CLASS : CC (Advanced)   *PARABOLA*   TEST-3

M.M.: 68   **Time: 60 Min**

PART-A
[SINGLE CORRECT CHOICE TYPE]

Q.1 to Q.4 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct.   **[4 × 3 = 12]**

Q.1 Let a ray of light is travelling along the line $4x + 3y - 2k = 0$ and strikes a parabola at $(1, p)$. After reflection it moves along the line $3x - 4y + k = 0$. If the directrix of the parabola is the line $3x - 4y + 15 = 0$ then the latus rectum of the parabola is

(A) 4   (B) 6   (C) 8   (D) 16

Q.2 A chord PQ is a normal to the parabola $y^2 = 4ax$ at P and subtends a right angle at the vertex. If $SQ = \lambda SP$ where S is the focus then the value of λ, is

(A) 1   (B) 2   (C) 3   (D) 4

Q.3 A ray of light moving parallel to the x-axis get reflected from a parabolic mirror whose equation is $(y - 2)^2 = 4(x + 1)$. After reflection, the ray must pass through the point

(A) $(0, 2)$ (B) $(2, 0)$ (C) $(0, -2)$ (D) $(-1, 2)$

Q.4 If the line $y - \sqrt{3}\,x + 3 = 0$ cut the parabola $y^2 = x + 2$ at A and B, then the value of $PA \cdot PB$ is [where $P = (\sqrt{3}, 0)$]

(A) $\dfrac{4(2 + \sqrt{3})}{3}$ (B) $\dfrac{4(2 + \sqrt{3})}{\sqrt{3}}$ (C) $\dfrac{4(2 - \sqrt{3})}{3}$ (D) $\dfrac{4(2 - \sqrt{3})}{\sqrt{3}}$

Q.5 to Q.14 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct.　　　**[10 × 3 = 30]**

Paragraph for question nos. 5 & 6

Normals are drawn from $P(4, 1)$ to the parabola $y^2 = 4x$. The tangents at the feet of normals to the parabola $y^2 = 4x$ form a triangle ABC.

Q.5 The centroid of triangle ABC is equal to

(A) $\left(-\dfrac{4}{3}, 0\right)$　　　(B) $\left(\dfrac{-1}{3}, \dfrac{1}{3}\right)$　　　(C) $\left(\dfrac{1}{3}, \dfrac{-1}{3}\right)$　　　(D) $\left(\dfrac{-2}{3}, 0\right)$

Q.6 The area of triangle ABC is equal to

(A) $\dfrac{\sqrt{5}}{4}$　　　(B) $\dfrac{\sqrt{5}}{8}$　　　(C) $\dfrac{\sqrt{5}}{2}$　　　(D) $\dfrac{\sqrt{5}}{16}$

Paragraph for question nos. 7 & 8

Consider a circle $S : (x + 2)^2 + y^2 = 1$ and a parabola $P : y^2 = 4x$

Q.7 If n is number of common tangents to both the curves and $m_1, m_2, \ldots\ldots m_n$ are the slopes of these tangents then $\displaystyle\sum_{i=1}^{n} m_i^2$ equals

(A) $\dfrac{5}{3}$　　　(B) $\dfrac{10}{3}$　　　(C) $\dfrac{20}{3}$　　　(D) 10

Q.8 If pair of tangents are drawn from each and every point on the circle S to the parabola P, then maximum perpendicular distance from the origin to the corresponding chord of contact is

(A) 3　　　(B) 4　　　(C) 5　　　(D) 6

Paragraph for question nos. 9 to 11

Consider the parabola $x^2 = 4y$ and circle $C : x^2 + (y-5)^2 = r^2$ $(r > 0)$. Given that the circle C touches the parabola at the points P and Q.

Q.9   Radius of the circle C is
(A) 2   (B) 3   (C) 4   (D) 5

Q.10   The distance between chord PQ and directrix of the parabola is

(A) $\dfrac{5}{3}$ (B) 3   (C) 4   (D) $\dfrac{4}{3}$

Q.11   The equation of the circle which passes through the vertex of the parabola $x^2 = 4y$ and touches it at the point $M(-4, 4)$, is
(A) $x^2 + y^2 - 10x - 18y = 0$ (B) $x^2 + y^2 - 8x - 16y = 0$
(C) $x^2 + y^2 + 8x = 0$ (D) $x^2 + y^2 - 8y = 0$

Paragraph for question nos. 12 to 14

Consider the following lines $L_1 : x = y + 1$, $L_2 : x + y = 5$ and $L_3 : y = 4$. Let L_1 is axis to a parabola, L_2 is tangent at the vertex to this parabola and L_3 is another tangent to this parabola at some point P. Let C be the circle circumscribing the triangle formed by tangent and normal at point P and axis of parabola.
The tangent and normal at the extremities of latus rectum of this parabola forms a quadrilateral $ABCD$.

