \dfrac{4y}{m} - 1 = 0$

$B^2 - 4AC > 0$

$\dfrac{1+m^2}{m^2} > 0 \Rightarrow m \in R - \{0\}$

222. Equation of circle of minimum radius which touches both the parabolas $y = x^2 + 2x + 4$ and $x = y^2 + 2y + 4$ is

a) $2x^2 + 2y^2 - 11x - 11y - 13 = 0$ b) $4x^2 + 4y^2 - 11x - 11y - 13 = 0$

c) $3x^2 + 3y^2 - 11x - 11y - 13 = 0$ d) $x^2 + y^2 - 11x - 11y - 13 = 0$

Key. B

Sol. Circle will be touching both parabolas. Circles centre will be on the common normal

223. If the normal at P(8, 2) on the curve xy = 16 meets the curve again at Q. Then angle subtended by PQ at the origin is

a) $\tan^{-1}\left(\dfrac{15}{4}\right)$ b) $\tan^{-1}\left(\dfrac{4}{15}\right)$ c) $\tan^{-1}\left(\dfrac{261}{55}\right)$ d) $\tan^{-1}\left(\dfrac{55}{261}\right)$

Key. A

Sol. If a normal cuts the hyperbola at point $\left(t, \dfrac{1}{t}\right)$ meets the curve again at $\left(ct^1, \dfrac{C}{t^1}\right)$ then

$t^3 t^1 = -1$

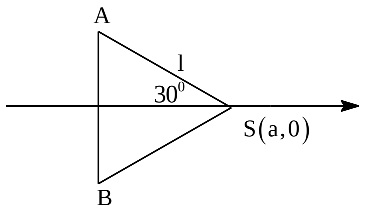
224. An equilateral triangle SAB is inscribed in the parabola $y^2 = 4ax$ having it's focus at 'S'. If the chord AB lies to the left of S, then the length of the side of this triangle is :

a) $3a\left(2-\sqrt{3}\right)$ b) $4a\left(2-\sqrt{3}\right)$

c) $2a\left(2-\sqrt{3}\right)$ d) $8a\left(2-\sqrt{3}\right)$

Key. B

Sol.



$A\left(a-l\cos 30^0, l\sin 30^0\right)$

Point 'A' lies on $y^2 = 4ax$

$\Rightarrow$ a quadratic in 'l'

225. Let the line lx + my = 1 cuts the parabola $y^2 = 4ax$ in the points A & B. Normals at A & B meet at a point C. Normal from C other than these two meet the parabola at a point D, then D =

a) $(a, 2a)$
b) $\left(\frac{4am}{l^2}, \frac{4a}{l}\right)$
c) $\left(\frac{2am^2}{l^2}, \frac{2a}{l}\right)$
d) $\left(\frac{4am^2}{l^2}, \frac{4am}{l}\right)$

Key. D

Sol. Conceptual

226. The normals to the parabola $y^2 = 4ax$ at points Q and R meet the parabola again at P. If T is the intersection point of the tangents to the parabola at Q and R, then the locus of the centroid of ΔTQR, is

a) $y^2 = 3a(x+2a)$
b) $y^2 = a(2x+3a)$
c) $y^2 = a(3x+2a)$
d) $y^2 = 2a(2x+3a)$

Key. C

Sol. Let $Q = \left(at_1^2, 2at_1\right)$

$R = \left(at_2^2, 2at_2\right)$

Normals at Q & R meet on parabola

Also $T = \left(at_1t_2, a\left(t_1+t_2\right)\right)$

Let (α, β) be centroid of ΔQRT

Then $3\alpha = a\left(t_1^2 + t_2^2 + t_1t_2\right) \& \beta = a\left(t_1+t_2\right)$

Eliminate $\left(t_1+t_2\right)$

227. The line x –y =1 intersects the parabola y^2 = 4x at A and B. Normals at A and B intersect at C. If D is the point other that A and B at which CD is normal to the parabola then the coordinate of D are

A) (4, 4) B) (4, –4) C) (1, 2) D) (16, –8)

Key. B

Sol. A, B, C be respectively $(t_1^2, 2t_1), (t_2^2, 2t_2), (t_3^2, 2t_3)$ since AB lie on $x-y=1$

$t_1^2 - 2t_1 = 1$, $t_2^2 - 2t_2 = 1$ subtracting $t_1 + t_2 - 2 = 0$ Now $t_1 + t_2 + t_3 = 0 \Rightarrow t_3 = -2$ so $D(4, -4)$

228. Radius of the largest circle which passes through the focus of the parabola x^2–2x– 4y+5=0 and contained in it is

A) $\sqrt{2}+1$ B) $4\sqrt{3}+1$ C) $\sqrt{3}-1$ D) 4

Key. D

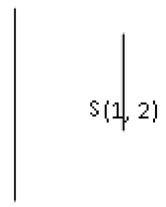
Sol. The parabola is $(x-1)^2 = 4(y-1)$

equation of circle $(x-1)^2 + (y-r-2)^2 = r^2$

solving with one $y^2 + \{4-2(r+2)\}y + 4r = 0$

It has equal roots D=0 $\Rightarrow$ r =4

229. The length of the normal chord at any point on the parabola $y^2 = 4ax$ which subtends a right angle at the vertex of the parabola is



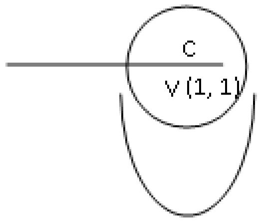
A) $6\sqrt{3}a$ B) $2\sqrt{3}a$ C) $\sqrt{3}a$ D) 2a

Key. A

Sol. $P(at^2, 2at), Q(at_1^{\,2}, 2at_1)$

So $t_1 = -t - \dfrac{2}{t}$ $\angle POQ = \dfrac{2}{t}.\dfrac{2}{t_1} = -1 \Rightarrow t_1 t = -4 \Rightarrow (-t - \dfrac{2}{t})t + 4 = 0$ $\Rightarrow$ $t^2 = 2 \Rightarrow t = \sqrt{2}$

$t_1 = -\dfrac{4}{t} = -2\sqrt{2}$ $\Rightarrow$ $PQ = \sqrt{a^2(t^2 - t_1^{\,2})^2 + 4a^2(t - t_1)^2} = 6\sqrt{3}a$



230. If P is a point (2,4) on the parabola $y^2 = 8x$ and PQ is a focal chord, the coordinate of the mirror image of Q with respect to tangent at P are given by

A) (6,4) B) $(-6,4)$ C) (2, 4) D) (6, 2)

Key. B

Sol. Tangent at extremities of focal chord intersect at right angle at directrix (let R)

$P(2t^2, 4t) \Rightarrow t = 1$

PQ is focal chord $t_1 t_2 = -1 \Rightarrow t_1 = -1 \Rightarrow Q(2, -4)$

Equation of tangent at 'P' ty = x+at^2 $\Rightarrow$ y = x + 2

Coordinate of R (put x = –2 $\Rightarrow$ y =0) $\Rightarrow$ (–2, 0)

R is the mid point of Q & Q^1(mirror image of Q) $\Rightarrow$ $Q^1 = (-6, 4)$

231. The locus of the mid point of chord of the circle $x^2 + y^2 = 9$ such that segment intercepted by the chord on the curve $y^2 - 4x - 4y = 0$ subtends the right angle at the origin.

A) $x^2 + y^2 - 4x - 4y = 0$ B) $x^2 + y^2 + 4x + 4y = 0$ C) $x^2 + 4x + 4y - 9 = 0$ D) None of these

Key. A

Sol. Let the mid point of chord of circle $x^2 + y^2 = 9$ is h, k

equation of chord of circle $hx + ky = h^2 + k^2$

equation of pair of lines joining the point of intersecting of chord and the parabola

with origin is $y^2 - 4(x + y).\dfrac{(hx + ky)}{(h^2 + k^2)} = 0$

Since the angle between these lines is 90^0 required locus is $x^2 + y^2 = 4(x + y)$

232. The locus of the centre of the circle passing through the vertex and the mid points of perpendicular chords from the vertex of the parabola $y^2 = 4ax$

A) $y^2 = 4a(x - 2a)$ B) $y^2 = a(x - 2a)$ C) $y^2 = 4a(x - a)$ D) $(x - a)^2 + y^2 = a^2$

Key. B

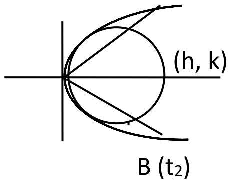
Sol. $t_1t_2 = -4$ $A(at_1^2, 2at_1)B(at_2^2, 2at_2)$

$P\left(\frac{at_1^2}{2}, at_1\right)$ $Q\left(\frac{at_2^2}{2}, at_2\right)$

C (h, k)

$h = \frac{a}{4}\left(t_1^2 + t_2^2\right), k = \frac{a}{2}\left(t_1 + t_2\right)$ $k^2 = \frac{a^2}{4}\left(t_1^2 + t_2^2 + 2t_1t_2\right) = a.\frac{a}{4}\left(t_1^2 + t_2^2\right) - 2a^2$

$k^2 + 2a^2 = a.h \Rightarrow y^2 = a(x - 2a)$



233. Tangents PA and PB are drawn to circle $(x+3)^2 + (y-2)^2 = 1$ from point P lying on $y^2 = 4x$, then the locus of circumcentre of ΔPAB is

A) $(y-1)^2 = 2x - 3$ B) $(y+1)^2 = 2x + 3$ C) $(y+1)^2 = 2x - 3$ D) $(y-1)^2 = 2x + 3$

Key. D

Sol. $p(t^2, 2t), C(-3, 2)$

APBC is a cyclic quadrilateral : Circum centre of ΔPAB is the midpoint of CP

$h = \frac{t^2 - 3}{2} \Rightarrow t^2 = 2h + 3$; $k = \frac{2t+2}{2} \Rightarrow t = k - 1$; locus $(y-1)^2 = 2x + 3$ Q

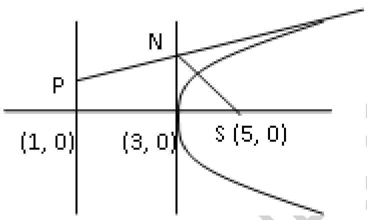
234. From any point P on the straight line $x = 1$ a tangent PQ is drawn to the parabola $y^2 - 8x + 24 = 0$, then the obcissae of N where N is the foot of the perpendicular drawn from A(5, 0) to PQ is

A) 1 B) 2 C) 3 D) 4

Key. C

Sol. $\angle$QNS = 90^0

x-coordinate of N = 3



235. If P(–3, 2) is one end of the focal chord PQ of the parabola $y^2+4x+4y = 0$ then the slope of the normal at Q is

A) –1/2 B) 1/2 C) 2 D) –2

Key. A

Sol. The equation of the tangent at (–3, 2) to the parabola $y^2+4x+4y = 0$ is

2y+2(x–3)+2(y+2) = 0 $\Rightarrow$ x+2y–1 = 0

The tangent at one end of the focal chord is parallel to the normal at the other end.

$\Rightarrow$ slope of normal at Q = slope of tangent at P = –1/2

236. The locus of the focus of the family of parabolas having directrix of slope m and touching the lines $x = a \, and \, y = b$ is

(a) $y + mx = am + b$ (b) $y + mx = am - b$ (c) $y - mx = am + b$ (d) $y - mx = am - b$

Key. A

Sol. Let the focus be (h, k)

Feet of the $\perp$ar from (h , k) on to targets are (a, k) (h, b)

Slope of directrix $=\dfrac{b-k}{h-a}$

$\Rightarrow \dfrac{b-k}{h-a}=m$

The locus is $y+mx=am+b$

237. A circle drawn on any focal chord of the parabola $y^2=4ax$ as diameter cuts the parabola and two points t and t^1 (other than exstremity of a focal chord). Then the value of $tt^1=$

(a) 2 (b) 3 (c) 1 (d) 4

Key. B

Sol. The circle whose diameter ends as $(at^2,2at)\left(\dfrac{a}{t^2},-\dfrac{2a}{t}\right)$ is

$$(x-at^2)\left(x-\frac{a}{t^2}\right)+\left(y-2at\right)\left(y+\frac{2a}{t}\right)=0 \quad \rightarrow (1)$$

Let t_1,t_2,t_3,t_4 be the points of intersection of (1) and parabola $y^2=4ax$ where t_1,t_2 are the ends of

diameter then $t_1t_2t_3t_4=\dfrac{-3a^2}{a^2}$

$t_3t_4=3$

238. Let S be the set of all possible values of the parameter "a" for which the points of intersection of the parabolas $y^2=3ax$ and $y=\dfrac{1}{2}(x^2+ax+5)$ are concyclic. Then S contains interval

(a) $(-\infty,2)$ (b) $(-2,0)$ (c) $(0,2)$ (d) $(2,\infty)$

Key. D

Sol. The family of curves passing through

The prints of intersection of two parabolas is

$$y^2-3ax+\lambda(x^2+ax+5-2y)=0 \quad \rightarrow (1)$$

Since (1) is circle

$a\in(-\infty,-2)\cup(2,\infty)$

239. The line x –y =1 intersects the parabola y^2 = 4x at A and B. Normals at A and B intersect at C. If D is the point other that A and B at which CD is normal to the parabola then the coordinate of D are

A) (4, 4) B) (4, –4) C) (1, 2) D) (16, –8)

Key. B

Sol. A, B, C be respectively $(t_1^2,2t_1),(t_2^2,2t_2),(t_3^2,2t_3)$ since AB lie on $x-y=1$

$t_1^2-2t_1=1$, $t_2^2-2t_2=1$ subtracting $t_1+t_2-2=0$ Now $t_1+t_2+t_3=0\Rightarrow t_3=-2$ so $D(4,-4)$

240. Radius of the largest circle which passes through the focus of the parabola x^2–2x– 4y+5=0 and contained in it is

A) $\sqrt{2}+1$ B) $4\sqrt{3}+1$ C) $\sqrt{3}-1$ D) 4

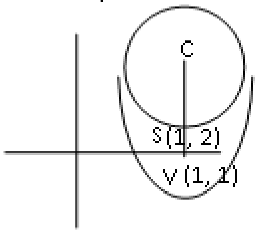
Key. D

Sol. The parabola is $(x-1)^2=4(y-1)$

equation of circle $(x-1)^2+(y-r-2)^2=r^2$

solving with one $y^2+\{4-2(r+2)\}y+4r=0$

It has equal roots D=0 $\Rightarrow$ r =4



241. The length of the normal chord at any point on the parabola $y^2 = 4ax$ which subtends a right angle at the vertex of the parabola is

A) $6\sqrt{3}a$ B) $2\sqrt{3}a$ C) $\sqrt{3}a$ D) 2a

Key. A

Sol. $P(at^2, 2at), Q(at_1^2, 2at_1)$

So $t_1 = -t - \frac{2}{t}$ $\angle POQ = \frac{2}{t}.\frac{2}{t_1} = -1 \Rightarrow t_1 t = -4 \Rightarrow (-t - \frac{2}{t})t + 4 = 0 \Rightarrow t^2 = 2 \Rightarrow t = \sqrt{2}$

$t_1 = -\frac{4}{t} = -2\sqrt{2} \Rightarrow PQ = \sqrt{a^2(t^2 - t_1^2)^2 + 4a^2(t - t_1)^2} = 6\sqrt{3}a$

242. If P is a point (2,4) on the parabola $y^2 = 8x$ and PQ is a focal chord, the coordinate of the mirror image of Q with respect to tangent at P are given by

A) (6,4) B) $(-6, 4)$ C) (2, 4) D) (6, 2)

Key. B

Sol. Tangent at extremities of focal chord intersect at right angle at directrix (let R)

$P(2t^2, 4t) \Rightarrow t = 1$

PQ is focal chord $t_1 t_2 = -1 \Rightarrow t_1 = -1 \Rightarrow Q(2, -4)$

Equation of tangent at 'P' ty = x+at^2 $\Rightarrow$ y = x + 2

Coordinate of R (put x = –2 $\Rightarrow$ y =0) $\Rightarrow$ (–2, 0)

R is the mid point of Q & Q^1(mirror image of Q) $\Rightarrow$ $Q^1 = (-6, 4)$

243. The locus of the mid point of chord of the circle $x^2 + y^2 = 9$ such that segment intercepted by the chord on the curve $y^2 - 4x - 4y = 0$ subtends the right angle at the origin.

A) $x^2 + y^2 - 4x - 4y = 0$ B) $x^2 + y^2 + 4x + 4y = 0$ C) $x^2 + 4x + 4y - 9 = 0$ D) None of these

Key. A

Sol. Let the mid point of chord of circle $x^2 + y^2 = 9$ is h, k

equation of chord of circle $hx + ky = h^2 + k^2$

equation of pair of lines joining the point of intersecting of chord and the parabola with origin is $y^2 - 4(x + y).\frac{(hx + ky)}{(h^2 + k^2)} = 0$

Since the angle between these lines is 90^0 required locus is $x^2 + y^2 = 4(x + y)$

244. The locus of the centre of the circle passing through the vertex and the mid points of perpendicular chords from the vertex of the parabola $y^2 = 4ax$

A) $y^2 = 4a(x - 2a)$ B) $y^2 = a(x - 2a)$ C) $y^2 = 4a(x - a)$ D) $(x - a)^2 + y^2 = a^2$

Key. B

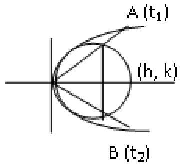
Sol. $t_1 t_2 = -4$ $A(at_1^2, 2at_1) B(at_2^2, 2at_2)$

$P\left(\frac{at_1^2}{2}, at_1\right)$ $Q\left(\frac{at_2^2}{2}, at_2\right)$

C (h, k)

$h=\frac{a}{4}\left(t_1^{\,2}+t_2^{\,2}\right), k=\frac{a}{2}\left(t_1+t_2\right)$ $\qquad k^2=\frac{a^2}{4}\left(t_1^{\,2}+t_2^{\,2}+2t_1t_2\right)=a.\frac{a}{4}\left(t_1^{\,2}+t_2^{\,2}\right)-2a^2$

$k^2+2a^2=a.h \Rightarrow y^2=a(x-2a)$



245. Tangents PA and PB are drawn to circle $(x+3)^2+(y-2)^2=1$ from point P lying on $y^2=4x$, then the locus of circumcentre of ΔPAB is

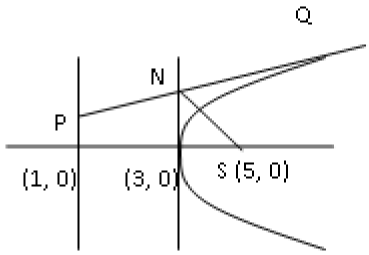
A) $(y-1)^2=2x-3$ B) $(y+1)^2=2x+3$ C) $(y+1)^2=2x-3$ D) $(y-1)^2=2x+3$

Key. D

Sol. $p(t^2,2t), C(-3,2)$

APBC is a cyclic quadrilateral : Circum centre of ΔPAB is the midpoint of CP

$h=\frac{t^2-3}{2}\Rightarrow t^2=2h+3$; $\quad k=\frac{2t+2}{2}\Rightarrow t=k-1$; $\quad$ locus $(y-1)^2=2x+3$



246. From any point P on the straight line $x=1$ a tangent PQ is drawn to the parabola $y^2-8x+24=0$, then the obcissae of N where N is the foot of the perpendicular drawn from A(5, 0) to PQ is

A) 1 B) 2 C) 3 D) 4

Key. C

Sol. $\angle QNS = 90^0$

x-coordinate of N = 3

247. If P(–3, 2) is one end of the focal chord PQ of the parabola $y^2+4x+4y=0$ then the slope of the normal at Q is

A) –1/2 B) 1/2 C) 2 D) –2

Key. A

Sol. The equation of the tangent at (–3, 2) to the parabola $y^2+4x+4y=0$ is

$2y+2(x-3)+2(y+2)=0 \Rightarrow x+2y-1=0$

The tangent at one end of the focal chord is parallel to the normal at the other end.

$\Rightarrow$ slope of normal at Q = slope of tangent at P = –1/2

248. A normal whose inclination is 30^0 to a parabola cuts it again at an angle of

(A) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (B) $\tan^{-1}\left(\frac{7}{\sqrt{3}}\right)$ (C) $\tan^{-1}(2\sqrt{3})$ (D) $\tan^{-1}\left(\frac{1}{2\sqrt{3}}\right)$

Key. D

Sol. The normal at $P(at_1^{\,2}, 2at_1)$ is $y + xt_1 = 2at_1 + at_1^{\,3}$ with slope say $\tan\alpha = -t_1 = \frac{1}{\sqrt{3}}$. If it meets curve at $Q(at_2^{\,2}, 2at_2)$ then $t_2 = -t_1 - \frac{2}{t_1} = \frac{7}{\sqrt{3}}$. Then angle θ between parabola (tangent at Q) and normal at P is given by $\tan\theta = \dfrac{-t_1 - \frac{1}{t_2}}{1 - \frac{t_1}{t_2}} = \dfrac{1}{2\sqrt{3}}$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{1}{2\sqrt{3}}\right)$$

249. The locus of the Orthocentre of the triangle formed by three tangents of the parabola $(4x-3)^2 = -64(2y+1)$ is

(A) $y = \frac{-5}{2}$ (B) $y = 1$ (C) $x = \frac{7}{4}$ (D) $y = \frac{3}{2}$

Key. D

Sol. The locus is directrix of the parabola

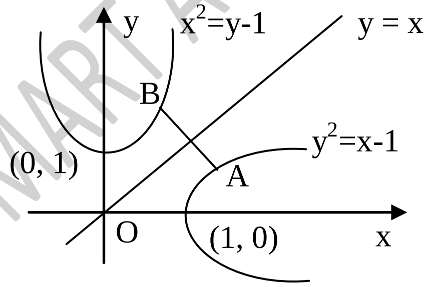
250. Minimum distance between the curves $y^2 = x - 1$ and $x^2 = y - 1$ is equal to

(A) $\frac{3\sqrt{2}}{4}$ (B) $\frac{5\sqrt{2}}{4}$ (C) $\frac{7\sqrt{2}}{4}$ (D) $\frac{\sqrt{2}}{4}$

Key. A

Sol. Both curves are symmetrical about the line y = x. If line AB is the line of shortest distance then at A and B slopes of curves should be equal to one. For $y^2 = x - 1 \Rightarrow \frac{dy}{dx} = \frac{1}{2y} = 1$

$\Rightarrow y = \frac{1}{2}$, $x = \frac{5}{4}$



$\Rightarrow B = \left(\frac{1}{2}, \frac{5}{4}\right)$, $A = \left(\frac{5}{4}, \frac{1}{2}\right)$

Hence minimum distance AB = $\sqrt{\left(\frac{5}{4} - \frac{1}{2}\right)^2 + \left(\frac{5}{4} - \frac{1}{2}\right)^2} = \frac{3\sqrt{2}}{4}$ units

251. If $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ are the feet of the three normals drawn from a point to the parabola $y^2 = 4ax$ then $\frac{x_1 - x_2}{y_3} + \frac{x_2 - x_3}{y_1} + \frac{x_3 - x_1}{y_2} =$

(A) 4a (B) 2a (C) a (D) 0

Key. D

Sol. $y_1 + y_2 + y_3 = 0$

252. Consider $y^2 = 8x$. If the normal at a point P on the parabola meets it again at a point Q, then the least distance of Q from the tangent at the vertex of the parabola is.

(A) 16 (B) 8 (C) 4 (D) 2

Key. A

Sol. Let $P(t_1) \& Q(t_2)$ be points on $y^2 = 8x$. Here $4a = 8$ or $a = 2$

Required distance = $z = at_2^2 = a\left(t_1^2 + \frac{4}{t_1^2} + 4\right) \quad \left(\because t_2 = -t_1 - \frac{2}{t_1}\right)$

Z is least if $\frac{dz}{dt_1} = 0$ or $t_1^2 = 2$ Least value of Z = 16

253. A parabola of latusrectum '4a' touches a fixed equal parabola, the axes of the two curves being parallel; the locus of the vertex of moving curve is parabola of latusrectum K then k=

(A) 2a (B) 4a (C) 8a (D) 16a

Key. C

Sol. Let the given parabola be $y^2 = 4ax$. ----(1)

If the vertex of moving parabola (α, β) its equation is

$(y - \beta)^2 = -4a(x - \alpha) - - - - - (2)$

Solving 1 and 2 $2y^2 - 2\beta y + \beta^2 - 4a\alpha = 0$

Since curve touch each other discriminant=0

$\Rightarrow \beta^2 = 8a\alpha$ *locus is* $y^2 = 8ax.$

$\therefore LR = 8a$

254. The locus of an end of latus rectum of all ellipses having a given major axis is

(A) A straight line (B) A parabola (C) An ellipse (D) A circle

Key. B

Sol. Let the given major axis have vertices $(-a, 0), (a, 0)$. If $P(x, y)$ is an end of the latusrectum then

$y = \frac{b^2}{a} = a(1 - e^2)$, x = ae

Now eliminate 'e'

255. Given the base of a triangle and the product of the tangents of base angles. Then the locus of the

Third vertex of the triangle is

(A) A straight line (B) A circle

(C) A parabola (D) An ellipse

Key. D

Sol. Take base vertices A (-a, 0) B (a, 0) and vertex C(x, y) given tanA tanB = k

$$\Rightarrow \frac{y}{a+x}.\frac{y}{a-x} = k \Rightarrow \frac{y^2}{a^2-x^2} = k.$$

256. The eccentricity of the conic defined by $\left|\sqrt{(x-1)^2+(y-2)^2}-\sqrt{(x-5)^2+(y-5)^2}\right|=3$

A) 5/2 B) 5/3 C) $\sqrt{2}$ D) $\sqrt{11}/3$

Key. B

Sol. Hyperbola for which (1, 2) and (5, 5) are foci and length of transverse axis 3.

$2ae=5$ and $2a=3$ $\therefore e=5/3$

Parabola

Multiple Correct Answer Type

1. If PQ and RS are normal chords of the parabola $y^2 = 8x$ and the points P, Q, R, S are concyclic then

A) Tangents at P and R meet on X-axis B) Tangents at P and R meet on Y-axis

C) PR is parallel to Y-axis D) PR is parallel to X-axis

Key. A,C

Sol. Equation of normal chords at $p\left(2t_1^2, 4t_1\right)$ and $R\left(2t_2^2, 4t_2\right)$ are $y + t_1x - 4t_1 - 2t_1^3 = 0$ and

$y + t_2x - 4t_2 - 2t_2^3 = 0$

Equation of curve through P, Q, R, S is

$\left(y + t_1x - 4t_1 - 2t_1^3\right)\left(y + t_2x - 4t_2 - 2t_2^3\right) + \lambda\left(y^2 - 8x\right) = 0$

P, Q, R, S are concyclic, $t_1 + t_2 = 0$ and $t_1t_2 = 1 + \lambda$

Points of intersection of tangents $\left(at_1t_2, a\left(t_1 + t_2\right)\right)$ lies on X-axis, slope of PR = $\dfrac{z}{t_1 + t_2}$

$\therefore$ PR is parallel to Y-axis.

2. A circle of radius $'r'$ touches the parabola $y^2 + 4x = 0$ at the vertex of the parabola. The centre of the circle lies to the left of the vertex and this circle lies completely within the parabola then exhaustive range of $'r'$ belongs to

A) $\left(1, \dfrac{5}{2}\right)$ B) $(0, 2)$ C) $\left(0, \dfrac{5}{2}\right)$ D) $(0, 3)$

Key. A,B,C

Sol. Equation of circle is $(x + r)^2 + y^2 = r^2$

Solving it with $y^2 + 4x = 0$, we get $x = 0$ & $x = 4 - 2r$

The circle lies completely inside the parabola, $4 - 2r$ is not less than zero $4 - 2r \geq 0 \Rightarrow r \leq 2$

3. Tangents are drawn $(-2, 0)$ to $y^2 = 8x$, radius of circle(s) that would touch these tangents and the corresponding chord of contact, can be equal to

A) $4\left(\sqrt{2} + 1\right)$ B) $4\left(\sqrt{2} - 1\right)$ C) $8\sqrt{2}$ D) None of these

Key. A,B

Sol. Point ‘p’ lies on the directrix of $y^2 = 8x$, slopes of PA and PB are 1 and –1 respectively

Equation of PA: $y = x + 2$, Equation of PB: $y = -x - 2$, Equation of AB: x=2

Let $(h, 0)$ be the centre and radius be 'r' $\Rightarrow \frac{|h+2|}{\sqrt{2}} = \frac{|h-2|}{1} = r$

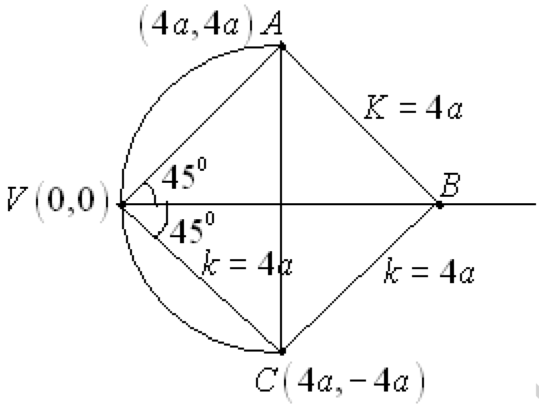
$\Rightarrow h^2 - 12h + 4 = 0 \Rightarrow h = 6 \pm 4\sqrt{2}$

$r = |h-2| = 4\left(\sqrt{2}-1\right), 4\left(\sqrt{2}+1\right)$

4. A square has one vertex at the vertex of the parabola $y^2 = 4ax$ and the diagonal through the vertex lies along the axis of the parabola. If the ends of the other diagonal lie on the parabola, the coordinates of the vertices of the square are

(A) (4a, 4a) (B) (4a, -4a) (C) (0, 0) (D) (8a, 0)

Key. A,B,C,D



Sol.

Let K be length of the side so that A(4a, 4a), B(4a, -4a); V(0, 0), B(8a, 0)

5. $A\left(at_1^2, 2at_1\right)$, $B\left(at_2^2, 2at_2\right)$, $C\left(at_3^2, 2at_3\right)$ be 3 points on the parabola $y^2 = 4ax$. If the orthocentre of $\Delta^{le} ABC$ is focus S of the parabola then

A. $t_1t_2 + t_3t_2 + t_3t_1 = -5$

B. $\frac{1}{t_1t_2} + \frac{1}{t_2t_3} + \frac{1}{t_3t_1} = -1$

C. If $t_1 = 0\, then\, t_2 + t_3 = 0$

D. $\left(1+t_1\right)\left(1+t_2\right)\left(1+t_3\right) = -4$

Key. A,B,C

Sol. Slope of AS = $\frac{2at_1}{t_1^2 - a} = \frac{2t_1}{t_1^2 - 1}$

Slope of BC = $\frac{2a(t_3 - t_2)}{a(t_3^2 - t_2^2)} = \frac{2}{t_3 + t_2}$

$\therefore \frac{2t_1}{t_1^2 - 1} \times \frac{2}{t_3 + t_2} = -1$

$4t_1 = t_3t_1^2 + t_1^2t_2 - t_3 - t_2$

$\therefore yt.t_1 = 0 \Rightarrow t_3 + t_2 = 0$. (C) is correct.

$t_1^2(t_2 + t_3) + 4t_1 = t_3 + t_2$

Ily, $t_2^2(t_1 + t_3) + 4t_2 = t_1 + t_3$

____

(-) $t_1^2 t_2 + t_1^2 t_3 - t_2^2 t_1 - t_2^2 t_3 + 4(t_1 - t_2) = t_2 - t_1$

$t_1 t_2 (t_1 - t_2) + t_3(t_1^2 - t_2^2) = +5(t_2 - t_1)$

$t_1 t_2 + t_3(t_1 + t_2) = -5$

$\therefore \sum t_1 t_2 = -5$ $\therefore$ (A) is true.

Now $t_1^2(t_2 + t_3) + 4t_1 = t_3 + t_2$

$t_1(t_1 t_2 + t_3 t_1) + 4t_1 = t_2 + t_3$

$t_1(-5 - t_2 t_3) + 4t_1 = t_2 + t_3$

$-t_1 t_2 t_3 - t_1 = t_2 + t_3$

$-t_1 t_2 t_3 = t_1 + t_2 + t_3$

$\therefore \frac{1}{t_1 t_2} + \frac{1}{t_2 t_3} + \frac{1}{t_3 t_1} = -1$ (B) is correct.

6. The normals at the points $P(t_1), Q(t_2)$ on the parabola $y^2 = 4ax$ intersect at $Q(t_3)$ on the parabola. Then which of the following is/are true?

A) $|t_3| \geq \sqrt{2}$ B) $|t_3| \geq 2\sqrt{2}$ C) $t_1 t_2 = 2$ D) $t_3 = -t_1 - t_2$

Key. B,C,D

Sol. Conceptual

7. Which of the following statements are true?

A) There exists a point in the plane of a parabola through which no real normal can be drawn to the parabola

B) If P, Q, R are co-normal points of a parabola then circum circle of ΔPQR passes through the vertex of the parabola and centroid of ΔPQR lies on the axis of the parabola

C) The circumcentre of triangle formed by three tangents of a parabola passes through focus of the parabola

D) Length of subnormal at any point on a parabola is equal to latusrectum of the parabola.

Key. B,C

Sol. Conceptual

8. Consider the parabola represented by the parametric equations $x = t^2 - 2t + 2; y = t^2 + 2t + 2$. Then which of the following is/are true?

A) Auxiliary circle of the parabola is $x + y = 4$ B) Vertex of the parabola is (2, 2)

C) Director circle of the parabola is $x + y = 6$ D) Focus of the parabola is (3, 3)

Key. A,B,D

Sol. $x = t^2 - 2t + 2; \; y = t^2 + 2t + 2$

$x + y = 2(t^2 + 2)$ and $y - x = 4t$

$$\frac{x+y}{2} = \frac{(y-x)^2}{16} + 2 \Rightarrow (y-x)^2 = 8(x+y-4)$$

$$\left(\frac{y-x}{\sqrt{2}}\right)^2 = 4\sqrt{2}\left(\frac{x+y-h}{\sqrt{2}}\right)$$

This is parabola for which $y = x$ is axis, $x + y = 4$ is tangent at vertex and length of latusrectum is $4\sqrt{2}$

9. The equations of the common tangents of the curves $x^2 + 4y^2 = 8$ and $y^2 = 4x$ are

A) $x + 2y + 4 = 0$ B) $x - 2y + 4 = 0$ C) $2x + y = 4$ D) $2x - y + 4 = 0$

Key. A,B

Sol. $\frac{x^2}{8} + \frac{y^2}{2} = 1, \; y^2 = 4x$

Any tangent to parabola is $y = mx + \frac{1}{m}$

If this line is tangent to ellipse then $\frac{1}{m^2} = 8m^2 + 2 \Rightarrow 8m^4 + 2m^2 - 1 = 0$

$$m^2 = \frac{-2 \pm \sqrt{4+32}}{16} = \frac{-2 \pm 6}{16}$$

$$\Rightarrow m^2 = \frac{1}{4} \Rightarrow m = \pm\frac{1}{2}$$

$y = \frac{x}{2} + 2$ or $y = -\frac{x}{2} - 2$

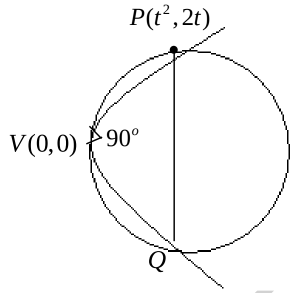
$x - 2y + 4 = 0$ or $x + 2y + 4 = 0$

10. Let PQ be a chord of the parabola $y^2 = 4x$. A circle is drawn with PQ as diameter passes through the vertex 'V' of the parabola. If area of triangle PVQ= 20 sq.units, then the coordinates of P are

a) (16, 8) b) (16, -8) c) (-16, 8) d) (-16, -8)

Key. A,B

Sol. slope of PV $= \dfrac{2t-0}{t^2-0} = \dfrac{2}{t}$



Equation of QV is $y = -\dfrac{t}{2}(x)$

On solving with $y^2 = 4x$, $Q = \left(\dfrac{16}{t^2}, \dfrac{-8}{t}\right)$

Area of ΔPVQ is $\dfrac{1}{2}.PV.VQ = 20$

$\Rightarrow PV.VQ = 40$

By solving above equation $t = \pm 4, \pm 1$

11. If $Ay^2 + By + Cx + D = 0$, (ABC $\neq$ 0) be the equation of parabola, then

a) the length of latusrectum is $\left|\dfrac{C}{A}\right|$

b) the axis of the parabola is a vertical line

c) Y- co-ordinate of the vertex is $-\dfrac{B}{2A}$

d) X- Co-ordinates of the vertex is $\left(\dfrac{B^2-4AD}{4AC}\right)$

Key. A,C,D

Sol. $-Cx = Ay^2 + By + D$

$-Cx = A\left(y^2 + \dfrac{B}{A}y + \dfrac{D}{A}\right)$

$x = -\dfrac{A}{C}\left(\left(y + \dfrac{B}{2A}\right)^2 + \dfrac{D}{A} - \dfrac{B^2}{4A^2}\right)$

$= -\dfrac{A}{C}\left[\left(y + \dfrac{B}{2A}\right)^2\right] - \dfrac{A}{C}\left[\dfrac{4AD - B^2}{4A^2}\right]$

$\Rightarrow \left(x + \dfrac{4AD - B^2}{4AC}\right) = -\dfrac{A}{C}\left(y + \dfrac{B}{2A}\right)^2$

$\Rightarrow \left(y + \dfrac{B}{2A}\right)^2 = -\dfrac{C}{A}\left(x + \dfrac{4AD - B^2}{4AC}\right)$

12. The focal chord to $y^2 = 16x$ is tangent to $(x-6)^2 + y^2 = 2$, then slope of focal chord is

A. 1 B. $\dfrac{1}{2}$ C. $-\dfrac{1}{2}$ D. -1

Key. A,D

Sol. $(x-6)^2+y^2=2$ $\rightarrow$ tangent is $y=m(x-6)+\sqrt{2m^2+2}$

It is passing through (4, 0) focus of parabola

$$0=-2m+\sqrt{2m^2+2}\Rightarrow 2m^2+2=4m^2$$

$$m^2=1\Rightarrow m=\pm 1$$

13. $A\left(at_1^2,2at_1\right)$, $B\left(at_2^2,2at_2\right)$, $C\left(at_3^2,2at_3\right)$ be 3 points on the parabola $y^2=4ax$. If the orthocentre of $\Delta^{le}ABC$ is focus S of the parabola then

A. $t_1t_2+t_3t_2+t_3t_1=-5$

B. $\frac{1}{t_1t_2}+\frac{1}{t_2t_3}+\frac{1}{t_3t_1}=-1$

C. If $t_1=0\, then\, t_2+t_3=0$

D. $\left(1+t_1\right)\left(1+t_2\right)\left(1+t_3\right)=-4$

Key. A,B,C

Sol. Slope of AS = $\frac{2at_1}{t_1^2-a}=\frac{2t_1}{t_1^2-1}$

Slope of BC = $\frac{2a(t_3-t_2)}{a(t_3^2-t_2^2)}=\frac{2}{t_3+t_2}$

$$\therefore \frac{2t_1}{t_1^2-1}\times\frac{2}{t_3+t_2}=-1$$

$$4t_1=t_3t_1^2+t_1^2t_2-t_3-t_2$$

$\therefore yt.t_1=0 \Rightarrow t_3+t_2=0$. (C) is correct.

$$t_1^2(t_2+t_3)+4t_1=t_3+t_2$$

Ily, $t_2^2(t_1+t_3)+4t_2=t_1+t_3$

(-) $t_1^2t_2+t_1^2t_3-t_2^2t_1-t_2^2t_3+4(t_1-t_2)=t_2-t_1$

$t_1t_2(t_1-t_2)+t_3(t_1^2-t_2^2)=+5(t_2-t_1)$

$t_1t_2+t_3(t_1+t_2)=-5$

$\therefore \sum t_1t_2=-5$ $\therefore$ (A) is true.

Now $t_1^2(t_2+t_3)+4t_1=t_3+t_2$

$t_1(t_1t_2+t_3t_1)+4t_1=t_2+t_3$

$t_1(-5-t_2t_3)+4t_1=t_2+t_3$

$-t_1t_2t_3-t_1=t_2+t_3$

$-t_1t_2t_3=t_1+t_2+t_3$

$\therefore \frac{1}{t_1t_2}+\frac{1}{t_2t_3}+\frac{1}{t_3t_1}=-1$ (B) is correct.

14. If the parabolas $y^2=4kx(k>0)\, and\, y^2=4(x-1)$ do not have a common normal other than the axis of parabola , then k$\in$

a) (0,1) b) $(2,\infty)$ c) $(3,\infty)$ d) $(0,\infty)$

Key. A,B,C

Sol. If the parabolas have a common normal of slope 'm' (the only allowed value of m is m=0) then it is given by $y=mx-2km-km^3$ and $y=m(x-1)-2m-m^3$

$$=mx-3m-m^3$$

$$\Rightarrow 2km+km^3=3m+m^3$$

$\Rightarrow m=0, m^2=\frac{3-2k}{k-1}$. If $m^2<0$ then the only common normal is the axis

$$\Rightarrow \frac{3-2k}{k-1}<o\Rightarrow (k-1)(2k-3)>0\Rightarrow k>\frac{3}{2}\ or\ k<1\,\&\,k>0$$

15. The points on axis of parabola $3x^2+4x-6y+8=0$ from which three distinct normals can be drawn to it are

a) $\left(\frac{-2}{3},2\right)$ b) $\left(\frac{-2}{3},3\right)$ c) $\left(\frac{-2}{3},4\right)$ d) $\left(\frac{-2}{3},1\right)$

Key. B,C

Sol. The parabola can be written as $\left(x+\frac{2}{3}\right)^2=2(y-\frac{10}{9})$ ie $X^2=2Y\left(X=x+\frac{2}{3},Y=y-\frac{10}{9}\right)$. A point on axis is $\left(\frac{-2}{3},Y\right)$ from which three normals can be drawn if $Y>1$

$$\Rightarrow y>\frac{19}{9}.$$

16. The tangents at the extremities of a focal chord of a parabola are

a) perpendicular
b) parallel
c) intersect on the directrix
d) intersect at the vertex

Key. A,C

Sol. For a focal chord we have t_1t_2 = -1 hence tangents are perpendicular and they intersect on the directrix

17. Let P, Q and R are three conormal points on the parabola $y^2=4ax$. Then the correct statement (s) is /are :

a) Algebraic sum of the slopes of the normals at P, Q and R is zero.
b) Algebraic sum of abscissa of the points P, Q and R is zero.
c) Centroid of the triangle PQR lies on the axis of the parabola
d) Circle circumscribing the triangle PQR passes through the vertex of parabola

Key. A,C,D

Sol. Equation of normal at $\left(am^2,-2am\right)$ is

$$y=mx-2am-am^3$$

$$\Rightarrow am^2+\left(2a-x\right)m+y=0$$

$$\sum m=0$$

$$\sum m_1m_2=\frac{2a-x}{a}$$

$$\sum m_1m_2m_3=-y/a$$

18. Which of the following statements are true for the curve

$$9x^2-24xy+16y^2-20x-15y-60=0$$

a) It represents a parabola
b) Length of latus rectum of the curve is 2
c) the equation of directrix of curve is $16x+12y+53=0$
d) the equation of axis of the curve is $3x-4y-35=0$

Key. A,B,C

Sol. Curve is $\left(\frac{3x-4y}{5}\right)^2=\left(\frac{20x+15+60}{25}\right)$

19. The equation of directrix of the parabola $x^2+4y-6x+k=0$ is $y+1=0$ then

(A) $k=17$ (B) $k=-17$

(C) Focus is $(3,-3)$ (D) vertex is $(3,-3)$

Key. A,C

Sol. $(x-3)^2=-4y-k+9=-4\left(y+\frac{k-9}{4}\right)$

Equation of the directrix is $y+\frac{k-9}{4}=1 \Rightarrow y=\frac{13-k}{4}$

$\frac{13-k}{4}=-1 \Rightarrow k=17$

Vertex is $\left(3,\frac{-k+9}{4}\right)=(3,-2)$

Focus $(3,-3)$

20. If a normal chord of $y^2 = 4ax$ subtends an angle $\pi/2$ at the vertex of the parabola then it's slope is equal to

(A) $\sqrt{2}$ (B) $-\sqrt{2}$

(C) 1 (D) – 1

Key. A,B

Sol. $t\left(-t-\frac{2}{t}\right)=-4$

$t = \pm\sqrt{2}$

slope = -t = m $\sqrt{2}$

21. Equation of common tangent of $y = x^2$, $y = -x^2 + 4x - 4$ is

(A) $y = 4(x - 1)$ (B) $y = 0$

(C) $y = -4(x - 1)$ (D) $y = -30x - 50$

Key. A,B

Sol. Let equation of any tangent to $y = x^2$ be

$y = mx - \frac{m^2}{4}$

$mx - \frac{m^2}{4} = -x^2 + 4x - 4$

$x^2 + x(m-4) - \frac{m^2}{4} + 4 = 0$

$\Delta = 0$ $m = 0$ & 4

equation is $y = 4x - 4$ & $y = 0$

22. Let A and B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then the slope of the line joining A and B can be

(A) $-\frac{1}{r}$ (B) $\frac{1}{r}$

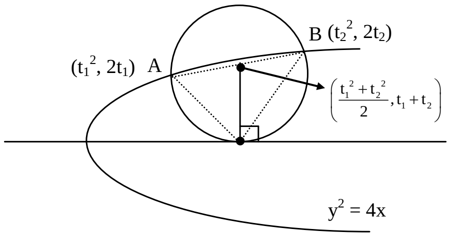
(C) $\frac{2}{r}$ (D) $-\frac{2}{r}$

Key. C,D

Sol. Slope of line AB

$$M = \frac{(t_2 - t_1)}{(t_2 - t_1)(t_2 + t_1)} = \left(\frac{2}{t_1 + t_2}\right) = \pm\frac{2}{r}$$

As $|t_1 + t_2| = r$



23. The points on axis of parabola $3x^2 + 4x - 6y + 8 = 0$ from which three distinct normals can be drawn to it are

a) $\left(\frac{-2}{3}, 2\right)$ b) $\left(\frac{-2}{3}, 3\right)$ c) $\left(\frac{-2}{3}, 4\right)$ d) $\left(\frac{-2}{3}, 1\right)$

Key. B,C

Sol. The parabola can be written as $\left(x + \frac{2}{3}\right)^2 = 2(y - \frac{10}{9})$ *ie* $X^2 = 2Y\left(X = x + \frac{2}{3}, Y = y - \frac{10}{9}\right)$. A point on axis is $\left(\frac{-2}{3}, Y\right)$ from which three normals can be drawn if $Y > 1$

24. If the parabolas $y^2 = 4kx (k > 0)$ *and* $y^2 = 4(x-1)$ do not have a common normal other than the axis of parabola , then k$\in$

a) (0,1) b) $(2, \infty)$ c) $(3, \infty)$ d) $(0, \infty)$

Key. A,B,C

Sol. If the parabolas have a common normal of slope 'm' (the only allowed value of m is m=0) then it is given by $y = mx - 2km - km^3$ and $y = m(x-1) - 2m - m^3$

$$= mx - 3m - m^3$$

$$\Rightarrow 2km + km^3 = 3m + m^3$$

$$\Rightarrow m = 0, m^2 = \frac{3-2k}{k-1}$$. If $m^2 < 0$ then the only common normal is the axis

$$\Rightarrow \frac{3-2k}{k-1} < o \Rightarrow (k-1)(2k-3) > 0 \Rightarrow k > \frac{3}{2} \; or \; k < 1 \,\&\, k > 0$$

25. The normals at the points $P(t_1), Q(t_2)$ on the parabola $y^2 = 4ax$ intersect at $Q(t_3)$ on the parabola. Then which of the following is/are true?

