SOLUTIONS

PART - I (PHYSICS)

1. **(b)** When force retards motion *i.e.*, F –(ve) so, work done –(ve)

2. **(a)** Given,
Radius of rod, r = 20 mm = 20×10^{-6}
Length of rod, $\ell = 2$m
Force, F = 62.8 kN
Young's modulus of steel, Y = 2×10^{11} N/m^2

$$\text{Strain} = \frac{\text{stress}}{Y} = \frac{\frac{62.8 \times 10^3}{\pi \times (0.02)^2}}{2 \times 10^{11}}$$

$$= \frac{62.8 \times 10^3}{3.14 \times 4 \times 10^{-4} \times 2 \times 10^{11}}$$

$= 2.5 \times 10^{-4} = 25 \times 10^{-5}$

3. **(a)** Slow isothermal expansion or compression of an ideal gas is reversible process, while the other given processes are irreversible in nature.

4. **(d)** Using equation, $= \frac{hc}{\lambda} - \phi$

$$KE_{\max} = \frac{hc}{\lambda} - \phi = \frac{hc}{500} - \phi \quad ...(i)$$

Again, $3KE_{\max} = \frac{hc}{200} - \phi \quad ...(ii)$

Dividing equation (ii) by (i),

$$\frac{3KE_{\max}}{KE_{\max}} = \frac{3}{1} = \frac{\frac{hc}{200} - \phi}{\frac{hc}{500} - \phi}$$

Putting the value of $hc = 1237.5$ and solving we get, work function, $\phi = 0.61$ eV.

5. **(b)** $Ee = mg$ or $E = mg/e$

6. **(b)** Bursting of helium balloon is irreversible and in this process $\Delta Q = 0$, so adiabatic.

7. **(a)** E = 500 V/m ΔV = 3000 V.
We know that electric field $|E| = 500 = \frac{\Delta V}{\Delta d}$

or $\Delta d = \frac{3000}{500} = 6\,\text{m}$

8. **(c)** In Bernoulli's theorem only law of conservation of energy is obeyed.

9. **(c)** $\omega = \frac{qB}{m}$

$$\Rightarrow \omega = \frac{q}{m} \mu_0 n I$$

$$\Rightarrow n = \frac{m\omega}{\mu_0 q I} = \frac{9.1 \times 10^{-31} \times 2\pi \times 10^8}{4\pi \times 10^{-7} \times 1.6 \times 10^{-19} \times 5}$$

$$= \frac{9.1}{16} \times 10^3 = 569$$

10. **(c)** ${}_1H^2 + {}_1H^2 \rightarrow {}_2He^4$
Total binding energy of two deuterium nuclei = 1.1×4 = 4.4 MeV
Binding energy of a (${}_2He^4$) nuclei = $4 \times 7 = 28$ MeV
Energy released in this process = 28 – 4.4 = 23.6 MeV

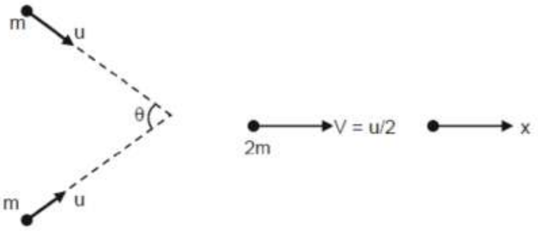
11. **(d)** Magnetic energy stored in an inductor

$$U = \frac{1}{2} L i^2 = \frac{1}{2} \times 4 \times 10^{-6} \times (2)^2$$

$= 8 \times 10^{-6}$ J = 8 μJ

12. **(a)**

13. **(d)**



According to linear momentum conservation along x-axis

$$2mu\cos\frac{\theta}{2} = 2m\frac{u}{2} \qquad \therefore \cos\frac{\theta}{2} = \frac{1}{2}$$

$$\frac{\theta}{2} = 60°$$

Hence θ = 120°

14. **(d)** The average value of magnetic field is zero. Also $u_E = u_B$

15. **(d)** For surface A,

$$\Rightarrow \frac{1}{v_1} = \frac{1}{-f} - \frac{1}{(-u)} = -1 + \frac{1}{3} = -\frac{2}{3}$$

$$\Rightarrow v_1 = -\frac{3}{2}\text{m}$$

For surface B,

$$\Rightarrow \frac{1}{v_2} = \frac{1}{-f} + \frac{1}{u} = -1 + \frac{1}{5} = -\frac{4}{5}$$

