Permutation & Combination

Single Correct Answer Type

1. Number of ordered triplets of natural number (a, b, c) for which abc ≤ 11 is

(A) 52 (B) 53 (C) 55 (D) 56

Key. D

Sol. abc = 1 in 1 ways
abc = 2, 3, 5, 7, 11 in 15 ways
abc = 4, 9 in 12 ways
abc = 8 in 10 ways
abc = 6, 10 in 18 ways
So, total number of solution is 56

2. A wooden cube with edge length 'n' (>2) units is painted red all over. By cutting parallel to faces, the cube is cut into n^3 smaller cubes each of unit edge length. If the number of smaller cubes with just one face painted Red is equal to the number of smaller cubes completely un painted, then n=

A) 2 B) 7 C) 8 D) 6

Key. C

Sol. Number of cubes obtained from one face which are painted on only one side = $(n-2)^2$

No. of cubes which are unpainted = $(n-2)^3$

$(n-2)^2 \times 6 = (n-2)^3$

$\Rightarrow n-2=6 \Rightarrow$ n= 8

3. Number of pairs of positive integers (p, q) whose LCM (Least common multiple) is 8100, is "K". Then number of ways of expressing K as a product of two coprime numbers is ____

A)2 B) 6 C) 4 D) 8

Key. A

Sol. L.C.M (p,q) = $2^2 3^4 .5^2$

$\mathbf{P = 2^{a_4} 3^{b_1} .5^{c_1}}$ $q = 2^{a_2} 3^{b_2} 5^{c_2}$

$\Rightarrow \max\{a_1, a_2\} = 2 \Rightarrow 5 \, ways$

$\Rightarrow \max\{b_1, b_2\} = 4 \Rightarrow$ 9 ways

$\Rightarrow \max\{c_1, c_2\} = 2 \Rightarrow 5 \, ways$

$\therefore K = 3^2.5^2 \text{can be expressed as} 1.3^2 5^2, 3^2.5^2$

4. When 32^{33} is divided by 34, it leaves the remainder

(A) 2 (B) 4
(C) 8 (D) 32

Key. D

Sol. $32^{33} = 2^{165} = 2 \times 16^{41} = 2 \times (17 - 1)^{41} = 2 \times (17k - 1) = 34k - 34 + 32$

So the remainder is 32.

5. The number of different words that can be formed using all the letters of the word 'SHASHANK' such that in any word the vowels are separated by atleast two consonants, is

(A) 2700 (B) 1800

(C) 900 (D) 600

Key. A

Sol. The letters other than vowels are SHSHNK which can be arranged in $\frac{6!}{2!\,2!}$ ways

Now in its each case, let the first A be placed in the r^{th} gap then the number of ways to place the 2nd A will be (7 - r - 1). So, the total number of ways = $\frac{6!}{2!\,2!}\sum_{r=1}^{5}(6-r)$

= $\frac{6!}{2!\,2!} \times (5 + 4 + 3 + 2 + 1) = 2700$.

6. The position vector of a point P is $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, when x, y, z $\in$ N and $\vec{a} = \hat{i} + \hat{j} + \hat{k}$. If $\vec{r}.\vec{a} = 10$, the number of possible position of P is

(A) 36 (B) 72

(C) 66 (D) $^{9}C_{2}$

Key. A

Sol. $\bar{r}.\bar{a} = 10$

$\Rightarrow$ x + y + z = 10, x, y, z $\in$ N

no. of solutions = $^{10-1}C_{3-1} = 36$

7. The number of divisors of $2^2.3^3.5^3.7^5$ of the form 2n + 1, n $\in$ N is

(A) 96 (B) 95

(C) 94 (D) 924

Key. B

Sol. Number of div. (3 + 1) (3 + 1) (5 + 1) – 1 = 95

8. The number of ways in which 5 identical balls can be kept in 10 identical boxes, if not more than one can go into a box, is

(A) $^{10}P_{5}$ (B) $\binom{10}{5}$

(C) 5 (D) 1

Key. D

Sol. one way

9. The number of ways of painting the six faces of a cube with six different given colours is

a) 1 b) 720 c) 30 d) 15

Key. C

Sol. First paint any colour on any face. Now the opposite face can be painted in 5 ways (with anyone of the remaining 5 colours). Now, the remaining 4 faces can be painted with the remaining 4 colours in (4-1)! ways. (circular permutations)

$\therefore$ Ans = $5\times(4-1)!=30$ ways.

10. A (1, 2) and B(5, 5) are two points. Starting from A, line segments of unit length are drawn either rightwards or upwards only, in each step, until B is reached. Then, the number of ways of connecting A and B in this manner is

a) 35 b) 40 c) 45 d) 50

Key. A

Sol. Given A(1,2) and B(5,5). Difference of x-coordinates = 5 – 1 = 4

$\therefore$ Exactly 4 rightward steps are needed.

Difference of y-coordinates = 5 – 2 = 3.

$\therefore$ Exactly 3 upward steps are needed.

<u>Note</u>: Order of the steps is immaterial.

Denote each rightward step by R and each upward step by U.

$\therefore$ The problem is arranging the letters RRRRUUU

No. of arrangements = $\dfrac{7!}{4!3!}$ = 35

11. Let the product of all the divisors of 1440 be P. If P is divisible by 24^x, then the maximum value of x is

a) 28 b) 30 c) 32 d) 36

Key. B

Sol. $1440 = 2^5.3^2.5^1$

No. of divisors = (5+1).(2+1).(1+1) = 36

Product of divisors = 1.2.3……. 480.720.1440. Here all the 36 divisors are written in the increasing order. They can be clubbed into 18 pairs, as shown below.

(1.1440). (2.720).(3.480) -----etc.

$\therefore$ Product of divisors = $(1440)^{18} = 2^{90}.3^{36}.5^{18} = (2^3.3)^{30}.3^6.5^{18} = 24^{30}.3^6.5^{18}$ which is divisible by 24^x

$\therefore$ Maximum value of x = 30

12. The number of 5-digit numbers which are divisible by 3 that can be formed by using the digits 1,2,3,4,5,6,7,8 and 9, when repetition of digits is allowed, is

a) 3^9 b) 4.3^8 c) 5.3^8 d) 7.3^8

Key. A

Sol. — — — — —(5 blanks)

1^{st} blank can be filled in 9 ways

2^{nd} blank can be filled in 9 ways ________ since repetition is allowed.

3^{rd} blank can be filled in 9 ways

4^{th} blank can be filled in 9 ways

Now, we have to fill the 5^{th} blank carefully such that the number is divisible by 3. Add the 4 numbers in the first 4 blanks.

If their sum is in the form 3n, then fill the last blank by 3, 6 or 9 so that the sum of all digits is divisible by 3.

If their sum is in the form 3n+1, than fill the last blank by 2, 5 or 8.

If their sum is in the form 3n+2, than fill the last blank by 1, 4 or 7.

Therefore, in any case, the last blank can be filled in 3 ways only.

$\therefore$ Ans = $9\times9\times9\times9\times3=3^9$.

13. The number of 4-digit numbers that can be formed by using the digits 1,2,3,4,5,6,7,8 and 9 such that the least digit used is 4, when repetition of digits is allowed, is

a) 617 b) 671 c) 716 d) 761

Key. B

Sol. Least digit used = 4

$\therefore$ We can use 4,5,6,7,8,9. But remember that at least one 4 must be used.

— — — —(4 blanks)

1^{st} blank can be filled in 6 ways.

2^{nd} blank can be filled in 6 ways.

3^{rd} blank can be filled in 6 ways.

4^{th} blank can be filled in 6 ways.

$\therefore$ 4 blanks can be filled in 6^4 ways. But out of these, some may contain no 4 at all. Let us find them.

— — — —(4 blanks)

Each blank can be filled in 5 ways (by 5, 6, 7, 8, or 9)

$\therefore$ 5^4 ways (no 4 at all)

$\therefore$ Ans = 6^4 - 5^4 (at least one 4)

= 671.

14. The number of arrangements of the letters of the word 'NAVA NAVA LAVANYAM' which begin with N and end with M is :

a) $\frac{\angle 16}{\angle 7(\angle 3)^2}$ b) $\frac{\angle 16}{\angle 7\angle 3}$ c) $\frac{\angle 14}{\angle 7\angle 3\angle 2}$ d) $\frac{\angle 14}{\angle 7\angle 3}$

Key. C

Sol. The word NAVA NAVA LAVANYAM consists of 16 letters out of which there are 7A's, 3V's, 3N's, and the other 3 are distinct put one N in the first place and M in the last place. In the remaining 14 letters there 7A's, 3V's and 2N's.

$\therefore$ No. of arrangements $=\frac{\angle 14}{\angle 7\angle 3\angle 2}$.

15. The number of bijections of a set consisting of 10 elements to itself is :

a) $\angle 10$ b) $\angle 10-10$ c) $\angle 9+10$ d) $\angle 10-2$

Key. A

Sol. Bijection from set – A to itself means permutation.

No. of permutations = $\angle 10$

16. Let $y = 2\sin x + \cos 2x\left(0 \le x \le 2\pi\right)$. All the points at which y is extremum are arranged in a row such that the points of maximum and minimum come alternately the number of such arrangements is :

a) 16 b) 8 c) 12 d) 24

Key. B

Sol. values of x at which as maximum and minimum are : $\frac{\pi}{6}, \frac{5\pi}{6}$

17. The position vector of a point P is $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ where x and y are positive integers and $\vec{a} = \vec{i} + \vec{j} + \vec{k}$. If $\vec{r}.\vec{a} = 10$, then the number of possible positions of P is :

a) 48 b) 72 c) 24 d) 36

Key. D

Sol. $\vec{r}.\vec{a} = x + y + z$

$\therefore$ x + y + z = 10 where x and y are positive integers.

$\therefore$ No. of positive integral solutions of

X + y + z = 10 is (10 - 1)C_{3-1} = 9C_2 = 36.

18. Two numbers ‘a’ & ‘b’ are chosen from the set of {1,2,3……3n}. In how many ways can these integers be selected such that $a^2 - b^2$ is divisible by 3

a) $\frac{3}{2}n\left(n+1\right)+n^2$ b) $\frac{3}{2}n\left(n-1\right)+n^2$ c) $\frac{1}{2}n\left(n+1\right)-n^2$ d) $\frac{1}{2}n\left(n-1\right)+n^2$

Key. B

Sol. $G_1: 3, 6, 9.......3n$

$G_2: 1, 4, 7.......\left(3n-2\right)$

$G_3: 2, 5, 8.......\left(3n-1\right)$

$a^2 - b^2 = \left(a-b\right)\left(a+b\right)$

Either a-b is divisible by 3 (or) a + b is divisible by 3 (or) both

$nc_2 + nc_2 + nc_2 + nc_1.nc_1$

$3\frac{n\left(n-1\right)}{2} + n^2$

19. The number of distinct rational numbers of the form p/q, where $p, q \in \left\{1, 2, 3, 4, 5, 6\right\}$ is

a) 23 b) 32 c) 36 d) 28

Key. A

Sol. $p = 1,\ q = 1, 2, 3, 4, 5, 6 \Rightarrow 6$

$p = 2,\ q = 1,3,4,5,6 \Rightarrow 3\ \left[Q\ (2,4),\ (2,6)\right]$

$p = 3,\ q = 1,2,4,5,6 \Rightarrow 4\ \left[Q\ (3,6)\right]$

$p = 4,\ q = 1,3,5,6 \Rightarrow 3\ \left[Q\ (4,6)\right]$

$p = 5,\ q = 1,2,3,4,6 \Rightarrow 5$

$p = 6,\ q = 1,5 \Rightarrow 2$

20. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent?

A) $6.8.{}^7C_4$ B) $7.{}^6C_4.{}^8C_4$ C) $8.{}^6C_4.{}^8C_4$ D) $6.7.{}^8C_4$

Key. B

Sol. There are $I-4, S-4, P-2, M-1$

Required number of words $= {}^8C_4 \times \frac{7!}{2! \times 4!} = {}^8C_4 \times 7 \times {}^6C_4$

21. The number of arrangements of $A_1, A_2, \ldots, A_{10}$ in a line so that A_1 is always above than A_2 ,is

A) $2 \times \lfloor 10$ B) $\frac{1}{2} \times \lfloor 10$ C) ${}^{10}P_2$ D) ${}^{10}C_2$

Key. B

Sol. Total number of arrangement in which A_1 is always above than $A_2 = \frac{1}{2}(10!)$

22. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five question. The number of choices available to him is

A) 140 B) 196 C) 280 D) 346

Key. B

Sol. Number of required ways $= {}^5C_4\ {}^8C_6 + {}^5C_5\ {}^8C_5 = 196$

23. The number of divisors of 9600 including 1 and 9600 are

A) 60 B) 58 C) 48 D) 46

Key. C

Sol.9600=$2^7 \times 3 \times 5^2$

Number of divisors = $8\times2\times3=48$

24. The total number of ways of dividing 15 different things into groups of 8,4 and 3 respectively is

A) $\frac{15!}{8!4!3!}$ B) $\frac{15!}{8!4!}$ C) $\frac{15!}{8!3!}$ D) $\frac{15!}{4!3!}$

Key. A

Sol. Required number of ways $=\frac{15!}{8!4!3!}$

25. The number of three digit numbers of the form xyz such that $x<y$ and $z\le y$ is

A) 276 B) 285 C) 240 D) 244

Key. A

Sol.If zero is included it will be at $^9C_2$ no. s

If zero is excluded $\begin{cases} x,\ y,\ z \ \text{all diff.} & \Rightarrow {}^9C_3\times2! \\ x=z<y & \Rightarrow {}^9C_2 \\ x<y=z & \Rightarrow {}^9C_2 \ \text{No's} \end{cases}$

Total number of ways = 276

Alternative

y can be from 2 to 9 so total number of ways = $\sum_{r=2}^{9}(r^2-1)=276$

26. Number of ways of selecting 6 shoes, out of 8 pairs of shoes, having exactly two pairs is

A) 1680 B) 240 C) 120 D) 3360

Key. A

Sol. Required number of ways = $^8C_2\times{}^6C_2\times2^2$ =1680.

27. In a polygon no three diagonals are concurrent. If the total number of points of intersection of diagonals interior to the polygon be 70 then the number of diagonals of the polygon is

A) 20 B) 28 C) 8 D) 36

Key. A

Sol. $^nC_4=70\Rightarrow n=8$

$\therefore$ no.q diagonals of poly sin= $\frac{n(n-3)}{2}$

28. From a group of persons the number of ways of selecting 5 persons is equal to that of 8 persons. The number of persons in the group is

A) 13 B) 40 C) 18 D) 21

Key. A

Sol. use ${}^{n}C_r = {}^{n}C_s \Leftrightarrow n = r + s \, or \, r = s$

29. The number of ways of painting the six faces of a cube with six different given colours is

a) 1 b) 720 c) 30 d) 15

Key. C

Sol. First paint any colour on any face. Now the opposite face can be painted in 5 ways (with anyone of the remaining 5 colours). Now, the remaining 4 faces can be painted with the remaining 4 colours in (4-1)! ways. (circular permutations)

$\therefore$ Ans = $5 \times (4-1)! = 30$ ways.

30. A (1, 2) and B(5, 5) are two points. Starting from A, line segments of unit length are drawn either rightwards or upwards only, in each step, until B is reached. Then, the number of ways of connecting A and B in this manner is

a) 35 b) 40 c) 45 d) 50

Key. A

Sol. Given A(1,2) and B(5,5). Difference of x-coordinates = 5 – 1 = 4

$\therefore$ Exactly 4 rightward steps are needed.

Difference of y-coordinates = 5 – 2 = 3.

$\therefore$ Exactly 3 upward steps are needed.

Note: Order of the steps is immaterial.

Denote each rightward step by R and each upward step by U.

$\therefore$ The problem is arranging the letters RRRRUUU

No. of arrangements = $\frac{7!}{4!3!}$ = 35

31. Let the product of all the divisors of 1440 be P. If P is divisible by 24^x, then the maximum value of x is

a) 28 b) 30 c) 32 d) 36

Key. B

Sol. $1440 = 2^5.3^2.5^1$

No. of divisors = (5+1).(2+1).(1+1) = 36

Product of divisors = 1.2.3……. 480.720.1440. Here all the 36 divisors are written in the increasing order. They can be clubbed into 18 pairs, as shown below.

(1.1440). (2.720).(3.480) -----etc.

$\therefore$ Product of divisors = $(1440)^{18} = 2^{90}.3^{36}.5^{18} = (2^3.3)^{30}.3^6.5^{18} = 24^{30}.3^6.5^{18}$ which is divisible by 24^x

$\therefore$ Maximum value of x = 30

32. The number of 5-digit numbers which are divisible by 3 that can be formed by using the digits 1,2,3,4,5,6,7,8 and 9, when repetition of digits is allowed, is

a) 3^9 b) 4.3^8 c) 5.3^8 d) 7.3^8

Key. A

Sol. — — — — —(5 blanks)

1^{st} blank can be filled in 9 ways

2^{nd} blank can be filled in 9 ways _______ since repetition is allowed.

3^{rd} blank can be filled in 9 ways

4^{th} blank can be filled in 9 ways

Now, we have to fill the 5^{th} blank carefully such that the number is divisible by 3. Add the 4 numbers in the first 4 blanks.

If their sum is in the form 3n, then fill the last blank by 3, 6 or 9 so that the sum of all digits is divisible by 3.

If their sum is in the form 3n+1, than fill the last blank by 2, 5 or 8.

If their sum is in the form 3n+2, than fill the last blank by 1, 4 or 7.

Therefore, in any case, the last blank can be filled in 3 ways only.

$\therefore$ Ans = $9\times9\times9\times9\times3=3^9$.

33. The number of 4-digit numbers that can be formed by using the digits 1,2,3,4,5,6,7,8 and 9 such that the least digit used is 4, when repetition of digits is allowed, is

a) 617 b) 671 c) 716 d) 761

Key. B

Sol. Least digit used = 4

$\therefore$ We can use 4,5,6,7,8,9. But remember that at least one 4 must be used.

— — — —(4 blanks)

1^{st} blank can be filled in 6 ways.

2^{nd} blank can be filled in 6 ways.

3^{rd} blank can be filled in 6 ways.

4^{th} blank can be filled in 6 ways.

$\therefore$ 4 blanks can be filled in 6^4 ways. But out of these, some may contain no 4 at all. Let us find them.

— — — —(4 blanks)

Each blank can be filled in 5 ways (by 5, 6, 7, 8, or 9)

$\therefore$ 5^4 ways (no 4 at all)

$\therefore$ Ans = $6^4 - 5^4$ (at least one 4)

= 671.

34. Let there be 9 fixed points on the circumference of a circle . Each of these points is joined to every one of the remaining 8 points by a straight line and the points are so positioned on the circumference that atmost 2 straight lines meet in any interior point of the circle . The number of such interior intersection points is

A) 126 B) 351 C) 756 D) 526

Key. A

Sol. Any interior intersection point corresponds to 4 of the fixed points , namely the 4 end points of the intersecting segments . Conversely, any 4 labled points determine a quadrilateral, the diagonals of which intersect once within the circle .

Number of interior intersection points $= {}^9C_4 = 126$

35. The sum of the integers lying between 1 and 100 and divisible by 3 or 5 or 7 is

A) 2838 B) 3468 C) 2738 D) 3368

Key. C

Sol. The integers divisible by 3 are 33 in numbers.
The integers divisible by 5 are 20 in numbers.
The integers divisible by 7 are 14 in numbers.
The integers divisible by both 3 and 5 are 6 in numbers.
The integers divisible by both 3 and 7 are 4 in numbers.
The integers divisible by both 5 and 7 are 2 in numbers.
There are no integers divisible by all three.
Hence the sum of numbers divisible by
3 or 5 or 7 is

$$= \frac{33}{2}(3+99) + \frac{20}{2}(5+95) + \frac{14}{2}(7+98) - \frac{6}{2}(15+90) - \frac{4}{2}(21+84) - 1(35+70)$$
$$= 33(51) + 950 + 735 - 315 - 210 - 105 = 2738$$

36. The digit at units place of the sum $(1!)^2 + (2!)^2 + (3!)^2 + \ldots\ldots + (2009!)^2$

A) 5 B) 0 C) 1 D) 7

Key. D

Sol. $(5!)^2, (6!)^2, (7!)^2$ And so on will contain 0 in units place

$\therefore 1+4+36+576 \Rightarrow$ Units place is 7

37. Total number of ways in which 256 identical balls can be placed in 16 numbered boxes (1,2,3,.,16) such that rth box contains at least r balls is $(1 \le r \le 16)$

A) ${}^{120}C_{15}$ B) ${}^{135}C_{15}$ C) ${}^{256}C_{15}$ D) ${}^{255}C_{15}$

Key. B

Sol. If we put minimum number of balls required in each box, then remaining 120 balls can be put in ${}^{135}C_{15}$ ways without any restriction.

38. There were two women participating in a chess tournament. Every participant played two games with the other participants. The number of games that the men played between themselves proved to exceed by 66 the number of games that the men played with the women. The number of participants is

A) 6 B) 11 C) 13 D) 12

Key. C

Sol. $\therefore 2.n_{C_2} - 2.2n = 66$ (By hypothesis)

$\Rightarrow n^2 - 5n - 66 = 0 \qquad \Rightarrow n = 11$

$\therefore$ Number of participants = 11 men + 2 women =13.

39. The number of ways in which a committee of 3 women and 4 men be chosen from 8 women and 7 men. If Mr. X refuses to serve on the committee if Mr Y is a member of the committee is

A) 420 B) 840 C) 1540 D) 1400

Key. D

Sol. The no.of ways of seleting 3 womes is ${}^8C_3$

Men selection both x, y are excluded = ${}^5C_4$

Only x is included = ${}^5C_3$

Only y is included = ${}^5C_3$

Hence the no.of ways is ${}^8C_3\left\{{}^5C_4 + 2\times{}^5C_3\right\} = 1400$

40. The number of arrangements of the letters of the word NAVA NAVA LAVANYAM which begin with N and end with M is

A) $\dfrac{\angle 16}{\angle 7(\angle 3)^2}$ B) $\dfrac{\angle 16}{\angle 7\angle 3}$ C) $\dfrac{\angle 14}{\angle 7\angle 3\angle 2}$ D) $\dfrac{\angle 14}{\angle 7\angle 3}$

Key. C

Sol. The word NAVA NAVA LAVANYAM consists of 16 letters out of which there are 7As, 3Vs, 3Ns, and the other 3 are distinct put one N is the first place and M in the last place. In the remaining 14 letters there 7As, 3Vs and 2Ns.

$\therefore$ No.of arrangements $= \dfrac{\angle 14}{\angle 7\angle 3\angle 2}$.

41. The number of different ways in which 8 persons can stand in a row so that between two particular persons A and B there are always two persons, is

A) $60(5!)$ B) $15(4!)\times(5!)$ C) $4!\times 5!$ D) $15(5!)$

Key. A

Sol. $2!\times {}^6P_2\times 5! = 60(5!)$

42. There are three piles of identical red, blue and green balls and each pile contains at least 10 balls. The number of ways of selecting 10 balls if twice as many red balls as green balls are to be selected is

A) 3 B) 4 C) 6 D) 8

Key. B

Sol. Let the number of green balls be x. Then the number of red balls is 2x. Let the number of blue balls be y. Then, $x + 2x + y = 10 \Rightarrow 3x + y = 10 \Rightarrow y = 10 - 3x$.

Clearly, x can take values 0, 1, 2, 3. The corresponding values of y are 10, 7, 4 and 1. Thus, the possibilities are (0, 10, 0), (2, 7, 1), (4, 4, 2) and (6, 1, 3), where (r, b, g) denotes the number of red, blue, green balls.

43. How many times is the digit 5 written when listing all numbers from 1 to 1,00,000 ?

A) 5×10^4 B) $1+10+100+1000+10,000$ C) 5×10^3 D) $1+10+100+1000$

Key. A

Sol. $= 5\times9^4\times1+{}^5C_2\times9^3\times2+{}^5C_3\times9^2\times3+{}^5C_4\times9\times4+{}^5C_5\times5$

$= 5\times10^4$

44. Given distinct lines $L_1, L_2, \ldots, L_{1000}$ in which all lines of the form of L_{4n} where n is the positive integer are parallel to each other. All lines L_{4n-3} are concurrent at a point. The maximum number of the points of intersection of pairs of line from the complete set $(L_1, L_2, \ldots\ldots, L_{1000})$ is

A) 437251 B) 437250 C) 437252 D) 437200

Key. A

Sol. 1000 lines intersects at ${}^{1000}C_2 = 499500$ points but 250 lines are parallel here ${}^{250}C_2 = 31125$ intersections we lost. Also 250 lies are concurrent so ${}^{250}C_2 - 1 = 31124$ more intersection lost.

45. In the next World Cup of cricket there will be 12 teams, divided equally in two groups. Teams of each group will play a match against each other. From each group 3 top teams will qualify for the next round. In this round each team will play against others once. Four top teams of this round will qualify for the semifinal round, where each team will play against the others once. Two top teams of this round will go to the final round, where they will play the best of three matches. The minimum number of matches in the next World Cup will be

A) 54 B) 53 C) 38 D) 58

Key. B

Sol. Use selection

46. The last digit of $(8217)^{1114}$ is

A) 9 B) 7 C) 3 D) 1

Key. A

Sol. 7^{4n+1} ends in $7, 7^{4n+2}$ ends in 9, 7^{4n+3} ends in $3. 7^{4n}$ ends in 1

Now $1114 = 4 \times 278 + 2$

Hence the last digit of $(8217)^{1114}$ is the same as the last digit of 7^{1114} = last digit of $7^2 = 9$

47. 20 persons are to be seated around a circular table. Out of these 20, two are brothers. Then number of arrangements in which there will be atleast three persons between the brothers is

A) $18 \times 20!$ B) $36 \times 18!$ C) $13 \times 18!$ D) $13! \times 18!$

Key. C

Sol. Consider exactly three persons between the brothers as reference now second brother can be placed in two ways (left or right), rest 18 in $18!$ Ways.

Similarly we have $2 \times 18!$ arrangements for exactly 4 persons or 5 persons or 6 persons or 7 persons or 8 persons in between the brothers. For exactly 9 persons in between them we have only 18! Ways.

These will also include 10 or 11 or 12 or 13 or 14, 15 persons in between the brothers.

So total number of arrangements $= 6 \times 2 \times 18! + 18! = 13 \times 18!$

Alternate

If one of the brothers is made reference point then remaining 18 persons

(excluding the second brother) can be seated in 18! Ways. For the second brother we have only

$19 - 6 = 13$ places to sit.

$\Rightarrow$ Total number of ways $= 13 \times 18!$

48. Four persons are selected at random out of 3 men, 2 women and 4 children. The probability that there are exactly 2 children in the selection is

A) 11/21 B) 9/21 C) 10/21 D) $8/21$

Key. C

Sol. Req. = 2 childrens and 2 others $= \dfrac{{}^4C_2 \times {}^5C_2}{{}^9C_4} = \dfrac{10}{21}$

49. The number of ways of painting the six faces of a cube with six different given colours is

A) 1 B) 720 C) 30 D) 15

Key. C

Sol. First paint any colour on any face. Now the opposite face can be painted in 5 ways (with anyone of the remaining 5 colours). Now, the remaining 4 faces can be painted with the the remaining 4 colours in $(4-1)!$ ways. (circular permutations)

$\therefore$ Ans $=5\times(4-1)!=30$ ways.

50. $A(1,2)$ and $B(5,5)$ are two points. Starting from A, line segments of unit length are drawn either rightwards or upwards only, in each step, until B is reached. Then, the number of ways of connecting A and B in this manner is

A) 35 B) 40 C) 45 D) 50

Key. A

Sol. Given $A(1,2)$ and $B(5,5)$. Difference of x coordinates $=5-1=4$

$\therefore$ Exactly 4 rightward steps are needed.

Difference of y coordinates $=5-2=3$.

$\therefore$ Exactly 3 upward steps are needed.

Note: Order of the steps is immaterial.

Denote each rightward step by R and each upward step by U.

$\therefore$ Product is arranging the letters RRRRUUU

No.of arrangements $=\frac{7!}{4!3!}=35$

51. Let the product of all the divisors of 1440 be P. If P is divisible by 24^x, then the maximum value of x is

A) 28 B) 30 C) 32 D) 36

Key. B

Sol. $1440=2^5.3^2.5^1$

No.of divisors $=(5+1).(2+1).(1+1)=36$

Product of divisors $=1.2.3......480.720.1440$. Here all the 36 divisors are written in the increasing order. They can be clubbed into 18 pairs, as shown below.

$(1.1440).(2.720).(3.480)$ .. etc.

$\therefore$ Product of divisors $= (1440)^{18} = 2^{90}.3^{36}.5^{18} = (2^3.3)^{30}.3^6.5^{18} = 24^{30}.3^6.5^{18}$ which is divisible by 24^x

$\therefore$ Maximum value of $x = 30$

52. The number of 5 digit numbers which are divisible by 3 that can be formed by using the digits 1,2,3,4,5,6,7,8 and 9, when repetition of digits is allowed, is

A) 3^9 B) 4.3^8 C) 5.3^8 D) 7.3^8

Key. A

Sol. ___ ___ ___ ___ ___ (5 blanks)

1st blank can be filled in 9 ways

2nd blank can be filled in 9 ways .... (since repetition is allowed.)

3rd blank can be filled in 9 ways

4th blank can be filled in 9 ways

Now, we have to fill the 5th blank carefully such that the number is divisible by 3. Add the 4 numbers in the first 4 blanks.

If their sum is in the form 3n, then fill the last blank by 3, 6 or 9 so that the sum of all digits is divisible by 3.

If their sum is in the form $3n+1$, then fill the last blank by 2, 5 or 8.

If their sum is in the form $3n+2$, then fill the last blank by 1, 4 or 7.

Therefore, in any case, the last blank can be filled in 3 ways only.

$\therefore$ Ans $= 9\times9\times9\times9\times3 = 3^9$.

53. There are 10 stations on a circular path. A train has to stop at 3 stations such that no two stations are adjacent. The number of such selections must be

A) 50 B) 84 C) 126 D) None of these

Key. A

Sol. Total selections $= {}^{10}C_3 = 120$

Number of selections in which 3 stations are adjacent = 10

Number of selections in which 2 stations are adjacent = 6

But there are 10 such pairs.

$\Rightarrow$ Total invalid selections $= 10 + 6\times10 = 70$

54. Let n and k be positive integers such that $n \geq \frac{k(k+1)}{2}$. The number of solutions $(x_1, x_2, \ldots x_k)$, $x_1 \geq 1, x_2 \geq 2, \ldots x_k \geq k$, all integers, satisfying $x_1 + x_2 + \ldots + x_k = n$, is

$\left(m = \dfrac{2n - k^2 + k - 2}{2}\right)$.

A) ${}^{m}C_k$ B) ${}^{m-1}C_k$ C) ${}^{m}C_{k-1}$ D) Zero

Key. C

Sol. Put $y_1 = x_1 - 1, y_2 = x_2 - 2....., y_k = x_k - k$

On adding, $y_1 + y_2 + ... + y_k = n - \dfrac{k(k+1)}{2}$ etc.

55. An n - digit number is a positive integer with exactly n - digits. Nine hundred distinct n - digit numbers are to be formed by using the digit 2, 5 and 7 only. The smallest value of n for which this is possible is

A) 6 B) 7 C) 8 D) 9

Key. B

Sol. We must have $3^n > 900$

The least n satisfying this is 7.