Q.12   The equation of circle C is
(A) $x^2 + y^2 - 2x = 31$ (B) $x^2 + y^2 - 2y = 31$
(C) $x^2 + y^2 - 2x - 2y = 31$ (D) $x^2 + y^2 + 2x + 2y = 31$

Q.13   Which of the following parabola is equal to the given parabola?

(A) $y^2 = 16\sqrt{2}\,x$ (B) $x^2 = -4\sqrt{2}\,y$ (C) $y^2 = -\sqrt{2}\,x$ (D) $y^2 = 8\sqrt{2}\,x$

Q.14   The area of quadrilateral $ABCD$ is equal to
(A) 8   (B) 16   (C) 32   (D) 64

[MULTIPLE CORRECT CHOICE TYPE]

Q.15 to Q.18 has four choices (A), (B), (C), (D) out of which **ONE OR MORE** may be correct. **[4 × 4 = 16]**

Q.15 The common tangents of the circle $(x + 1)^2 + y^2 = 1$ and the parabola $y^2 = 4x$ is(are)

(A) $x = 0$ 　　　(B) $y = \dfrac{1}{\sqrt{3}} x + \sqrt{3}$ 　(C) $y = \dfrac{-1}{\sqrt{3}} x - \sqrt{3}$ 　(D) $y = x$

Q.16 Suppose two tangents drawn to a parabola at points A and B on it are $x + y + 2 = 0$ and $x - y + 4 = 0$. While the normals drawn at A and B meet at C(1,2) lying on its axis, then
(A) Equation of axis of parabola is $x - 4y + 7 = 0$.

(B) Length of Latus rectum of parabola is $\sqrt{17}$.
(C) Equation of axis of parabola is $x + 4y - 9 = 0$

(D) Length of Latus rectum of parabola is $\sqrt{15}$.

Q.17 Let PQ be a chord of parabola $y^2 = 4x$. A circle drawn with PQ as a diameter passes through the vertex V of the parabola. If area $(\Delta PVQ) = 20$, then the coordinates of P are
(A) $(16, 8)$ 　　　(B) $(16, -8)$ 　　　(C) $(-16, 8)$ 　　　(D) $(-16, -8)$

Q.18 The circle $x^2 + y^2 - 4x = 6$ and the parabola $y^2 = 9x$ intersect at P and Q in the first and the fourth quadrants respectively. Tangents to the circle at P and Q intersect the x-axis at R and tangents to the parabola at P and Q intersect the x-axis at S. Then
(A) length of PQ equals 8.

(B) radius of circle described on RS as diameter equals $\dfrac{7}{2}$.

(C) $(1, -1)$ is a point on the common chord of circle and parabola.

(D) area of triangle PRS equals $\dfrac{27}{2}$.

PART-C
[INTEGER TYPE]

Q.1 & Q.2 are "Integer Type" questions. (The answer to each of the questions **are <u>Single digits</u>**) **[2 × 5 = 10]**

Q.1 Chord of the curve $3x^2 - y^2 - 2x + 4y = 0$, which subtend a right angle at the origin, always passes through vertex of a concave up parabola whose axis is parallel to y-axis and length of latus rectum is 8. If extremity of latus rectum of parabola is $L_1(\alpha, \beta)$, $L_2(\gamma, \delta)$, then find $(\alpha + \beta + \gamma + \delta)$.

Q.2 Consider a parabola $y^2 = -16x$ and P(x, y) be any point lying on it. If maximum and minimum value of
$z = \dfrac{2(y - 1)}{2x - 1}$ are M and m respectively, then find $|M + m|$.

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ANSWER KEY
PART-A

Q.1	A	Q.2	C	Q.3	A	Q.4	A	Q.5	D
Q.6	C	Q.7	B	Q.8	A	Q.9	C	Q.10	C
Q.11	B	Q.12	A	Q.13	D	Q.14	D	Q.15	ABC
Q.16	AB	Q.17	AB	Q.18	BC				

PART-C

Q.1	2	Q.2	2