$$\Rightarrow v_2 = -\frac{5}{4}\text{m}$$

$$\therefore v_1 - v_2 = 0.25\text{ m}$$

Magnification of A $= \frac{v_1}{u} = \frac{3/2}{3} = \frac{1}{2}$

$\therefore$ Height of $A = \frac{1}{2} \times 2 = 1\,m$

Magnification of $B = \frac{v_2}{u} = \frac{5/4}{5} = \frac{1}{4}$

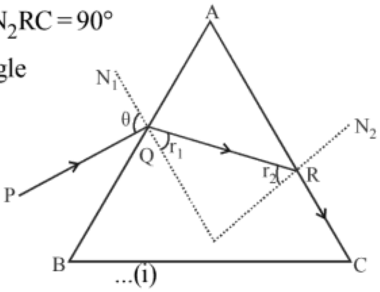
$\therefore$ Height of $B = \frac{1}{4} \times 2 = 0.5\,m$

16. **(c)** $\mu = \tan i$

$\Rightarrow i = \tan^{-1}(\mu) = \tan^{-1}(\sqrt{3}) = 60°$.

17. **(c)** When $r_2 = C$, $\angle N_2RC = 90°$

Where C = critical angle



As $\sin C = \frac{1}{\mu} = \sin r_2$

Applying Snell's law at 'R'

$\mu \sin r_2 = 1 \sin 90°$ ...(i)

Applying Snell's law at 'Q'

$1 \times \sin\theta = \mu \sin r_1$ ...(ii)

But $r_1 = A - r_2$

So, $\sin\theta = \mu \sin(A - r_2)$

$\sin\theta = \mu \sin A \cos r_2 - \cos A$ ...(iii) [using (i)]

From (i)

$\cos r_2 = \sqrt{1 - \sin^2 r_2} = \sqrt{1 - \frac{1}{\mu^2}}$ ...(iv)

By eq. (iii) and (iv)

$\sin\theta = \mu \sin A \sqrt{1 - \frac{1}{\mu^2}} - \cos A$

on further solving we can show for ray not to transmitted through face AC

$$\theta = \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$$

So, for transmission through face AC

$$\theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$$

18. **(a)** Density of nuclei,

$$\rho = \frac{A \times 1.6 \times 10^{-27}}{\frac{4}{3}\pi \times \left(1.5 \times 10^{-15}\right)^3 \times A} = 0.113 \times 10^{18}\ kg/m^3$$

Density of water $= \rho_w = 10^3\ kg/m^3$

So, $\frac{\rho}{\rho_w} = 11.31 \times 10^{13}\ kg/m^3$

19. **(c)** $mg = 6\pi\eta r v$

$2mg = 6\pi\eta . 2^{1/3} r v'$

$v' = 2^{2/3} v = 4^{1/3} v = \sqrt[3]{4} v$

20. **(b)** Given :

Capacitance, $C = 40\,\mu F = 40 \times 10^{-6}\,F$

Frequency, $f = 50$ Hz $\therefore \omega = 2\pi f = 100\pi$

$\varepsilon_{rms} = 200$ V $\quad \therefore I_{rms} = \frac{\varepsilon_{rms}}{X_C} = \frac{\varepsilon_{rms}}{\frac{1}{C\omega}}$

$= 200 \times 40 \times 10^{-6} \times 2\pi \times 50 = 2.5$ A.

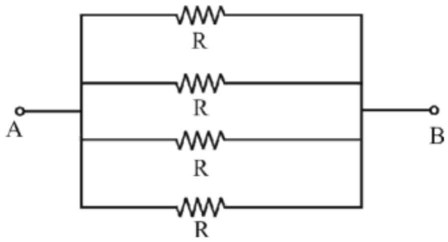
21. **(d)** Amplitude of electric field (E) and Magnetic field (B) of an electromagnetic wave are related by the relation

$\frac{E}{B} = c \Rightarrow E = Bc$

$\Rightarrow E = 5 \times 10^{-8} \times 3 \times 10^{8} = 15\ N/C \Rightarrow \vec{E} = 15\hat{i}\ V/m$

22. **(a)** Resistance of each wire in this case will be same as l & A are same and made of same material.

$\therefore R_1 = R_2 = R_3 = R_4 = R$



In parallel, $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$

$\therefore R_P = \frac{R}{4} \Rightarrow 0.25 = \frac{R}{4}$

$\therefore R = 1\,\Omega$

When arranged in series then equivalent resistance.

$R_s = R + R + R + R = 4R$

$\therefore R_s = 4 \times 1 = 4\Omega$

23. **(a)** Coefficient of static friction $= \frac{\text{force of friction}}{\text{normal reaction}}$

Therefore, coefficient of static friction depends upon the normal reaction.