A) $|t_3| \geq \sqrt{2}$ B) $|t_3| \geq 2\sqrt{2}$ C) $t_1 t_2 = 2$ D) $t_3 = -t_1 - t_2$

Key. B,C,D

Sol. Conceptual

26. Which of the following statements are true?

A) There exists a point in the plane of a parabola through which no real normal can be drawn to the parabola

B) If P, Q, R are co-normal points of a parabola then circum circle of ΔPQR passes through the vertex of the parabola and centroid of ΔPQR lies on the axis of the parabola

C) The circumcentre of triangle formed by three tangents of a parabola passes through focus of the parabola

D) Length of subnormal at any point on a parabola is equal to latusrectum of the parabola.

Key. B,C

Sol. Conceptual

27. Consider the parabola represented by the parametric equations $x = t^2 - 2t + 2; y = t^2 + 2t + 2$. Then which of the following is/are true?

A) Auxiliary circle of the parabola is $x + y = 4$ B) Vertex of the parabola is (2, 2)

C) Director circle of the parabola is $x + y = 6$ D) Focus of the parabola is (3, 3)

Key. A,B,D

Sol. $x = t^2 - 2t + 2;\ \ y = t^2 + 2t + 2$

$x + y = 2(t^2 + 2)$ and $y - x = 4t$

$$\frac{x+y}{2} = \frac{(y-x)^2}{16} + 2 \Rightarrow (y-x)^2 = 8(x+y-4)$$

$$\left(\frac{y-x}{\sqrt{2}}\right)^2 = 4\sqrt{2}\left(\frac{x+y-h}{\sqrt{2}}\right)$$

This is parabola for which $y = x$ is axis, $x + y = 4$ is tangent at vertex and length of latusrectum is $4\sqrt{2}$

28. The equations of the common tangents of the curves $x^2 + 4y^2 = 8$ and $y^2 = 4x$ are

A) $x + 2y + 4 = 0$ B) $x - 2y + 4 = 0$ C) $2x + y = 4$ D) $2x - y + 4 = 0$

Key. A,B

Sol. $\frac{x^2}{8} + \frac{y^2}{2} = 1,\ \ y^2 = 4x$

Any tangent to parabola is $y = mx + \frac{1}{m}$

If this line is tangent to ellipse then $\frac{1}{m^2} = 8m^2 + 2 \Rightarrow 8m^4 + 2m^2 - 1 = 0$

$$m^2 = \frac{-2 \pm \sqrt{4+32}}{16} = \frac{-2 \pm 6}{16}$$

$$\Rightarrow m^2 = \frac{1}{4} \Rightarrow m = \pm\frac{1}{2}$$

$y = \frac{x}{2} + 2$ or $y = -\frac{x}{2} - 2$

$x - 2y + 4 = 0$ or $x + 2y + 4 = 0$

29. The equation of a tangent to the parabola $y^2 = 8x$ which makes an angle 45º with the line $y = 3x + 5$ is

(A) $2x + y + 1 = 0$ (B) $y = 2x + 1$

(C) $x – 2y + 8 = 0$ (D) $x + 2y – 8 = 0$

Key. A,C

Sol. Equation of tangent in terms of slope of $y^2 = 8x$ is $y = mx + \frac{2}{m}$ …..(i)

Q Angle between equation. (i) and $y = 3x + 5$ is 45º, then

$\Rightarrow \left|\frac{m-3}{1+3m}\right| = \tan 45° = 1$

$\Rightarrow \pm (m – 3) = 1 + 3m$

Taking '+' sign, then $m – 3 = 1 + 3m$

$\therefore m = –2$

and taking '–' sign, then s

$– m + 3 = 1 + 3m$

$\therefore m = \frac{1}{2}$

Now, from eq. (i) equation of tangents are

$y = -2x - 1$ and $y = \frac{x}{2} + 4$

or $2x + y + 1 = 0$ and $x – 2y + 8 = 0$

30. Consider a circle with its centre lying on the focus of the parabola $y^2 = 2px$, such that it touches the directrix of the parabola. Then a point of intersection of the circle and the parabola is

(A) $\left(\frac{p}{2}, p\right)$ (B) $\left(\frac{p}{2}, -p\right)$

(C) $\left(-\frac{p}{2}, p\right)$ (D) $\left(-\frac{p}{2}, -p\right)$

Key. A,B

Sol. Focus of the parabola is $\left(\frac{p}{2}, 0\right)$. since the circle touches directrix $x = -\frac{p}{2}$ of the parabola, the radius of the circle $= \frac{p}{2} + \frac{p}{2} = p$

$\Rightarrow$ equation of the circle is

$\left(x - \frac{p}{2}\right)^2 + y^2 = p^2$

$\Rightarrow x^2 + y^2 - px - \frac{3p^2}{4} = 0$

this circle meets the parabola $y^2 = 2px$ at points whose abscissae are given by

$$x^2 + 2px - px - \frac{3p^2}{4} = 0$$

$\Rightarrow\ x = \frac{p}{2},\ x = -\frac{3p}{4}$. But $x = -\frac{3p}{4}$ is not possible on parabola, $y^2 = 2px$.

$\therefore\ x = \frac{p}{2}$

31. Let $y^2 - 5y + 3x + k = 0$ be a parabola, then

(A) its latus rectum is least when k = 1

(B) its latus rectum is independent of k

(C) the line y = 2x + 1 will touch the parabola if $k = \frac{73}{16}$

(D) $y = \frac{5}{2}$ is the only normal to the parabola whose slope is zero.

Key. B,C,D

Sol. The equation to the parabola can be written as

$$\left(y - \frac{5}{2}\right)^2 = -3\left(x - \frac{25-4k}{12}\right)$$

$\Rightarrow$ The length of the latus rectum is 3 and the horizontal normal is y = 2x + 1 is a tangent, the quadratic $(2x + 1)^2 - 5(2x + 1) + 3x + k = 0$ must have equal roots, which is $4x^2 - 3x + k - 4 = 0$. Now roots are equal if $b^2 = 4ac$,

$\Rightarrow 9 = 16(k - 4)$, etc.

Since $y = \frac{5}{2}$ is normal at the vertex.

32. If f(x + y) = f(x).f(y) for all x, y and f(1) = 2 and $a_r = f(r)$, for $r \in N$, then the coordinates of a point on the parabola $y^2 = 8x$ whose focal distance is 4, may be

(A) (a_1, a_2) (B) $(a_1, -a_2)$

(C) (a_1, a_1) (D) (a_2, a_2)

Key. A,B

Sol. Given parabola is $y^2 = 8x$ …..(i)

Here, a = 2. Let $P(2t^2, 4t)$ be a point on parabola

Eq. (i), and S be the focus

Given, SP = 4

$\therefore\ a(1 + t^2) = 4$

$2(1 + t^2) = 4 \Rightarrow t = \pm 1$

$\therefore\ P \equiv (2, 4)$ or $(2, -4)$

Given f(x + y) = f(x)f(y) for all x and y

Given, f(1) = 2 …..(ii)

From eq. (ii) $f(2) = f(1 + 1) = f(1).f(1) = 2^2 = 4$

Similarly $f(n) = 2^n$

Q $a_r = f(r)$

$\therefore\ a_1 = f(1) = 2,$

$a_2 = f(2) = 4$

Hence, $P \equiv (a_1, a_2)$ or $(a_1, -a_2)$.

33. The line x + y +2 = 0 is a tangent to a parabola at point A, intersect the directrix at B and tangent at vertex at C respectively. The focus of parabola is S (2, 0). Then

a) CS is perpendicular to AB   b) AC. BC = CS^2

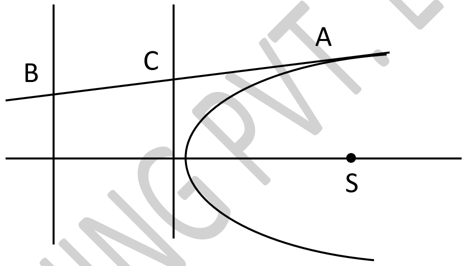
c) AC. BC = 8   d) AC = BC

Key: A, B, C

Hint: For parabola $y^2 = 4ax$, eq. of tangent at A $(at^2, 2at)$ is $ty = x + at^2$

$$\therefore C(0, at),\ B\left(-a, at - \frac{a}{t}\right)$$

$$AC = at\sqrt{1+t^2};\quad BC = \frac{a}{t}\sqrt{1+t^2};\quad CS = a^2\left(1+t^2\right)$$



$$\Rightarrow AC.BC = (CS)^2$$

$$slope\ of\ CS \times slope\ of\ AB = \frac{0-at}{a-0} \times \frac{1}{t} = -1$$

$$hence\ CS\ is\ perpendicular\ to\ AB$$

$$CS = \frac{|2+2|}{\sqrt{2}} = 2\sqrt{2}$$

34. The equation of tangent to the parabola $y^2 = 8x$ which makes an angle 45° with the line y = 3x + 5 is

(A) 2x + y + 1 = 0   (B) y = 2x + 1

(C) x – 2y + 8 = 0   (D) x + 2y – 8 = 0

Key : A, C

Sol : Equation of tangent in term of slope of

$y^2 = 8x$ is

$$y = mx + \frac{2}{m} \quad \text{....(i)}$$

$\because$ angle between eq. (i) and y = 3x + 5 is 45°, then

$$\Rightarrow \left|\frac{m-3}{1+3m}\right| = \tan 45° = 1$$

$$\Rightarrow \pm(m-3) = 1 + 3m$$

taking '+' sing, then m = -3 = 1 + 3m

$\therefore$ m = -2

and tanking '-' sign, then

$$-m + 3 = 1 + 3m$$

$$\therefore m = \frac{1}{2}$$

Now, from eq. (i) equation of tangents are

$y=-2x-1$ and $y=\frac{x}{2}+4$

or $2x+y+1=0$ and $x-2y+8=0$

35. consider a circle with its center lying on the focus of the parabola $y^2 = 2px$, Such that it touches the directrix of the parabola. Then a point of intersection of the circle and the parabola is

a) ($p/2$,p) b) (p/2,-p) c) (-p/2,p) d) (-p/2,-p)

Key. A,B

Sol. Focus of the parabola is $\left(p/_2,0\right)$, since the circle touches directrix x = $-p/_2$ of the parabola, the radius of the circle

$\Rightarrow p/_2 + p/_2 = p$

$\Rightarrow$ eq of the circle is $\left(x-p/_2\right)^2 + y^2 = p^2$

$x^2+y^2 - px - \frac{3p^2}{4} = 0$

This circle meets the parabola $y^2=2px$ at points whose abscissa are given by

$x^2+2px-px - 3p^2/_4 = 0$

$x=p/_2$, $x=-3p/_4$, But $x=-3p/_4$ is not possible on parabola, $y^2=2px$ there fore $x=p/_2$, choice (a) (b)

36. Let $y^2-5y+3x+k=0$ be a parabola, then

a) its lactus rectum is least when k = 1

b) its lactus rectum is independent of k

c) The line y = 2x+1 will touch the parabola if k =73/16

d) y = 5/2 is the only normal to the parabola whose slope is zero

Key. A,B,C,D

Sol. The eq to the parabola can be written as

$\left(y-5/_2\right)^2=-3\left(x-\frac{25-4k}{12}\right)$

$\Rightarrow$ The length of the latus rectum is 3 and the horizontal normal is y = 2x + 1 is a tangent , the quadratic

$(2x+1)^2-5(2x+1)+3x+k = 0$

Must have equal roots, which is $4x^2-3x+k-4=0$

Now roots are equal is $B^2 = 4AC$

$\Rightarrow 9=16(k-4), \text{etc}$

$\Rightarrow (b)(c)(d)$ is also correct

Since $y = 5/_2$ is normal at the vertex

37. let $y^2 = 4ax$ be parabola and (α,β) be a point from where three normals are drawn to parabola, Then

a) If two normals are coincidental, then $27a\beta^2 = 4(\alpha-2a)^3$

b) If two normals are coincidental, then $4a\beta^2 = 27(\alpha-2a)^3$

c) If these three normals cut the axis of the parabola at points whose distances from vertex are in A.P, then $27a\beta^2 = 2(\alpha-2a)^3$

d) If these three normals cut the axis of parabola points whose distances from vertex are in A.P, then $2a\beta^2 = 27(\alpha-2a)^3$

Key. A,C

Sol. $y = tx + 2at + at^3$

(α, β) lies on normal

$\Rightarrow at^3 + t(2a-\alpha) - \beta = 0$

If two normals coincident then roots of above

Eq will be t_1, t_1, t_2

$2t_1 + t_2 = 0 \qquad (1)$

$t_1^2 t_2 = \frac{\beta}{a} \qquad (2)$

$t_1^2 + 2t_1t_2 = 2-\frac{\alpha}{a} \qquad (3)$

Formula (1) & (2) $\Rightarrow -2t_1^3 = \frac{\beta}{a}$

Formula (1) & (3) $-3t_1^3 = 2 - \frac{\alpha}{a}$

Eliminate t_1 to get $27\,a\beta^2 = 4(\alpha-2a)^3$

Let $y = mx - 2am - am^3$ be the eq of normal (α, β) lies on normal

$\beta = mx - 2am - am^3 \qquad - (4)$

To get distance of point of intersection of normal and axis from origin, put y = 0

$\Rightarrow x = \frac{2am + am^3}{m} = 2a + am^2$

There are three normals with slope m_1, m_2 and m_3 which are the roots of eq (4),

Distance are , $2a+am_1^2$, $2a + am_2^2$, $2a + am_3^2$ Now these distances are in A.P.

$m_1^2 + m_3^2 = 2m_2^2 \qquad (5)$

Of course, $m_1, m_2 \,\&\, m_3$ are roots of(4) and thus $am^3 + m(2\alpha-\beta) + \beta = 0$

$\Rightarrow m_1 + m_2 + m_3 = 0$

$m_1\, m_2\, m_3 = \frac{\beta}{a} \qquad (6)$

$m_1m_2 + m_2m_3 + m_3m_1 = \frac{2a-\alpha}{a}$

From (2) & (6) $27a\beta^2 = 2\left(\alpha - 2a\right)^3$

38. The focal chord to $y^2 = 16x$ is tangent to $\left(x-6\right)^2 + y^2 = 2$, then slope of focal chord is

A. 1 B. $\frac{1}{2}$

C. $-\frac{1}{2}$ D. -1

Key. A,D

Sol. $(x-6)^2 + y^2 = 2 \rightarrow$ tangent is $y = m(x-6) + \sqrt{2m^2+2}$

It is passing through (4, 0) focus of parabola

$0 = -2m + \sqrt{2m^2+2} \Rightarrow 2m^2 + 2 = 4m^2$

$m^2 = 1 \Rightarrow m = \pm 1$

39. The focus of parabola is (2,3) and it is touching the coordinate axes. Then

A. equation of axis of parabola is 3x – 2y = 0

B. length of latus rectum of parabola is $\frac{24}{\sqrt{13}}$

C. equation of tangent at vertex is 3x + 2y – 6 = 0

D. equation of directrix is 3x + 2y = 0

Key. B,C,D

Sol. Parabola touches x-axis at (2,0) and y-axis at (0,3) tangent at vertex is line joining (2,0) & (0,3) which is 3x + 2y = 6

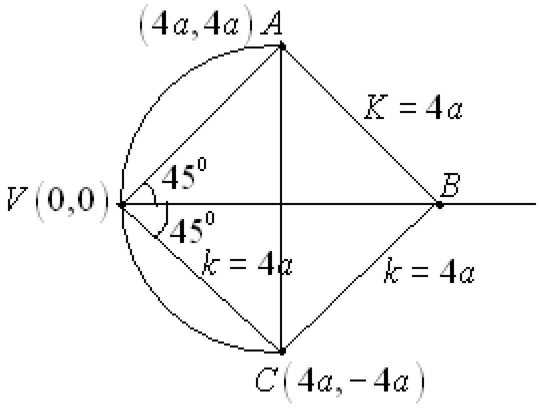
(0,0) lies on directrix. Hence directrix is 3x + 2y = 0.

40. A square has one vertex at the vertex of the parabola $y^2 = 4ax$ and the diagonal through the vertex lies along the axis of the parabola. If the ends of the other diagonal lie on the parabola, the coordinates of the vertices of the square are

(A) (4a, 4a) (B) (4a, -4a) (C) (0, 0) (D) (8a, 0)

Key. A,B,C,D

Sol.



Let K be length of the side so that A(4a, 4a), B(4a, -4a); V(0, 0), B(8a, 0)

41. If the parabolas $y^2 = 4kx(k>0)\, and\ y^2 = 4(x-1)$ do not have a common normal other than the axis of parabola , then k$\in$

a) (0,1) b) $(2,\infty)$ c) $(3,\infty)$ d) $(0,\infty)$

Key. A,B,C

Sol. If the parabolas have a common normal of slope 'm' (the only allowed value of m is m=0) then it is given by $y = mx - 2km - km^3$ and $y = m(x-1) - 2m - m^3$

$$= mx - 3m - m^3$$

$$\Rightarrow 2km + km^3 = 3m + m^3$$

$\Rightarrow m = 0, m^2 = \dfrac{3-2k}{k-1}$. If $m^2 < 0$ then the only common normal is the axis

$$\Rightarrow \frac{3-2k}{k-1} < o \Rightarrow (k-1)(2k-3) > 0 \Rightarrow k > \frac{3}{2}\ or\ k < 1 \,\&\, k > 0$$

42. The points on axis of parabola $3x^2 + 4x - 6y + 8 = 0$ from which three distinct normals can be drawn to it are

a) $\left(\dfrac{-2}{3}, 2\right)$ b) $\left(\dfrac{-2}{3}, 3\right)$ c) $\left(\dfrac{-2}{3}, 4\right)$ d) $\left(\dfrac{-2}{3}, 1\right)$

Key. B,C

Sol. The parabola can be written as $\left(x + \dfrac{2}{3}\right)^2 = 2(y - \dfrac{10}{9})\ ie\ X^2 = 2Y\left(X = x + \dfrac{2}{3}, Y = y - \dfrac{10}{9}\right)$. A point on axis is $\left(\dfrac{-2}{3}, Y\right)$ from which three normals can be drawn if $Y > 1$

$$\Rightarrow y > \frac{19}{9}.$$

43. The normals at the points $P(t_1), Q(t_2)$ on the parabola $y^2 = 4ax$ intersect at $Q(t_3)$ on the parabola. Then which of the following is/are true?

A) $|t_3| \ge \sqrt{2}$ B) $|t_3| \ge 2\sqrt{2}$ C) $t_1 t_2 = 2$ D) $t_3 = -t_1 - t_2$

Key. B,C,D

Sol. Conceptual

44. Which of the following statements are true?

A) There exists a point in the plane of a parabola through which no real normal can be drawn to the parabola

B) If P, Q, R are co-normal points of a parabola then circum circle of ΔPQR passes through the vertex of the parabola and centroid of ΔPQR lies on the axis of the parabola

C) The circumcentre of triangle formed by three tangents of a parabola passes through focus of the parabola

D) Length of subnormal at any point on a parabola is equal to latusrectum of the parabola.

Key. B

Sol. Conceptual

45. Consider the parabola represented by the parametric equations $x = t^2 - 2t + 2; y = t^2 + 2t + 2$. Then which of the following is/are true?

A) Auxiliary circle of the parabola is $x + y = 4$ B) Vertex of the parabola is (2, 2)

C) Director circle of the parabola is $x + y = 6$ D) Focus of the parabola is (3, 3)

Key. A,B,D

Sol. $x = t^2 - 2t + 2; \; y = t^2 + 2t + 2$

$x + y = 2(t^2 + 2)$ and $y - x = 4t$

$$\frac{x+y}{2} = \frac{(y-x)^2}{16} + 2 \Rightarrow (y-x)^2 = 8(x+y-4)$$

$$\left(\frac{y-x}{\sqrt{2}}\right)^2 = 4\sqrt{2}\left(\frac{x+y-h}{\sqrt{2}}\right)$$

This is parabola for which $y = x$ is axis, $x + y = 4$ is tangent at vertex and length of latusrectum is $4\sqrt{2}$

46. The equation of a conic is $y^2 + 2ax + 2by + c = 0$, then

A) It is an ellipse B) it is a parabola C) Its latus-rectum = *a* D) Its latus rectum = 2*a*

Key. B,D

Sol. $(y+b)^2 = -2a\left(x - \frac{b^2}{2a} + \frac{c}{2a}\right)$, it is a parabola. Latusrectum = 2*a*.

47. Let P, Q and R are three conormal points on the parabola $y^2 = 4ax$. Then the correct statement (s) is /are :

a) Algebraic sum of the slopes of the normals at P, Q and R is zero.

b) Algebraic sum of abscissa of the points P, Q and R is zero.

c) Centroid of the triangle PQR lies on the axis of the parabola

d) Circle circumscribing the triangle PQR passes through the vertex of parabola

Key. A,C,D

Sol. Equation of normal at $\left(am^2, -2am\right)$ is

$$y = mx - 2am - am^3$$

$$\Rightarrow am^2 + \left(2a - x\right)m + y = 0$$

$$\sum m = 0$$

$$\sum m_1 m_2 = \frac{2a - x}{a}$$

$$\sum m_1 m_2 m_3 = -y/a$$

48. Which of the following statements are true for the curve
$9x^2 - 24xy + 16y^2 - 20x - 15y - 60 = 0$
a) It represents a parabola
b) Length of latus rectum of the curve is 2
c) the equation of directrix of curve is $16x + 12y + 53 = 0$
d) the equation of axis of the curve is $3x - 4y - 35 = 0$

Key. A,B,C

Sol. Curve is $\left(\frac{3x-4y}{5}\right)^2 = \left(\frac{20x+15+60}{25}\right)$

49. Given the parabola $y^2 = 4ax$ and the points $A(at^2, 2at)$, $B(at^{-2}, 2at^{-1})$, $C\left(\frac{4a}{t^2}, \frac{4a}{t}\right)$, $D\left(a\left(t+\frac{2}{t}\right)^2, -2a\left(t+\frac{2}{t}\right)\right)$ then make all the correct alternative
A) AB is a focal chord
B) AD is a normal chord
C) normals at A, C intersect on the parabola
D) Tangents at A, B intersect at 90^0 on the directrix

Key. A,B,C,D

Sol. From standard result, we can prove them.

50. The points on axis of parabola $3x^2 + 4x - 6y + 8 = 0$ from which three distinct normals can be drawn to it are
(A) $\left(\frac{-2}{3}, 2\right)$ (B) $\left(\frac{-2}{3}, 3\right)$ (C) $\left(\frac{-2}{3}, 4\right)$ (D) $\left(\frac{-2}{3}, 1\right)$

Key. B,C

Sol. The parabola can be written as $\left(x+\frac{2}{3}\right)^2 = 2(y-\frac{10}{9})$ *ie* $X^2 = 2Y\left(X = x+\frac{2}{3}, Y = y-\frac{10}{9}\right)$. A point on axis is $\left(\frac{-2}{3}, Y\right)$ from which three normals can be drawn if $Y > 1$ $\Rightarrow y > \frac{19}{9}$.

51. Which of the following statements are true?
(A) There exists a point in the plane of a parabola through which no real normal can be drawn to the parabola
(B) If P, Q, R are co-normal points of a parabola then circum circle of ΔPQR passes through the vertex of the parabola and centroid of ΔPQR lies on the axis of the parabola
(C) The circumcentre of triangle formed by three tangents of a parabola passes through focus of the parabola
(D) Length of subnormal at any point on a parabola is equal to latusrectum of the parabola.

Key. B

Sol. Conceptual

52. Consider the parabola represented by the parametric equations $x = t^2 - 2t + 2; y = t^2 + 2t + 2$. Then which of the following is/are not true?

(A) Locus of feet of perpendiculars from the focus of the parabola to any tangent of parabola is $x+y=4$

(B) Vertex of the parabola is (2, 2)

(C) Orthoptic locus of the parabola is $x+y=6$

(D) Length of latus rectum of the parabola is $2\sqrt{2}$

Key. C,D

Sol. $x=t^2-2t+2;\ y=t^2+2t+2$

$x+y=2(t^2+2)$ and $y-x=4t$

$\frac{x+y}{2}=\frac{(y-x)^2}{16}+2 \Rightarrow (y-x)^2=8(x+y-4)$

$\left(\frac{y-x}{\sqrt{2}}\right)^2=4\sqrt{2}\left(\frac{x+y-4}{\sqrt{2}}\right)$

This is parabola for which $y=x$ is axis, $x+y=4$ is tangent at vertex and length of latusrectum is $4\sqrt{2}$

53. Let PQ be a chord of the parabola $y^2=4x$. A circle is drawn with PQ as diameter passes through the vertex 'V' of the parabola. If area of triangle PVQ= 20 sq.units, then the coordinates of P are

(A) (16, 8) (B) (16, -8) (C) (-16, 8) (D) (-16, -8)

Key. A,B

Sol. slope of PV = $\frac{2t-0}{t^2-0}=\frac{2}{t}$

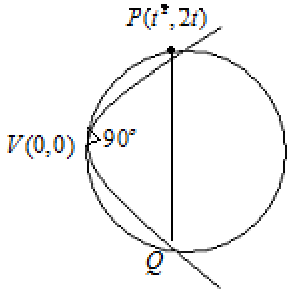
Equation of QV is $y=-\frac{t}{2}(x)$

On solving with $y^2=4x$, $Q=\left(\frac{16}{t^2},\frac{-8}{t}\right)$

Area of ΔPVQ is $\frac{1}{2}.PV.VQ$ = 20

$\Rightarrow PV.VQ$ = 40

By solving above equation $t=\pm 4,\pm 1$



Parabola

Assertion Reasoning Type

a) Both A and R are true and R is correct explanation of A.

b) Both A and R are true but R is not correct explanation of A.

c) A is true, R is false. d) A is false, R is true.

1. STATEMENT -1: PQ is a chord of the parabola $x^2+2x+12y-11=0$ through (-1, -2) and a circle is described on PQ as diameter, then the circle touches the line $y=4$
STATEMENT-2: If a circle is described on a focal chord of a parabola as diameter, then it touches the directrix.

Key. A

Sol. Conceptual

2. STATEMENT -1: Through the vertex 'O' of the parabola $y^2=4x$ chords OP & OQ are drawn at right angles to one another. For all positions of the line through P and Q cuts the axis of parabola at a fixed point
STATEMENT-2: Any point on the axis of the parabola $y^2=4ax$ is of the form $(x_1,0)$ and the line $L_1+\lambda L_2=0$, where $L_1 \& L_2$ are two fixed non – parallel lines is passes through a fixed point.

Key. A

Sol. Conceptual

3. Assertion A : If $a^2>8b^2$, then a point can be found such that the two tangents from it to the parabola $y^2=4ax$ are normals to the parabola $x^2=4by$

Reason R : The equations $ty=x+at^2, x+sy=2bs+bs^3$ where 'S' and 't' are parameters representing same lines for two pair values of 's' and 't' of $a^2>8b^2$

Key. A

Sol. Any tangent to the parabola $y^2=4ax$ is

$ty = x + at^2$ at $(at^2, 2at)$

While any normal to the parabola

$x^2 = 4by$ at $(2bs, bs^2)$

$x + ys = 2bs + bs^3$

If these are same then

$$\frac{t}{s}=\frac{-1}{1}=\frac{2at^2}{-2bs-bs^3}$$

$\Rightarrow s=-t$ from first two ratios on equating first and third ratio, we get

$$1=\frac{at^2}{-2bt-bt^2}$$

This should have two distinct roots

$a^2-8b^2>0 \Rightarrow A$ is true .

In concluding this , we have used the correct reason R.

Thus A is true and R is correct explanation

4. Statement - 1: *A* is the vertex of $y^2 = 4ax$ and *B*, *C* are points on the parabola such that $AB = AC$ and $\underline{|BAC} = \pi/2$. Then area of ΔABC is $16a^2$.

Statement - 2: *A* is the vertex of $y^2 = 4ax$ and *B*, *C* are points on the parabola such that ΔABC is equilateral. Then area of ΔABC is $24\sqrt{3}a^2$.

Key. C

Sol. Conceptual

5. Statement - 1: All chords of a parabola which subtend a right angle at the vertex of the parabola pass through a fixed point on the axis of the parabola.

Statement - 2: If $L_1 = 0$ and $L_2 = 0$ are the equations of two fixed straight lines which are not parallel then $L_1 + \lambda L_2 = 0, \lambda \in R$ represents a family of concurrent lines.

Key. A

Sol. Conceptual

6. Statement - 1: Product of the lengths of the perpendicular drawn from the points (4, 2) and $(4, -6)$ to any tangent of $\frac{(x-4)^2}{9} + \frac{(y+2)^2}{25} = 1$ is 9.

Statement - 2: Product of the lengths of the perpendicular drawn from the foci of an ellipse to any tangent of the ellipse is square on the semi minor axis of the ellipse.

Key. A

Sol. Conceptual

7. Statement - 1: The equation $x^2 \cos^2\theta + y^2 \cot^2\theta = 1$ represents a family of confocal ellipses.

Statement - 2: The equation $x^2 \sec^2\theta - y^2 \text{cosec}^2\theta = 1$ represents a family of confocal hyperbolas.

Key. B

Sol. Conceptual

8. Assertion (A): The distance of a focus of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ to an asymptote of the hyperbola is *b*.

Reason (R): The product of distances of any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ from the asymptotes of the hyperbola is $\frac{a^2b^2}{a^2+b^2}$.

Key. B

Sol. Conceptual

9. Assertion (A): Maximum area of the triangle whose vertices lie on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b>0)$ is $\frac{3\sqrt{3}ab}{4}$.

Reason (R): The area of a triangle inscribed in a circle of given radius is maximum when the triangle is equilateral.

Key. A

Sol. Conceptual

10. Assertion (A): The length of the chord of the parabola $y^2=x$ which is bisected at (2, 1) is $2\sqrt{5}$.

Reason (R): Length of the chord joining the points t_1, t_2 on the parabola $y^2=4ax$ is $|a(t_1-t_2)|\sqrt{(t_1+t_2)^2+4}$.

Key. A

Sol. Conceptual

11. Assertion (A): The locus of point of intersection of normals at the end points of a focal chord of $y^2=4ax$ is a parabola whose directrix is $x=13a/4$.

Reason (R): Locus of the foot of the perpendicular from the focus of conic $x^2+y^2=(x\cos\theta+y\sin\theta-2p)^2, p>0$ to any tangent of the conic is $x\cos\theta+y\sin\theta=p$.

Key. D

Sol. Conceptual

12. Statement - 1: If $\bar{u}$ and $\bar{v}$ are unit vectors inclined at angle 'α' and '$\bar{x}$' is a unit vector bisecting the angle between them, then $\bar{x}=\frac{\bar{u}+\bar{v}}{2\sin\alpha/2}$

Statement - 2: If ABC is an isosceles triangle with AB = AC = 1, then the vector representing bisector of angle 'A' is given by $\overline{AD}=\frac{\overline{AB}+\overline{AC}}{2}$

Key. D

Sol. In an isosceles triangle ABC in which AB=AC, the median and bisector from 'A' must be same line

$\Rightarrow$ Reason 'R' is true

Now $\overline{AD}=\frac{\bar{u}+\bar{v}}{2}$ and $\left|\overline{AD}\right|^2=\frac{1}{4}\left(|\bar{u}|^2+|\bar{v}|^2+2|\bar{u}||\bar{v}|\cos\alpha\right)$

$$=\frac{1}{4}(1+1+2\cos\alpha)$$

$$\Rightarrow\left|\overline{AD}\right|=\cos\alpha/2$$

$\Rightarrow$ Unit vector along AD is $\quad \bar{x} = \dfrac{\bar{u} + \bar{v}}{2\cos\alpha/2}$

13. Statement - 1: The lines $\dfrac{x-1}{1} = \dfrac{y}{-1} = \dfrac{z+1}{1}$ and $\dfrac{x-2}{1} = \dfrac{y+1}{2} = \dfrac{z}{3}$ are coplanar and equation of the plane containing them is 5x+ 2y – 3z-8 = 0.

Statement - 2: The line $\dfrac{x-2}{1} = \dfrac{y+1}{2} = \dfrac{z}{3}$ is perpendicular to the plane 3x+6y +9z-8=0 and parallel to the plane x+y-z=0

Key. B

Sol. $\begin{vmatrix} 1-2 & 0+1 & -1-0 \\ 1 & -1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow$ given lines are coplanar

Equation of the plane is $\begin{vmatrix} x-1 & y & z+1 \\ 1 & -1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = 0$

i.e., 5x + 2y – 3z - 8 = 0

Since $\dfrac{1}{3} = \dfrac{2}{6} = \dfrac{3}{9} \Rightarrow \dfrac{x-2}{1} = \dfrac{y+1}{2} = \dfrac{z}{3}$ is perpendicular to the plane

And also 1(1) + 2(1) +3 (-1) = 0

$$\Rightarrow \frac{x-2}{1} = \frac{y+1}{2} = \frac{z}{3} \text{ is parallel to } x+y-z=0$$

14. Statement - 1: The normals at three points P, Q, R of the parabola $y^2 = 4ax$ meet in (h, k) then centroid of ΔPQR lies on x = 0

Statement - 2: If A, B, C are three co.normal points of the parabola $y^2 = 4ax$ then centroid of ΔABC lies on axis of the parabola

Key. D

Sol. Let $P\left(at_1^2, 2at_1\right)$, $Q\left(at_2^2, 2at_2\right)$, $R\left(at_3^2, 2at_3\right)$ be the three points on parabola.

Equation of the normal at't' is $y + xt = 2at + at^3$

The above normal is passing through (h, k)

$$\Rightarrow k + ht = 2at + at^3$$

$$\Rightarrow at^3 + (2a-h)t - k = 0$$

The above is a cubic equation in 't'. whose roots are $t_1, t_2, t_3 \quad \Rightarrow \sum t_1 = 0$

Centroid of ΔPQR is $\left(\dfrac{a\left(t_1^2 + t_2^2 + t_3^2\right)}{3}, \dfrac{a}{3}\left(t_1 + t_2 + t_3\right) \right)$

$$= \left(\frac{a\left(t_1^2 + t_2^2 + t_3^2\right)}{3}, 0 \right)$$

Centroid lies on y = 0

Hence 'A' is false and 'R' is correct

15. Statement - 1: Let 'a' and 'b' be the segments of the focal chord and 2*l* is its latusrectum then

$$a^3 + l^3 > 2b^3$$

Statement - 2: A.M > G.M > H.M

Key. B

Sol. W.K.T semi latusrectum is the H.M between focal segments

$\Rightarrow$ '*l*' is H.M of a,b

$\Rightarrow$ w.k.t G.M >H.M

$$\sqrt{ab} > \frac{2ab}{a+b} \Rightarrow \sqrt{ab} > l$$

$$\Rightarrow ab > l^2$$

Now $A.M > G.M$

$$\frac{a^3 + b^3}{2} > \sqrt{a^3 b^3} > l^3 \Rightarrow a^3 + b^3 > 2l^3$$

16. Assertion (A): Length of the chord of contact of tangents drawn from (x_1, y_1) to $y^2 = 4ax$ is

$$\frac{\sqrt{(y_1^2 - 4ax_1)(y_1^2 + 4a^2)}}{|a|}$$

Reason (R): If the line $y = mx + c$ cuts any curve $y = f(x)$ in $A(x_1, y_1); B(x_2, y_2)$ then AB is given by $|x_1 - x_2|\sqrt{1+m^2}$

Key. A

Sol. Conceptual

17. Assertion (A): The normal at any point of a parabola is equally inclined to the focal distance of the point and the axis of the parabola

Reason (R): The inclination of shortest normal chord of $y^2 = 4ax$ to x-axis is $Tan^{-1}\sqrt{2}$

Key. B

Sol. Conceptual

18. Assertion (A): The area of the triangle whose vertices are $A(1,2,3); B(-2,1,-4); C(3,4,-2)$ is $\frac{\sqrt{1218}}{2}$ square units.

Reason (R): If A is area of $\Delta ABC; A_x, A_y, A_z$ are areas of projections of ΔABC on yz, zx, xy planes respectively then area of $\Delta ABC = \sqrt{A_x^2 + A_y^2 + A_z^2}$

Key. A

Sol. Conceptual

19. Assertion (A): Two straightlines in space which are neither parallel nor intersecting are called as skew lines.

Reason (R): If θ is angle between $\bar{r} = \bar{a} + \lambda\bar{b}$ and $\bar{r}\cdot\bar{n} = d$ then $\cos\theta = \frac{\bar{b}\cdot\bar{n}}{|\bar{b}||\bar{n}|}$

Key. C

Sol. Conceptual

20. Statement I : The point of intersection of the lines joining A(2,3), B(-1,2) and C(-2,1),D(3,4)is an internal point of $\overline{AB}$

Statement II : A(2,3),B(-1,2) are on opposite sides of the line through C(-2,1) and D(3,4)

Key. A

Sol. Conceptual

21. Statement I: If $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0, \frac{a_1}{b_1} \neq \frac{a_2}{b_2} \neq \frac{a_3}{b_3}$ then lines

$a_1x + b_1y + c_1 = 0, a_2x + b_2y + c_2 = 0 \text{ and } a_3x + b_3y + c_3 = 0$ are concurrent

Statement II : Area of triangle formed by

$a_1x + b_1y + c_1 = 0, a_2x + b_2y + c_2 = 0 \text{ and } a_3x + b_3y + c_3 = 0$ is

$$\frac{1}{2}\left\|\begin{matrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{matrix}\right\|$$

Key. C

Sol. Conceptual

22. Statement I : Through $P(\lambda, \lambda+1)$, no tangent can be drawn to the parabola $y^2 = 4(x+1)$ if $\lambda > 3$

Statement II : $P(\lambda, \lambda+1)$ is an interior point of $y^2 = 4(x+1)$ if $\lambda \in (-1,3)$

Key. D

Sol. Conceptual

23. Statement I : Latus rectum of any parabola is a focal chord of minimum length

Statement II : If SP,SQ are segments of a focal chord of a parabola $y^2 = 4ax(a > 0)$ then SP+SQ$\geq$4a

Key. A

Sol. Conceptual

24. Statement I: The normals at the points where $t = \pm\sqrt{2}$, on the parabola $y^2 = 4ax$ subtends a right angle at the vertex of the parabola.

Statement II: For a normal chord of the parabola $y^2 = 4ax$ having ends t_1, t_2 we always have $t_1.t_2 = 2$.

Key. C

Sol. When P(t_1) ,Q(t_2) are the ends of a normal chord then the product of slopes of AP and AQ is

$$\frac{2}{t_1} \text{x} \frac{2}{t_2} = \frac{4}{t_1\left(-t_1 - \frac{2}{t_1}\right)} = \frac{-4}{t_1^2 + 2} = -1 \text{ when } t_1 = \pm\sqrt{2}$$

$\therefore$ Statement I is true

Also statement II is false

25. Consider the lines L_1: 2x + 3y + p – 3 = 0, L_2: 2x + 3y + p + 3 = 0, where ‘p’ is a real number, and C : $x^2 + y^2 + 6x - 10y + 30 = 0$

Statement I: If the line L_1 is a chord of the circle C, then the line L_2 is not always a diameter of circle.

Statement II: If the line L_1 is a diameter of the circle C, then the line L_2 is not a chord of circle C.

Key. D

Sol. Conceptual

26. Statement I: The foci of the hyperbola $xy = 36$ are $\left(6\sqrt{2}, 6\sqrt{2}\right)$ and $\left(-6\sqrt{2}, -6\sqrt{2}\right)$.

Statement II: The foci of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are $\left(\pm\sqrt{a^2 - b^2}, 0\right)$.

Key. C

Sol. Foci of xy = 36 are $\left(c\sqrt{2}, c\sqrt{2}\right)$ and $\left(-c\sqrt{2}, -c\sqrt{2}\right)$ = $\left(6\sqrt{2}, 6\sqrt{2}\right), \left(-6\sqrt{2}, -6\sqrt{2}\right)$

$\therefore$ statement I is true

The foci of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are $\left(\pm\sqrt{a^2 + b^2}, 0\right)$

Statement II is false

27. Statement I: The radius of the largest circle with center (1, 0) that can be inscribed in the ellipse $x^2 + 4y^2 = 16$ is $\sqrt{\frac{11}{3}}$.

Statement II: The normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at a point ‘θ’ is

$$\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$$

Key. A

Sol. Equation of any normal to $\frac{x^2}{16}+\frac{y^2}{4}=1$ is $(4\sec\theta)$ x-(2cosec θ) y = 12

Putting (1,0) we have 4sec θ = 12 $\Rightarrow \cos\theta=\frac{1}{3},\sin\theta=\frac{2\sqrt{2}}{3}$

Hence the point of contact is $\left(\frac{4}{3},\frac{4\sqrt{2}}{3}\right)$

Req rad. = $=\sqrt{\left(\frac{4}{3}-1\right)^2+\left(\frac{4\sqrt{2}}{3}\right)^2}=\sqrt{\frac{11}{3}}$

Statement I is true

Statement II is also true but not a correct explanation of statement –I

28. Statement –1 : the parametric coordinates of any point on the parabola $y^2=4ax$ can be taken as $\left(a\sin^2 t,2a\sin t\right)$

Statement – 2 : If t is a paratmeter, $\left(a\sin^2 t,2a\sin t\right)$ point satisfies the equation $y^2=4ax$.

Key. D

Sol. It doesn’t contain all the points of $y^2=4ax$

29. Statement – 1 : The sum of eccentric angles of four co-normal points of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is an odd multiple of π (π radian = 180^0)

Statement – 2 : The sum of the eccentric angles of the points in which a circle cuts an ellipse is an even multiple of π (π radius = 180)

Key. B

Sol. Conceptual

30. Statement – 1 : Locus of ‘z’ given by $\left|z-\left(3+2i\right)\right|=\left|z\cos\left(\frac{\pi}{4}-\arg z\right)\right|$ represents a parabola

Statement – 2 : If distance of a variable point from a fixed point is equal to the distance from a fixed line, it represents a parabola

Key. C

Sol. If the point lies on the directrix, its pair of straight line

31. Statement – 1 : If a conic circumscribe a quadrilateral, the ratio of the product of the perpendiculars from any point ‘p’ of the conic up on two opposite side of the quadrilateral to the product of perpendiculars from ‘p’ upon the other sides is the same for all positions of ‘p’

Statement – 2 : Equation of conic passing through the angular points formed by lines L = 0, M = 0, N = 0 and R = 0, which are the four sides of a quadrilateral taken in order is $LN = \mu MR$

Key. A

Sol. $LN = \mu RM$ is a conic passing through angular points and L is proportional to the perpendicular from any point (x, y) on the line L = 0

32. Statement-I: The curve $y = \frac{-x^2}{2} + x + 1$ is symmetric about the line $x = 1$, because

Statement-II: A parabola is symmetric about its axis

Key. A

Sol. Conceptual

33. Statement-I: In a central conic any 4 co-normal points can lie on a rectangular hyperbola, because

Statement-II: In a central conic sum of the eccentric angles of any 4 conormal points is always an odd multiple of π

Key. B

Sol. Let normals at $(x_i, y_i), i = 1,2,3,4$ be concurrent at (h, k)

Normal at (x_1, y_1) $\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$ passes through (h, k)

$$\Rightarrow \frac{a^2h}{x_1} - \frac{b^2k}{y_1} = a^2 - b^2$$

$$\Rightarrow x_1y_1(a^2 - b^2) - a^2hy_1 + b^2kx_1 = 0$$

$\Rightarrow (x_i, y_i)$ satisfy an equation of the type $xy(a^2 - b^2) - a^2hy + b^2kx = 0$

Which represents a hyperbola

34. Statement-I: In a ΔABC, if base BC is fixed and perimeter of the triangle is also fixed, Then vertex A moves on an ellipse, because

Statement-II: Locus of a moving point is an ellipse if sum of its distances from two fixed points is a positive constant (where all the points are coplanar)

Key. C

Sol. Conceptual

35. Statement-I: For a parabola latus rectum is the shortest focal choral, because

Statement-II: The length of a focal chord of a parabola inclined at an angle $'\theta'$ with its axis is given by $4a\sec^2\theta$

Key. C

Sol. Length of focal chord of a parabola inclined at an angle $'\theta'$ with its axies is given by $4a\,\text{cosec}^2\theta$

36. Statement I: If the normal at t_1, to the parabola $y^2 = 4ax$ cuts the curve again at t_2 then $t_2^2 \geq 8$.

Statement II : Equation to the tangent at 't' on $y^2 = 4ax$ is $yt = x + at^2$,

Key. A

Sol. $t_2 = -t_1 \dfrac{-2}{t_1} \Rightarrow t_1^{\,2} + t_1 t_2 + 2 = 0 \Rightarrow t_2^{\,2} - 4.1.2 \geq o \Rightarrow t_2^{\,2} \geq 8$

37. Statement I: If a & b are lengths of segments of a focal chord of parabola $(a \neq b)$, and 2c is the length of latus rectum, then $a^3 + b^3 > 2c^3$.

Statement II: $AM > GM > HM$.

Key. A

Sol. $\dfrac{2ab}{a+b} = c \Rightarrow a, c, b,$ all in HP

$\therefore GM > HM \Rightarrow \sqrt{ab} > c$

$\therefore AM > GM \Rightarrow \dfrac{a^3 + b^3}{2} > \sqrt{a^3 b^3} > c^3 \Rightarrow a^3 + b^3 > 2c^3$

38. Statement I : The locus of centre of the circle described on any focal chord of a parabola $y^2 = 4ax$ as diameter is, $y^2 = 2a(x+a)$.

Statement II: If $\left(at_1^2, 2at_1\right) \& \left(at_2^2, 2at_2\right)$ are the extremities of a focal chord of $y^2 = 4ax$, then, $t_1 t_2 = -1$

Key. D

Sol. circle locus is , $y^2 = 2a(x-a)$

39. Statement I: The angle of intersection between the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 \&$ the circle $x^2 + y^2 = ab$ is $\tan^{-1}\left(\dfrac{b-a}{\sqrt{ab}}\right)$.

Statement II : The point of intersection of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 \& x^2 + y^2 = ab$ is $\left(\sqrt{\dfrac{ab}{a+b}}, \sqrt{\dfrac{ab}{a+b}}\right)$

Key. C

Sol. POI is , $\left(\sqrt{\frac{a^2b}{a+b}},\sqrt{\frac{ab^2}{a+b}}\right)$, with

$$m_1=\frac{-b^2}{a^2}\sqrt{\frac{a}{b}}4m_2=-\sqrt{\frac{a}{b}}.\Rightarrow\theta=\tan^{-1}\left(\frac{m_1-m_2}{1+m_1m_2}\right)=\tan^{-1}\left(\frac{b-a}{\sqrt{ab}}\right).$$

40. STATEMENT-1: Through $(h,h+1)$ there cannot be more than one normal to $y^2=4x$ if $h<2$

STATEMENT-2 : $(h,h+1)$ lies out side the parabola for all $h\neq 1$

Key. B

Sol. $y+tx=2t+t^3$

It passes through (*h, h + 1*)

$h+1+th=2t+t^3$

$\Rightarrow$ $t^3-t(h-2)-(h+1)=0$

$f'(t)=3t^2-(h-2)$

$f'(t)=0$ $3t^2=h-2<0$

$\therefore$ t will have imaginary root $\therefore$ Only one real root

$(h+1)^2-4h>0$ $\therefore$ $(h,h+1)$ out side the parabola.

41. STATEMENT-1 : The equation of the director circle to $4x^2-3y^2=12$ is $x^2+y^2=1$.

STATEMENT-2 : Director circle is the locus of the point of intersection of mutually $\perp^r$ tangents to the hyperbola

Key. D

Sol. $4x^2-3y^2=12$

$\Rightarrow$ $\frac{x^2}{3}-\frac{y^2}{4}=1$

$a^2<b^2$

$\therefore$ Director circle does not exist.