56. The total number of positive integral solutions of the inequality $15 < x_1 + x_2 + x_3 \le 20$ is

A) 685 B) 785 C) 1125 D) 570

Key. A

Sol. $x_1 + x_2 + x_3 = 16 + r, r = 0,1,2,3,4$

Number of positive integral solutions $= {}^{15+r}C_2$

Required number of solutions = $= \sum_{r=0}^{4} (15 + r)_{C_2}$

57. Messages are conveyed by arranging four white, one blue and three red flags on a pole. Flags of the same colour are alike. If a message is transmitted by the order in which the colours are arranged. The total number of messages that can be conveyed if exactly six flags are used is

A) 45 B) 65 C) 125 D) 185

Key. D

Sol. 4 alike and 2 others alike $(4W, 2R) = \dfrac{6!}{4!2!}$

4 alike and 2 others different $\left(4W, 1R, 1B\right) = \frac{6!}{4!}$

3 alike, 3 others alike $\left(3W, 3R\right) = \frac{6!}{3!3!}$

3 alike, 2 others alike, 1 different $\left(3W, 1B, 2R, \ (or) \ 3R, 1B, 2W\right)$

$= 2C_1 \times \frac{6!}{3!2!}$

58. There are 12 pairs of shoes in a box. Then the possible number of ways of picking 7 shoes so that there are exactly two pairs of shoes are

A) 63360 B) 63300 C) 63260 D) 63060

Key. A

Sol. Total number of ways of picking up 7 shoes with 2 pairs is ${}^{12}C_2 \times {}^{10}C_3 \times 2^3$

59. If a, b, c are three natural numbers in AP and $a+b+c=21$ then the possible number of values of the ordered triplet (a,b,c) is

A) 15 B) 14 C) 13 D) 17

Key. C

Sol. $a+a+d+a+2d=21$ or $a+d=7$

$\therefore a+c=14$ and $b=7$.

The number of positive integral solutions of $(a+c=14)$ is 13.

60. In how many ways can 10 persons take seats in a row of 20 fixed seats so that each person has exactly one neighbour ?

a) $\frac{84\times 12!}{(2!)^5}$ b) $\frac{462\times 10!}{(2!)^5}$ c) $\frac{84\times 10!}{(2!)^5}$ d) $\frac{462\times 12!}{(2!)^5}$

Key. B

Sol. ${}^{11}C_5 \times \frac{10!}{(2!)^5 \times 5!} \times 5!$

61. The number of homogenous products of degree 3 from 4 variables is equal to

a) 20 b) 16 c) 12 d) 4

Key. A

Sol. $a+b+c+d=3$

no.of products ${}^{4+3-1}C_3 = 20$

62. Five digit numbers are formed by using the numbers 0,1,2,3,4and 5 with repetition of the same digit in any number, then the number of numbers that are divisible by 3 is

a) 1080 b) 2160 c) 540 d) 4320

Key. B

Sol. $\underset{5\times6\times6\times6\times2}{\downarrow\downarrow\downarrow\downarrow\downarrow}=2160$

63. There are 2010 chairs round the table numbered from 1 to 2010.The numbers of ways in which 5 persons can be seated in any five of these chairs so that the number of empty chairs between any two consecutive persons must be same, is

a) $402\times5!$ b) $804\times5!$ c) $201\times5!$ d) 0

Key. A

Sol. $\frac{2010}{5}=402$

64. The number of positive integer solutions of the equation xyz = 105 so that $x\neq y\neq z$, is

a) 24 b) 27 c) 6 d) 12

Key. A

Sol. $3!+{}^3C_2\times3!$

65. The number of non-congruent rectangles that can be formed on chessboard is

a) 28 b) 36 c) 8 d) 20

Key. B

Sol. ${}^8C_2+8=36$

66. The number of ways of writing 4096 as the product of three positive integers is

a) 19 b) 91 c) 72 d) 18

Key. A

Sol. $1+\frac{18}{3}+\frac{72}{6}=19$

67. How many ways are there to form a three-letter sequence using the letters a,b,c,d,e,f containing e when repetition of the letters is allowed

a) 90 b) 91 c) 92 d) 89

Key. B

Sol. $6\times6+5\times6+5\times5=91$

68. How many integers between l and 10,000 has exactly one 8 and one 9

a) 4×3 b) $4\times3\times8\times7$ c) $2\times4\times3\times8^2$ d) $4\times3\times8^2$

Key. D

Sol. Conceptual

69. How many times is the digit 5 written when listing all numbers from 1 to 1,00,000 ?

a) 5×10^4 b) $1+10+100+1000+10,000$

c) 5×10^3 d) $1+10+100+1000$

Key. A

Sol. $=5\times9^4\times1+{}^5C_2\times9^3\times2+{}^5C_3\times9^2\times3+{}^5C_4\times9\times4+{}^5C_5\times5$

$=5\times10^4$

70. Number of arrangements of SYSTEMATIC in which each S is immediately followed by a vowel

a) ${}^3C_2\,{}^8P_6$ b) $4\times3\times\,{}^8P_2$ c) ${}^4P_2\times\,{}^8P_6$ d) ${}^3P_2\,{}^8P_6$

Key. A

Sol. Conceptual

71. Let N be the number of 7-digit numbers the sum of whose digits is even. The number of +ve divisors of N is

a) 64 b) 72 c) 88 d) 126

Key. D

Sol. $N=\frac{9\times10^6}{2}=2^5.3^2.5^6$

No of divisions N is $6\times3\times7=126$

72. There are 15 different apples and 10 different pears. How many ways are there for Jack to pick an apple or a pear and then Jill to pick an apple and a pear.

a) 23×150 b) 33×150 c) 43×150 d) 53×150

Key. A

Sol. If Jack Pick an apple in ${}^{15}C_1$ ways then Jill in ${}^{14}C_1.{}^{10}C_1$. If Jack pick a pear in ${}^{10}C_1$ way then Jill in ${}^{15}C_1.{}^{9}C_1$

$\therefore$ Total no. of ways $={}^{15}C_1{}^{14}C_1{}^{10}C_1+{}^{10}C_1{}^{15}C_1{}^{9}C_1$

$=150(23)$

73. Let $A=\{0,1,2,3,...9\}$ be a set consisting of different digits. The number of ways in which a nine digit number can be made in which,1 and 2 are present and 1 is always ahead of 2 and repetition of digits is not allowed.

a) $7!\left(\frac{65}{2}\right)$ b) $9!\left(\frac{65}{2}\right)$ c) $8!\left(\frac{65}{2}\right)$ d) $10!\left(\frac{65}{2}\right)$

Key. C

Sol.

$$\frac{{}^9P_2{}^8P_7-{}^7C_1\times8!}{2}\qquad\frac{1}{2}\begin{pmatrix}\text{Total number of permutations of nine numbers in which 1\& 2 are present -}\\ \text{Number of permutations in which 0 occupies first place and containing 1 \& 2}\end{pmatrix}$$

74. A bag contains 4 red and 3 blue balls. Two drawings of two balls are made. The probability that the first drawing gives 2 red balls and the second drawing gives two blue balls if the balls are not returned to the bag after the first draw is

A) 2/49 B) 3/35 C) 3/10 D) ¼

Key. B

Sol. $\frac{4c_2}{7c_2}\times\frac{3c_2}{5c_2}=\frac{4\times3}{7\times6}\times\frac{3}{10}=\frac{3}{35}$

75. Total number of integers $'n'$ such that $2\le n\le2000$ and $H.C.F$ of $'n'$ and 36 is one, is equal to

A) 666 B) 667 C) 665 D) 668

Key. A

Sol. $36=2^2\times3^2$

From 2 to 2000 number of multiples of 2 are $\left[\frac{2000}{2}\right]=1000$

From 2 to 2000 number of multiples of 3 are $\left[\frac{2000}{3}\right] = 666$

From 2 to 2000 number of multiples of 6 are $\left[\frac{2000}{6}\right] = 333$

$\therefore$ Number of possible 'n' are $= 1999 - [1000 + 666 - 333]$

76. A flight of stairs has 10 steps. A person can go up the steps one at a time, two at a time or any combination of 1s and 2s. The total number of ways in which the person can go up the stairs is

a) 75 b) 79 c) 85 d) 89

Key : D

Sol :

A flight of stairs has 10 steps. A person can go up the steps one at a time, two at a time or any combination of 1s and 2s.

Let x + 2y = 10

where x is the number of times he takes single steps and y is the number of times he takes two steps.

Cases	Total no. of ways
I : When x = 0 and y = 5	$\frac{5!}{5!} = 1(2\,2\,2\,2\,2)$
II : When x = 2 and y = 4	$\frac{6!}{2!\cdot 4!} = 15(1\,1\,2\,2\,2\,2)$
III : When x = 4 and y = 3	$\frac{7!}{4!\cdot 3!} = 35(1\,1\,1\,1\,2\,2\,2)$
IV : When x = 6 and y = 2	$\frac{8!}{2!\cdot 6!} = 28(1\,1\,1\,1\,1\,1\,2\,2)$

V : When x = 8 and $y = 1\,{}^{9}C_1 = 9(1\,1\,1\,1\,1\,1\,1\,1\,2)$

VI : When x = 10 and $y = 0\,1(1\,1\,1\,1\,1\,1\,1\,1\,1\,1)$

Hence, total no. of ways

= 1+ 15 + 35 + 28 + 9 + 1 = 89.

77. The number of ways of forming an arrangement of 5 letters from the letters of the word "IITJEE" is

a) 60 b) 96 c) 120 d) 180

KEY : D

HINT

Number of arrangements in which 2 are identical of one kind, two identical of another kind and one letter different from the remaining two letters is $2C_1 \times \frac{5!}{(2!)^2} = 60.$ Number of arrangements in which 2 are identical of one kind and the rest are different is

$2C_1 \times \frac{5!}{2!} = 120$

78. How many combinations can be made up of 3 hens, 4 ducks and 2 geese so that each combination has hens, ducks and geese? (birds of same kind all different)

1) 305 2) 315 3) 320 4) 325

KEY : 2

SOL : $\left(2^3-1\right)\left(2^4-1\right)\left(2^2-1\right)$

79. Let $A=\left\{x_1,x_2,x_3....x_7\right\},B=\left\{y_1y_2y_3\right\}$. The total number of functions $f:A\to B$ that are onto and there are exactly three elements x in A such that $f\left(x\right)=y_2$, is equal to

(A) $14.^7C_2$ (B) $14.^7C_3$

(C) $7.^7C_2$ (D) $7.^7C_3$

Key: B

Hint: $A=\left\{x_1,x_2,x_3,x_4,x_5,x_6,x_7\right\}$, $B=\left\{y_1,y_2,y_3\right\}$

$f:A\to B$ is onto $\ni f\left(x\right)=y_2$

Exactly 3 elements x in is y_2. This can be done

In $^7C_3$ ways

Remain A four elements in B 2 elements

$\therefore 2^4-{}^2C_1\left(2-1\right)^4=14$

Total no.of onto functions $={}^7C_3\times 14$

80. The number of times the digit 3 will be written when listing the integers from 1 to 1000, is

(a) 269 (b) 300 (c) 271 (d) 302

Key: b

Hint: Since 3 does not occur in 1000, we have to count the number of times 3 occurs when we list the integers from 1 to 999. Any number between 1 and 999 is of the form xyz where $0\le x,y,z\le 9$. Let us first count the number in which 3 occurs exactly once. Since 3 can occur at one place in $^3C_1$ ways, there are $^3C_1$ (9 × 9) = 3 × 9^2 such numbers. Next, 3 can occur in exactly two places in ($^3C_2$)(9) = 3 × 9 such numbers. Lastly, 3 can occur in all three digits in one number only.

Hence, the number of times 3 occurs is

1 × (3 × 9^2) + 2 × (3 × 9) + 3 × 1 = 300.

81. The number of three digit numbers with three distinct digits such that one of the digits is the arithmetic mean of the other two is

A) 120 B) 180 C) 112 D) 104

KEY : C

HINT: If the number is $100a+10b+c$ where $1\le a\le 9$

$0\le b,c\le 9$ then $a=\dfrac{b+c}{2}$ or $b=\dfrac{a+c}{2}$ or $c=\dfrac{a+b}{2}$, where a, b, c are distinct

82. The number of ways of selecting 10 balls out of an unlimited number of white, red, blue and green balls is

(A) 270 (B) 84

(C) 286 (D) 86

Key. C

Sol. Let x_1, x_2, x_3 and x_4 be the number of white, red, blue and green balls that are selected. Then $x_1 + x_2 + x_3 + x_4 = 10$. The required number of ways

= coefficient of y^{10} in $(1 + y + y^2 + y^3 + \ldots)^4$

= coefficient of y^{10} in $(1 - y)^{-4}$

= coefficient of y^{10} in $(1 + {}^4C_1y + {}^5C_2y^2 + {}^6C_3y^3 + \ldots.)$

= ${}^{13}C_3=\dfrac{13\times 12\times 11}{2\times 3}=286$

83. If the equation $ax^2 - bx + 12 = 0$ where a and b are +ve integers not exceeding 10, has roots both greater than 2 then the number of ordered pair (a, b) is

(A) 0 (B) 1

(C) 3 (D) 5

Key. B

Sol. Imposing the conditions; $\dfrac{b}{2a} > 2$, $b^2 \ge 48a$ and f(2) i.e., $2a - b + 12 > 0$ there is only one solution for $(a, b) \equiv (1, 7)$

84. The number of different ordered triplets (a, b, c), a, b, c$\in$I such that these can represent sides of a triangle whose perimeter is 21, is

(A) 12 (B) 31

(C) 55 (D) 91

Key. C

Sol. $a + b + c = 21 \Rightarrow b + c > a \Rightarrow a + b + c > 2a \Rightarrow 2a < 21 \Rightarrow a \le 10$. So $1 \le a, b, c \le 10$.

The cases when a > b > c are (10, 9, 2), (10, 8, 3), (10, 7, 4), (10, 6, 5), (9, 8, 4), (9, 7, 5) and (8, 7, 6). So, number of cases when a, b, c are all distinct is $7 \times 3! = 42$.

The cases when a = b > c or a > b = c are (10, 10, 1), (9, 9, 3), (8, 8, 5) and (9, 6, 6). So number of cases when two same and 1 different is $4 \times 3!/2! = 12$.

The cases when a = b = c is (7, 7, 7). The total number of ordered triplets = 42+12 + 1=55.

85. The number of different permutations of all the letters of the word 'PERMUTATION' such that any two consecutive letters in the arrangement are neither both vowels nor both identical is

(A) $63 \times 6! \times 5!$ (B) $57 \times 5! \times 5!$

(C) $33 \times 6! \times 5!$ (D) $7 \times 7! \times 5!$

Key. B

Sol. The letters other than vowels are : PRMTTN

Number of permutations with no two vowels together is $\frac{6!}{2!} \times {}^7C_5 \times 5!$

Further among these permutations the number of cases in which T's are together is $5! \times {}^6C_5 \times 5!$

So the required number = $\frac{6!}{2!}\ {}^7C_5 \times 5! - 5! \times {}^6C_5 \times 5! = 57 \times (5!)^2$

86. The least positive integral value of x which satisfies the inequality ${}^{10}C_{x-1} > 2.\ {}^{10}C_x$ is:

(A) 7 (B)8 (C)9 (D) 10

Key. B

Sol. $10 \geq x-1 \Rightarrow x \leq 11$ and $10 \geq x$

$\therefore \quad x \leq 10$

$\because\ {}^{10}C_{x-1} > 2.{}^{10}C_x 1 > 2.\frac{{}^{10}C_x}{{}^{10}C_{x-1}}$

$\Rightarrow \quad 1 > 2.\frac{10-x+1}{x}$

$\Rightarrow \quad x > 22-2x; \Rightarrow x > \frac{22}{3}$

$\Rightarrow \quad x > 7\frac{1}{3} \ \therefore\ x = 8$

87. A man moves one unit distance for each step he takes. He always moves forward, backward, up or down either parallel to the x-axis or y-axis.. He starts at the point (0,0) and reaches the point (1 ,1) at the end of six steps. The number of ways he can do it is

A. 280 B. 300 C. 360 D. 420

Key. B

Sol. Let E, W, N, S stand for one unit movement along +ve, -ve, x-direction, +ve, -ve, y-direction respectively. The sequence of 6 steps are.

ENNNSS with $\frac{6!}{3!2!} = 60\,ways$

EEWNNS with $\frac{6!}{2!2!} = 180\,ways$

EEEWWN with $\frac{6!}{3!2!} = 60\,ways$

The desired number $= 60 + 180 + 60 = 300$

88. A wooden cube with edge length ‘n’ (>2) units is painted red all over. By cutting parallel to faces, the cube is cut into n^3 smaller cubes each of unit edge length. If the number of smaller cubes with just one face painted Red is equal to the number of smaller cubes completely un painted, then n=

A) 2 B) 7 C) 8 D) 6

Key. C

Sol. Number of cubes obtained from one face which are painted on only one side = $(n-2)^2$

No. of cubes which are unpainted = $(n-2)^3$

$(n-2)^2 \times 6 = (n-2)^3$

$\Rightarrow n-2=6 \Rightarrow$ n= 8

89. Number of pairs of positive integers (p, q) whose LCM (Least common multiple) is 8100, is "K". Then number of ways of expressing K as a product of two coprime numbers is ____

A)2 B) 6 C) 4 D) 8

Key. A

Sol. L.C.M (p,q) = $2^2 3^4.5^2$

$\mathbf{P = 2^{a_4} 3^{b_1}.5^{c_1}}$ $q = 2^{a_2} 3^{b_2} 5^{c_2}$

$\Rightarrow \max\{a_1, a_2\} = 2 \Rightarrow 5\, ways$

$\Rightarrow \max\{b_1, b_2\} = 4 \Rightarrow$ 9 ways

$\Rightarrow \max\{c_1, c_2\} = 2 \Rightarrow 5\, ways$

$\therefore K = 3^2.5^2$ can be expressed as $1.3^2 5^2, 3^2.5^2$

90) There are 6 balls of different colours and 6 boxes of colours same as those of the balls. The number of ways in which the balls, one each in a box could be placed such that a ball does not go to a box of its own colour is

1) 275 2) 265 3) 285 4) 305

Key. 2

Sol. $\lfloor 6 \left[1 - \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} - \frac{1}{\lfloor 3} + \frac{1}{\lfloor 4} - \frac{1}{\lfloor 5} + \frac{1}{\lfloor 6}\right]$

91) The number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty is

1) 38 2) 21 3) 5 4) ${}^8C_3$

Key. 2

Sol. Let the number of balls in the 3 boxes be x, y, z respectively, then $x+y+z=8, x,y,z \geq 1$

$\therefore$Required number ${}^{n+r-1}C_r = {}^{3+3-1}C_5 = {}^7C_5 = {}^7C_2 = 21$.

92) Four boys picked 30 apples.The number of ways in which they can divide them if all the apples are identical, is

1) 5630 2) 4260 3) 5456 4) 5556

Key. 3

Sol. We know , the number of ways to divide n identical things among r persons = ${}^{n+r-1}C_{r-1}$
Here n=30,r=4

$\therefore$ Required number of ways = ${}^{30+4-1}C_{4-1} = {}^{33}C_3 = \dfrac{33\times 32\times 31}{3\times 2} = 5456$.

93) If N is the number of positive integral solutions of $x_1x_2x_3x_4 = 770$. Then N =

1) 256 2) 729 3) 900 4) 770

Key. 1

Sol. 770 = 2 x 5 x 7 x 11
We can assign 2 to x, or x_2 or x_3 or x_4. Thus 2 can be assigned in 4 ways similarly each of 5,7,11 can be assigned in 4 ways
$\therefore$ No. of solutions = 4 x 4 x 4 x 4 = 256

94) Five balls of different colours are to be placed in three boxes of different sizes. Each box can hold all five balls. In how many ways can we place the balls so that no box remains empty ?

1) 20 2) 100 3) 150 4) 200

Key. 3

Sol.

$$\bar{3}\ \bar{1}\ \bar{1}\quad {}^5C_3 \cdot {}^2C_1 \cdot {}^1C_1 \left(\frac{3!}{2!}\right)$$

$$2\ 2\ 1\quad {}^5C_2 \cdot {}^3C_2 \cdot {}^1C_1 \left(\frac{3!}{2!}\right)$$

95) The number of point (x, y, z) in space, whose each coordinate is a negative integer such that $x+y+z+12=0$, is :

1) 55 2) 110 3) 75 4) 100

Key. 1

Sol. $x+y+z+12=0, x, y, z$ are negative integers.

Let $x=-a, y=-b, z=-c, a, b, c$ are positive integers.

Then required number of points (x, y, z)

= number of positive integral solutions of $a+b+c=12$

= coefficient of x^{12} in $\left(x+x^2+\ldots\right)^3$

= coefficient of x^9 in $(1-x)^{-3}$

= ${}^{11}C_2 = \dfrac{11\times 10}{2} = 55$

96) The position vector of a point P is $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, where $x\in N, y\in N, z\in N$ and $\vec{a} = \vec{i} + \vec{j} + \vec{k}$. If $\vec{r}\cdot\vec{a} = 10$, the number of possible position of P is

1) 36 2) 72 3) 66 4) 56

Key. 1

Sol. $\vec{r}.\vec{a} = 10 \Rightarrow x + y + z = 10$

Number of positive integral solution of above equation = Coeff. of x^{10} in

$\left(x + x^2 + x^3 + ...\right)^3 = 36$

97) At an election, a voter may vote for any number of candidates not grater than the number to be elected. There are ten candidates and 4 are to be elected. If a voter votes for at least one candidate, then the number of ways in which he can vote, is

1) 6210 2) 385 3) 1110 4) 5040

Key. 2

Sol. ${}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$

98) The number of 3-digited numbers abc such that b < c is

1) 450 2) 405 3) 400 4) 410

Key. 2

Sol.

a	b	c

$9 \times {}^{10}C_2$

99) In how many ways can 5 books be selected out of 10 books, if two specific books are never selected ?

1) 56 2) 65 3) 58 4) 66

Key. 1

Sol. ${}^{8}C_5$

100) The number of ordered pairs of integers (x, y) satisfying the equation $x^2 + 6x + y^2 = 4$ is

1) 2 2) 4 3) 6 4) 8

Key. 4

Sol. $x^2 + 6x + y^2 = 4 \Rightarrow (x+3)^2 + y^2 = 13$

This equation satisfied by ordered pairs $(0,\ \pm 2)\ (-6,\ \pm 2)\ (-5,\ \pm 3)\ (-1,\ \pm 3)$

101) The number of functions *f* from the set $A = \{0,1,2\}$ into the set $B = \{0,1,2,3,4,5,6,7\}$ such that $f(i) \le f(j)$ for i < j and i, $j \in A$ is

1) ${}^{8}C_3$ 2) ${}^{8}C_3 + 2\left({}^{8}C_2\right)$ 3) ${}^{10}C_3$ 4) ${}^{8}C_3 + {}^{10}C_3$

Key. 3

Sol. A function $f : A \to B$ such that $f(0) \le f(1) \le f(2)$ falls in one of the following 4 cases

1. $f(0) \le f(1) \le f(2)$

there are ${}^{8}C_3$ functions in this case

2. $f(0) = f(1) < f(2)$

there are ${}^{8}C_2$ functions in this case

3. $f(0) < f(1) = f(2)$

there are ${}^{8}C_2$ functions in this case

4. $f(0) = f(1) = f(2)$

there are ${}^{8}C_1$ functions in this case

the reg. number of functions

${}^{8}C_3 + {}^{8}C_2 + {}^{8}C_2 + {}^{8}C_1$

${}^{9}C_3 + {}^{9}C_2 = {}^{10}C_3$

102) ABCD is a quadrilateral. 3, 4, 5 and 6 points are marked on the sides. AB, BC, CD and DA respectively. The number of triangles with vertices on different sides is

1) 270 2) 220 3) 282 4) 342

Key. 4

Sol. The required no. of triangles

$= {}^{3}C_1 \times {}^{4}C_1 \times {}^{5}C_1 + {}^{3}C_1 \times {}^{4}C_1 \times {}^{6}C_1 + {}^{3}C_1 \times {}^{5}C_1 \times {}^{6}C_1 + {}^{4}C_1 \times {}^{5}C_1 \times {}^{6}C_1 = 342.$

103) A father with 8 children takes them 3 at a time to the Zoological gardens, as often as he can without taking the same 3 children together more than once. The number of times he will go the garden is

1) 336 2) 112 3) 56 4) 21

Key. 3

Sol. ${}^{8}C_3$

104) If a polygon has 65 diagonals, then the number of sides of the polygon is

1) 25 2) 20 3) 15 4) 13

Key. 4

Sol. $\frac{n(n-3)}{2} = 65 \Rightarrow n = 13$

105) The number of positive terms in the sequence

$x_n = \frac{195}{4\ {}^{n}P_n} - \frac{{}^{n+3}P_3}{{}^{n+1}P_{n+1}}, n \in N$ is

1) 2 2) 3 3) 4 4) 5

Key. 3

Sol. We have, $x_n = \frac{195}{4\ {}^nP_n} - \frac{{}^{n+3}P_3}{{}^{n+1}P_{n+1}}, n \in N$

$= \frac{195}{4.n!} - \frac{(n+3)(n+2)(n+1)}{(n+1)!}$

$= \frac{171-4n^2-20n}{4.n!}$

$\therefore x_n$ is positive

$\therefore \frac{171-4n^2-20n}{4.n!} > 0 \Rightarrow 4n^2+20n-171<0$

Which is true for *n* = 1,2,3,4.

Hence, the given sequence has 4 positive terms.

106. There are 3 rows containing 2 seats in each row. The number of ways in which 3 persons can be seated such that no row remains empty is p then $\frac{p}{16}$ =

A) 1 B) 2 C) 3 D) 4

Key. C

Sol. R_1 , R_2, R_3

contain 1 person each

${}^2C_1.{}^2C_1.{}^2C_1\ 3! = P$

$\frac{8.6}{16} = \frac{48}{16} = 3$

107. Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of n sides. If $T_{n+1} - T_n = 21$, then n =

A) 7 B) 6 C) 8 D) 1

Key. A

Sol. ${}^{(n+1)}C_3 - {}^nC_3 = 21 \Rightarrow n = 7$

108. No of ways of arranging the letters of the word BANANA so that letters of the same kind are together is _____

A) 5 B)6 C) 4 D) 1

Key. B

Sol. B NN AAA can be arranged in 3!

109. No of ways of selecting 4 letters from the letters of the words EQUATION so that "E" "Q" "U" always occur and A never occurs is

A) 1 B) 2 C) 3 D) 4

Key. D

Sol. 1 letter is to be selected from TION

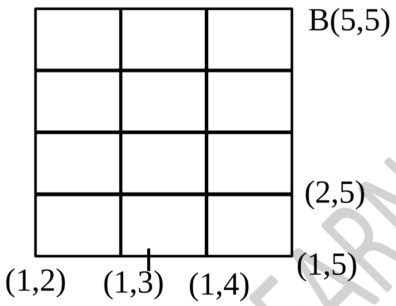
110. A (1, 2) and B(5, 5) are two points. Starting from A, line segments of unit length are drawn either rightwards or upwards only, in each step, until B is reached. Then, the number of ways of connecting A and B in this manner is

A) 35 B) 40 C) 45 D) 50

Key. A

Sol.

$$^{4+5-2}C_3 = {}^7C_3 = 35$$



111. Let the product of all the divisors of 1440 be P. If P is divisible by 24^x, then the maximum value of x is

A) 28 B) 30 C) 32 D) 36

Key. B

Sol. Product of all division of N= $N^{\frac{T.N.D}{2}}$

$$1440 = 2^5.3^2.5$$

$$T.N.D = 6.3.2 = 36$$

$$= \left(2^5.3^2.5\right)^{\frac{36}{2}}$$

$$= \left(2^{90}.3^{36}.5^{18}\right)$$

$$= \left(2^3.3\right)^{30}.3^6.5^{18}$$

$$= 24^{30}.3^6.5^{18}$$

112. 12 small sticks of length 1 cm each are distributed into three children A,B and C. These children join the sticks in the form of line segments individually. If 'n' is the number of ways in which the sticks can be distributed to the children so that the line segments joined by them form a triangle then the value of 'n' is

A) 10 B) 9 C) 8 D) 12

Key. A

Sol. Let x, y, z be the number of sticks received by the children A, B and C. Then the line segments formed by them form a triangle iff $1 \le x \le 5, 1 \le y \le 5, 1 \le z \le 5$

$\therefore$ n is the number of positive integral solutions of the equation $x + y + z = 12$ Where $x, y, z \le 5.$

$\Rightarrow n =$ Coefficient of x^{12} in $\left(x + ... + x^5\right)^3$ = coefficient of x^9 in $\left(1 + x + x^2 + x^3 + x^4\right)^3$

= coefficient of x^9 in $\left(1 + x^5\right)^3 \left(1 - x\right)^{-3}$

= coefficient of x^9 $\left(1 - 3x^5 + 3x^{10} - x^{15}\right)\left(1 - x\right)^{-3}$

$= 1 \times {}^{11}C_2 - 3 \times {}^{6}C_2$

$= 55 - 45 = 10$

113. Using the points from an 4×2 array of equally spaced points how many distinct non-degenerate triangles (i.e. triangles with non zero area) can be constructed?