24. **(a)** If m_1, m_2 are masses and u_1, u_2 are velocity then by conservation of momentum $m_1u_1 + m_2u_2 = 0$

or $|m_1u_1| = |m_2u_2|$

25. **(b)** Given,

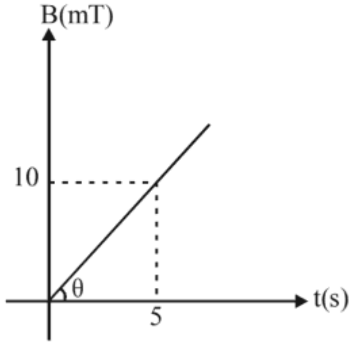
Area of metallic plate, $A = 4\,m^2$

Induced emf,

$\varepsilon = \left|\frac{d\phi}{dt}\right| = \frac{d(BA)}{dt} = \frac{AdB}{dt}$

But $\frac{dB}{dt}$ = Slope of B-t curve

$= \tan\theta = \frac{10}{5} = 2 \Rightarrow \frac{dB}{dt} = 2mT$



$\therefore \varepsilon = 4 \times 2 = 8$ mV

26. **(d)** Let the velocity of the particle be v m/s.
Momentum of the particle (p) = mv
Kinetic energy of the particle

$$(E) = \frac{1}{2} mv^2 = \frac{1}{2}.\frac{(mv)^2}{m} \Rightarrow E = \frac{p^2}{2m}$$

27. **(d)** When work is done upon a system by a conservative force then its potential energy increases.

28. **(b)** $W = \int_0^{x_1} F\,dx = \int_0^{x_1} c\,x\,dx = \left[\frac{1}{2}c\,x^2\right]_0^{x_1}$

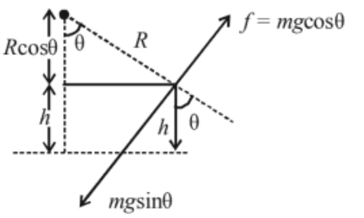
$$= \frac{1}{2}c(x_1^2 - 0) = \frac{1}{2}c\,x_1^2$$

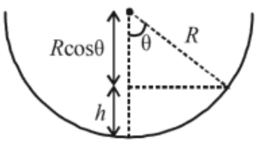
29. **(c)** Bulk modulus, $B = -\frac{\Delta P}{\frac{\Delta V}{V}} \Rightarrow \Delta P = -B\frac{\Delta V}{V}$

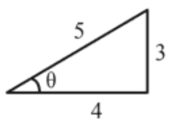
$|\Delta P| = +3 \times 10^{10} \times 0.02 = 6 \times 10^8$

30. **(a)** For balancing, $mg\sin\theta = f = \mu mg\cos\theta$

$$\Rightarrow \tan\theta = \mu = \frac{3}{4} = 0.75$$







$$h = R - R\cos\theta = R - R\left(\frac{4}{5}\right) = \frac{R}{5}$$

$\therefore h = \frac{R}{5} = 0.2$ m [$\because$ radius, $R = 1$m]

31. **(a)** You precise measurement, we always take P and Q approx equal and small.

32. **(a)** Dielectric constant of conductor = ∞. So $K = \infty$

Now, $C_f = \frac{A\epsilon_0}{d - t + t/k} = \frac{A\epsilon_0}{d - \frac{d}{2} + \frac{d}{\infty}}$

$= \frac{A\epsilon_0}{\frac{d}{2}} = 2\frac{A\epsilon_0}{d} = 2C_i$. So, $\frac{C_f}{C_i} = \frac{2}{1}$ $\left(\because C_i = \frac{\epsilon_0 A}{d}\right)$

33. **(b)** $\eta = 10^{-2}$ poise

$v = 18$ km/h $= \frac{18000}{3600} = 5$ m/s

$l = 5$ m

Strain rate $= \frac{v}{l}$

Coefficient of viscosity, $\eta = \frac{\text{shearing stress}}{\text{strain rate}}$

$\therefore$ Shearing stress $= \eta \times$ strain rate

$$= 10^{-2} \times \frac{5}{5} = 10^{-2}\ \text{Nm}^{-2}$$

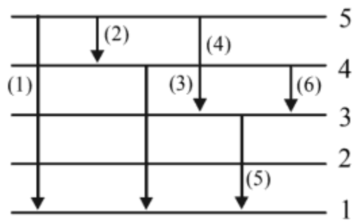
34. **(b)** When terminal velocity is reached then body moves with constant velocity hence, accelesation is zero.