42. STATEMENT-1 : The condition on a & b for which two distinct chords of the ellipse $\frac{x^2}{2a^2}+\frac{y^2}{2b^2}=1$ passing through $(a,-b)$ are bisected by $x+y=b$ is $a^2+6ab-7b^2>0$.

STATEMENT-2 : Equation of the chord of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose mid point (x_1,y_1) is of the form $T=S_1$. i.e. $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}-1=\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1$.

Key. A

Sol. Let the mid point $(t, b-t)\frac{tx}{2a^2}+\frac{(b-t)y}{2b^2}=\frac{t^2}{2a^2}+\frac{(b-t)}{2b^2}$

It passes through $(a,-b)\frac{ta}{2a^2}-\frac{b(b-t)}{2b^2}=\frac{t^2}{2a^2}+\frac{(b-t)^2}{2b^2}$ $t^2(a^2+b^2)-ab(3a+b)t+2a^2b^2=0$ For real t, $a^2b^2(3a+b)^2-4(a^2+b^2)2a^2b^2>0$

$9a^2+6ab+b^2-8a^2-8b^2>0$

$a^2+6ab-7b^2>0$

43. STATEMENT-1: If $x^2y^3=6,\ \ x>0, y>0$ then the least value of $3x+4y$ is 10

STATEMENT-2 : Least value of 3x + 4y occurs when $9x=8y$.

Key. A

Sol. Consider $\frac{3}{2}x \,\&\, \frac{4}{3}y$ having weights 2 & 3 respectively

$$\therefore \qquad \frac{3x+4y}{5}\geq\left(\frac{9x^2}{4}.\frac{64y^3}{27}\right)^{\frac{1}{5}}$$

$$\Rightarrow \qquad \frac{3x+4y}{5}\geq\left(\frac{9}{4}.\frac{64}{27},\frac{6}{2}\right)^{\frac{1}{5}}\geq 2$$

$$3x+4y\geq 10$$

Least value occurs when $\frac{3x}{2}=\frac{4y}{3}\Rightarrow 9x=8y$

44. Statement - 1: The parametric coordinates of any points on the parabola $y^2=4ax$ can be taken as $\left(a\sin^2 t, 2a\sin t\right)$.

Statement - 2: If 't' is a parameter, $\left(a\sin^2 t, 2a\sin t\right)$ point satisfies the equation $y^2=4ax$

Key. D

Sol. It does not contain all the points of $y^2=4ax$

45. Statement - 1: The angle of intersection, of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the circle $x^2+y^2=ab$ is $\tan^{-1}\left(\frac{b-a}{\sqrt{ab}}\right)$.

Statement - 2: The point of intersection, of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $x^2+y^2=ab$ is $\left(\sqrt{\frac{ab}{a+b}},\sqrt{\frac{ab}{a+b}}\right)$

Key. C

Sol. point of intersection is $\left(\sqrt{\frac{a^2b}{a+b}},\sqrt{\frac{ab^2}{a+b}}\right)$ with $m_1=\frac{-b^2}{a^2}.\sqrt{a/b}, m_2=-\sqrt{a/b}$ $\Rightarrow \theta=\tan^{-1}\left(\frac{b-a}{\sqrt{ab}}\right)$

46. Statement - 1: If the point (x , y) lies on the curve $2x^2 + y^2 - 24y + 80 = 0$ then the maximum value of $x^2 + y^2$ is 400.

Statement - 2: The point (x , y) is at a distance of $\sqrt{x^2 + y^2}$ from origin.

Key. A

Sol. given equation represents ellipse $\frac{x^2}{32} + \frac{(y-12)^2}{64} = 1$; The maximum value of $\sqrt{x^2 + y^2}$ is the distance between (0, 0) & (0, 20).

47. Consider $\frac{x^2}{25} + \frac{y^2}{9} = 1$ and $x^2 + y^2 = 9$

Statement - 1: There can be some points on the ellipse, from which normals drawn are tangents to the given circle.

Statement - 2: If the radius of the circle is greater than the difference of length of semi major axis and semi minor axis of the ellipse which can be tangent to the circle.

Key. C

Sol. Conceptual

48. Statement - 1: *A* is the vertex of $y^2 = 4ax$ and *B*, *C* are points on the parabola such that $AB = AC$ and $\underline{|BAC} = \pi/2$. Then area of ΔABC is $16a^2$.

Statement - 2: *A* is the vertex of $y^2 = 4ax$ and *B*, *C* are points on the parabola such that ΔABC is equilateral. Then area of ΔABC is $24\sqrt{3}a^2$.

Key. C

Sol. Conceptual

49. Statement - 1: All chords of a parabola which subtend a right angle at the vertex of the parabola pass through a fixed point on the axis of the parabola.

Statement - 2: If $L_1 = 0$ and $L_2 = 0$ are the equations of two fixed straight lines which are not parallel then $L_1 + \lambda L_2 = 0, \lambda \in R$ represents a family of concurrent lines.

Key. A

Sol. Conceptual

50. Statement - 1: Product of the lengths of the perpendicular drawn from the points (4, 2) and $(4,-6)$ to any tangent of $\frac{(x-4)^2}{9} + \frac{(y+2)^2}{25} = 1$ is 9.

Statement - 2: Product of the lengths of the perpendicular drawn from the foci of an ellipse to any tangent of the ellipse is square on the semi minor axis of the ellipse.

Key. A

Sol. Conceptual

51. Statement - 1: The equation $x^2 \cos^2\theta + y^2 \cot^2\theta = 1$ represents a family of confocal ellipses.

Statement - 2: The equation $x^2 \sec^2\theta - y^2 \operatorname{cosec}^2\theta = 1$ represents a family of confocal hyperbolas.

Key. B

Sol. Conceptual

52. Assertion (A): The distance of a focus of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ to an asymptote of the hyperbola is *b*.

Reason (R): The product of distances of any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ from the asymptotes of the hyperbola is $\frac{a^2b^2}{a^2+b^2}$.

Key. B

Sol. Conceptual

53. Assertion (A): Maximum area of the triangle whose vertices lie on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > b > 0)$ is $\frac{3\sqrt{3}ab}{4}$.

Reason (R): The area of a triangle inscribed in a circle of given radius is maximum when the triangle is equilateral.

Key. A

Sol. Conceptual

54. Assertion (A): The length of the chord of the parabola $y^2 = x$ which is bisected at (2, 1) is $2\sqrt{5}$.

Reason (R): Length of the chord joining the points t_1, t_2 on the parabola $y^2 = 4ax$ is $|a(t_1 - t_2)|\sqrt{(t_1+t_2)^2 + 4}$.

Key. A

Sol. Conceptual

55. Assertion (A): The locus of point of intersection of normals at the end points of a focal chord of $y^2 = 4ax$ is a parabola whose directrix is $x = 13a/4$.

Reason (R): Locus of the foot of the perpendicular from the focus of conic $x^2 + y^2 = (x\cos\theta + y\sin\theta - 2p)^2, p > 0$ to any tangent of the conic is $x\cos\theta + y\sin\theta = p$.

Key. D

Sol. Conceptual

56. Statement - 1: The parametric coordinates of any points on the parabola $y^2 = 4ax$ can be taken as $\left(a\sin^2 t, 2a\sin t\right)$.

Statement - 2: If 't' is a parameter, $\left(a\sin^2 t, 2a\sin t\right)$ point satisfies the equation $y^2 = 4ax$

Key. D

Sol. It does not contain all the points of $y^2 = 4ax$

57. Statement - 1: The angle of intersection, of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the circle $x^2+y^2=ab$ is $\tan^{-1}\left(\frac{b-a}{\sqrt{ab}}\right)$.

Statement - 2: The point of intersection, of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $x^2+y^2=ab$ is $\left(\sqrt{\frac{ab}{a+b}}, \sqrt{\frac{ab}{a+b}}\right)$

Key. C

Sol. point of intersection is $\left(\sqrt{\frac{a^2b}{a+b}}, \sqrt{\frac{ab^2}{a+b}}\right)$ with $m_1 = \frac{-b^2}{a^2}.\sqrt{a/b}, m_2 = -\sqrt{a/b} \Rightarrow \theta = \tan^{-1}\left(\frac{b-a}{\sqrt{ab}}\right)$

58. Statement - 1: If the point (x , y) lies on the curve $2x^2+y^2-24y+80=0$ then the maximum value of x^2+y^2 is 400.

Statement - 2: The point (x , y) is at a distance of $\sqrt{x^2+y^2}$ from origin.

Key. A

Sol. given equation represents ellipse $\frac{x^2}{32}+\frac{(y-12)^2}{64}=1$; The maximum value of $\sqrt{x^2+y^2}$ is the distance between (0, 0) & (0, 20).

59. Consider $\frac{x^2}{25}+\frac{y^2}{9}=1$ and $x^2+y^2=9$

Statement - 1: There can be some points on the ellipse, from which normals drawn are tangents to the given circle.

Statement - 2: If the radius of the circle is greater than the difference of length of semi major axis and semi minor axis of the ellipse which can be tangent to the circle.

Key. D

Sol. conceptual

60. STATEMENT- 1

The minimum distance between the parabola $y^2 = 4x$ and the circle $x^2 + y^2 - 54y + 704 = 0$ is $\sqrt{522}$.

STATEMENT 2

Shortest distance between two non intersecting curves occurs along the common normal.

Key: D

61. Let $S:\frac{x^2}{9}-\frac{y^2}{16}=1 \quad C:x^2+y^2=7$

Statement I : Tangents drawn from any point $\left(\sqrt{7}\cos\theta,\sqrt{7}\sin\theta\right)$ ($0\le\theta\le 2\pi$) to S are perpendicular.

Statement II : Two common tangents can be drawn to S and C

Key: D

Hint: If $a<b$ perpendicular tangents cannot be drawn to $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

62. STATEMENT – 1: Slopes of tangents drawn from (4, 10) to $y^2=9x$ are $\frac{1}{4},\frac{9}{4}$.

STATEMENT – 2: Every parabola is symmetric about its directrix.

Key. C

Sol. $y=mx+\frac{a}{m}$

$10=4m-1.\frac{9}{4}$

$\Rightarrow 16m^2-40m+9=0$

$m_1+m_2=\frac{5}{2},m_1m_2=\frac{9}{16}$

$m_1=\frac{1}{4},m_2=\frac{9}{4}$

Every parabola is symmetric about its axis only.

63. STATEMENT – 1 : In ellipse the sum of the distances between the foci is always less than the sum of the focal distances of any point on it.

STATEMENT – 2 : The eccentricity of any ellipse is less than 1.

Key. A

Sol. distance between foci is 2a e. Sum of the focal distance is 2a.

$ae<a,e<1.$

64. STATEMENT - 1: The maximum no. of common normals of $y^2=4ax\ \&\ x^2=4ay$ can have is 5.

STATEMENT – 2: The polynomial of the gradient of the normal is of fifth degree.

Key. A

Sol. $2a+\frac{a}{m^2}=-2am-am^3$

$\Rightarrow 2m^2+1=-2m^3-m^5$

$\Rightarrow m^5+2m^3+2m^2+1=0$

65. STATEMENT – 1 : The locus of mid points of the chord of the parabola $y^2=4x$ which subtend a right angle at the vertex is $y^2=2x-8$

STATEMENT – 2 : Chord PQ joining points $'t_1'\, and\, 't_2'$ on $y^2=4x$ subtends a right angle at the vertex if $t_1t_2=-4$

Key. A

Sol. The chord joining $P(t_1^{\,2}, 2t_1)$ *and* $Q(t_2^{\,2}, 2t_2)$ subtends a right angle at the vertex O(0,0) => $OP \perp OQ$

$$\Rightarrow \frac{2t_1}{t_1^{\,2}}.\frac{2t_2}{t_2^{\,2}} = -1 \Rightarrow t_1.t_2 = -4$$

The mid point $(x_1, y_1) = \left(\frac{t_1^{\,2} + t_2^{\,2}}{2}, t_1 + t_2\right)$

$$\Rightarrow t_1 + t_2 = y_1 \quad \& \quad t_1^{\,2} + t_2^{\,2} = 2x_1$$

Eliminating t_1, t_2 gives $y_1^{\,2} + 8 = 2x_1$

66. Assertion A : 'A' is a point on the parabola $y^2 = 4ax$. The normal at 'A' cuts the parabola again at point 'B' If AB subtends a right angle at the vertex of the parabola, then slope of AB is $\frac{1}{\sqrt{2}}$

Reason R : If normal at $\left(at_1^2, 2at_1\right)$ meets the parabola again at $\left(at_2^2, 2at_2\right)$ then $t_2 = -t_1 - \frac{2}{t_1}$

Key. D

Sol. A is false, since if AB is a normal chord and A is $\left(at_1^2, 2at_1\right)$ then B is $\left(at_2^2, 2at_2\right)$ where

$$t_2 = -t_1 - \frac{2}{t_1} \qquad (1)$$

Now slope $OA = \frac{2at_1}{at_1^2} = \frac{2}{t_1}$

Slope OB = $\frac{2}{t_2}$

Since AB subtends 90^0

$$\Rightarrow t_1\ t_2 = -4 \qquad (2)$$

Now slope $AB = \frac{2}{t_1 + t_2} = \frac{2}{-2/t_1} = -t_1$

Substituting the value of t_2 is from (2) in (1)

we get $t_1 = \sqrt{2}$

$\Rightarrow$ slope of AB is $-\sqrt{2}$

The true reason R is a standard result

67. Assertion A : Three normals are drawn from the point 'P' with slopes m_1, m_2, m_3 to the parabola $y^2 = 4x$. If locus of 'P' with $m_1 m_2 = \alpha$ is a part of the parabola itself then $\alpha = 2$

Reason R : If normals at $(x_1, y_1), (x_2, y_2)$ and (y_3, y_3) are concurrent then $y_1 + y_2 + y_3 = 0$

Key. B

Sol. $\Rightarrow m_1 + m_2 + m_3 = 0 \qquad (1)$

$\Rightarrow m_1m_2 + m_2m_3 + m_3m_1 = \frac{2a-h}{a}$ (2)

$\Rightarrow m_1m_2m_3 = \frac{-k}{a}$ (3)

Where m_1 m_2, m_3 are three values of m which represent three slope of normals which may go through (h, k)

We are also given $m_1m_2 = \alpha$ (4)

We easily eliminate m_1, m_2, m_3 in (1) (2) (3) and (4) to get locus of (h, k) as (a = 1)

$y^2 = \alpha^2 x + \left(\alpha^3 - 2\alpha^2\right)$

Which is same as the given parabola $y^2 = 4ax \Rightarrow \alpha = 2$

Thus (a) is true

The reason R is also true but is not sufficient to deduce $\alpha = 2$

68. Statement 1: The curve $y = -\frac{x^2}{2} + x + 1$ is symmetrical with respect to the line $x = 1$

Statement 2: A parabola is symmetric about its axis

A)Statement I is True, Statement II is True and Statement II is correct explanation of Statement I

B)Statement I is True, Statement II is True but Statement II is not correct explanation of Statement I

C)Statement I is True, Statement II is False

D)Statement I is False, Statement II is True

Key. A

Sol. Statement -2 is true, equation in statement -1 is $(x-1)^2 = -2(y-3/2)$ which is a parabola with axis $x-1 = 0$, using statement-2, statement -1 is also True.

69. Statement 1: The tangents at the extremities of a focal chord of a parabola intersect on its directrix.
Statement 2: The locus of the point of intersection of perpendicular tangents to the parabola is its directrix.

A)Statement I is True, Statement II is True and Statement II is correct explanation of Statement I
B)Statement I is True, Statement II is True but Statement II is not correct explanation of Statement I
C)Statement I is True, Statement II is False
D)Statement I is False, Statement II is True

Key. A
Sol. Statement -2 is true, equations of the perpendicular tangents to the statement-1 is True.
Parabola $y^2 = 4ax$ are $y = mx + a/m, y = m'x + a/m'$ where $mm' = -1$ they intersect at the point for which

$$(m-m')x = \frac{a}{m'} - \frac{a}{m} = \frac{a(m-m')}{mm'} \Rightarrow x = -a$$

Which is the directrix of the parabola.

Since the extremities of a focal chord are $(at^2, 2at)$ and $(at'^2, 2at')$ where $tt' = -1$ and the slopes of the tangents at those points are $1/t$ and $1/t'$

Whose product is-1, the tangents are perpendicular and hence by statement-2 they intersect on the directrix.

70. STATEMENT–1

For any value of θ ($\theta \neq \frac{n\pi}{2}$, $n \in I$) the chord joining the points ($a\tan^2\theta$, $2a\tan\theta$) and ($4a\cot^2\theta$, $-4a\cot\theta$), on the parabola $y^2 = 4ax$ can not subtend a right angles at the vertex of the parabola.

because

STATEMENT–2

Any focal chord of a parabola can not subtend a right angles at the vertex of the parabola.

Key. B

Sol. For the chord to subtend right angles at vertex $t_1t_2 = -4$. Here $t_1t_2 = (\tan\theta)(-2\cot\theta) = -2$. It is also not a focal chord as for that $t_1t_2 = -1$.

71. Statement I: If the normal at t_1, to the parabola $y^2 = 4ax$ cuts the curve again at t_2 then $t_2^2 \geq 8$.

Statement II : Equation to the tangent at 't' on $y^2 = 4ax$ is $yt = x + at^2$,

Key. A

Sol. $t_2 = -t_1 \frac{-2}{t_1} \Rightarrow t_1^2 + t_1t_2 + 2 = 0 \Rightarrow t_2^2 - 4.1.2 \geq o \Rightarrow t_2^2 \geq 8$

72. Statement I: If a & b are lengths of segments of a focal chord of parabola $(a \neq b)$, and 2c is the length of latus rectum, then $a^3 + b^3 > 2c^3$.

Statement II: $AM > GM > HM$.

Key. A

Sol. $\frac{2ab}{a+b} = c \Rightarrow a, c, b,$ all in HP

$\therefore GM > HM \Rightarrow \sqrt{ab} > c$

$\therefore AM > GM \Rightarrow \frac{a^3 + b^3}{2} > \sqrt{a^3b^3} > c^3 \Rightarrow a^3 + b^3 > 2c^3$

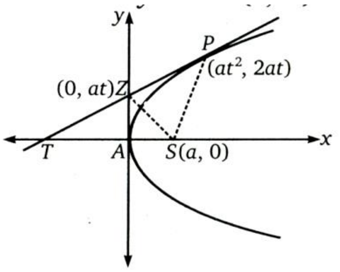
73. Statement I : The locus of centre of the circle described on any focal chord of a parabola $y^2 = 4ax$ as diameter is, $y^2 = 2a(x+a)$.

Statement II: If $(at_1^2, 2at_1)$ & $(at_2^2, 2at_2)$ are the extremities of a focal chord of $y^2 = 4ax$, then, $t_1t_2 = -1$

Key. D

Sol. circle locus is , $y^2 = 2a(x-a)$

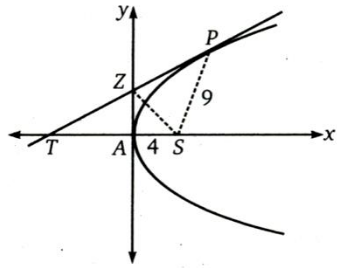
74. Statement–1 : In the given figure, AS = 4, SP = 9,then SZ = 6.



Statement–2 : If SZ be perpendicular to the tangent at a point P of a parabola, then Z lies on the tangent at the vertex and SZ^2 = AS . SP, where A is the vertex of the parabola.

Key. A

Sol. Let $P(at^2, 2at)$ be any point on the parabola $y^2 = 4ax$, then the equation of the tangent at P is $yt = x + at^2$. It cuts y–axis at (0, at).



Clearly SZ perpendicular PT

Now, $SZ = \sqrt{a^2 + a^2t^2} = a\sqrt{1+t^2}$

$SP = \sqrt{\left(at^2 - a\right)^2 + \left(2at - 0\right)^2}$

$\Rightarrow$ SP= $a(t^2 + 1)$ and AS = a

$\therefore$ SZ = $a^2(1 + t^2)$ and AS . SP = $a^2(t^2 + 1)$

Clearly SZ^2 = AS . SP

$\therefore$ From the figure

$\Rightarrow$ $SZ^2 = (4)(9) \Rightarrow SZ = 6$

75. **Statement I :** Through $P(\lambda, \lambda+1)$, no tangent can be drawn to the parabola $y^2 = 4(x+1)$ if $\lambda > 3$

Statement II : $P(\lambda, \lambda+1)$ is an interior point of $y^2 = 4(x+1)$ if $\lambda \in (-1, 3)$

Key. D

Sol. $(\lambda, \lambda+1)$ is an interior point of $y^2 = 4(x+1)$ *if* $(\lambda+1)^2 - 4(\lambda+1) < 0$

$\Leftrightarrow \lambda^2 - 2\lambda - 3 < 0$

$\Leftrightarrow \lambda \in (-1, 3)$

76. **Statement I :** Latus rectum of any parabola is a focal chord of minimum length

Statement II : If SP,SQ are segments of a focal chord of a parabola $y^2 = 4ax (a > 0)$ then SP+SQ≥ 4a

Key. A

Sol. Conceptual

77. Statement –1 : the parametric coordinates of any point on the parabola $y^2 = 4ax$ can be taken as

$$\left(a\sin^2 t, 2a\sin t\right)$$

Statement – 2 : If t is a paratmeter, $\left(a\sin^2 t, 2a\sin t\right)$ point satisfies the equation $y^2 = 4ax$.

Key. D

Sol. It doesn't contain all the points of $y^2 = 4ax$

78. Statement – 1 : Locus of 'z' given by $\left|z-(3+2i)\right| = \left|z\cos\left(\frac{\pi}{4}-\arg z\right)\right|$ represents a parabola

Statement – 2 : If distance of a variable point from a fixed point is equal to the distance from a fixed line, it represents a parabola

Key. C

Sol. If the point lies on the directrix, its pair of straight line

79. Statement-I: The curve $y = \frac{-x^2}{2} + x + 1$ is symmetric about the line $x = 1$, because

Statement-II: A parabola is symmetric about its axis

Key. A

Sol. Conceptual

80. Statement-I: For a parabola latus rectum is the shortest focal choral, because

Statement-II: The length of a focal chord of a parabola inclined at an angle $'\theta'$ with its axis is given by $4a\sec^2\theta$

Key. C

Sol. Length of focal chord of a parabola inclined at an angle $'\theta'$ with its axies is given by $4a\,\mathrm{cosec}^2\theta$

81. Let point A lie on parabola $y^2 = 8x$ and point B lie on circle $(x-4)^2+(y-2)^2 = 1$

STATEMENT 1 : Minimum value of length AB is $\sqrt{8}$.

STATEMENT 2 : For minimum value of length AB normal to the parabola at A should pass through the centre of the circle.

Key. D

Sol. Equation of normal to the parabola $y = mx-4m-2m^3$

it passes through (4, 2) $\Rightarrow m = -1 \Rightarrow$ equation of normal $y = -x+6$

solving $y^2 = 8x$, $y = -x+6 \Rightarrow y = 4, -12$; Point A (2, 4) or (18, –12)

Minimum distance of AB $= \sqrt{(2-4)^2+(4-2)^2} - 1 = \sqrt{8} - 1$

82. STATEMENT 1 : If the normals at the end points of a variable chord PQ of the parabola $y^2 - 4y - 2x = 0$ are perpendicular then the locus of point of intersection of the tangent at P and Q will be $2x + 5 = 0$

STATEMENT 2 : Two perpendicular tangents of a parabola always intersect on its directrix.

Key. A

Sol. The parabola is $(y-2)^2 = 2(x+2)$

The normal of P & Q are perpendicular then the tangents at P and Q will also perpendicular.

Equation of directrix → $x+2+(1/2) = 0$

83. STATEMENT 1 : Length of latusractum of parabola $(4x+3y+1)^2 = 4(3x-4y+3)$ is 4.

STATEMENT 2 : Length of latusractum of parabola $y^2 = 4ax$ is 4a.

Key. D

Sol. $\left(\frac{4x+3y+1}{5}\right)^2 = \frac{4}{5}\left(\frac{3x-4y+3}{5}\right) \Rightarrow$ length of latus rectum = 4/5

84. STATEMENT 1 : The normal at $P(ap^2, 2ap)$ meets the parabola $y^2 = 4ax$ again at

Q(aq^2, 2aq) such that the lines joining the origin to P and Q are at right angle then $p^2 = 2$.

STATEMENT 2 : The normal at (ap^2, 2ap) meets the parabola $y^2 = 4ax$ again at Q(aq^2, 2aq) then q = –p–(2/p).

Key. A

Sol. $q=-p-\frac{2}{p} \Rightarrow$ OP is perpendicular to OQ $\frac{2ap-0}{ap^2-0}.\frac{2aq-0}{aq^2-0}=-1 \Rightarrow pq=-4$

$p\left(-p-\frac{2}{p}\right)=-4 \Rightarrow p^2=2$

85. Let point A lie on parabola $y^2 = 8x$ and point B lie on circle $(x-4)^2+(y-2)^2 = 1$

STATEMENT 1 : Minimum value of length AB is $\sqrt{8}$.

STATEMENT 2 : For minimum value of length AB normal to the parabola at A should pass through the centre of the circle.

Key. D

Sol. Equation of normal to the parabola $y = mx-4m-2m^3$

it passes through (4, 2) ⇒ m = –1 ⇒ equation of normal y = –x+6

solving $y^2 = 8x$, y = –x+6 ⇒ y = 4, –12 ; Point A (2, 4) or (18, –12)

Minimum distance of AB $=\sqrt{(2-4)^2+(4-2)^2}-1=\sqrt{8}-1$

86. STATEMENT 1 : If the normals at the end points of a variable chord PQ of the parabola $y^2-4y-2x = 0$ are perpendicular then the locus of point of intersection of the tangent at P and Q will be 2x + 5 = 0

STATEMENT 2 : Two perpendicular tangents of a parabola always intersect on its directrix.

Key. A

Sol. The parabola is $(y-2)^2 = 2(x+2)$

The normal of P & Q are perpendicular then the tangents at P and Q will also perpendicular.

Equation of directrix → x+2+(1/2) = 0

87. STATEMENT 1 : Length of latusractum of parabola $(4x+3y+1)^2 = 4(3x-4y+3)$ is 4.

STATEMENT 2 : Length of latusractum of parabola $y^2 = 4ax$ is 4a.

Key. D

Sol. $\left(\frac{4x+3y+1}{5}\right)^2=\frac{4}{5}\left(\frac{3x-4y+3}{5}\right) \Rightarrow$ length of latus rectum = 4/5

88. STATEMENT 1 : The normal at P(ap^2, 2ap) meets the parabola $y^2 = 4ax$ again at Q(aq^2, 2aq) such that the lines joining the origin to P and Q are at right angle then $p^2 = 2$.

STATEMENT 2 : The normal at (ap^2, 2ap) meets the parabola $y^2 = 4ax$ again at Q(aq^2, 2aq) then q = –p–(2/p).

Key. A

Sol. $q=-p-\frac{2}{p} \Rightarrow$ OP is perpendicular to OQ $\frac{2ap-0}{ap^2-0}.\frac{2aq-0}{aq^2-0}=-1 \Rightarrow pq=-4$

$p\left(-p-\frac{2}{p}\right)=-4 \Rightarrow p^2=2$

Parabola

Comprehension Type

Paragraph – 1

Let R (h, k) be the middle point of the chord PQ of the parabola $y^2 = 4ax$, then its equation will be

$ky - 2ax + 2ah - k^2 = 0$

The locus of the mid-point of chords of the parabola which

1. Subtend a constant angle α at the vertex is $\left(y^2 - 2ax + 8a^2\right)^2 \tan^2\alpha = \lambda a^2(4ax - y^2)$, where $\lambda =$

(A) 4 (B) 8

(C) 16 (D) 32

Key. (C)

Sol. Let $P(h,k)$ be the mid-point, equation of chord through mid point (h,k)

Equation of chord through mid point is $ky - 2ax + 2ah - k^2 = 0$

Combined equation of OA and OB will be

$$y^2 - 4ax\frac{(ky - 2ax)}{k^2 - 2ah} = 0$$

$$\tan\alpha = \frac{4a\sqrt{4ah - k^2}}{k^2 - 2ah + 8a^2}$$

$$(k^2 - 2h + 8a^2)^2 \tan^2\alpha = 16a^2(4ah - k^2)$$

$$(y^2 - 2ax + 8a^2)^2 \tan^2\alpha = 16a^2(4ax - y^2)$$

2. Are such that the focal distances of their extremities are in the ratio 2: 1 is

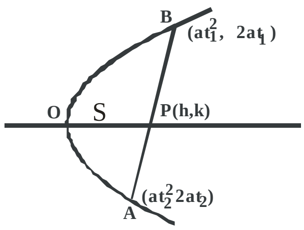
$9(y^2 - 2ax)^2 = \lambda a^2 (2x - a)(4x + a)$ where $\lambda =$

(A) 4 (B) 8

(C) 16 (D) 12

Key. (A)

Sol.



$$t_1^2 + t_2^2 = \frac{2h}{a} \qquad \ldots.. \qquad (1)$$

$$t_1 + t_2 = \frac{k}{a}$$

$$\frac{SB}{SA} = \frac{(t_1^2+1)}{t_2^2+1} = \frac{2}{1} \quad \ldots.. \quad (2)$$

Solving all the equations, we get

$$9\left(k^2 - 2ah\right) = 4a^2(2h-a)(4h+a)$$

Paragraph – 2

The normal at any point (x_1, y_1) of curve is a line perpendicular to tangent at the point (x_1, y_1). In case of parabola $y^2 = 4ax$ the equation of normal is $y = mx - 2am - am^3$ (m is slope of normal). In case of rectangular hyperbola $x y = c^2$ the equation of normal at (ct, c/t) is $xt^3 - yt - ct^4 + c = 0$. The shortest distance between any two curves always exist along the common normal.

3. If normal at (5, 3) of rectangular hyperbola $xy - y - 2x - 2 = 0$ intersect it again at a point

   (A) (–1, 0) (B) (–1, 1) (C) (0, –2) (D) (3/4, -14)

Key. (D)

Sol. $xy - y - 2x - 2 = 0$

$(x - 1)(y - 2) = 4$

$XY = 4$

Normal at (ct, c/t) intersect it again at $(ct', c/t')$ then $t' = -1/t^3$

$2t = 4$

$t = 2$

$(X', Y') \equiv \left(-\frac{1}{4}, -16\right)$

$(x', y') \equiv (3/4, -14)$

4. The shortest distance between the parabola $2y^2 = 2x - 1$, $2x^2 = 2y - 1$ is

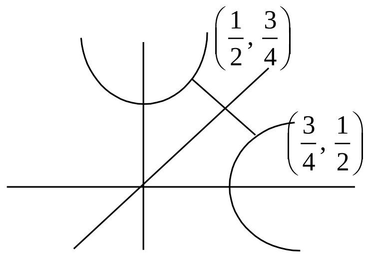
   (A) $2\sqrt{2}$ (B) $\frac{1}{2\sqrt{2}}$ (C) 4 (D) $\sqrt{\frac{36}{5}}$

Key. (B)

Sol. $2y\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{2y} = 1 \Rightarrow y = \frac{1}{2}$

$d = \sqrt{\frac{1}{16} + \frac{1}{16}} = \frac{1}{2\sqrt{2}}$



5. Number of normals drawn from $\left(\frac{7}{6},4\right)$ to parabola $y^2 = 2x - 1$ is

(A) 1 (B) 2 (C) 3 (D) 4

Key. (A)

Sol. $y^2 = 2(x - \frac{1}{2})$

$Y^2 = 2X$

For 3 normals X > 1

x > 3/2

$\Rightarrow$ only one normal can be drawn.

Paragraph – 3

Conic posses enormous properties which can be proved by taking their standard forms. Unlike circle these properties rarely follow by geometrical considerations. Most of the properties of conic are proved analytically. For example, the properties of a parabola can be proved by taking its standard equation $y^2 = 4ax$ and a point $\left(at^2, 2at\right)$ on it

6. If the tangent and normal at any point 'P' on the parabola whose focus is S, meets its axis in T and G respectively, then

a) PG = GT
b) S is mid-point of T and G
c) ST = 2SG
d) none of these

Key. B

7. The angle between the tangents drawn at the extremeties of a focal chord must be

a) 30° b) 60° c) 90° d) 120°

Key. C

8. If the tangent at any point 'p' meets the directrix at K, then $\angle KSP$ must be

a) 30° b) 60° c) 90° d) None of these

Key. C

Sol. 6.(b) The equation of tangents at $P\left(at^2, 2at\right)$ of the parabola $y^2 = 4ax$ is $ty = x + at^2$

On putting y = 0 we get x = $-at^2$

$\Rightarrow OT = at^2$ (leaving sign)

$\Rightarrow ST = a + at^2$

Again equation of normal at P is y + tx = 2at + at^3

On putting y = 0, we get x = 2at + at^2

Thus S is the Mid point of T and G hence (b) is true and (a)(c) and (d) are ruled out.

7. If $\left(at_1^{\,2}, 2at_1\right)$, $\left(at_2^2, 2at_2\right)$ are extrematics of focal chord , then $t_1 t_2$ = -1

Now slopes of tangents at these points are $1/t_1$ and $1/t_2$

$\Rightarrow$ product of slopes = $\dfrac{1}{t_1\ t_2} = -1$

$\Rightarrow$ Tangents are perpendicular

8. Let 'P' be $\left(at^2, 2at\right)$ then equation of tangent at P is ty = x + at^2

$\Rightarrow$ K is $\left(-a, \dfrac{a+at^2}{t}\right)$

$\Rightarrow$ Slope KS = $\dfrac{\dfrac{at\ at^2}{t}}{-a\ -a} = \dfrac{1-t_2}{2t}$

and slope SP = $\dfrac{2at\ -0}{at^2-a} = \dfrac{2t}{t^2-1}$

Product of slopes = -1

$\Rightarrow \angle KSP = 90^0$

Paragraph – 4

We know that general equation of second degree, ie $ax^2+2hy+by^2+2gx+2fy+C=0$ represents conic sections if $\Delta \neq 0$ Where $\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$. As a special case this represents a parabola of $\Delta \neq 0$ and $h^2 = ab$. Alternatively a parabola is defined as the locus of a point which is equidistant from a point and from a line. They are respectively called focus and directrix of the parabola.

9. The equation $x^2+4xy+4y^2+4x+4y+\lambda=0$ will represent a parabola

a) for all values of λ

b) for all except for one value of λ

c) for no values of λ

d) None of these

Key. A

10. The equation $x^2+2xy+y^2+2x+\lambda=0$ will represent a parabola λ

a) for all values of λ b) for all except for one value of for λ

c) for no values of λ d) None of these

Key. A

11. The equation $\lambda x^2+4xy+y^2+\lambda x+3y+2=0$ represents a parabola if λ is

a) -4 b) 4 c) 0 d) none of these

Key. B

Sol. 9. (a) $\Delta=\begin{vmatrix}1&2&2\\2&4&2\\2&2&\lambda\end{vmatrix}=-4\neq 0\text{for all }\lambda$

The second degree terms are forming a perfect square

$\Rightarrow$ Given equation represents a parabola for all λ

11.(a)) $\Delta=\begin{vmatrix}1&1&1\\1&1&0\\1&0&\lambda\end{vmatrix}=-1$ for all λ also $h^2=ab$

$\Rightarrow$ Given equation represents a parabola for all λ

$\Rightarrow$ (a) is correct

Paragraph – 5

Normals at three points P, Q, R on the parabola $y^2=4ax$ meet at (α,β), then

12. The centroid of the Triangle PQR must be

a) $\left(\frac{\alpha-2a}{3},0\right)$ b) $\left(\frac{2\alpha-4a}{3},0\right)$ c) $\left(\frac{\alpha}{3},\frac{\beta}{3}\right)$ d) None of these

Key. B

13. The orthocenter of the Triangle PQR must be at

a) $\left(\alpha+6a,\frac{\beta}{2}\right)$ b) $\left(\alpha-3a,\frac{\beta}{2}\right)$ c) $\left(\alpha-6a,\frac{-1}{2}\beta\right)$ d) None of these

Key. C

14. The circum center of ΔPQR must be

a) $\left(\frac{\alpha+2a}{2},-\frac{\beta}{4}\right)$ b) $\left(\frac{\alpha+2a}{4},\frac{\beta}{4}\right)$ c) $\left(\frac{\alpha}{4},\frac{\beta}{4}\right)$ d) None of these

Key. A

Sol. 12. $\Rightarrow t_1+t_2+t_3=0,\qquad t_1t_2+t_2t_3+t_3t_1=\frac{2a-\alpha}{a}$

$t_1+t_2+t_3=\frac{\beta}{a}$

If P, Q, R are $\left(at_1^2,\,2at_1\right),\left(at_2^2,\,2at_2\right)$ and $\left(at_3^2,\,2at_3\right)$

Then centroid of ΔPQR

$=\left(\frac{at_1^2+at_2^2+at_3^2}{3},\frac{2at_1+2at_2+2at_3}{3}\right)$

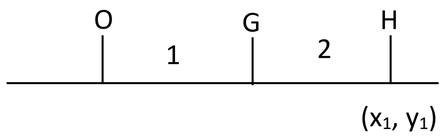
$=\left(\frac{2\alpha-4a}{3},O\right)$

$\Rightarrow$ (b) is correct

13. (c)The orthocenter of ΔPQR can be easily found as $\left(-4a,-a\,\left(t_1t_2+t_2t_3+t_3t_1\right)\right)$

$=\left(\alpha-6a,\ \beta/2\right)\Rightarrow$ (c) is correct

14. Using $\frac{OG}{GH}=1/2$ and last question



$\frac{2\alpha-4a}{3}=\frac{1+(\alpha-6a)+2\times x_1}{3}$

$O=\frac{1\left(-\beta/2\right)+2y_1}{3}$

$\Rightarrow$ Circum center $\left(x_1,y_1\right)$ is $\left(\frac{\alpha+2a}{4},\ \beta/4\right)$

$\Rightarrow$ (a) is correct

Paragraph – 6

Normals are drawn at P,Q,R lying on $y^2=4x$ which intersect at $(3,0)$ then

15. Area of $\Delta^{le}PQR$ is

A. -2 sq. units B. 1 Sq. unit C. $\frac{1}{2}$ Sq. unit D. 4 Sq. Units

Key. A

16. Radius of circum circle of $\Delta^{le}PQR$ is

A. $\dfrac{1}{2}$ B. $\dfrac{5}{2}$ C. $\dfrac{3}{2}$ D. 2

Key. B

17. Circum centre of $\Delta^{le} PQR$ is

A. $\left(\dfrac{1}{2},0\right)$ B. $\left(\dfrac{3}{2},0\right)$ C. $\left(\dfrac{5}{2},0\right)$ D. $(0,0)$

Key. C

Sol. Equation of normals to $y^2 = 4x$

$y + xt = 2at + at^3$

$y + xt = 2t + t^3$

It is drawn from (3,0)

$3t = 2t + t^3$

$\Rightarrow t = 0 \quad 1 = t^2 \Rightarrow t = +1, t = -1$

$\therefore P(0,0) \quad Q(1,2) \quad R(1,-2)$

15. Area of triangle PQR = $\dfrac{1}{2}\begin{vmatrix} 1 & 1 \\ 2 & -2 \end{vmatrix} = \dfrac{4}{2} = 2$ sq. units

16. Ans. B

17. $\left(\dfrac{5}{2},0\right)$

Paragraph – 7

$ABCD$ is a square with $A = (-4,0), B = (4,0)$ and other vertices of the square lie above the x-axis. Let *O* be the origin and O^1 be the mid point of CD. A rectangular hyperbola passes through the points C, D, O and its transverse axis is along the straight line OO^1.

18. The centre of the hyperbola is

A) $(0,4)$ B) $(0,3)$ C) $(0,5)$ D) $(0,2)$

Key. B

19. One of the asymptotes of the hyperbola is

A) $2x+y=3$ B) $y=2x+3$ C) $y=x+3$ D) $y=4-x$

Key. C

20. The area of the larger region bounded by the hyperbola and the square is

A) $20+8\log 3$ B) $44-9\log 3$ C) $44+8\log 3$ D) $44+9\log 3$

Key. D

Sol. 18 – 20

The equation of Hyperbola is $(y-3)^2-x^2=9$

Paragraph – 8

Consider the conic defined by $x^2+y^2=(3x+4y+10)^2$.

21. If (α,β) is the centre of the conic then $4\alpha+3\beta=$

A) –8 B) –10 C) –6 D) –9

Key. B

22. If (p,q) is a vertex of the conic then $2p-q=$

A) –1 B) 1 C) –3 D) 2

Key. A

23. The number of points through which a pair of real perpendicular tangents can be drawn to the conic is

A) infinite B) 1 C) 0 D) 4

Key. C

Sol. 21 – 23

The given equation can be expressed as $\sqrt{x^2+y^2}=5\frac{|3x+4y+10|}{5}$

Hence it is Hyperbola with eccentricity 5.
Focus is (0, 0)
Directrix is $3x+4y+10=0$
And hence the axis is $4x-3y=0$

Paragraph – 9

Consider the parabola $y^2=4x$. Let $A=(-1,0)$ and $B=(0,1)$. *F* is the focus of the parabola. Answer the following questions

24. If $P(\alpha,\beta)$ is a point on the parabola such that $\left||PA|-|PB|\right|$ is maximum then $\alpha+\beta=$

A) 4 B) $5\sqrt{2}$ C) 3 D) $4\sqrt{3}$

Key. C

25. If $P(\alpha, \beta)$ is a point on the parabola such that $\left\| PA \right| - \left| PB \right\|$ is minimum then a value of $2\alpha + \beta$ is

A) 4 B) 3 C) $4\sqrt{2}$ D) $2\sqrt{3}$

Key. A

26. If $L = (4,3)$ and $Q(a,b)$ is a point on the parabola such that $|FQ| + |QL|$ is least then $a + b =$

A) 6 B) 19/2 C) 20/3 D) 21/4

Key. D

Sol. 24 – 26:

24. $|PA - PB|$ is max when P, A, B are collinear and P divides AB externally

Equation of AB is $-x + y = 1$. i.e., $y = x + 1$

$(x+1)^2 = 4x \Rightarrow x = 1$

$\therefore \overrightarrow{AB}$ intersect parabola at (1, 2)

25. Minimum value of $|PA - PB| = 0$. i.e., P lies on the perpendicular bisector of AB which is $y = -x$.

This line meets the parabola at (0, 0), $(4, -4)$.

26. (4, 3) lies inside the parabola $y^2 = 4x$

$|FQ| + |QL|$ is least when LQ is a diameter of the parabola.

Paragraph – 10

The length of latusrectum of a parabola which does not meet the X-axis is 1. The parabola passes through the point (0, 3) and it is symmetric with respect to the line $x = 1$. B is the point of intersection of the line $y = 11$ and parabola and the point B lies in first quadrant.

Then answer the following questions.

27. Sum of the co-ordinates of focus of the parabola is

A) 11/2 B) 13/4 C) 9/2 D) 7/4

Key. B

Sol. Conceptual

28. Magnitude of cross product of vectors $\overrightarrow{OA}, \overrightarrow{OB}$ is

A) 3/2 B) 4 C) 3 D) 5/2

Key. C

Sol. Conceptual

29. The area bounded by the parabola and the line $y = 3$ is

A) 4/3 B) 5/3 C) 7/3 D) 28/3

Key. A

Sol. Conceptual

Paragraph – 11

Consider the circles $x^2+y^2=a^2$ and $x^2+y^2=b^2$ where $b>a>0$. Let $A(-a,0);B(a,0)$. A parabola passes through the points A, B and its directrix is a tangent to $x^2+y^2=b^2$. If the locus of focus of the parabola is a conic then

30. The eccentricity of the conic is

A) $2a/b$ B) b/a C) a/b D) 1

Key. C

31. The foci of the conic are

A) $(\pm 2a,0)$ B) $(0,\pm a)$ C) $(\pm a,2a)$ D) $(\pm a,0)$

Key. D

32. Area of triangle formed by a latusrectum and the lines joining the end points of the latusrectum and the centre of the conic is

A) $\frac{a}{b}\left(b^2-a^2\right)$ B) $2ab$ C) $ab/2$ D) $4ab/3$

Key. A

Sol. 30 – 32:

$x^2+y^2=a^2;\ x^2+y^2=b^2;\ b>a>0,\ A=(-a,0);\ \ B=(a,0)$

Let (h,k) be a point on the locus. Any tangent to circle $x^2+y^2=b^2$ is $x\cos\theta+y\sin\theta=b$

$\therefore$ Equation of parabola is $\sqrt{(x-h)^2+(y-K)^2}=|x\cos\theta+y\sin\theta-b|$

i.e., $(x-h)^2+(y-K)^2=(x\cos\theta+y\sin\theta-b)^2$

The points $(\pm a,0)$ satisfy this equation

$\therefore (a-h)^2+K^2=(a\cos\theta-b)^2$ --- (1)

$(a+h)^2+K^2=(a\cos\theta+b)^2$ ---- (2)

(2) – (1) $\Rightarrow h=b\cos\theta$

$\therefore$ Required locus is $(a+x)^2+y^2=\left(\frac{ax}{b}+b\right)^2$

i.e., $\frac{x^2}{b^2}+\frac{y^2}{b^2-a^2}=1$ which is an ellipse.

Paragraph – 12

A quadratic polynomial y = f(x) with absolute term '3' neither touches nor intersect abscissa axis and is symmetric about the line x = 1. The coefficient of leading term of the polynomial is unity. A point $A(x_1,y_1)$ with abscissa x_1 = 1 and a point $B(x_2,y_2)$ with ordinate y_2 = 11 are given in a cartesian rectangular system of coordinates oxy in the first quadrant on the curve y = f(x) where 'O' is origin.