A) 1056 B)1064 C) 1060 D)1024

Key. A

Sol. ${}^{20}C_3 - \left[{}^{5}C_3.4 + {}^{4}C_3.5 + 4\left({}^{3}C_3 + {}^{4}C_3\right)\right] = 1060$

114. A guard of 12 men is formed from a group of ‘n’ soldiers. It is found that 2 particular soldiers A and B are 3 times as often together on guard as 3 particulars soldiers C, D and E. Then (n-24) = is equal to

A) 8 B) 7 C) 2 D) 6

Key. A

Sol. ${}^{n-2}C_{12-2} = 3.{}^{n-3}C_{12-3}$

${}^{n-2}C_{10} = 3.{}^{n-3}C_{9}$

$\frac{n-2}{10} = 3 \Rightarrow n = 32$

115. Number of pairs of positive integers (p,q) whose LCM (Least common multiple) is 8100, is “K”. Then number of ways of expressing K as a product of two coprime numbers is ____

A)4 B) 6 C) 2 D) 8

Key. C

Sol. L.C.M (p,q) = $2^2 3^4 .5^2$

$\mathbf{P = 2^{a_4} 3^{b_1}.5^{c_1}}$ $q = 2^{a_2} 3^{b_2} 5^{c_2}$

$\Rightarrow \max\{a_1, a_2\} = 2 \Rightarrow 5\,ways$

$\Rightarrow \max\{b_1, b_2\} = 4$ $\Rightarrow$ 9 ways

$\Rightarrow \max\{c_1, c_2\} = 2 \Rightarrow 5\,ways$

$\therefore K = 3^2.5^2$ can be expressed as $1.3^2 5^2, 3^2.5^2$

116. A wooden cube with edge length ‘n’ (>2) units is painted red all over. By cutting parallel to faces, the cube is cut into n^3 smaller cubes each of unit edge length. If the number of smaller cubes with just one face painted Red is equal to the number of smaller cubes completely un painted, then n=

A) 8 B) 7 C) 2 D) 6

Key. A

Sol. Number of cubes obtained from one face which are painted on only one side = $(n-2)^2$

No. of cubes which are unpainted = $(n-2)^3$

$$(n-2)^2\times 6=(n-2)^3$$

$\Rightarrow n-2=6 \Rightarrow$ n= 8

117. In a cross word puzzle, 40 words are to be guessed, of which 7 words have each an alternative solution. If k is the number of solutions of the cross word puzzle then 130 -k is ____

A)4 B) 6 C) 2 D) 8

Key. C

Sol. Each one of 7 words can be filled in 2 ways i.e 2^7=128 ways
Rest of 33 words can be filled in only one way.
$\therefore$ total ways = 128

118. Number of 4 digit numbers having the sum of the digits equal to 9 is $^{11}C_r$ then least value of r =

A) 1 B) 2 C) 3 D) 4

Key. C

Sol. $x_1+x_2+x_3+x_4=9 \quad x\neq 0$

coefficient of x^9 in $\left(x+x^2+.x^9\right)\left(1+x+.+x^8\right)^3$

coefficient of x^8 in $\left(1+x+.+x^8\right)^3$ $={}^{8+4-1}C_8={}^{11}C_3$

119. The number of ways in which 4 married couples can be sit four on each side of a long table so that males on one side and females on the other side and no wife is in front of her husband is k then k is _____

A) 7 B) 9 C) 10 D) 11

Key. B

Sol. $K=4!\left(\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right)=12-4+1=9$

120. Consider $S=\{1,2,3,4,.......,10\}$. Then sum of all products of numbers by taking two or more from S is (11! – k) then $\left[\frac{k}{11}\right]$ where [ ] is G.I. F is

A) 1 B) 2 C) 4 D) 5

Key. D

Sol. $(x+1)(x+2)...(x+10)=x^{10}+(\Sigma 1)x^9+(\Sigma 1.2)x^8+....+1.2.3...10.$

take $x=1$

$$11!=1+\frac{10.11}{2}+(\Sigma 1.2+\Sigma 1.23+.....)$$

$$\Rightarrow \Sigma 1.2+\Sigma 1.2.3+....=11!-56$$

$$\therefore K=56$$

121. There are 7 cars available to transport 27 students. Then at least one car has to accommodate.

A) 4 or more passengers B) 5 or more passengers

C) 6 or more passengers D) 7 or more passengers

Key. A

Sol. $27 = 7(3) + 6$

$\therefore$ at least one car has to accomodate 4 or more passager.

122. Let $x_1x_2x_3x_4x_5x_4x_3x_2x_1$ be a nine digit palindrome such that either the sequence $(x_1, x_2, x_3, x_4, x_5)$ is a strictly ascending or strictly descending. Then the number of such palindromes is

A) $9 \times {}^9P_4$ B) $3 \times {}^9P_5$ C) $9 \times {}^9C_5$ D) $3 \times {}^9C_5$

Key. D

Sol. Strictly descending $\rightarrow {}^{10}C_5$

Strictly ascending $\rightarrow {}^9C_5$ (because zero can't be at x_1)

$${}^{10}C_5 + {}^9C_5$$
$$= 2.{}^9C_4 + {}^9C_4$$
$$= 3.{}^9C_4$$
$$= 3.{}^9C_5$$

123. $(1^2 + 1)1! + (2^2 + 1)2! + (3^2 + 1)3! + \ldots$ upto 15 terms =

(A) 16! (B) $15 \times 14!$ (C) $15 \times 16!$ (D) $15 \times 17!$

Key. C

Sol. $T = (n^2 + 1)n!$

$= [(n+2)(n+1) - 3(n+1) + 2]n!$

$= (n+2)! - 3(n+1)! + 2(n!)$

124. Let a = $a_1a_2a_3$ and b = $b_1b_2b_3$ be two three digit numbers. How many pairs of 'a' and 'b' can be formed so that 'a' can be subtracted from 'b' without borrowing?

(A) $55 \times (45)^2$ (B) $9! \times 10! \times 10!$ (C) $45 \times (55)^2$ (D) $(45)^3$

Key. C

Sol. Number of cases = $(1 + 2 + 3 + \ldots + 9)(1 + 2 + 3 + \ldots + 10)^2 = 45(55)^2$

125. The number of positive divisors of $(2008)^8$ that are less than $(2008)^4$ are

(A) 28 (B) 112 (C) 224 (D) 56

Key. B

Sol. $(2008)^8 = 2^{24} \times 251^8$ has $25 \times 9 = 225$ positive divisors, including $(2008)^4 = \sqrt{(2008)^8}$. There is a one to one correspondence between the positive divisors less than $(2008)^4$ and those larger than $(2008)^4$. It follows that there are $\frac{1}{2}(225 - 1)$ = 112 positive divisors less than $(2008)^4$.

126. In the next world cup of cricket their will be 12 teams, divided equally in two groups. Teams of each group will play a match against each other. From each group 3 top teams will qualify for the next round. In this round each team will play against others once. Four top teams of this round will qualify for the semifinal round, where each team will play against the others

once. Two top teams of this round will go into the final round, where they play one match. The minimum number of matches in the next world cup will be

1) 54 2) 53 3) 52 4) 55

Key. 3

Sol. No. of matches in the first round = $6C_2 + 6C_2$

No. of matches in the second round = $6C_2$

No. of matches in the semifinal round = $4C_2$

Total = 15+15+15+6+1 = 52

127. In a polygon, no three diagonals are concurrent. If the total number of points of intersection of diagonals interior to the polygon be 70. Then the number of diagonals of the polygon is

1) 20 2) 28 3) 8 4) 32

Key. 1

Sol. Given, $nC_4 = 70 \Rightarrow n = 8$

No. of diagonals = $nC_2 - n$ = 28-8 = 20

128. Let A be a set of $n(\geq 3)$ distinct elements. The number of triplets (x,y,z) of the elements of A in which at least two coordinates are equal is

1) ${}^n p_3$ 2) $n^3 - {}^nC_3$ 3) $3n^2 - 2n$ 4) $3n^2\left(n-1\right)$

Key. 3

Sol. Total no. of triplets without restriction = n^3

No. of triplets with different coordinates = nP_3

Required ways = $n^3 - nP_3$

= $n^3 - n(n-1)(n-2)$

= $n^3 - n^3 + 3n^2 - 2n$

= $3n^2 - 2n$

129. A shopkeeper sells three varieties of perfumes and he has a large number of bottles of the same size of each variety in his stock. There are 5 places in a row in his showcase. The number of ways of displaying all the three varieties of perfumes in the showcase is

1) 6 2) 50 3) 150 4) none of these

Key. 3

Sol. Required ways = $3^5 - 3.2^5 + 3$

= 150

130. Madhuri has 10 friends among whom two are married to each other. She wishes to invite 5 of them for a party. If the married couple refuse to attend separately. Then the number of different ways in which she can invite five friends is

1) ${}^8C_5$ 2) $2\times {}^8p_3$ 3) ${}^{10}C_5 - 2\times {}^8C_4$ 4) ${}^8C_6$

Key. 3

Sol. Total no. of ways = $10C_5$

Couple separately attend = $2.8C_4$

$\therefore$ required ways = $10C_5 - 2.8C_4$

131. In a plane these are two families of lines $y = x + r$, $y = -x + r$, where $r \in \{0,1,2,3,4\}$.

The number of squares of diagonals of the length 2 units formed by the lines is

1) 9 2) 16 3) 25 4) 36

Key. 1

Sol. Required ways = 3×3 = 9.

132. The number of proper divisors of $2^p.6^q.15^r$ is

1) $(p+1)(q+1)(r+1)-2$ 2) $(p+q)(q+r)r-2$

3) $(p+q+1)(q+r+1)(r+1)-2$ 4) $(p+1)(q+1)(r+1)$

Key. 3

Sol. $2^p.6^q.15^r = 2^{p+q}.3^{q+r}.5^r$

No. of proper divisors = $(p+q+1)(q+r+1)(r+1)-2$

133. The number of positive integers $\leq$ 100000 which contain exactly one 2, one 5 and one 7 in its decimal representation is

1) 2940 2) 7350 3) 2157 4) 1582

Key. 1

Sol. Take 5 gaps --------

The digit '2' can occupy any of 5 places

The digit '5' ca occupy any of 4 places

The digit '7' can occupy any of 3 places

Remaining 2 places in 7×7 ways

Total ways = 5×4×3×7×7=2940

134. If $E = \frac{1}{4}.\frac{2}{6}.\frac{3}{8}.\frac{4}{10}....\frac{30}{62}.\frac{31}{64} = 8^x$, then value of x is

1) -7 2) -9 3) -10 4) -12

Key. 4

Sol. $E = \frac{31!}{2^{31}(32)^!} = 2^{-36} = \left(2^3\right)^{-12} = 8^{-12}$

$\therefore x = -12$

135. The remainder when $x = 1!+2!+3!+4!+.....+100!$ is divided by 240, is

1) 153 2) 33 3) 73 4) 187

Key. 1

Sol. For $r \geq 6, r!$ is divisible by 240.

$\therefore$ remainder = $1!+2!+3!+4!+5! = 153$

136. If $x, y \in (0,30)$ such that $\left[\frac{x}{3}\right]+\left[\frac{3x}{2}\right]+\left[\frac{y}{2}\right]+\left[\frac{3y}{4}\right] = \frac{11}{6}x+\frac{5}{4}y$ (where [x] denotes greatest integer $\leq x$). Then the number of ordered pairs (x,y) is

1) 0 2) 2 3) 4 4) 28

Key. 4

Sol. From the given condition, $\left\{\frac{x}{3}\right\}+\left\{\frac{3x}{2}\right\}+\left\{\frac{y}{2}\right\}+\left\{\frac{3y}{4}\right\}=0$

$\Rightarrow \frac{x}{3},\frac{3x}{2},\frac{y}{2},\frac{3y}{4}$ must be integers

$\therefore x=6,12,18,24$

y= 4, 8, 12, 16, 20, 24, 28

$\therefore$ No. of order pairs = 4×7=28

137. Let X be a set containing n elements. The no. of all the ordered triplets (A, B, C) such that C is a subset of B and B is a proper subset of A where $A \subseteq X$, is

(A) 4^n (B) 3^n

(C) $4^n - 3^n$ (D) $3^n - 2^n$

Key. C

Sol. $C \subseteq B \subset A \subseteq X$

The no. of ways = $\sum_{i=0}^{n} {}^nC_i\left[\sum_{j=0}^{i-1} {}^iC_j\left[\sum_{k=0}^{j} {}^jC_k\right]\right]$

= $\sum_{i=0}^{n} {}^nC_i \sum_{j=0}^{i-1} {}^iC_j\, 2^j = \sum_{i=0}^{n} {}^nC_i(3^i - 2^i) = 4^n - 3^n$

138. Number of ways of giving away 10 different gifts to 5 students so that each get atleast one gift and a particular student gets exactly 4 gifts

(A) 393120 (B) 327600

(C) $10_{C_4}\left(\frac{6!}{2!2!}\times\frac{4!}{2!2!}+\frac{6!}{3!3!}\times 4!\right)$ (D) $10_{C_6}\left(1080+480\right)$

Key. B,C,D

Sol. Number of ways $=10_{C_4}\left(\frac{6!}{2!2!}\times\frac{4!}{2!2!}+\frac{6!}{3!3!}\times 4!\right)=327600$

139. If $\sum_{r=0}^{n-1}\left(\frac{n_{Cr}}{n_{Cr}+n_{Cr+1}}\right)^3=\frac{4}{5}$, then $n=$

(A) 5 (B) 4 (C) 3 (D) 6

Key. B

SOL. L.H.S $=\sum_{r=0}^{n-1}\left(\frac{n_{C_r}}{n_{C_r}+n_{C_{r+1}}}\right)^3=\sum_{r=0}^{n-1}\left(\frac{n_{C_r}}{n_{C_r}\left(1+\frac{n_{C_{r+1}}}{n_{C_r}}\right)}\right)^3$

$=\sum_{r=0}^{n-1}\left(\frac{r+1}{n+1}\right)^3=\frac{n^2\left(n+1\right)^2}{4\left(n+1\right)^3}=\frac{n^2}{4\left(n+1\right)}$

NOW $\frac{n^2}{4\left(n+1\right)}=\frac{4}{5} \Rightarrow 5n^2-16n-16=0$

$n=4$ or $n=-20$

ANS: N = 4

140. The number of ways of painting the six faces of a cube with six different given colours is

a) 1 b) 720 c) 30 d) 15

Key. C

Sol. First paint any colour on any face. Now the opposite face can be painted in 5 ways (with anyone of the remaining 5 colours). Now, the remaining 4 faces can be painted with the remaining 4 colours in (4-1)! ways. (circular permutations)

$\therefore$ Ans = $5\times(4-1)!=30$ ways.

141. A (1, 2) and B(5, 5) are two points. Starting from A, line segments of unit length are drawn either rightwards or upwards only, in each step, until B is reached. Then, the number of ways of connecting A and B in this manner is

a) 35 b) 40 c) 45 d) 50

Key. A

Sol. Given A(1,2) and B(5,5). Difference of x-coordinates = 5 – 1 = 4

$\therefore$ Exactly 4 rightward steps are needed.

Difference of y-coordinates = 5 – 2 = 3.

$\therefore$ Exactly 3 upward steps are needed.

<u>Note</u>: Order of the steps is immaterial.

Denote each rightward step by R and each upward step by U.

$\therefore$ The problem is arranging the letters RRRRUUU

No. of arrangements = $\dfrac{7!}{4!3!}$ = 35

142. Let the product of all the divisors of 1440 be P. If P is divisible by 24^x, then the maximum value of x is

a) 28 b) 30 c) 32 d) 36

Key. B

Sol. $1440 = 2^5.3^2.5^1$

No. of divisors = (5+1).(2+1).(1+1) = 36

Product of divisors = 1.2.3……. 480.720.1440. Here all the 36 divisors are written in the increasing order. They can be clubbed into 18 pairs, as shown below.

(1.1440). (2.720).(3.480) -----etc.

$\therefore$ Product of divisors = $(1440)^{18} = 2^{90}.3^{36}.5^{18} = (2^3.3)^{30}.3^6.5^{18} = 24^{30}.3^6.5^{18}$ which is divisible by 24^x

$\therefore$ Maximum value of x = 30

143. The number of 5-digit numbers which are divisible by 3 that can be formed by using the digits 1,2,3,4,5,6,7,8 and 9, when repetition of digits is allowed, is

a) 3^9 b) 4.3^8 c) 5.3^8 d) 7.3^8

Key. A

Sol. — — — — —(5 blanks)

1st blank can be filled in 9 ways

2nd blank can be filled in 9 ways _______ since repetition is allowed.

3rd blank can be filled in 9 ways

4th blank can be filled in 9 ways

Now, we have to fill the 5th blank carefully such that the number is divisible by 3. Add the 4 numbers in the first 4 blanks.

If their sum is in the form 3n, then fill the last blank by 3, 6 or 9 so that the sum of all digits is divisible by 3.

If their sum is in the form 3n+1, than fill the last blank by 2, 5 or 8.

If their sum is in the form 3n+2, than fill the last blank by 1, 4 or 7.

Therefore, in any case, the last blank can be filled in 3 ways only.

$\therefore$ Ans = $9\times9\times9\times9\times3=3^9$.

144. The number of 4-digit numbers that can be formed by using the digits 1,2,3,4,5,6,7,8 and 9 such that the least digit used is 4, when repetition of digits is allowed, is

a) 617 b) 671 c) 716 d) 761

Key. B

Sol. Least digit used = 4

$\therefore$ We can use 4,5,6,7,8,9. But remember that at least one 4 must be used.

————(4 blanks)

1st blank can be filled in 6 ways.

2nd blank can be filled in 6 ways.

3rd blank can be filled in 6 ways.

4th blank can be filled in 6 ways.

$\therefore$ 4 blanks can be filled in 6^4 ways. But out of these, some may contain no 4 at all. Let us find them.

————(4 blanks)

Each blank can be filled in 5 ways (by 5, 6, 7, 8, or 9)

$\therefore$ 5^4 ways (no 4 at all)

$\therefore$ Ans = 6^4 - 5^4 (at least one 4)

= 671.

145. The number of arrangements of the letters of the word 'NAVA NAVA LAVANYAM' which begin with N and end with M is :

a) $\dfrac{\angle 16}{\angle 7\left(\angle 3\right)^2}$ b) $\dfrac{\angle 16}{\angle 7\angle 3}$ c) $\dfrac{\angle 14}{\angle 7\angle 3\angle 2}$ d) $\dfrac{\angle 14}{\angle 7\angle 3}$

Key. C

Sol. The word NAVA NAVA LAVANYAM consists of 16 letters out of which there are 7A's, 3V's, 3N's, and the other 3 are distinct put one N in the first place and M in the last place. In the remaining 14 letters there 7A's, 3V's and 2N's.

$\therefore$ No. of arrangements $= \dfrac{\angle 14}{\angle 7\angle 3\angle 2}$.

146. The number of bijections of a set consisting of 10 elements to itself is :

a) $\angle 10$ b) $\angle 10 - 10$ c) $\angle 9 + 10$ d) $\angle 10 - 2$

Key. A

Sol. Bijection from set – A to itself means permutation.

No. of permutations = $\angle 10$

147. Let $y = 2\sin x + \cos 2x\left(0 \le x \le 2\pi\right)$. All the points at which y is extremum are arranged in a row such that the points of maximum and minimum come alternately the number of such arrangements is :

a) 16 b) 8 c) 12 d) 24

Key. B

Sol. values of x at which as maximum and minimum are : $\dfrac{\pi}{6}, \dfrac{5\pi}{6}\ \dfrac{\pi}{2}, \dfrac{3\pi}{2}$

148. The position vector of a point P is $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ where x and y are positive integers and $\vec{a} = \vec{i} + \vec{j} + \vec{k}$. If $\vec{r}.\vec{a} = 10$, then the number of possible positions of P is :

a) 48 b) 72 c) 24 d) 36

Key. D

Sol. $\vec{r}.\vec{a} = x + y + z$

$\therefore$ x + y + z = 10 where x and y are positive integers.

$\therefore$ No. of positive integral solutions of

X + y + z = 10 is (10 - 1)C_{3-1} = $9C_2$ = 36.

149. In the expansion of $(1+x)^m(1-x)^n$, the coefficients of x and x^2 are respectively 3 and -6. Then m equals :

a) 6 b) 9 c)12 d) 24

Key. C

Sol. $(1+x)^m(1-x)^n = \left(1+mC_1x+mC_2x^2+..........+mC_mx^m\right)\left(1-nC_1x+nC_2x^2-....\right)$

Coefficient of $x = mC_1 - nC_1 = 3$

$\therefore m-n=3$ -------- (1)

Coefficient of x^2 = $mC_2 + nC_2 - mC_1.nC_1$ = - 6

$\therefore \frac{m(m-1)}{2}+\frac{n(n-1)}{2}-mn=-6$ ----------- (2)

From (1) and (2) $\frac{(n+3)(n+2)}{2}+\frac{n(n-1)}{2}-n(n+3)=-6$

$n^2+5n+6+n^2-2n^2-6n=-12$

$-2n=-18$, n = 9

$\therefore m=n+3=12$

150. The number of ways of arranging 6 players to throw the cricket ball so that the oldest player may not throw first is

(A) 120 (B) 600
(C) 720 (D) 7156

Key. B

Sol. For the first place 5 players (excluding the oldest) and for the remaining places 5(including the oldest) players are available.

$\therefore$ no. of ways = 5 ×5×4×3×2×1 = 600

151. A game is played by three players. The loser has to triple the money of each of the other players has. Three games are played and each one loses a game. At the end all have the same amount namely Rs. 54. The amount the first loser has at the beginning is

(A) Rs. 120 (B) Rs. 112
(C) Rs. 110 (D) Rs. 90

Key. C

Sol. Let players name is A, B, C and they lose the games in order of A, B, C

	Amount after 3rd game	Amount before 3rd game	Amount before 2nd game	Amount before 1st game
A	54	$\frac{54}{3}=18$	$\frac{18}{3}=6$	$6+\frac{2}{3}(42)+\frac{2}{3}(114)=110$
B	54	$\frac{54}{3}=18$	$18+\frac{2}{3}(18)+\frac{2}{3}(126)=114$	$\frac{114}{3}=38$
C	54	$54+\frac{2}{3}(54)+\frac{2}{3}(54)=126$	$\frac{126}{3}=42$	$\frac{42}{3}=14$

Ans. (C) Rs. 110

152. A positive integer n is of the form n = $2^\alpha 3^\beta$, where $\alpha \geq 1$, $\beta \geq 1$. If n has 12 positive divisors and 2n has 15 positive divisors, then the number of positive divisors of 3n is

(A) 15 (B) 16
(C) 18 (D) 20

Key. B

Sol. n = 2^{α}. 3^{β}

no. of divisors = (α + 1) (β + 1) = 12 ... (i)

2n = $2^{\alpha+1}$ 3^{β}

No. of divisors = (α + 2) (β + 1) = 15 ... (ii)

$\Rightarrow \dfrac{\alpha+2}{\alpha+1}=\dfrac{5}{4}$ $\Rightarrow 4\alpha + 8 = 5\alpha + 5 \Rightarrow \alpha = 3$

$\Rightarrow \beta = 2 \Rightarrow 3n = 2^3\ 3^3$

No. of divisors = (3 + 1) (3 + 1) = 16

Ans. (B) 16

153. Two numbers 'a' & 'b' are chosen from the set of {1,2,3……3n}. In how many ways can these integers be selected such that a^2-b^2 is divisible by 3

a) $\frac{3}{2}n(n+1)+n^2$ b) $\frac{3}{2}n(n-1)+n^2$ c) $\frac{1}{2}n(n+1)-n^2$ d) $\frac{1}{2}n(n-1)+n^2$

Key. B

Sol. $G_1:3,6,9.......3n$

$G_2:1,4,7.......(3n-2)$

$G_3:2,5,8.......(3n-1)$

$a^2-b^2=(a-b)(a+b)$

Either a-b is divisible by 3 (or) a + b is divisible by 3 (or) both

$nc_2+nc_2+nc_2+nc_1.nc_1$

$3\dfrac{n(n-1)}{2}+n^2$

154. The number of distinct rational numbers of the form p/q, where $p,q\in\{1,2,3,4,5,6\}$ is

a) 23 b) 32 c) 36 d) 28

Key. A

Sol. $p=1,\ q=1,2,3,4,5,6\Rightarrow 6$

$p=2,\ q=1,3,4,5,6\Rightarrow 3\ [\text{Q}\ (2,4),\ (2,6)]$

$p=3,\ q=1,2,4,5,6\Rightarrow 4\ [\text{Q}\ (3,6)]$

$p=4,\ q=1,3,5,6\Rightarrow 3\ [\text{Q}\ (4,6)]$

$p=5,\ q=1,2,3,4,6\Rightarrow 5$

$p=6,\ q=1,5\Rightarrow 2$

155. The number of divisors of 1029, 847 and 122 are in

a) A.P b) G.P c) H.P d) none of these

Ans. **a**

Sol. We have 1029 = 3.7^3

It has numbered of divisors = (1 + 1) (1 + 3) = 8

Similarly 847 = 7.11^2, number of divisors = 2.3 = 6

and 122 = 2.61, number of divisors = 2.2 = 4

Clearly 8, 6, 4 are in A.P.

156. The number of homogenous products of degree 3 from 4 variables is equal to

a) 20 b) 16 c) 12 d) 4

Key. A

Sol. $a+b+c+d=3$

no.of products $\overset{4+3-1}{C}_3 = 20$

157. Five digit numbers are formed by using the numbers 0,1,2,3,4and 5 with repetition of the same digit in any number, then the number of numbers that are divisible by 3 is

a) 1080 b) 2160 c) 540 d) 4320

Key. B

Sol. $\underset{5\times6\times6\times6\times2}{\overline{\downarrow}\overline{\downarrow}\overline{\downarrow}\overline{\downarrow}\overline{\downarrow}} = 2160$

158. There are 2010 chairs round the table numbered from 1 to 2010.The numbers of ways in which 5 persons can be seated in any five of these chairs so that the number of empty chairs between any two consecutive persons must be same, is

a) $402\times5!$ b) $804\times5!$ c) $201\times5!$ d) 0

Key. A

Sol. $\dfrac{2010}{5} = 402$

159. The number of positive integer solutions of the equation xyz = 105 so that $x \neq y \neq z$, is

a) 24 b) 27 c) 6 d) 12

Key. A

Sol. $3! + {}^3C_2 \times 3!$

160. The number of non-congruent rectangles that can be formed on chessboard is

a) 28 b) 36 c) 8 d) 20

Key. B

Sol. ${}^8C_2 + 8 = 36$

161. The number of ways of writing 4096 as the product of three positive integers is

a) 19 b) 91 c) 72 d) 18

Key. A

Sol. $1+\dfrac{18}{3}+\dfrac{72}{6}=19$

162. Let $x_1, x_2, \ldots\ldots, x_k$ are the total divisors of positive integer n. If $\sum_{i=1}^{k} x_i = 93$ and $\sum_{i=1}^{k} \dfrac{1}{x_i} = \dfrac{93}{50}$ then the value of k is

a) 4 b) 3 c) 6 d) 7

Key. C

Sol. $\left(\sum x_i\right) / \left(\sum \dfrac{1}{x_i}\right) = n$

163. Number of positive integer n, less than 17, for which $n!+(n+1)!+(n+2)!$ is an integral multiple of 49 is

A) 0 B) 3 C) 5 D) 2

Key. C

Sol. $n!+(n+1)!+(n+2)!=n!(n+2)^2$

$\Rightarrow$ Either 7 divides $(n+2)$ or 49 devides n!

$\Rightarrow n=5,12,14,15,16$

164. The no.of rational numbers lying in the interval (2002, 2003) all whose digits after the decimal point are non-zero and are in decreasing order

A) $\sum_{i=1}^{9} {}^9P_i$ B) $\sum_{i=1}^{10} {}^9P_i$ C) 2^9-1 D) $2^{10}-1$

Key. C

Sol. A rational number of the desired category is of the form 2002.

$x_1x_2...x_K\,(1\le K\le 9 \text{ and } 9\ge x_1>x_2>...>x_K\ge 1)$

total $={}^9C_1+{}^9C_2+......+{}^9C_9=2^9-1$

165. Let $S=\{1,2,3,...n\}$. If X denote the set of all subsets of S containing exactly two elements, then the value of $\sum_{A\in X}(\min A)$ is

A) ${}^{n+1}C_3$ B) ${}^nC_3$ C) ${}^nC_2$ D) ${}^nC_1$

Key. A

Sol. $\sum_{A\in X}\min(A)\ =1(n-1)+2(n-2)+......+(n-1)1$

$\sum_{r=1}^{n-1} r(n-r)={}^{n+1}C_3$

166. 5 different marbles are placed in 5 different boxes randomly. Find the probability that exactly 2 boxes remain empty, given that each box can hold any number of marbles

a) $\frac{150}{5^5}$ b) $\frac{160}{5^5}$ c) $\frac{170}{5^5}$ d) $\frac{180}{5^5}$

Key. A

Sol. 2 empty boxes can be selected in ${}^5C_2$ ways and 5 marbles can be placed in remaining 3 boxes in groups of 221 or 311 in $3!\left(\frac{5!}{2!2!2!}+\frac{5!}{3!2!}\right)=150$

167. There are 6 red ball, and 6 green balls in a bag 5 balls are drawn out at random and placed in a red box. The remaining 7 balls are put in green box. If the probability that the number of red balls in the green box plus the number of green balls in red box is not prime number is $\frac{p}{q}$ where p, q are relatively prime the value of (p + q) is

a) 36 b) 37 c) 38 d) 39

Key. B

Sol. $12 \begin{cases} 6G \\ 6R \end{cases} \longrightarrow 5 \text{ drawn}$

Red Box	**Green Box**
5R 0G	1R 6G *
4R 1G 2R	5G
3R 2G 3R	4G
2R 3G 4R	3G
1R 4G 5R	2G *
0R 5G 5G	6R

Let E be the desired event

$$P(E) = \frac{{}^6C_0 . {}^6C_5 + {}^6C_4 . {}^6C_1}{{}^{12}C_5} = \frac{{}^6C_1 + {}^6C_4 . {}^6C_1}{11. \quad 9. \quad 8} = \frac{4}{33}$$

168. The number of positive integer solutions for $x_1x_2x_3x_4$ = 504 is _____

a)200 b)400 c)600 d) 800

Key. D

Sol. $x_1x_2......x_k = p_1^{\alpha_1} p_2^{\alpha_2} ......p_n^{a_n}$

Total positive integer solutions $\alpha_1 + k - 1_{c_{k-1}} \times \alpha_2 + k - 1_{c_{k-1}} \times .... \times \alpha_n + k - 1_{c_{k-1}}$

169. The number of integral solutions of the equation $x_1.x_2.x_3.x_4.x_5 = 2310$ are

$(A) 5^5$ (B) 6.5^5 (C) 16.5^5 (D) 5^6

Key. C

Sol. $x_1 \; x_2 \; x_3 \; x_4 \; x_5 = 2310 = 3' \, 7' \, 10' \, 11'$ each of 3, 7, 10, 11 can be distributed at 5 places in 5 ways

+ ve integral sols are 5^5

i) Two are negative and 3 positive then ${}^5C_3 5^5$ ways

ii) Four are negative and 1 positive then ${}^5C_4 5^5$ ways.

