CLASS : CC (Advanced)   ***ELLIPSE & HYPERBOLA***   **TEST-4**

M.M.: 63   **Time: 60 Min**

PART-A
[SINGLE CORRECT CHOICE TYPE]

Q.1 to Q.8 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct.   **[8 × 3 = 24]**

Q.1   If the line $y = ax + b$ is tangent to the curve $f(x) = x - x^3$ at the point $(-1, 0)$ then the eccentricity of the

ellipse $\dfrac{x^2}{a^4} + \dfrac{y^2}{b^2} = 1$ is

(A) $\dfrac{1}{2}$ (B) $\dfrac{1}{\sqrt{2}}$ (C) $\dfrac{\sqrt{3}}{2}$ (D) $\dfrac{2}{3}$

Q.2   Let LL' be the latus rectum through the focus of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ and A' be the farther

vertex. If $\Delta A'LL'$ is equilateral, then the eccentricity of the hyperbola is

(A) $\sqrt{3}$ (B) $\sqrt{3} + 1$ (C) $\dfrac{\sqrt{3} + 1}{\sqrt{2}}$ (D) $\dfrac{\sqrt{3} + 1}{\sqrt{3}}$

Q.3   The curve $y = (|x| - 1)\, \text{sgn}(x - 1)$ divides $\dfrac{9x^2}{64} + \dfrac{4y^2}{25} = \dfrac{1}{\pi}$ into two parts of areas A_1 and A_2 $(A_1 < A_2)$, then

[**Note :** sgn(k) denotes signum function of k.]

(A) $A_1 = \dfrac{7}{6}$ (B) $A_1 : A_2 = 7 : 13$ (C) $A_2 = \dfrac{13}{6}$ (D) $A_2 - A_1 = 1$

Q.4 The locus of centre of circle touching the circles $(x+3)^2 + y^2 = 16$ and $(x-3)^2 + y^2 = 4$ externally is

(A) $\dfrac{x^2}{8} - y^2 = 1$ 　　(B) $x^2 - \dfrac{y^2}{8} = 1$ 　　(C) $\dfrac{x^2}{4} - y^2 = 1$ 　　(D) $x^2 - \dfrac{y^2}{4} = 1$

Q.5 If normal at any point P on the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ $(a>b>0)$ meet the major and minor axes at Q and R respectively so that $3PQ = 2PR$, then the eccentricity of ellipse is equal to

(A) $\dfrac{1}{\sqrt{3}}$ 　　　　(B) $\sqrt{\dfrac{2}{3}}$ 　　　　(C) $\dfrac{\sqrt{3}}{2}$ 　　　　(D) $\dfrac{1}{\sqrt{2}}$

Q.6 If the ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{b^2} = 1$ and the hyperbola $\dfrac{x^2}{9} - \dfrac{y^2}{7} = 1$ intersects orthogonally, then b is equal to

(A) $\sqrt{3}$ 　　　　(B) 3 　　　　(C) $2\sqrt{3}$ 　　　　(D) 9

Q.7 The point on the ellipse $x^2 + 2y^2 = 6$ which is nearest to the line $x - y = 7$ is
(A) $(2, 1)$ 　　　　(B) $(2, -1)$ 　　　　(C) $(-2, 1)$ 　　　　(D) $(-2, -1)$

Q.8 If the line $\dfrac{x}{a} + \dfrac{y}{b} = \sqrt{2}$ touches the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ $(a>b)$, then the eccentric angle of the point of contact is

(A) $\dfrac{\pi}{4}$ 　　　　(B) $\dfrac{3\pi}{4}$ 　　　　(C) $\dfrac{5\pi}{4}$ 　　　　(D) $\dfrac{7\pi}{4}$

Q.9 to Q.11 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct. **[3 × 3 = 9]**

Paragraph for question nos. 9 to 11

Let the line $2mx + y\sqrt{1-m^2} = 1$ touches a fixed ellipse whose axes are coordinate axes, for infinitely many real values of m.