Now answer the following questions:

33. Vertex of the quadratic polynomial
a) (1, 1) b) (2, 3) c) (1, 2) d) (0, 0)

Key. C

34. The graph of y = f(x) represents a parabola whose focus is
a) $\left(1,\frac{7}{4}\right)$ b) $\left(1,\frac{5}{4}\right)$ c) $\left(1,\frac{5}{2}\right)$ d) $\left(1,\frac{9}{4}\right)$

Key. D

35. The scalar product of the vectors $\overline{OA}$ and $\overline{OB}$ is
a) -18 b) 26 c) 22 d)-22

Key. B

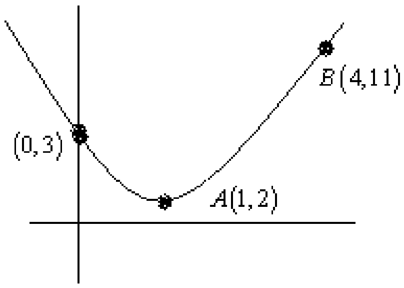
Sol. 33Q, 34Q and 35Q

Let $y=ax^2+bx+c$ where c = 3 and a = 1 $\Rightarrow$ curve is completely above x-axis

$\therefore f(x)=y=x^2+bx+3 \Rightarrow$ Line of symmetry being x = 1

$\therefore$ minima occurs at x = 1 $\Rightarrow \therefore f'(1)=0 \Rightarrow 2x+b=0$ at x = 1 $\Rightarrow$ b = -2

Hence $f(x)=x^2-2x+3$



Paragraph – 13

Passage – I

Let S be a given fixed point (focus); 'l' is given fixed line. P is a movable point such that $\frac{SP}{PM}=e$ where e = 1 then locus of P is called a parabola

36. A Normal drawn at P cuts the parabola $y^2=4ax$ at Q and PQ subtends a right angle at the focus of the parabola then its length is
(a) $a\sqrt{5}$ (b) $5a\sqrt{5}$ (c) $6a\sqrt{3}$ (d) $7a\sqrt{5}$

Key. B

Sol. $\frac{2t_1}{(t_1^2-1)}\cdot\frac{2t_2}{(t_2^2-1)}=-1$

$t_2=-t_1-\frac{2}{t_1}$

37. If tangents at A and B on $y^2=4ax$ intersect on $x+16a=0$ then AB always pass through

(a) $(-16a,0)$ (b) $(16a,0)$ (c) $(a,0)$ (d) $(-a,0)$

Key. B

Sol. Point of intersection of tangents at $A(t_1);B(t_2)$ is $(at_1t_2,a(t_1+t_2))=(-16a,k)$

$t_1t_2=-16;t_2=\frac{-16}{t_1}$

Equation of AB is

$y(t_1+t_2)=2x+2at_1t_2$

$y(t_1-\frac{16}{t_1})=(2x-32a)$

$(2x-32a)-(t_1-\frac{16}{t_1})y=0$

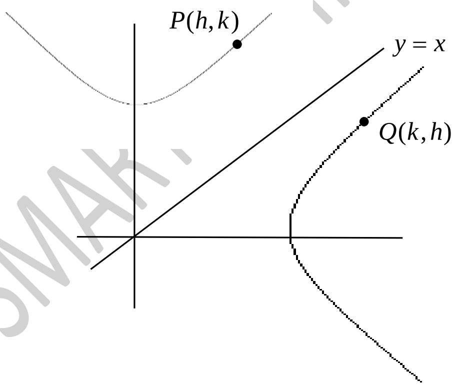
38. The equation of the smallest circle touching both the parabolas $y^2=4(x-2)$ and $x^2=4(y-2)$ is

(a) $x^2+y^2-10x-10y+6=0$ (b) $x^2+y^2-5x-5y+12=0$

(c) $x^2+y^2-2x-2y-1=0$ (d) $x^2+y^2-x-y-3=0$

Key. B

Sol.



Centre of circle is Mid point of $(3,2)(2,3)$

Paragraph – 14

If $\bar{a},\bar{b},\bar{c}$ are any three vectors then

$$\bar{a}\times(\bar{b}\times\bar{c})=(\bar{a}\cdot\bar{c})\bar{b}-(\bar{a}\cdot\bar{b})\bar{c};(\bar{a}\times\bar{b})\cdot(\bar{c}\times\bar{d})=\begin{vmatrix}\bar{a}\cdot\bar{c} & \bar{b}\cdot\bar{c}\\ \bar{a}\cdot\bar{d} & \bar{b}\cdot\bar{d}\end{vmatrix}$$

39. The value of 'a' so that the volume of the parallelepiped formed by vectors $i+aj+k;\ j+ak;ai+k$ becomes minimum is

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{-1}{\sqrt{3}}$ (c) 1 (d) $\pm\frac{1}{\sqrt{3}}$

Key. A

Sol. $V=\begin{vmatrix}1 & a & 1\\ 0 & 1 & a\\ a & 0 & 1\end{vmatrix}=1+a^3-a=0;V\ is\ Minimum;\ \frac{dv}{da}=0$

$a=\pm\frac{1}{\sqrt{3}}$

40. Let $\bar{a}=2i+3j+4k;\bar{b}=i+5j+2k;\bar{c}=3i+15j+6k$ then the value of $\begin{vmatrix}\bar{a}\cdot\bar{a} & \bar{a}\cdot\bar{b} & \bar{a}\cdot\bar{c}\\ \bar{b}\cdot\bar{a} & \bar{b}\cdot\bar{b} & \bar{b}\cdot\bar{c}\\ \bar{c}\cdot\bar{a} & \bar{c}\cdot\bar{b} & \bar{c}\cdot\bar{c}\end{vmatrix}$ is

(a) 429 (b) 0 (c) 1 (d) – 5

Key. B

Sol. $\begin{vmatrix}\bar{a}\cdot\bar{a} & \bar{a}\cdot\bar{b} & \bar{a}\cdot\bar{c}\\ \bar{b}\cdot\bar{a} & \bar{b}\cdot\bar{b} & \bar{b}\cdot\bar{c}\\ \bar{c}\cdot\bar{a} & \bar{c}\cdot\bar{b} & \bar{c}\cdot\bar{c}\end{vmatrix}=[\bar{a}\,\bar{b}\,\bar{c}]^2$

41. If $\bar{a}=i+j+k;\bar{b}=4i+3j+4k;\bar{c}=i+\alpha j+\beta k$ are linearly dependent vectors; $|\bar{c}|=\sqrt{3}$ then

(a) $\beta=-1;\alpha=1$ (b) $\alpha=1;\beta=\pm1$ (c) $\alpha=-1;\beta=\pm1$ (d) $\alpha=\pm1;\beta=1$

Key. D

Sol. $\begin{vmatrix}1 & 1 & 1\\ 4 & 3 & 4\\ 1 & \alpha & \beta\end{vmatrix}=0;\sqrt{1+\alpha^2+\beta^2}=\sqrt{3}$

$\alpha^2+\beta^2=2$

$1-\alpha(0)+\beta(-1)=0$

$\beta=1$

Paragraph – 15

If $\alpha, \beta, \gamma, \delta$ are eccentric angles of 4 – points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ the normals at which are concurrent then

42. $\alpha+\beta+\lambda+\delta$ =

A. $2n\pi, n\in z$

B. $(2n+1)\frac{\pi}{2}, n\in z$

C. $(2n+1)\pi, n\in z$

D. $(2n+1)\frac{\pi}{4}, n\in z$

Key. C

43. $\cos(\alpha+\beta)+\cos(\alpha+\lambda)+\cos(\alpha+\delta)+\cos(\beta+\gamma)+\cos(\beta+\delta)+\cos(\lambda+\delta)=$

A. 6 B. 3 C. 0 D. 1

Key. C

44. $\sin(\alpha+\beta)+\sin(\beta+\lambda)+\sin(\lambda+\delta)=$

A. O B. 1 C. -1 D. 2

Key. A

Sol.

Let Z=cisθ $\frac{1}{Z}=\cos\theta-i\sin\theta$

$$2\cos\theta=Z+\frac{1}{Z}, \quad \cos\theta=\frac{Z^2+1}{2Z}$$

$$\sin\theta=\frac{Z^2+1}{2iZ}$$

Equation of normal is $\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$

It is drawn from (x_1, y_1).

$$\frac{ax_1}{\cos\theta}-\frac{by_1}{\sin\theta}=a^2-b^2$$

$$\frac{ax_1}{\left(\frac{Z^2+1}{2t}\right)}-\frac{by_1}{\frac{Z^2-1}{2\,iZ}}=a^2-b^2$$

$$(a^2-b^2)Z^4-2(ax_1-iby_1)Z^3+2(ax_1+iby_1)Z-(a^2-b^2)=0 \to (1)$$

42. Roots are Z_1, Z_2, Z_3, Z_4

$$Z_1 Z_2 Z_3 Z_4 = -1 \qquad cis\alpha.cis\beta.cis\gamma.cis\delta = -1$$

$$cis(\alpha+\beta+\gamma+\delta) = -1$$

$$\cos(\alpha+\beta+\gamma+\delta) = -1, \quad \sin(\alpha+\beta+\gamma+\delta) = 0$$

$$\alpha+\beta+\gamma+\delta = (2n+1)\pi$$

43. $\sum Z_1 Z_2 = 0$

$$\sum cis\alpha.cis\beta = 0$$

$$\sum cis(\alpha+\beta) = 0$$

$$\cos(\alpha+\beta)+\cos(\alpha+\gamma)+\cos(\alpha+\delta)+\cos(\beta+\gamma)+\cos(\beta+\delta)+\cos(\gamma+\delta) = 0$$

44. Ily, $\sin(\alpha+\beta)+\sin(\alpha+\gamma)+\sin(\alpha+\delta)+\sin(\beta+\gamma)+\sin(\beta+\delta)+\sin(\gamma+\delta) = 0$

$$\sin(\alpha+\beta) = \sin(\gamma+\delta)$$

$$\sin(\beta+\gamma) = \sin(\alpha+\delta)$$

$$\sin(\gamma+\alpha) = \sin(\beta+\delta)$$

$$2(\sin(\alpha+\beta)+\sin(\beta+\gamma)+\sin(\gamma+\delta)) = 0$$

$$\sin(\alpha+\beta)+\sin(\beta+\gamma)+\sin(\gamma+\delta) = 0$$

Paragraph – 16

Normals are drawn at P,Q,R lying on $y^2 = 4x$ which intersect at $(3,0)$ then

45. Area of $\Delta^{le} PQR$ is

A. -2 sq. units

B. 1 Sq. unit

C. $\frac{1}{2}$ Sq. unit

D. 4 Sq. units

Key. A

46. Radius of circum circle of $\Delta^{le} PQR$ is

A. $\frac{1}{2}$ B. $\frac{5}{2}$ C. $\frac{3}{2}$ D. 2

Key. B

47. Circum centre of $\Delta^{le} PQR$ is

A. $\left(\frac{1}{2},0\right)$ B. $\left(\frac{3}{2},0\right)$ C. $\left(\frac{5}{2},0\right)$ D. $(0,0)$

Key. C

Sol.

Equation of normals to $y^2 = 4x$

$$y + xt = 2at + at^3$$

$$y + xt = 2t + t^3$$

It is drawn from (3,0)

$$3t = 2t + t^3$$

$$\Rightarrow t = 0 \quad 1 = t^2 \Rightarrow t = +1,\ t = -1$$

$$\therefore P(0,0) \quad Q(1,2) \quad R(1,-2)$$

45. Area of triangle PQR = $\frac{1}{2}\begin{vmatrix} 1 & 1 \\ 2 & -2 \end{vmatrix} = \frac{4}{2} = 2$ sq. units

46. Ans. B

47. $\left(\frac{5}{2},0\right)$

Paragraph – 17

A parabola is drawn through two given points A(1,0) and B(-1,0) such that its directrix always touches the circle $x^2 + y^2 = 4.$ Then

48. The equation of directrix is of the form

a) $x\cos\alpha + y\sin\alpha = 1$ b) $x\cos\alpha + y\sin\alpha = 2$ c) $x\cos\alpha + y\sin\alpha = 3$ d) $x\tan\alpha + y\sec\alpha = 2$

Key. B

49. The locus of focus of the parabola is

a) $\frac{x^2}{4}+\frac{y^2}{3}=1$ b) $\frac{x^2}{4}+\frac{y^2}{5}=1$ c) $\frac{x^2}{3}+\frac{y^2}{4}=1$ d) $\frac{x^2}{5}+\frac{y^2}{4}=1$

Key. A

50. The maximum possible length of semi latus rectum is

a) $2+\sqrt{3}$ b) $3+\sqrt{3}$ c) $4+\sqrt{3}$ d) $1+\sqrt{3}$

Key. A

Sol. 48 TO 50

Any point on circle $x^2+y^2=4$ is $(2\cos\alpha, 2\sin\alpha)$

$\therefore$ equation of directrix is $x(\cos\alpha)+y(\sin\alpha)-2=0.$

Let focus be (x_1, y_1) .Then as A(1,0), B(-1,0) lie on parabola we must have

$$\left.\begin{array}{l}(x_1-1)^2+y_1^2=(\cos\alpha-2)^2\\(x_1+1)^2+y_1^2=(\cos\alpha+2)^2\end{array}\right\}\Rightarrow x_1=2\cos\alpha \ ,.\ y_1=\pm\sqrt{3}\sin\alpha$$

$\therefore$ locus of focus is $\frac{x^2}{4}+\frac{y^2}{3}=1$ and focus is of the form $(2\cos\alpha, \pm\sqrt{3}\sin\alpha)$.

$\therefore$ length of semi latus rectum of parabola = $\perp^r$ distance from focus to directrix

$\left|2\pm\sqrt{3}\right|\sin^2\alpha$

Hence maximum possible length = $2+\sqrt{3}$

Paragraph – 18

A point $P(x, y)$ in a plane is called lattice point if $x, y\in Z$ and a rational point if $x, y\in Q.$ Every lattice point is then a rational point .

Answer the following

51. The number of lattice points inside the circle $x^2+y^2=16$ is

a) 16 b) 45 c) 28 d) 36

Key. B

52. A rational point on $x^2+y^2=1$ is of the form

a) $\left(\frac{m-n}{m+n}, \frac{2\sqrt{mn}}{m+n}\right), m, n\in Z, m+n\neq 0$ b) $\left(\frac{m^2-n^2}{m^2+n^2}, \frac{2mn}{m^2+n^2}\right), m, n\in Q, m^2+n^2\neq 0$

c) $\left(\frac{m}{m+n},\frac{n}{m+n}\right), m,n\in Q, m+n\neq 0$ d) $\left(\frac{2\sqrt{mn}}{m+n},\frac{m-n}{m+n}\right), m,n\in Z, m+n\neq 0$

Key. B

53. For a circle whose centre is not a rational point , maximum number of rational points on it is

a) 1 b) 2 c) 3 d) 4

Key. B

Sol. 51 to 53

For $x^2+y^2=16$, a point (x,y) is internal if $-4<x<4, -4<y<4$ *and* $x^2+y^2-16<0$

$x=0 \Rightarrow y=-3,-2,-1,0,1,2,3 \to 7$

$x=\pm1 \Rightarrow y=-3,-2,-1,0,1,2,3 \to 14$

$x=\pm2 \Rightarrow y=-3,-2,-1,0,1,2,3 \to 14$

$x=\pm3 \Rightarrow y=-2,-1,0,1,2 \to 10$

Total=45

$x=\frac{m^2-n^2}{m^2+n^2}, y=\frac{2mn}{m^2+n^2} \Rightarrow x^2+y^2=1.$

$As\ m,n\in Q$, $\left(\frac{m^2-n^2}{m^2+n^2},\frac{2mn}{m^2+n^2}\right)$ is a rational point, others are not.

Paragraph – 19

A curve $y=f(x)$ passes through (2,0) and slope of tangent at any point $P(x,y)$ on the curve is $\frac{(x+1)^2+y-3}{x+1}$, then

54. The curve is

a) a parabola b)a circle c) an ellipse d) a hyperbola

Key. A

55. Area bounded between $y=|f(x)|$, x-axis and $|x|=3$

a) 20 b) 21 c) $\frac{62}{3}$ d) $\frac{52}{3}$

Key. C

56. The number of points at which $y=x|f(x)|$ is not differentiable is

a) 1 b) 2 c) 0 d) 3

Key. A

Sol. 54 to 56

Given $\frac{dy}{dx}-\frac{y}{x+1}=x+1-\frac{3}{x+1} \Rightarrow y\left(\frac{1}{x+1}\right)=\int\left(1-\frac{3}{(x+1)^2}\right)dx$

$\Rightarrow y=(x+1)(x+c)+3$ *But* $(2,0)$ lies on this curve

$\therefore$ c=-3 .Hence curve is $y = x^2 - 2x$, a parabola

Area bounded by $y = |x^2 - 2x|,\ x - axis, |x| = 3\ is$

$$= \int_{-3}^{0} (x^2 - 2x)dx + \int_{0}^{2} (2x - x^2)dx + \int_{2}^{3} (x^2 - 2x)dx = 62/3.$$

Paragraph – 20

Let P, Q, R be three points on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and let P', Q', R' be their corresponding points on it's auxiliary circle, then

57. The maximum area of the triangle PQR is

a) $\frac{3\sqrt{3}}{4}ab$ b) $\frac{3\sqrt{3}}{2}ab$ c) $\frac{\sqrt{3}}{4}ab$ d) πab

Key. A

Sol. Let P = (a cosα , b sin α) P[1] = (a cos α , a sin α)

P = (a cos β , b sin β) Q[1] = (a cos β , a sin β)

R= (a cos γ , b sin γ) R[1] = (a cosγ ,a sin γ).

Area of ΔPQR is 2ab $\sin\frac{\alpha-\beta}{2}\sin\frac{\beta-\gamma}{2}\sin\frac{\gamma-\alpha}{2}$ its max value is 2ab

$$\left(\frac{\sqrt{3}}{2}\right)\left(\sqrt{3}/2\right)\left(\sqrt{3}/2\right) = \frac{3\sqrt{3}ab}{4}$$

58. $\frac{Area\,of\ \Delta PQR}{Area\,of\ \Delta P'Q'R'} =$

a) $\frac{a}{b}$ b) $\frac{b}{a}$

c) $\frac{1}{2}$ d) depends on points taken

Key. B

Sol. $$\frac{\text{Area of }\Delta PQR}{\text{Area of }\Delta P^1Q^1R^1} = \frac{\frac{1}{2}ab\begin{vmatrix}\cos\alpha-\cos\gamma & \sin\alpha-\sin\gamma\\ \cos\alpha-\cos\beta & \sin\alpha-\sin\beta\end{vmatrix}}{\frac{1}{2}a^2\begin{vmatrix}\cos\alpha-\cos\gamma & \sin\alpha-\sin\gamma\\ \cos\alpha-\cos\beta & \sin\alpha-\sin\beta\end{vmatrix}} = \frac{b}{a}$$

59. When the area of triangle PQR is maximum, the centroid of triangle $P'Q'R'$ lies at

a) one focus b) one vertex c) centre d) on one directrix

Key. C

Sol. Area of ΔPQR is max when $\alpha - \beta = \beta - \gamma = \gamma - d = 120^0$ is $\Delta P^1Q^1R^1$ is equilateral hence its centroid is (0,0) centre of the ellipse

Paragraph – 21

Let C : $y = x^2 - 3$, D: $y = kx^2$, L_1 : $x = a$, L_2 : $x = 1$, $(a \neq 0)$

60. If the parabolas C and D intersect at a point A on the line L_1, then the tangent line L at A to the parabola D is

a) $2\left(a^2 - 3\right)x - ay + a^3 - 3a = 0$
b) $2\left(a^2 - 3\right)x - ay + a^3 + 3a = 0$
c) $\left(a^2 - 3\right)x - 2ay - 2a^3 + 6a = 0$
d) $2\left(a^2 - 3\right)x - ay - a^3 + 3a = 0$

Key. D

Sol. $A = \left(a, a^2 - 3\right)$ Equation of tangent L is $S_1 = 0$ is $2(a^2-3)x - ay - a^3 + 3a = 0$

61. If the line L meets the parabola C at a point B on the line :L_2, other than A then '*a*' is equal to

a) – 3 b) – 2 c) 2 d) 30

Key. B

Sol. The line L meets the parabola C : $y = x^2 - 3$ at the Points for which $x^2 - 3 = \dfrac{2\left(a^2 - 3\right)}{a}x - a^2 + 3$

$\Rightarrow$ $(x-a)(ax+6-a^2) = 0$ But $x = 1$ and $x \neq a$

$$x = \frac{a^2 - 6}{a} = 1 \Rightarrow a = -2, 3$$

62. If a > 0, the angle subtended by the chord AB at the vertex of the parabola C is

a) $\tan^{-1}(5/7)$ b) $\tan^{-1}(1/2)$ c) $\tan^{-1}(2)$ d) $\tan^{-1}(1/8)$

Key. B

Sol. If a>o, then a = 3, A= (3,6), B = (1,-2) equation of C is $y = x^2 - 3$ or $x^2 = y + 3$

Vertex 'O' of the parabola C is (0,-3) slope OA = 3 , slope OB= 1

Paragraph – 22

Let the normal at P to the hyperbola $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$ meets the transverse axis in G and conjugate axis in 'g' and CF be perpendicular to the normal, from the centre C then

63. The value of product of PF and PG is

a) 9 b) 16 c) 25 d) 7

Key. B

Sol. Conceptual

64. The value of the product of PF and Pg is

a) 9 b) 16 c) 25 d) 7

Key. A

Sol. Conceptual

65. The ratio of SG to SP, (where S is the focus of the hyperbola) is

a) 5/4 b) 5/9 c) 5/3 d) 3/5

Key. C

Sol. Conceptual

Paragraph – 23

The equation of normal to a parabola $y^2 = 4ax$ is $y = mx - 2am - am^3$. From this conclude that three normals real or imaginary can be drawn from point 'p'.

66. The locus of 'p' such that two normals make complementary angles with axis of parabola is:

a) $y^2 = a(x+a)$ b) $y^2 = a(x-a)$

c) $y^2 = a(y-a)$ d) None

Key. B

67. The locus of 'p' if one normal is bisector of other two is

a) $27ay^2 = (2x-a)(x-5a)^2$ b) $y^2 = (x-2a)(x-5a)^2$

c) $x^2 = (y-2a)(y-5a)^2$ d) None

Key. A

68. The locus of a point 'p' if the sum of the angles made by the normals with the axis is a constant is

a) A straight line b) A parabola

c) A circle d) An ellipse

Key. A

Sol. $y = mx - 2ax - ax^3$

$ax^3 + (2a - x)m + y = 0$

$\sum m = 0, \sum m_1 m_2 = \frac{2a - x}{a}, \sum m_1 m_2 m_3 = \frac{-y}{a}$

66. $m_1 m_2 = 1$

67. $\frac{m_1 - m_2}{1 + m_1 m_2} = \frac{m_2 - m_3}{1 + m_2 m_3}$ & $am_2^3 + (2a - x)m_2 + y = 0$

68. $\theta_1 + \theta_2 + \theta_3 = K \quad \Rightarrow \tan(\theta_1 + \theta_2 + \theta_3) = \frac{S_1 - S_3}{1 - S_2} = K$

Paragraph – 24

If 'P' is any point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. S_1 and S_2 are foci of the ellipse

69. Locus of incentre of triangle PS_1S_2 will be

a) a straight line b) a circle c) a parabola d) an ellipse

Key. D

70. If $e=\frac{1}{2}$ and $\underline{|P}S_1S_2=\alpha, \underline{|P}S_2S_1=\beta, \underline{|S_1}PS_2=\gamma$, then $\cot\frac{\alpha}{2}, \cot\frac{\gamma}{2}$ and $\cot\frac{\beta}{2}$ are in

a) A.P b) G.P c) H.P d) None

Key. A

71. Maximum area of the triangle PS_1S_2 is equal to

a) b^2e sq.units b) a^2e sq.units

c) ab sq.units d) abe sq.units

Key. D

Sol. 69. $\frac{PS_2}{S_2G}=\frac{PS_1}{GS_1}=\frac{PS_2+PS_1}{S_2G+GS_1}=\frac{2a}{2ae}=\frac{1}{e}$

So $PI:IG=1:e$

70. $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{1-e}{1+e}=\frac{1}{3}$

71. Base S_1S_2 fixed and PS_2+PS_2 is fixed, Hence area will be maximum if $PS_1=PS_2$

Paragraph – 25

A straight line drawn through the point $p(-1,2)$ meets the hyperbola $xy=c^2$ at the points A and B (points A and B lie on the same side of P)

72. A point Q is chosen on this line such that PA, PQ and PB are in A.P, then locus of point Q is.

a) $x=y(1+2x)$ b) $x=y(1+x)$

c) $2x=y(1+2x)$ d) None of these

Key. C

73. If PA, PQ and PB are in G.P., then locus of Q is

a) $xy - y + 2x - c^2 = 0$ b) $xy + y - 2x + c^2 = 0$

c) $xy + y + 2x + c^2 = 0$ d) $xy - y - 2x - c^2 = 0$

Key. B

74. If PA, PQ and PB are in H.P. then locus of Q is

a) $2x - y = 2c^2$ b) $x - 2y = 2c^2$

c) $2x + y + 2c^2 = 0$ d) $x + 2y = 2c^2$

Key. A

Sol. 72. $x = \gamma\cos\theta - 1,\ y = \gamma\sin\theta + 2$

$xy = c^2$

$\Rightarrow \sin\theta\cos\theta\gamma^2 + (2\cos\theta - \sin\theta)\gamma - 2 - c^2 = 0$

$\dfrac{PA + PB}{2} = PQ \Rightarrow -\dfrac{2\cos\theta - \sin\theta}{2\sin\theta\cos\theta} = \gamma$

73. $(PA)(PB) = \dfrac{-(2 + c^2)}{\sin\theta\cos\theta} = \gamma^2$

74. $\dfrac{2}{PQ} = \dfrac{2}{\gamma} = \dfrac{PA + PB}{PA.PB} = \dfrac{\sin\theta - 2\cos\theta}{-(2 + c^2)}$

Paragraph – 26

Consider the conic defined by the equation :

$$\left|\sqrt{(x-1)^2 + (y-2)^2} - \sqrt{(x-5)^2 + (y-5)^2}\right| = 3$$

75. The equation of an axis of the conic is

a) $6x + 8y = 45$ b) $3x - 4y - 5 = 0$

c) $8x + 6y = 45$ d) $3x + 4y + 5 = 0$

Key. C

Sol. Given equation represents a hyperbola having foci $S(1,2)$ and $S'(5,5)$ & $2a = 3$

transverse axis : line SS' : $3x - 4y + 5 = 0$

Conjugate axis : perpendicular bisector of $SS':8x+6y=45$

76. The distance between the directrices of the conic is

a) $9/5$ b) $3/5$

c) $5/3$ d) $5/9$

Key. A

Sol. Distance between diretrices = $=\frac{2a}{e}=\frac{3}{5/3}=\frac{9}{5}$

77. The eccentricity of the conic conjugate to the given one, is

a) $5/3$ b) $5/4$ c) $5/2$ d) 5

Key. B

Sol. let e' be the ecc. of conjugate hyperbola then $\frac{1}{e^2}+\frac{1}{e'^2}=1 \Rightarrow e'^2=\frac{25}{16}$

Paragraph – 27

An ellipse E has its centre C (1,3), focus at S (6, 3) and passes through the point P (4, 7). Then

78. The product of the perpendicular distances of foci from tangent at P to the ellipse, is

a) 20 b) 45 c) 40 d) 60

Key. A

79. The point of intersection of the lines joining each focus to the foot of the perpendicular from the other focus upon the tangent at P, is

a) $\left(\frac{5}{3},5\right)$ b) $\left(\frac{4}{3},3\right)$

c) $\left(\frac{8}{3},3\right)$ d) $\left(\frac{10}{3},5\right)$

Key. D

80. If the normal at a variable point on the ellipse (E) meets its axes in Q and R, then the locus of the midpoint of QR is a conic with eccentricity =

a) $3/\sqrt{10}$ b) $\sqrt{5}/3$

c) $3/\sqrt{5}$ d) $\sqrt{10}/3$

Key. B

Sol. $CS=ae=5$

$S'=(-4,5)$

$PS+PS'=2a=6\sqrt{5}$

$\Rightarrow e=\frac{\sqrt{5}}{3}$

86. Product $= b^2$

Paragraph – 28

A quadratic polynomial $y = f(x)$ with constant term 3 neither touches nor intersects the abscissa axis and is symmetric about the line $x = 1$. The coefficient of the leading term of the polynomial is unity. Now answer the following questions:

81. Vertex of the quadratic polynomial is

a) $(1,1)$ b) $(2,3)$ c) $(1,2)$ d) (5, 7)

Key. C

82. The area bounded by the curve $y = f(x)$ and a line $y = 3,$ is

a) $\frac{4}{3}$ b) $\frac{5}{3}$ c) $\frac{7}{3}$ d) $\frac{28}{3}$

Key. A

83. The graph of $y = f(x)$ represents a parabola whose focus has the co-ordinates

a) $\left(1, \frac{7}{4}\right)$ b) $\left(1, \frac{5}{4}\right)$ c) $\left(1, \frac{5}{2}\right)$ d) $\left(1, \frac{9}{4}\right)$

Key. D

Sol. 15,16,17

Let $y = ax^2 + bx + c,$ where $c = 3,$ and $a = 1,$ therefore, the curve lies completely above the $x - axis.$

$\therefore \; f(x) = y = x^2 + bx + c$. Line of symmetry being 1, therefore minima occurs at $x = 1$.

$\therefore \; f^1(1) = 0 \Rightarrow 2x + b = 0$ at $x = 1$

$b = -2$

Hence, $f(x) = x^2 - 2x + 3$

Vertex is $(1,2)$.

If $y = 3,$ then $x^2 - 2x = 0 \Rightarrow x = 0$ or 2

Hence, the area bounded = $\int_0^2 3 - \left(x^2 - 2x + 3\right) dx$

$$= \int_0^2 \left(2x - x^2\right) dx = \left[x^2 - \frac{x^3}{3}\right]_0^2 = 4 - \frac{8}{3} = \frac{4}{3}.$$

Paragraph – 29

If the axis of the rectangular hyperbola $x^2 - y^2 = a^2$ are rotated through an angle of $\frac{\pi}{4}$ in clock wise direction, then the equation $x^2 - y^2 = a^2$ reduces to $xy = c^2$ where $c = \frac{a}{\sqrt{2}}$. Parametric equation of $xy = c^2$ are $x = ct, y = \frac{c}{t}$ Where 't' is the parameter.

Answer the following.

84. If $t_1 \& t_2$ are the roots of the equation $x^2 - 8x + 4 = 0$, then, the point of intersection of tangents at $t_1 \& t_2$ on $xy = c^2$ is.

a) (c, c) b) $\left(c, \frac{c}{2}\right)$ c) $\left(c, \frac{c}{4}\right)$ d) $\left(\frac{c}{4}, \frac{c}{4}\right)$

Key. C

Sol. Conceptual

85. If $A(t_1), B(t_2), c(t_3)$ are three points on $xy = c^2$, then , area of triangle ABC is

a) $c^2(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)$ b) $\frac{c^2}{2t_1t_2t_3}(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)$

c) $\frac{c^2}{t_1t_2t_3}(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)$ d) $2c^2t_1t_2t_3(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)$

Key. B

Sol. Conceptual

86. If the normal drawn at $P(t = 1)$ to $xy = 1$ cuts the curve again at Q, then, length of PQ is

a) 1 b) $2\sqrt{2}$ c) $3\sqrt{2}$ d) $4\sqrt{2}$

Key. B

Sol. Conceptual

Paragraph – 30

The equation $ax^2 + 2hxy + by^2 = 1, h^2 \neq ab$ represents ellipse or a hyperbola accordingly as $h^2 < ab(or)h^2 > ab$. The length of the axis of the conic are related with the roots of the quadratic $(ab - h^2)t^2 - (a + b)t + 1 = 0$. If t_1, t_2 are positive, then, lengths of the axes are $2\sqrt{t_1} \& 2\sqrt{t_2}$. If $t_1 > 0$ & $t_2 < 0$, then, lengths of the transverse and conjugtate axes are $2\sqrt{t_1} \& 2\sqrt{-t_2}$. The equation to the axes of the conic are $(at_1 - 1)x + ht_1y = 0 \& (at_2 - 1)x + ht_2y = 0$.

Answer the following.

87. The eccentricity of the conic $x^2 + xy + y + y^2 = 1$ is

a) $\frac{1}{\sqrt{3}}$ b) $\frac{3}{5}$ c) $\frac{\sqrt{2}}{3}$ d) $\frac{2}{\sqrt{6}}$

Key. D

Sol. Conceptual

88. Area enclosed by the ellipse $5x^2-6xy+5y^2=8$ is,

a) $\pi\sqrt{2}$ b) 2π c) $\pi\sqrt{3}$ d) $\frac{4\pi}{3}$.

Key. B

Sol. Conceptual

89. If the line $\frac{x}{a}+\frac{y}{b}=1$ is the transverse axis of the hyperbola $(x+1)^2+4(x+1)(y-1)+(y-1)^2=4,$ then, $a+b=$

a) 0 b) -1 c) 2 d) -3.

Key. A

Sol. Conceptual

Paragraph – 31

A sequence of ellipse $E_1, E_2, ...E_n$ is constructed as follows : Ellipse E_n is drawn so as to touch ellipse E_{n-1} as the extremities of the major axis of E_{n-1} and to have its foci at the extremities of the minor axis of E_{n-1}.

90. If E_n is independent of n then the eccentricity of the ellipse E_{n-2}.

(A) $\frac{3-\sqrt{5}}{2}$ (B) $\frac{\sqrt{5}-1}{2}$

(C) $\frac{2-\sqrt{3}}{2}$ (D) $\frac{\sqrt{3}-1}{2}$

Key. B

Sol. $\frac{x^2}{a_n^2}+\frac{y^2}{b_n^2}=1, \quad a_n > b_n$

$b_n^2 = a_n^2\left(1-b_n^2\right)$ ...(i)

$b_n = b_{n-1}$ .....(ii), $a_{n-1}=a_n b_n$ ....(iii)

For $E_{n-1}, a_{n-1}=b_{n-1}^2\left(1-e_{n-1}^2\right)$

From (i) & (ii) $b_{n-1}^2 = an^2\left(1-e_{n-1}^2\right)$

$\therefore \quad a_n^2 b_n^2 = a_n^2\left(1-e_n^2\right)\left(1-e_{n-1}^2\right)$

Let all the eccentricities are e

$\therefore \quad e^2=\left(1-e^2\right)^2 \Rightarrow e^4-3e^2+1=0$

$$e^2 = \frac{3 \pm \sqrt{5}}{2} \Rightarrow e = \frac{\sqrt{5}-1}{2}$$

91. If eccentricity of ellipse E_n is e_n then locus of $\left(e_n^2, e_{n-1}^2\right)$ is a

(A) parabola
(B) An ellipse
(C) Circle
(D) A rectangular hyperbola

Key. D

Sol. $e_n^2 = \left(1-e_n^2\right)\left(1-e_{n-1}^2\right)$

$\Rightarrow \quad h = (1-h)(1-k) \qquad h = e_n^2$

$\Rightarrow \quad x = 1 - x - y + xy \qquad k = e_{n-1}^2$

$\Rightarrow \quad xy - 2x - y + 1 = 0$

A rectangular hyperbola.

92. If eccentricity of E_n is independent of n then the locus of the mid point of chords of slope - 1 of E_n (If axis of E_n is along y-axis)

(A) $\left(\sqrt{5}-1\right)x = 2y$
(B) $\left(\sqrt{5}+1\right)x = 2y$
(C) $\left(3-\sqrt{5}\right)x = 2y$
(D) $\left(3+\sqrt{5}\right)x = 2y$

Key. B

Sol. $T = S_1 \Rightarrow \frac{xx_1}{a_n^2} + \frac{yy_1}{b_n^2} = \frac{x_1^2}{a_n^2} + \frac{y_1^2}{b_n^2} - \frac{b_n^2 x_1}{a_n^2 y_1} = -1$

If eccentricity of E_n is independent of n

$e = \frac{\sqrt{5}-1}{2} \Rightarrow e^2 = \frac{3-\sqrt{5}}{2}$

$b_n^2 x_1 = a_n^2 y_1$

$x_1 = \left(1 - \frac{\left(3-\sqrt{5}\right)}{2}\right) y_1 \Rightarrow 2x_1 = \left(\sqrt{5}-1\right) y_1$

$\Rightarrow 2x_1\left(\sqrt{5}+1\right) = 4y_1$

$2y = x\left(\sqrt{5}+1\right)$

Paragraph – 32

If the second degree curve representing by a hyperbola S = 0. The difference between the equations of hyperbola and pair of asymptotes is constant. That is A $\equiv S + \lambda = 0$ where λ is constant. By using the condition of pair of straight lines, we get λ. If the equation of conjugate hyperbola of S is represented by S_1. Also asymptotes is the arithmetic mean of S and S_1.

93. Pair of asymptotes of the hyperbola $xy - 3y - 2x = 0$ is

(A) $xy-3y-2x+2=0$ (B) $xy-3y-2x+4=0$

(C) $xy-3y-2x+6=0$ (D) $xy-3y-2x+12=0$

Key. C

Sol. Pair of asymptotes $xy-3y-2x+\lambda=0$ for pair of straight lines

$$0+2.\left(-\frac{3}{2}\right)(-1).\frac{1}{2}-0-0-\lambda\left(\frac{1}{2}\right)^2=0$$

$$\frac{3}{2}=\frac{\lambda}{4}\Rightarrow\lambda=6$$

$$xy-3y-2x+6=0$$

94. If the angle between the asymptotes of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{\pi}{3}$. Then the eccentricity of conjugate hyperbola is

(A) $\sqrt{2}$ (B) 2

(C) $\frac{2}{\sqrt{3}}$ (D) $\frac{4}{\sqrt{3}}$

Key. B

Sol. $2\tan^{-1}\left(\frac{b}{a}\right)=\frac{\pi}{3}$

$$\frac{b}{a}=\frac{1}{\sqrt{3}}$$

$$e^2=1+\frac{1}{3}=\frac{4}{3}$$

$$\frac{1}{e'^2}+\frac{1}{e^2}=1$$

$$\Rightarrow \quad \frac{1}{e'^2}+\frac{3}{4}=1$$

$$\Rightarrow \quad \frac{1}{e'^2}=\frac{1}{4}\Rightarrow e'=2$$

95. A variable chord $x\cos\theta+y\sin\theta=p$ of $\frac{x^2}{a^2}-\frac{y^2}{2a^2}=1$ subtends a right angle at the origin. This chord always touches a curve whose radius is

(A) a (B) $\frac{a}{\sqrt{2}}$

(C) $a\sqrt{2}$ (D) $2a\sqrt{2}$

Key. C

Sol. $\frac{x^2}{a^2}-\frac{y^2}{2a^2}=\left(\frac{x\cos\theta+4\sin\theta}{p}\right)^2$

$$\Rightarrow \quad \frac{1}{a^2}-\frac{\cos^2\theta}{p^2}+\left(-\frac{1}{2a^2}-\frac{\sin^2\theta}{p^2}\right)=0 \quad \Rightarrow \quad \frac{1}{2a^2}=\frac{1}{p^2}\Rightarrow p=a\sqrt{2}$$

$x\cos\theta + y\sin\theta = a\sqrt{2}$ will always touch $x^2 + y^2 = 2a^2$

Paragraph – 33

If a sequence or series is not a direct form of an AP, GP, etc. Then its nth term can not be determined. In such cases, we use the following steps to find the nth term (T_n) of the given sequence.

Step – I : Find the differences between the successive terms of the given sequence. If these differences are in AP, then take $T_n = an^2 + bn + c$, where a,b,c are constants.

Step – II : If the successive differences found in step I are in GP with common ratio r, then take $T_n = a + bn + cr^{n-1}$, where a, b, c are constants.

Step – III : If the second successive differences (Differences of the differences) in step I are in AP, then take $T_n = an^3 + bn^2 + cn + d$, where a, b, c, d are constants.

Step – IV : If the second successive differences (Differences of the differences) in step I are in GP, then take $T_n = an^2 + bn + c + dr^{n-1}$, where a, b, c, d are constants.

Now let sequences :

A : 1, 6, 18, 40, 75, 126, ….. B : 1, 1, 6, 26, 91, 291, …. C : ln 2 ln 4, ln 32, ln 1024 …..

96. If the nth term of the sequence *A* is $T_n = an^3 + bn^2 + cn + d$ then the value $6a + 2b - d$ is

(A) ln 2 (B) 2

(C) ln 8 (D) 4

Key. D

Sol. $T_n = an^3 + bn^2 + cn + d$

$T_1 = a + b + c + d = 1$

$T_2 = 8a + 4b + 2c + d = 6$

$6a + 2b - d = 4$

97. For the sequence 1, 1, 6, 26, 91, 291, …….. Find the S_{50} where $S_{50} = \sum_{r=1}^{50} T_r$

(A) $\frac{5}{8}\left(3^{50} - 1\right) - 3075$ (B) $\frac{5}{8}\left(3^{50} - 1\right) - 5075$

(C) $\frac{5}{8}\left(3^{50} - 1\right) - 1275$ (D) None of these

Key. A

Sol. $T_n = \frac{5}{4}3^{n-1} - \frac{5n}{2} + \frac{9}{4}$

$$S_{50} = \frac{5}{4}\left(1 + 3 + ... + 3^{49}\right) - \frac{5}{2}\left(1 + 2 + ... + 50\right) + 50.\frac{9}{4}$$

$$= \frac{5}{4}\left(\frac{3^{50} - 1}{2}\right) - \frac{5}{2}.\frac{50.51}{2} + \frac{450}{4}$$

$$=\frac{5}{8}\left(3^{50}-1\right)-\frac{125.51}{2}+\frac{450}{4}$$

$$=\frac{5}{8}\left(3^{50}-1\right)-3075$$

98. The sum of the series $1.n+2.(n-1)+3.(n-2)+....+n.1$

(A) $\frac{n(n+1)(n+2)}{6}$ (B) $\frac{n(n+1)(n+2)}{3}$

(C) $\frac{n(n+1)(2n+1)}{6}$ (D) $\frac{n(n+1)(2n+1)}{3}$

Key. A

Sol. $\sum_{r=1}^{n} r(n-r+1)=\sum_{r=1}^{n}(n+1)r-\sum_{r=1}^{n}r^2$

$$=(n+1)\sum n-\sum n^2$$

$$=\frac{(n+1)^2 n}{2}-\frac{n(n+1)(2n+1)}{6}$$

$$=\frac{n(n+1)}{6}(3n+3-2n-1) = \frac{n(n+1)(n+2)}{6}$$

Paragraph – 34

Normal to parabola $y^2=4ax$ at the point $(at^2,2at)$ is given by $xt+y=2at+at^3$. If it is passes through point (h, k) then $at^3+t(2a-h)-k=0$ ...(1)

If t_1,t_2,t_3 be roots of (1) then three points P, Q, R are $(at_1^2,2at_1),(at_2^2,2at_2),(at_3^2,2at_3)$ from which normals pass through the point (h, k) points P, Q, R are called co-normal points.

Putting 2at = y, the ordinates of P, Q, R are the roots of $y^3+4a(2a-h)y-8a^2k=0$ ...(2)

Let the circle through P, Q, R be $x^2+y^2+2gx+2fy+c=0$

Eliminating x from equations of the parabola and circle, we have

$\frac{y^4}{16a^2}+y^2+2g\frac{y^2}{4a}+2fy+c=0$, i.e $y^4+y^2(16a^2+8ag)+32a^2fy+16a^2c=0$ ...(3)

Equation (3) gives the ordinates of the points of intersection of the parabola and the circle. Three roots of equation (3) are the same as the roots of equation (2). Let these identical roots be y_1,y_2,y_3 and let the fourth root be y_4.

Then from (3), $y_1+y_2+y_3+y_4=0$

Also, from (2), $y_1+y_2+y_3=0$, so we get $y_4=0$

As, one root of equation (3) is zero $\Rightarrow c=0$.

And the equation (3) becomes, $y^3+y(16a^2+8ag)+32a^2f=0$ ... (4)

Equation (2) and (4) being identical, we have

$4a(2a-h)=16a^2+8ag$ and $-8a^2k=32a^2f$

Solving which we can find g and f and hence the equation of the desired circle.

99. If the normal at Q & R on $y^2=4ax$ meet the parabola at the same point P then the locus of the circumcentre of ΔPQR is

(A) a straight line (B) a circle

(C) another parabola (D) hyperbola

Key. C

Sol. Then tangents at Q & R meet at $(2a,-at_3)$ if t_3 is the parameter for P.

$P=(at_3^2,2at_3)$

$\therefore 2h=2a+at_3^2$

$2K=at_3$

$\therefore 2h=2a+\frac{4K^2}{a}$

$2a(h-a)=4K^2 \Rightarrow 2y^2=a(x-a)$

100. The equation of the circle through the feet of the normals drawn to $y^2=4ax$ from (h, k) is

(A) $x^2+y^2=a^2$ (B) $x^2+y^2+2ax=0$

(C) $x^2+y^2-2ax+(K+a)y=0$ (D) $x^2+y^2-(h+2a)x-\frac{1}{2}ky=0$

Key. D

Sol. $2g=-(h+2a)$

$2f=-\frac{K}{2}$

$x^2+y^2-(h+2a)x-\frac{1}{2}K\ y-0$

101. The circle through co-normal points of a parabola passes through

(A) focus (B) Vertex

(C) one end of latusrectum (D) Point of intersection of axis and directrix

Key. B

Sol. The circle passes through the vertex of the parabola.

Paragraph – 35

Suppose than an ellipse and a circle are respectively given by the equation

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ..(1) and

$x^2+y^2+2gx+2fy+c=0$ ...(2)

The equation, $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}-1\right)+\lambda(x^2+y^2+2gx+2fy+c)=0$ ...(3)

Represents a curve which passes through the common points of the ellipse (1) and the circle (2).

We can choose λ so that the equation (3) represents a pair of straight lines. In general we get three values of λ, indicating three pair of straight lines can be drawn through the points. Also when (3) represents a pair of straight lines they are parallel to the lines $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\lambda(x^2+y^2)=0$ ¸which represents a pair of lines equally inclined to axes (the term containing xy is absent). Hence two straight lines through the points of intersection of an ellipse and any circle make equal angles with the axes. Above description can be applied identically for a hyperbola and a circle.

102. The radius of the circle passing through the point of intersection of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ & $x^2-y^2=0$ is

(A) $\frac{ab}{\sqrt{a^2+b^2}}$ (B) $\frac{\sqrt{2}\,ab}{\sqrt{a^2+b^2}}$

(C) $\frac{a^2-b^2}{\sqrt{a^2+b^2}}$ (D) $\frac{a^2+b^2}{\sqrt{a^2+b^2}}$

Key. B

Sol. $x^2+y^2=\frac{a^2b^2}{a^2+b^2}$

$\therefore$ radius of the circle

$\sqrt{\frac{2a^2b^2}{a^2+b^2}}=\frac{\sqrt{2}ab}{\sqrt{a^2+b^2}}$

103. Suppose two lines are drawn through the common points of intersection of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ & $x^2+y^2+2gx+2fy+c=0$. If these lines are inclined at an angle α,β to $x-$ axis then

(A) $\alpha=\beta$ (B) $\alpha+\beta=\frac{\pi}{2}$

(C) $\alpha+\beta=\pi$ (D) $\alpha+\beta=2\tan^{-1}\left(\frac{b}{a}\right)$

Key. C

Sol. As the lines joining common point of intersection must be equally inclined to the axis

$\tan\alpha=-\tan\beta\Rightarrow\alpha+\beta=T_1$

104. The no. of pair of St. lines through the points of intersection of $x^2-y^2=1$ and $x^2+y^2-4x-5=0$.

(A) 0 (B) 1

(C) 2 (D) 3

Key. C

Sol. Any curve through their point of intersection

$x^2+y^2-4x-5+\lambda(x^2-y^2-1)\Rightarrow(1+\lambda)x^2+(1-\lambda)y^2-4x-5-\lambda=0$

$(1+\lambda)(1-\lambda)(-5-\lambda)+0-(1+\lambda).0-(1-\lambda).4+(5+\lambda).0=0$

$(\lambda-1)(\lambda+3)^2=0\Rightarrow\lambda=1,-3$

$\therefore$ Two pair of St.lines can be drawn.

Paragraph – 36

Consider a hyperbola x y = 4 and a line $y+2x=4$. O is the centre of hyperbola. Tangent at any point P of hyperbola intersect the coordinate axes at A and B

105. Locus of circum centre of triangle OAB is

A) an ellipse with eccentricity $1/\sqrt{2}$ B) an ellipse with eccentricity $1/\sqrt{3}$

C) a hyperbola with eccentricity $\sqrt{2}$ D) a circle

Key. C

106. Shortest distance between the line and hyperbola is

A) $8\sqrt{2}/\sqrt{5}$ B) $\frac{4\left(\sqrt{2}-1\right)}{\sqrt{5}}$ C) $\frac{2\sqrt{2}}{\sqrt{5}}$ D) $\frac{4\left(\sqrt{2}+1\right)}{\sqrt{5}}$

Key. B

107. Let the given line intersects the x-axis at R . If a line through R, intersect the hyperbolas at S and T then, then minimum value of RS$\times$RT is ___

A) 2 B) 4 C) 6 D) 8

Key. D

Sol. 105,106&107 -P-1.