$\therefore$ Total no. of ways $= 5^5 \left(1 + {}^5C_3 + {}^5C_4\right) = 16.5^5$

170. Number of rectangles excluding squares from a rectangle of size 7x 4

(a) 220 (b) 216 (c) 208 (d) 202

Key. A

Sol. $\left(\frac{7 \times (7+1)}{2} \times \frac{4 \times (4+1)}{2}\right) - (4 \times 7 + 3 \times 6 + 2 \times 5 + 1 \times 4) = 220$

171. How many ways are there to form a three-letter sequence using the letters a, b, c, d, e, f containing e when repetition of the letters is allowed

a) 90 b) 91 c) 92 d) 89

Key. B

Sol. $6 \times 6 + 5 \times 6 + 5 \times 5 = 91$

172. How many times is the digit 5 written when listing all numbers from 1 to 1,00,000 ?

a) 5×10^4 b) $1 + 10 + 100 + 1000 + 10,000$

c) 5×10^3 d) $1 + 10 + 100 + 1000$

Key. A

Sol. $=5\times9^4\times1+{}^5C_2\times9^3\times2+{}^5C_3\times9^2\times3+{}^5C_4\times9\times4+{}^5C_5\times5$

$=5\times10^4$

173. Let N be the number of 7-digit numbers the sum of whose digits is even. The number of +ve divisors of N is

a) 64 b) 72 c) 88 d) 126

Key. D

Sol. $N=\dfrac{9\times10^6}{2}=2^5.3^2.5^6$

No of divisions N is $6\times3\times7=126$

174. There are 15 different apples and 10 different pears. How many ways are there for Jack to pick an apple or a pear and then Jill to pick an apple and a pear.

a) 23×150 b) 33×150 c) 43×150 d) 53×150

Key. A

Sol. If Jack Pick an apple in ${}^{15}C_1$ ways then Jill in ${}^{14}C_1.{}^{10}C_1$. If Jack pick a pear in ${}^{10}C_1$ way then Jill in ${}^{15}C_1.{}^{9}C_1$

$\therefore$ Total no. of ways $={}^{15}C_1{}^{14}C_1{}^{10}C_1+{}^{10}C_1{}^{15}C_1{}^{9}C_1$

$=150(23)$

175. Let $A=\{0,1,2,3,...9\}$ be a set consisting of different digits. The number of ways in which a nine digit number can be made in which,1 and 2 are present and 1 is always ahead of 2 and repetition of digits is not allowed.

a) $7!\left(\dfrac{65}{2}\right)$ b) $9!\left(\dfrac{65}{2}\right)$

c) $8!\left(\dfrac{65}{2}\right)$ d) $10!\left(\dfrac{65}{2}\right)$

Key. C

Sol

$$\frac{{}^9P_2\,{}^8P_7-{}^7C_1\times8!}{2}\qquad \frac{1}{2}\left(\begin{array}{l}\text{Total number of permutations of nine numbers in which 1\& 2 are present -}\\ \text{Number of permutations in which 0 occupies first place and containing 1 \& 2}\end{array}\right)$$

176. The number of divisors of $2^2.3^3.5^3.7^5$ of the form 2n + 1, n $\in$ N is

(A) 96 (B) 95

(C) 94 (D) 924

Key. B

Sol. Number of div. (3 + 1) (3 + 1) (5 + 1) – 1 = 95

177. The number of ways in which 5 identical balls can be kept in 10 identical boxes, if not more than one can go into a box, is

(A) ${}^{10}P_5$ (B) $\binom{10}{5}$

(C) 5 (D) 1

Key. D

Sol. one way

178. In how many number of ways can 10 students be divided into three teams, one containing four students and the other two, three each?

a) 1050 b) 2100

c) 4200 d) $10p_4 \times 6p_3$

Key. B

Sol. $10C_4 \times \frac{6!}{(3!)^2 2!} = 210 \times 10 = 2100$

179. Total number of divisors of $3^5.5^7.7^9$ which are of the form $4\lambda+1,\ \lambda \ge 0$, is

a) 30 b) 60 c) 120 d) 240

Key. D

Sol. Any positive integral power of 5 is of the form $4\lambda+1$. Even power of 3 and 7 are of the form $4\lambda+1$ and odd powers of 3 and 7 are of the form $4\lambda-1$. The required number $=8(3\times5+3\times5)$

180. How many different 5 letter sequences can be made using the letters A, B, C, D with repetition such that the sequence does not include the word BAD?

a) 48 b) 550 c) 976 d) 1024

Key. C

Sol. Number of sequences that can be formed = 4^5

Number of sequences that include BAD = $3\times4^2 = 48$

Required number $= 4^5 - 48 = 976$

181. The number of seven digit integers, with sum of the digits equal to 9 and using at least one of the digits
1, 2, 3 only is

a) 63 b) 36 c) 28 d) 21

Key. C

Sol. Number of 7 digit numbers with 3,1,1,1,1,1,1 $= \frac{7!}{6!} = 7$

Number of 7 digit numbers with 2,2,1,1,1,1,1 $= \frac{7!}{5!\,2!} = 21$

182. Total number of arrangements that can be formed with the letters of the word "NARAYANA" such that the vowels occupy even positions is

a) 12 b) 24 c) 48 d) 96

Key. A

Sol. $\frac{4!}{2!} = 12$

183. The number of positive integral solutions of $x_1x_2x_3x_4x_5 = 840$ is

a) 625 b) 3125 c) 3750 d) 4375

Key. D

Sol. $840 = 2^3\times3\times5\times7$. Therefore the number of solutions = $5^3\times(5C_1 + 5C_1\times4C_1 + 5C_3) = 4375$

184. There are two balls in an urn whose colours are not known. (Each ball can be either black or white). A white ball is put in the urn. A ball is now drawn from the bag at random. The probability that the ball drawn is white is

A) $\frac{1}{3}$ B) $\frac{1}{4}$ C) $\frac{3}{4}$ D) $\frac{2}{3}$

Key. D

Sol. The two balls in the bag can be WW or BB or WB.
All cubes are equally likely.

185. If n identical dice are rolled simultaneously, the number of distinct throws is

(A) ${}^{n+5}C_5$. (B) $\frac{6^n - 6}{n} + 6$

(C) 6^n (D) $\frac{6^n - 6}{n}$

Key. A

Sol. The number of distinct throws when exactly
r $(1 \le r \le 6)$ numbers appear will be
${}^6C_r \times$ (the number of ways of putting n identical things into r distinct boxes with no box empty)
$= {}^6C_r \times {}^{n-1}C_{r-1}$

The total number of distinct throws $= \sum_{r=1}^{6} {}^6C_r \; {}^{n-1}C_{r-1}$

$= \sum_{r=1}^{6} {}^6C_r \; {}^{n-1}C_{n-r} = {}^{n+5}C_n = {}^{n+5}C_5$

186. Number of points having position vector $a\bar{i} + b\bar{j} + c\bar{k}$ when a, b, c $\in \{1,2,3,4,5\}$ such that $2^a + 3^b + 5^c$ is divisible by 4 is

a) 140 b) 70 c) 100 d) 120

Key. B

Sol. 4m= $2^a + 3^b + 5^c = 2^a + (4-1)^b + (1+4)^c$

$= 2^a + 4k + (-1)^b + (1)^c$

$\therefore a = 1,\ b = even \quad c = any$ number

$a \ne 1,\ b = odd \quad c = any\ number$

$\therefore$ Required number of ways = $1\times2\times5 + 4\times3\times5 = 70$

187. Number of 4 digit positive integers if the product of their digits is divisible by 3 is

a) 2700 b) 6628 c) 7704 d) 5464

Key. C

Sol. Product will be divisible by 3, if at least one digit is 0 ,3, 6, 9

188. In how many ways two distinct numbers n_1 and n_2 can be selected from the set $\{1,2,3,4,.....100\}$ so that $7^{n_1} + 3^{n_2}$ is a multiple of 5 is

a) 1625 b) 625 c) 12525 d) 1825

Key. D

Sol.

$7^1 = 7 \qquad 3^1 = 3$

$7^2 = 49 \qquad 3^2 = 9$

$7^3 = 343 \qquad 3^3 = 27$

$7^4 = 2401 \qquad 3^4 = 81$

ie. $7^{4\lambda}$ is always end with 1, $7^{4\lambda-1}$ ends with 3, $7^{4\lambda-2}$ ends with 9, $7^{4\lambda-3}$ ends with 7.

Similarly $3^{4\lambda}$ ends with 1, $3^{4\lambda-1}$ ends with 7, $3^{4\lambda-2}$ ends with 9, $3^{4\lambda-3}$ ends with 3

we will get a number divisible by 5 only when if its end digit is 'O' (or) 5

189. A six digit number is formed using all the six digits 2,3,4,5,7,8, then number of such digits that are divisible by 11 is …

a) 36 b) 720 c) 180 d) 72

Key. D

Sol. Sum of the digits is odd places (or) sum of the digits in even places are equal (or) differ by multiple of 11

190. Let N be a natural number if its first digit (from the left) is deleted, it gets reduced to $\frac{N}{57}$. The sum of all the digits of N is …

a) 15 b) 18 c) 24 d) 30

Key. A

Sol. $N = a_n\, a_{n-1}\, a_{n-2} \ldots a_2\, a_1\, a_0$

$$\frac{N}{57} = a_{n-1}.a_{n-2} \ldots a_2\, a_1\, a_0$$

$$a_n 10^n = 56\left(a_0 + 10a_1 + 100a_2 \ldots . + 10^{n-1} a_{n-1}\right)$$

$$\Rightarrow 56 \;\; divides \;\; a_n 10^n$$

$$\Rightarrow a_n = 7, \;\; n \geq 3$$

$$\Rightarrow 5^3 = a_0 + 10a_1 + 10^2 a_2$$

$\Rightarrow$ The required N= 7125 (or) 71250 (or) 712500 etc

$\Rightarrow$ sum of digits = 15

191. An unlimited number of coupons bearing the letters A, B and C are available, then the number of ways of choosing 10 of these coupons so that they can't used to spell BAC

a) $3\left(2^{10}-1\right)$ b) $2\left(3^{10}-1\right)$ c) $2^{10}-1$ d) 2^{10}

Key. A

Sol. <u>Case: I</u>

When all the selected coupons bear the same letter.

One letter can be selected from three letters in ${}^3c_1$ ways

$\Rightarrow$ Total number of ways of choosing 10 coupons bearing the same letter is ${}^3c_1 \times 1 = 3$ <u>Case</u> : II

When selected coupons bear two letters only

The number of ways of selecting two letters from 3 is ${}^3c_2$

$\Rightarrow$ The number of ways in which selected coupons bear two letters only = ${}^3c_2\left(2^{10}-2\right)$

Hence required number of ways = $3 + {}^3c_2\left(2^{10}-2\right) = 3\left(2^{10}-1\right)$

192. The integers from 1 to 1000 are written in order around a circle. Starting at 1, every fifteenth numbers is marked (ie. 1,16,31 etc). This process is continued until a number is reached which has already been marked, then unmarked numbers are ….

a) 200 b) 400 c) 600 d) 800

Key. D

Sol. In 1st round all the integers, which leaves the remainder 1 when divided by 15, will be marked

Last number of this category is 991

Next number to be marked is (991+15-1000) =6 again, second round of integers which leaves the remainder '6' when divided by 15 will be marked.

Last number of this category is 996

Next number to be marked is (996+15-1000) = 11

Thus third round of integers which leaves the remainder 11 when divided by 15, will be marked .

last number of this category is 986

Next number to be marked is 986+15-1000 = 1 which is already been marked. $\Rightarrow$

Marked number = 200

193. Five distinct letters are to be transmitted through a communication channel. A total number of 15 blanks is to be inserted between the two letters with at least three between every two. The total number of ways in which this can be done is .

a) 1200 b) 1800 c) 2400 d) 3000

Key. C

Sol. For $1 \le i \le 4$, *Let* $x_i(\ge 3)$ be the number of blanks between i^{th} *and* $(i+1)^{th}$ letters. Then

$$x_1 + x_2 + x_3 + x_4 = 15 \quad ....(1)$$

The no. of solutions of (1) $= coeff\ \ of\ x^{15}\ in\ \left(x^3 + x^4 + ...\right)^4$

$$= coeff\ \ of\ x^3\ in\ (1-x)^{-4}$$

$$= coeff\ \ of\ x^3\ in\ \left[1 + 4c_1x + 5c_2x^2 + 6c_3x^3 + ...\right]$$

$$= {}^6c_3$$

= 20

But 5 letters can be permuted in $\lfloor 5 = 120$ ways

$\Rightarrow \text{Re}\,qd\ \ no.of\ \ arrangements = (120)(20) = 2400$

194. A train having 12 stations enroute has to be stoped at 4 stations. The number of ways it can be stopped if no two stoppings stations are consecutive

(a) ${}^8C_4$ (b) ${}^9C_4$ (c) ${}^{12}C_4 - {}^8C_4 + {}^4C_4$ (d) ${}^{12}C_4 - {}^{10}C_4 + {}^8C_4 - {}^6C_4$

Key. B

Sol. consider 4 Identical things (AAAA) and 8 other Identical things (BB….B) to be arranged in a row so that no two A's are together __B__ B__B__B__ B__B__ B__B

9 gaps between B's

no. of ways of doing it is ${}^9C_4$ this is the final Ans. For each such arrangement we have one way of stoping the train at 4 stations (position of A's)

195. The number of permutations of the letters of the word HONOLULU taken 4 at a time is

(a) 354 (b) 314 (c) 220 (d) 124

Key. A

Sol. LL,OO,UU,H,N,

(i) all diff ${}^5C_4.4! = 120$

(ii) 2 same, 2 diff ${}^3C_1.{}^4C_2.\dfrac{4!}{2!} = 216$

(iii) 2 same of one kind, 2 same of other kind ${}^3C_2.\dfrac{4!}{2!2!} = 18$

196. The number of ways in which two Americans, two British, one Chinese, one Duteh and one Indian can sit on a round table so that persons of the same nationality are separated.

a) 344 b) 246 c) 336 d) 384

Key. C

Sol. Total 6!. n(A) = n(A_1A_2 together) = 5! 2! = 240

n(B) = 2(B_1B_2 together) = 5! 2! =240

$n(A\cup B)=n(A)+n(B)-n(A\cap B)=240+240-96=384$

Hence $n(\overline{A}\cap\overline{B})=Total-n(A\cup B)=6!-384=336$

197. Twelve boys and 2 girls are to be seated in a row such that there are at least 3 boys between the two girls. Then the number of ways it can be done is

(a) 110. 12! (b) ${}^{14}C_2.12!$ (c) ${}^{9}C_2.12!2!$ (d) ${}^{11}C_2.12!$

Key. A

Sol. The girls sit together in (1, 2) or (2, 3) …… seated in 2 x 13 = 26 ways

If one boy sits between the girls they can be seated in 2 x 12 = 24 ways

If two boy sits between the girls they can be seated in 2 x 12 = 24 ways

The desired number is $14!-(26+24+22)12!=110.12!$

198. Number of ordered pair (x, y) such that LCM of x, y is $2^3 3^4 5^6$ is

(a) 140 (b) 819 (c) 72 (d) 5184

Key. B

Sol. $x=2^{a_1}3^{b_1}5^{c_1}$

$y=2^{a_2}3^{b_2}5^{c_3}$

(a_1, a_2) = (0, 3) (1, 3) (2, 3) (3, 3) (3, 2) (3, 1) (3, 0) – 7 ways

(b_1, b_2) = (0, 4) (1, 4) (2, 3) (3, 4) (4, 4) (4, 3) (4, 2) (4, 1) (4, 0) – 9 ways

(c_1, c_2) = (0, 6) (1, 6) (2, 6) (3, 6) (4, 6) (5, 6) (6, 6) (6, 5) (6, 4) (6, 3) (6, 2) (6, 1) (6, 0) – 13 ways

no. of ordered pair = 7 x 9 x 13

199. The number of ways in which 6 boys and 6 girls can be seated at a round table so that no two girls sit together and two particular girls do not sit next to a particular boy is

(a) 6! .4! (b) 2. 5! .4! (c) 2. 6!. 4! (d) 5! .4!

Key. C

Sol. (i) Placing boys in circular table (ii) but of 6 places between boys placing 2 particular girls to any 4 places which is not adjacent to particular boy (iii) remaining 4 girls to remaining 4 places

5!. 4. 3. 4! = 2.6! 4!

200. The number of times digit 1 will be written when listing the intergers from 1 to 1000

(a) 269 (b) 270 (c) 300 (d) 301

Key. D

Sol. $000\leftrightarrow 999$ total no. of digits used 3000, every digit used 300 times, one more time 1 used in writing 1000.

201. Total number of even divisors of 1323000 which are divisible by 105 is

(a) 36 (b) 48 (c) 54 (d) 64

Key. C

Sol. 1323000 = $2^3\ 3^3\ 7^2\ 5^3$

D = $2^a\ 3^b\ 7^c\ 5^d$ then $a\geq 1, b\geq 1, c\geq 1, d\geq 1$ for any even divisor divisible by 105

Hence required ans are is 3 x 3 x 2 x 3 = 54

202. The letters of the word COCHIN are permuted and all the permutations are arranged in alphabetical order as in English dictionary. The number of words that appear before the word COCHIN is

a) 360 b) 192 c) 96 d) 48

Key. C

Sol. C C……… 4! Words

C H……… 4! Words

C I……… 4! Words

C N……… 4! Words

The next word is COCHIN

There are $4(4!) = 96$ words before COCHIN.

203. The number of ways in which 20 different pearls of two colours can be set alternately on a necklace, there being 10 pearls of each colour, is

a) $9! \times 10!$ b) $5(9!)^2$ c) $(9!)^2$ d) $(10!)^2$

Key. B

Sol. Ten pearls of one colour can be arranged in $\frac{1}{2}.(10-1)!$ ways

The number of arrangements of 10 pearls of the other colour in 10 places between the pearls of the first colour $=10!.2!$

$\therefore$ The required number of ways $=\frac{1}{2}\times 9!\times 10!.2!$

204. The number numbers between 100 and 1000 which are neither divisible by 5 nor divisible by 3 is (excluding 100 and 1000)

a) 460 b) 439 c) 560 d) 440

Key. A

Sol.

$$\begin{array}{llllll} \text{Divisible by} & 5 & \text{is} & 199 & \left.\right\} & \\ \text{Divisible by} & 3 & \text{is} & 300 & \left.\right\} & 499 \\ \text{Divisible by} & 15 & \text{is} & 60 & & 60 \\ \hline & & & & & 439 \end{array}$$

Neither divisible by 5 nor divisible by 3 is $899-439=460$

205. There are thirty volumes of a magazine on the book shelf. The number of ways in which, they can be arranged, so that two particular volumes are kept side by side and another two volumes are not side by side, is

a) $(57)(27!)$ b) $(56)(27!)$ c) $(54)(28!)$ d) $(56)(28!)$

Key. C

Sol. Let us consider the case in which the first two particular volumes out of 30 are always together and the other two particular volumes are anywhere.

$\therefore$ Required Ways $=(29)!\ 2!$

Now, let us remove all those cases from above in which the other two particular books are always together, we are automatically left with the arrangement in which the first two volumes are always together and the other two volumes are never together.

206. The number of ways of choosing n objects out of 3n+1 objects of which 'n' are identical and (2n+1) are distinct is
(A) 2^{2n} (B) 2^{2n+1} (C) 2^{2n-1} (D) $2.\,2^{n}$

Key. A

Sol. If we choose $k\left(0 \le k \le n\right)$ identical objects, then we must choose n-k distiact objects. This can be done in ${}^{2n+1}C_{n-k}$ ways.

Thus the required number of ways

$= \sum_{k=0}^{n} {}^{2n+1}C_{n-k} = 2^{2n}$

207. The number of ways in which a committee of 3 women and 4 men be chosen from 8 women and 7 men. If Mr. X refuses to serve on the committee if Mr Y is a member of the committee is
(A) 420 (B) 840 (C) 1540 (D) 1400

Key. D

Sol. The no.of ways of seleting 3 womes is ${}^{8}C_{3}$

Men selection both x, y are excluded = ${}^{5}C_{4}$

Only x is included = ${}^{5}C_{3}$

Only y is included = ${}^{5}C_{3}$

Hence the no.of ways is ${}^{8}C_{3}\left\{{}^{5}C_{4} + 2\times {}^{5}C_{3}\right\} = 1400$

208. The number of 5 digit numbers that contain 7 exactly once is
(A) 41 (9^3) (B) 37 (9^3) (C) 7 (9^4) (D) 41 (9^4)

Key. A

Sol. 5 digit numbers having 7 in 1^{st} place = 9^4
In 2^{nd}, 3^{rd}, 4^{th}, 5^{th} places is $4 \times 8 \times 9^3$
Total number of 5 digit numbers having 7 exactly once is = 41 (9^3)

209. The number of ways of arranging the letters AAAAABBBCCCDEEF in a row, if the letters C are separated from one another is

1) $\dfrac{12!}{5!3!2!}\times 13!$ 2) $\dfrac{12!}{5!3!2!}\times {}^{13}p_3$ 3) $\dfrac{12!}{5!3!2!}\times {}^{13}C_3$ 4) $\dfrac{15!}{5!3!2!2!}$

Key. 3

Sol. Other than Cs are arranged in 3Cs are arranged in $\dfrac{12!}{5!3!2!}$ ways. In the 13 places between them,3c's can be arranged in $\dfrac{{}^{13}P_3}{3!}$ ways.

210. The number of 4 digit numbers, that can be formed by the digits 3,4,5,6,7,8,0, no digit is being repeated, is :

1) 720 2) 840 3) 280 4) 640

Key. 1

Sol. ${}^{7}P_4 - {}^{6}P_3$

211. The number of different 7 digit numbers that can be written using only the three digits 1, 2 and 3 with the condition that the digit 2 occurs twice in each number is

1) ${}^{7}P_{2}2^{5}$ 2) ${}^{7}C_{2}2^{5}$ 3) ${}^{7}C_{2}5^{2}$ 4) ${}^{7}C_{3}5^{3}$

Key. 2

Sol. Other than 2, remaining five place are to be filled by 1 & 3

$\therefore$ No. of ways for five places = 2^5

For 2, selecting 2 places out of 7 = ${}^{7}C_{2}$

$\therefore$ Required no. of ways = ${}^{7}C_{2}2^{5}$

212. On a new year day every student of a class sends a card to every other student. The postman delivers 600 cards. The number of students in the class are

1) 42 2) 34 3) 25 4) 35

Key. 3

Sol. Let n be the number of students.

Now number of ways in which two students can be selected out of n students is ${}^{n}C_{2}$.

$\therefore$ number of pairs of students = ${}^{n}C_{2}$

But for each pair of students, number of cards sent is (since if there are two students A and B, A will send a card to B and B will send a card to A).

$\therefore$ For ${}^{n}C_{2}$ pairs, number of cards sent = $2.{}^{n}C_{2}$.

According to the question , $2.{}^{n}C_{2}$ =600

$2.\dfrac{n(n-1)}{2!} = 600$ or $n^2 - n - 600 = 0$

Or, $(n-25)(n+24) = 0 \therefore n = 25, -24$

But $n \neq -24 \therefore n = 25$

213. The letters of the word LOGARITHM are arranged in all possible ways. The number of arrangements in which the relative positions of the vowels and consonants are not changed is

1) 4320 2) 720 3) 4200 4) 3420

Key. 1

Sol. $LGRTHM$; $OAI - 6!.3! = 4320$

214. Ten different letters of alphabet are given. Words with 5 letters are formed from these given letters then the number of words which have at least one letter repeated is

1) 69760 2) 30240 3) 99748 4) 88620

Key. 1

Sol. No.of ways = $10^5 - {}^{10}P_5$ =100000-30,240 = 69,760

215. Let $f : A \to A$ be an invertible function where $A = \{1, 2, 3, 4, 5, 6\}$ The number of these functions in which at least three elements have self image is

1) 40 2) 56 3) 16 4) 3

Key. 2

Sol. $n_{c_r} r!\left(\frac{1}{0!}-\frac{1}{1!}+\frac{1}{2!}-........\right)$

Required functions = $6_{c_3}(2)+6_{c_4}(1)+6_{c_5}(0)+6_{c_6}$

216. The number of permutations that can be formed with the letters of the word SRINATHDUBE. So that a vowel occupies the central place is

1) 10! 2) $4.10!$ 3) $4!.7!$ 4) $7!.10!$

Key. 2

Sol. Central place can be filled with any one of 4 vowels

$^4P_1 \times 10! = 4 \times 10!$

217. The number of natural numbers of 10 digits with distinct digits is

1) $9^{10}-1$ 2) $10^{10}-9^{10}$ 3) $9!$ 4) $9.9!$

Key. 4

Sol. $\overline{9}\ \overline{9}\ \overline{8}\ \overline{7}\ \overline{6}\ \overline{5}\ \overline{4}\ \overline{3}\ \overline{2}$

218. The number of ordered pairs $(m, n), m, n \in \{1, 2, ...50\}$ such that $6^n + 9^m$ is multiple of 5

1) 2500 2) 1250 3) 625 4) 500

Key. 2

Sol. All the numbers of the form 6^n will end with 6 9^m will end with 9, if m is odd, and will end with 1, if m is even so $6^n + 9^m$ will end with 5 if n is any number and m is odd.
So ordered pairs will be 50 x 25 = 1250

219. Sum of the even divisors of 1512 is

1) 4800 2) 4600 3) 4480 4) 320

Key. 3

Sol. Sum of even divisors of $2^{\alpha_1}.p_1^{\alpha_2}.p_2^{\alpha_3} ----- p_k^{\alpha_{k+1}}$ is

$$2.\frac{2^{\alpha_1}-1}{2-1}\times\frac{p_1^{\alpha_2+1}-1}{p_1-1}\times\frac{p_2^{\alpha_3+1}-1}{p_2-1}\times-----\frac{p_k^{\alpha_k+1}-1}{p_k-1}$$

220. A man has 7 relatives, 4 women and 3 men. His wife also has 7 relatives, 3 women and 4 men. The number of ways in which they can invite 3 men and 3 women so that they both invite three each is

1) 485 2) 584 3) 720 4) 1024

Key. 1

Sol. $^4C_3 \cdot {}^4C_3 + {}^3C_1 \cdot {}^4C_2 \cdot {}^4C_2 \cdot {}^3C_1 + {}^3C_2 \cdot {}^4C_1 \cdot {}^4C_1 \cdot {}^3C_2 + {}^3C_3 \cdot {}^3C_3 = 485$

221. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions .The number of choices available to him is .

1) 140 2) 196 3) 280 4) 346

Key. 2

Sol. The number of choices available to him is

$$= {}^5C_4 \times {}^8C_6 + {}^5C_5 \times {}^8C_5$$

$$= \frac{5!}{4!1!} \times \frac{8!}{6! \times 2!} + \frac{5!}{5!0!} \times \frac{8!}{5!3!} = 5 \times 4 \times 7 + 8 \times 7 = 140 + 56 = 196$$

222. In how many ways 25 apples can be divided in to 5 sets each set containing equal number.

1) $\frac{\lfloor 25}{\lfloor 3 \lfloor 5}$ 2) $\frac{\lfloor 25}{5(\lfloor 5)^6}$ 3) $\frac{\lfloor 25}{(\lfloor 5)^5 \lfloor 5}$ 4) $\frac{\lfloor 25}{\lfloor 20 \lfloor 5}$

Key. 3

Sol. $\frac{\lfloor mn}{(\lfloor m)^n \lfloor n}$

223. The number of ways in which TEN examination papers can be arranged so that the best and worst papers do not come together is

1) 2093400 2) 2903040 3) 2903004 4) 2903404

Key. 2

Sol. $8! \times {}^9P_2 = 40320 \times 72 = 2903040$

224. The number of ways in which the six faces of a cube be painted with six different colours is

1) 6 2) 6! 3) ${}^6C_2$ 4) 30

Key. 4

Sol. No. of ways = $1.5.\lfloor 3 = 5(6) = 30$

225. The number of ways of dividing 20 persons into 10 couples is :

1) $\frac{20!}{(2!)^{10}}$ 2) ${}^{20}C_{10}$ 3) $\frac{20!}{2^{10}}$ 4) $\frac{20!}{10! \times 2^{10}}$

Key. 4

Sol. Here, problem is of group formation i.e., order of couples is not in consideration.

$\therefore$ required number of ways = ${}^{20}C_2 \cdot {}^{18}C_2 \ldots\ldots {}^4C_2 \cdot {}^2C_2 \cdot \frac{1}{10!} = \frac{20!}{10! \times 2^{10}}$

Permutation & Combination

Multiple Correct Answer Type

1. A contest consists of ranking 10 songs of which 6 are Indian classic and 4 are western songs. Number of ways of ranking so that

A) There are exactly 3 Indian classic songs in top 5 is $(5!)^3$

B) Top rank goes to Indian classic song is 6(9!)

C) The ranks of all western songs are consecutive is 4! 7!

D) The 6 Indian classic songs are in a specified order is ${}^{10}P_4$

Key. A,B,C,D

Sol. A) ${}^{6}C_3.{}^{4}C_2.5!.5! = (5!)^3$

B) $6C_1.9!$

C) $(6+1)!4!$

D) ${}^{10}P_4$

2. A contest consists of ranking 10 songs of which 6 are Indian classic and 4 are western songs. Number of ways of ranking so that

A) There are exactly 3 Indian classic songs in top 5 is $(5!)^3$

B) Top rank goes to Indian classic song is 6(9!)

C) The ranks of all western songs are consecutive is 4! 7!

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Key. A,B,C,D

Sol. A) ${}^{6}C_3.{}^{4}C_2.5!.5! = (5!)^3$

B) $6C_1.9!$

C) $(6+1)!4!$

D) ${}^{10}P_4$

3. Thirteen persons are sitting in a row. Number of ways in which four persons can be selected so that no two of them are consecutive is equal to ____

a) number of ways in which all the letters of the word "M A R R I A G E" are permutated if no two vowels are never together.

b) number of numbers lying between 100 and 1000 using only the digits 1,2,3,4,5,6,7 without repetition.

c) number of ways in which 4 alike chocolates can be distributed among 10 children so that each child getting at most one chocolate.

d) number of triangles can be formed by joining 12 points in a plane, of which 5 are collinear

Sol : ans: b,c,d

$x_1+x_2+x_3+x_4+x_5=9,\ \ x_1,x_5\ge 0$

$x_2,x_3,x_4\ge 1$, number of solutions are 210

a) $5\times12\times12=720$ b) ${}^7P_3=210$ c) ${}^{10}C_4=210$ d) ${}^{12}C_3-{}^5C_3=210$

4. The value of $\sum_{k=0}^{7}\left[\frac{\binom{7}{k}}{\binom{14}{k}}\sum_{r=k}^{14}\binom{r}{k}\binom{14}{r}\right]$, where $\binom{n}{r}$ denotes ${}^nC_r$, is

(A) 6^7 (B) greater than 7^6

(C) 8^7 (D) greater than 7^8

KEY : A,B

HINT The number of ways in which we can choose 2 distinct integers from 1 to 200 so that the difference between them is atmost 20 is

(A) 3790 (B) ${}^{200}C_2-{}^{180}C_2$

(C) ${}^{180}C_1\times20+\frac{19\times20}{2}$ (D) ${}^{180}C_2$

Key: A,B,C

Hint: For any no. choosen from [1,180] there are 20 ways to select the second no. and from [181,199] there are 19, 18, …..1, ways resp. to select the second no. hence required no.of ways = 20 x 180 + (19 + 18 + …….+1) = 3790

5. ${}^{11}C_{10}.{}^9C_1+{}^{11}C_9.{}^9C_2+\ldots\ldots\ldots\ldots+{}^{11}C_2.{}^9C_9=$

A) ${}^{20}C_{11}$ B) ${}^{20}C_8$ C) ${}^{20}C_9-1$

D) Number of different ways of exchanging 11 books of A with the 9 books of B

Key. C,D

Sol. $(1+x)^{11}={}^{11}C_0+{}^{11}C_1x+{}^{11}C_2x^2+\cdots\cdots+{}^{11}C_{11}x^{11}$ --(I)

$(1+x)^9={}^9C_0+{}^9C_1x+{}^9C_2x^2+\cdots\cdots+{}^9C_9x^9$ --(II)

On multiply (I) & (II) and compare coefficient of x^{11} on both sides and put $x=1$

${}^{20}C_{11}={}^{11}C_{11}{}^9C_0+{}^{11}C_{10}{}^9C_1+\cdots\cdots+{}^{11}C_2{}^9C_9$

$\therefore {}^{20}C_9-1={}^{11}C_{10}{}^9C_1+\cdots\cdots+{}^{11}C_2{}^9C_9$

6. If $10!=2^p3^q5^r7^s$ Then

A) $p+q=12$ B) $qr=8$ C) $p+q+r+s=15$ D) $r-s=3$

Key. A,B,C

Sol. $10!=2^8.3^4.5^2.7^1$

7. A contest consists of ranking 10 songs of which 6 are Indian classic and 4 are western songs. Number of ways of ranking so that

A) There are exactly 3 Indian classic songs in top 5 is $(5!)^3$

B) Top rank goes to Indian classic song is 6(9!)

C) The ranks of all western songs are consecutive is 4! 7!

