Class : CC (Advanced) *COMPLEX NUMBER* **TEST-7**

M.M.: 70 **Time: 60 Min**

PART-A
[SINGLE CORRECT CHOICE TYPE]

Q.1 to Q.10 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct. **[$10 \times 3 = 30$]**

Q.1 Let z_i, $i = 1, 2, 3, 4, 5, 6$ be the roots of equation $z^6 + z^4 = 2$, then $\sum_{i=1}^{6} |z_i|^4$ is equal to

(A) 4 (B) 6 (C) 8 (D) 10

Q.2 If m and M denotes the minimum and maximum value of $|2z+1|$ where $|z-2i| \le 1$ then $(m+M)^2$ is equal to

(A) 17 (B) 34 (C) 51 (D) 68

Q.3 Number of solutions of the equation $z^3 + \frac{3(\bar{z})^2}{|z|} = 0$, where $z \in C$ is equal to

(A) 2 (B) 3 (C) 5 (D) 6

Q.4 If $|z_1 + iz_2| = |z_1| + |z_2|$ and $|z_1| = 3$ and $|z_2| = 4$, then area of triangle ABC, if affix of A, B and C are z_1, z_2 and $\frac{z_2 - iz_1}{1-i}$ respectively, is equal to

(A) $\frac{5}{2}$ (B) 0 (C) $\frac{25}{2}$ (D) $\frac{25}{4}$

Q.5 If $|z_1| = 2$ and $(1-i)z_2 + (1+i)\bar{z}_2 = 8\sqrt{2}$, then the minimum value of $|z_1 - z_2|$ equals

(A) 4 (B) 3 (C) 2 (D) 1

Q.6 Image of the point whose affix is $\frac{2-i}{3+i}$, in the line $(1+i)z + (1-i)\bar{z} = 0$ is the point whose affix is

(A) $\frac{1+i}{2}$ (B) $\frac{1-i}{2}$ (C) $\frac{-1+i}{2}$ (D) $\frac{-1-i}{2}$

Q.7 If z_1, z_2, z_3 and z_4 are the complex roots of the equation $z^4+3z^2+1=0$, then the value of $\prod_{i=1}^{4}\left(4+z_i^2\right)$ is

(A) 9 (B) 12 (C) 25 (D) 27

Q.8 If $|z-1-2i|=r,\ \ 0<r<5$ and a and b be the corresponding complex numbers z for which $|z-5+i|$ is minimum and maximum respectively, then $\arg\left(\frac{a-5+i}{b-1-2i}\right)$ is equal to

(A) 0 (B) $\frac{\pi}{2}$ (C) π (D) $\frac{\pi}{6}$

Q.9 If z is a complex number lying in the fourth quadrant of argand plane and $\left|\frac{kz}{k+1}+2i\right|>\sqrt{2}$ is true for all real values of k $(k\neq-1)$, then range of arg(z) is

(A) $\left(\frac{-\pi}{8},0\right)$ (B) $\left(\frac{-\pi}{6},0\right)$ (C) $\left(\frac{-\pi}{4},0\right)$ (D) $\left(\frac{-\pi}{3},0\right)$

Q.10 If $\arg\left(\frac{z_1-\frac{z}{|z|}}{\frac{z}{|z|}}\right)=\frac{\pi}{2}$ and $\left|\frac{z}{|z|}-z_1\right|=3$, then $|z_1|$ is equal to

(A) $\sqrt{26}$ (B) $\sqrt{10}$ (C) $\sqrt{3}$ (D) $2\sqrt{2}$

[PARAGRAPH TYPE]

Q.11 to Q.15 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct. **[5 × 3 = 15]**

Paragraph for question nos. 11 to 13

Let z and ω be two complex numbers satisfying $z+\overline{\omega}=z^2$ and $\omega+\overline{z}=\omega^2$.

Q.11 Number of ordered pairs (z, ω) is equal to
(A) 1 (B) 2 (C) 3 (D) 4

Q.12 Let z_r and ω_r, r = 1, 2, 3, ... n be the solutions of the equations, then $\sum z_r^2+\sum \omega_r^2$ is equal to
(A) 4 (B) 6 (C) 8 (D) 10

Q.13 The sum of an infinite terms of G.P. $|z|+\dfrac{|z|}{|\omega+1|}+\dfrac{|z|}{|\omega+1|^2}+\dfrac{|z|}{|\omega+1|^3}+.....\infty$ is equal to
(A) 1 (B) 3 (C) 5 (D) not possible

Paragraph for question nos. 14 & 15

Let $z_1 = 3 + i0$ and $z_2 = 7 + i0$ $\left(i=\sqrt{-1}\right)$ represents two points M and N respectively on complex plane. Let the curve C_1 be the locus of point P(z) satisfying $|z-z_1|^2+|z-z_2|^2=10$ and the curve C_2 be the locus of point Q(z) satisfying $|z-z_1|^2+|z-z_2|^2=16$.

Q.14 The locus of point from which tangents drawn to C_1 and C_2 are perpendicular is
(A) $|z-3|=2$ (B) $|z-5|=\sqrt{5}$ (C) $|z-5|=3$ (D) $|z-5|=4$

Q.15 The least distance between two curves C_1 and C_2 is
(A) 1 (B) 2 (C) 3 (D) 4

PART-D
[INTEGER TYPE]

Q.1 to Q.5 are "Integer Type" questions. (The answer to each of the questions **are upto <u>4 digits</u>**) **[5 × 5 = 25]**

Q.1 Let ω ($\omega \neq 1$) be a cube root of unity such that $(1 + \omega^2)^8 = a + b\omega$ where $a, b \in R$, then find the value of $| 2a^{-1} + 2b^{-1} |$.

Q.2 Let $z_1, z_2, z_3 \in C$ such that $| z_1 | = | z_2 | = | z_3 | = 1$. If $z_1 + z_2 + z_3 \neq 0$ and $z_1^2 + z_2^2 + z_3^2 = 0$, then find the value of $| z_1 + z_2 + z_3 |$.

Q.3 Let $A(z_1)$ and $B(z_2)$ be lying on the curve $z - 3 - 4i = \dfrac{25}{\bar{z} - 3 + 4i}$ where $|z_1|$ is maximum. Now, $A(z_1)$ is rotated about the origin in anticlockwise direction through 90° reaching at $P(z_0)$. If A, B and P are collinear then find the value of $\left(|z_0 - z_1| \cdot |z_0 - z_2|\right)$.

Q.4 If a, b and c are different odd integers, then find the minimum value of $\dfrac{1}{2}| a + b\omega + c\omega^2 |^2$ where ω is an imaginary cube root of unity.

Q.5 Let C be the set of complex numbers and $z \in C$. If $S = \{z : |z + \bar{z}| + 2|z - \bar{z}| = 4$ and $|z|$ is minimum$\}$ then area of the polygon formed by all the points in S taking as a vertices is $\dfrac{p}{q}$ where $p, q \in N$. Find the least value of $(p - q)$.

Class : CC (Advanced) ***COMPLEX NUMBER*** **TEST-7**

ANSWER KEY
PART-A

Q.1	D	Q.2	D	Q.3	C	Q.4	D	Q.5	C
Q.6	C	Q.7	C	Q.8	A	Q.9	C	Q.10	B
Q.11	B	Q.12	C	Q.13	B	Q.14	B	Q.15	A

PART-D

Q.1	0004	Q.2	0002	Q.3	0100	Q.4	0006	Q.5	7