Let $\left(2t, 2/t\right)$ be a point on the hyperbola. Equation of the tangent at this point $x+yt^2=8t$.

A=(8t , 0) , B=(0 , 8/t)

Locus of circumcentre of triangle OAB is its eccentricity is $=\sqrt{2}$

Shortest distance exist along the common normal. $t^2=1/2\Rightarrow t=1/\sqrt{2}$, foot of the perpendicular is $\left(\sqrt{2},2\sqrt{2}\right)$; shortest distance is $\frac{4\left(\sqrt{2}-1\right)}{\sqrt{5}}$. Let R(2, 0) & $S\left(2+\cos\theta, r\sin\theta\right)$ lies on hyperbola $|r_1r_2|=8/|\sin2\theta|$; minimum of $RS\times RT$ is 8

Paragraph – 37

The chord AC of the parabola $y^2=4ax$ subtends an angle of 90^{O} at points B and D on the parabola. If A,B,C and D are represented by t_1,t_2,t_3 and t_4 then

108. Value of $\left|\frac{t_2+t_4}{t_1+t_3}\right|=$ __

A) 0 B) 1 C) 2 D) 4

Key. B

109. Minimum value of $|t_1 - t_3| =$ ___

A) 0 B) 1 C) 2 D) 4

Key. D

110. The y-coordinate of the mid point of the points of intersection of the tangents at A,C and B,D is __

A) 0 B) 1 C) 2 D) 4

Key. A

Sol. 108,109&110-P-II.

We have $(t_2+t_3)(t_2+t_1) = -4$; $t_2^2 + t_2(t_1+t_3) + t_1t_3 + 4 = 0 \to (1)$

$(t_1+t_3)^2 - 4(t_1t_3+4) \ge 0$; $(t_1-t_3)^2 \ge 16$; $t_2 + t_4 = -1(t_1+t_3)$ & $t_2t_4 = t_1t_3 + 4$

Because t_2 and t_4 are y-coordinates of intersection points of the tangents at (A,C) and (B,D) then $y_1 == t_1 + t_3,\ y_2 = t_2 + t_4 \Rightarrow y_1 + y_2 = 0$

Paragraph – 38

The points P,Q,R are taken on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with eccentricities $\theta,\ \theta+\alpha,\ \theta+2\alpha$ then

111. Area of the triangle PQR is independent of

A) θ B) α C) θ & α both D) none

Key. A

112. If the area of triangle PQR is maximum, then

A) $\alpha = \pi/3$ B) $\alpha = \pi/2$ C) $\alpha = 2\pi/3$ D) none

Key. C

113. If A_1 be the area of triangle PQR and A_2 be the area of the triangle formed by corresponding points on the auxiliary circle then $\frac{A_1}{A_2}$ is ___.

A) 1 B) a/b C) b/a D) none

Key. C

Sol. 111,112 & 113

(111) a ; 112) c ; 113) c)

$$A_1 = \Delta = \frac{1}{2}\begin{vmatrix} a\cos\theta & b\sin\theta & 1 \\ a\cos(\theta+\alpha) & b\sin(\theta+\alpha) & 1 \\ a\cos(\theta+2\alpha) & b\sin(\theta+2\alpha) & 1 \end{vmatrix} = ab(1-\cos\alpha)\sin\alpha$$

Δ is max $\Rightarrow \alpha = 2\pi/3$;

$$A_1 = \frac{1}{2}\begin{vmatrix} a\cos\alpha & b\sin\alpha & 1 \\ a\cos\beta & b\sin\beta & 1 \\ a\cos\gamma & b\sin\gamma & 1 \end{vmatrix} = A_2 = \frac{1}{2}\begin{vmatrix} a\cos\alpha & b\sin\alpha & 1 \\ a\cos\beta & b\sin\beta & 1 \\ a\cos\gamma & b\sin\gamma & 1 \end{vmatrix} \quad \therefore A_1/A_2 = b/a$$

Paragraph – 39

P is any point of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. S and S′ are foci and e is the eccentricity of ellipse. $\angle PSS' = \alpha$ and $\angle PS'S = \beta$

114. $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}$ is equal to

(A) $\frac{2e}{1-e}$ (B) $\frac{1+e}{1-e}$

(C) $\frac{1-e}{1+e}$ (D) $\frac{2e}{1+e}$

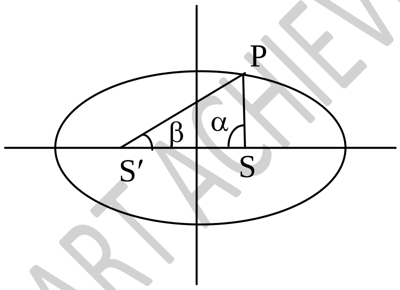
Key. C

Sol. $\frac{PS}{\sin\beta}=\frac{PS'}{\sin\alpha}=\frac{2ae}{\sin(\pi-(\alpha+\beta)}$

or, $\frac{2a}{\sin\alpha+\sin\beta}=\frac{2ae}{\sin(\alpha+\beta)}$

or, $\frac{1}{e}=\frac{2\sin\frac{\alpha+\beta}{2}.\cos\frac{\alpha-\beta}{2}}{2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha+\beta}{2}}$

$\therefore \frac{1-e}{1+e}=\tan\frac{\alpha}{2}\tan\frac{\beta}{2}$



115. Locus of incentre of triangle PSS′ is

(A) an ellipse (B) hyperbola

(C) parabola (D) circle

Key. A

Sol. y - 0 = tan $\frac{\beta}{2}$ (x + ae) ... (i)

y - 0 = -tan $\frac{\alpha}{2}$ (x - ae) ... (ii)

or, $y^2 = -\left(\frac{1-e}{1+e}\right)$ $[x^2 - a^2e^2]$

or, $\left(\dfrac{1-e}{1+e}\right)x^2 + y^2 = \left(\dfrac{1-e}{1+e}\right)a^2e^2$

or, $\dfrac{x^2}{a^2e^2} + \dfrac{y^2}{\left(\dfrac{1-e}{1+e}\right)a^2e^2} = 1$

which is clearly an ellipse.

116. Eccentricity of conic, which is locus of incentre of triangle PSS′

(A) $\sqrt{\dfrac{e}{1+e}}$ (B) $\sqrt{\dfrac{2e}{1+e}}$

(C) $\sqrt{\dfrac{2e}{1-e}}$ (D) $\sqrt{\dfrac{e}{1-e}}$

Key. B

Sol. $e' = \sqrt{1-\dfrac{1-e}{1+e}}$

$= \sqrt{\dfrac{2e}{1+e}}$

Paragraph – 40

Any point on the parabola $y^2 = 4ax$ can be considered as $x = at^2$, $y = 2at$ (where t is parameter) equation of tangent at $(at^2, 2at)$ is given by $ty = x + at^2$, if t_1, t_2 are parameters corresponding to two points on the parabola $y^2 = 4ax$, point of intersection of tangents at these point is given by $[at_1t_2, a(t_1 + t_2)]$. Now answer the following questions.

117. Locus of point of intersection of tangents to a parabola $y^2 = 4ax$ whose chord of contact subtends an angle $\pi/3$ at the vertex

(A) $3x^2 + 4y^2 + 40ax + 48a^2 = 0$ (B) $3x^2 - 4y^2 + 40ax + 48a^2 = 0$

(C) $4x^2 + 3y^2 + 40ax + 48a^2 = 0$ (D) $4x^2 - 3y^2 + 40ax + 48a^2 = 0$

Key. B

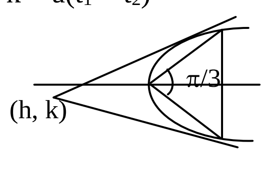
Sol. $\tan \pi/3 = \left|\dfrac{\dfrac{2}{t_1} - \dfrac{2}{t_2}}{1 + \dfrac{4}{t_1 t_2}}\right|$

or, $3(4 + t_1 t_2)^2 = 4[(t_1 + t_2)^2 - 4t_1t_2]$

or, $3\left[4 + \dfrac{h}{a}\right]^2 = 4\left[\left(\dfrac{k}{a}\right)^2 - 4\dfrac{h}{a}\right]$

$h = at_1 t_2$

$k = a(t_1 + t_2)$



or, $3x^2 - 4y^2 + 40ax + 48a^2 = 0$

118. TP and TQ are any two tangents to a parabola and the tangent at a third point R cuts them in P′ and Q′ then $\frac{TP'}{TP}+\frac{TQ'}{TQ}$ is equal to

(A) 1 (B) 2
(C) 1/4 (D) 1/2

Key. A

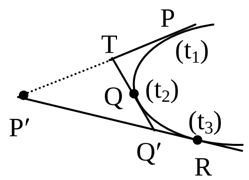
Sol. $TP = a(t_1 - t_2)\sqrt{1+t_1^2}$

$TP' = a(t_2 - t_3)\sqrt{1+t_1^2}$ $\frac{TP'}{TP}=\frac{t_2-t_3}{t_1-t_2}$

Similarly $\frac{TQ'}{TQ}=\frac{t_1-t_3}{t_2-t_1}$

$\frac{TP'}{TP}+\frac{TQ'}{TQ}=1$

$T \equiv (at_1 t_2, a(t_1 + t_2))$
$P' \equiv (at_1 t_3, a(t_1 + t_3))$
$Q' \equiv (at_2 t_3, a(t_2 + t_3))$



119. Locus of point of intersection of tangents drawn at extremities of normal chord of the parabola $y^2 = 4x$ is

(A) $\frac{4}{x^2} + y + 2 = 0$ (B) $\frac{4}{y^2} + x + 1 = 0$

(C) $\frac{4}{y^2} + x + 2 = 0$ (D) $\frac{4}{x^2} + y + 1 = 0$

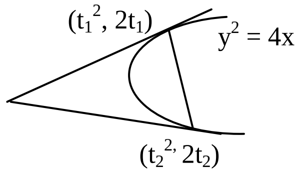
Key. C

Sol. $h = t_1t_2 = t_1\left(-t_1 - \frac{2}{t_1}\right) = -t_1^2 - 2$

$k = t_1 + t_2 = t_1 + \left(-t_1-\frac{2}{t_1}\right)=\frac{-2}{t_1}$

$\therefore\ h = -\left(\frac{-2}{k}\right)^2-2$

Hence locus is $\frac{4}{y^2}+x+2 = 0$



Paragraph – 41

A circle C whose radius is 1 unit, touches the x-axis at point A. The centre Q of C lies in first quadrant. The tangent from origin O to the circle touches it at T and a point P lies on it such that ΔOAP is a right angled triangle at A and its perimeter is 8 units.

120. The length of QP is

A) $\frac{1}{2}$ B) $\frac{4}{3}$ C) $\frac{5}{3}$ D) $\frac{5}{2}$

Key. C

121. Equation of circle C is

A) $(x-2)^2 + (y-1)^2 = 1$ B) $\left\{x-\left(2+\sqrt{3}\right)\right\}^2 + (y-1)^2 = 1$

C) $\left(x-\sqrt{3}\right)^2 + (y-1)^2 = 1$ D) none of these

Key. A

122. Equation of tangent OT is

A) $4x-3y=0$ B) $x-\sqrt{3}y=0$

C) $y-\sqrt{3}x=0$ D) $x+\sqrt{3}y=0$

Key. A

Sol. <u>Solutions for 33-35</u>

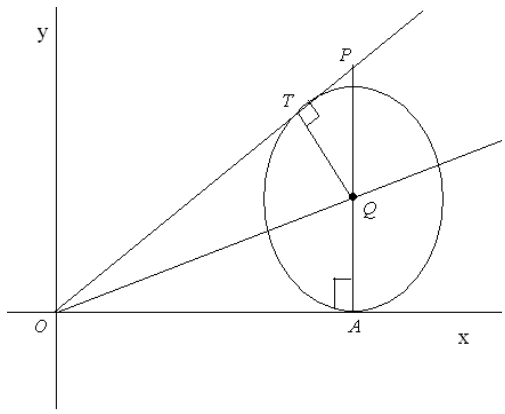
Given $QT = QA = 1$

Let $PQ = x$, then $PT = \sqrt{x^2-1}$

Then ΔTQP and ΔAPO are similar triangles

Then $OT = OA = \frac{x+1}{\sqrt{x^2-1}}$

$$\Rightarrow 1 + x + \frac{2(x+1)}{\sqrt{x^2-1}} + \sqrt{x^2-1} = 8 \Rightarrow x = \frac{5}{3}$$



$AP = \frac{8}{3}, OP = \frac{10}{3}$; and OA=2

$\therefore Q = (2,1)$

Equation of the circle is $(x-2)^2 + (y-1)^2 = 1$

Coordinates of P are $\left(2,\frac{8}{3}\right)$

$\therefore$ equation of OT is 4x-3y=0

Paragraph – 42

At times the methods of coordinates becomes effective in solving problems of properties of triangles. We may choose one vertex of the triangle as origin and one side passing through this vertex as x-axis. Thus without loss of generality, we can assume that every triangle ABC has a vertex situated at (0 ,0) another at (x, 0) and third one at (h, k).

123. If in $\Delta ABC, AC=3, BC=4$ medians AD and BE are perpendicular then area of ΔABC ______ sq.units.

a) $\sqrt{7}$ b) $\sqrt{11}$

c) $2\sqrt{2}$ d) $2\sqrt{11}$

Key. B

Sol. Take B as origin, BC as x-axis and take A as (h,k) C (4,0).

Area of $\Delta ABC=\frac{1}{2}4\times k=2k$ -----(1)

D = (2,0) and $E\left(\frac{h+4}{2},\frac{k}{2}\right)$

$\because AD\perp BE$ slope of AD × slope of BE = -1

$\Rightarrow k^2+(h+4)(h-2)=0\rightarrow(2)$

Also AC = 3 $\Rightarrow (h-4)^2+k^2=9\rightarrow(3)$

(2) - (3) $\Rightarrow h=\frac{3}{2}$ and $k^2=\frac{11}{4}$

$k=\frac{\sqrt{11}}{2}$

From (1): Area of $\Delta ABC=\sqrt{11}$

124. Suppose the bisector AD of the interior angle A of ΔABC divides side BC into segments BD=4; DC=2 then

a) $b>c$ and c<4 b) $2<b<6$ and c<1

c) $2<b<6$ and 4<c<12 d) $b<c$ and c>4

Key. C

Sol. Now AD is the bisector

$\frac{AB}{AC}=\frac{BD}{DC}\Rightarrow c=2b$

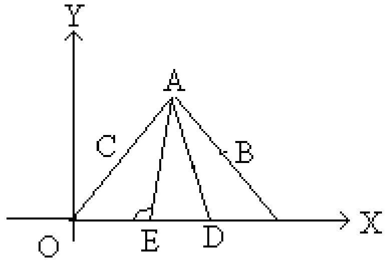
$b+c>a\Rightarrow b+c>6$

$\therefore b>2$

Again $\frac{b^2+4b^2-3b}{4b^2}<1$

$\Rightarrow b<6$

$\therefore 2<b<b$ and consequently $4<c<12$



125. If in the above question (34), altitude $AE>\sqrt{10}$ and suppose lengths of AB and AC are integers, then b will be

a) 3
b) 6
c) 4 or 5
d) 3 or 6

Key. C

Sol. Now $c^2=h^2+k^2$ and $b^2=(h-b)^2+k^2$

$c^2-b^2=12h-3b \Rightarrow h=\frac{b^2+12}{4}$

Given that $k^2>10 \Rightarrow c^2-h^2>10$

$$\Rightarrow 4b^2-\left(\frac{b^2+12}{4}\right)^2>10$$

$$\Rightarrow b^2+\left(20-\sqrt{96},20+\sqrt{96}\right)$$

B is either 4 or 5

Paragraph – 43

$ABCD$ is a square with $A=(-4,0), B=(4,0)$ and other vertices of the square lie above the x-axis. Let O be the origin and O^1 be the mid point of CD. A rectangular hyperbola passes through the points C,D,O and its transverse axis is along the straight line OO^1.

126. The centre of the hyperbola is

A) $(0,4)$
B) $(0,3)$
C) $(0,5)$
D) $(0,2)$

Key. B

127. One of the asymptotes of the hyperbola is

A) $2x+y=3$
B) $y=2x+3$
C) $y=x+3$
D) $y=4-x$

Key. C

128. The area of the larger region bounded by the hyperbola and the square is

A) $20+8\log 3$
B) $44-9\log 3$
C) $44+8\log 3$
D) $44+9\log 3$

Key. D

Sol. $(132-134)$

The equation of Hyperbola is $(y-3)^2 - x^2 = 9$

Paragraph – 44

Consider the conic defined by $x^2 + y^2 = (3x+4y+10)^2$.

129. If (α, β) is the centre of the conic then $4\alpha + 3\beta =$

A) –8 B) –10 C) –6 D) –9

Key. B

130. If (p, q) is a vertex of the conic then $2p - q =$

A) –1 B) 1 C) –3 D) 2

Key. A

131. The number of points through which a pair of real perpendicular tangents can be drawn to the conic is

A) infinite B) 1 C) 0 D) 4

Key. C

Sol. (129 – 131)

The given equation can be expressed as $\sqrt{x^2 + y^2} = 5\frac{|3x+4y+10|}{5}$

Hence it is Hyperbola with eccentricity 5.

Focus is (0, 0)

Directrix is $3x + 4y + 10 = 0$

And hence the axis is $4x - 3y = 0$

Paragraph – 45

Consider the parabola $y^2 = 4x$. Let $A = (-1, 0)$ and $B = (0, 1)$. *F* is the focus of the parabola. Answer the following questions

132. If $P(\alpha, \beta)$ is a point on the parabola such that $\left\| PA \right| - \left| PB \right\|$ is maximum then $\alpha + \beta =$

A) 4 B) $5\sqrt{2}$ C) 3 D) $4\sqrt{3}$

Key. C

133. If $P(\alpha, \beta)$ is a point on the parabola such that $\left\| PA \right| - \left| PB \right\|$ is minimum then a value of $2\alpha + \beta$ is

A) 4 B) 3 C) $4\sqrt{2}$ D) $2\sqrt{3}$

Key. A

134. If $L = (4, 3)$ and $Q(a, b)$ is a point on the parabola such that $|FQ| + |QL|$ is least then $a + b =$

A) 6 B) 19/2 C) 20/3 D) 21/4

Key. D

Sol. 132 – 134:

132. $|PA-PB|$ is max when P, A, B are collinear and P divides AB externally

Equation of AB is $-x+y=1$. i.e., $y=x+1$

$(x+1)^2=4x \Rightarrow x=1$

$\therefore \overleftrightarrow{AB}$ intersect parabola at (1, 2)

133. Minimum value of $|PA-PB|=0$. i.e., P lies on the perpendicular bisector of AB which is $y=-x$.

This line meets the parabola at (0, 0), $(4,-4)$.

134. (4, 3) lies inside the parabola $y^2=4x$

$|FQ|+|QL|$ is least when LQ is a diameter of the parabola.

Paragraph – 46

The length of latusrectum of a parabola which does not meet the X-axis is 1. The parabola passes through the point (0, 3) and it is symmetric with respect to the line $x=1$. B is the point of intersection of the line $y=11$ and parabola and the point B lies in first quadrant.

Then answer the following questions.

135. Sum of the co-ordinates of focus of the parabola is

A) 11/2 B) 13/4 C) 9/2 D) 7/4

Key. B

Sol. Conceptual

136. Magnitude of cross product of vectors $\overrightarrow{OA}, \overrightarrow{OB}$ is

A) 3/2 B) 4 C) 3 D) 5/2

Key. C

Sol. Conceptual

137. The area bounded by the parabola and the line $y=3$ is

A) 4/3 B) 5/3 C) 7/3 D) 28/3

Key. A

Sol. Conceptual

Paragraph – 47

Consider the circles $x^2+y^2=a^2$ and $x^2+y^2=b^2$ where $b>a>0$. Let $A(-a,0); B(a,0)$. A parabola passes through the points A, B and its directrix is a tangent to $x^2+y^2=b^2$. If the locus of focus of the parabola is a conic then

138. The eccentricity of the conic is

A) $2a/b$ B) b/a C) a/b D) 1

Key. C

139. The foci of the conic are

A) $(\pm 2a, 0)$ B) $(0, \pm a)$ C) $(\pm a, 2a)$ D) $(\pm a, 0)$

Key. D

140. Area of triangle formed by a latusrectum and the lines joining the end points of the latusrectum and the centre of the conic is

A) $\frac{a}{b}\left(b^2-a^2\right)$ B) $2ab$ C) $ab/2$ D) $4ab/3$

Key. A

Sol. 138 – 140:

$x^2+y^2=a^2;\ x^2+y^2=b^2;\ b>a>0,\ A=(-a,0);\ \ B=(a,0)$

Let (h,k) be a point on the locus. Any tangent to circle $x^2+y^2=b^2$ is $x\cos\theta+y\sin\theta=b$

$\therefore$ Equation of parabola is $\sqrt{(x-h)^2+(y-K)^2}=\left|x\cos\theta+y\sin\theta-b\right|$

i.e., $(x-h)^2+(y-K)^2=(x\cos\theta+y\sin\theta-b)^2$

The points $(\pm a,0)$ satisfy this equation

$\therefore (a-h)^2+K^2=(a\cos\theta-b)^2$ --- (1)

$(a+h)^2+K^2=(a\cos\theta+b)^2$ ---- (2)

(2) – (1) $\Rightarrow h=b\cos\theta$

$\therefore$ Required locus is $(a+x)^2+y^2=\left(\frac{ax}{b}+b\right)^2$

i.e., $\frac{x^2}{b^2}+\frac{y^2}{b^2-a^2}=1$ which is an ellipse.

Paragraph – 48

Consider a hyperbola x y = 4 and a line $y+2x=4$. O is the centre of hyperbola. Tangent at any point P of hyperbola intersect the coordinate axes at A and B

141. Locus of circum centre of triangle OAB is

A) an ellipse with eccentricity $1/\sqrt{2}$ B) an ellipse with eccentricity $1/\sqrt{3}$

C) a hyperbola with eccentricity $\sqrt{2}$ D) a circle

Key. C

142. Shortest distance between the line and hyperbola is

A) $8\sqrt{2}/\sqrt{5}$ B) $\frac{4\left(\sqrt{2}-1\right)}{\sqrt{5}}$ C) $\frac{2\sqrt{2}}{\sqrt{5}}$ D) $\frac{4\left(\sqrt{2}+1\right)}{\sqrt{5}}$

Key. B

143. Let the given line intersects the x-axis at R . If a line through R, intersect the hyperbolas at S and T then, then minimum value of RS$\times$RT is ___

A) 2 B) 4 C) 6 D) 8

Key. D

Sol. 141,142&143

(141) c ; 142) b ; 143) d) Let $\left(2t, \frac{2}{t}\right)$ be a point on the hyperbola. Equation of the tangent at this point $x + yt^2 = 8t$. A=(8t , 0) , B=(0 , 8/t)

Locus of circumcentre of triangle OAB is its eccentricity is $= \sqrt{2}$

Shortest distance exist along the common normal. $t^2 = \frac{1}{2} \Rightarrow t = \frac{1}{\sqrt{2}}$, foot of the perpendicular is $\left(\sqrt{2}, 2\sqrt{2}\right)$; shortest distance is $\frac{4\left(\sqrt{2}-1\right)}{\sqrt{5}}$. Let R(2, 0) & $S\left(2+\cos\theta, r\sin\theta\right)$ lies on hyperbola $|r_1 r_2| = \frac{8}{|\sin 2\theta|}$; minimum of $RS \times RT$ is 8

Paragraph – 49

The chord AC of the parabola $y^2 = 4ax$ subtends an angle of 90^O at points B and D on the parabola. If A,B,C and D are represented by t_1, t_2, t_3 and t_4 then

144. Value of $\left|\frac{t_2 + t_4}{t_1 + t_3}\right| = \_\_$

A) 0 B) 1 C) 2 D) 4

Key. B

145. Minimum value of $|t_1 - t_3| = \_\_\_$

A) 0 B) 1 C) 2 D) 4

Key. D

146. The y-coordinate of the mid point of the points of intersection of the tangents at A,C and B,D is __

A) 0 B) 1 C) 2 D) 4

Key. A

Sol. 144,145&146

We have $\left(t_2 + t_3\right)\left(t_2 + t_1\right) = -4$; $t_2^{\,2} + t_2\left(t_1 + t_3\right) + t_1 t_3 + 4 = 0 \rightarrow (1)$

$\left(t_1 + t_3\right)^2 - 4\left(t_1 t_3 + 4\right) \ge 0$; $\left(t_1 - t_3\right)^2 \ge 16$; $t_2 + t_4 = -1\left(t_1 + t_3\right)$ $\& \, t_2 t_4 = t_1 t_3 + 4$

Because t_2 and t_4 are y-coordinates of intersection points of the tangents at (A,C) and (B,D) then $y_1 == t_1 + t_3,\ y_2 = t_2 + t_4 \Rightarrow y_1 + y_2 = 0$

Paragraph – 50

The points P,Q,R are taken on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with eccentricities $\theta,\ \theta + \alpha,\ \theta + 2\alpha$ then

147. Area of the triangle PQR is independent of

A) θ B) α C) $\theta \,\&\, \alpha$ both D) none

Key. A

148. If the area of triangle PQR is maximum, then

A) $\alpha = \pi/3$ B) $\alpha = \pi/2$ C) $\alpha = 2\pi/3$ D) none

Key. C

149. If A_1 be the area of triangle PQR and A_2 be the area of the triangle formed by corresponding points on the auxiliary circle then $\frac{A_1}{A_2}$ is ___.

A) 1 B) a/b C) b/a D) none

Key. C

Sol. 147,148 & 149

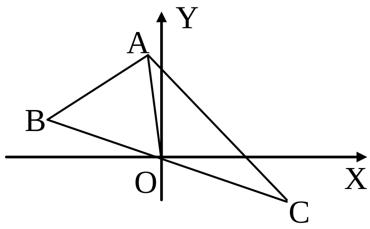
$$A_1 = \Delta = \frac{1}{2}\begin{vmatrix} a\cos\theta & b\sin\theta & 1 \\ a\cos(\theta+\alpha) & b\sin(\theta+\alpha) & 1 \\ a\cos(\theta+2\alpha) & b\sin(\theta+2\alpha) & 1 \end{vmatrix} = ab(1-\cos\alpha)\sin\alpha$$

Δ is max $\Rightarrow \alpha = 2\pi/3$;

$$A_1 = \frac{1}{2}\begin{vmatrix} a\cos\alpha & b\sin\alpha & 1 \\ a\cos\beta & b\sin\beta & 1 \\ a\cos\gamma & b\sin\gamma & 1 \end{vmatrix} = A_2 = \frac{1}{2}\begin{vmatrix} a\cos\alpha & b\sin\alpha & 1 \\ a\cos\beta & b\sin\beta & 1 \\ a\cos\gamma & b\sin\gamma & 1 \end{vmatrix} \quad \therefore A_1/A_2 = b/a$$

Paragraph – 51

In the adjacent figure AO (O being origin) is the median through the vertex A of the triangle ABC.



Now, considering two upward parabolas P_1 and P_2;

P_1 : $y = x^2 + 2px + q$, $(p, q \in R)$ is passing through A and has its vertex at B

P_2 : $y = ax^2 + 2bx + 1$, $(a, b \in R)$ is passing through A and has its vertex at C.

Answer the following:

150. Which of the following is correct?

(A) $p^2 - b^2 > q - a$ (B) $(p^2 - q)(b^2 - a) > 0$

(C) $p^2 + b^2 = q + a$ (D) $\frac{a}{b^2} + q > p^2 + 1$

Key. D

Sol. As mid–point of BC is at origin,

$y_B + y_C = 0 \Rightarrow -(p^2 - q) - \left(\frac{b^2 - a}{a}\right) = 0$

$\Rightarrow ap^2 - qa + b^2 - a = 0$ ... (i)

Also $D_1 . D_2 < 0 \Rightarrow (p^2 - q)(b^2 - a) < 0$

$\Rightarrow p^2b^2 - ap^2 - qb^2 + aq < 0$ ... (ii)

Adding (i) & (ii) we get, $p^2b^2 + b^2 - qb^2 - a < 0$

$\Rightarrow (p^2 + 1) b^2 < a + qb^2$.

151. The product of roots of the equation $x^2 + 2px + q = 0$ must lie in the interval

(A) (0, 1/4) (B) (1/4, 1/2)

(C) $(1, \infty)$ (D) (1/2, 4)

Key. C

Sol. Clearly $y_1(0) > y_2(0)$

So, $q > 1$.

152. Which of the following is not correct?

(A) the sum of all the roots of the equation $(x^2 + 2px + q)(ax^2 + 2bx + 1) = 0$ is zero

(B) $ab < 0$

(C) $pq > 0$

(D) $ap + b \neq 0$

Key. D

Sol. $x_B = -x_C \Rightarrow -p = \frac{b}{a} \Rightarrow ap + b = 0$.

Paragraph – 52

Given two parabolas $y^2 = 4ax$ & $x^2 = 4by$. Then

153. Equation of their common tangents if a = b

(A) $x + y + 2a = 0$ (B) $x - y + a = 0$

(C) $x + y - a = 0$ (D) $x + y + a = 0$

Key. D

SOL. Equation of any tangent is $y = mx + a/m$ ... (i)

Equation of any tangent to $x^2 = 4by$ is $x = my + b/m \Rightarrow \; y = \frac{1}{m}x - \frac{b}{m^2}$ ... (ii)

(i) & (ii) are identical equation

$\therefore$ $1/1 = m/(1/m) = (1/m)/(-1/m^2)$ (as a = b)

$\Rightarrow m = -1$

$\therefore$ equation is $y = -x + a/(-1)$

$\Rightarrow y = -x - a$

$\Rightarrow x + y + a = 0$

154. Which of the following statement is true

(A) Point of contact of common tangent is at the vertex of parabola

(B) Point of contact of common tangent is at the end of latus rectum of both parabola

(C) Point of contact of common tangent is at the end of latus rectum of $y^2 = 4ax$ only

(D) Point of contact of common tangent is at the end of latus rectum of parabola $x^2 = 4by$ only

Key. B

Sol. for $y^2 = 4ax$, the equation of tangent at (x_1, y_1) is

$yy_1 = 2a(x + x_1)$

the equation of common tangent is $y = -x - a$

$\therefore$ comparing both equation $\frac{y_1}{1} = \frac{2a}{-1} = \frac{2ax_1}{-a} \Rightarrow x_1 = a,\ y_1 = -2a$

which is the end of latus-rectum of parabola $y^2 = 4ax$

155. If tangents drawn from any point to the parabola $y^2 = 4ax$ becomes normal to $x^2 = 4by$ then
(A) $a^2 > 8b^2$ (B) $b^2 > 8a^2$
(C) $a^2 < 8b^2$ (D) $b^2 < 8a^2$

Key. A

Sol. Normal to the parabola
$x = my – 2bm – bm^3$
if it becomes tangent to
$y^2 = 4ax$
then $2b + bm^2 = am$
$\Rightarrow bm^2 – am + 2b = 0$
for m to be real and distinct
$\mathbf{A^2 > 8B^2}$

Paragraph – 53

The equation of the normal to the parabola $y^2 = 4ax$ at a point (x_1, y_1) is $y - y_1 = \dfrac{-y_1}{2a}(x - x_1)$.
The equation of the normal to the parabola $y^2 = 4ax$ at $(at^2, 2at)$ is $y + tx = 2at + at^3$.
The equation of normal of slope m to the parabola $y^2 = 4ax$ is $y = mx - 2am - am^3$ at the point $(am^2, - 2am)$.

156. If the normal at a point $P(at^2 , 2at)$ to the parabola $y^2 = 4ax$ subtends a right angle at the vertex of the parabola, then t^2 is equal to
(A) 1 (B) 2
(C) 3 (D) 4

Key. B

Sol. The equation of the normal to the parabola $y^2 = 4ax$ at P is $y + tx = 2at + at^3$. Suppose it meets the parabola at Q. If O is the vertex of the parabola, then the combined equation of OP and OQ is a homogenous equation of second degree given by
$$y^2 = 4ax\left(\frac{y+tx}{2at+at^3}\right)$$
$\Rightarrow y^2 (2at + at^3) = 4ax (y + tx)$
If OP anbd OQ are at right angles, then the coefficient of x^2 + coefficient of $y^2 = 0$
$\Rightarrow 4at - 2at - at^3 = 0 \Rightarrow t^2 = 2$

157. The equations of the normals at the ends of the latus rectum of the parabola $y^2 = 4ax$ is
(A) $x - y - 3a = 0$ (B) $x + y - 3a = 0$
(C) $x + y + 3a = 0$ (D) $x - y + 3a = 0$

Key. B

Sol. The coordinates of the ends of the latus of the parabola $y^2 = 4ax$ are (a, 2a) and (a, - 2a) is
$y - 2a = (-2a/2a) (x - a)$ or $x + y - 3a = 0$

158. If the normals at two points P and Q of a parabola $y^2 = 4ax$ intersect at a third point R on the curve, then the product of the ordinates of P and Q is
(A) $9a^2$ (B) $10 a^2$

(C) $8a^2$ (D) a^2 .

Key. C

Sol. Let P $(at_1^2, 2at_1)$ and Q $(at_2^2, 2at_2)$ be two points on the parabola $y^2 = 4ax$.

It is given that the normal at P and Q intersect at R on the parabola.

$\therefore t_1t_2 = 2$

So product of the ordinates at P and Q

$= (2at_1)(2at_2) = 4a^2 \times t_1t_2 = 8a^2$

Paragraph – 54

Consider the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with centre "O" where a >b>0. Tangent at any point P on the ellipse meets the co-ordinate axis at X and Y and N is the root of the perpendicular from the origin on the tangent at P. Minimum length of XY is 24 and maximum length of PN is 8.

159. The eccentricity of the ellipse is

a) 2/5 b) 3/5 c) $\frac{\sqrt{3}}{2}$ d) ¾

160. maximum area of an isosceles triangle inscribed in the ellipse of one of its vertex coincides with one end of the major axis of the ellipse is

a) $196\sqrt{3}$ b) $96\sqrt{3}$ c) 96 d) $3\sqrt{96}$

161. Maximum area of the triangle OPN is

a) 96 b) $96\sqrt{3}$ c) $196\sqrt{3}$ d) 48

Sol. 159. (c) $a+b=24$, $a-b=8$

160. (b) $\frac{3\sqrt{3}}{4}ab$

161. (d) $\frac{a^2-b^2}{4}$

Paragraph – 55

Let $f(x) = \sin x - x\cos x,\ x \in R$.

162. The least positive value of x, for which f(x) = 0, lies in quadrant

(A) I (B) II (C) III (D) IV

163. The set of all values of $x \in (0, 2\pi)$, for which f(x) > 0, is

(A) $(0, \pi)$ (B) $(\pi, 2\pi)$ (C) $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ (D) none of these

164. If α is the least positive value for which $\tan\alpha = \alpha$, then the area bounded by y = f(x), x-axis, x = 0 and
x = 2π is

(A) 4 (B) $4(1 - \cos\alpha)$ (C) $4(1 + \cos\alpha)$ (D) none of these

Sol. 162. (C)

$f(x) = 0 \Rightarrow \tan x = x$ (we can divide by cos x, as $x = \pi/2$, does not satisfy f(x) = 0).

$\Rightarrow$ x lies in the IIIrd quadrant.

(can be seen using graphs of y = tan x and y = x)

163. (D)

For $x \in \left(0, \frac{\pi}{2}\right)$, f(x) > 0, as here tan x > x.

at $x = \frac{\pi}{2}$, obviously f(x) > 0.

For $x \in \left(\frac{\pi}{2}, \pi\right]$, sin x is positive, while x cos x is negative and hence here also f(x) > 0.

If α is the least positive value for which tan x = x, then f(x) > 0 for $x \in (\pi, \alpha)$.

for $x \in [\alpha, 2\pi), f(x) \le 0$

Thus required set is $(0, \alpha)$.

164. (D)

Required area = $\int_0^{\alpha} f(x)dx - \int_{\alpha}^{2\pi} f(x)dx = -(2\cos x + x\sin x)_0^{\alpha} + (2\cos x + x\sin x)_{\alpha}^{2\pi}$

$= -(2\cos\alpha + \alpha\sin\alpha) + (2) + (2) - (2\cos\alpha + \alpha\sin\alpha)$

$= 4 - 2(2\cos\alpha + \alpha\sin\alpha) = 4 - 4\cos\alpha - 2\alpha^2.\cos\alpha$ (as $\sin\alpha = \alpha\cos\alpha$)

$= 4 - 2(2 + \alpha^2)\cos\alpha$

Paragraph – 56

To find out the lengths and positions of the axes of the conic whose equation is $ax^2 + 2hxy + by^2 = 1$ ---(1), where the axes of co-ordinates being rectangular, consider a circle of radius 'r' with its' centre at the centre of the conic, whose equation is $\frac{x^2 + y^2}{r^2} = 1$---(2). Subtracting (2) from (1), we obtain $\left(a - \frac{1}{r^2}\right)x^2 + 2hxy + \left(b - \frac{1}{r^2}\right)y^2 = 0$. ---(3), which represents a pair of straight lines through origin and the intersection of (1) and (2). Theses straight lines will be coincident when and only when they lie along the axes of the conic, the condition for which is $\left(a - \frac{1}{r^2}\right)\left(b - \frac{1}{r^2}\right) = h^2$ ----(4). If r_1^2 and r_2^2 be the root and both be +ve, then the conic is an ellipse with $2r_1$ and $2r_2$ as the length of its axes.

Given a conic $5x^2 - 6xy + 5y^2 + 22x - 26y + 29 = 0$, the axes being rectangular. Now answer the following questions.

165. Length of major axis is

a) 4 b) 6 c) 3 d) 2

166. Lengths of minor axis is

a) 3 b) 4 c) 2 d) 1

167. Equation of major and minor axes respectively are

a) $2x+y-1=0, x-2y+3=0$ b) $2x-y+3=0, x+2y+4=0$

c) $x-y+5=0, x+y+3=0$ d) $x-y+3=0, x+y-1=0$

Sol. 165-167. (A) (C) (D) The centre is given by

$5x-3y+11=0,$

$-3x+5y-13=0,$

from which we find x = -1, y = 2.

On transeferring the origin to this point we find that the equation of the conic becomes

$5x^2-6xy+5y^2-8=0$,

that is $\frac{5}{8}x^2-2\left(\frac{3}{8}\right)xy+\frac{5}{8}y^2=1,$

so that $a=\frac{5}{8}, h=-\frac{3}{8}, b=\frac{5}{8}$

the lengths of the semi-axes are then given by

$\left(\frac{5}{8}-\frac{1}{r^2}\right)\left(\frac{5}{8}-\frac{1}{r^2}\right)=\frac{9}{64}$

$\therefore r^3=4 \; or \; L$

these re of course the equations of the axes of the ellipse referred to the new axes of coordinates. The equation of the major axis referred to the original axes will be

$(x+1)-(y-2)=0,$ that is $x-y+3=0,$ and of the minor axis.

$(x+1)+(y-2)=0,$ That is

$x+y-1=0,$ and of the minor axis.

Paragraph – 57

A conic “c” satisfies the differential equation, $\left(1+y^2\right)dx-xydy=0$ and passes through the point (1,0) An ellipse ‘’E’ which is confocal with ”c” having its eccentricity equal to $\sqrt{\frac{2}{3}}$

168. Length of the latus rectum of the conic “C” is

a)1 b)2 c)3 d)4

169. Equation of the ellipse “E” is

a) $\frac{x^2}{9}+\frac{y^2}{4}=1$ b) $\frac{x^2}{4}+\frac{y^2}{9}=1$ c) $\frac{x^2}{1}+\frac{y^2}{3}=1$ d) $\frac{x^2}{3}+\frac{y^2}{1}=1$

170. Locus of the point of intersection of the perpendicular tangents to the ellipse E is

a) $x^2+y^2=4$ b) $x^2+y^2=8$ c) $x^2+y^2=10$ d) $x^2+y^2=12$

Sol. 168 – 170. (B) (D) (A)

$\left(1+y^2\right)dx=xydy$

$2\log x = \log\left(1+y^2\right)+1$

$x=1, y=0 \Rightarrow c=0$

$eqn^n of\ 'c'\ is\ x^2+y^2$

$e=\sqrt{2}$

168. $2a=2$

169. $b^2=a^2\left(1-e^2\right)=1$

ellipse $\frac{x^2}{3}+\frac{y^2}{1}=1$

170. $x^2+y^2=4$

Paragraph – 58

Consider the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with centre "O" where a >b>0. Tangent at any point P on the ellipse meets the co-ordinate axis at X and Y and N is the root of the perpendicular from the origin on the tangent at P. Minimum length of XY is 24 and maximum length of PN is 8.

171. The eccentricity of the ellipse is

a) 2/5 b) 3/5 c) $\frac{\sqrt{3}}{2}$ d) ¾

172. maximum area of an isosceles triangle inscribed in the ellipse of one of its vertex coincides with one end of the major axis of the ellipse is

a) $196\sqrt{3}$ b) $96\sqrt{3}$ c) 96 d) $3\sqrt{96}$

173. Maximum area of the triangle OPN is

a) 96 b) $96\sqrt{3}$ c) $196\sqrt{3}$ d) 48

Sol. 171. (C) $a+b=24$, $a-b=8$

172. (B) $\frac{3\sqrt{3}}{4}ab$

173. (D) $\frac{a^2-b^2}{4}$

Paragraph – 59

A point P moves such that sum of the slopes of the normals drawn from it to the hyperbola xy = 16 is equal to the sum of ordinates of feet of normals. The locus of P is a curve C.

174. The equation of the curve C is

(A) $x^2 = 4y$ (B) $x^2 = 16y$

(C) $x^2 = 12y$ (D) $y^2 = 8x$

175. If the tangent to the curve C cuts the co–ordinate axis in A and B, then the locus of the middle point of AB is

(A) $x^2 = 4y$ (B) $x^2 = 2y$

(C) $x^2 + 2y = 0$ (D) $x^2 + 4y = 0$

176. Area of the equilateral triangle inscribed in a curve C having one vertex is the vertex of curve C.

(A) $772\sqrt{3}$ sq. units (B) $776\sqrt{3}$ sq. units

(C) $760\sqrt{3}$ sq. units (D) $768\sqrt{3}$ sq. units

Sol. 174. (b)

Any point on the hyperbola xy = 16 is $\left(4t, \frac{4}{t}\right)$ of the normal passes through P(h, k), then

$k - 4/t = t^2(h - 4t)$

$\Rightarrow$ $4t^4 - t^3h + tk - 4 = 0$

$\therefore$ $\sum t_1 = \frac{h}{4}$

$\sum t_1t_2 = 0$

$\sum t_1t_2t_3 = -\frac{k}{4}$ and $t_1t_2t_3t_4 = -1$

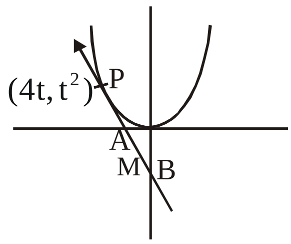
$\therefore$ $\frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} + \frac{1}{t_4} = \frac{k}{4}$ $\Rightarrow$ $y_1 + y_2 + y_3 + y_4 = k$

from questions

$t_1^2 + t_2^2 + t_3^2 + t_4^2 = \frac{h^2}{16} = k$

$\Rightarrow$ Locus of (h, k) is $x^2 = 16y$.

175. (c)



$x^2 = 16y$

Equation of tangent of P is

$x \cdot 4t = \frac{16(y + t^2)}{2}$

$4tx = 8y + 8t^2$

$tx = 2y + 2t^2$

$A = (2t, 0), B = (0, -t^2)$

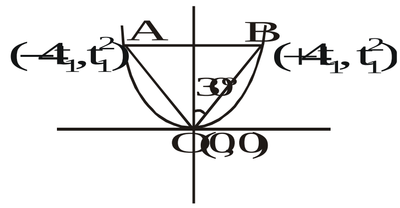
$M(h, k)$ is the middle point of AB.

$h = t, k = -\frac{t^2}{2} \Rightarrow 2k = -h^2$

Locus of $M(h, k)$ is $x^2 + 2y = 0$.

176. (d)

$$\tan 30^\circ = \frac{4t_1}{t_1^2} = \frac{4}{t_1}$$



$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{4}{t_1} \Rightarrow t_1 = 4\sqrt{3}$$

$$AB = 8t_1 = 32\sqrt{3}$$

Area of $\Delta OAB = \frac{\sqrt{3}}{4} \times 32\sqrt{3} \times 32\sqrt{3} = 768\sqrt{3}$ sq.units

Paragraph – 60

An ellipse whose major axis is parallel to x –axis such that the segments of the focal chords are 1 and 3 units. The lines $ax + by + c = 0$ are the chords of the ellipse such that a, b, c are in A.P. and bisected by the point at which they intersect. The equation of its auxiliary circle is $x^2 + y^2 + 2\alpha x + 2\beta y - 2\alpha - 1 = 0$ then

177. The centre of the ellipse is

A) $(1,1)$ B) $(1,2)$ C) $(1,-2)$ D) $(-2,1)$

Key. C
Sol. Conceptual

178. Equation of the auxiliary circle is

A) $x^2 + y^2 - 2x + 4y + 1 = 0$ B) $x^2 + y^2 + 2x + 2y - 3 = 0$

C) $x^2 + y^2 + 2x + 4y + 1 = 0$ D) $x^2 + y^2 - 4x + 2y - 3 = 0$

Key. A
Sol. Conceptual

179. Length of major and minor axis are

A) $4, 2\sqrt{3}$ B) $4, \sqrt{3}$ C) $2, \sqrt{3}$ D) $3, 2\sqrt{3}$

Key. A
Sol. Conceptual

Paragraph – 61

A parabola touches both the axes, and its focus is (3, 4). Then answer the following

180. Equation of axis of this parabola is

a) 3x – 4y + 7 = 0 b) 3x – 4y = 0

c) 4x – 3y = 0 d) None of these

181. Length of its latus rectum is

a) $\frac{24}{5}$ b) $\frac{48}{5}$ c) $\frac{13}{5}$ d) 48

182. The point where it touches the Y-axis is

a) $\left(0, \frac{25}{4}\right)$ b) $\left(0, \frac{24}{5}\right)$ c) $\left(0, \frac{25}{3}\right)$ d) $\left(0, \frac{4}{3}\right)$

KEY : A-B-A

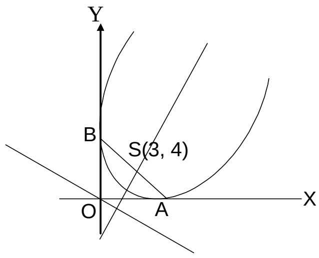
HINT

180. X-axis & Y-axis are $\perp$ er tangents to the parabola. Hence their point of intersection (0, 0) lies on the directrix.

Hence Eq. of directrix is y = mx.

Let A(h, 0), B(0, k) are points of contact.

AB must be a focal chord.



$\angle OSA = 90^0$ & $\angle OSB = 90^0$

$\therefore \frac{4}{3} \times \frac{4}{3-h} = -1$

16 = –9 + 3h

$h = \frac{25}{3}$

Similarly we get $k = \frac{25}{4}$

$\therefore A\left(\frac{25}{3}, 0\right), B\left(0, \frac{25}{4}\right)$

Since B is a point on parabola, $SB^2 = BM^2$ i.e. $3^2 + \left(4 - \frac{25}{4}\right)^2 = \left(\frac{\left|m.0 - \frac{25}{4}\right|}{\sqrt{m^2+1}}\right)^2$,

On solving we get

$m = \pm \frac{4}{3}$

But since directrix make obstuse angle with X-axis

$m = -\frac{4}{3}$

Eq. of directrix is $y - 4 = \frac{3}{4}(x - 3)$

Eq. of axis is

$\Rightarrow\ 3x - 4y + 7 = 0$

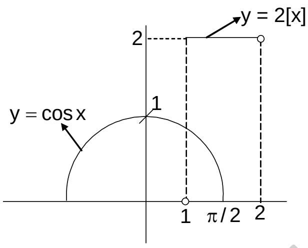
$y = \frac{-4}{3}x \Rightarrow\ 4x + 3y = 0$

Distance b/n focus and directrix is 2a

$2a = \frac{24}{5}$

Latus rectum 4a = 48/5

182.