D) The 6 Indian classic songs are in a specified order is ${}^{10}P_4$

Key. A,B,C,D

Sol. A) ${}^6C_3.{}^4C_2.5!.5! = (5!)^3$

B) $6C_1.9!$

C) $(6+1)!4!$

D) ${}^{10}P_4$

8. Using the elements –3, –2, –1 0, 1, 2, 3

A) The number of 3 × 3 matrices having trace 0 is $37(7^6)$

B) The number of 3 × 3 matrices is 7^9

C) The number of 3 × 3 skew symmetric matrices is 7^3

D) The number of 3× 3 symmetirc matrices is 7^6

Key. A,B,C,D

Sol. A) $a_{11}+a_{22}+a_{33}=0$ remaining '6' elements can be filled in 7^6 ways

$$\left.\begin{matrix}(-3,0,3),(-2,0,2),(-1,0,1)\\(-3,1,2),(3,-1,-2)\end{matrix}\right\}\rightarrow 3!.5=30$$

$$(-2,1,1)(2,-1,-1)\rightarrow 3.2=6$$

$$(0,0,0)\rightarrow\frac{1}{37}$$

B) Each of 9 elements can be filled in 7 ways

C) $\begin{bmatrix}0 & - & -\\ & 0 & -\\ & & 0\end{bmatrix}$ 3 elements can be filled 7^3 ways

D) a_{11}, a_{22}, a_{33} filled in 7 ways, also a_{12} filled in 7 ways then a_{21} filled in one way and so on

$\begin{bmatrix}- & - & -\\ & - & -\\ & & -\end{bmatrix}$ 6 elements can be filled 7^6 ways

9. The number of ways in which five different books to be distributed among 3 persons so that each person gets at least one book, is equal to the number of ways in which

(A) 5 persons are allotted 3 different residential flats so that each person is allotted at most one flat and no two persons are allotted the same flat

(B) number of parallelograms (some of which may be overlapping) formed by one set of 6 parallel lines and other set of 5 parallel lines that goes in other direction

(C) 5 different toys are to be distributed among 3 children, so that each child gets at least one toy

(D) 3 professors of mathematics are assigned five different lectures to be delivered, so that each professor gets at least one lecture

Key. B,C,D

Sol. No. of ways =Total number of onto function from

A = {1, 2, 3, 4, 5} to B = {a, b, c}

10. Consider the following statements.

(i) In a 12 storeyed house, 10 people enter the lift cabin at ground floor. It is known that they will leave lift in groups of particular 2, 3 and 5 people at different storey. The number of ways this can be done if the lift does not stop at first and second floors is 720.

(ii) Each of three ladies have brought their one child for admission to a school. The principal wants to interview the six persons one by one, subject to the condition that no mother is interviewed before her child. The number of ways in which interviews can be arranged is 90.

(iii) The number of ways in which one can put three balls numbered 1, 2, 3 in three boxes labelled a, b, c such that at most one box is empty is equal to 18.

(iv) A box contains 5 different red balls and 6 different white balls. The total number of ways in which 4 ball can be selected, taking atleast 1 ball of each colour is 310.

(A) statements (i), (ii) are correct. (B) statements (ii) and (iv) are correct.

(C) statements (i) and (iii) are correct. (D) All statements are correct.

Key. A,B

Sol. (A) The number of ways = ${}^{10}C_3 \times 3! = 720$

(B) Each lady and her child can be arranged in a fixed order only.

$\therefore$ The total number of ways in which interview can be held = $\dfrac{6!}{2!2!2!} = 90$

(C) Case I: No box empty.

Then the number of ways = 3! = 6

Case II: If one of the boxes is empty, then number of ways = ${}^3C_1(2^3 - 2) = 18$.

$\therefore$ total number of ways = 6 + 18= 24

(D) Total – (All red) – (All white)

$${}^{11}C_4 - {}^5C_4 - {}^6C_4 = 330 - 5 - 15 = 310$$

11. Let for $n \in N$, $f(n) = \sum_{r=0}^{n} (-1)^r \dfrac{{}^nC_r\, 2^{r+1}}{(r+1)(r+2)}$ then

(A) $f(2n) = f(2n + 1)$ (B) $f(n) = f(n + 1)$

(C) $f(2n) = f(2n - 1)$ (D) $f(2011) = f(2012)$

Key. C,D

Sol. $f(n) = \dfrac{1}{2(n+1)(n+2)}\sum_{r=0}^{n} {}^{n+2}C_{r+2}(-2)^{r+2} = \dfrac{(1-2)^{n+2}-1+2(n+2)}{2(n+1)(n+2)}$

$$= \begin{cases} \dfrac{1}{n+2} \text{ if } n = \text{odd} \\ \dfrac{1}{n+1} \text{ if } n = \text{even} \end{cases}$$

12. A fair coin is tossed n times. Let a_n denotes the no. of cases in which no two heads occur consecutively, then

(A) $a_1 = 2$ (B) $a_2 = 3$

(C) $a_5 = 14$ (D) $a_8 = 55$

Key. A,B,D

Sol. The cases for a_1 {H, T} i.e., $a_1 = 2$

The case for a_2 : {HT, TH, TT}, $a_2 = 3$

for $n \geq 3$, If the first outcome is H then next just T and then a_{n-2} . If the first outcome is T then a_{n-1} should follow.

So, $a_n = 1 \times 1 \times a_{n-2} + 1 \times a_{n-1} \Rightarrow a_n = a_{n-2} + a_{n-1}$

So, $a_3 = a_1 + a_2 = 5$, $a_4 = 3 + 5 = 8$ and so on.

13. Four balls numbered 1, 2, 3, 4 are to be placed into five boxes numbered 1, 2, 3, 4, 5, such that exactly one box remains empty and no ball goes to its own numbered box. The no. of ways is

(A) $5!\sum_{r=0}^{5}\dfrac{(-1)^r}{r!}$ (B) $4!\sum_{r=0}^{4}\dfrac{(-1)^r}{r!}$

(C) $4!\sum_{r=0}^{4}\dfrac{(-1)^r}{r!} + 5!\sum_{r=0}^{5}\dfrac{(-1)^r}{r!}$ (D) 54

Key. C

Sol. Let us consider a dummy ball numbered 5.

Case I: When it goes to box no. 5 then the required ways is same as derangement of 4 which is $4!\sum_{r=0}^{4}\dfrac{(-1)^r}{r!} = 9$

Case II: When it does not go the box no. 5 then the required ways = 5! $\sum_{r=0}^{5}\dfrac{(-1)^r}{r!} = 44$

So total no. of ways = 9 + 44 = 53

14. If X = 144, then

a) no. of divisors (including 1 and X) of X = 15

b) sum of divisors (including 1 and X) of X = 403

c) product of divisors (including 1 and X) of X = 12^{15}

d) sum of reciprocals of divisors (including 1 and X) of X = $\dfrac{403}{144}$

Key. A,B,C,D

Sol. $144 = 2^4 . 3^2$

a) no. of divisors (4+1). (2+1) = 15

b) Sum of divisors $(1+2+2^2+2^3+2^4)(1+3+3^2) = 403$

c) Product of divisors $(144)^{\frac{15}{2}} = (12)^{15}$

d) Sum of reciprocals of divisors = $\frac{\text{sum of divisiors}}{144}$ = $\frac{403}{144}$

15. Letters of the word SUDESH can be arranged in

a) 120 ways when two vowels are together

b) 180 ways when two vowels occupy in alphabetical order

c) 24 ways when vowels and consonants occupy their respective places

d) 240 ways when vowels do not occur together

Key. A,B,C,D

Sol. $(a)\ (2!)\frac{5!}{2!}$

$(b)\frac{6!}{2!}$

$(c)\frac{4!}{2!}(2!)$

$(d)360-120$

16. Let f(n) denote the number of ways in which n letters go into n envelops so that no letter is in the correct envelope, (where n>5), then $f(n)-nf(n-1)=$

a) $f(n-2)-(n-2)f(n-3)$ b) $f(n-1)-(n-1)f(n-2)$

c) $(n-3)f(n-4)-f(n-3)$ d) $(n-4)f(n-5)-f(n-4)$

Key. A,C

Sol. we know that $f(n)=(n-1)\{f(n-1)+f(n-2)\}$

17. The number of interior points that can be formed when diagonals of convex polygon of n-vertices, intersect if no three diagonals pass through the same interior point, is

a) ${}^nC_4$ b) ${}^nC_2$ c) ${}^nC_{n-4}$ d) ${}^nC_{n-2}$

Key. A,C

Sol. Each quadrilateral gives one point of intersection

18. The number of isosceles triangles with integer sides if no side exceeds 2008 is

a) $(1004)^2$ if equal sides do not exceed 1004

b) $2(1004)^2$ if equal sides exceed 1004

c) $3(1004)^2$ if equal sides have any length ≤ 2008

d) $(2008)^2$ if equal sides have any length ≤ 2008

Key. A,B,C

Sol. If the sides are a, a, b then the triangle forms only when $2a>b$ .so for any $a\varepsilon N$, b can change from 1 to 2a -1 when $a\leq 1004$ then number of triangles = 1+3+5+..+(2(1004)-1)= $(1004)^2$ and if $1005\leq a\leq 2008,$ b cam take any value from 1to 2008. but a has 1004 possibilities hence number of triangles = $1004\times 2008=2(1004)^2$

$\therefore$ Total number of isosceles triangles $=3(1004)^2$

19. Which of the following is/are true

a) $\overset{6}{5}-{}^5C_1.\overset{6}{4}+{}^5C_2.\overset{6}{3}-{}^5C_3.\overset{6}{2}+{}^5C_4.\overset{6}{1}={}^6C_2.\lfloor\underline{5}$ b)

$\overset{6}{5}-{}^6C_1.\overset{5}{5}+{}^6C_2.\overset{5}{4}-{}^6C_3.\overset{5}{3}+{}^6C_4.\overset{5}{2}-{}^6C_1.\overset{5}{1}=0$

c) $\overset{6}{6}-{}^6C_1.\overset{6}{5}+{}^6C_2.\overset{6}{4}-{}^6C_3.\overset{6}{3}+{}^6C_4.\overset{6}{2}-{}^6C_5.\overset{6}{1}=720$ d)

$\overset{5}{6}-{}^6C_1.\overset{5}{5}+{}^6C_2.\overset{5}{4}-{}^6C_3.\overset{5}{3}+{}^6C_4.\overset{5}{2}-{}^6C_5.\overset{5}{1}={}^5C_2.\lfloor\underline{6}$

Key. A,C

Sol. 1) Number of on to functions from a set containing 6 elements to a set containing 5 elements $={}^6C_2.\lfloor\underline{5}$

3) Number of on to functions from a set containing 6 elements to a set containing 6 elements $=\lfloor\underline{6}=720$

20. Let $S=\{1,2,3,.....,n\}$ and f_n be the no.of those subsets of S which do not contain consecutive elements of S, then

A) $f_n=\dfrac{n(n-1)(n-2)}{6}$ B) $f_n=2f_{n-1}$ C) $f_n=f_{n-1}+f_{n-2}$ D) $f_4=8$

Key. C,D

Sol. Let $n=4$, then the subsets of {1, 2, 3, 4} which do not contain consecutive elements of this set and $\phi,\{1\},\{2\},\{3\},\{4\},\{1,3\},\{1,4\},\{2,4\}$

$f_4=8$ similarly $f=3,\ f_3=5$

21. The cube of any whole number when divided by 9 may yield the reminder

A) 0 B) 2 C) 1 D) 8

Key. A,C,D

Sol. Any whole number is either 9K, 9K+1, ……, 9K+8

When we cube them reminder will be 0, 1 or 8 only

22. The no.of integers from 1 to 10^5 which contain exactly one 3, exactly one 4 and exactly one 5 must be

A) more than 2000 B) more than 3000 C) 2940 D) 3270

Key. A,C

Sol. Required number is $5\times4\times3\times7\times7=2940$

23. If p,q,r,s,t be distinct primes and $N=pq^2r^3st$, then

A) N has 96 divisors

B) N can be written as a product of two positive integers in 96 ways

C) N can be written as a product of two positive integers in 48 ways

D) N can not be divisible by 13!

Key. A,C,D

Sol. No.of divisors $=(1+1)(2+1)(3+1)(1+1)(1+1)=96$

Since there are 6 primes which are ≤ 13 and N contain only five distinct primes, N can not be div. By $13!$.

24. Which of the following will not be true

a) The last two digits of 3^{100} will be 73 b) The last two digits of 3^{50} will be 51

c) The last two digits of 3^{50} will be 49 d) The last three digits of 3^{50} will be 249

Key. A,B

Sol. $3^{100}=9^{50}=\left(10-1\right)^{50}=10^{50}-{}^{50}C_1 10^{49}+.........{}^{50}C_{49}10+{}^{50}C_{50}$

= a multiple of 100 + 1

$3^{50} = (10-1)^{25} = 10^{25} - {}^{25}C_1(10)^{24} + ... + {}^{25}C_{24}10 - {}^{25}C_{25}$

= a multiple of 100 + 249

25. 10 distinct balls are arranged in a row. The number of ways of selecting three of these balls so that no two of them are next to each other is

(A) $\frac{1}{6} \times 8 \times 7 \times 6$ (B) ${}^8C_3$

(C) ${}^7C_3 + {}^7C_2$ (D) none of these

Key. A,B,C

Sol. Required number of ways = $\frac{1}{6}(10-20)(10-3)(10-4)$

$\frac{1}{6} \times 8 \times 7 \times 6 = {}^8C_3$

26. The no. of words formed with or without meaning, each of 3 vowels and 2 consonants from the letters of the word INVOLUTE is written in the form of $2^a.3^b.5^c.7^d$ then

a) $a=6$ b) $b=2$ c) $c=1$ d) d=0

Key. A,B,C,D

Sol. Number of ways selecting 3 vowels and 2 consonants and arranging them is

${}^4C_3.{}^4C_2.5! = 2^6.3^2.5^1$

27. Triangles are formed by joining vertices of a octagon then number of triangle

(A) In which exactly one side common with the side of octagon is 32

(B) In which atmost one side common with the side of polygon is 48

(C) At least one side common with the side polygon 50

(D) Total number of triangle 56

Key. A,B,D

Sol. Total number of triangle = ${}^8C_3$ = 56

Number of triangle having exactly one side common with the polygon = 8×4 = 32

Number of triangle having exactly two side common with the polygon = 8

Number of triangle having no side common with the polygon = 16

28. The letters of the word "ARRANGE" are arranged in all possible ways. Let m be the number of arrangements in which the two A's are together and the two R's are not together and n be the number of arrangements in which neither the two A's nor the two R's are together. Then

a) m + n = 900 b) m + n = 1260 c) n – m = 780 d) n – m = 420

Key. A,D

Sol. $m = 240$ & $n = 660$

29. Suppose $A_1, A_2, ..., A_{20}$ are the vertices of a 20-sided regular polygon. Triangles with vertices among the vertices of the polygon are formed. Let m be the number of non-isosceles (Scalene) triangles that can be formed one of whose sides is a side of the polygon and n be the number of non-isosceles triangles that can be formed none of whose sides is a side of the polygon. Then

a) n =2m b) m + n = 960 c) m + n = 500 d) n – m = 320

Key. A,B,D

Sol. Number of isosceles triangles $= 20 \times 9 = 180$

$m = 20 \times 16 = 320$

$n = 20C_3 - (180 + 320) = 640$

30. If p is an odd prime number, then $f(p) = (2p-1)C_{(p-1)} - 1$ is divisible by

a) $p-1$ b) p c) p^2 d) $p+1$

Key. B,C

Sol. $\sum_{r=1}^{n} (pC_r)^2 = 2f(p) \Rightarrow p^2 / 2f(p) \Rightarrow p^2 / f(p)$

31. Consider the set of all positive integers n for which $f(n) = n! + (n+1)! + (n+2)!$ is divisible by 49.

a) The number of integers n in (1, 15) is 3 b) The number of integers n in (5, 17) is 4

c) The number of integers n in (1, 20) is 8 d) The number of integers n in (1, 20) is 9

Key. A,B,C

Sol. $f(n) = (n!)(1 + n + 1 + (n+1)(n+2)) = (n+2)^2 (n!)$

$49 / f(n) \Rightarrow 7 / n+2 \text{ or } 49 / n!$

32. The sum of all three digited numbers that can be formed from the digits 1 to 9 and when the middle digit is perfect square is

a) 1,34,055 (When repetitions are allowed) b) 1,70,555 (When repetitions are allowed)

c) 8,73,74 (When repetitions are not allowed) d) 93,387 (When repetitions are not allowed)

Key. A,D

Sol. When repetitions are not allowed

$^7p_1(101)\left(\sum 9 - 1\right) + {}^8p_2 \times 10 + {}^7p_1(101)\left(\sum 9 - 4\right) + {}^8p_2 \times 40$ +

$^7p_1(101)\left(\sum 9 - 9\right) + {}^8p_2 \times 90$ = 93,387

Permutation & Combination

Assertion Reasoning Type

a) Statement –1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for Statement – 1

b) Statement –1 is True, Statement – 2 is True; Statement – 2 is NOT a correct explanation for Statement – 1

c) Statement –1 is True, Statement – 2 is False

d) Statement –1 is False, Statement – 2 is True

1. $K \in R, n \in N$ and r is a whole number such that $n \ge r$

STATEMENT – 1: If $(n-1)_{C_r} = (K^2-3)n_{C_{r+1}}$ then the only integer value of K is 2

Because

STATEMENT – 2: $0 < \frac{(n-1)_{C_r}}{n_{C_{r+1}}} = \frac{r+1}{n} \le 1$

KEY : D

2. STATEMENT – 1 : $\left((106)^{85}+(155)^{50}\right)-\left((50)^{155}+(85)^{106}\right)$ is divisible by 7

Because

STATEMENT – 2 : 105,154,49,84 are divisible by7

KEY : A

3. Assertion (A) : The number of ways of arranging the letters of the word ASSOCIATION such that the two S's come together and two I' s are not together is $\frac{8!}{2!2!} \times {}^9P_2$

Reason (R) : If in the given n things p things are alike of one kind and q things are alike of second kind, then the number of ways of arranging all the n things is $\frac{n!}{p!q!}$

Key. D

Sol. That two S's as one unit and keep I's away. We can arrange them in $\frac{8!}{2!2!}$ ways

Now there 9 place and we can arrange the two I's is then 9 places it ways. Thus answer is $\frac{8!}{2!2!} \times {}^9C_2$

Assertion is wrong . But reason is correct

4. Assertion (A) : The number of divisors of the number 18225×10^5 including 1 and the given number is 336

Reason (R) : If p things are alike of one kind, q things are alike of second kind and r things are alike of third kind, then the number of ways of selecting any number of things (including no item) out of them is $(p+1)(q+1)(r+1)$

Key. A

Sol. $6^2 \times 10^3 \times 15^4 = 2^5 \times 3^6 \times 5^7$

Therefore the number of divisors = $(5+1)(6+1)(7+1)=336$

Assertion is correct Reason explains is

5. Statement-1 : The number of ways of partitioning the set $\{a,b,c,d\}$ into one or more non empty subsets is 16. Because

Statement-2 : The number of ways of partitioning a set of $(m+n)$ members into two subsets of m and n members is ${}^{m+n}C_m$ if $m \neq n$ and $\frac{1}{2}\,{}^{2m}C_m$ if $m=n$

Key. D

Sol.

Partitioning	Number of ways
4 members	1
1 + 3 members	4
2 + 2 members	3
1 + 1 + 2 members	6
1 + 1 + 1 +1 members 1	
	15 ways

6. Statement-1 : Let N be the number of 3-digit numbers with distinct digits so that the digits in any number are neither increasing nor decreasing order. Then the sum of the divisors of N is 1064

Statement-2 : If p_1, p_2, p_3 are distinct primes, then the sum of the divisors of $N = p_1^{\alpha_1}\cdot p_2^{\alpha_2}\cdot p_3^{\alpha_3}$ is $\frac{\left(p_1^{\alpha_1+1}-1\right)\left(p_2^{\alpha_2+1}-1\right)\left(p_3^{\alpha_3+1}-1\right)}{(p_1-1)(p_2-1)(p_3-1)}$

Key. A

Sol. $N = 9\cdot 9\cdot 8 - {}^{9}C_3 - {}^{10}C_3 = 648-84-120$

$= 444 = 2^2\cdot 3^1\cdot 37^1$

$$\frac{(2^3-1)(3^2-1)(37^2-1)}{(2-1)(3-1)(37-1)} = 7\cdot 4\cdot 38 = 1064$$

7. STATEMENT-I: $n^n - {}^nc_1(n-1)^n + {}^nc_2(n-2)^n - {}^nc_3(n-3)^n + ... + (-1)^{n-1}\,{}^nc_{n-1} = n!$

STATEMENT-II: If A and B have the same number of elements then No. of onto functions from A to B = No.of one – one functions from A to B.

Key. A

Sol. No. of onto functions from a set of n elements to a set of r elements.

$= r^n - {}^rc_1(r-1)^n + {}^rc_2(r-2)^n + {}^rc_3(r-3)^n + ----- + (-1)^{r-1}\,{}^rc_{r-1}$

No. of one-one functions from a set of n elements to another set of n elements = n!

$\therefore$ Ans = A.

8. STATEMENT-I: If P is a natural number having number of divisors (including unity and P) equal to 105 then $\{\sqrt{P}\}=0$ where $\{x\}$ stands for fractional part of x.

STATEMENT-II : $2^2.3^4.5^6$ is one of such numbers P.

Key. B

Sol. If $P = a^x.b^y.c^z ---$, where *a,b,c* etc are prime factors, then we know that no. of divisors of $P = (x+1).(y+1).(z+1)---$ etc = 105.

$\Rightarrow x+1, y+1, z+1, ---$ all must be odd

$\Rightarrow x, y, z, ---$ all must be even

$\Rightarrow P$ is a perfect square

$\therefore$ Statement-I is true.

Statement-II is also true, but it is not the correct explanation.

9. STATEMENT -1: The number of selections of four letters from the letters of word PARALLEL is 15.

STATEMENT-2: Coefficient of x^2 in the expansion of $(1+x)^6$ is 15

Key. A

Sol. The number of divisors of $1400 = (3+1)(2+1)(1+1) = 24$

$\therefore$ No. of ways of writing as product of two numbers $= \frac{24}{2} = 12$

10. STATEMENT–1:

The number of positive integral solutions of the equation $x_1x_2x_3x_4x_5 = 1050$ is 1875.

because

STATEMENT–2:

The total number of divisor of 1050 is 25.

Key. C

Sol. $x_1x_2x_3x_4x_5 = 1050 = 2 \times 3 \times 5^2 \times 7$

Thus 5^2 can as sign in ${}^5C_1 + {}^5C_2 = 15$ ways

We can assign 2, 3, or 7 to any. of 5 variables.

Hence req. number of solutions. $= 5 \times 5 \times 5 \times 15 = 1875$

11. Statement – 1 : The number of selections of four letters taken from the word PARALLEL must be 15

Because

Statement – 2 : Coefficient of x^4 in the expansion of $(1-x)^{-3}$ is 15 $(|x|<1)$

Key. D

Sol. $1'p,\ 2'A, 1R, 3'L, 1E$

4 diff : $5c_4 = 5$

3 alike of 1 kind & 1 diff = $1c_1.4c_1 = 4$

2 alike of 1 kind & 2 diff = $2c_1.4c_2 = 2.6 = 12$

2 alike of 1 kind & 2 diff of 2nd kind = $2c_2 = 1$

Total = 22

12. Statement – 1 : If $f:\{1,2,3,4,5\} \rightarrow \{1,2,3,4,5\}$ then the number of onto functions such that $f(i) \neq i$ is 42

Statement – 2 : If n things are arranged in row, the number of ways in which they can be de-arranged so that no one of them occupies its original place is

$$n!\left(1-\frac{1}{1!}+\frac{1}{2!}+.......(-1)^n\frac{1}{n!}\right)$$

Key. D

Sol. Conceptual

13. Statement – 1 : Number of ways of distribution of 12 identical balls into 3 identical boxes is 19

Because

Statement – 2 : Number of ways of distribution of *n* identical objects among r persons, each one of whom can receive any number of objects is $n+r-1\ c_{r-1}$

Key. B

Sol. Total 12 identical in 3 distinct

$$12+3-1_{C_{3-1}}=91 \qquad ie.(x+y+z=91)$$

Case (i) When each box contains equal number

$$x=y=z=4=1way$$

Case (ii) When two boxes contains equal number

$$2x+z=12\Rightarrow(x=6,z=0)(x=5,z=2),(x=3,z=6)$$

$$(x=2,z=8)(x=1,z=10),(x=0,z=12)$$

$$3c_2.6=18 \text{ ways } =\frac{18}{\left(\frac{3!}{2!}\right)}=6\,ways$$

Case (iii) distinct number

$$\text{Total }-(1+18)=72=\frac{72}{3!}=12$$

$$\therefore Total=1+6+12=19$$

14. Assertion (A): If a,b,c are positive integers such that $a+b+c\le 8$, then the no.of possible values of the ordered triplets (a,b,c) is 56.

Reason (R): The no.of ways in which *n* identical things can be distributed into *r* different groups is ${}^{n-1}C_{r-1}$.

Key. C

Sol. Conceptual

15. Statement-1: If n is the odd, integer number of ways in which three numbers are AP can be selected from 1, 2, 3, ………. n is $\frac{(n-1)^2}{4}$

Statement-2: AM of two odd numbers or two even number is an integer.

Key. A

Sol. by selecting two odd numbers or two even numbers let (a, c), we will have an AP $a, \frac{a+c}{2}, c$

$1,3,5,......., n \to \frac{n+1}{2}$ odd no $\Rightarrow \frac{n+1}{^2C_2}$ ways to select two no's

$2,4,6,......., n-1 \to \frac{n-1}{2}$ even no's $\Rightarrow \frac{n-1}{^2C_2}$ ways to select two no's

Total no.of AP's $\frac{n+1}{^2C_2} + \frac{n-1}{^2C_2}$

16. Statement-1: The number of divisors of 10! is 270

Statement-2: The number of divisors of $(p_1)^{\alpha_1}(p_2)^{\alpha_1}(p_3)^{\alpha_3}$ are $(\alpha_1+1)(\alpha_2+1)(\alpha_3+1)$ where p_1, p_2, p_3 are prime integers and $\alpha_1, \alpha_2, \alpha_3$ natural numbers.

Key. C

Sol. $10! = 2^8\, 3^4\, 5^2 > 1$ ➔ no. of division = 9 x 5 x 3 x 2 = 270

17. Statement-1: If maths books have 4 different volumes each volume have 5 copies and physics book have 5 different volumes each have 4 copies then total no. of selecting at least one book of each subject is $(6^4 - 1)(5^5 - 1)$

Statement-2: If p things are alike of one kind, q things are alike of other kind then no.of ways of selecting at least one of each kind is p.q.

Key. B

Sol. $5M_1, 5M_2, 5M_3, 5M_4, 4P_1, 4P_2, 4P_3, 4P_4, 4P_5$

no.of ways of selecting at least one maths is 6.6.6.6 – 1

no.of ways of selecting atleast one plysics book is 5.5.5.5.5 – 1

18. STATEMENT-1 : The number of ways of selecting 5 students from 12 students (of which six are boys and six are girls), such that in the selection there are at least three girls is $^6C_3 \times {}^9C_2$.

STATEMENT-2 : If a work has two independent parts, of which first can be done in m way, and for each choice of first part, the second part can be done in n ways, then the work can be completed in $m \times n$ ways.

Key. D

Sol. Reason (R) is true, known as the rule of product.

Assertion (A) is not true as the two parts of the work are not independent. Three girls can be chosen out of six girls in $^6C_3$ ways, but after this choosing 3 students out of remaining nine students depends on the first part.

19. STATEMENT-1: The number of ways of arrangement of n boys and n girls in a circle such that no two boys are consecutive, is $((n-1)!)^2$.

STATEMENT-2 : The number of ways of arrangement n distinct objects in a circle is $(n-1)!$.

Key. D

Sol. Reason (R) is true as on fixing one object anywhere in the circle, the remaining $n-1$ objects can be arranged in $(n-1)!$ ways.

Reason(R) is false, as after arranging boys on the circle in $(n-1)!$ ways, girls can be arranged in between the boys in $n!$ ways (for any arrangement of boys).

Hence, number of arrangement is $n!(n-1)!$.

Permutation & Combination

Comprehension Type

Paragraph – 1

10-digit numbers are formed by using all the digits 0,1,2,3,4,5,6,7,8 and 9 such that they are divisible by 11111**.**

1. The digit in the ten's place, in the smallest of such numbers, is

A) 9 B) 8 C) 6 D) 7

Key. C

2. The digit in the unit's place, in the greatest of such numbers, is

A) 2 B) 3 C) 4 D) 1

Key. C

3. The total number of such numbers is

A) 3456 B) 5634 C) 6543 D) 4365

Key. A

Sol. Let a b c d e f g h i j be one of such numbers where a b c d e f g h i j is some permutation of the digits 0,1,2,3,4,5,6,7,8,9 where $a \neq 0$.

Sum of digits of the number = 0+1+2+3+4+5+6+7+8+9 = 45, which is divisible by 9 and hence the number is divisible by 9. But it is divisible by 11,111 also and 11,111 is not divisible by 3 or 9. Therefore, the number is divisible by $11,111 \times 9 = 99,999$.

And a b c d e f g h i j = a b c d e $\times$ 10^5 + f g h i j

= a b c d e $\times$ (99,999+1) + f g h i j

= a b c d e $\times$ 99,999 + a b c d e + f g h i j is divisible by 99,999.

$\Rightarrow$ a b c d e + f g h i j is divisible by 99,999.

But a b c d e < 99,999

And f g h i j < 99,999

$\Rightarrow$ a b c d e + f g h i j < 2 $\times$ 99,999

$\therefore$ a b c d e + f g h i j = 99,999

$\Rightarrow$ e + j = d + i = c + h = b + g = a + f = 9

1.

a	b	c	d	e	f	g	h	i	j
1	0	2	3	4	8	9	7	6	5

For smallest number a must be 1 (since a can not be 0) and hence f = 8.

Then, b = 0 $\Rightarrow$ g = 9

Then, c = 2 $\Rightarrow$ h = 7

Then, d = 3 $\Rightarrow$ i = 6

Then, e = 4 $\Rightarrow$ j = 5

` $\therefore$ The smallest of such numbers is 1023489765 and the digit in the ten's place is 6.

2.

a	b	c	d	e	f	g	h	i	j

9	8	7	6	5	0	1	2	3	4

For greatest number a = 9 $\Rightarrow$ f = 0

Then, b = 8 $\Rightarrow$ g = 1

Then, c = 7 $\Rightarrow$ h = 2

Then, d = 6 $\Rightarrow$ i = 3

Then, e = 5 $\Rightarrow$ j = 4

$\therefore$ The greatest of such numbers is 9876501234 and the digit in the units place is 4.

3.

a	b	c	d	e	f	g	h	i	j
9	8	6	4	2	1	1	1	1	1

The blank 'a' can be filled in 9 ways (except 0).

Then blank f can be filled in only one way (by 9-a).

Now, blank 'b' can be filled by any of the remaining 8 digits.

Then blank 'g' can be filled in only one way (by 9-b)

Now, blank 'c' can be filled by any of the remaining 6 digits.

Then blank 'h' can be filled in only one way (by 9-c).

Now, blank 'd' can be filled by any of the remaining 4 digits.

Then blank 'i' can be filled in only one way (by 9-d).

Now, blank 'e' can be filled by any of the remaining 2 digits.

Then blank 'j' can be filled in only one way (by 9-e).

$\therefore$ The total number of such numbers = $9\times1\times8\times1\times6\times1\times4\times1\times2\times1$

= 3456.

