CLASS: CC (Advanced) *Straight line and circle* **TEST-1**

M.M.: 64 **Time: 60 Min**

PART-A
[SINGLE CORRECT CHOICE TYPE]

Q.1 to Q.9 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct. **[9 × 3 = 27]**

Q.1 The least positive integral value of 'b' for which the point $(2b + 3, b^2)$ lies above the line $3x - 4y - a(a - 2) = 0 \ \forall\ a \in R$, is
(A) 1 (B) 2 (C) 3 (D) 4

Q.2 The line $3x - 4y + 7 = 0$ is rotated through an angle of $\frac{\pi}{4}$ in the clockwise direction about $P(-1, 1)$. The equation of line in its new position is
(A) $x + 7y - 6 = 0$ (B) $5y - x - 6 = 0$ (C) $7y + 13x + 6 = 0$ (D) $x + y = 0$

Q.3 The equations of perpendicular bisectors of two sides AB and AC of a triangle ABC are $x + y + 1 = 0$ and $x - y + 1 = 0$ respectively. If circumradius of ΔABC is 2 units and the locus of vertex A is $x^2 + y^2 + gx + c = 0$, then $(g^2 + c^2)$, is equal to
(A) 4 (B) 5 (C) 9 (D) 13

Q.4 If the circles $x^2 + y^2 + 2ax + 2by + c = 0$ and $x^2 + y^2 + 2bx + 2ay + c = 0$ where $c > 0$, have exactly one point in common then the value of $\frac{(a+b)^2}{2c}$ is
(A) 1 (B) $\sqrt{2}$ (C) 2 (D) 1/2

Q.5 The locus of the centres of circles passing through the origin and intersecting the circle $x^2 + y^2 - 5x + 3y = 1$ orthogonally, is
(A) circle of radius 1 (B) circle of radius $\frac{1}{2}$ (C) straight line of slope $\frac{3}{5}$ (D) straight line of slope $\frac{5}{3}$

Q.6 If circular arcs $\overset{\frown}{AC}$ and $\overset{\frown}{BC}$ have centres at $B(-\alpha, 0)$ and $A(\alpha, 0)$ respectively and equation of circle which touches both arcs $\overset{\frown}{AC}$ and $\overset{\frown}{BC}$ and line AB is $(x-a)^2 + (y-b)^2 = r^2$, $r > 0$, if length of arc $BC = 8\pi$, the value of $|a + b + r - \alpha|$ equals
(A) 2 (B) 3 (C) 5 (D) 6

Q.7 The number of integral value(s) of p for which the point $\left([2p-4], \{3p^2 - 2p + 1\}\right)$ lies inside the circle $x^2 + y^2 = 9$ is(are)
[**Note :** [y] and {y} denote gretest integer and fractional part functions of y respectively.]
(A) 2 (B) 3 (C) 4 (D) 5

Q.8 The members of a family of circles are given by the equation $2(x^2 + y^2) + \lambda x - (1 + \lambda^2)y = 10$. The number of circles belonging to the family that are cut orthogonally by the fixed circle $x^2 + y^2 + 4x + 6y + 3 = 0$, is
(A) 2 (B) 1 (C) 0 (D) 3

Q.9 Given 2 points A (– 2, 0) and B (0, 4), then sum of abscissa and ordinate of the point C on the line $x - y = 0$ so that perimeter of ΔABC is least is
(A) – 2 (B) 0 (C) 1 (D) 2

[PARAGRAPH TYPE]

Q.10 to Q.14 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct. **[5 × 3 = 15]**

Paragraph for question nos. 10 & 11

A straight line $L : 4x - 4y + 3 = 0$ is rotated in clockwise about the point where the line cuts the y-axis and a circle S_1 whose centre is $\left(\lambda, \frac{3}{4}\right)$ touches both the lines L and L_1 (L_1 is the line obtained after rotation) and the x-axis.

Q.10 The value of $[\lambda]$ is equal to

(A) 1 (B) 2 (C) 8 (D) 9

[**Note :** [k] denotes greatest integral value of k.]

Q.11 If area of the triangle formed by the lines L_1, angle bisector between L & L_1 and the x-axis is $\frac{p}{q}$, $p, q \in N$ then least value of $(p + q)$ equals

(A) 3 (B) 17 (C) 41 (D) 56

Paragraph for question nos. 12 to 14

A straight line L with negative slope passes through the point (9, 4) and cuts the positive coordinate axes at points P and Q respectively.

Q.12 The minimum value of OP + OQ, as L varies, where O is the origin is

(A) 18 (B) 25 (C) 36 (D) 49

Q.13 The area of triangle OPQ, when OP + OQ becomes minimum (where O is the origin) is

(A) 75 (B) 90 (C) 125 (D) 150

Q.14 Let R be a moving point on xy plane such that OPRQ becomes a rectangle then the locus of R, as L varies is

(A) $\frac{x}{9} + \frac{4}{y} = \frac{1}{2}$ (B) $\frac{x}{9} + \frac{4}{y} = 1$ (C) $\frac{9}{x} + \frac{4}{y} = 1$ (D) $\frac{4}{x} + \frac{9}{y} = 1$

[MULTIPLE CORRECT CHOICE TYPE]

Q.15 to Q.17 has four choices (A), (B), (C), (D) out of which **ONE OR MORE** may be correct. **[3 × 4 = 12]**

Q.15 Consider the circle $x^2 + y^2 - 10x - 6y + 30 = 0$. Let O be the centre of the circle and tangent at A(7, 3) and B(5, 1) meet at C. Let $S = 0$ represents family of circles passing through A and B, then

(A) area of quadrilateral OACB equals 4.

(B) the radical axis for the family of circles $S = 0$ is $x + y = 10$.

(C) the smallest possible circle of the family $S = 0$ is $x^2 + y^2 - 12x - 4y + 38 = 0$.

(D) the coordinates of point C are (7, 1).

Q.16 Let P(x, y) satisfy $x^2 + y^2 + 8x - 10y = 40$.

If $p = \max\left(\sqrt{(x+2)^2 + (y-3)^2}\right)$ and $q = \min\left(\sqrt{(x+2)^2 + (y-3)^2}\right)$, then

(A) $p + q = 18$ (B) $p + q = \sqrt{2}$ (C) $p - q = 4\sqrt{2}$ (D) $p\,q = 73$

Q.17 The lines $x + y = 1$, $(k - 1)x + (k^2 - 7)y - 5 = 0$ and $(k - 2)x + (2k - 5)y = 0$ are

(A) concurrent for three values of k

(B) concurrent for exactly one value of k

(C) concurrent for no value of k

(D) parallel for $k = 3$

PART-D

[INTEGER TYPE]

Q.1 & Q.2 are "Integer Type" questions. (The answer to each of the questions **are upto 4 digits**) **[2 × 5 = 10]**

Q.1 Let Q and R be the images of a point P(–3, 1) in the mirror $x^2 - y^2 - x + y = 0$. If S is the area of triangle PQR, then find the value of $\left[\sqrt{S}\right]$. [**Note :** [k] denotes greatest integer less than or equal to k.]

Q.2 The line $x + 2y + 3 = 0$ cuts the circle $x^2 + y^2 + 4x + 4y - 1 = 0$ at points P and Q and the line $2x + 3y + \lambda = 0$ cuts the circle $x^2 + y^2 + 6x + 2y - 7 = 0$ at points R and S. If P, Q, R and S are concyclic, then find the value of λ.

CLASS : CC (Advanced) *Straight line and circle* **WORKSHEET-1**

M.M.: 64 **Time: 60 Min**

PART-A

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