Paragraph – 62

The normal at any point (x_1, y_1) of curve is a line perpendicular to tangent at the point (x_1, y_1). In case of parabola $y^2 = 4ax$ the equation of normal is $y = mx - 2am - am^3$ (m is slope of normal). In case of rectangular hyperbola $xy = c^2$ the equation of normal at (ct, c/t) is $xt^3 - yt - ct^4 + c = 0$. The shortest distance between any two curve always exist along the common normal.

183. If normal at (5, 3) of rectangular hyperbola $xy - y - 2x - 2 = 0$ intersect it again at a point

(A) (–1, 0) (B) (–1, 1)

(C) (0, –2) (D) (3/4, -14)

184. The shortest distance between the parabola $2y^2 = 2x - 1$, $2x^2 = 2y - 1$ is

(A) $2\sqrt{2}$ (B) $\frac{1}{2\sqrt{2}}$

(C) 4 (D) $\sqrt{\frac{36}{5}}$

185. Number of normals drawn from $\left(\frac{7}{6}, 4\right)$ to parabola $y^2 = 2x - 1$ is

(A) 1 (B) 2

(C) 3 (D) 4

KEY : D-B-A

HINT

183. $xy - y - 2x - 2 = 0$

$(x - 1)(y - 2) = 4$

$XY = 4$

Normal at (ct, c/t) intersect it again at (ct′, c/t′) then $t' = -1/t^3$

$2t = 4$

$t = 2$

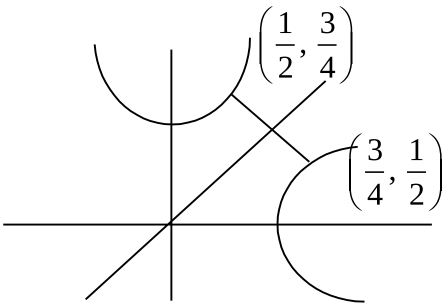
$(X', Y') \equiv \left(-\frac{1}{4}, -16\right)$

$(x', y') \equiv (3/4, -14)$

184. $2y\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{2y} = 1 \Rightarrow y = \frac{1}{2}$

$d = \sqrt{\frac{1}{16} + \frac{1}{16}} = \frac{1}{2\sqrt{2}}$



185. $y^2 = 2(x - \frac{1}{2})$

$Y^2 = 2X$

For 3 normals $X > 1$

$x > 3/2$

$\Rightarrow$ only one normal can be drawn.

Paragraph – 63

A variable line $y = l(x)$ intersects the parabola $y = x^2$ at points P and Q whose x-coordinates are α and β respectively with $\alpha < \beta$. The area of the figure enclosed by the segment PQ and the parabola is always equal to $\frac{4}{3}$. The variable segment PQ has its middle point as M

186. The value of $(\beta - \alpha)$ is equal to

(A) 1 (B) 2

(C) 4 (D) 8

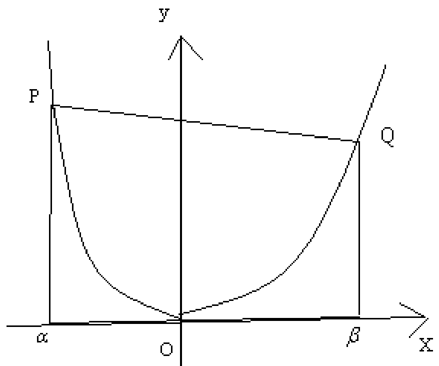
Key: B

Hint: $y = x^2$

Any two points on it can be taken as

$P(\alpha, \alpha^2), Q(\beta, \beta^2)$

Eq. of line PQ is $y - \alpha^2 = (\beta + \alpha)(x - \alpha)$



i.e., $y=\alpha^2+(\beta+\alpha)x-(\beta+\alpha)\alpha$

$y=(\beta+\alpha)x-\alpha\beta$

Req. Area - $\int_{\alpha}^{\beta}\left(\left((\alpha+\beta)x-\alpha\beta\right)-x^2\right)dx$

$\frac{4}{3}=\left((\alpha+\beta)\frac{x^2}{2}-\alpha\beta x-\frac{x^3}{3}\right)_{\alpha}^{\beta}$

on simplification we get $(\beta-\alpha)^3=8$

$\therefore \beta-\alpha=2$

187. Equation of the locus of the mid point of PQ is

(A) $y=1+x^2$ (B) $y=1+4x^2$

(C) $4y=1+x^2$ (D) $2y=1+x^2$

Key: A

Hint: Mid point of $PQ=\left(\frac{\alpha+\beta}{2},\frac{\alpha^2+\beta^2}{2}\right)$

$\therefore x=\frac{\alpha+\beta}{2}, y=\frac{\alpha^2+\beta^2}{2}$

$2x=\alpha+\beta$, $2y=\alpha^2+\beta^2$

$2y=\frac{(\alpha+\beta)^2+(\alpha-\beta)^2}{2}$

$=\frac{4x^2+4}{2}$

Ans : A

188. Area of the region enclosed between the locus of M and the pair of tangents of it from the origin, is

(A) $\frac{8}{3}$ (B) 2

(C) $\frac{4}{3}$ (D) $\frac{2}{3}$

Key: D

Hint: Pair of tangents from origin are

$y=2x$ & $y=-2x$

Req. Area = $\int_0^1 \left(x^2+1\right)-\left(2x\right)dx$

$$=2\left(\frac{\left(x-1\right)^3}{3}\right)_0^1=2\left(\frac{1}{3}\right)=\frac{2}{3}$$

Paragraph – 64

$H : x^2 - y^2 = 9$, $P : y^2 = 4(x - 5)$, $L : x = 9$.

189. If L is a chord of contact of the hyperbola H, then the equation of the corresponding pair of tangents

(a) $9x^2-8y^2+18x-9=0$ (b) $9x^2-8y^2-18x+9=0$

(c) $9x^2-8y^2-18x-9=0$ (d) $9x^2-8y^2+18x+9=0$

Key: b

Hint: Let R(h, k) be the point of intersection of the tangents to H at the extremities of the chord L : x = 9 then equation of L is hx – ky = 9 $\Rightarrow$ h = 1, k = 0.

$\therefore$ coordinates of R are (1, 0).

Equation of the pair of tangents from R to H is

$(x^2 - y^2 - 9)(1 - 9) = (x - 9)^2$ $(SS_1 = T^2)$

$\Rightarrow$ $9x^2 - 8y^2 - 18x + 9 = 0$.

190. If R is the point of intersection of the tangents to H at the extremities of the chord L, then equation of the chord of contact of R with respect to the parabola P is

(a) x = 7 (b) x = 9 (c) y = 7 (d) y = 9

Key: b

Hint: Now equation of the chord of contact of R(1, 0) with respect to the parabola P: $y^2 = 4(x - 5)$ is

$y \times 0 = 2(x + 1) - 20 \Rightarrow x = 9$

191. If the chord of contact of R (as in Q.No. 59) with respect to the parabola P meets the parabola at T and T', S is the focus of the parabola, then area of the triangle STT' is equal to

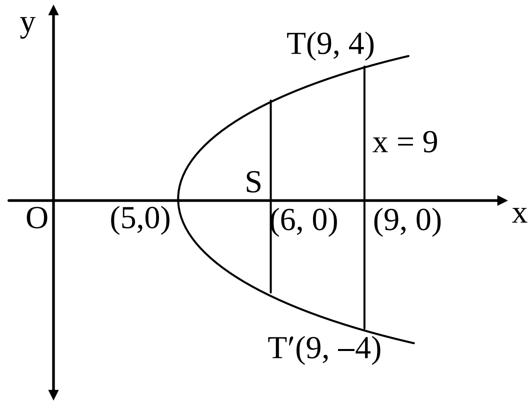
(a) 8 sq. units (b) 9 sq. units (c) 12 sq. units (d) 16 sq. units

Key: c

Hint: Coordinates of T and T' are (9, 4), (9, –4).

Coordinates of focus S of P are (6, 0).

Area of $\Delta SST'$ = 4 × 3 = 12 sq. units.



Paragraph – 65

If a source of light is placed at the fixed point of a parabola and if the parabola is a reflecting surface, then the ray will bounce back in a line parallel to the axis of the parabola.

On the basis of above information, answer the following questions:

192. A ray of light is coming along the line y = 2 from the positive direction of x-axis and strikes a concave mirror whose intersection with the x – y plane is a parabola $y^2 = 8x$, then the equation of the reflected ray is

(A) 2x + 5y = 4 (B) 3x + 2y = 6

(C) 4x + 3y = 8 (D) 5x + 4y = 10

193. A ray of light moving parallel to the x-axis gets reflected from a parabolic mirror whose equation is $y^2 + 10y - 4x + 17 = 0$, After reflection, the ray must pass through the point

(A) (–2, –5) (B) (–1, –5)

(C) (–3, –5) (D) (–4, –5)

194. Two rays of light coming along the lines y = 1 and y = –2 form the positive direction of x–axis and strikes a concave mirror whose intersection with the x – y plane is a parabola $y^2 = x$ at A and B respectively. The reflected rays pass through a fixed point C, then the area of the triangle ABC is

(A) $\frac{21}{8}$ sq unit (B) $\frac{19}{2}$ sq unit

(C) $\frac{17}{2}$ sq unit (D) $\frac{15}{2}$ sq unit

Key : C – B – A

Sol :

192. Point of intersection of y = 2 and $y^2 = 8x$ is $P\left(\frac{1}{2}, 2\right)$ and focus of the parabola is S(2, 0)

$\therefore$ Equation of the reflected ray is $y - 0 = \frac{2-0}{\frac{1}{2}-2}(x-2)$

$\Rightarrow \quad y = -\frac{4}{3}(x-2)$

$\Rightarrow \quad 4x + 3y = 8$

193. Q $y^2 + 10y - 4x + 17 = 0$

$\Rightarrow \quad (y + 5)^2 - 25 - 4x + 17 = 0$

$\Rightarrow \quad (y + 5)^2 = 4x + 8$

$\Rightarrow \quad (y + 5)^2 = 4(x + 2)$

Let $\quad y + 5 = Y,\ x + 2 = X$

ten $\quad Y^2 = 4X$

focus is X = 1, Y = 0

i.e, (–1, –5)

After reflection, the ray must pass through focus (–1, –5)

194. Solving y = 1 and $y^2 = x$

Then $A \equiv (1, 1)$

and solving $\quad y = -2$

and $\quad y^2 = x$

then $\quad B \equiv (4, -2)$

Q After reflection both reflected rays pass through focus of the parabola $y^2 = x$

ie. $\quad C \equiv \left(\frac{1}{4}, 0\right)$

$\therefore$ Required area

Paragraph – 66

A, B, C, D are consecutive vertices of a rectangle whose area is 2006. An ellipse with area 2006π passes through A and C and has foci at B and D.

195. The perimeter of the rectangle is

A) $8\sqrt{2006}$ B) $8\sqrt{1003}$ C) $6\sqrt{1003}$ D) $6\sqrt{2006}$

196. The eccentricity of the ellipse is

A) $\sqrt{\frac{2006}{4009}}$ B) $\sqrt{\frac{3009}{4012}}$ C) $\frac{3}{11}$ D) $\sqrt{\frac{2006}{2009}}$

197. The radius of director circle of the ellipse is

A) $\sqrt{5015}$ B) $\sqrt{4014}$ C) $\sqrt{3003}$ D) $\sqrt{2009}$

Key: B-B-A

Hint: Question nos: 205 – 207

Let 2a, 2b respectively be the lengths of major axis and minor axis of the ellipse. Let the dimensions of the rectangle be x, y then by hypothesis ab = 2006 = xy and $x^2 + y^2 = 4(a^2 - b^2)$.

Paragraph – 67

If the equation of the curve is of the form $y = mx + c + \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + ...$ then y = mx + c will be an asymptote of the given curve, and

(a) The curve lies above the asymptote if

(I) $A \neq 0$ and A and x have same signs

or (II) A = 0, B = 0, and C and x have same signs

(b) The curve lies below the asymptote if

(I) and A and x have opposite signs

or (II) $A \neq 0$, B < 0

or (III) A = 0, B = 0, $C \neq 0$ and C and x have opposite signs.

Now answer the following questions

198. Asymptote of the curve $y^3 = x^2(x-a)$

A) $y = x + \frac{a}{3}$ B) $y = x - \frac{a}{3}$ C) y = ax D) y = –ax

199. Asymptote of the curve $y^5 = x^5 + 2x^4$ is

a) y = x b) 5x + 5y = 2 c) x + y = 2 d) 5x – 5y + 2 = 0

200. In question number 59 if curve lies above the asympote then

a) x < 0 b) x > 0 c) x = 0 d) nothing can be said

Key: B-B-A

Hint: The curve is,

$$y^2 = x^2(x-a) = x^3\left(1-\frac{a}{x}\right)$$

$$\Rightarrow \quad y = x\left(1-\frac{a}{x}\right)^{1/3}$$

$$\Rightarrow \quad y = x\left(1-\frac{1}{3}\frac{a}{x}-\frac{1}{9}\frac{a^2}{x^2}....\right)$$

$$\Rightarrow \quad y = x-\frac{a}{3}-\frac{1}{9}\frac{a^2}{x}....$$ which is of the form

which is of the form

The given curve is

$$y^5 = x^5 + 2x^4$$

or $y^5 = x^5\left(1+\frac{2}{x}\right)$

or $y = x\left(1+\frac{2}{x}\right)^{1/5}$

or $y = x\left(1+\frac{2}{5}\cdot\frac{1}{x}-\frac{8}{25}\cdot\frac{1}{x^2}...\right)$

$$= x + \frac{2}{5} - \frac{8}{25x} + ...$$

The asymptote is ; $y = x + \frac{2}{5}$

(i) Now if $A = -\frac{8}{25}$ and x have the same sign x > 0.

then the curve lies above the asymptote.

Paragraph – 68

In a $\Delta\, ABC$ B(2, 4) , C(6, 4) and A lies on a curve S such that $\tan\frac{B}{2}\tan\frac{C}{2}=\frac{1}{2}$

201. Let a line passing through C and perpendicular to BC intersects the curve S at P and Q. If R is the mid point of BC then area of ΔPQR is

(A) $\frac{18}{3}sq.u$ (B) $\frac{8}{3}sq.u$ (C) $\frac{32}{3}sq.u$ (D) $\frac{26}{3}sq.u$

202. From the data of the above problem the radius of the circle passing through P, B, C is

(A) $\frac{10}{3}units$ (B) $\frac{9}{4}units$ (C) $\frac{16}{3}units$ (D) $\frac{7}{4}units$

203. The eccentricity of the hyperbola whose transverse axis lies along the line through B, C and passes through B, C and (0, 2) is

(A) $\frac{\sqrt{19}}{4}$ (B) $\frac{\sqrt{17}}{2}$ (C) $\sqrt{\frac{7}{3}}$ (D) $\frac{2}{\sqrt{3}}$

Key: C-A-D

Hint: 201, 202, 203

. Given $\frac{S-a}{S}=\frac{1}{2}\Rightarrow b+c=3a\Rightarrow BA+CA=12$

$\therefore$ A lies on ellipse whose foci are B and C

Centre of ellipse = (4, 4) and major axis parallel to x – axis

$\Rightarrow$ Length of major axis = 12 units

$\therefore 12e=4\Rightarrow e=\frac{1}{3}$

$\Rightarrow$ Length of minor axis = $8\sqrt{2}$ units

201. PQ is the latus rectum of the ellipse.

Area = $2\left(\frac{1}{2}\times k_1 e\times\frac{k_2^{\ 2}}{k_1}\right)=ek_2^{\ 2}=\frac{32}{3}sq.units$

202. $\Delta\, PBC$ is right angled at $\angle C$.

203. Equation of Hyperbola is $\frac{(x-4)^2}{4}-\frac{(y-4)^2}{4(e^2-1)}=1$

It passes through (0, 2) $\Rightarrow e=\frac{2}{\sqrt{3}}$

Paragraph – 69

Let R (h, k) be the middle point of the chord PQ of the parabola $y^2=4ax$, then its equation will be $ky-2ax+2ah-k^2=0$

The locus of the mid-point of chords of the parabola which

204. Subtend a constant angle α at the vertex is $\left(y^2-2ax+8a^2\right)^2\tan^2\alpha=\lambda a^2(4ax-y^2)$, where $\lambda=$

(A) 4 (B) 8
(C) 16 (D) 32

Key. C

Sol. Let $P\left(h,k\right)$ be the mid-point, equation of chord through mid point $\left(h,k\right)$

Equation of chord through mid point is $ky-2ax+2ah-k^2=0$

Combined equation of OA and OB will be

$$y^2-4ax\frac{(ky-2ax)}{k^2-2ah}=0$$

$$\tan\alpha=\frac{4a\sqrt{4ah-k^2}}{k^2-2ah+8a^2}$$

$(k^2 - 2h + 8a^2)^2 \tan^2\alpha = 16a^2(4ah - k^2)$

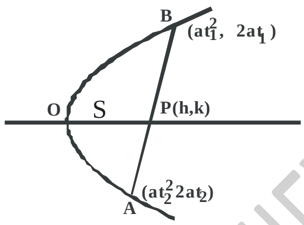
$(y^2-2ax+8a^2)^2\tan^2\alpha=16a^2(4ax-y^2)$

205. Are such that the focal distances of their extremities are in the ratio 2 : 1 is $9(y^2 - 2ax)^2 = \lambda a^2(2x - a)(4x + a)$ where $\lambda =$

(A) 4 (B) 8
(C) 16 (D) 12

Key. A

Sol.



$$t_1^2+t_2^2=\frac{2h}{a} \quad ..... \quad (1)$$

$$t_1+t_2=\frac{k}{a}$$

$$\frac{SB}{SA}=\frac{(t_1^2+1)}{t_2^2+1}=\frac{2}{1} \quad ..... \quad (2)$$

Solving all the equations, we get

$9\left(k^2-2ah\right)=4a^2(2h-a)(4h+a)$

Paragraph – 70

If $P:y^2=8x$ is a parabola having angle between two chords AP and AQ is $\tan^{-1}\left(\sqrt{2}\right)$, where A is vertex and PQ is a chord. Now answer the following

206. If PQ is a focal chord, then $\left|PQ\right|$ is equal to

A) $\frac{3}{2}$ B) $\frac{9}{2}$ C) 3 D) 9

Key. D

Sol. $P=\left(at_1^2,\ 2at_1\right),\ Q=\left(at_2^2,\ 2at_2\right)$

$$PQ=a\left(t_1+\frac{1}{t_1}\right)^2=2\frac{9}{2}=9$$

$$\text{Tan}\theta=\frac{2\left|t_2-t_1\right|}{t_1t_2+4}\Rightarrow\left|t_2-t_1\right|=\frac{3}{\sqrt{2}}\left(\because t_1t_2=-1\right)\ \Rightarrow\left|t_1+\frac{1}{t_1}\right|=\frac{3}{\sqrt{2}}$$

207. The chord PQ always touches an ellipse E whose major axis is

A) $16\sqrt{3}$ B) $18\sqrt{3}$ C) $12\sqrt{3}$ D) $8\sqrt{3}$

Key. A

Sol. Equation of PQ, $y=\frac{2}{t_1+t_2}x+\frac{4t_1t_2}{t_1+t_2}$

Let $\frac{2}{t_1+t_2}=m,\ \frac{2\left(t_2-t_1\right)}{4+t_1t_2}=\sqrt{2}$

$$\Rightarrow t_1t_2=-8\pm\sqrt{48+\frac{8}{m^2}}=\frac{-8m\pm\sqrt{48m^2+8}}{m}$$

$$\frac{4t_1t_2}{t_1+t_2}=2m\left(\frac{-8m\pm\sqrt{48m^2+8}}{m}\right)\Rightarrow-16m\pm\sqrt{192m^2+32}$$

$$\therefore y=mx-16m\pm\sqrt{192m^2+32}$$

$$y=m\left(x-16\right)\pm\sqrt{192m^2+32}$$

$$\therefore E:\frac{\left(x-16\right)^2}{192}+\frac{y^2}{32}=1$$

$$\therefore 2a=2\sqrt{192}=16\sqrt{3}$$

208. The radius of the director circle of ellipse E is

A) $4\sqrt{14}$ B) $14\sqrt{14}$ C) $8\sqrt{14}$ D) $16\sqrt{14}$

Key. A

Sol. $r^2=a^2+b^2=192+32\Rightarrow r=4\sqrt{14}$

Paragraph – 71

Tangent is drawn to the parabola $y^2 = 4x$ at the point P which is the upper end of the latus rectum.

209. Image of the parabola $y^2 = 4x$ in the tangent line at the point P is

(A) $(x+4)^2 = 16y$ (B) $(x+2)^2 = 8(y-2)$
(C) $(x+1)^2 = 4(y-1)$ (D) $(x-2)^2 = 2(y-2)$

Key. C

210. Radius of the circle touching the parabola $y^2 = 4x$ at the point P and passing through its focus, is

(A) 1 (B) $\sqrt{2}$ C) $\sqrt{3}$ D) 2

Key. B

211. Area enclosed by the tangent and normal at 'P' to the parabola and x-axis is

(A) 2/3 (B) 4/3 C) 14/3 D) 4

Key. D

Sol. 209. Here P(1, 2) $\Rightarrow$ Equation of tangent at P is : y = x + 1

Now to find image of the parabola, find image of a variable point on the parabola and then take its locus.

Let the variable point on $y^2 = 4x$ is A(t^2, 2t)

Now image of A w.r.to y = x + 1 is (2t - 1, t^2 + 1)

Hence h = 2t - 1, k = t^2 + 1

Locus(h, k) is $(x+1)^2 = 4(y-1)$

210. By using family of circle equation of the circle is

$(x-1)^2 + (y-2)^2 + \lambda(x-y+1) = 0$

If this circle passes through (1, 0), then $\lambda = -2$

Then equation of the circle is $(x-2)^2 + (y-1)^2 = 2$

211. Area enclosed by parabola, x-axis and y = x+1 is figure

(area of triangle AOB = 1/2)

$$\frac{1}{2} + \int_0^1 (x+1-\sqrt{4x})dx = \frac{2}{3}$$

Paragraph – 72

Tangent is drawn to the parabola $y^2 = 4x$ at the point P which is the upper end of the latus rectum.

212. Image of the parabola $y^2 = 4x$ in the tangent line at the point P is

(A) $(x+4)^2 = 16y$ (B) $(x+2)^2 = 8(y-2)$
(C) $(x+1)^2 = 4(y-1)$ (D) $(x-2)^2 = 2(y-2)$

Key. C

213. Radius of the circle touching the parabola $y^2 = 4x$ at the point P and passing through its focus, is

(A) 1 (B) $\sqrt{2}$ C) $\sqrt{3}$ D) 2

Key. B

214. Area enclosed by the tangent and normal at 'P' to the parabola and x-axis is

(A) 2/3 (B) 4/3 C) 14/3 D) 4

Key. D

Sol. 212. Here P(1, 2) $\Rightarrow$ Equation of tangent at P is : y = x + 1

Now to find image of the parabola, find image of a variable point on the parabola and then take its locus.

Let the variable point on $y^2 = 4x$ is A(t^2, 2t)

Now image of A w.r.to y = x + 1 is (2t - 1, $t^2 + 1$)

Hence h = 2t - 1, k = t^2 + 1

Locus(h, k) is $(x+1)^2 = 4(y-1)$

213. By using family of circle equation of the circle is

$$(x-1)^2 + (y-2)^2 + \lambda(x-y+1) = 0$$

If this circle passes through (1, 0), then $\lambda = -2$

Then equation of the circle is $(x-2)^2 + (y-1)^2 = 2$

214. Area enclosed by parabola, x-axis and y = x+1 is figure

(area of triangle AOB = 1/2)

$$\frac{1}{2} + \int_0^1 (x+1-\sqrt{4x})dx = \frac{2}{3}$$

Paragraph – 73

Normals at P,Q & R on the parabola $y^2 = 4ax$ meet at (α, β) , then

215. Centriod of triangle PQR must be

(A) $\left(\frac{\alpha - 2a}{3}, 0\right)$ (B) $\left(\frac{2\alpha - 4a}{3}, 0\right)$ (C) $\left(\frac{\alpha}{3}, \frac{\beta}{3}\right)$ (D) $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$

Key. 2

216. Ortho centre of triangle PQR must be at

(A) $\left(\alpha + 6a, \beta/2\right)$ (B) $\left(\alpha - 3a, \beta/2\right)$ (C) $\left(\alpha - 6a, \beta/2\right)$ (D) $\left(\alpha + 3a, \beta/2\right)$

Key. 3

217. Circum centre of triangle PQR must be

(A) $\left(\frac{\alpha+2a}{2}, -\beta/4\right)$ (B) $\left(\frac{\alpha+2a}{4}, \beta/4\right)$ (C) $\left(\frac{\alpha}{4}, \beta/4\right)$ (D) $\left(\alpha/3, \beta/3\right)$

Key. 1

Sol. 215-217: Any normal is $y+tx=2at+at^3$ thru (α,β)

$$\Rightarrow at^3+(2a-\alpha)t-\beta=0$$

$$\Rightarrow t_1+t_2+t_3=0 \qquad \sum t_1t_2=\frac{2a-\alpha}{\alpha} \qquad t_1t_2t_3=\frac{\beta}{a}$$

$$\Rightarrow \text{Centroid} = \left(\frac{a}{3}\sum t_1^2, \frac{2a}{3}\sum t_1\right) \equiv \left(\frac{2\alpha-4a}{3}, 0\right)$$

Also orthocenter $\equiv\left(\alpha-6a, \frac{\beta}{2}\right)$

$\Rightarrow$ Circumcentre $\left(\frac{\alpha+2a}{4}, \frac{-\beta}{4}\right)$ (Centriod divides orthocenter and ciurcum centre in 2:1 ratio)

Paragraph – 74

Consider the curve C : $y^2 - 8x - 4y + 28 = 0$. tangents TP and TQ are drawn on C at P(5, 6) and Q (5, –2). Further normals at P and Q meet at R.

218. Circumcentre of ΔPQR is

(A) (5, 3) (B) (5, 4)
(C) (5, 2) (D) (5, 6)

Key. C

219. Area of quadrilateral TPRQ is

(A) 8 (B) 16
(C) 32 (D) 64

Key. C

220. Angle between pair of tangents drawn at the end points of the chord y + 5t = tx + 2 of curve C. $t\in R$ is

(A) 30° (B) 45°
(C) 60° (D) 90°

Key. D

Sol. 218–220. Given curve is a parabola $(y-2)^2 = 8(x-3)$ whose focus is (5, 2)

P(5, 6) and Q(5, –2) are the coordinates of the end points of the latus–rectum of the parabola.

$\therefore$ Normals at P and Q are perpendicular to each other meeting on the axis of the parabola.

$\therefore$ ΔPQR is right angled at R.

$\Rightarrow$ circumcentre of ΔPQR is focus of the parabola i.e. (5, 2)

Obviously quadrilateral TPRQ is a square

area = $\frac{8}{\sqrt{2}}.\frac{8}{\sqrt{2}}=32$.

Also y + 5t = tx + 2 is a focal chord of the given parabola.

$\Rightarrow$ Angle between pair of tangent = $\pi/2$.

Paragraph – 75

A quadratic polynomial $y = f(x)$ with constant term 3 neither touches nor intersects the abscissa axis and is symmetric about the line $x = 1.$ The coefficient of the leading term of the polynomial is unity. Now answer the following questions:

221 Vertex of the quadratic polynomial is

a) $(1,1)$ b) $(2,3)$ c) $(1,2)$ d) (5, 7)

Key. C

222. The area bounded by the curve $y = f(x)$ and a line $y = 3,$ is

a) $\frac{4}{3}$ b) $\frac{5}{3}$ c) $\frac{7}{3}$ d) $\frac{28}{3}$

Key. A

223. The graph of $y = f(x)$ represents a parabola whose focus has the co-ordinates

a) $\left(1, \frac{7}{4}\right)$ b) $\left(1, \frac{5}{4}\right)$ c) $\left(1, \frac{5}{2}\right)$ d) $\left(1, \frac{9}{4}\right)$

Key. D

Sol. 221,222,223

Let $y = ax^2 + bx + c,$ where $c = 3,$ and $a = 1,$ therefore, the curve lies completely above the $x - axis.$

$\therefore\ f(x) = y = x^2 + bx + c$. Line of symmetry being 1, therefore minima occurs at $x = 1$.

$\therefore\ f^1(1) = 0 \Rightarrow 2x + b = 0$ at $x = 1$

$b = -2$

Hence, $f(x) = x^2 - 2x + 3$

Vertex is $(1,2)$.

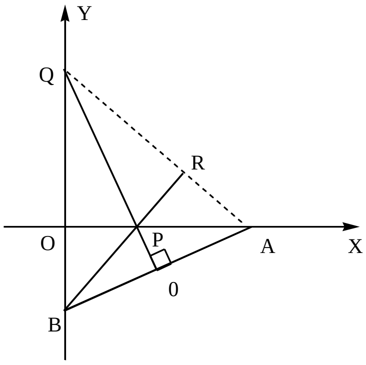
If $y = 3,$ then $x^2 - 2x = 0 \Rightarrow x = 0 \text{ or } 2$

Hence, the area bounded = $\int_0^2 3 - (x^2 - 2x + 3)dx$

$$= \int_0^2 (2x - x^2)\,dx = \left[x^2 - \frac{x^3}{3}\right]_0^2 = 4 - \frac{8}{3} = \frac{4}{3}.$$

Paragraph – 76

Let a line L_1 cuts the coordinate axes at points A and B respectively. The line L_1 is such that it makes intercepts 7 units, 5 units with positive direction of x-axis and negative direction of y-axis respectively. A variable line PQ is drawn perpendicular to AB cutting the x-axis in P and the y-axis in Q. The lines AQ and BP intersect at the point R(as shown in the following figure)



Let T be a point on the path traced by R such that it is farthest from the origin. A parabola C_1 with vertex at T and axis parallel to x-axis is drawn with length of the latusrectum 2 units opening towards positive direction of x-axis. A light ray coming from infinity having the equation $y=\lambda(\lambda \in R^+, be\, a\ parameter)$ reflected from the parabolic mirror C_1 after touching it at point M. The reflected ray passes through a fixed point S.

224. The coordinates of point T will be
a) (7,5) b) (7,-5) c) (-7,5) d) (-7,-5)

Key. B

225. The equation of the parabola C_1 will be
a) $y^2+10y-2x+39=0$ b) $y^2+10y+2x+9=0$ c) $y^2+11y-2x+9=0$ d) $y^2+10y+2x-39=0$

Key. A

226. The abscissa of the point M will always be
a) greater than 19 b) greater than or equals to 19
c) greater than or equal to 7 d) greater than 7

Key. A

Sol. 224. Locus of R is a circle on A(7,0), B(0,-5) as a diameter $x^2+y^2-7x+5y=0$ hence T = (7,-5)

225. Equation of parabola C_1 is $(y+5)^2 = 2(x-7)$

226. Point of contact of the light ray M will have abscissae (since $\lambda \in R^+$) $2x-14 > 25 \Rightarrow$

$$x > \frac{39}{2} \approx 20$$

Paragraph – 77

The general form of a parabola is given by (Perpendicular distance from axis)^2 = $\pm$ L.R(Perpendicular distance from T.V) Now, let us consider the parabola $9x^2-24xy+16y^2-20x-15y-60=0$

227. The equation of the axis is given by
a) $4y-3x=0$ b) $4y+3x=0$ c) $3x-4y+5=0$ d) $3x-4y+7=0$

Key. A

228. Length of the latus rectum will be
a) 1 unit b) 2 units c) 3 units d) 4 units

Key. A

229. The co- ordinates of vertex are
a) $\left(\frac{48}{25},\frac{36}{25}\right)$ b) $\left(\frac{48}{25},\frac{-36}{25}\right)$ c) $\left(\frac{-48}{25},\frac{36}{25}\right)$ d) $\left(\frac{-48}{25},\frac{-36}{25}\right)$

Key. D

Sol. 227. The given equation can be rewritten as

$$\left(\frac{3x-4y}{5}\right)^2 = 1.\left(\frac{4x+3y+12}{5}\right)$$

Hence by the given general form, we can say that equation of its axis is 3x-4y = 0, and equation of T.V. is 4x+3y+12 = 0

228. Comparing equation (i) with the given general form we get L-R = 1 unit

229. Vertex is the intersection point of its axis (3x-4y = 0) and its T.V. (4x+3y+12z = 0) which an solving gives $\left(\frac{-48}{25}, \frac{-36}{25}\right)$.

Paragraph – 78

A parabola is drawn through two given points A(1,0) and B(-1,0) such that its directrix always touches the circle $x^2 + y^2 = 4$. Then

230. The equation of directrix is of the form

a) $x\cos\alpha + y\sin\alpha = 1$ b) $x\cos\alpha + y\sin\alpha = 2$ c) $x\cos\alpha + y\sin\alpha = 3$ d) $x\tan\alpha + y\sec\alpha = 2$

Key. B

231. The locus of focus of the parabola is

a) $\frac{x^2}{4} + \frac{y^2}{3} = 1$ b) $\frac{x^2}{4} + \frac{y^2}{5} = 1$ c) $\frac{x^2}{3} + \frac{y^2}{4} = 1$ d) $\frac{x^2}{5} + \frac{y^2}{4} = 1$

Key. A

232. The maximum possible length of semi latus rectum is

a) $2+\sqrt{3}$ b) $3+\sqrt{3}$ c) $4+\sqrt{3}$ d) $1+\sqrt{3}$

Key. A

Sol. **230 TO 232**

Any point on circle $x^2 + y^2 = 4$ is $(2\cos\alpha, 2\sin\alpha)$

$\therefore$ equation of directrix is $x(\cos\alpha) + y(\sin\alpha) - 2 = 0.$

Let focus be (x_1, y_1) .Then as A(1,0), B(-1,0) lie on parabola we must have

$$\left.\begin{array}{l}(x_1-1)^2 + y_1^2 = (\cos\alpha - 2)^2 \\ (x_1+1)^2 + y_1^2 = (\cos\alpha + 2)^2\end{array}\right\} \Rightarrow x_1 = 2\cos\alpha \ ,.\ y_1 = \pm\sqrt{3}\sin\alpha$$

$\therefore$ locus of focus is $\frac{x^2}{4} + \frac{y^2}{3} = 1$ and focus is of the form $(2\cos\alpha, \pm\sqrt{3}\sin\alpha)$.

$\therefore$ length of semi latus rectum of parabola = $\perp^r$ distance from focus to directrix

$\left|2 \pm \sqrt{3}\right|\sin^2\alpha$

Hence maximum possible length = $2+\sqrt{3}$

Paragraph – 79

Consider the parabola $y^2 = 4x$. Let $A=(-1,0)$ and $B=(0,1)$. *F* is the focus of the parabola. Answer the following questions

233. If $P(\alpha,\beta)$ is a point on the parabola such that $\left||PA|-|PB|\right|$ is maximum then $\alpha+\beta=$

A) 4 B) $5\sqrt{2}$ C) 3 D) $4\sqrt{3}$

Key. C

234. If $P(\alpha,\beta)$ is a point on the parabola such that $\left||PA|-|PB|\right|$ is minimum then a value of $2\alpha+\beta$ is

A) 4 B) 3 C) $4\sqrt{2}$ D) $2\sqrt{3}$

Key. A

235. If $L=(4,3)$ and $Q(a,b)$ is a point on the parabola such that $|FQ|+|QL|$ is least then $a+b=$

A) 6 B) 19/2 C) 20/3 D) 21/4

Key. D

Sol. 233. $|PA-PB|$ is max when P, A, B are collinear and *P* divides AB externally

Equation of AB is $-x+y=1$. i.e., $y=x+1$

$(x+1)^2 = 4x \Rightarrow x=1$

$\therefore AB$ intersect parabola at (1, 2)

234. Minimum value of $|PA-PB|=0$. i.e., *P* lies on the perpendicular bisector of AB which is $y=-x$.

This line meets the parabola at (0, 0), $(4,-4)$.

235. (4, 3) lies inside the parabola $y^2=4x$

$|FQ|+|QL|$ is least when LQ is a diameter of the parabola.

Paragraph – 80

The equation of normal to a parabola $y^2 = 4ax$ is $y = mx - 2am - am^3$. From this conclude that three normals real or imaginary can be drawn from point ‘p’.

236. The locus of ‘p’ such that two normals make complementary angles with axis of parabola is:

a) $y^2 = a(x+a)$ b) $y^2 = a(x-a)$

c) $y^2 = a(y-a)$ d) None

Key. B

237. The locus of ‘p’ if one normal is bisector of other two is

a) $27ay^2 = (2x-a)(x-5a)^2$ b) $y^2 = (x-2a)(x-5a)^2$

c) $x^2 = (y-2a)(y-5a)^2$ d) None

Key. A

238. The locus of a point 'p' if the sum of the angles made by the normals with the axis is a constant is

a) A straight line b) A parabola c) A circle d) An ellipse

Key. A

Sol. $y = mx - 2ax - ax^3$

$ax^3 + (2a - x)m + y = 0$

$\sum m = 0, \sum m_1 m_2 = \frac{2a - x}{a}, \sum m_1 m_2 m_3 = \frac{-y}{a}$

236. $m_1 m_2 = 1$

237. $\frac{m_1 - m_2}{1 + m_1 m_2} = \frac{m_2 - m_3}{1 + m_2 m_3} \& am_2^3 + (2a - x)m_2 + y = 0$

238. $\theta_1 + \theta_2 + \theta_3 = K \quad \Rightarrow \tan(\theta_1 + \theta_2 + \theta_3) = \frac{S_1 - S_3}{1 - S_2} = K$

Paragraph – 81

Normals at three points *P, Q, R* on the parabola $y^2 = 4ax$ meet at (α, β).

239

a) $\left(\frac{\alpha - 2a}{3}, 0\right)$ b) $\left(\frac{2\alpha - 4a}{3}, 0\right)$

c) $\left(\frac{\alpha}{3}, \frac{\beta}{3}\right)$ d) $\left(\frac{4\alpha - 2a}{3}, 0\right)$

Key. B

240. The orthocentre of the triangle *PQR* must be

a) $(\alpha + 6a, \beta/2)$ b) $(\alpha - 3a, \beta/2)$

c) $(\alpha - 6a, -\beta/2)$ d) $(\alpha + 3a, -\beta/2)$

Key. C

241. The circumcentre of ΔPQR must be

a) $\left(\frac{\alpha + 2a}{2}, \frac{\beta}{4}\right)$ b) $\left(\frac{\alpha + 2a}{4}, \frac{\beta}{4}\right)$

c) $\left(\frac{\alpha}{4}, \frac{\beta}{4}\right)$ d) $\left(\frac{\alpha + 2a}{2}, -\frac{\beta}{4}\right)$

Key. A

Sol. Normal at t passes through (α, β).

$\Rightarrow at^3 + (2a - \alpha)t - \beta = 0$.

If t_1, t_2, t_3 are roots, then they correspond to the points P, Q, R.

$t_1 + t_2 + t_3 = 0$.

$$t_1t_2 + t_2t_3 + t_3t_1 = \frac{2a-\alpha}{a}$$

$$t_1t_2t_3 = \frac{\beta}{a}.$$

239. Centroid $= \left[\frac{a}{3}\left(t_1^2+t_2^2+t_3^2\right), \frac{2a}{3}\left(t_1+t_2+t_3\right)\right]$

$$= \left[\frac{a}{3}\left\{\left(t_1+t_2+t_3\right)^2 - 2\left(t_1t_2+t_2t_3+t_3t_1\right)\right\}, \frac{2a}{3}\left(t_1+t_2+t_3\right)\right]$$

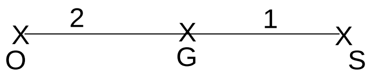
$$= \left[\frac{2\alpha-4a}{3}, 0\right].$$

240. Orthocentre

$$= \left[-a\left(t_1t_2+t_2t_3+t_3t_1+4\right), \frac{a}{2}\left\{\left(t_1+t_2+t_3\right)\left(t_1t_2+t_2t_3+t_3t_1\right)+4\left(t_1+t_2+t_3\right)-t_1t_2t_3\right\}\right].$$

$$= \left[\alpha-6a, -\frac{\beta}{2}\right].$$

241.



$$S = \left[\frac{\alpha+2a}{2}, \frac{\beta}{4}\right].$$

Paragraph – 82

If a circle is drawn of focal distance of any point P, lying on a parabola, as diameter, it touches the tangent at vertex of the parabola at the point where the tangent to the parabola at point P meets the circle.

Now considering a parabola with focus at S(–1, –1). Let 3x–y–8 = 0 be the equation of tangent to the parabola at point P(7, 13). Then

242. The foot of perpendicular from focus upon the tangent to the parabola is

A) $\left(\frac{13}{5}, \frac{1}{5}\right)$ B) (2, –2) C) (–2, 2) D) $\left(\frac{1}{5}, \frac{13}{5}\right)$

Key. B

243. Slope of the normal to the circle through the point found in the previous questions is

A) 4 B) 8 C) –4 D) –8

Key. B

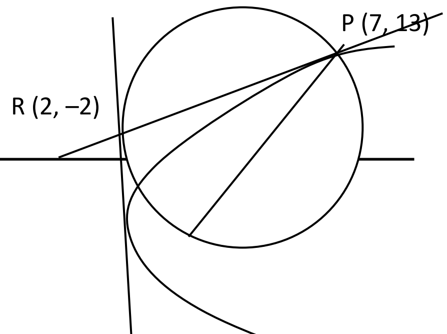
244. Equation of tangent at vertex to the parabola is

A) x–8y+14 = 0 B) 8x–y+14 = 0 C) x+8y+14 = 0 D) 8x+y+14 = 0

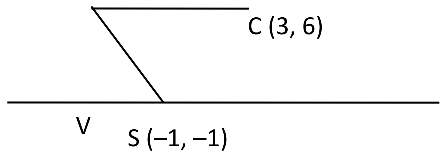
Key. C

Sol. 242.

$$\frac{\alpha+1}{3} = \frac{\beta+1}{-1} = -\frac{(-3+1-8)}{10}$$



$\Rightarrow (\alpha, \beta) = (2, -2)$

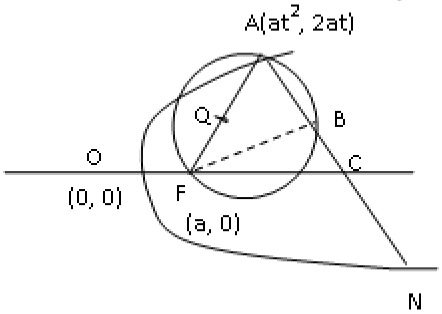


243. Normal at (2, –2) will pass through the centre of circle. Centre = mid point of (7, 13) and (–1, –1)
C = (3, 6)
Slope of RC = $\dfrac{6+2}{3-2} = 8$

244. Tangent at vertex will be perpendicular to RC and passing through R.
$y+2 = -(1/8)(x-2) \Rightarrow x+8y+14 = 0$

Paragraph – 83

Let A(at^2, 2at) be a point on the parabola
$y^2 = 4ax$ where $t \neq 0$, $a > 0$ and F(a, 0).
The normal at A is AN which meets x-axis at C.
If the circle S with AF as diameter and centre Q meets AN at B then answer the following.



245. OF, AB, AF are in
   A) A.P. B) G.P. C) H.P. D) None

Key. B

246. Locus of the point B is
   A) $ay^2 = x-a$ B) circle C) $y^2 = a(x-a)$ D) $2y = x-a$

Key. C

247. If $\dfrac{\text{area of quadrilateral QBCF}}{\text{area of circle S}} = \lambda$ then maximum value of λ is
   A) $\dfrac{3}{2\pi}$ B) $\dfrac{1}{2}$ C) $\dfrac{4}{3\pi}$ D) none

Key. A

Sol. 245. Normal at A is $y+xt = 2at + at^3$ ………… (1)
$\angle FBA = 90^0 \Rightarrow$ B is the foot of perpendicular drawn from F(a, 0) on equation (1)
$\dfrac{x-a}{t} = \dfrac{y-0}{1} = \dfrac{-(0+at-2at-at^3)}{1+t^2}$
B = ($a(1+t^2)$, at) ; OF = a, AB = $a\sqrt{1+t^2}$, AF = $a(1+t^2)$
$\therefore$ OF, AB, AF are in G.P.

246. B($a+at^2$, at) = (x, y) ; eliminating t we get $y^2 = a(x-a)$

247. Point C ($2a+at^2$, 0)

QBCF is a trapezium $\Rightarrow \dfrac{Area\ of\ QBCF}{Area\ of\ circles}=\dfrac{\frac{1}{2}\left[\frac{a}{2}\left(1+t^2\right)+a(1+t^2)\right]at}{\pi a^2\left(\frac{1+t^2}{2}\right)^2}=\dfrac{3}{\pi\left(t+\frac{1}{t}\right)}\le\dfrac{3}{2\pi}$

Paragraph – 84

From a point $p(h,k)$ in general three normals can be drawn to the parabola $y^2=4ax$. If t_1,t_2,t_3 are the parameters associated with the feet of normals, then t_1,t_2,t_3 are the roots of the equation $at^3+(2a-h)t-k=0$ moreover from the line $x=-a$ two perpendicular tangents can be drawn to the parabola.

248. If the feet $Q(at_1^2,2at_1)$ and $R(at_2^2,2at_2)$ are the ends of a focal chord of the parabola, then the locus of $p(h,k)$ is

(a) $y^2=a(x-2a)$ (b) $y^2=a(x-a)$ (c) $y^2=a(x-3a)$ (d) $y^2=3a(x-a)$

Key. C

249. If the tangents at the feet $Q(at_1^2,2at_1)$ and $R(at_2^2,2at_2)$ to the parabola meet on the line $x=-a$ then t_1,t_2 are the roots of the equation

(a) $t^2-t_3t+1=0$ (b) $t^2+t_3t+1=0$ (c) $t^2-t_3t-1=0$ (d) $t^2+t_3t-1=0$

Key. D

250. If $p(h,k)$ is a vertex of the square comprising normals to the parabola from p and tangents from the directrix then (h, k) is the same as

(a) (a , 0) (b) (2a, 0) (c) (3a, 0) (d) (4a, 0)

Key. C

Sol. 248. Normal at t is given by

$y+xt=2at+at^3$

If it passes through (h,k) then

$k+ht=2at+at^3$

$at^3+(2a-h)t-k=0 \quad \rightarrow (1)$

Let t_1,t_2,t_3 be the feet of the normal

Then $t_1+t_2+t_3=0,\ t_1t_2+t_2t_3+t_3t_1=\dfrac{2a-h}{a},\ t_1t_2t_3=\dfrac{k}{a}$

Normal at t_1 is $y+xt_1=2at_1+at_1^3 \quad \rightarrow (1)$

Normal at t_2 is $y+xt_2=2at_2+at_2^3 \quad \rightarrow (2)$

Find the locus of the point of intersection of (1) & (2)

Paragraph – 85

If a circle is drawn of focal distance of any point P, lying on a parabola, as diameter, it touches the tangent at vertex of the parabola at the point where the tangent to the parabola at point P meets the circle.