Paragraph – 2

A is a set containing n elements. A subset S_1 of A is chosen. The set A is reconstructed by replacing the elements of S_1. Again, a subset S_2 of A is chosen and again the set is reconstructed by replacing the elements of S_2. The number of ways of choosing S_1 or S_2 where

4. S_1 and S_2 have one element common is

(A) 3^{n-1} (B) $n \cdot 3^{n-1}$

(C) 2^{n-1} (D) n

KEY : A

SOL : Required number of ways = ${}^nC_1 \cdot (3)^{n-1}$

5. $S_1 \cup S_2 = A$ is

(A) 3^n (B) $n \cdot 3^n$

(C) 4^n (D) 4^{n-1}

KEY : A

SOL : Each element $\in S_1 \cup S_2$ in 3 ways

6. S_1 is a subset of S_2 is

(A) 4^{n-1} (B) 3^{n+1}

(C) 4^n (D) 3^n

KEY : B

SOL : If S_2 has r elements then S_1 and S_2 can be choosen in ${}^{n}C_{r}\, 2^{r}$ ways.

Paragraph – 3

Define a function $\phi : N \to N$ as follows : $\phi(1) = 1, \phi\left(P^{n}\right) = P^{n-1}(P-1)$ if P is prime and $n \in N$ and $\phi(mn) = \phi(m)\phi(n)$ if m & n are relatively prime natural numers.

7. $\phi(8n+4)$ where $n \in N$ is equal to

(A) $\phi(4n+2)$ (B) $\phi(2n+1)$

(C) $2\phi(2n+1)$ (D) $4\phi(2n+1)$

Key: C

Hint: $Q(1) = 1, \theta\left(P^{n}\right) = P^{n-1}(p-1), \phi(mn) = \phi(m).\phi(n)$

$$\phi(8n+4) = \phi(4(2n+1)) = \phi(4).\phi(2n+1)$$

$$= \phi\left(2^{2}\right).\phi(2n+1)$$

$$= 2.\phi(2n+1)$$

8. The number of natural numbers 'n' such that $\phi(n)$ is odd is

(A) 1 (B) 2

(C) 3 (D) 4

Key: B

Hint: $\phi(n)$ is odd.

$\Rightarrow \phi\left(p^{n}\right)$ is odd

$\Rightarrow P^{n-1}(P-1)$ is odd

$\therefore$ p is prime. The only value p can take is P = 2

$\therefore \phi\left(2^{n}\right)$ is odd

$\Rightarrow 2^{n-1}(2-1) = 2^{n-1}$ is odd

$\Rightarrow n-1 = 0$

$\Rightarrow n = 1$

$\therefore \phi(1) = 1 = \phi(2)$

9. If $\phi\left(7^{n}\right) = 2058$ where $n \in N,$ then the value of n is

(A) 3 (B) 4

(C) 5 (D) 6

Key: B

Hint: $\phi\left(7^{n}\right) = 2058$

$7^{n-1}(7-1) = 2058$

$7^{n-1} = 343$

$n-1 = 3$

$n = 4$

Paragraph – 4

If a set A has n elements then the number of subsets of A containing exactly r elements is $^{n}C_{r}$ *. The number of all subsets of A is* 2^{n}*. Now answer the following questions. A set A has 7 elements. A subset P of A is selected. After noting the elements they are placed back in A. Again subset Q is selected. Then the number of ways of selecting P and Q such that*

10. P, Q have no common element is

A) 2835 B) 128 C) 3432 D) 2187

Key. D

11. P and Q have exactly 3 elements is common is

A) 2835 B) 128 C) 3432 D) 2187

Key. A

12. P and Q have equal number of elements is (P and Q may be null sets)

A) 2835 B) 128 C) 3432 D) 2187

Key. C

Sol. 10. Each of the 7 elements have 3 choice. They are

$x \in P \text{ and } x \notin Q$

$\text{or } x \notin P \text{ and } x \in Q$

$\text{or } x \notin P \text{ and } x \notin Q$

For each element there are three of above choices

Hence the number of ways of selection P or Q is $= 3^7$ = 2187

11. As in both sets three elements are common so, three elements can be choosen in ${}^7C_3$ ways . And rest of the elements can be choosen in any of the above three ways in 3^4 ways. So, total number of ways = ${}^7C_3 \times 3^4$=2835

12. Let P & Q have same number of elements i.e $\mathrm{r}\left(0 \le \mathrm{r} \le 7\right)$

$\Longrightarrow$ Total number of ways in which P & Q have same number of elements

$$= \sum_{r=0}^{7} {}^7C_r \cdot {}^7C_r = \left({}^7C_0\right)^2 + \left({}^7C_1\right)^2 + \left({}^7C_2\right)^2 + \cdots\cdots + \left({}^7C_7\right)^2$$

= 3432

Paragraph – 5

10-digit numbers are formed by using all the digits 0,1,2,3,4,5,6,7,8 and 9 such that they are divisible by 11111.

13. The digit in the ten's place, in the smallest of such numbers, is

A) 9 B) 8 C) 7 D) 6

Key. D

14. The digit in the unit's place, in the greatest of such numbers, is

A) 4 B) 3 C) 2 D) 1

Key. A

15. The total number of such numbers is

A) 6543 B) 5634 C) 3456 D) 4365

Key. C

Sol. Let a b c d e f g h i j be one of such numbers where a b c d e f g h i j is some permutation of the digits 0,1,2,3,4,5,6,7,8,9 where $a \neq 0$.

Sum of digits of the number = 0+1+2+3+4+5+6+7+8+9 = 45, which is divisible by 9 and hence the number is divisible by 9. But it is divisible by 11,111 also and 11,111 is not divisible by 3 or 9. Therefore, the number is divisible by $11,111 \times 9 = 99,999$.

And a b c d e f g h i j = a b c d e $\times$ 10^5 + f g h i j

= a b c d e $\times$ (99,999+1) + f g h i j

= a b c d e $\times$ 99,999 + a b c d e + f g h i j is divisible by 99,999.

$\Rightarrow$ a b c d e + f g h i j is divisible by 99,999.

But a b c d e < 99,999

And f g h i j < 99,999

$\Rightarrow$ a b c d e + f g h i j < 2 $\times$ 99,999

$\therefore$ a b c d e + f g h i j = 99,999

$\Rightarrow$ e + j = d + i = c + h = b + g = a + f = 9

13.

a	b	c	d	e	f	g	h	i	j
1	0	2	3	4	8	9	7	6	5

For smallest number a must be 1 (since a can not be 0) and hence f = 8.

Then, b = 0 $\Rightarrow$ g = 9

Then, c = 2 $\Rightarrow$ h = 7

Then, d = 3 $\Rightarrow$ i = 6

Then, e = 4 $\Rightarrow$ j = 5

` $\therefore$ The smallest of such numbers is 1023489765 and the digit in the ten's place is 6.

14.

a	b	c	d	e	f	g	h	i	j
9	8	7	6	5	0	1	2	3	4

For greatest number a = 9 $\Rightarrow$ f = 0

Then, b = 8 $\Rightarrow$ g = 1

Then, c = 7 $\Rightarrow$ h = 2

Then, d = 6 $\Rightarrow$ i = 3

Then, e = 5 $\Rightarrow$ j = 4

$\therefore$ The greatest of such numbers is 9876501234 and the digit in the units place is 4.

15.

a	b	c	d	e	f	g	h	i	j
9	8	6	4	2	1	1	1	1	1

The blank 'a' can be filled in 9 ways (except 0).

Then blank f can be filled in only one way (by 9-a).

Now, blank 'b' can be filled by any of the remaining 8 digits.

Then blank 'g' can be filled in only one way (by 9-b)

Now, blank 'c' can be filled by any of the remaining 6 digits.

Then blank 'h' can be filled in only one way (by 9-c).

Now, blank 'd' can be filled by any of the remaining 4 digits.

Then blank 'i' can be filled in only one way (by 9-d).

Now, blank 'e' can be filled by any of the remaining 2 digits.

Then blank 'j' can be filled in only one way (by 9-e).

$\therefore$ The total number of such numbers = $9\times1\times8\times1\times6\times1\times4\times1\times2\times1$

= 3456.

Paragraph – 6

Two numbers x and y are drawn without replacement from the set of the first 15 natural numbers. The number of ways of drawing them such that

16. x^3+y^3 is divisible by 3

A) 21 B) 33 C) 35 D) 69

Key. C

17. x^2-y^2 is divisible by 5

A) 21 B) 33 C) 35 D) 69

Key. B

18. x^4-y^4 is divisible by 5

A) 57 B) 64 C) 69 D) 72

Key. C

Sol. 16. The natural numbers are written in rows

1	4	7	10	13
2	5	8	11	14
3	6	9	12	15

x^3+y^3 is divisible by 3 if and only if and only if $x+y$ is divisible by 3. The numbers x and y are taken one from row 1 and other row 2 or both from row 3.

The desired number is $^5C_1\times{}^5C_1+{}^5C_2=25+10=35$

17. The numbers are written is rows

1	6	11
2	7	12
3	8	13
4	9	14
5	10	15

$x^2-y^2=(x+y)(x-y)$ is divisible by $5\Rightarrow$ both x and y are from any of these rows or one from row 1 and the other from row 4 or one from row 2 and other from row $3\Rightarrow$ desired number is

$$5\times{}^3C_2+2\left({}^3C_1\right)^2=15+18=33$$

18. $x^4-y^4=(x^2+y^2)(x-y)(x+y)$ is divisible by $5\Rightarrow$ Both x and y are from any one row or one from any one row and the other from other row of the first 4 rows. The desired number is $5\times{}^3C_2+6\left({}^3C_1\right)^2=15+54=69$

Paragraph – 7

Let A, B, C, D, E be the smallest positive integers having 10, 12, 15, 16, 20 positive divisors respectively. Then

19. A + B =

A) 108 B) 110 C) 126 D) 130

Key. A

20. C + D =

A) 350 B) 354 C) 380 D) 420

Key. B

21. A + E =

A) 288 B) 320 C) 350 D) 380

Key. A

Sol. $10 = 2 \cdot 5 \Rightarrow A = 2^4 \cdot 3 = 48$

$12 = 2 \cdot 2 \cdot 3 \Rightarrow B = 2^2 \cdot 3 \cdot 5 = 60$

$15 = 3 \cdot 5 \Rightarrow C = 2^4 \cdot 3^2 = 144$

$16 = 2 \cdot 2 \cdot 2 \cdot 2 \Rightarrow D = 2 \cdot 3 \cdot 5 \cdot 7 = 210$

$20 = 2 \cdot 2 \cdot 5 \Rightarrow E = 2^4 \cdot 3 \cdot 5 = 240$

19. A + B = 48 + 60 = 108

20. C + D = 144 + 210 = 354

21. A + E = 48+240 = 288

Paragraph – 8

A square of n units by n units is divided into n^2 squares each of area 1 sq. unit, by horizontal and vertical lines.

22. Total no. of shortest ways to reach from the corner to opposite corner along horizontal and vertical of square, equal to

(A) $\frac{2n!}{n!n!}$ (B) $\frac{2(n+1)!}{n!n!}$

(C) $\frac{(2n+2)!}{(n+1)!(n+1)!}$ (D) $\frac{(2n+2)!}{(n+1)!(n-1)!}$

Key. A

23. No. of ways in which four points out of total points formed by intersection of horizontal and vertical lines, can be selected to form a square is

(A) $\frac{n^2(n+1)^2}{2}$ (B) $\frac{n(n+1)^2}{2}$

(C) $\frac{n^2(n+1)}{2}$ (D) $\frac{n(n+1)}{2}$

Key. C

24. No. of squares having its sides horizontal are

(A) $\sum n^3$ (B) $\sum n$

(C) $\sum n^2$ (D) $\sum n^4$

Key. C

Sol. 22. (A)

We have to travel n horizontal and n vertical units which can be selected in $\frac{2n!}{n!n!}$ ways

23. (C)
Total no. of points well be $(n + 1)^2$
No. of horizontal lines = n + 1
No. of vertical lines = n + 1
24. (C)
Total no. of such squares = $1^2 + 2^2 + 3^2 + \ldots\ldots + n^2$

Paragraph – 9

If $x_1 + x_2 + x_3 \ldots.. + x_r = n$

Then number of solutions of equation ${}^{n+r-1}C_n$ when x_i are (i = 1, 2, 3 …. r) non-negative integers and ${}^{n-1}C_{r-1}$ when x_i are (i = 1, 2, 3 …. r) positive integers

25. If a, b, c be three natural numbers in A.P. then number of solution of a + b + c = 21 is
(A) 15 (B) 14
(C) 13 (D) 16

Key. C

26. Number of ways of distributing 22, identical toys among 4 children when each child must get odd number of toys is equal to
(A) ${}^{8}c_3$ (B) ${}^{12}c_9$
(C) ${}^{21}c_3$ (D) ${}^{25}c_{22}$

Key. B

Sol. 25. as a, b, c are in A.P. $b = \frac{a+c}{2}$

$(a+c) + \frac{a+c}{2} = 21$

$\Rightarrow$ a + c = 14

$\Rightarrow$ number of solution = 13

26. $x_1 + x_2 + x_3 + x_4 = 22$
$x_i = 2n_i + 1$
$x_i \in I^+ \cup \{0\}$
i = 1,2,3,4
$n_1 + n_2 + n_3 + n_4 = 9$

number of solutions = $^{12}C_9$.

Paragraph – 10

10-digit numbers are formed by using all the digits 0,1,2,3,4,5,6,7,8 and 9 such that they are divisible by 11111.

27. The digit in the ten's place, in the smallest of such numbers, is
a) 9 b) 8 c) 7 d) 6

Key. D

28. The digit in the unit's place, in the greatest of such numbers, is
a) 4 b) 3 c) 2 d) 1

Key. A

29. The total number of such numbers is
a) 6543 b) 5634 c) 3456 d) 4365

Key. C

Sol. Let a b c d e f g h i j be one of such numbers where a b c d e f g h i j is some permutation of the digits 0,1,2,3,4,5,6,7,8,9 where $a \neq 0$.

Sum of digits of the number = 0+1+2+3+4+5+6+7+8+9 = 45, which is divisible by 9 and hence the number is divisible by 9. But it is divisible by 11,111 also and 11,111 is not divisible by 3 or 9. Therefore, the number is divisible by $11,111 \times 9 = 99,999$.

And a b c d e f g h i j = a b c d e $\times$ 10^5 + f g h i j
= a b c d e $\times$ (99,999+1) + f g h i j
= a b c d e $\times$ 99,999 + a b c d e + f g h i j is divisible by 99,999.

$\Rightarrow$ a b c d e + f g h i j is divisible by 99,999.

But a b c d e < 99,999

And f g h i j < 99,999

$\Rightarrow$ a b c d e + f g h i j < 2 $\times$ 99,999

$\therefore$ a b c d e + f g h i j = 99,999

$\Rightarrow$ e + j = d + i = c + h = b + g = a + f = 9

27.

a	b	c	d	e	f	g	h	i	j
1	0	2	3	4	8	9	7	6	5

For smallest number a must be 1 (since a can not be 0) and hence f = 8.

Then, b = 0 $\Rightarrow$ g = 9

Then, c = 2 $\Rightarrow$ h = 7

Then, d = 3 $\Rightarrow$ i = 6

Then, e = 4 $\Rightarrow$ j = 5

` $\therefore$ The smallest of such numbers is 1023489765 and the digit in the ten's place is 6.

28.

a	b	c	d	e	f	g	h	i	j
9	8	7	6	5	0	1	2	3	4

For greatest number a = 9 $\Rightarrow$ f = 0

Then, b = 8 $\Rightarrow$ g = 1

Then, c = 7 $\Rightarrow$ h = 2

Then, d = 6 $\Rightarrow$ i = 3

Then, e = 5 $\Rightarrow$ j = 4

$\therefore$ The greatest of such numbers is 9876501234 and the digit in the units place is 4.

29.

a	b	c	d	e	f	g	h	i	j
9	8	6	4	2	1	1	1	1	1

The blank 'a' can be filled in 9 ways (except 0).
Then blank f can be filled in only one way (by 9-a).
Now, blank 'b' can be filled by any of the remaining 8 digits.
Then blank 'g' can be filled in only one way (by 9-b)
Now, blank 'c' can be filled by any of the remaining 6 digits.
Then blank 'h' can be filled in only one way (by 9-c).
Now, blank 'd' can be filled by any of the remaining 4 digits.
Then blank 'i' can be filled in only one way (by 9-d).
Now, blank 'e' can be filled by any of the remaining 2 digits.
Then blank 'j' can be filled in only one way (by 9-e).
$\therefore$ The total number of such numbers = $9\times1\times8\times1\times6\times1\times4\times1\times2\times1$
= 3456.

Paragraph – 11

D_1, D_2,-----,D_{1000} are 1000 doors and P_1, P_2,-------P_{1000} are 1000 persons. Initially all the doors are closed. P_1 opens all the doors. Then, P_2 closes D_2, D_4, D_6---- D_{998}, D_{1000}. Then P_3 changes the status of
D_3, D_6, D_9, D_{12},-----etc.(doors having numbers which are multiples of 3). Changing the status of a door means closing it if it is open and opening it if it is closed. Then P_4 changes the status of D_4, D_8, D_{12}, D_{16},-----etc (doors having numbers which are multiples of 4). And so on until lastly P_{1000} changes the status of D_{1000}.

30. Finally, how many doors are open?
a) 30 b) 31 c) 32 d) 33

Key. B

31. What is the greatest number of consecutive doors that are closed finally?
a) 56 b) 58 c) 60 d) 62

Key. C

32. The door having the greatest number that is finally open is
a) D_{960} b) D_{961} c) D_{962} d) D_{963}

Key. B

Sol. 30. Consider any door, for example, D_{72}. It is operated by
$P_1,P_2,P_3,P_4,P_6,P_8,P_9,P_{12},P_{18},P_{24},P_{36},P_{72}$, (Remember that D_m is operated by P_n if m is a multiple of n)
Here 1,2,3,4,6,8,9,12,18,24,36,72 are all the factors of 72. Initially all the doors are closed. Therefore, if odd numbers of persons operate it, it will be finally open. Otherwise it will be closed finally.
$\therefore D_m$ will be finally open, if m has an odd number of factors. And, we know that m has an odd number of factors if and only if m is a perfect square.
$\therefore$ $1^2, 2^2, 3^2, 4^2$,------, 31^2 are the numbers of the doors that are open finally.
$\therefore$ No. of doors finally open = 31.

31. $D_1, D_4, D_9, D_{16}, D_{25}$,-----,$D_{900}, D_{961}$ are the 31doors that are open finally.
$\therefore$ $D_{901}, D_{902}, D_{903}$, -----,$D_{960}$ are the 60 consecutive doors that are closed and 60 is clearly greatest.

32. Ans: D_{961}

Paragraph – 12

The sides of a triangle a, b, c be positive integers and given $a \le b \le c$. If c is given, then

33. The number of triangle that can be formed when c is odd are _____

a) $\frac{(c+1)^2}{4}$ b) $\frac{3c-1}{2}$ c) $\frac{1}{4}c(c+2)$ d) $\frac{1}{2}(3c-2)$

Key. A

34. The number of triangle that can be formed when c is even are ______

a) $\frac{(c+1)^2}{4}$ b) $\frac{3c-1}{2}$ c) $\frac{1}{4}c(c+2)$ d) $\frac{1}{2}(3c-2)$

Key. C

35. The no.of isosceles or equletent triangle that can be formed when c is odd is ____

a) $\frac{(c+1)^2}{4}$ b) $\frac{3c-1}{2}$ c) $\frac{1}{4}c(c+2)$ d) $\frac{1}{2}(3c-2)$

Key. B

Sol. 33. No.of Δ les when c is odd (let C = 2m + 1)

$$=(2m+1)+(2m-1)+.....+1=(m+1)^2=\frac{(C+1)^2}{4}$$

34. No.of triangles when C is even (let C = 2m)

$$=(2m)+(2m-2)+.....+2=m(m+1)=\frac{1}{4}C(C+2)$$

35. No.of isosceles or equilateral Δ les when C is odd are

$$(2m+1)+1+1+.....+1=3m+1=\frac{3C-1}{2}$$

Paragraph – 13

There are 'n' intermediate stations on a railway line from one terminus to another. In how many ways can the train stop at 3 of these intermediate stations, if

36. All the three stations are consecutive

a) (n + 2) b) (n + 1) c) (n – 1) d) (n – 2)

Key. D

37. Atleast two of the stations are consecutive

a) (n + 2) (n - 1) b) (n – 2) (n – 1) c) $(n-2)^2$ d) None

Key. C

38. No two of these stations are consecutive

a) n_{c_3} b) $(n-2)_{c_3}$ c) $\frac{(n-2)(n-3)}{6}$ d) none

Key. B

Sol. 36. $(s_1,s_2,s_3),(s_2,s_3,s_4)......(s_{n-2},s_{n-1},s_n)=(n-2)$

37. (n – 2) ways (n – 1) ways - (n – 2) = $(n-2)^2$

38. $n_{c_3}-(n-2)^2=(n-2)_{c_3}$

Paragraph – 14

A is a set containing ‘n’ elements. A subset ‘P’ of ‘A’ is chosen at random. The set A is reconstructed by replacing the elements of ‘P’. A subset Q is again chosen at random. Then the number of ways of selecting P & Q so that

39. P = Q

a) 3^n b) 2^n c) $n.3^{n-1}$ d) $3n$

Key. B

40. $P \cap Q$ contains just one element

a) 3^n b) 2^n c) $n.3^{n-1}$ d) $3n$

Key. C

41. $P \cup Q$ contains just one element

a) 3^n b) 2^n c) $n.3^{n-1}$ d) $3n$

Key. D

Sol. 39. If P contains r elements

Then number of ways of selecting P is nc_5

Q $P = Q \quad \sum_{r=0}^{n} nc_r = 2^n$

40. P can be nc_r ways

$Q / P \cap Q$ contains just one element

$rc_1.\left(n - rc_0 + n - rc_1 + ...n - rc_{n-r}\right)$

$\Rightarrow nc_r\left[rc_1.\{n - rc_0 + n - rc_1 + ...n - rc_{n-1}\}\right]$

$\dfrac{n}{r}.n-1c_{r-1}.\ r.2^{n-r}$

$\Rightarrow \sum_{r=1}^{n} n.n-1c_{r-1}.2^{n-r} = n.3^{n-1}$

41. $nc_1 + nc_0.nc_1 + nc_1.nc_0 = 3n$

Paragraph – 15

$A = \{a_1, a_2, ...a_n\}$,

$A \times A = \left\{\left(a_i, a_j\right); a_i, a_j \varepsilon A, 1 \le i, j \le n\right\}$ $A * A = \left\{\{a_i, a_j\} : a_i, a_j \varepsilon A, 1 \le i, j \le n\right\}$

42. Number of functions defined form $A \times A \to A$

a) n^{n^2} b) $n^{(n-1)^2}$ c) $n^{(n+1)^2}$ d) n^{2n}

Key. A

43. Number of functions defined from $A * A \to A$

a) $n^{\frac{(n+1)^2}{2}}$ b) $n^{\frac{n^2}{2}}$ c) $n^{\frac{n(n+1)}{2}}$ d) $n^{\frac{n(n-1)}{2}}$

Key. C or D

Sol. 42. Number of elements in $A\times A=n^2$

43. Number of elements in $A*A=\dfrac{n(n+1)}{2}$

Paragraph – 16

For a finite set A, let $|A|$ denote the number of elements in the Set A. Also Let F denote the set of all functions $f:\{1,2,....,n\}\to\{1,2,...,k\},(n\geq 3,k\geq 2)$ satisfying $f(i)\neq f(i+1)$ for every i, $1\leq j\leq n-1$

44. $|F|=$

a) $k^n(k-1)$ b) $k(k-1)^n$ c) $k^{n-1}(k-1)$ d) $k(k-1)^{n-1}$

Key. D

45. If $c(n,k)$ denote the number of functions in F satisfying $f(n)\neq f(1)$, then for $n\geq 4, C(n,k)$

a) $k(k-1)^{n-1}-c(n-1,k)$ b) $k(k-1)^n-c(n-1,k-1)$

c) $k^{n-1}(k-1)^n-c(n-1,k)$ d) $k^n(k-1)-c(n-1,k)$

Key. A

46. For $n\geq k, c(n,k)$, where $c(n,k)$ has the same meaning as in question no.37, equals.

a) $k^n+(-1)^n(k-1)$ b) $(k-1)^n+(-1)^{n-1}(k-1)$

c) $(k-1)^n+(-1)^n(k-1)$ d) $k^n+(-1)^{n-1}(k-1)$

Key. C

Sol. 44. The image of the element 1 can be chosen in k ways and for each of the remaining $(n-1)$ elements, the image can be defined in $(k-1)$ ways, since $f(i)\neq f(i+1)$

$\therefore$ Total number of mapping in $F=k(k-1)^{n-1}$

45. Out of the total number of mappings in F, the number of mapping which satisfy $f(n)=f(1)$ is same as the number of mappings which satisfy $f(n-1)\neq f(1)$ and this number is $C(n-1,k)$

$\therefore\ C(n,k)=|F|-C(n-1,k)$

46. $C(n,k)=k(k-1)^{n-1}-c(n-1,k)$

$=(k-1)^n+(k-1)^{n-1}-C(n-1,k)$

$C(n,k)-(k-1)^n=(-1)\left\{C(n-1,k)-(k-1)^{n-1}\right\}$

$=(-1)^{n-3}\left\{c(3,k)-(k-1)^3\right\}$

but $c(3,k)$ = number of mappings f in F for which $f(3)\neq f(1)$

$\therefore C(3,k)=k(k-1)(k-2)$

$$\therefore C(n,k)-(k-1)^n=(-1)^{n-1}(k-1)\left\{k(k-2)-(k-1)^2\right\}$$

$$(-1)^n(k-1)$$

$$\therefore c(n,k)=(k-1)^n+(-1)^n(k-1)$$

Paragraph – 17

Given are six 0`s, five 1`s and four 2`s. consider all possible permutations of all these numbers. [A permutation can have its leading digit 0].

47. How many permutations have the first 0 preceeding the first 1?

a) ${}^{15}C_4\times{}^{10}C_5$ b) ${}^{15}C_5\times{}^{10}C_4$ c) ${}^{15}C_6\times{}^{10}C_5$ d) ${}^{15}C_5\times{}^{10}C_5$

Key. A

48. In how many permutations does the first 0 preceed the first 1 and the first 1 preceed first 2.

a) ${}^{14}C_5\times{}^{8}C_6$ b) ${}^{14}C_5\times{}^{8}C_4$ c) ${}^{14}C_6\times{}^{8}C_4$ d) ${}^{14}C_6\times{}^{8}C_6$

Key. B

49. The no. of permutations in which all 2`s are together but no two of the zeroes are together is :

a) 42 b) 40 c) 84 d) 80

Key. A

Sol. 47. The no. of ways of arranging 2`s is ${}^{15}C_4$. Fill the first empty position left after arranging the 2`s with a 0(1 way) and pick the remaining five places the position the remaining five zeros $\to{}^{10}C_5$ ways.

$\therefore\ {}^{15}C_4\times1\times{}^{10}C_5$

48. Put a) in the first position, (1 way). Pick five other positions for the remaining O`s (${}^{14}c_5$ ways), put a 1 in the first of the remaining positions ( 1 way), then arrange the remaining four 1`s (${}^{8}C_4$ ways)

$$\therefore {}^{14}C_5\times{}^{8}C_4$$

Paragraph – 18

Let S be the set of the first 18 positive integers.

50. The number of ways of selecting three numbers from S such that they form an A.P. is

a) 60 b) 64 c) 72 d) 80

Key. C

51. The number of ways of selecting two numbers form S such that the sum of their cubes is divisible by 3 is

a) 21 b) 31 c) 45 d) 51

Key. D

52. The number of ways of selecting 3 numbers from S such that either they are all consecutive or no two of them are consecutive is

a) 560 b) 576 c) 625 d) 800

Key. B

Sol. 50. Number of ways of selecting 2 even integers $=9C_2$

Number of ways of selecting 2 odd integers $=9C_2$

$\therefore$ Total number of ways of selecting two integers so that their sum is even $=2\times 9C_2=72$

51. a^3+b^3 is divisible by 3 only if 3 divides a + b.

$\therefore$ Required number $=6C_1\times 6C_1+6C_2=51$

52. Number of ways of selecting 3 numbers which are consecutive = 16

Number of ways of selecting 3 numbers no two of which are consecutive =

$$\binom{18-3+1}{3}=\binom{16}{3}=560$$

Paragraph – 19

Let $\theta=\left(a_1,a_2,a_3,..........,a_n\right)$ be a given arrangement of n distinct objects $a_1,a_2,a_3,..........,a_n$. A derangement of θ is an arrangement of these n objects in which none of the objects occupies its original position. Let D_n be the number of derangements of the permutation θ.

53. 16. D_n is equal to

a) $(n-1)D_{n-1}+D_{n-2}$ b) $D_{n-1}+(n-1)D_{n-2}$

c) $n\left(D_{n-1}+D_{n-2}\right)$ d) $(n-1)\left(D_{n-1}+D_{n-2}\right)$

Key. D

54. The relation between D_n and D_{n-1} is given by

a) $D_n-nD_{n-1}=(-1)^n$ b) $D_n-(n-1)D_{n-1}=(-1)^{n-1}$

c) $D_n-nD_{n-1}=(-1)^{n-1}$ d) $D_n-D_{n-1}=(-1)^{n-1}$

Key. A

55. There are 5 different colour balls and 5 boxes of colours same as those of the balls. The number of ways in which one can place the balls into the boxes, one each in a box, so that no ball goes to a box of its own colour is

a) 40 b) 44 c) 45 d) 60

Key. B

Sol. 53. For every choice of $r=1,2,....(n-1)$, when the n th object a_n goes to the rth place, there are $D_{n-1}+D_{n-2}$ ways of the other $(n-1)$ objects $a_1,a_2,.......a_{n-1}$ to be deranged.

Hence $D_n=(n-1)\left(D_{n-1}+D_{n-2}\right)$

54. $D_n-nD_{n-1}=(-1)\left(D_{n-1}-(n-1)D_{n-2}\right)$

By implied induction on n, we obtain

$$D_n-nD_{n-1}=(-1)^{n-2}\left(D_2-2D_1\right)\ where\, D_1=0\ and\ D_2=1$$
$$=(-1)^n$$

55. $\dfrac{D_n}{n!}-\dfrac{D_{n-1}}{(n-1)}=\dfrac{(-1)}{n!}$ gives $\dfrac{D_n}{n!}=\sum_{r=2}^{n}\dfrac{D_r}{r!}-\dfrac{D_{r-1}}{(r-1)!}\quad=\sum_{r=2}^{n}\dfrac{(-1)^r}{r!}$

$$D_n=n!\sum_{r=0}^{n}\frac{(-1)^r}{r!}$$

$$\therefore D_5=(5!)\left(\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right)=44$$

Paragraph – 20

Let 'S' be the set of first 18 natural numbers. The number of ways of selecting from 'S'

56. Three numbers such that they are all consecutive (or) no two of them are consecutive is
a) 576 b) 600 c) 640 d) 800

Key. A

57. Three numbers such that they from an AP is
a) 60 b) 64 c) 72 d) 80

Key. C

58. Two number such that the sum of their cubes is divisible by 3 is …
a) 21 b) 31 c) 45 d) 51

Key. D

Sol. 56. NO. of ways of selecting 3 consecutive numbers = 16
No. of ways selecting 3 numbers such that no two are consecutive = 560
$\Rightarrow$ Required number of ways = 560 +16 =576
57. If a, b, c are in AP $\Rightarrow$ 2b = a + c
$\therefore$ Both a, c are odd (or) both are even
$\Rightarrow$ number of ways = $2\left({}^{9}c_2\right)$ = 72
58.