Q.9 The equation of auxiliary circle of the ellipse is

(A) $x^2 + y^2 = \dfrac{1}{4}$ 　　(B) $x^2 + y^2 = 1$ 　　(C) $x^2 + y^2 = \dfrac{3}{4}$ 　　(D) $x^2 + y^2 = \dfrac{1}{2}$

Q.10 The foci of ellipse are

(A) $(0, \pm\sqrt{3})$ 　　(B) $(\pm\sqrt{3}, 0)$ 　　(C) $\left(\pm\dfrac{\sqrt{3}}{2}, 0\right)$ 　　(D) $\left(0, \pm\dfrac{\sqrt{3}}{2}\right)$

Q.11 The locus of point of intersection of perpendicular tangents of the ellipse is

(A) $x^2 + y^2 = 2$ 　　(B) $x^2 + y^2 = \dfrac{3}{2}$ 　　(C) $x^2 + y^2 = \dfrac{5}{4}$ 　　(D) $x^2 + y^2 = \dfrac{7}{4}$

[MULTIPLE CORRECT CHOICE TYPE]

Q.12 to Q.16 has four choices (A), (B), (C), (D) out of which **ONE OR MORE** may be correct. **[5 × 4 = 20]**

Q.12 A normal to the hyperbola $x^2 - 4y^2 = 4$ meets the x and y axes at A and B respectively. The locus of point of intersection of the straight lines drawn through A and B perpendicular to the x and y axes respectively is a hyperbola. Then

(A) eccentricity equals $\sqrt{3}$
(B) length of latus rectum equals 20.
(C) equation of auxiliary circle equals $x^2 + y^2 = 25$.
(D) distance between foci equals $5\sqrt{5}$

Q.13 If a pair of variable straight lines $x^2 + 4y^2 + \alpha xy = 0$ (α is real parameter) cuts the ellipse $x^2 + 4y^2 = 4$ at two points A and B, then the locus of the point of intersection of tangents at A and B of ellipse is
(A) $x - 2y = 0$ 　　(B) $2x - y = 0$ 　　(C) $x + 2y = 0$ 　　(D) $2x + y = 0$

Q.14 Let P be a point on the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$. The tangent drawn at P meets the coordinate axes at

A and B, then
(A) minimum area of $\triangle OAB$ is 6. (B) minimum area of $\triangle OAB$ is 12.

(C) minimum length of AB is 5. (D) minimum length of AB is $\sqrt{13}$.

(where O is origin)

Q.15 Tangents are drawn from points on the line $3x + 4y = 24$ to the curve $x^2 + \dfrac{y^2}{4} = 1$.

The point of concurrency of the chords of contact lie on the curve
(A) $16x - 3y = 0$ (B) $9y^2 = 32x$ (C) $24x + 24y = 19$ (D) $y = 2x$

Q.16 Two lines are drawn from point $P(\alpha, \beta)$ which touches $y^2 = 8x$ at A, B and touches $\dfrac{x^2}{4} + \dfrac{y^2}{6} = 1$ at

C, D, then
(A) $\alpha + \beta = -4$ (B) $\alpha\beta = 4$

(C) Area of $\triangle PAB = 128\sqrt{2}$ (D) Area of $\triangle PAB = 32\sqrt{2}$

PART-D
[INTEGER TYPE]

Q.1 to Q.2 are "Integer Type" questions. (The answer to each of the questions **are upto 4 digits**) **[2 × 5 = 10]**

Q.1 If the locus of centres of the circles which cut the ellipse $\dfrac{x^2}{8} + \dfrac{y^2}{6} = 1$, at $P\left(2, \sqrt{3}\right)$ orthogonally is

$\sqrt{a}\, x + 2y = b\sqrt{c}$, where $a, b, c \in N$ and b, c are relatively prime, then find the value of $\left|\dfrac{a+b+c}{2}\right|$.

Q.2 Consider an ellipse $E : \dfrac{(3x-4y+5)^2}{100} + \dfrac{(4x+3y-10)^2}{50} = 1$. M and m are lengths of major and minor axes respectively. If p_1 and p_2 are perpendicular distances on major and minor axes from a point P in the x-y plane such that $m \le p_1 + p_2 \le M$, then P lies in the region whose area is Δ. Find the sum of the digits in Δ.

CLASS : CC (Advanced)　　　　*ELLIPSE & HYPERBOLA*　　　　**TEST-4**

ANSWER KEY
PART-A

Q.1	C	Q.2	D	Q.3	B	Q.4	B	Q.5	A
Q.6	B	Q.7	B	Q.8	A	Q.9	B	Q.10	D
Q.11	C	Q.12	BD	Q.13	AC	Q.14	AC	Q.15	ABC
Q.16	AD								

PART-D

Q.1	5	Q.2	7