Now considering a parabola with focus at S(–1, –1). Let 3x–y–8 = 0 be the equation of tangent to the parabola at point P(7, 13). Then

251. The foot of perpendicular from focus upon the tangent to the parabola is

A) $\left(\dfrac{13}{5},\dfrac{1}{5}\right)$ B) (2, –2) C) (–2, 2) D) $\left(\dfrac{1}{5},\dfrac{13}{5}\right)$

Key. B

252. Slope of the normal to the circle through the point found in the previous questions is
A) 4 B) 8 C) –4 D) –8

Key. B

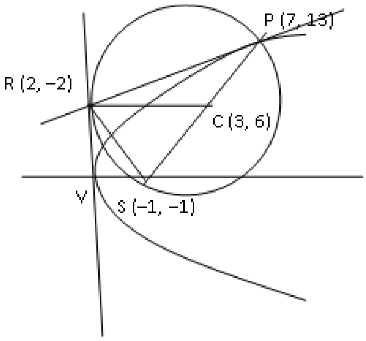
253. Equation of tangent at vertex to the parabola is
A) x–8y+14 = 0 B) 8x–y+14 = 0 C) x+8y+14 = 0 D) 8x+y+14 = 0

Key. C

Sol. 251.

$$\frac{\alpha+1}{3}=\frac{\beta+1}{-1}=-\frac{(-3+1-8)}{10}$$

$$\Rightarrow(\alpha,\beta)=(2,-2)$$



252

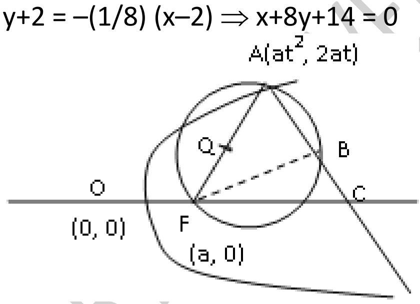
circle. Centre = mid point of (7, 13) and (–1, –1)

C = (3, 6)

Slope of RC = $\frac{6+2}{3-2}=8$

253. Tangent at vertex will be perpendicular to RC and passing through R.

y+2 = –(1/8) (x–2) $\Rightarrow$ x+8y+14 = 0



Paragraph – 86

Let A(at^2, 2at) be a point on the parabola $y^2 = 4ax$ where $t \neq 0$, a > 0 and F(a, 0).
The normal at A is AN which meets x-axis at C.
If the circle S with AF as diameter and centre Q meets AN at B then answer the following.

256. OF, AB, AF are in
A) A.P. B) G.P. C) H.P. D) None

Key. B

257. Locus of the point B is
A) $ay^2 = x–a$ B) circle C) $y^2 = a(x–a)$ D) 2y = x–a

Key. C

258. If $\frac{area\ of\ quadrilateral\ QBCF}{area\ of\ circle\ S}=\lambda$ then maximum value of λ is

A) $\frac{3}{2\pi}$ B) $\frac{1}{2}$ C) $\frac{4}{3\pi}$ D) none

Key. A

Sol. 256. Normal at A is $y+xt = 2at + at^3$ ………… (1)

$\angle FBA = 90^0 \Rightarrow$ B is the foot of perpendicular drawn from F(a, 0) on equation (1)

$$\frac{x-a}{t}=\frac{y-0}{1}=\frac{-(0+at-2at-at^3)}{1+t^2}$$

$B = (a(1+t^2), at)$; OF = a, AB = $a\sqrt{1+t^2}$, AF = $a(1+t^2)$

$\therefore$ OF, AB, AF are in G.P.

257. $B(a+at^2, at) = (x, y)$; eliminating t we get $y^2 = a(x-a)$

258. Point C $(2a+at^2, 0)$

QBCF is a trapezium $\Rightarrow$ $$\frac{Area\ of\ QBCF}{Area\ of\ circles}=\frac{\frac{1}{2}\left[\frac{a}{2}(1+t^2)+a(1+t^2)\right]at}{\pi a^2\left(\frac{1+t^2}{2}\right)^2}=\frac{3}{\pi\left(t+\frac{1}{t}\right)}\le\frac{3}{2\pi}$$

Paragraph – 87

Let C : $y = x^2 - 3$, D: $y = kx^2$, L_1 : $x = a$, L_2 : $x = 1$, $(a \neq 0)$

259. If the parabolas C and D intersect at a point A on the line L_1, then the tangent line L at A to the parabola D is

(A) $2(a^2-3)x - ay + a^3 - 3a = 0$ (B) $2(a^2-3)x - ay + a^3 + 3a = 0$

(C) $(a^2-3)x - 2ay - 2a^3 + 6a = 0$ (D) $2(a^2-3)x - ay - a^3 + 3a = 0$

Key. D

260. If the line L meets the parabola C at a point B on the line :L_2, other than A then ‘a’ is equal to

(A) – 3 (B) – 2 (C) 2 (D) 1

Key. B

261. If a > 0, the angle subtended by the chord AB at the vertex of the parabola C is

(A) $\tan^{-1}(5/7)$ (B) $\tan^{-1}(1/2)$ (C) $\tan^{-1}(2)$ (D) $\tan^{-1}(1/8)$

Key. B

Sol. 259. $A = (a, a^2-3)$ Equation of tangent L is $S_1 = 0$ is $2(a^2-3)x - ay - a^3 + 3a = 0$

260. The line L meets the parabola C : $y= x^2-3$ at the Points for which

$x^2-3 = \frac{2(a^2-3)}{a}x - a^2 + 3 \Rightarrow (x-a)(ax+6-a^2) = 0$ But $x = 1$ and $x \neq a$

$$x = \frac{a^2-6}{a} = 1 \Rightarrow a = -2, 3$$

261. If a>0, then a = 3, A= (3,6), B = (1,-2) equation of C is y = x^2 -3 or x^2 = y+3

Vertex 'O' of the parabola C is (0,-3) slope OA = 3 , slope OB= 1

Paragraph – 88

If a source of light is placed at the fixed point of a parabola and if the parabola is a reflecting surface, then the ray will bounce back in a line parallel to the axis of the parabola. Then answer the following questions.

262. A ray of light is coming along the line y=2 from the positive direction of x-axis and strikes a concave mirror whose intersection with x-y plane is a parabola $y^2 = 8x$, then the equation of the reflected ray is

(A) 2x + 5y = 4 (B) 3x + 2y – 6 = 0 (C) 4x + 3y – 8 = 0 (D) 5x + 4y =10

Key. C

263. A ray of light moving parallel to the x-axis gets reflected from a parabolic mirror whose equation is $y^2 + 10y - 4x + 17 = 0$. After reflection, the ray must pass through the point

(A) (-2, -5) (B) (-1, -5) (C) (-3, -5) (D) (-4, -5)

Key. B

264. Two rays of light coming along the lines y = 1 ; y = -2 from the positive direction of x-axis and strikes a concave mirror whose intersection with the x-y plane is a parabola $y^2 = x$ at A and B respectively. The reflected rays pass through a fixed point C. Then the area of triangle ABC is

(A) $\frac{21}{8}$ sq.units (B) $\frac{19}{2}$ sq.units (C) $\frac{17}{2}$ sq.units (D) $\frac{15}{2}$ sq.units

Key. A

Sol. 262. focus of $y^2 = 8x$ is S(2, 0), put y=2 in $y^2 = 8x$

$x = \frac{1}{2}$ $\therefore$ point of intersection is $P\left(\frac{1}{2}, 2\right)$

Equation of SP is $4x + 3y - 8 = 0$

263. That point is focus. Find the focus

264. Focus is $\left(\frac{1}{4}, 0\right)$ & the points of intersection of $y^2 = x$ &

y = 1; y = - 2 are A(1, 1) B(4, -2)

Area of triangle ABC = $\frac{21}{8}$

Paragraph – 89

Consider the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$ where $b > a > 0$. Let $A(-a, 0); B(a, 0)$. A parabola passes through the points A, B and its directrix is a tangent to $x^2 + y^2 = b^2$. If the locus of focus of the parabola is a conic then

265. The eccentricity of the conic is

A) $2a/b$ B) b/a C) a/b D) 1

Key. C

266. The foci of the conic are
A) $(\pm 2a, 0)$ B) $(0, \pm a)$ C) $(\pm a, 2a)$ D) $(\pm a, 0)$

Key. D

267. Area of triangle formed by a latusrectum and the lines joining the end points of the latusrectum and the centre of the conic is
A) $\frac{a}{b}\left(b^2 - a^2\right)$ B) $2ab$ C) $ab/2$ D) $4ab/3$

Key. A

Sol. **265 –267:**

$x^2 + y^2 = a^2$; $x^2 + y^2 = b^2$; $b > a > 0$, $A = (-a, 0)$; $B = (a, 0)$

Let (h, k) be a point on the locus. Any tangent to circle $x^2 + y^2 = b^2$ is
$x\cos\theta + y\sin\theta = b$

$\therefore$ Equation of parabola is $\sqrt{(x-h)^2 + (y-K)^2} = |x\cos\theta + y\sin\theta - b|$

i.e., $(x-h)^2 + (y-K)^2 = (x\cos\theta + y\sin\theta - b)^2$

The points $(\pm a, 0)$ satisfy this equation

$\therefore (a-h)^2 + K^2 = (a\cos\theta - b)^2$ --- (1)

$(a+h)^2 + K^2 = (a\cos\theta + b)^2$ ---- (2)

$(2) - (1) \Rightarrow h = b\cos\theta$

$\therefore$ Required locus is $(a+x)^2 + y^2 = \left(\frac{ax}{b} + b\right)^2$

i.e., $\frac{x^2}{b^2} + \frac{y^2}{b^2 - a^2} = 1$ which is an ellipse.

Parabola

Integer Answer Type

1. If parabola with focus $\left(\frac{2}{5}, \frac{4}{5}\right)$ touches X and Y axis at A and B respectively, then area of ΔOAB is, where 'o' is origin.

Key. 1

Sol. The parabola touches x-axis at $A(a, 0)$ and y-axis at $B(0, b)$, then focus is the point of intersection of circles with diameter OA and OB.

2. Consider the parabola $y^2 = 4x$. Let P and Q be two points $(4, -4)$ and $(9, 6)$ on the parabola. Let R be a moving point on the arc of the parabola between P and Q. If the maximum area of ΔRPQ is 'S' then $(4S)^{\frac{1}{3}}$ equals

Key. 5

Sol. Let $R = \left(t_1^2, 2t\right)$ be a point on the parabola.

Perpendicular distance of R to PQ is maximum for $t = \frac{1}{2}$

Maximum area $S = \frac{125}{4} \Rightarrow (4S)^{\frac{1}{3}} = 5$

3. Two tangents are drawn from point (1, 4) to the parabola $y^2 = 4x$. Angle between these tangents is $\frac{\pi}{K}$ then K = ...

Key. 3

Sol. $y = mx + \frac{1}{m}$ is tangent to the parabola $y^2 = 4x$

$\Rightarrow y^2 = m + \frac{1}{m} \Rightarrow m^2 - 4m + 1 = 0$

$m_1 + m_2 = 4, m_1 m_2 = 1$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \sqrt{3}$

$\theta = \frac{\pi}{3}$

4. If the line $x - 1 = 0$ is the directrix of the parabola $y^2 - kx + 8 = 0$, then *k*(>0) is

Key. 4

Sol. $y^2 = kx - 8$

$y^2 = k\left(x - 8/K\right)$

$r\left(8/K, 0\right); 4a = K$

$a = \frac{K}{4}$

We know that $rz = a$

$\Rightarrow \left|\frac{8}{K} - 1\right| = \frac{K}{4}$

$\Rightarrow \frac{8}{K} - \frac{K}{4} = 1$

$\Rightarrow 32 - K^2 = 4K$

$\Rightarrow K^2 + 4K - 32 = 0$

$\Rightarrow K\left(K+8\right) - 4\left(K+8\right) = 0$

K = 4 or K = -8 X

5. If $x + y = k$ is normal to $y^2 = 12x$, then *k* is

Key. 9

Sol. Equation of normal of the parabola $y^2 = 12x$ is $y = mx - 2am - am^3$

Where a = 3

$\Rightarrow y = mx - 6m - 3m^3 ....(1)$

Given $y = -x + K ....(2)$

By comparing : $m = -1, K = -6m - 3m^3$

$K = -6\left(-1\right) - 3\left(-1\right)^3$

= 6+3 = 9

6. The number of distinct normals, which can be drawn from the point (2, 8) to the parabola $y^2 = 6x$ is ....

Key. 1

Sol. Any normal will be $y + tx = 3t + \frac{3}{2}t^3$, it passes through (2, 8), so $3t^3 + 2t - 16 = 0$

Let $f(t) = 3t^3 + 2t - 16$

$\Rightarrow f'(t) = 9t^2 + 2 < 0$, So only one normal.

7. Tangents are drawn from the points on the parabola $y^2 = -8(x+4)$ to the parabola $y^2 = 4x$, if locus of mid point of chord of contact is again a parabola, with length of latus rectum λ, then 5λ is ....

Key. 8

Sol. Let (x_1, y_1) be a point on $y^2 = -8(x + 4)$

equation of chord of contact is

$2x - y_1 y + 2x_1 = 0$, if p(h, k) be its mid point, then its equation will be

$2x - ky + k^2 - 2h = 0$

Compare both $k = y_1$, $2x_1 = k^2 - 2h$

So, $k^2 = -4(k^2 - 2h + 8) \Rightarrow k^2 = \frac{8}{5}(h - 4)$

So, $\lambda = \frac{8}{5}$

8. If e is the eccentricity of the hyperbola $(5x - 10)^2 + (5y + 15)^2 = (12x - 5y + 1)^2$ then $\frac{25e}{13}$ is equal to ....

Key. 5

Sol. Equation can be rewritten as $\sqrt{(x-2)^2+(y+3)^2} = \frac{13}{5}\left|\frac{12x-5y+1}{13}\right|$

So, e = $\frac{13}{5}$.

9. If a variable tangent of the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$, cuts the circle $x^2 + y^2 = 4$ at point A, B and locus of mid point of AB is $9x^2 - 4y^2 - \lambda (x^2 + y^2)^2 = 0$ then λ is ....

Key. 1

Sol. Equation of chord of circle with mid point (h, k) is xh + xk = $h^2 + k^2$ or

$y = \left(\frac{-h}{k}\right)x + \frac{h^2+k^2}{k}$, it touches the hyperbola

10. If the angle between the asymptotes of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{\pi}{3}$. Then the eccentricity of conjugate hyperbola is

Key. 2

Sol. $2\tan^{-1}\left(\frac{b}{a}\right)=\frac{\pi}{3}$

$\frac{b}{a}=\frac{1}{\sqrt{3}}$

$e^2 = 1+\frac{1}{3}=\frac{4}{3}$

$\frac{1}{e'^2}+\frac{1}{e^2}=1$

$\Rightarrow \quad \frac{1}{e'^2}+\frac{3}{4}=1$

$\Rightarrow \quad \frac{1}{e'^2}=\frac{1}{4} \Rightarrow e'=2$

11. If α, β be the roots $x^2 + px - q = 0$ and γ, δ be the roots $x^2 + px + r = 0$, $q + r \neq 0$ then $\frac{(\alpha-\gamma)(\alpha-\delta)}{(\beta-\gamma)(\beta-\delta)}$

Key. 1

Sol. Conceptual

12. The equation $2^{2x}+(a-1)2^{x+1}+a=0$ has roots of opposite signs then $[a]$ is (where $[.]$ denotes greatest integer function)

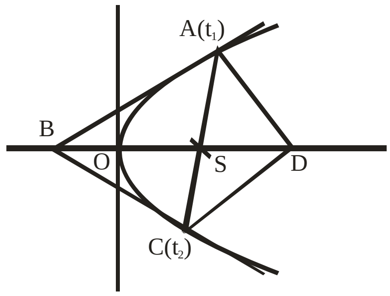
Key. 0

Sol. $a \in \left(0, \frac{1}{3}\right)$

13. Tangent and normal at the ends A and C of focal chord AC of parabola $y^2 = 4x$ intersect at B and D, then minimum area of ABCD is

Key. 8

Sol. $t_1t_2 = -1$



Clearly ABCD is a rectangle

Co-ordinate of B(t_1t_2, ($t_1 + t_2$))

$$AB = |(t_2 - t_1)|\sqrt{1+t_1^2}$$

$$BC = (t_2 - t_1)\sqrt{1+t_2^2}$$

Area = AB × BC = $(t_2 - t_1)^2 \sqrt{(1+t_1^2)(1+t_2^2)}$

$$= \left(t_1 + \frac{1}{t_1}\right)^3$$

Least value = 8.

14. The minimum distance of $4x^2 + y^2 + 4x - 4y + 5 = 0$ from the line $-4x + 3y = 3$ is

Key. 1

Sol. The given curve represents the point $\left(-\frac{1}{2}, 2\right)$

$\therefore$ minimum distance = 1..

15. A = (- 3,0) and B = (3 ,0) are the extremities of the base AB of triangle PAB. If the vertex P varies such that the internal bisector of angle APB of the triangle divides the opposite side AB into two segments AD and BD such that AD : BD = 2 : 1 , then the maximum value of the length of the altitude of the triangle drawn through the vertex P is

Key. 1

Sol. The point E dividing $\overline{AB}$ externally in the ratio 2 : 1 is (9 , 0) . Since P lies on the circle described on $\overline{DE}$ as diameter, coordinates of P are of the form $(5+4\cos\theta, 4\sin\theta)$

$\therefore$ maximum length of the altitude drawn from P to the base $AB = |4\sin\theta|_{\max} = 4$

16. The tangents drawn from the origin to the circle $x^2 + y^2 - 2rx - 2hy + h^2 = 0$ are perpendicular then sum of all possible values of $\frac{h}{r}$ is ___

Key. 0

Sol. Combined equation of the tangents drawn from (0, 0)to the circle is

$\left(x^2+y^2-2rx-2hy+h^2\right)h^2=\left(-rx-hy+h^2\right)^2$ here coefficient of

$x^2+\text{coffecient of } y^2=0 \Rightarrow \left(h^2-r^2\right)+\left(h^2-r^2\right)=0$

$\Rightarrow \frac{h}{r}=\pm 1$

17. All terms of an A.P. are natural numbers. The sum of its first nine terms lies between 200 and 220. If the second term is 12, then first term is

Key. 8

Sol. According to the given condition

$200 < \frac{9}{2}\left(2(12-d)+8d\right)<220$

$\Rightarrow d=4$

$\therefore$ First term = 12 – d = 8

18. Coordinates of the vertices $B \,\&\, C$ are $(2,0)$ and $(8,0)$ respectively. The vertex $'A'$ is varying in such a way that $4\tan\frac{B}{2}\tan\frac{C}{2}=1$. If the locus of $'A'$ is an ellipse then the length of its semi major axis is

Key. 5

Sol. $4\tan\frac{B}{2}\tan\frac{C}{2}=1$

$\Rightarrow \sqrt{\frac{(s-c)(s-a)(s-a)(s-b)}{s(s-b)s(s-c)}}=\frac{1}{4}$

$\Rightarrow \frac{s-a}{s}=\frac{1}{4} \Rightarrow \frac{25-a}{a}=\frac{5}{3} \Rightarrow b+c=\frac{5}{3}\times 6=10$

$\left(\because\ a=\overline{BC}=6\right)$

$\therefore$ Locus of A is

$\frac{(x-5)^2}{25}+\frac{y^2}{16}=1$

19. A parabola is drawn through two given points $A(1,0)$ and $B(-1,0)$ such that its directrix always touches the circle $x^2+y^2=4$. If the maximum possible length of semi latus–rectum is $'k'$ then $[k]$ is (where [.] denotes greatest integer function)

Key. 3

Sol. Any point on circle $x^2+y^2=4$ is $(2\cos\alpha, 2\sin\alpha)$

$\therefore$ Equation of directrix is $x(\cos\alpha)+y(\sin\alpha)-2=0$

Let focus be (x_1, y_1). Then as $A(1,0)$, $B(-1,0)$ lie on parabola we must have

$$\left.\begin{array}{l}(x_1-1)^2+y_1^2=(\cos\alpha-2)^2\\(x_1+1)^2+y_1^2=(\cos\alpha+2)^2\end{array}\right\}\Rightarrow x_1=2\cos\alpha,\ y_1=\pm\sqrt{3}\sin\alpha$$

$\therefore$ Length of semi latus–rectum of parabola $=\perp^r$ distance from focus to directrix

$\left|2\pm\sqrt{3}\right|\sin^2\alpha$

Hence, maximum possible length $=2+\sqrt{3}$

20. A line passing through $(21,30)$ and normal to the curve $y=2\sqrt{x}$. If m is slope of the normal then $m+6=$

KEY : 1

SOL : Equation of the normal is $y=mx-2m-m^3$

If it pass through $(21,30)$ we have $30=21m-2m-m^3\Rightarrow m^3-19m+30=0$

Then $m=-5,2,3$

But if $m=2$ or 3 then the point where the normal meets the curve will be $(am^2,-2am)$ where the curve does not exist. Therefore $m=-5$

$\therefore m+6=1$

21. Let P, Q be two points on the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ whose eccentric angles differ by a right angle. Tangents are drawn at P and Q to meet at R. If the chord PQ divides the joint of C and R in the ratio m : n (C being centre of ellipse), then find m+n(m:n is in simplified form).

Key: 2

Hint: Let P be $(5\cos\theta,4\sin\theta)$; Q be $(-5\sin\theta,4\cos\theta)$

Equation of tangent at P $\frac{x}{5}\cos\theta+\frac{y}{4}\sin\theta=1$ ……(i)

Equation of tangent at Q $-\frac{x}{5}\sin\theta+\frac{y}{4}\cos\theta=1$ …….(ii)

Solving (i) and (ii) $\Rightarrow R=\left(5(\cos\theta-\sin\theta),4(\sin\theta+\cos\theta)\right)$

$\therefore$ m : n is 1 : 1

$\Rightarrow$ m + n = 2

Alternate :

Let P(5,0), Q(0,4)

$\Rightarrow$ R(5,4)

Intersection of CR and PQ is $\left(\frac{5}{2},2\right)$, which is mid poi8nt of CR

$\Rightarrow m:n=1:1\Rightarrow m+n=2$

22. $\sqrt{x}+\sqrt{y}=1$ is a part of the parabola whose length of latus rectum is √k, then find k.

Key. 2

Sol. $(y - x - 1)^2 = 4x \Rightarrow x^2 - 2xy + y^2 - 2x - 2y + 1 = 0$

$\Rightarrow \quad (x - y + \lambda)^2 = 2x + 2y - 1 + 2\lambda x - 2\lambda y + \lambda^2, \lambda \in R$

we choose λ such that

$x - y + \lambda = 0$ and $2x + 2y - 1 + 2\lambda x - 2\lambda y + \lambda^2 = 0$ are perpendicular lines

$\Rightarrow \quad \lambda = 0$ now solve it.

23. $\sqrt{x} + \sqrt{y} = 1$ is a part of the parabola whose length of latus rectum is $\sqrt{k}$, then find k.

Key. 2

Sol. $(y - x - 1)^2 = 4x \Rightarrow x^2 - 2xy + y^2 - 2x - 2y + 1 = 0$

$\Rightarrow \quad (x - y + \lambda)^2 = 2x + 2y - 1 + 2\lambda x - 2\lambda y + \lambda^2, \lambda \in R$

we choose λ such that

$x - y + \lambda = 0$ and $2x + 2y - 1 + 2\lambda x - 2\lambda y + \lambda^2 = 0$ are perpendicular lines

$\Rightarrow \quad \lambda = 0$ now solve it.

24. If AFB is a focal chord of the parabola $y^2 = 4ax$ and AF = 3, FB = 6 , then the latus–rectum of the parabola is equal to

Key. 8

Sol. $\frac{1}{AF} + \frac{1}{FB} = \frac{1}{a}$

$\Rightarrow \frac{1}{a} = \frac{1}{3} + \frac{1}{6} = \frac{1}{2} \Rightarrow a = 2 \Rightarrow LR = 8$

25. If $2x + 3y = \alpha, x - y = \beta$ and $kx + 15y = r$ are 3 consecutive normal's of parabola $y^2 = \lambda x$ then value of k is

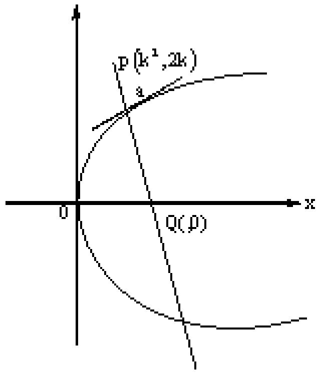
Key. 5

Sol. $t_1 + t_2 + t_3 = 0 \Rightarrow \left(\frac{-2}{3}\right) + 1 + \left(\frac{-k}{15}\right) = 0$

$\Rightarrow \frac{k}{15} = \frac{1}{3} \Rightarrow k = 5$

26. The locus of the mid – point of the portion of the normal to the parabola y^2 =16x intercepted between the curve and the axis is another parabola whose latus rectum is

Key. 4



Sol.

Consider the parabola $y^2 = 4ax$

We have to find the locus of R(h, k), since Q has ordinate 'O', ordinate of P is 2k

Also P is on the curve, then abscissa of P is k^2 / a

Now PQ is normal to curve

Slope of tangent to curve at any point $\frac{dy}{dx} = \frac{2a}{y}$

Hence slope of normal at point P is $-\frac{k}{a}$

Also slope of normal joining P and R(h, k) is $\frac{\frac{2k-k}{k^2}}{\frac{k^2}{a}-h}$

Hence comparing slopes $\frac{2k-k}{\frac{k^2}{a}-h} = -\frac{k}{a}$

Or $y^2 = a(x-a)$

For $y^2 = 16x,\ a = 4$, hence locus us $y^2 = 4(x-a)$

27. If the line $x-1=0$ is the directrix of the parabola $y^2 - kx + 8 = 0$, then *k*(>0) is

Key. 4

Sol. $y^2 = kx - 8$

$y^2 = k\left(x - 8/K\right)$

$r\left(8/K, 0\right); 4a = K$

$a = \frac{K}{4}$

We know that $rz = a$

$\Rightarrow \left|\frac{8}{K} - 1\right| = \frac{K}{4}$

$\Rightarrow \frac{8}{K} - \frac{K}{4} = 1$

$\Rightarrow 32 - K^2 = 4K$

$\Rightarrow K^2 + 4K - 32 = 0$

$\Rightarrow K(K+8) - 4(K+8) = 0$

K = 4 or K = -8 X

28. If $x + y = k$ is normal to $y^2 = 12x$, then *k* is

Key. 9

Sol. Equation of normal of the parabola $y^2 = 12x$ is $y = mx - 2am - am^3$

Where a = 3

$\Rightarrow y = mx - 6m - 3m^3 ....(1)$

Given $y = -x + K...........(2)$

By comparing : $m = -1, K = -6m - 3m^3$

$K = -6(-1) - 3(-1)^3$

= 6+3 = 9

29. Two tangents are drawn from point (1, 4) to the parabola $y^2 = 4x$. Angle between these tangents is $\frac{\pi}{K}$ then K = ...

Key. 3

Sol. $y = mx + \frac{1}{m}$ is tangent to the parabola $y^2 = 4x$

$\Rightarrow y^2 = m + \frac{1}{m} \Rightarrow m^2 - 4m + 1 = 0$

$m_1 + m_2 = 4, m_1 m_2 = 1$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \sqrt{3}$

$\theta = \frac{\pi}{3}$

30. The shortest distance between parabolas $y^2 = x - 1$ and $x^2 = y - 1$ is $\frac{3\sqrt{2}}{k}$, then numerical value of *k* is

Key. 4

Sol. Both the curves are symmetrical about the line $y=x$. Distance between any pair of points = 2{distance of $\left(t,t^2+1\right)$ from $y=x$]

$$2\left[\frac{t^2+1-t}{\sqrt{2}}\right]=\sqrt{2}\left[t^2-t+1\right]$$

Min $t^2-t+1=-\left[\frac{1-4}{4}\right]=\frac{3}{4}$

$$\Rightarrow \frac{3\sqrt{2}}{4}$$

31. Through the vertex 'O' of parabola $y^2=4x$, chords OP and OQ are drawn at right angles to one another. Then the locus of middle point of PQ is a parabola with Latus rectum λ, then λ equals

Key. 2

Sol. $h=\frac{t_1^2+t_2^2}{2}$

$$k=\frac{2\left(t_1+t_2\right)}{2}=t_1+t_2$$

Also $t_1t_2=-4 \Rightarrow 2h+\left(t_1+t_2\right)^2-2t_1t_2$

$$2h=k^2+8$$

$$y^2=2\left(x-4\right)$$

$\therefore$ Latus rectum = 2

32. Points A,B,C lie on parabola $y^2=4ax$. Tangents to A,B,C taken in pairs intersect at P,Q, R. Then $\frac{ar\Delta ABC}{ar\Delta PQR}$ is

Key. 2

Sol. $ar\Delta ABC=\frac{1}{2}\begin{vmatrix} at_1^2 & at_2^2 & at_3^2 \\ 2at_1 & 2at_2 & 2at_3 \\ 1 & 1 & 1 \end{vmatrix}=a^2\left|\left(t_2-t_1\right)\left(t_3-t_2\right)\left(t_1-t_3\right)\right|$

Also $\Delta PQR=\frac{1}{2}\begin{vmatrix} at_1t_2 & at_2t_3 & at_3t_1 \\ a\left(t_1+t_2\right) & a\left(t_2+t_3\right) & a\left(t_3+t_1\right) \\ 1 & 1 & 1 \end{vmatrix}=\frac{a^2}{2}\left|\left(t_3-t_1\right)\left(t_1-t_2\right)\left(t_2-t_3\right)\right|$

$$\Rightarrow \frac{ar\Delta ABC}{ar\Delta PQR}=2$$

Parabola

Matrix-Match Type

1. Consider the parabola $(x-1)^2+(y-2)^2=\frac{(12x-5y+3)^2}{169}$.

	Column - I		Column – II
(a)	Locus of point of intersection of perpendicular tangent	p.	$12x-5y-2=0$
(b)	Locus of foot of perpendicular from focus upon any tangent	q.	$5x+12y-29=0$
(c)	Line along which minimum length of focal chord occurs	r.	$12x-5y+3=0$
(d)	Line about which parabola is symmetrical	s.	$24x-10y+1=0$

Key. a$\rightarrow$r, b$\rightarrow$s, c$\rightarrow$p, d$\rightarrow$q

Sol. Focus (1, 2) directrix = $12x-5y+3=0$

(a) Locus of point of intersection of perpendicular tangents is directrix = 12x-5y+3=0.

(b) Locus of point of perpendicular from focus upon any tangent is the line parallel to directrix and passing through vertex = 24x-10y+1=0.

(c) Required line in the line parallel to directrix and passing through focus = 12x-5y-2=0.

(d) Required line is the line perpendicular to directrix and passing through focus = 5x+12y-29=0.

2. Match the following

	COLUMN-I	COLUMN-II
(A)	If the distances of two points P & Q lie on the parabola $y^2=4ax$ from the focus S of the same parabola are 4 & 9, then the distance of the point of intersection R of tangents at P & Q from the focus is equal to	P) 8
(B)	The normal chord of a parabola $y^2=4ax$ at the point whose ordinate is equal to the abscissa, then angle subtended by normal chord at the focus, is $\operatorname{cosec}^{-1}$ (?)	Q)4
(C)	The distance between a tangent to the parabola $y^2=4Ax$ (A > 0) and parallel normal with gradient 1, is $(\sqrt{p})A$, then p = ?	R)6
(D)	If the normal to a parabola $y^2=4ax$ at P meets the curve again at Q and if PQ & the normal at Q makes angles α & β respectively, then $\lvert 2\tan\alpha(\tan\alpha+\tan\beta)\rvert$ equals to	S) 1

Key. A-R, B-S, C-P, D-Q

Sol. -(A)

$Let \quad P(t_1) \,\&\, Q(t_2)$

$\Rightarrow \quad R(at_1t_2, a(t_1+t_2))$

$\Rightarrow 4 = at_1{}^2 + a \quad \& \quad 9 = at_2{}^2 + a$

$\& \quad SR = a\sqrt{1+t_1{}^2}\sqrt{1+t_2{}^2}$

$\Rightarrow SR = 6$

Sol-(B)

Equation of normal at A(4a, 4a) is y + x = 12a

This normal intersects the parabola at B(9a, - 6a) & focus S(a, 0)

Hence angle ASB is $\pi/2 = \operatorname{cosec}^{-1}(1)$

Sol-(C)

Equation of tangent y = mx + A/m $\Rightarrow$ y = x + A

Equation of normal y = mx – 2Am - Am^3 $\Rightarrow$ y = x – 3A

$\Rightarrow$ Distance between these two lines is = $\sqrt{8}$

Sol-(D)

$Let \quad P(t_1) \Rightarrow Q(t_2 = -t_1 - \frac{2}{t_1})$

$\Rightarrow$ Slope of PQ = - $t_1 = tan\alpha$

& slope of normal at Q = - $t_2 = -t_1 - \frac{2}{t_1} = \tan\beta$

$\Rightarrow |2\tan\alpha(\tan\alpha + \tan\beta)| = 4$

3. Match the following

Consider the parabola $y^2 = 12x$

	COLUMN-I	COLUMN-II
(A)	Tangent and normal at the extremities of the latus rectum intersect the x-axis at T & G respectively. The coordinates of middle point of T & G are	P) (0,0)
(B)	Variable chords of the parabola passing through a fixed point K on the axis, such that sum of the reciprocals of two parts of the chord through K, is a constant. Coordinates of K are	Q) (3, 0)
(C)	All variable chords of the parabola subtending a right angle at the origin are concurrent at the point	R) (6, 0)
(D)	AB & CD are the chords of a parabola which intersect at a point E on the axis. The radical axis of the two circles described on AB & CD as diameter always passes through the point	S) (12, 0)

Key. A-Q, B-Q, C-S, D-P

Sol. (A)

Equation of tangent at (3, 6) : y = x + 3 $\Rightarrow$ T(- 3, 0)

Equation of normal at (3, 6) : y = - x + 9 $\Rightarrow$ G(9, 0)

Hence middle point (3, 0)

Sol-(B)

Point is obviously focus (3, 0)

Sol-(C)

If variable chord is PQ, then

$$\begin{array}{l} Let \quad P(t_1)\,\&\,Q(t_2) \\ \Rightarrow \quad t_1t_2=-4 \end{array} \quad \Rightarrow \quad \text{Chords are concurrent at } (4a, 0) \Rightarrow (12, 0)$$

Sol-(D)

$$Let \quad A(t_1)\,\&\,B(t_2), C(t_3)\,\&\,D(t_4)$$

If AB & CD intersect at a point E on the axis, then by solving the equations of AB & CD we get the relation $t_1t_2=t_3t_4$

Now equations of the circles with AB & CD as diameters are

$$(x-at_1{}^2)(x-at_2{}^2)+(y-2at_1)(y-2at_2)=0$$
$$(x-at_3{}^2)(x-at_4{}^2)+(y-2at_3)(y-2at_4)=0$$

If we solve these two circles, then equation of their radical axis is of the form y = mx

4. Match the following

Column – I	Column II
A. An ellipse passing through the origin has the foci (3,4) (6, 8) then length of minor axis is	p. 8
B. If PQ is focal chord of ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ which passes Through S = (3, 0) and PS =2 then length of chord (PQ) is	q. $10\sqrt{2}$
C. If the line y = x+ k touches the ellipse $9x^2+16y^2=144$ then The difference of values of k is	r. 10
D. Sum of the distances of a point on the ellipse $\frac{x^2}{9}+\frac{y^2}{16}=1$ from the foci.	s. 12

Key. A-q,B-r, C-r, D-p

Sol. Conceptual

5. *P* is a point on the ellipse $9x^2+25y^2=225$. The tangent at *P* meet the X-axis, Y-axis at T, t respectively and the normal at *P* meet the X-axis, Y-axis at G, g respectively. *C* is the centre of the ellipse and *F* is the foot of the perpendicular from *C* to normal at *P*.

Column – I	Column – II
a) $\lvert PF\rvert \times \lvert PG\rvert =$	p) 25
b) $\lvert PF\rvert \times \lvert Pg\rvert =$	q) 16
c) $\lvert CG\rvert \times \lvert CT\rvert =$	r) 9
d) $\lvert Ct\rvert \times \lvert Cg\rvert =$	s) 24

Key. . a) r; b) p; c) q; d) q

Sol. Conceptual

6.

Column – I	Column – II
a) A tangent to the ellipse $\frac{x^2}{27}+\frac{y^2}{48}=1$ has slope $-\frac{4}{3}$ and the tangent cuts the axes of the ellipse at A, B. Area of ΔOAB is (*O* is the origin)	p) 36
b) Product of perpendiculars drawn from the points $(\pm 3, 0)$ to the line $y = mx - \sqrt{25m^2+16}$ is	q) $10\sqrt{2}$
c) An ellipse passing through (0, 0) has its foci at (3, 4) and (6, 8). Length of its minor axis is	r) 24
d) If *e* is the eccentricity of the conic $\sqrt{x^2+y^2}+\sqrt{(x+3)^2+(y-4)^2}=10$, then $72e =$	s) 16

Key. a) p; b) s; c) q; d) p

Sol. Conceptual

7. The normals at four points $(x_i, y_i),\ i = 1, 2, 3, 4$ on the hyperbola $xy = 4$ are concurrent at the point (α, β)

Column – I	Column – II
a) $y_1 + y_2 + y_3 + y_4 =$	p) 0
b) $\sum_{1\le i<j\le 4} x_i x_j =$	q) –16
c) $x_1 x_2 x_3 x_4 =$	r) $-\beta$
d) $y_1 y_2 y_3 y_4 =$	s) β

Key. a) s; b) p; c) q; d) q

Sol. Conceptual

	Column - I	Column – II
8.	(a) The angle between two diagonals of a cube is	(p) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(b) In a regular tetrahedron, the angle between any two faces is (q) $\frac{\pi}{2}$

(c) If $\bar{a},\bar{b},\bar{c}$ are three mutually perpendicular vectors of equal magnitude then $(\bar{a}+\bar{b}+\bar{c},\bar{a})$ is (r) $\cos^{-1}\left(\frac{2}{3}\right)$

(d) If $\bar{a},\bar{b},\bar{c}$ are three unit vectors such that $\bar{b},\bar{c}$ are non parallel and $\bar{a}\times(\bar{b}\times\bar{c})$ is parallel to $\bar{b}$ then $(\bar{a},\bar{b})$ is (s) $\cos^{-1}\frac{1}{3}$

Key. $a \to s; b \to s; c \to p; d \to q$

Sol. Conceptual

Column - I	Column – II
9. (a) Length of perpendicular from (1, 2, 3) to the line $\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$ is	(p) $\sqrt{11}$
(b) Length of perpendicular from (2, 3, 7) to $3x-y-z=7$ is	(q) 1
(c) Distance of (1, -2, 3) from the plane $x-y+z=5$ measured along a line parallel to $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$ is	(r) 7
(d) The distance of the point of intersection of the line $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}$ and the plane $x+y+z=17$ from $A(3,4,5)$ is	(s) 3

Key. $a \to r; b \to p; c \to q; d \to s$

Sol. Conceptual

10. (a) Length of latusrectum of parabola $9x^2-24xy+16y^2-20x-15y-60=0$ is (p) -1

(b) PQ is a variable focal chord of $y^2=3x$ whose vertex is A, then the length of latusrectum of locus of centroid of ΔAPQ is (q) 1

(c) The tangents at $P(t_1);Q(t_2)\,wrt\;\; y^2=4ax$ makes complimentary angles with x-axis than t_1t_2 is (r) 2

(d) The number of points of intersection of $x^2+y^2+2x=0$ with $y^2=4x$ are (s) 3

Key. $a \to q; b \to q; c \to q; d \to q$

Sol. Conceptual

11. AB is a chord of the parabola $y^2 = 4x$ such that the normals at A and B intersect at the point $C(9,6)$

Column-I	Column-II
A. Length (AB)	p. 20
B. Area of $\Delta^{le} ABC$	q. $\frac{4}{\sqrt{13}}$
C. Distance of origin from the line through (AB)	r. $\sqrt{13}$
D. The area bounded by the coordinate axes and the line through (AB)	s. $\frac{4}{3}$

Key. A-r, B-p, C-q, D-s

Sol. Conceptual

12. Match the following

Column – I	Column II
A. An ellipse passing through the origin has the foci (3,4) (6, 8) then length of minor axis is	p. 8
B. If PQ is focal chord of ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ which passes Through S = (3, 0) and PS =2 then length of chord (PQ) is	q. $10\sqrt{2}$
C. If the line y = x+ k touches the ellipse $9x^2 + 16y^2 = 144$ then The difference of values of k is	r. 10
D. Sum of the distances of a point on the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$ from the foci.	s. 12

Key. A-q, B-r, C-r, D-p

Sol. Conceptual

13. The normals at four points $(x_i, y_i), i=1,2,3,4$ on the hyperbola xy=16 are concurrent at the point (α,β)

	Column I		Column II
(A)	$x_1x_2x_3x_4 =$	(P)	β
(B)	$y_1y_2y_3y_4 =$	(Q)	0
(C)	$y_1+y_2+y_3+y_4 =$	(R)	- 256
(D)	$\sum\sum_{1\le i<j\le 4} y_iy_j$	(S)	$-\beta$

Key. A –R; B – R; C – P; D – Q

Sol. Conceptual

14. From the point (3, 0) three normals are drawn to the parabola $y^2 = 4x$ which meet the parabola in the points P, Q and R. Then

	Column I		Column II
(A)	Area of triangle PQR =	(P)	2
(B)	*Circumradius of triangle PQR =*	(Q)	$\left(\frac{2}{3},0\right)$
(C)	centroid of triangle PQR =	(R)	$(-3,0)$
(D)	orthocenter of triangle PQR =	(S)	$5/2$

Key. A –P; B – S; C – Q; D – R

Sol. Conceptual

15.

	Column I		Column II
(A)	The locus of the midpoints of chords of an ellipse which are drawn through an end of minor axis, is	(P)	circle
(B)	The locus of an end of latus-recturm of all ellipses having a given major axis, is	(Q)	parabola
(C)	The locus of the foot of perpendicular from a focus of an ellipse on any tangent to it	(R)	ellipse

(D)	The locus of the midpoints of the portions of lines (drawn through a given point) between the co-ordinate axes	(S)	hyperbola

Key. A – R; B – Q; C – P; D – S

Sol. a) Let BC be a chord of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ let B = (0,b) and midpoint be $M=(\alpha,\beta)$ then

$C-(2\alpha,2\beta-b)$ will lie on the ellipse

b) $L\left(ae,\frac{b^2}{a}\right)$ given 2a = constant

$$\alpha=ae\Rightarrow e=\frac{\alpha}{a}$$

$$\beta=\frac{b^2}{a}=a\left(1-e^2\right)$$

$$\Rightarrow\beta=a\left(1-\frac{\alpha^2}{a^2}\right)$$

$$\Rightarrow\alpha^2=a^2-a\beta$$

c) Auxilary circle

d) $\frac{x}{a}+\frac{y}{b}=1$ let midpoint of (a,0) & (o, b), be (α,β) then $2\alpha=a,2\beta=b$ let all lines pass through a given point (h, k), then $\frac{h}{a}+\frac{k}{b}=1.$

16. Observe the following lists:

List – I	List – II
(A) The locus of mid-points of chords of an ellipse which are drawn through an end of minor axis, is	p) hyperbola
(B) The locus of an end of latus rectum of all ellipses having a given major axis is	q) circle
(C) The locus of the foot of perpendicular from a focus of the ellipse on any tangent is	r) parabola
	s) ellipse
(D) A variable line is drawn through a fixed point cuts axes at A and B. The locus of the mid point of AB is	

Key. A – s , B – r , C – q , D – p

Sol. (A) The chord wit mid point (h,k)

$$\frac{hx}{a^2}+\frac{Ky}{b^2}=\frac{h^2}{a^2}+\frac{K^2}{b^2}$$

$\therefore$ Locus is $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=\dfrac{y}{b}$

(B) (h,k) be the end of the latus rectum.

$h=ae, K=a(1-e^2)$

$h^2=-a(K-a)\Rightarrow x^2=-a(y-a)$ parabola.

(C) $y-mx=\sqrt{a^2m^2+b^2}$

$\therefore x^2+y^2=a^2$

(D) $\dfrac{x}{a}+\dfrac{y}{b}=1$ passes through (α,β) $\dfrac{\alpha}{a}+\dfrac{\beta}{b}=1$

$h=\dfrac{a}{2}, K=\dfrac{b}{2}\Rightarrow \dfrac{h}{\alpha}+\dfrac{\beta}{K}=2$

$\dfrac{\alpha}{x}+\dfrac{\beta}{y}=2\Rightarrow\left(x-\dfrac{\alpha}{2}\right)\left(y-\dfrac{\beta}{2}\right)=\dfrac{\alpha\beta}{4}$

17. Observe the following lists :

List – I	List – II
(A) If two distinct chords of a parabola $y^2=4ax$ passing through the point $(a,2a)$ are bisected by the line $x+y=1$, then the length of the latus rectum can be	p) -1
(B) The parabola $y=x^2-5x+4$ cuts the $x-axis$ at P and Q. A circle is drawn through P and Q so that the origin lies outside it. The length of tangent to the circle from the origin is equal to	q) 0
(C) If $y+b=m_1(x+a)$ and $y+b=m_2(x+a)$ are two tangents to $y^2=4ax$ then m_1m_2 is equal to	r) 1
(D) If the point $(h,-1)$ is exterior to both the parabolas $y^2=\vert x\vert$,then the integral part of h can be equal to	s) 2

Key. A – r, s, B – s, C – p , D – p,q

Sol. (A) Any point is $(t,1-t)$

The chord with this as mid point

$y(1-t)-2a(x+t)=(1-t)^2-4at$

$\Rightarrow(1-t)^2=2a(1-a)\ge 0\Rightarrow 0<a\le 1$

$\therefore LR\in(0,4]$

(B) $P\equiv(1,0), Q\equiv(4,0)$

$(x-1)(x-4)+y^2+\lambda y=0$

The length of the tangent from (0, 0) is $\sqrt{4}=2$

(C) $m_1m_2=-1$

(D) $1-|h|>0 \Rightarrow -1<h<1$

18. Observe the following lists :

List – I	List – II
(A) If three unequal number a, b, c are A.P. and $b-c, c-b$, a are in GP., then $\dfrac{a^3+b^3+c^3}{3abc}$ is equal to	p) 4
(B) Let x be the arithmetic mean and y, z be two geometric means between any two positive numbers, then $\dfrac{y^3+z^3}{xyz}$ is equal to	q) 1
(C) If $a_1, a_2, a_3 -----, a_{50}$ are 50 distinct numbers in A.P and $a_1^{\ 2}-a_2^{\ 2}+a_3^{\ 2}------a_{50}^{\ 2}=\left(\dfrac{5}{7}\right)^n\left(a_1^{\ 2}-a_{50}^{\ 2}\right)$, $(n \in N)$ then n =	r) 2
(D) $\lim\limits_{n\to\infty} \tan\left\{\sum\limits_{r=1}^{n}\tan^{-1}\left(\dfrac{1}{2r^2}\right)\right\}$ is equal to	s) 3

Key. A – r , B – r , C – r , D – q

Sol. (A) $(b-a)=(c-b)$ and $(c-b)^2=a(b-a)$

$\Rightarrow (b-a)^2=a(b-a) \Rightarrow b=2a, c=3a$

$\therefore a:b:c=1:2:3$

(B) $x=\dfrac{a+b}{2}, b=ar^3 \Rightarrow r=\left(\dfrac{b}{a}\right)^{\frac{1}{3}}$

$$\frac{y^3+z^3}{myz}=\frac{a+b}{\dfrac{a+b}{2}}=2$$

(C)

$$a_1^{\ 2}-a_2^{\ 2}+a_3^{\ 2}------a_{50}^{\ 2}=\left(a_1+a_2\right)\left(a_1-a_2\right)+\left(a_3+a_4\right)\left(a_3-a_4\right)+------+\left(a_{49}+a_{50}\right)\left(a_{49}-a_{50}\right)$$

$$=-d\left[a_1+a_2+----+a_{50}\right]=-\frac{25}{49}\left(a_{50}-a_1\right)\left(a_{50}+a_1\right)$$

$$=\left(\frac{25}{49}\right)\left(a_1^{\ 2}-a_{50}^{\ 2}\right)$$

(D) $\tan^{-1}\left(\dfrac{1}{2r^2}\right)=\tan^{-1}\left(\dfrac{2}{4r^2}\right)=\tan^{-1}\left(\dfrac{2r+1-(2r-1)}{1+(2r+1)(2r-1)}\right)$

$= \tan^{-1}(2r+1) - \tan^{-1}(2r-1)$

19.	(A)	$y = -2x + 12$ is a normal to the parabola $y^2 = 4x$ at the point whose distance from the focus of the parabola is	(p)	2
	(B)	Length of the latus rectum of a parabola whose focus is (2, 0) and directrix $3x + 4y + 4 = 0$, is	(q)	3
	(C)	If $2x + 3y = \alpha$, $x - y = \beta$ and $kx + 15y = \gamma$ are the three concurrent normals of parabola $y^2 = \lambda x$, the value of k is	(r)	4
	(D)	If two distinct chords of a parabola $y^2 = 4ax$, passing through (a, 2a) are bisected on the line $x + y = 1$, then length of the latus rectum can be	(s)	5

Key. (A–s), (B–r), (C–s), (D–p, q)

Sol. (A) We know $y = mx - 2am - am^3$ is a normal to the parabola $y^2 = 4ax$ at the point $(am^2, -2am)$

The given equation can be written as

$y = -2x - 2a(-2) - (-2)^3 a$

Which represents a normal to the parabola corresponding to $m = -2$ at the point (4a, 4a) whose distance from the focus (a, 0) is

$\sqrt{(4a-a)^2 + (4a)^2} = \sqrt{(9a^2 + 16a^2)} = 5a = 5$

(B) Length of the latusrectum = 4a

= 2(distance from the focus of the directrix)

$= 2 \times \frac{|3\times 2 + 4\times 0 + 4|}{\sqrt{(9+16)}} = \frac{2\times 10}{5} = 4$ unit

(C) We know that, $m_1 + m_2 + m_3 = 0$

$\Rightarrow m_3 = -m_1 - m_2 = \frac{2}{3} - 1 = -\frac{1}{3} = -\frac{k}{15} \Rightarrow k = 5$

(D) Any point on the line $x + y = 1$ can be taken as $(t, 1-t)$. Equation of the chord, with this as mid point is

$y(1-t) - 2a(x+t) = (1-t)^2 - 4at$, it passes through (a, 2a).