R_1	1	4	7	10	13	16
R_2	2	5	8	11	14	17
R_3	3	6	9	12	15	18

$x^3 + y^3$ is divisible by '3'
$\Rightarrow$ we can select both x, y from R_3 (or) one element from R_1 and another element from R_2
and it can be done is ${}^{6}c_2 + {}^{6}c_1 \times {}^{6}c_1$ = 15+36 =51

Paragraph – 21

A is a set containing 'n' elements. A subset P of A is chose at random. The set A is reconstructed by replacing the elements of 'P' A subset Q of 'A' is again chosen at random. Then the number of ways of selecting 'P' and Q such that

59. $P \cup Q = A$
a) 2^n b) $3^n - 2$ c) 3^n d) 4^n

Key. C

60. $P \cup Q$ contains just one element is
a) 3^n b) $3^n - 1$ c) $3n$ d) $2^n - 1$

Key. C

61. $P \cap Q$ contains just two elements is
a) ${}^{n}c_2 2^{n-2}$ b) 3^{n-2} c) ${}^{n+1}c_2 3^{n-2}$ d) ${}^{n}c_2 3^{n-2}$

Key. D

Sol. 59. Let 'P' be constructed by selecting 'r' elements from 'n' elements of set 'A'. This can be done in ${}^{n}c_r$ ways. In order to construct Q such that $P \cup Q = A$
We must include n-r elements of set A in Q and any number of elements from 'r' elements selected for P This can be done in $1 \times \left({}^{r}c_0 + {}^{r}c_1 + {}^{r}c_2 + \dots + {}^{r}c_r\right) = 2^r$
Number of ways of selecting P and Q such that $P \cup Q = A$ is ${}^{n}c_r\left(2^r\right)$

but 'r' varies from 0 to n

$$\Rightarrow reqd \quad no.of \;\; ways = \sum_{r=0}^{n} {}^{n}c_r 2^r = (1+2)^n = 3^n$$

60. Suppose 'P' is subset containing 'r' elements

$\Rightarrow$ It can be selected in ${}^{n}c_r$ ways

Since $P \cap Q$ contains excectly two elements

$\Rightarrow$ It can be selected in ${}^{r}c_2 \left({}^{n-r}c_0 + {}^{n-r}c_1 + ... + {}^{n-r}c_{n-r} \right) = {}^{r}c_2 2^{n-r}$

Number of ways of selecting P and Q such that 'P' contains 'r' elements and $P \cap Q$ Contains exactly two elements is

${}^{n}c_r 2^{n-r}$

$= \dfrac{n(n-1)}{2} . {}^{n-2}c_{r-2} 2^{n-r}$

Since 'r' cab take any value from o to n

$$\Rightarrow \text{Re}\, qd \;\; no.of \;\; ways = \sum_{r=2}^{n} \frac{n(n-1)}{2} . {}^{n-2}c_{r-2} 2^{n-r}$$

$$= {}^{n}c_2 \sum_{r=2}^{n} {}^{n-2}c_{r-2} 2^{n-r}$$

$$= {}^{n}c_2 (3)^n$$

Paragraph – 22

When letters of any word are written in all possible ways using all letters of given word and these words are arranged as in a dictionary then the position of the word in that list in called the rank of the word.

62. What is the rank of word 'PARALLEL'

(a) 2353 (b) 2629 (c) 2593 (d) 2623

Key. D

63. If list of all words is made by using the letters of the word IITJEE in dictionary order then the word at 141 th position is

(a) JEE IIT (b) JII TEE (c) JIE TIE (d) JII EET

Key. B

64. If all numbers starting from 1 are listed in increasing order by using the digits 0, 1, 2, 3, 4, 5, 6 then the number whose rank is 8000 is

(a) 32605 (b) 32116 (c) 32216 (d) 33316

Key. C

Sol. 62-64.

62. AAELLLPR

$A - - - - \dfrac{7!}{3!}$

$E - - - - \dfrac{7!}{3!2!}$

$L - - - - \dfrac{7!}{2!2!}$

$PAA - - - - \dfrac{5!}{3!}$

$PAE----\frac{5!}{3!}$

$PAL----\frac{5!}{2!}$

$PARAE----\frac{3!}{3!}$

$PARALE----\frac{2!}{2!}$

$PARALLEL----\frac{1}{2623}$

63. E E I I J T

$E----\frac{5!}{2!}=60$

$I----\frac{5!}{2!}=60$

$JE----\frac{4!}{2!}=12$

$JIE----3!=6$

$JIIE----2!=2$

$JIITEE----1!=\frac{1}{141}$

64. One digit number = 6
2 digit number = 6x7 = 42
3 digit number = 6 x 7 x 7 = 294
4 digit number = 6 x 7 x 7 x 7 = 2058
5 digit number starting with 1 -------- = 7^4 = 2401
5 digit number starting with 2 -------- = 7^4 = 2401
5 digit number starting with 30 -------- = 7^3 = 343
5 digit number starting with 31 -------- = 7^3 = 343
5 digit number starting with 320 -------- = 7^2 = 49
5 digit number starting with 321 -------- = 7^2 = 49
5 digit number starting with 3220 -------- = 7 = 7
5 digit number starting with 32216 -------- = 7 = 7

8000

Paragraph – 23

Consider all possible permutations of 6 Identical Red balls, 5 Identical yellow balls, and 4 different blue balls.

65. The number of permutations in which all blue balls are together but no two yellow balls are together.

(a) $7.{}^8C_5$ (b) $7.4!{}^8C_5$ (c) $7!4!{}^8P_5$ (d) $7.4!{}^7C_5$

Key. B

66. The number of permutation in which all Blue balls are preceding the first yellow ball

(a) ${}^{15}C_6.4!$ (b) $\frac{10!}{4!}$ (c) $\frac{15!}{6!5!}$ (d) ${}^{15}C_4.4!$

Key. A

67. The number of permutations in which the first Red ball precedes the first yellow ball and the first yellow ball precedes the first blue ball

(a) ${}^{14}C_6.{}^{8}C_4\,4!$ (b) ${}^{15}C_5.{}^{10}C_4\,4!$ (c) ${}^{14}C_5.{}^{8}C_4$ (d) ${}^{14}C_5.{}^{8}C_4\,4!$

Key. D

Sol. 65. 6R, 1 GB can be arranged in $\frac{7!}{6!}.4!$

And ${}^{8}C_5$ ways 5 yellow balls can be placed in between then. Hence ans in $\frac{7!}{6!}.4!\ {}^{8}C_5$

66. First of all placing 6 Reds in 15 places $\to {}^{15}C_6$ after that remaining first 4 unfilled places place the blue and last 5 unifled placed place ${}^{15}C_6.4!.1$

Step I place one Red a first place and remaining in an of 14 places $\to {}^{14}C_5$

Step II place one yellow in first vacant place and remaining in 8 places $\to {}^{8}C_4$

Step III place 4 blues in 4 vacant places → 4 ! Ans ${}^{14}C_4.{}^{8}C_4.4!$

67. a) ${}^{7}C_3 - 3$

b) ${}^{6}C_2 + 2.{}^{4}C_2 + 2\,{}^{6}C_1.{}^{4}C_1$

c) $6^3 - 5^3$

d) ${}^{4}C_2.{}^{5}C_2 + {}^{4}C_3.{}^{5}C_1 + {}^{4}C_4.{}^{5}C_0 = 81$

Paragraph – 24

Let P be a prime number and n be a natural number, the exponent of P in $n!$ is denoted by $E_p(n!)$ and is given by

$E_p(n!) = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + ...+ \left[\frac{n}{p^k}\right]$ such that

$p^k \le n < p^{k+1}$ And $[y]$ denotes the integral part of y. If we isolate the power of each prime number contained in any number N then we can write N as

$N = 2^{\alpha_1}.3^{\alpha_2}.5^{\alpha_3}.....$ where $\alpha_i\ (i = 1,.....n)$ is natural number

On the basis of above data (information) answer the following questions.

68. The number of zeroes at the end of 107! is

a) 102 b) 25 c) 13 d) 21

Key. B

69. The last non zero digit in 11! is

a) 6 b) 3 c) 8 d) 4

Key. C

70. The maximum value of n, for which 33! is divisible by 2^n is

a) 33 b) 30 c) 32 d) 31

Key. D

Sol. 68. $E_2(107)!$

$= 53 + 26 + 13 + 6 + 3 + 1 = 102$

$E_5(107)! = 21 + 4 = 25$

$\therefore\ E_{10}(107!) = \min.\text{of } (25, 102) = 25$

69. $E_2(11!) = 5 + 2 + 1 = 8$; $E_3(11!) = 3 + 1 = 4$; $E_5(11!) = 2$; $E_7(11!) = 1$;

$E_{11}(11!) = 1$

$11! = 2^8\ 3^4\ 5^2\ 7^1\ 11^1 = 10^2\ 2^6\ 3^4\ 7^1\ 11^1$

$\therefore$ last non zero digit in $11!$ is the unit digit in the product of $4 \times 1 \times 7 \times 1$. (as 2^6 ends with 4 and 3^4 ends with 1 and 11 end with 1)

$\therefore$ last non zero digit in 11! is the number 8.

Alternative Method.

$10! = 3628800$

$\therefore 11! = 10! \times 11 = 3628800 \times 11$

Whose last non zero digit is $1 \times 8 = 8$.

70. $E_2(33!) = 16 + 8 + 4 + 2 + 1 = 31$

$\therefore\ E_2(33!) = 31 = n$

Paragraph – 25

There are n intermediate stations on a railway line from one terminus to another. In how many ways can the train stop at 3 of these intermediate station if

71. All the three stations are consecutive

a) $n-2$ b) $n-1$ c) $n-3$ d) $(n-1)(n-2)$

Key. A

72. At least two of the stations are consecutive

a) $(n-1)(n-2)$ b) $(n-2)^2$ c) $(n-3)^2$ d) $(n-1)$

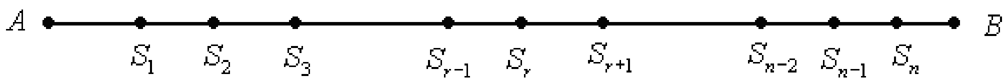
Key. B

73. No two of these stations are consecutive

a) ${}^{n-1}C_3$ b) ${}^{n-4}C_3$ c) ${}^{n-2}C_3$ d) ${}^{n-6}C_3$

Key. C

Sol. 71.



The number of triples of consecutive station, viz.

$S_1S_2S_3, S_2S_3S_4, S_3S_4S_5 \ldots\ldots\ldots\ldots S_{n-2}S_{n-1}S_n$ is $(n-2)$.

72. The total number of consecutive pair of station, viz.

$S_1S_2, S_2S_3, \ldots\ldots\ldots S_{n-1}S_n$ is $(n-1)$

Each of the above pair can be associated with a third station in $(n-2)$ ways. Thus, choosing a pair of stations and any third station can be done in $(n-1)(n-2)$ ways. The above count also includes the case of three consecutive stations. However, we can see that each such case has been counted twice. For example, the pair S_4S_5 combined with S_6 and the pair S_5S_6 combined with S_4 are identical.

Hence, subtracting the excess counting, the number of ways in which three stations can be chosen so that at least two of them are consecutive.

$$=(n-1)(n-2)-(n-2)=(n-2)^2.$$

73. Without restriction, the train can stop at any three stations in ${}^nC_3$ ways.

Hence, the number of ways the train can stop so that no two stations are consecutive

$$={}^nC_3-(n-2)^2=\frac{n(n-1)(n-2)}{1.2.3}-(n-2)^2$$

$$=(n-2)\left(\frac{n^2-n-6n+12}{6}\right)=\frac{(n-2)(n-3)(n-4)}{6}={}^{n-2}C_3$$

Paragraph – 26

If there are two jobs such that one of them can be completed in m ways, and when it has been completed in any one of these m ways, second job can be completed in n ways; then the two jobs in succession can be completed in $m\times n$ ways

74. Find the number of ways in which n distinct balls can be put into three boxes so that no two boxes remain empty.

a) 3^n b) 3^n-1 c) 3^n-2 d) 3^n-3

Key. D

75. The number of ways of wearing 5 different rings on 4 fingers of one hand is

a) 4824 b) 5060 c) 6720 d) 480

Key. C

76. A letter lock consists of three rings each marked with fifteen different letters. It is found that a man could open the lock only after he makes half the number of possible unsuccessful attempts to open the lock. If each attempt takes 10 sec, the time he must have spent is not less than

a) $4\frac{1}{2}hr$ b) $5\frac{1}{2}hr$ c) $6\frac{1}{2}hr$ d) $9\ hr$

Key. A

Sol. 74. The number of possible unsuccessful attempts $=15^3-1$

If T is the time taken by the man to unlock, then

$$T=\frac{1}{2}\left(15^3-1\right)\frac{10}{360}hr=\left(\frac{375}{8}-\frac{1}{72}\right)hr$$

Which is greater than $4\frac{1}{2}$ hr but less than 5 hr.

Paragraph – 27

Suppose there are 5 mangoes, 4 apples and 3 oranges in a bag, fruits of same variety being identical. Then

77. The number of ways can a selection of fruits be made is
(A) 120 (B) 119 (C) $2^{12 -1}$ (D) 121

Key. A

78. The number of ways can a selection of fruits be made if at least 2 mangoes be included is
(A) 80 (B) 120 (C) 2^7 (26) (D) $2^{12}-1$

Key. A

79. The number of ways can a selection of fruits be made if atleast one fruit each kind is to be included is
(A) $\left(2^5-1\right)\left(2^4-1\right)\left(2^3-1\right)$ (B) 120
(C) 80 (D) 60

Key. D

Sol. 77. Req selections = 6x5x4-1 = 119
78.Req. selections = 4x5x4 = 80
79. Req. selections = 5x4x3 = 60

Permutation & Combination

Integer Answer Type

1. If n > 1 is the smallest integer with the property that $n^2(n-1)$ is divisible by 2009, then the integral part of $\frac{n}{8} =$

Key. 5

Sol. Therefore, 41 must divide $n^2(n-1)$, which implies that 41 is a factor of either n or $n-1$. In particular, $n \ge 41$. For n = 41, neither n nor $(n-1)$ is divisible by 7. For n = 42, n^2 is divisible by 7^2, since n is divisible by 7. Therefore n = 42 is the smallest integer.

2. The number of onto functions which are non decreasing from $A = \{1, 2, 3, 4, 5\}$ to $B = \{7, 8, 9\}$ is __.

Key. 6

Sol. Images of 1,2,3,4,5 are respectively.

7,7,7,8,9; 7,7,8,8,9; 7,7,8,9,9

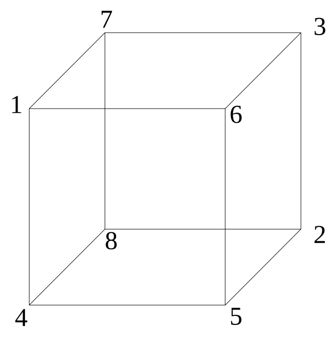
7,8,8,8,9; 7,8,8,9,9; 7,8,9,9,9

$\therefore$ there are six functions.

3. Put numbers 1, 2, 3, 4, 5, 6, 7, 8 at the vertices of a cube, such that the sum of any three numbers on any face is not less than 10. The minimum sum of the four number on a face is k, then k/2 is equal to

Key. 8

Sol. Suppose that the four numbers on face of the cube is a_1, a_2, a_3, a_4 such that their sum reaches the minimum and $a_1 < a_2 < a_3 < a_4$.
Since the maximum sum of any three numbers less than 5 is 9, we have $a_4 \ge 6$ and $a_1 + a_2 + a_3 + a_4 \ge 16$.
As seen in figure, we have
2 + 3 + 5 + 6 = 16



and that means minimum sum of four numbers on a face is 16.

4. If a and b are positive integers and a + 11b is divisible by 13 and a + 13b is divisible by 11. Then minimum value of a + b – 20 is

Key. (8)

Sol. a + 11b = $13I_1$
a + 13b = $11I_2$
and proceed

5. A cricket player played $n(n>1)$ matches during his career and made a total of $\frac{(n+1)(2^{n+1}-n-2)}{4}$ runs. If the player made $k.2^{n-k+1}$ runs in the k^{th} match $(1 \le k \le n)$.
Then the value of 'n' is

Key. 7

Sol. Where $S = 1.2^n + 2.2^{n-1} + 3.2^{n-2} + ... + n.2$

$\frac{1}{2}S = 2^{n-1} + 2.2^{n-2} + ... + (n-1).2 + n$

Subtracting, $\frac{1}{2}S = (2^n + 2^{n-1} + ... + 2) - n = 2(2^n - 1) - n$

$\therefore\ S = 2(2^{n+1} - 2) - 2n = 2(2^{n+1} - n - 2)$

Hence, $\frac{n+1}{4} = 2\ (i.e.,)\ n = 7$

6. A cricket player played n (n > 1) matches during his career and made a total of $\frac{(n+1)(2^{n+1}-n-2)}{4}$ runs. If the player made $k.2^{n-k+1}$ runs in the k th match $(1 \le k \le n)$,find n.

ANS : 7

HINT : $\left(\frac{n+1}{4}\right)(2^{n+1} - n - 2) = \sum_{k=1}^{n} k.2^{n+1-k} = S$ where

$S = 1.2^n + 2.2^{n-1} + 3.2^{n-2} + ... + n.2$

$\frac{1}{2}S = \quad 2^{n-1} + 2.2^{n-2} + ... + (n-1).2 + n$

Subtracting , $\frac{1}{2}S = (2^n + 2^{n-1} + ... + 2) - n = 2(2^n - 1) - n$

$\therefore S = 2(2^{n+1} - 2) - 2n = 2(2^{n+1} - n - 2)$

Hence, $\frac{n+1}{4} = 2\ (i.e)\ n = 7$

7. Let $S = \{1, 2, 3, \_\_ n\}$, If X denote the set of all subsets of S containing exactly two elements, then the value of $\sum_{A \in X} (\min A)$ is ${}^{n+1}C_{\lambda}$ then λ = ____

Key. 3

Sol. There are exactly $(n-1)$ sub sets of S containing two elements having 1 as least element; exactly (n-2) subsets of S having 2 as least element and so on.

Thus $\sum_{A \in X} \min(A) = (1)(n-1) + 2(n-2) + \_\_\_ + (n-1)(1)$

$= \sum_{r=1}^{n-1} r(n-r) = n+1_{C_3}$

8. There are 6 balls of each of the four colours: red, white, yellow and black in a bag (balls of same colour are identical). Let n be the total number of different ways of drawing 6 balls one by one without replacement such that no two consecutive balls are of the same colour and no colour is missing in the draw. Find the non–zero digit of n.

Key. 6

Sol. $n = 4 \times 3^5 - {}^4C_1 \,.\, 3.2^5 + {}^4C_2 \,.\, 2 \,.\, 1^5 = 600$

9. Let P be the product of distances of any vertex of a regular decagon inscribed in a unit circle from the remaining vertices then the sum of digits occuring in P is

Key. 1

Sol. If 1, α_1, α_2, ..., α_9 are the 10th roots of unity then $(1-\alpha_1)(1-\alpha_2)\ldots(1-\alpha_9) = 10 \Rightarrow |1-\alpha_1|\,|1-\alpha_2| \ldots |1-\alpha_9| = 10.$ So, P = 10.

10. The number of numbers from 1 to 10^6 (both inclusive) in which two consecutive digits are same is equal to 402128 + K where K is a single digit number then K must be equal to ______.

Key. 2

Sol. No.of n digit numbers in which no two consecutive digits are same = 9^n

$\Rightarrow$ no.of numbers from 1 to 10^6 in which no two consecutive digits are same

$= \sum_{n=1}^{6} 9^n = 597870$

Required number $= 10^6 - 597870 = 402130 = 402128 + 2$

$K = 2$

11. The number of ways of distributing 3 identical physics books and 3 identical mathematics books among three students such that each student gets at least one book is 50 + K, where K is single digit number, then K is _____.

Key. 5

Sol. $n(A) = {}^{3+2-1}C_{2-1} \times {}^{3+2-1}C_{2-1} = 16$

$n(B) = n(C) = 16$

$n(A \cap B) = {}^{3+1-1}C_{1-1}\, {}^{3+1-1}C_{1-1} = 1 = n(B \cap C) = n(C \cap A)$

$n(A \cap B \cap C) = 0$

Required no.of ways $= 100 - (16+16+16-1-1-1+0) = 55 = 50+5$

$\therefore K = 5$

12. The number of polynomials of the form $x^3 + ax^2 + bx + c$ which are divisible by $x^2 + 1$ where $a, b, c \in \{1, 2, 3, \ldots, 10\}$ is 10K, then K is ______.

Key. 1

Sol. $x^2 + 1 = (x+i)(x-i)$

$b = 1, a = c$

No.of ways of choosing $a, b, c = 10 = 10 \times 1$

$\therefore K = 1$

13. Consider $n \times n$ graph paper where n is a natural number. Consider the right angled isosceles triangles whose vertices are integer points of this graph and whose sides forming right angle are parallel to x and y axes. If the no.of such triangles is $\frac{2}{K} n(n+1)(2n+1)$, then K is ______.

Key. 3

Sol. Required no.of triangles $= 4\left[n^2+(n-1)^2+...+1^2\right] = \frac{2}{3}n(n+1)(2n+1)$

$\therefore K = 3$

14. The number of ways of arranging 11 objects $A, B, C, D, E, F, \alpha, \alpha, \alpha, \beta, \beta$ so that every β lie between two α (not necessarily adjacent) is $K \times 6! \times {}^{11}C_5$, then K is ____.

Key. 3

Sol. There are three major ways $\alpha\alpha\beta\beta\alpha, \alpha\beta\beta\alpha\alpha$ and $\alpha\beta\alpha\beta\alpha$

Each major way has six empty spaces. The number of ways of putting letters at these empty spaces must be non-negative integer function of $x_1 + x_2 + ... + x_6 = 6$

$= {}^{6+6-1}C_{6-1} = {}^{11}C_5$

No.of arrangements is $= 3 \times {}^{11}C_5 \times 6! \Rightarrow K + 3$

15. The no.of positive integer solutions of $x + y + z = 10$, where x, y, z are unequal is (20 + K) then K is

Key. 4

Sol. $x < y < z$, these are 1 2 7, 3 6 1, 1 4 5, 2 3 5

total $= 3! \times 4 = 24 = 20 + 4$

$\therefore K = 4$

16. The total number of ways of selecting 5 letters of word INDEPENDENT is x then sum of digits of x

Key. 9

Sol. Ans : 72 , 7 + 2 = 9

17. Five balls of different colours are to be placed in 3 boxes of different sizes each box can hold all 5 balls. Then number of ways we can place the balls so that no box remaining empty is x then sum of digits of x.

Key. 6

Sol. Number onto functions from set of 5 elements onto set of 3 elements = 150

18. If one test (on screening paper basis) was conducted on Batch A, maximum number of marks is (90 × 3) = 270. 4 students get the marks lower than 80. Coaching institute decide to inform their guardians, that is why their result card were sent to their home. The number of ways, in which all the letters were put in wrong envelopes, is

Key. 9

Sol. The number of ways in which all the letters are in wrong envelopes

$$= 4!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right)$$

$$= 4!\left(\frac{1}{2} - \frac{1}{6} + \frac{1}{24}\right) = 12 - 4 + 1 = 9$$

19. The exponent of 7 in ${}^{100}C_{50}$ is :

Key. 0

Sol. $${}^{100}C_{50} = \frac{100!}{50!50!}$$

Exponent of 7 in 100 ! = 16

Exponent of 7 in 50! $\left[\frac{50}{7}\right]+\left[\frac{50}{7^2}\right]=8$

Exponent of 7 in $(50!)^2 = 16$

$\therefore$ Exponent of 7 in ${}^{100}C_{50} = 16 - 16 = 0$

20. The unit digit in 1! + 2! + 3! + ….. + 49! is

Key. 3

Sol. 1! +2! + 3! + 4! = 33

5! = 120, 6! = 720, 7! = 5040

8! = 40320, 9! = 326880

Thus the two digit of

1! + 2! + …. + 9! = 1

Also note that n! is divisible by 100 for all n $\geq$ 10.

$\therefore$ term digits of 10! + 11! + ….. + 49! = 0

$\therefore$ term digits of 1! + 2! + ……+ 49! = 1.

21. Nine hundred distinct n-digit number s are to be formed using exactly the three digits. 2, 5 and 7. The smallest value of n for which this is possible is.

Key. n = 7

Sol. For n = 6

$3\times3\times3\times3\times3\times3 = 729 < 900$

For n = 7

$3\times3\times3\times3\times3\times3\times3 = 2187 > 900$

For n = 8

Number of n-digits formed > 900

Since the least n is required.

$\therefore$ n = 7.

22. The number of divisors of the form 2n + 1 (n ≥ 1) of the integer 120 is

Key. 3

Sol. 240 is divisible by 4n + 2 or 120 is divisible by 2n + 1.

Number of the form (2n+1), n $\in I$ are all odd natural numbers.

Thus we have to find al odd numbers dividing 120.

These numbers are 1, 3, 5, 15.

Hence, number of divisors = 4.

23. If number of numbers greater than 3000, which can be formed by using the digits 0, 1, 2, 3, 4, 5 without repetition, is n then $\frac{n}{230}$ is equal to

Key. 6

Sol. No. of 4 digit numbers = 3 × 5 × 4 × 3 = 180

No. of 5 digit numbers = 5 × 5 × 4 × 3 × 2 = 600

No. of 6 digit numbers = $5 \times 5 \times 4 \times 3 \times 2 = 600$

$$n = 1380$$

$$\Rightarrow \quad \frac{n}{230} = 6$$

24. Nine hundred distinct n-digit numbers are to be formed using only the 3 digits 2, 5, 7. The smallest value of n for which this is possible is

Key. 7

Sol. $3^n \geq 900 \Rightarrow n \geq 7$

25. Out of 5 apples, 10 mangoes and 15 oranges, the number of ways of distributing 15 fruits each to two persons, is n then $\frac{n}{22}$ is equal to

Key. 3

Sol. $x_1 + x_2 + x_3 = 15$

$0 \leq x_1 \leq 5,\ 0 \leq x_2 \leq 10,\ 0 \leq x_3 \leq 15$

n = co-efficient of $x^{15}(1 - x^6)\ (1 - x^{11})\ (1 - x^{16})\ (1 - x)^{-3}$

$n = 66$

$$\frac{n}{22} = 3$$

26. The number of different ordered triplets (a, b, c), a, b, $c \in I$ such that these can represent sides of a triangle whose perimeter is 21, is 9k+10,then k is _____.

Key. 5

Sol. $a + b + c = 21 \Rightarrow b + c > a \Rightarrow a + b + c > 2a \Rightarrow 2a < 21 \Rightarrow a \leq 10$. So $1 \leq a, b, c \leq 10$.
The cases when $a > b > c$ are (10, 9, 2), (10, 8, 3), (10, 7, 4), (10, 6, 5), (9, 8, 4), (9, 7, 5) and (8, 7, 6). So, number of cases when a, b, c are all distinct is $7 \times 3! = 42$.
The cases when $a = b > c$ or $a > b = c$ are (10, 10, 1), (9, 9, 3), (8, 8, 5) and (9, 6, 6). So number of cases when two same and 1 different is $4 \times 3!/2! = 12$.
The cases when $a = b = c$ is (7, 7, 7). The total number of ordered triplets = 42+12 + 1=55.

27. Considering the rectangular hyperbola xy = 15! The number of points (α, β) lying on it, where $\alpha, \beta \in N$ and α divides β, is 12γ then the value of γ is _____.

Key. 8

Sol. The largest number whose perfect square can be made with 15! is $2^5\ 3^3\ 5^1\ 7^1$
So that number of ways of selecting x will be
$(1 + 5)\ (1 + 3)\ (1 + 1)\ (1 + 1) = 96.$

Permutation & Combination

Matrix-Match Type

1. Match the following:

There are 2 Indian couples, 2 American couples and one unmarried person

	Column -I		Column II
(A)	The total number of ways in which they can sit in a row such that an Indian wife and an American wife are always on either side of the unmarried person, is	(p)	22680
(B)	The total number of ways in which they can sit in a row such that the unmarried person always occupy the middle position, is	(q)	5760
(C)	The total number of ways in which they can sit around a circular table such that an Indian wife and an American wife are always on either side of the unmarried person, is	(r)	40320
(D)	If all the nine persons are to be interviewed one by one then the total number of ways of arranging their interviews such that no wife gives interview before her husband, is	(s)	24320

Key. (A) $\rightarrow$(r); (B) $\rightarrow$(r); (C) $\rightarrow$(q); (D) $\rightarrow$(p)

Sol. A) One Indian wife and one American wife can be selected in ${}^2C_1 \times {}^2C_1$ ways and keeping an unmarried person in between these two wives the total number of linear arrangements are ${}^2C_1 \times {}^2C_1 \times \lfloor 7 \times \lfloor 2 = 40320$

B) Required number of ways $= \lfloor 8 = 40320$

C) Required number of ways $= \lfloor (7-1) \times \lfloor 2 \times {}^2C_1 \times {}^2C_1 = 5760$

D) Number of ways in which interviews can be arranged

$= 9 \times {}^8C_2 \times {}^6C_2 \times {}^4C_2 \times {}^2C_2 = 22680$

2. There are 2 Indian couples, 2 American couples and one unmarried person

	Column I		Column II
(A)	The total number of ways in which they can sit in a row such that an Indian wife and an American wife are always on either side of the unmarried person, is	(p)	22680
(B)	The total number of ways in which they can sit in a row such that the unmarried person always occupy the middle position, is	(q)	5760
(C)	The total number of ways in which they can sit around a circular table such that an Indian wife and an American wife are always on either side of the unmarried person, is	(r)	40320

(D)	If all the nine persons are to be interviewed one by one then the total number of ways of arranging their interviews such that no wife gives interview before her husband, is	(s)	24320

3. (A – r),
(B – r),
(C – q),
(D – p),

22. (A) One Indian wife and one American wife can be selected in ${}^2C_1 \times {}^2C_1$ ways and keeping an unmarried person in between these two wives the total number of linear arrangements are ${}^2C_1 \times {}^2C_1 \times \lfloor\underline{7} \times \lfloor\underline{2} = 40320$

(B) Required number of ways $= \lfloor\underline{8} = 40320$

(C) Required number of ways $= \lfloor\underline{(7-1)} \times \lfloor\underline{2} \times {}^2C_1 \times {}^2C_1 = 5760$

(D) Number of ways in which interviews can be arranged

$= 9 \times {}^8C_2 \times {}^6C_2 \times {}^4C_2 \times {}^2C_2 = 22680$

4. Match the following:

Consider all possible permutations of the letters of the word M A S T E R B L A S T E R S

	Column – I		Column – II
A)	The number of permutations containing the word RAAT is	p)	$\dfrac{(7!)^2}{3!(2!)^4}$
B)	The number of permutations in which S occurs in first place and R occurs in the last place is	q)	$\dfrac{11! \times 4!}{3! \times (2!)^2}$
C)	The number of permutations in which none of the letters S, T, R occur in first 7 positions is	r)	$\dfrac{11!}{3! \times 2!}$
D)	The number of permutations in which the letters A, S, R occur in even positions is	s)	$\dfrac{12!}{(2!)^4}$

KEY : A – r, B – s, C – p, D - p

HINT : $AA, SSS, TT, EE, RR, M, B, L$

A) Take RAAT as one unit. Therefore 10 + 1 = 11 units can be arranged in $\dfrac{11!}{3! \times 2!}$ ways.