So, $t^2 - 2t + 2a^2 - 2a + 1 = 0$, this should have two distinct real roots so,

$a^2 - a < 0$, $0 < a < 1$, so, length of latusrectum < 4.

20. Equation of a parabola is $(3x - 4y + 1)^2 = 20|4x + 3y - 7|$, then

	Column I		Column II
(A)	equation of directrix	(p)	$3x - 4y + 1 = 0$
(B)	equation of axis	(q)	$4x + 3y - 2 = 0$
(C)	equation of tangent at the vertex	(r)	$4x + 3y - 12 = 0$
(D)	equation of latus rectum	(s)	$4x + 3y - 7 = 0$

Key.(A-r, q), (B-p), (C-s), (D-q, r)

Sol. Conceptual

21. The parabola $y^2 = 4ax$ has a chord AB joining points $A(at_1^2, 2at_1)$ and $B(at_2^2, 2at_2)$.

	Column I		Column II
(A)	If AB is a normal chord then	(p)	$t_2 = -t_1 + 2$
(B)	If AB is a focal chord then	(q)	$t_2 = \frac{-4}{t_1}$
(C)	If AB subtends 90º at point (0, 0) then	(r)	$t_2 = \frac{-1}{t_1}$
(D)	If AB is inclined at 45º to the axis of parabola then	(s)	$t_2 = -t_1 - \frac{2}{t_1}$

Key. (A-s), (B-r), (C-q), (D-p)

Sol. (A) If AB is a normal chord $t_2 = -t_1 - \frac{2}{t_1}$

(B) If AB is a focal chord $t_1 t_2 = -1$

(C) If AB subtend 90° at (0, 0) then

$t_1 t_2 = -4$

(D) $\tan 45° = \frac{2a(t_1 - t_2)}{a(t_1 - t_2)(t_1 + t_2)} = \frac{2}{t_1 + t_2}$

$\therefore 1 = \frac{2}{t_1 + t_2} \Rightarrow t_1 + t_2 = 2$

22.

(A)	If the co-ordinates of a point are (4 tan ϕ, 3 sec ϕ) where ϕ is a parameter then the points lies on a conic section whose eccentricity is	(p)	$\sqrt{3}$
(B)	The eccentricity of conic whose conjugate diameter are $y = -x$ & $y = 3x$ is	(q)	$\frac{\sqrt{3}}{2}$
(C)	If AB is a latus rectum of a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ such that ΔOAB (0 is origin) is an equilateral triangle the eccentricity of hyperbola e is	(r)	$\frac{5}{3}$
(D)	If the foci of the ellipse $\frac{x^2}{k^2a^2} + \frac{y^2}{a^2} = 1$ and the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$ coincide then one of the value of k is equal to	(s)	$\sqrt{\frac{2}{3}}$

Key. (A-r), (B-s), (C-q), (D-p)

Sol. (A) Equation of curve is $y^2/9 - x^2/16 = 1$

$\Rightarrow e = 5/3$

(B) We have $F_1P + F_2P = 2a = 10 \Rightarrow a = 5$

$(F_1B_1)(F_2B_2) = b^2 = 16 \Rightarrow b = 4$

e = $\frac{3}{5}$

(C) $\frac{3}{a^2}-\frac{1}{9}=1$

$\frac{1}{a^2}=\frac{10}{27} \Rightarrow a^2=\frac{27}{10}$

Hence e = $\sqrt{\frac{13}{3}}$

(D) focii of the ellipse $\left(\pm a\sqrt{k^2-1},\, 0\right)$, focii of hyperbola $\left(\pm\sqrt{2}\, a,\, 0\right)$ equating the both focii we get k = ± $\sqrt{3}$, one of the values of k = $\sqrt{3}$.

23. If $y = m_i x + \frac{1}{m_i}, (i = 1,2,3)$ represent three Straight lines whose slopes roots of the equation $2m^3 - 3m^2 - 3m + 2 = 0$, then

	Column I		Column II
(A)	Algebraic sum of the intercepts made by the lines on x-axis is,	p)	$\frac{4\sqrt{2}+9\sqrt{5}}{4}$
(B)	Algebraic sum of the intercepts made by the lines on y-axis is,	(q)	$\frac{3}{2}$
(C)	Sum of the distances of the lines from origin is	(r)	$\frac{-21}{4}$
(D)	Sum of the lengths of the lines intercepted between the coordinate axes is	(s)	$\frac{5\sqrt{2}+9\sqrt{5}}{10}$
		(t)	0

Key. $A - r$
$B - q$
$C - s$
$D - p$

Sol. M = -1, ½, 2

a) $\sum \frac{-1}{M_i^2} = \frac{-21}{4}$

b) $\sum \frac{1}{M_i} = \frac{3}{2}$

c) $\sum\left|\frac{-1/M_i}{\sqrt{1+M_i^2}}\right| = \frac{5\sqrt{2}+9\sqrt{5}}{10}$

d) $\sum\sqrt{\left(\frac{1}{M_i^2}\right)^2+\left(\frac{1}{M_i}\right)^2} = \frac{4\sqrt{2}+9\sqrt{5}}{10}$ 40. B- p,q,r,s

C-p,q,r

D- q,r,s,t

The other vertices of the triangle are (5,2 $\sqrt{5}$) and $\left(5,-2\sqrt{5}\right)$

Therefore , the centroid is $\left(\frac{10}{3},0\right)$;

the circumcenter is $\left(\frac{9}{2},0\right)$

and the incenter is (3,0).

24. A triangle ABC is inscribed in the parabola $y^2=4x$ with A as vertex and the orthocenter of the triangle as the focus of the parabola

	Column I		Column II
(A)	The distance of the centroid of the triangle from the vertex A is not more than	(p)	1
(B)	The distance of the circumcenter of the triangle from the vertex A is more than	(q)	2
(C)	The distance of the incenter of the triangle from the vertex A is not less than	(r)	3
(D)	Distance between the incenter and the circum center of the triangle is less than	(s)	4
		(t)	5

Key.
$A-s,t$
$B-p,q,r,s$
$C-p,q,r$
$D-q,r,s,t$

Sol. Conceptual

25. Match the following:

	Column-I		Column-II
a)	Locus of centre of circles touching $x^2+y^2-4x-4y=0$ internally and $x^2+y^2-6x-6y+17=0$ externally is,	p)	Straight line
b)	The locus of the point $(3h-2,3k)$ where (h,k) lies on the circle $x^2+y^2-2x-4y-4=0$ is	q)	Circle
c)	Locus of centres of the circles touching the two circles $x^2+y^2+2x=0$ and $x^2+y^2-6x+5=0$ externally is	r)	Ellipse
d)	emities of a diagonal of a rectangle are $(0,0)$ and $(4,4)$. The locus of the extremities of the diagonal is	s)	Part of Hyperbola

Key. A-r, C-s, B-a, D-q

Sol. $a \to r, b \to q, c \to s, d \to q$

a) $sp + s^1 p = 2a$

b) $\alpha = 32 - 2, \beta = 3k \Rightarrow \dfrac{\alpha+2}{3} = h, \dfrac{\beta}{3} = k$

c) $\left|sp - s^1 p\right| = 2a$

d) Locus is a circle with the given diagonal as diameter

26. Consider the following linear equations in x and y

ax + by + c = 0

bx + cy + a = 0

cx + ay + b = 0

Match the condition in column I with statement in column II

	Column – I		Column – II
(A)	a + b + c = 0 and $a^2 + b^2 + c^2 = ab + bc + ca$	(P)	Lines are identical
(B)	a + b + c = 0 and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(Q)	Lines represent the whole of the xy plane
(C)	a + b + c $\neq$ 0 and $a^2 + b^2 + c^2 = ab + bc + ca$	(R)	Lines are different and passing through a fixed point
(D)	a + b + c $\neq$ 0 and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(S)	Lines are sides of a triangle

Key. A – q, B – r, C – p, D – s

Sol. Conceptual

27. *P* is a point on the ellipse $9x^2+25y^2=225$. The tangent at *P* meet the X-axis, Y-axis at T, t respectively and the normal at *P* meet the X-axis, Y-axis at G, g respectively. *C* is the centre of the ellipse and *F* is the foot of the perpendicular from *C* to normal at *P*.

Column – I	Column – II
a) $\lvert PF\rvert \times \lvert PG\rvert =$	p) 25
b) $\lvert PF\rvert \times \lvert Pg\rvert =$	q) 16
c) $\lvert CG\rvert \times \lvert CT\rvert =$	r) 9
d) $\lvert Ct\rvert \times \lvert Cg\rvert =$	s) 24

Key. a) r; b) p; c) q; d) q

Sol. Conceptual

28.

Column – I	Column – II
a) A tangent to the ellipse $\frac{x^2}{27}+\frac{y^2}{48}=1$ has slope $-\frac{4}{3}$ and the tangent cuts the axes of the ellipse at A, B. Area of ΔOAB is (*O* is the origin)	p) 36
b) Product of perpendiculars drawn from the points $(\pm 3, 0)$ to the line $y=mx-\sqrt{25m^2+16}$ is	q) $10\sqrt{2}$
c) An ellipse passing through (0, 0) has its foci at (3, 4) and (6, 8). Length of its minor axis is	r) 24
d) If *e* is the eccentricity of the conic $\sqrt{x^2+y^2}+\sqrt{(x+3)^2+(y-4)^2}=10$, then $72e=$	s) 16

Key. a) p; b) s; c) q; d) p

Sol. Conceptual

29. The normals at four points $(x_i, y_i),\ i=1,2,3,4$ on the hyperbola $xy=4$ are concurrent at the point (α, β)

Column – I	Column – II
a) $y_1+y_2+y_3+y_4=$	p) 0
b) $\sum_{1\le i<j\le 4} x_i x_j =$	q) –16
c) $x_1x_2x_3x_4=$	r) $-\beta$
d) $y_1y_2y_3y_4=$	s) β

Key. a) s; b) p; c) q; d) q

Sol. Conceptual

30. Match the statements/expressions in Column I with the open intervals in Column II

	Column I		Column II
(A)	The x-coordinate of points on the axis of the parabola $y^2 - 4x - 2y + 5 = 0$ from which all the three normals to the parabola are real is	(p)	4
(B)	The x-coordinate of points on the axis of the parabola $4y^2 - 32x + 4y + 65 = 0$ from which all the three normals to the parabola are real is	(q)	5
(C)	The x-coordinate of points on the axis of the parabola $4y^2 - 16x - 4y + 41 = 0$ from which all the three normals to the parabola are real is	(r)	6
(D)	The x-coordinate of the point on the axis of the parabola from which all the three normals drawn to the parabola $y^2 - 2y - 8x + 17 = 0$ are real is	(s)	7
		(t)	8

Key. (A –p, q, r, s, t), (B –s, t), (C –q, r, s, t), (D –s, t)

Sol. If three normals drawn to any parabola $y^2 = 4ax$ from a given point (h, k) be real, then $h > 2a$.

(A) Q $y^2 - 4x - 2y + 5 = 0$

$\Rightarrow (y-1)^2 = 4(x-1)$

Let $y - 1 = Y$ and $x - 1 = X$

$\therefore\ Y^2 = 4X$

On comparing with $Y^2 = 4aX$

$\therefore\ a = 1$

According to question $X > 2a$

$\Rightarrow \quad x - 1 > 2$

or $x > 3$

$\therefore\ x = 4, 5, 6, 7, 8$ (P,Q,R,S,T)

(B) Q $4y^2 - 32x + 4y + 65 = 0$

$\Rightarrow 4(y^2 + y) = 32x - 65$

$$\Rightarrow 4\left(\left(y+\frac{1}{2}\right)^2 - \frac{1}{4}\right) = 32x - 65$$

$$\Rightarrow 4\left(y+\frac{1}{2}\right)^2 = 32x - 64$$

$$\text{or } \left(y+\frac{1}{2}\right)^2 = 8(x-2)$$

Let $y + \frac{1}{2} = Y$ and $x - 2 = Y$

$\therefore\ Y^2 = 8X.$

On comparing with $y^2 = 4ax$

$\therefore\ a = 2$

According to question X > 2a

$\Rightarrow$ x – 2 > 4

$\therefore$ x > 6

$\therefore$ x = 7, 8 (S, T)

(C) Q $4y^2 - 16x - 4y + 41 = 0$

$\Rightarrow \quad 4(y^2 - y) = 16x - 41$

$$\Rightarrow \quad 4\left\{\left(y-\frac{1}{2}\right)^2 - \frac{1}{4}\right\} = 16x - 41$$

$$\Rightarrow \quad 4\left(y-\frac{1}{2}\right)^2 = 16x - 40$$

$$\text{or} \quad \left(y-\frac{1}{2}\right) = 4\left(x-\frac{5}{2}\right)$$

Let $\quad y - \frac{1}{2} = Y$ and $x - \frac{5}{2} = X$

$\therefore \quad Y^2 = 4X$

On comparing with $Y^2 = 4aX$

$\therefore \quad a = 1$

According to question

X > 2a

$$\Rightarrow \quad x - \frac{5}{2} > 2$$

$$\text{or} \quad x > \frac{9}{2}$$

$\therefore$ x = 5, 6, 7, 8 (Q, R, S, T)

31.	(A)	$y = -2x + 12$ is a normal to the parabola $y^2 = 4x$ at the point whose distance from the focus of the parabola is	(p)	2
	(B)	Length of the latus rectum of a parabola whose focus is (2, 0) and directrix $3x + 4y + 4 = 0$, is	(q)	3
	(C)	If $2x + 3y = \alpha$, $x - y = \beta$ and $kx + 15y = \gamma$ are the three concurrent normals of parabola $y^2 = \lambda x$, the value of k is	(r)	4
	(D)	If two distinct chords of a parabola $y^2 = 4ax$, passing through (a, 2a) are bisected on the line $x + y = 1$, then length of the latus rectum can be	(s)	5

Key. (A–s), (B–r), (C–s), (D–p, q)

Sol. (A) We know $y = mx - 2am - am^3$ is a normal to the parabola $y^2 = 4ax$ at the point $(am^2, -2am)$

The given equation can be written as

$y = -2x - 2a(-2) - (-2)^3 a$

Which represents a normal to the parabola corresponding to m = –2 at the point (4a, 4a) whose distance from the focus (a, 0) is

$$\sqrt{(4a-a)^2(4a)^2} = \sqrt{(9a^2+16a^2)} = 5a = 5$$

(B) Length of the latusrectum = 4a

= 2(distance from the focus of the directrix)

$= 2 \times \frac{|3\times2+4\times0+4|}{\sqrt{(9+16)}} = \frac{2\times10}{5}$ = 4 unit

(C) We know that, $m_1 + m_2 + m_3 = 0$

$\Rightarrow m_3 = -m_1 - m_2 = \frac{2}{3} - 1 = -\frac{1}{3} = -\frac{k}{15} \Rightarrow k = 5$

(D) Any point on the line x + y = 1 can be taken as (t, 1 – t). Equation of the chord, with this as mid point is

$y(1-t) - 2a(x+t) = (1-t)^2 - 4at$, it passes through (a, 2a).

So, $t^2 - 2t + 2a^2 - 2a + 1 = 0$, this should have two distinct real roots so,

$a^2 - a < 0$, $0 < a < 1$, so, length of latusrectum < 4.

32. A triangle ABC is inscribed in the parabola $y^2=4x$ with A as vertex and the orthocenter of the triangle as the focus of the parabola

	Column I		Column II
(A)	The distance of the centroid of the triangle from the vertex A is not more than	(p)	1
(B)	The distance of the circumcenter of the triangle from the vertex A is more than	(q)	2
(C)	The distance of the incenter of the triangle from the vertex A is not less than	(r)	3
(D)	Distance between the incenter and the circum center of the triangle is less than	(s)	4
		(t)	5

KEY : *A-s,t*

B- p,q,r,s

C-p,q,r

D- q,r,s,t

Sol. The other vertices of the triangle are $(5, 2\sqrt{5})$ and $\left(5, -2\sqrt{5}\right)$

Therefore , the centroid is $\left(\frac{10}{3}, 0\right)$;

the circumcenter is $\left(\frac{9}{2}, 0\right)$

and the incenter is (3,0).

33.

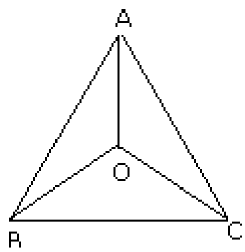
	Column-I		Column-II
A)	The distance of the point (1,-2,3) from the plane x-y+z-5=0 measured parallel to $\frac{x}{2}=\frac{y}{3}=\frac{z-1}{-6}$	p)	0
B)	If the straight lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{4-z}{K}$ and $\frac{x-1}{K}=\frac{y-4}{2}=\frac{z-5}{1}$ intersect then K is equal to	q)	1
C)	The shortest distance between any two opposite edges of the tetrahedron formed by the planes $y+z=0, z+x=0, x+y=0$ and $x+y+z=\sqrt{6}$ is	r)	-3
D)	If θ is the angle between line x=y=z and the plane x+y+z=4 then $\tan\frac{\theta}{2}$ is	s)	2

Key. A) q B) p,r C) s D) q

Sol. A) point on the line is (1+2λ,-2+3λ,3-6λ)

B) $\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -K \\ K & 2 & 1 \end{vmatrix}=0$

C) Vertices of a tetrahedron are O(0,0,0), A$(\sqrt{6},\sqrt{6},-\sqrt{6})$, $B(\sqrt{6},-\sqrt{6},\sqrt{6})$, $C(-\sqrt{6},\sqrt{6},\sqrt{6})$ find the shortest distance between the lines $\overrightarrow{AO}$ & $\overrightarrow{BC}$



D) line is perpendicular to the plane $\theta=\frac{\pi}{2}$

34. Match the following:-

LIST-I		LIST-II	
A)	Radius of the largest circle which passes through the focus of the parabola $y^2=4x$ and contained in it, is	p)	16
B)	The shortest distance between parabola $y^2=4x$ and $y^2=2x-6$ is d then d^2 is	q)	5
C)	The harmonic mean of the segments of a focal chord of a parabola $y^2=12x$ is	r)	6
D)	Tangents drawn from P to the parabola $y^2=16x$ meet at A and B are perpendicular then the least value of AB is	s)	4

Key. A) s B) q C) r D) p

Sol. A) $(x-h)^2+y^2=(h-1)^2$

$y^2=4x$

$(x-h)^2+4x=(h-1)^2$

Apply $\Delta=0$

B) Find the common normal to

$y^2=4x$, $y^2=2x-6$

$y=m(x-3)-m-\frac{1}{2}m^3$ Apply $c=-2am-am^3$

$-4m-\frac{1}{2}m^3=-2m-m^3$

$\frac{m^3}{2}-2m=0$ $m=0, m=\pm 2$

C) length of the semi latusrectum is the H.M between segments of the focal chord.

D) Tangents at the ends of focal chord are perpendicular

35. Match the Following:

Column I		Column II	
(A)	Number of mutually perpendicular tangents that can be drawn from the curve $y=\left\|\left|1-e^x\right|-2\right\|$ to the parabola $x^2=-4y$	(p)	2
(B)	Locus of vertex of parabola whose focus is (1, 2) and latus rectum is of 12 unit is $(x-1)^2+(y-2)^2=a^2$ then a =	(q)	4
(C)	A line drawn through the focus F and parallel to tangent at $P(1,2\sqrt{2})$ on the	(r)	3

	parabola $y^2 = 8x$ cut the line $y = 2\sqrt{2}$ at Q then PQ is equal to		
(D)	A movable parabola touches the x and y-axes at (1, 0) and (0, 1) then radius of locus of focus of parabola	(s)	0
		(t)	5

Key: (A -p), (B -r), (C -r), (D -s)

Hint: (A) Two tangent can be drawn because curve $y = \left|\left|1-e^x\right|-2\right|$ intersect the line y = 1 at two points

(B) $(x-1)^2 + (y-2)^2 = 3^2 \Rightarrow a = 3$

(C) PQ = a + x = 2 + 1 = 3

(D) Locus of focus of parabola is $2x^2 - 2x + 2y^2 - 2y + 1 = 0$

$\therefore$ radius is zero

36. Column I (equation of pair of curves) — Column II (equation of a common tangent)

Column I		Column II	
(A)	$xy = -4$ and $x^2 + 16y = 0$	(p)	$x + 2y + 1 = 0$
(B)	$x^2 = 8y$ and $y^2 = x$	(q)	$x - y + 4 = 0$
(C)	$7x^2 + 25y^2 - 175 = 0$ and $x^2 + y^2 - 16 = 0$	(r)	$x + y + 4\sqrt{2} = 0$
(D)	$x^2 + y^2 - 4 = 0$ and $x^2 + y^2 - 8x + 15 = 0$	(s)	$y = \frac{1}{\sqrt{15}}(x - 8)$

Key: (A-q), (B-p), (C-r), (D-s)

Hint: For A $\rightarrow$ 5 × 6 × 6 × 2 = 360

For B $\rightarrow$ $^5C_2 \left[\frac{4!}{2!2!} + \frac{4!}{3!} \times 2\right] + {}^5C_1 \left[\frac{3!}{2!} \times 2 + 1\right]$ = 175

For C $\rightarrow$ $^5C_3 \times \frac{4!}{2!} + {}^5C_2\ [9 \times 2 + 6]$ = 360

37. A triangle ABC is inscribed in the parabola $y^2=4x$ with A as vertex and the orthocenter of the triangle as the focus of the parabola

Column I		Column II	
(A)	The distance of the centroid of the triangle from the vertex A is not more than	(p)	1
(B)	The distance of the circumcenter of the triangle from the vertex A is more than	(q)	2
(C)	The distance of the incenter of the triangle from the vertex A is not less than	(r)	3

(D)	Distance between the incenter and the circum center of the triangle is less than	(s)	4
		(t)	5

Key: A→s,t; B→p,q,r,sC→p,q,r; B→q,r,s,t

Hint: The other vertices of the triangle are $(5, 2\sqrt{5})$ and $\left(5, -2\sqrt{5}\right)$

Therefore , the centroid is $\left(\frac{10}{3}, 0\right)$;

the circumcenter is $\left(\frac{9}{2}, 0\right)$

and the incenter is (3,0).

38.

	Column I		Column II
(A)	Suppose F_1, F_2 are the foci of the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and P is a point on the ellipse such that $PF_1:PF_2 = 2:1$. Then area of the triangle PF_1,F_2 exceeds	(p)	1
(B)	The straight line $\frac{x}{4}+\frac{y}{3}=1$ intersects the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ at two points A and B. The number of points P on the ellipse such that area of triangle PAB is 3 is not less than	(q)	2
(C)	The number of roots of the equation $x\sin x = 1 \, in \, (0, 2\pi) \, is \; less\, than$	(r)	3
(D)	The number of solutions of $\sin^5 x + \cos^3 x = 1$ in the interval $[-\pi, \pi]$ is less than	(s)	4
		(t)	5

KEY A-p, q, r, B-p, q, C-r, s ,t, D- r, s, t

Hint (A) $PF_1 = 4, PF_2 = 2 \; and \; F_1F_2 = 2\sqrt{5}$.. So the triangle PF_1F_2 is right angled at P and its area is 4

(B) If the points P and O lie on either side of AB, then the distance of P from AB is less than $\frac{6}{5}$ which is the height of the triangle with base AB. So P and O lie to the same side of AB.

C) The equation x sin x = 1 has precisely one root in each of the intervals $\left(0,\frac{\pi}{2}\right)$ *and* $\left(\frac{\pi}{2},\pi\right)$ *and no root in* $(\pi,2\pi)$

D) There are only two solutions (i.e) $x=\frac{\pi}{2}$ and $x = 0$ in the interval $[-\pi,\pi]$

39. Consider the ellipse $(3x-6)^2+(3y-9)^2=\frac{4}{169}(5x+12y+6)^2$.

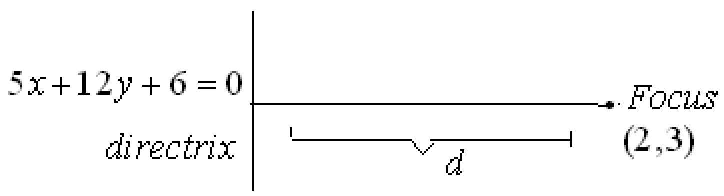
Column I contains the distances associated with this ellipse and Column II gives their value.

Match the expressions/statements in column I with those in column II.

	Column – I		Column – II
(A)	The length of major axis	(P)	$\frac{72}{5}$
(B)	The length of minor axis	(Q)	$\frac{16}{\sqrt{5}}$
(C)	The length of latus rectum	(R)	$\frac{16}{3}$
(D)	The distance between the directrices	(S)	$\frac{48}{5}$

KEY : A-S, B-Q, C-R, D-P

Sol. Rewrite the equation as $(x-2)^2+(y-3)^2=\frac{4}{9}\left[\frac{5x+12y+6}{13}\right]^2$



$$d=\frac{5.2+12.3+6}{\sqrt{5^2+12^2}}=\frac{52}{13}=4$$

Also $e=\frac{2}{3}$

Length of major axis $= \dfrac{2e}{1-e^2}\ d = \dfrac{2.2/3}{1-4/9} \times 4 = \dfrac{48}{5}$

Length of minor axis = (Length of major axis) $\sqrt{1-e^2} = \dfrac{16}{\sqrt{5}}$

Length of latusrectum = (Length of major axis) $\left(1-e^2\right) = \dfrac{16}{3}$

Distance between the directrices = (Length of major axis) $\times$ 1/e = 72/5

40. Match the following:

	Column-I		Column-II
a)	The equation of the axis of the parabola $9y^2 - 16x - 12y - 57 = 0$ is	p)	10
b)	The parametric equation of a parabola is $x = t^2 + 1;\ y = 2t + 1$: Then the equation of Directrix is	q)	7
c)	A point P on the ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$ has the eccentric angle $\dfrac{\pi}{8}$. The sum of the distances of P from the two foci is	r)	$3x - 2 = 0$
d)	The foci of the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{b^2} = 1$ and the hyperbola $\dfrac{x^2}{144} - \dfrac{y^2}{81} = \dfrac{1}{25}$ coincide then the value of $b^2 =$	s)	$3y - 2 = 0$
		t)	$x = 0$

Key: $a \to s, b \to t, c \to p, d \to q$

Hint: Conceptual

41. Match the following:

Column -I		Column -II	
(A)	The normal chord at a point on the parabola $y^2 = 4x$ subtends a right angle at the vertex, then t^2 is	(p)	4
(B)	The area of the triangle inscribed in the curve $y^2 = 4x$, the parameter of coordinates whose vertices are $1, 2$ and 4 is	(q)	2
(C)	The number of distinct normal possible from $\left(\dfrac{11}{4}, \dfrac{1}{4}\right)$ to the parabola $y^2 = 4x$ is	(r)	3
(D)	The normal at $(a, 2a)$ on $y^2 = 4ax$ meets the curve again at $\left(at^2, 2at\right)$, then the value of $\lvert t-1 \rvert$ is	(s)	6

Key. (A) $\to$(q); (B) $\to$(s); (C) $\to$(q) ; (D) $\to$(p)

Sol. A) Equation of normal is $y=-tx+2at+at^3 at P(t)$

It intersect the curve again at point $Q(t_1)$ on the parabola such that $t_2=-t-\frac{2}{t}$

Again slope of OP is $\frac{2}{t}=M_{OP}$

Also, slope of OQ is $\frac{2}{t_1}=M_{OQ}$

Since $M_{OP}.M_{OQ}=-1=\frac{4}{tt_1} \Rightarrow tt_1=-4$

$t\left(-t-\frac{2}{t}\right)=-4 \Rightarrow t^2=2$

B) $P(1,2)$, $Q(4,4)$, $R(16,8)$

Now, $ar(\Delta PQR)=6$ sq.units

C) Equation of normal from any point $P(am^2,-2m)$ is $y=mx-2am-am^3$

It passes through $\left(\frac{11}{4},\frac{1}{4}\right) \Rightarrow 4m^2+8m-11m+1=0 \Rightarrow 4m^2-3m+1=0$

Now, $f(m)=4m^2-3m \Rightarrow f'(m)=12m^2-3=0 \Rightarrow =m\pm\frac{1}{2}$

Since $f\left(\frac{1}{2}\right)f\left(\frac{-1}{2}\right)<0$ has 3 normals are possible

D) Since, normal at $P(t_1)$ if meets the curve again at (t_2), then $t_2=-t_1-\frac{2}{t_1}$

Such that here normal at $P(1)$ meets the curve again at $Q(t)$

$\Rightarrow t=-1-\frac{1}{2}=-3 \Rightarrow |t-1|=4$

42.

	Column I		Column II
(A)	The coordinates of the point on the parabola $y=x^2+7x+2$, which is nearest to the straight line y = 3x – 3 are	(p)	(2,1)
(B)	y = x + 2 is a tangent to the parabola $y^2=8x$. The point on this line, the other tangent from which is perpendicular to this tangent is	(q)	(–2, 0)
(C)	The point on the ellipse $x^2+2y^2=6$ whose distance from the line x + y = 7 is least is	(r)	$\left(2,\frac{2}{\sqrt{3}}\right)$

(D)	The foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ are S and S'. P is a point on the ellipse whose eccentric angle is $\pi/3$. The incentre of the triangle SPS' is	(s)	(–2,–8)
		(t)	(2, 2)

Key. A – s ; B – q ; C – p ; D – r

Sol. (A) Any point on the parabola is (x, x^2+7x+2) Its distance from the line

y = 3x – 3 is given by

$$P=\left|\frac{3x-(x^2+7x+2)-3}{\sqrt{9+1}}\right|$$

$$=\left|\frac{x^2+4x+5}{\sqrt{10}}\right|$$

$$=\frac{x^2+4x+5}{\sqrt{10}}(\text{as } x^2+4x+5>0 \text{ for all } x\in R)$$

$\frac{dP}{dx}=0\Rightarrow x=-2.$ So, the required point is $(-2,-8)$

(B) Let (x_1y_1) be a point on y = x + 2

Therefore, $y_1=x_1+2$

Equation of the line perpendicular to the given line through (x_1, y_1) is

$$y-(x_1+2)=-(x-x_1)\text{i.e., } y=-x+2(x_1+1)$$

If this line is a tangent to $y^2=8x, c=\frac{a}{m}$ gives

$$2(x_1+1)=\frac{2}{-1}\text{i.e., } x_1+1=-1\Rightarrow x_1=-2$$

Hence, $y_1=0$

Therefore, the required point is (–2, 0)

(C) Given equation of ellipse $\frac{x^2}{6}+\frac{y^2}{3}=1$. Slope of the tangent at any point

$P(x_1, y_1)$ to $\frac{x^2}{6}+\frac{y^2}{3}=1$ is given by $2x+4y\frac{dy}{dx}=0$

$\Rightarrow x=2y$

$\because \dfrac{dy}{dx} = \dfrac{-x}{2y} = -1$

Putting x = 2y in the equation of the ellipse we have y = 1. Evidently, the point lies in the first quadrant

Therefore, y = 1 and x = 2

Hence, required point is (2, 1)

(D) The coordinates of the point P are $\left(\dfrac{5}{2}, \dfrac{3\sqrt{3}}{2}\right)$. Since $e = \sqrt{1 - \dfrac{9}{25}} = \dfrac{4}{5}$, so, the coordinates of the foci are S(4, 0) and $S'(-4,0)$ and $SS' = 8$.

Also, $SP = a - ex_1 = 5 - \dfrac{4}{5} \times \dfrac{5}{2} = 3$

And $S'P = a + ex_1 = 7$

Therefore, the coordinates of the incentre (x_1, y_1) are

$$x_1 = \frac{7 \times 4 + 3 \times -4 + 8 \times \frac{5}{2}}{7 + 3 + 8} = 2$$

$$y_1 = \frac{7 \times 0 + 3 \times 0 + 8 \times \frac{3\sqrt{3}}{2}}{7 + 3 + 8} = \frac{2}{\sqrt{3}}$$

43. AB is a chord of the parabola $y^2 = 4x$ such that the normals at A and B intersect at the point $C(9,6)$

Column-I	Column-II
A. Length (AB)	p. 20
B. Area of $\Delta^{le} ABC$	q. $\dfrac{4}{\sqrt{13}}$
C. Distance of origin from the line through (AB)	r. $\sqrt{13}$
D. The area bounded by the coordinate axes and the line through (AB)	s. $\dfrac{4}{3}$

Key. A-r, B-p,C-q,D-s

Sol. Conceptual

44. Match the following

Consider the parabola $y^2 = 12x$

	COLUMN-I	COLUMN-II
(A)	Tangent and normal at the extremities of the latus rectum intersect the x-axis at T & G respectively. The coordinates of middle point of T & G are	P) (0,0)
(B)	Variable chords of the parabola passing through a fixed point K on the axis, such that sum of the reciprocals of two parts of the chord through K, is a constant. Coordinates of K are	Q) (3, 0)
(C)	All variable chords of the parabola subtending a right angle at the origin are concurrent at the point	R) (6, 0)
(D)	AB & CD are the chords of a parabola which intersect at a point E on the axis. The radical axis of the two circles described on AB & CD as diameter always passes through the point	S) (12, 0)

Key. A-Q, B-Q, C-S, D-P

Sol. Sol-(A)

Equation of tangent at (3, 6) : y = x + 3 $\Rightarrow$ T(- 3, 0)

Equation of normal at (3, 6) : y = - x + 9 $\Rightarrow$ G(9, 0)

Hence middle point (3, 0)

Sol-(B)

Point is obviously focus (3, 0)

Sol-(C)

If variable chord is PQ, then

$Let \quad P(t_1) \,\&\, Q(t_2)$
$\Rightarrow \quad t_1 t_2 = -4$ $\Rightarrow$ Chords are concurrent at (4a, 0) $\Rightarrow$ (12, 0)

Sol-(D)

$Let \quad A(t_1) \,\&\, B(t_2), C(t_3) \,\&\, D(t_4)$

If AB & CD intersect at a point E on the axis, then by solving the equations of AB & CD we get the relation $t_1 t_2 = t_3 t_4$

Now equations of the circles with AB & CD as diameters are

$$(x - at_1^2)(x - at_2^2) + (y - 2at_1)(y - 2at_2) = 0$$
$$(x - at_3^2)(x - at_4^2) + (y - 2at_3)(y - 2at_4) = 0$$

If we solve these two circles, then equation of their radical axis is of the form y = mx

45. Consider the parabola $(x-1)^2 + (y-2)^2 = \dfrac{(12x - 5y + 3)^2}{169}$.

	Column - I		Column – II

(a)	Locus of point of intersection of perpendicular tangent	p.	$12x-5y-2=0$
(b)	Locus of foot of perpendicular from focus upon any tangent	q.	$5x+12y-29=0$
(c)	Line along which minimum length of focal chord occurs	r.	$12x-5y+3=0$
(d)	Line about which parabola is symmetrical	s.	$24x-10y+1=0$

Key. a $\to$ r, b $\to$ s, c $\to$ p, d $\to$ q

Sol. Focus (1, 2) directrix = $12x-5y+3=0$

(a) Locus of point of intersection of perpendicular tangents is directrix = 12x-5y+3=0.
(b) Locus of point of perpendicular from focus upon any tangent is the line parallel to directrix and passing through vertex = 24x-10y+1=0.
(c) Required line in the line parallel to directrix and passing through focus = 12x-5y-2=0.
(d) Required line is the line perpendicular to directrix and passing through focus = 5x+12y-29=0.

46. The parabola $y^2=4ax$ has a chord AB joining the points $A\left[at_1^2, 2at_1\right]$ and $B\left[at_2^2, 2at_2\right]$.

Match the following:

Column I		Column II	
(a)	If AB is a normal chord at A, then	(p)	$t_2=-t_1+2$
(b)	If AB is a focal chord, then	(q)	$t_2=\frac{-4}{t_1}$
(c)	If AB subtends 90^0 at $(0,0)$, then	(r)	$t_2=\frac{-1}{t_1}$
(d)	If AB is inclined at 45^0 with the axis of parabola in anti clock wise sense wrt positive direction of x-axis, then	(s)	$t_2=-t_1-\frac{2}{t_1}$

Key. A – s; B – r; C – q; D – p

Sol. (a) $t_2=-t_1-\frac{2}{t_1} \Rightarrow (a)\to(s)$

(b) AB is a focal chord $\Rightarrow t_1t_2=-1$ $(b)\to(r)$

(c) AB subtends 90^0 $\Rightarrow \frac{-2}{t_1}\times\frac{-2}{t_2}=-1 \Rightarrow t_1t_2=-4$ $(c)\to(q)$

(d) $\Rightarrow \dfrac{2}{t_1+t_2}=1 \;\; t_1+t_2=2 \quad (d)\to(p)$

47. Match the following:
Given the parabola $y^2 = 4x$ then mach the points in list I with the no.of distinet real normals that can be drawn from the point to the parabola

	Column I		Column II
(A)	(1,2)	(P)	0
(B)	*(2,3)*	(Q)	1
(C)	(8,1)	(R)	2
(D)	$\left(8,4\sqrt{2}\right)$	(S)	3

Key. A –Q; B – Q; C – S; D – R

Sol. for the curve $27y^2 - 4(x-2)^3 = 0$

A) (1,2) lies to the left hence 1 real normal

B) (2,3) lies to the left hence 1 real normal

C) (8,1) lies to the right hence 3 distinct real normal

D) $\left(8,4\sqrt{2}\right)$ lies on the curve hence 2 distinct real normals are possible.

48. From the point (3, 0) three normals are drawn to the parabola $y^2 = 4x$ which meet the parabola in the points P, Q and R. Then

	Column I		Column I
(A)	Area of triangle PQR =	(P)	2
(B)	*Circumradius of triangle PQR =*	(Q)	$\left(\frac{2}{3},0\right)$
(C)	centroid of triangle PQR =	(R)	$(-3,0)$
(D)	orthocenter of triangle PQR =	(S)	$5/2$

Key. A –P; B – S; C – Q; D – R

Sol. Conceptual

49. Match the following

A) The locus of point of intersection of the perpendicular tangents of parabola $(y-3)^2 = -4(x-1)$ is — P) $4x-4y+3=0$

B) The parametric equation of a parabola is $x=t^2+1$, $y=2t+1$, then the Cartesian form of its directrix is — Q) $x-2=0$

C) The directrix and focus of a parabola are — R) x =0

$x+2y+10=0$ and $(2,1)$ respectively, then the equation of the tangent at the vertex is

D) The lines which is / are tangent to the parabola $y^2=3x$ S) x+2y+3=0

Key. A-Q, B-R, C-S, D-P, R, S

Sol. 1) Point of intersection of perpendicular tangents lie on directrix

$x-1=+1 \Rightarrow x=2$

2) equation of parabola $(y-1)^2=4(x-1)$

equation of directrix $x-1=-1 \Rightarrow x=0$

3) equation of latus rectum $(x-2)+2(y-1)=0 \Rightarrow x+2y-4=0$

equation of tangent at vertex $\frac{x+2y-4+x+2y+10}{2}=0$ x+2y+3 = 0

4) equation of tangent will be of form $y = mx + \frac{3}{4m}$

$m = -1/2 \rightarrow x+2y+3=0$

$m = 1 \rightarrow 4x-4y+3=0$ & $x=0$

50. Match the following

A) The circle with centre at (0, 5) touches the parabola $x^2 = 4y$. Then radius of the circle is P) 3

B) The length of the chord of the parabola $y^2 = x$ which is bisected at (2, 1) is Q) 4

C) The $x-2y+1=0$ is a tangent to the parabola $3y^2 = kx$ then k equal to R) 2

D) If AFB is a focal chord of the parabola $y^2 = 4ax$ and AF = 3, FB = 6 then a equal to S) $2\sqrt{5}$

Key. A-Q, B-S, C-P, D-R

Sol. i) $x^2+(y-5)^2=r^2$, $x^2=4y$

$\Rightarrow 4y+(y-5)^2-r^2=0$ will have equal roots $\Rightarrow r=4$

ii) $S_1 = S_{11} \Rightarrow 2y = x$

length of the chord = $2.\sqrt{2^2+1^2}=2\sqrt{5}$

iii) $3y^2 = kx$ $x=1-2y$

$3y^2 = k(1-2y)$ should have equal roots

$D=0 \Rightarrow k=3$

iv) $a=\frac{l_1l_2}{l_1+l_2}=3$

51. Match the following

A) The circle with centre at (0, 5) touches the parabola $x^2 = 4y$. Then radius of the circle is P) 3

B) The length of the chord of the parabola $y^2 = x$ which is bisected at (2, 1) is Q) 4

C) The $x-2y+1=0$ is a tangent to the parabola $3y^2 = kx$ then k equal to R) 2

D) If AFB is a focal chord of the parabola $y^2 = 4ax$ and AF = 3, FB = 6 then a equal to — S) $2\sqrt{5}$

Key. A-Q, B-S, C-P, D-R

Sol. i) $x^2+(y-5)^2 = r^2$, $x^2 = 4y$

$\Rightarrow 4y+(y-5)^2-r^2 = 0$ will have equal roots $\Rightarrow r = 4$

ii) $S_1 = S_{11} \Rightarrow 2y = x$

length of the chord = $2.\sqrt{2^2+1^2} = 2\sqrt{5}$

iii) $3y^2 = kx$ $x = 1-2y$

$3y^2 = k(1-2y)$ should have equal roots

$D = 0 \Rightarrow k = 3$

iv) $a = \dfrac{l_1 l_2}{l_1 + l_2} = 3$

52. Match the following

A) The locus of point of intersection of the perpendicular tangents of parabola $(y-3)^2 = -4(x-1)$ is — P) $4x-4y+3=0$

B) The parametric equation of a parabola is $x = t^2+1$, $y = 2t+1$, then the Cartesian form of its directrix is — Q) $x-2=0$

C) The directrix and focus of a parabola are $x+2y+10=0$ and $(2,1)$ respectively, then the equation of the tangent at the vertex is — R) x =0

D) The lines which is / are tangent to the parabola $y^2 = 3x$ — S) x+2y+3=0

Key. A-Q, B-R, C-S, D-P, R, S

Sol. 1) Point of intersection of perpendicular tangents lie on directrix

$x-1 = +1 \Rightarrow x = 2$

2) equation of parabola $(y-1)^2 = 4(x-1)$

equation of directrix $x-1 = -1 \Rightarrow x = 0$

3) equation of latus rectum $(x-2)+2(y-1) = 0 \Rightarrow x+2y-4 = 0$

equation of tangent at vertex $\dfrac{x+2y-4+x+2y+10}{2} = 0$ $x+2y+3 = 0$

4) equation of tangent will be of form $y = mx + \dfrac{3}{4m}$

$m = -1/2 \rightarrow x+2y+3 = 0$

$m = 1 \rightarrow 4x-4y+3 = 0$ & $x = 0$

53. Match the following

Column - I	Column - II
A) The locus of point of intersection of the perpendicular tangents of parabola $(y-3)^2 = -4(x-1)$ is	P) $4x-4y+3=0$
B) The parametric equation of a parabola is $x = t^2+1$, $y = 2t+1$, then the Cartesian form of its directrix is	Q) $x-2=0$
C) The directrix and focus of a parabola are $x+2y+10=0$ and $(2,1)$ respectively, then the	R) x =0

equation of the tangent at the vertex is

D) The lines which is / are tangent to the parabola $y^2 = 3x$

S) x+2y+3=0

T) x+2y-4=0

Key. A-Q, B-R, C-S, D-P,R,S

Sol. A) Point of intersection of perpendicular tangents lie on directrix

$x-1 = 1 \Rightarrow x = 2$

B) equation of parabola $(y-1)^2 = 4(x-1)$

equation of directrix $x-1 = -1 \Rightarrow x = 0$

C) equation of latus rectum $(x-2)+2(y-1) = 0 \Rightarrow x+2y-4 = 0$

equation of tangent at vertex $\dfrac{x+2y-4+x+2y+10}{2} = 0 \Rightarrow x+2y+3 = 0$

D) equation of tangent will be of form $y = mx + \dfrac{3}{4m}$

$m = -1/2 \Rightarrow x+2y+3 = 0$

$m = 1 \Rightarrow 4x-4y+3 = 0$ & $x = 0$

54. Let P be a parabola which touches x – axis at (1,0) and y – axis at (0,2) then

Column I		Column II	
(A)	Vertex of P is	(P)	$x+2y=0$
(B)	Focus of P is	(Q)	$2x-y-\frac{6}{5}=0$
(C)	Equation of axis of P is	(R)	$\left(\frac{16}{25},\frac{2}{25}\right)$
(D)	Equation of directrix P is	(S)	$\left(\frac{4}{5},\frac{2}{5}\right)$

Key. (A-r), (B-s), (C-q), (D-p)

Sol. Origin lies on the direction x, line joining (1,0), (0,2) is parallel to the directrix

It's eqn. is x+2y=0. also vertex of the parabola is $\left(\frac{16}{25},\frac{2}{25}\right)$, focus is $\left(\frac{4}{5},\frac{2}{5}\right)$