B) After fixing S in first position and R in last position the remaining 12 letters can be arranged in remaining 12 positions in $\dfrac{12!}{(2!)^4}$ ways

C) First 7 positions can be filled with $A's, E's, M, B, L$ in $\frac{7!}{(2!)^2}$ ways. The remaining 7 positions can be filled with $S's, T's, R's$ in $\frac{7!}{3!\times(2!)^2}$ ways.

D) 7 even positions can be filled with $A's, S's, R's$ in $\frac{7!}{(2!)^2\times 3!}$ ways. 7 odd positions can be filled with $T's, E's, M, B, L$ in $\frac{7!}{(2!)^2}$

5. Four digit natural number is formed using, the digits from the set {0, 1, 2, 3, 4, 5}, repetition of digits is allowed

Column I (Conditions)		Column II (Number of natural numbers)	
(A)	Number formed is multiples of 3	(p)	480
(B)	number formed contains exactly two different digits	(q)	540
(C)	Numbers formed contains exactly three different digits	(r)	360
(D)	Number formed is odd	(s)	175

Key: (A-r), (B-s), (C-r), (D-q)

Hint: For A $\rightarrow 5 \times 6 \times 6 \times 2 = 360$

For B $\rightarrow {}^5C_2\left[\frac{4!}{2!2!}+\frac{4!}{3!}\times 2\right]+{}^5C_1\left[\frac{3!}{2!}\times 2+1\right] = 175$

For C $\rightarrow {}^5C_3 \times \frac{4!}{2!} + {}^5C_2\ [9 \times 2 + 6] = 360$

6. Letters of the word *INDIANOIL* are arranged at random. Probability that the word formed

Column I		Column II	
(A)	Contains the word INDIAN	(P)	$\frac{1}{{}^9C_5}$
(B)	Contains the word OIL	(Q)	$\frac{1}{({}^5C_2)({}^7C_2)(9!)}$
(C)	Begins with I and ends with L	(R)	$\frac{1}{24}$
(D)	Has vowels at the odd places	(S)	$\frac{1}{({}^7C_3)({}^9C_2)}$

Key: A-S, B-R, C-Q, D-P

Hint: Total number of ways of arranging letters of the word INDIANOIL is $\frac{9!}{3!2!}$.

(A) Treating INDIAN as a single object we can permute INDIAN, O, I and L in 4! ways.

$\therefore$ probability of the required event = $\dfrac{4!3!2!}{9!} = \dfrac{1}{\left({}^7C_3\right)\left({}^9C_2\right)}$

(B) We can permute OIL, I, N, D, I, A, N in $\dfrac{7!}{2!2!}$ ways.

$\therefore$ probability of the required event = $\dfrac{\lfloor 7 \lfloor 3 \lfloor 2}{\lfloor 2 \lfloor 2 \lfloor 9} = \dfrac{\lfloor 7 \lfloor 3}{\lfloor 2 \lfloor 9}$

(C) Fixing an I at the first place and L at the last place, we can permute the remaining letters viz. A, D, I, I, N, N, O in $\dfrac{7!}{2!2!}$ ways.

$\therefore$ probability of the required event is = $\dfrac{1}{24}$

(D) Vowels can be arranged at odd places viz 1^{st}, 3^{rd}, 5^{th}, 7^{th} and 9^{th} in $\dfrac{5!}{3!}$ ways.

The remaining letters can be arranged at 4 even places in $\dfrac{4!}{2!}$ ways.

$\therefore$ probability of the required event = $\dfrac{5!4!}{3!2!} \times \dfrac{3!2!}{9!} = \dfrac{1}{{}^9C_5}$

7. 20 Identical balls have to be distributed among 4 jugglers. The number of ways in which these balls can be distributed such that

	Column I		Column II
(A)	All the jugglers get at least one ball is	(p)	885
(B)	All the jugglers get at least one ball and no one gets more than 10 balls is	(q)	1
(C)	All the jugglers get odd number of balls is	(r)	969
(D)	All of them get equal number of balls is	(s)	165

Key: A$\rightarrow$r; B$\rightarrow$pC$\rightarrow$s; D$\rightarrow$q

Hint: A) Coefficient of a^{20} in $\left(a + a^2 + a^3 + ...\right)^4 = a^4(1- a)$

Or coefficient of x^{16} in $(1- a)^{-4} = {}^{19}C_{16} = {}^{19}C_3 = 969$

B) If one variable exceed 10. Let x>10 then

$3 \le y + z + w \le 9 (excluded\ \ zero)$

$\Rightarrow$ the number of positive integral solution

= The(If one variable exceed 10)sum of coefficient of

$a^3, a^4, a^5, a^6, a^7, a^8, a^9$ in $\left(a + a^2 + a^3 + ...\right)^3$

= The sum of coefficient of $a^0, a, a^2 a^3, a^4, a^5, a^6 \ in \ \left(1 + a + a^2 + ...\right)^3$

$= 1 + {}^3C_1 + {}^4C_2 + {}^5C_3 + {}^6C_4 + {}^7C_5 + {}^8C_6$

$= 1 + 3 + 6 + 10 + 15 + 21 + 28 = 84$

Hence the number of positive integral solutions i.e., no variable may exceed 10; zero value excluded

$= 969 - 84 = 885$

C) The coefficient of $a^{20} \ in \ \left(a + a^3 + a^5 + ...\right)^4$

Or the coefficient of $a^{16} \ in \ \left(1 + a^2 + a^4 + ...\right)^4$

Þ The coefficient of $a^{16} \ in \ \left(1 - a^2\right)^{-4} = {}^{11}C_8 = {}^{11}C_3 = 165$

D) All of them get 5 balls

Total number of ways =1

8. Match the Following:

Column I		Column II	
(A)	Number of ways to select n objects from 3n objects of which n are identical and rest are different is $k^{2k-1} + \frac{1}{k}\frac{(kn)!}{(n!)^2}$, k is	(p)	3
(B)	Number of interior point when diagonals of a convex polygon of n side intersect if no three diagonal pass through the same interior point is ${}^nC_\lambda$, then λ is	(q)	2
(C)	Five digit number of different digit can be made in which digit are in descending order is ${}^{10}C_\mu$ then μ is	(r)	4
(D)	Number of term in expansion of $\left(1+3^{1/3}\right)^6$ which are free from radical sign	(s)	5
		(t)	1

Key: (A -q), (B -r), (C -s), (D -p)

Hint: (A) Required number of selection $= {}^{2n}C_0 + {}^{2n}C_1 + ... + {}^{2n}C_n = 2^{2n-1} + \frac{1}{2}\frac{(2n)!}{(n!)^2}$

(B) ${}^nC_4$ (each quadrilateral gives one point of intersection)

(C) $x_4 > x_3 > x_2 > x_1 > x_0$

${}^{10}C_5$ (5 distinct digits selection)

(D) Terms is involving $3^0, 3^{1/3}, 3^{2/3} \to 3$

9. Match the following

Column - I *Column - II*

A)	If n be the numbers between 500 and 4000 can be formed with the digits 2,3,4,5,6 when repetition is not allowed, then n is divisible by	P)	2
B)	If n be the number of even numbers between 200 and 3000 can be formed with the digit 0,1,2,3,4 when repetition is not allowed, then n is divisible by	Q)	3
C)	If n be the number of words that can be made by arranging the letters of the word ROORKEE that neither begin with R nor end with E, then n is divisible by	R)	5
D)	In a class tournament when the participants where to play one game with another, two class players fell ill, having played 3 games each. If the total number of games played is 84, the number of participants at the beginning was k then k is divisible by	S)	11
		T)	17

Key. A - PQ ; B -QT;C-PQRS ;D-Q,R

Sol. A) i) When number is of three digit

5 or 6 only		

First place can be filled in 2 ways, second place can be filled in 4 ways third place can be filled in 3 ways.

$\therefore$ Number of ways $= 2 \times 4 \times 3 = 24$

ii) When number is of four digit

2 or 3 only			

First place can be filled in 2 ways, second place can be filled in 4 ways and third place can be filled in 3 ways and fouth place can be filled in 2 ways.

$\therefore$ Number of ways

$\therefore$ Total number of ways = 24+48=72

$\therefore n = 72 = 2 \times 2 \times 2 \times 3 \times 3$ (P, Q)

B) Case I) : When number of three digits

a) The three digit number with '0' at unit place, first place can be filled in 3 ways and second place can be filled in 3 ways

		O

$\therefore$ Number of ways $= 3 \times 3 = 9$

b) The three digit number with 2 or 4 at unit place. first place can be filled in 2 ways and second place can be filled in 3 ways

		2 or 4

$\therefore$ Number of ways $= 2 \times 3 \times 2 = 12$

case II: When number of four digits

a) The four digit number with '0' at unit place

			0

First place can be filled in 2 ways (1 or 2), second place can be filled in 3 ways and third place can be filled in 2 ways.

$\therefore$ Number of ways $= 2 \times 3 \times 2 = 12$

b) The four digit number with '2' at unit place

			2

First place can be filled in 1 way, second place can be filled in 3 ways and third place can be filled in 2 ways.

$\therefore$ Number of ways $= 1 \times 3 \times 2 = 6$

c) The four digit number with '4' at unit place

			4

first place can be filled in 2 ways (1 or 2), second place can be filled in 3 ways and third place can be filled in 2 ways

$\therefore$ Number of ways $= 2 \times 3 \times 2 = 12$

$\therefore$ Total number of ways $= 9 + 12 + 12 + 6 + 12 = 51$

$\therefore$ n=51= 3×17 (Q , T)

C) ROORKEE

Number of ways $= \dfrac{7!}{2!2!2!} = 630$

Number of words begin with R $= \dfrac{6!}{2!2!} = 180$

Number of words end with E $= \dfrac{6!}{2!2!} = 180$

and number of words begin with

R and end with E $= \dfrac{5!}{2!} = 60$

$\therefore$ Required number of words

$= 630 - 180 - 180 + 60 = 330$

$\therefore$ n = 330= $2 \times 3 \times 5 \times 11$ (P,Q, R,S)

D) Total number of games = ${}^{k-2}C_2 + 6 = 84$

$$^{k-2}C_2 = 78$$

$$k = 15$$

10. Match the following

Column - I *Column - II*

A)	Number of ways selecting 8 balls out of an unlimited collection of Red , blue ,green and yellow balls is	P)	$^{21}C_3$
B)	Number of 4 digit numbers having the sum of the digits equal to 9 is	Q)	Number of ways of distributing 12 apples to four people with each one getting at least one
C)	Number of ways of arranging 3 identical red balls 20 identical white balls in a row so that no two red balls are together is	R)	6
D)	Exponent of 3 in 17! is	S)	$^{11}C_3$

Key. A- Q,S; B-Q,S; C-P; D-R

Sol. A) Non-negative integral solutions of

$R+B+G+Y=8$ $\quad {}^{8+4-1}C_{4-1} = {}^{11}C_3$

B) $D_1+D_2+D_3+D_4=9 \;\; D_1 \neq 0$

$D_1-1+D_2+D_3+D_4=8$

${}^{8+4-1}C_{4-1} = {}^{11}C_3$

C) Select '3' places out of of 21 places= ${}^{21}C_3$

D) $\left[\frac{17}{3}\right]+\left[\frac{17}{3^2}\right]=5+1=6$

11. Match the following

Column - I

Column - II

A)	The number of ways, in which 12 red balls, 12 black balls and 12 white balls be given to 2 children each 18, is	P)	125
B)	The number of ways of forming one team having 5 numbers choosen from 5 boys and 5 girls, so that girls are in majority and atleast one boy is there in the team.	Q)	127
C)	Six bundles of books are to be kept in 6 boxes one in each box. If 2	R)	135

	of the boxes are too small for three of the bundles, the number of ways keeping the bundles in the boxes is		
D)	A bag contains 6 black, 6 blue, 6 red, 6 green and 6 white balls. The balls are numbered 1 to 6 in each colour. The number of ways of drawing 2 balls from the bag such that the balls are of the same colour or the numbers on them are same, is	S)	144

Key. A-Q, B-P, C-S; D-R

Sol. A-Q: Coefficient of x^{18} in $(1+x+\cdots+x^{12})^3$

$$=(1-x^{13})^3(1-x)^{-3}$$

$$=(1-3x^{13})\left(1+{}^3C_1x+\left({}^4C_2\right)x^2+\cdots\right.$$

Which is ${}^{20}C_{18}-3\,{}^7C_5={}^{20}C_2-3\,{}^7C_2=190-63=127$

B-P : A team of 5 can be prepared in having 3G & 2B and 4G & 1B

Total ways = ${}^5C_3\times{}^5C_2+{}^5C_4\times{}^5C_1=125$

C-S : We first fill the small boxes with 2 of the three bundles and the remaining boxes with the remaining bundles.

The desired number is ${}^3P_2\cdot 4!=6\cdot 24=144$

D-R : We choose any colour and take 2 balls or take any number from 1-6 and choose 2 colours.

The desired number is

$${}^5C_1\,{}^6C_2+{}^6C_1\,{}^5C_2=75+60=135$$

12. Match the following

Column - I

Column - II

A)	If 6 letter words are formed using the letters of the word NUMBER and the words are arranged in dictionary order, then the rank of the word NUMBER is	P)	576
B)	Four identical dice are rolled once. The number of ways of getting only prime numbers on them, is	Q)	12
C)	The largest integer n, such that 100! Is divisible by 100^n, is	R)	15
D)	The number of 4-digit numbers which contain not more than two different digits, is	S)	469

Key. A-S, B-R, C-Q; D-P

Sol. A-S; The letters are BEMNRU

Starting with	B, 5!	=120 words
Starting with	E, 5!	= 120 words
Starting with	M, 5!	= 120 words
Starting with	NB, 4!	= 24 words
Starting with	NE, 4!	= 24 words
Starting with	NM, 4!	= 24 words
Starting with	NR, 4!	= 24 words
Starting with	$NUB, 3!$	= 6 words
Starting with	$NUE, 3!$	= 6 words
	NUMBER	=1 words

$\therefore$ The desired rank is $120\times 3+24\times 4+2\times 6+1=469$

B-R : Primes : 2, 3, 5

$aaaa$	—	${}^3C_1=3$
$aaab$	—	${}^3C_2\times 2=6$
$aabb$	—	${}^3C_2=3$
$aabc$	—	${}^3C_3\times 3=3$

Total = 15

C- Q: Number should be divisible by 10^{2n}, so here we have to determine number of zeroes in the value of 100!, each zero comes from the combination of 2 & 5

Power of 5 = $\left[\frac{100}{5}\right]+\left[\frac{100}{5^2}\right]=24$

$\therefore$ No. is divisible by 10^{24} or 100^{12}

D-P : So, in 4 digit number of either all digits are same, digits are from two distinct digits.

All digits are same the number of ways = 9

Two different digits are used (excluding zero) = ${}^9C_2 \times \left(2^4-2\right)=504$

Two different digits are used (including zero) = ${}^9C_1 \times \left(2^3-1\right)=63$

$\therefore$ Total ways = 9 + 504 + 63 = 576

13. digit numbers are formed using the digits 0, 1, 2, 3, 4, 5. Answer the following *Column - I* *Column - II*

A)	How many of them are divisible by 3 if repetition is not allowed	P)	216
B)	How many of them are divisible by 3 if repetition of digits is allowed	Q)	108
C)	How many of them are divisible by 3 but not by 2 if repetition is not allowed	R)	1000
D)	Number of 4 digit numbers divisible by 5 (without repetition)	S)	42

Key. A – p;B – r;C – q;D -s

Sol. A) A 5 - digit number is divisible by 3 if the sum of the digits is divisible by 3. We leave either ‘0’ or ‘3’. Leaving ‘0’ we get 5! number; we leaving 3 we get 4 x 4! numbers

Hence answer is 5! + 4.4! = 216

(B) Find the first place with any digit after than ‘0’

This can be done in 4 ways. The next 3 places can be filles each is 5 ways. Units place can be filled only in 2 ways. The answer is 4 x 5^3 x 2 = 1000

(C) Leaving 0 we use 1, 2, 3, 4, 5

Fill units place with an odd digit - 3 ways

Fill the remaining places in 4! ways

Then we get 3 x 4! = 72

Leaving 3 : Fill the units place with odd digit - 2 ways Fill the 1^{st}, 2^{nd}, 3^{rd}, 4^{th} places in 3, 3, 2, 1 ways respectively.

Thus the answer is 72 + 36 = 108

D-S ;

Numbers of 4 digits divisible by 5

$\square\ \square\ \square\ \boxed{10} \rightarrow {}^4P_3 = 24$

$\square\ \square\ \square\ \boxed{5} \rightarrow {}^4P_3$

$\boxed{10}\ \square\ \square\ \boxed{5} \rightarrow {}^3P_2$

$= 24 - 6 = 18$

$\therefore$ Total : 24+18=42

14. Let n(P) represents the number of points P(α, β) lying on the rectangular hyperbola xy = 15!, under the conditions given in column I, match the value of n(P) given in column II.

	Column –I		Column –II
(A)	$\alpha, \beta \in I$	(p)	32
(B)	$\alpha, \beta \in I^+$ and HCF $(\alpha, \beta) = 1$	(q)	64
(C)	$\alpha, \beta \in I^+$ and α divides β	(r)	96
(D)	$\alpha, \beta \in I^+$ and HCF $(\alpha, \beta) = 35$	(s)	4032
		(t)	8064

Key. (A–t), (B–q), (C–r), (D–p)

Sol. $xy = 15! = 2^{11}\, 3^6\, 5^3\, 7^2\, 11^1\, 13^1$

(A) No. of the integral solutions = no. of ways of fixing x

= the no. of factors of 15!

= (1 + 11) (1 + 6) (1 + 3) (1 + 2) (1 + 1) (1 + 1) = 4032.

⇒ Total no. of integral solutions = 2 × 4032 = 8064

(B) HCF (α, β) = 1. So identical primes should not be separated

So, no. of solutions = 2^6 = 64

(C) The largest number whose perfect square can be made with 15! is $2^5\, 3^3\, 5^{-1}\, 7^1$

So the no. of ways of selecting x will be

(1 + 5) (1 + 3) (1 + 1) (1 + 1) = 96

(D) Let $\alpha = 35\alpha_1$ and $\beta = 35\beta$ where HCF $(\alpha_1, \beta_1) = 1$

Now, $\alpha\beta = 15! \Rightarrow \alpha_1\beta_1 = 2^{11}\, 3^6\, 5^1\, 11^1\, 13^1$

So, no. of solutions = 2^5 = 32.

15. Consider all possible permutations of letters of the word ENDEANOEL.

Column – I Column – II

(A)	The number of permutations containing the word ENDEA is	(P)	$\lfloor 5$
(B)	The number of permutations in which the letter E occurs in the first and the last positions is	(Q)	$2\lfloor 5$
(C)	The number of permutations in which non of the letters D, L, N occurs in the last five positions	(R)	$7\lfloor 5$
(D)	The number of permutations in which the letters A, E, O occur only in odd positions is	(S)	$21\lfloor 5$

Key. A – P; B – S; C – Q; D – Q

Sol. Number of arrangements = ${}^5P_5 = 5!$ $\therefore$ A – p

Number of arrangements = $\frac{7!}{2!} = 21\,5!$ $\therefore$ B – s

Number of arrangements = $\frac{4!}{2!} \times \frac{5!}{3!} = 2 \times 5!$ $\therefore$ C – q

Number of arrangement = $\frac{5!}{3!} \times \frac{4!}{2!} = 2 \times 5!$ $\therefore$ D – q

16. Consider all possible permutations of the letters of the word ENDEANOEL. Match the Statements/Expressions in Column I with the Statements/Expressions in Column II.

Column – I	Column – II
a) The number of permutations containing the word ENDEA, is	p) $5!$
b) The number of permutations in which the letter E occurs in the first and the last positions, is	q) $2 \times 5!$
c) The number of permutations in which none of the letters D, L, N occur in the last five positions, is	r) $7 \times 5!$
d) The number of permutations in which the letters A, E, O occur only in odd positions, is	s) $21 \times 5!$

Key. $a \to p;\ b \to s;$
$c \to q; d \to q$

Sol. ENDEANOEL

a) Consider ENDEA as a single unit

$\Rightarrow$ENDA, N, O,E, L $\Rightarrow \lfloor 5$

b) After filling E's at first and last positions remaining letters are N, D, A, N, O, E, L

$$\Rightarrow \frac{\lfloor 7}{\lfloor 2} = 21\lfloor 5$$

c) D, L, M, N can't be present in the last 5 positions

$\Rightarrow$they occupy 1st four positons, for which no of ways $= \frac{\lfloor 4}{\lfloor 2} = 12$

And the remaining 5 letters : E, E, E, A, O will occupy last 5 positions in $\frac{\lfloor 5}{\lfloor 3}$ ways

$\Rightarrow$required no.of ways $= 12 \times \frac{\lfloor 5}{\lfloor 3} = 2\lfloor 5$

d) A,E,O $\Rightarrow$ A, E,E,E,O

In fact there are available only 5 odd positions

17. There are 2 Indian couples, 2 American couples and one unmarried person

Column-I	Column-II
a) The total number of ways in which they can sit in a row such that an Indian wife and American wife are always on either side of the unmarried person, is	p) 22680
b) The total numbers of ways in which they can sit in row such that an unmarried person always occupy the middle position is	q) 5760
c) The total number of ways in which they can sit round a circular table such that an Indian wife and an American wife are always on either side of the unmarried person, is	r) 40320
d) If all the nine persons are to be interviewed one by one then the total number of ways of arranging their interviews such that no wife gives interview before her husband, is	s) 24320

Key. a) r; b) r; c) q; d) p

Sol. a) one Indian wife and one American wife can be selected in ${}^2C_1 \times {}^2C_1$ ways and keeping an unmarried person in between these two wives the total number of linear arrangements are ${}^2C_1 \times {}^2C_1 \times \lfloor\underline{7} \times \lfloor\underline{2} = 40320$

b) Required number of ways $\lfloor\underline{8} = 40320$

c) Required number of ways $\lfloor\underline{(7-1)} \times \lfloor\underline{2} \times {}^2C_1 \times {}^2C_1 = 5760$

d) Number of ways in which interviews can be arranged $= 9 \times {}^8C_2 \times {}^6C_2 \times {}^4C_2 \times {}^2C_2 = 22680$

18. **Match the following: -**

Column-I	Column-II
a) The number of positive unequal integral solutions of the equation $x+y+z+t=20$	p) 504
b) The number of zeros at the end of $\lfloor\underline{100}$ is	q) 36
c) Number of congruent triangles that can be formed using the vertices of a regular polygon of 72 vertices such that the number of vertices of the polygon between any two consecutive vertices of triangle must be same, is	r) 24
d) The number of ways in which the letters of the word " SUNDAY" be arranged so that they neither begin with s nor end with Y, is	s) 552

Key. a) s; b) r; c) r; d) p

Sol. a)We can assume that $x<y<z<t$ without loss of genterality. Now put $x_1 = x, x_2 = y-x, x_3 = z-y$ and $x_4 = t-z$, Then $x_1, x_2, x_3, x_4 \geq 1$ and the given equation becomes $4x_1 + 3x_2 + 2x_3 + x_4 = 20$.The number of positive integer solutions of this equation = 552

b) $\lfloor\underline{100} = 2^{97} \times 3^b \times 5^{24} \times 7^d \times ....$

c) $\frac{72}{3} = 24$

d) $6! - 2(5!) + 4! = 504$

19.

	Column I		Column II
(A)	In a polygon the number of diagonals is 54. The number of side of the polygon is	(p)	3
(B)	The number of divisors of the form 4n + 2 (n ≥ 0) of the integer 240 is	(q)	12
(C)	The total number of selection of at least one and of most n things from (2n+1) different thing is 63. Then the value of n + 1 is	(r)	4
(D)	The number of distinct rational number x such that 0<x<1 and x = $\frac{p}{q}$ where p, q $\in$ {1, 2, 3, 4, 5, 6} is K. then K =	(s)	11

Key. (A)-q; (B)-r; (C)-r; (D)-s

Sol. Let n be the number of sides.

$\therefore$ number of diagonals = ${}^nC_2 - n$ = 54 (given)

$$\Rightarrow \frac{n(n-1)}{2} - = 54$$

$$\Rightarrow n^2 - n - 2n = 108$$

$$\Rightarrow n^2 - 3n - 108$$

$$\Rightarrow n^2 - 3n - 108 = 0$$

$$\Rightarrow (n-12)(n+9) = 0 \Rightarrow n = 12$$

($\because$ n cannot be -ve)

(B) The number of divisors of the form $4n + 2(n \geq 0)$ of the integer 240 is 0, 6, 10, 30.

These are 4 in numbers .

$\therefore B - p, B - r$

(C) $63 = {}^{2n+1}C_1 + {}^{2n+1}C_2 + ... + {}^{2n+1}C_n$

$\therefore \; 64 = {}^{2n+1}C_0 + {}^{2n+1}C_1 + ... + {}^{2n+1}C_n$

$$= \frac{1}{2}\left({}^{2n+1}C_0 + {}^{2n+1}C_1 + ... + {}^{2n+1}C_{2n+1}\right) = \frac{1}{2}.2^{n+1} \Rightarrow 128 = 2^{2n+1}$$

$$\Rightarrow \quad 2^7 = 2^{2n+1}$$

$\Rightarrow \quad 7 = 2n + 1 \Rightarrow n = 3$

$\therefore \quad n + 1 = 3 + 1 = 4$

$\therefore \quad c - p, c - r$

(d) As $0 < x < x1,$ we have $p < q.$

The number of rational numbers

$= 5 + 4 + 3 + 2 + 1 = 15.$

When p, q have a common factor, we get some rational numbers, which are not different from those already counted. There are 4 such numbers

$\frac{2}{4}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}.$

$\therefore$ the required number of rational numbers

$= 15 - 4 = 11$

$\therefore \quad K = 11 \Rightarrow K + 1 = 12$

$\therefore$ D-q, D-s

20.

	Column I		Column II
(A)	The maximum number of points at which 5 straight lines interact is	(p)	47
(B)	The number of distinct divisors of $2^4.3^5.5^3$ is	(q)	120
(C)	In how many ways can at least one selection be made out of 2 mangoes, 3 apples and 3 oranges ?	(r)	240
(D)	In a students union meeting in a school 16 students stand up. Each shake hands with each other exactly once. Total number of handshakes is	(s)	10

Key. (A)- s; (B)-q; (C)-p; (D)-q

Sol. (A) Maximum number of points of intersection = ${}^5C_2 = \frac{5 \times 4}{1 \times 2} = 10. \quad \therefore$ A – s

(B) The number of distinct divisors of $2^4.3^5.5^3$ =

$(4+1)(5+1)(3+1) = 5 \times 6 \times 4 = 120$

$\therefore$ B – q, B – r

(C) Reqd. number of ways

= (2 + 1) (3+1) (3+1) -1 = 48 -1 = 47

$\therefore$ C – p

(D) Number of handshakes = ${}^{16}C_2 = \frac{16\times15}{2} = 120$

$\therefore$ D –q, D –r.

21. Consider the equation $x+y+z+p=13$, where x, y,z and p are all integers.

	COLUMN – I		COLUMN - II
A	The number of non-negative integral solutions is	P	20
B	The number of positive integral solutions is	Q	200
C	The number of solutions which belong to [1, 4] is	R	220
D	The number of solutions in which $x\ge1,\ y\ge2, z\ge3\ \ and\ \ p\ge4$ is	S	560
		T	$11C_2 + 11C_3$

Key.
$A \to s;$
$B \to r,t;$
$C \to p;$
$D \to p;$

Sol. $(A)\ \binom{13+3}{3} = \binom{16}{3} = 560$

$(B)\ \binom{13-4+3}{3} = \binom{12}{3} = 220$

$(C)\ \binom{13-10+3}{3} = \binom{6}{3} = 20$

(D) Coefficient of $x^{13}\ in\ \left(x+x^2+x^3+x^4\right)^4$

= coefficient of $x^9\ in\ \left(1+x+x^2+x^3\right)^4$

= coefficient of $x^9\ in\ (1+x)^4\left(1+x^2\right)^4$

= 4 + 16 = 20

22. Match the following

Four dice are rolled .Then the number of ways in which

Column-I		**Column-II**
a)	No die shows 3 is	p) 671
b)	At least one die shows 3 is	q) 125
c)	Sum of the upturned faces is 10 is	r) 80
d)	Sum of the upturned faces is 11 is	s) 625
		t) 104

Key. A $\to$ s ; B $\to$ p; C $\to$ r; D $\to$ t

Sol. a) 5^4

b) $6^4 - 5^4$

c) $coeff\ of\ x^{10}\ in\ \left(x+x^2+x^3+x^4+x^5+x^6\right)^4$

$= coeff\ of\ x^6\ in\ \left(1+x+x^2+x^3+x^4+x^5\right)^4$

$= coeff\ of\ x^6\ in\ \left(\dfrac{1-x^6}{1-x}\right)^4$

$= coeff\ of\ x^6\ in\ \left(1-x^6\right)^4 (1-x)^{-4}$

$= coeff\ of\ x^6\ in\ \left(1-4x^6\right)^4 \left(1+{}^4c_1 x+{}^5c_2 x^2+{}^6c_3 x^3+...+{}^9c_6 x^6\right)$

$= {}^9c_6 - 4$

$= 122$

d) $coeff\ \ of\ x^{11} in\left(x+x^2+x^3+x^4+x^5+x^6\right)^4$

$= coeff\ \ of\ x^7 in\left(1-x^6\right)^4 (1-x)^{-4}$

$= coeff\ \ of\ x^7 in\left(1-4x^6+...\right)\left(1-{}^4c_1 x+{}^5c_2 x^2+{}^6c_3 x^3+{}^9c_6 x^6+{}^{10}c_7 x^7\right)^{-4}$

$= {}^{10}c_7 - 16$

23. Consider all possible permutations of the letters of the word " RACHITIHCAR"

Column I		Column II	
(A)	The number of words containing the word ACHIT is	(P)	56700
(B)	The number of words beginning with 'RA' and ending with 'AR' is	(Q)	**630**
(C)	The number of words in which vowels occur at the odd places is	(R)	45360
(D)	The number of words in which the word IIT appears is	(S)	2520

Key. (A- s), (B-q), (C-p), (D-r)

Sol. a) ACHIT, R, R, I, H, C, A $\to \dfrac{7!}{2!} = 2520$

b) C, C, H, H, I, I, T can be arranged in $\dfrac{7!}{2!2!2!} = 630$ ways

c) 4 out of 6 odd places can be selected in ${}^6C_4$ ways and A, A, I, I can be arranged in $\dfrac{4!}{2!2!}$

hence it is ${}^6C_4 \times \dfrac{4!}{2!2!} \times \dfrac{7!}{2!2!2!} = 56700$

d) IIT, R, R, A, A, C, C, H, H $\to \dfrac{9!}{2!2!2!} = 45360$