35. **(c)** Velocity of efflux

$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 0.2} = 2\ \text{ms}^{-1}$

PART - II (CHEMISTRY)

36. **(d)**



Total radiations are = 6

37. **(b)**

Species	Bond order
He_2^-	0.5
Be_2	0
He_2^+	0.5
O_2^{2-}	1

Be_2 does not exist due to zero bond order.

38. **(c)** For isothermal reversible expansion.

$$w = -nRT\ln\frac{V_2}{V_1}$$

39. **(b)** $CO_2 + H_2O \rightleftharpoons H_2CO_3 \rightleftharpoons H^+ + HCO_3^-$

If CO_2 escapes, the equilibrium will shift to LHS and $[H^+]$ concentration will decrease

40. **(c)** For the reaction

$$2BaO_2(s) \rightleftharpoons 2BaO(s) + O_2(g);\ \Delta H = +\text{ ve}.$$

At equilibrium $K_p = P_{O_2}$

Hence, the value of equilibrium constant depends only upon partial pressure of O_2. Further on increasing temperature formation of O_2 increases as this is an endothermic reaction. Hence, pressure of O_2 is dependent on temperature.

41. **(b)** $P_{total} = P_A^o \times X_A + P_B^o \times X_B$
$= 80.0 \times 0.4 + 120.0 \times 0.6 = 104$ mm Hg
The observed P_{total} is 100 mm Hg which is less than 104 mm Hg. Hence the solution shows negative deviation.

42. **(b)** $\Delta T_b = K_b \dfrac{W_B}{M_B \times W_A} \times 1000$;

$\Delta T_f = K_f \dfrac{W_B}{M_B \times W_A} \times 1000$;

$$\frac{\Delta T_b}{\Delta T_f} = \frac{K_b}{K_f} = \frac{\Delta T_b}{-0.186} = \frac{0.512}{1.86} = 0.0512°C.$$

43. **(b)** The oxidation potential

$\propto \dfrac{1}{\text{Concentration of ions}}$ and reduction potential

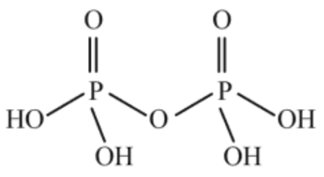
$\propto$ concentration of ions. The cell voltage can be increased by decreasing the concentration of ions around anode or by increasing the concentration of ions around cathode

44. **(c)** We know that the activation energy of chemical reaction is given by formula $= \dfrac{k_2}{k_1} = \dfrac{E_a}{2.303R}\left[\dfrac{T_2 - T_1}{T_1 T_2}\right]$,

where k_1 is the rate constant at temperature T_1 and k_2 is the rate constant at temperature T_2 and E_a is the activation energy. Therefore activation energy of chemical reaction is determined by evaluating rate constant at two different temperatures.

45. **(d)** Enzymes are specific biological catalysts possessing well - defined active sites.

46. **(d)**

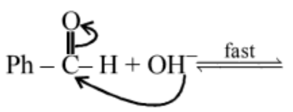


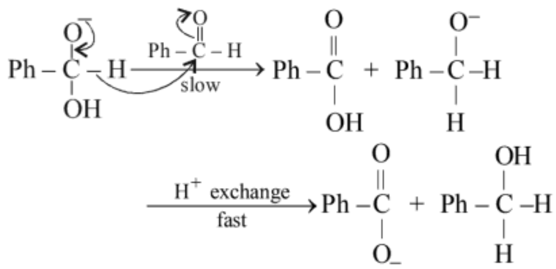
Pyrophosphoric acid ($H_4P_2O_7$)

Oxidation State :
Each P atom is bound to one oxygen = –1
2OH = –1 × 2 = – 2; 1 Oxygen = – 2
Total = –5; P = + 5.

47. **(b)** Mn^{2+} (d^5)is more stable than Mn^{3+} (d^4), thus $E^-_{Mn^{3+}/Mn^{2+}} = +ve$

48. **(a)**





49. **(c)** Inversion in configuration occurs in S_N2 reactions.

50. **(b)** The structure of CaC_2 is $Ca^{2+}[:C \equiv C:]^{2-}$
i.e, one σ and two π bonds

51. **(d)** $\Delta H = \Delta U + \Delta n_g RT$
For the reaction $\Delta n_g = 12 - 15 = -3$
$\Delta H - \Delta U = -3 \times 8.314 \times 300$
$= -7482 \text{ J mol}^{-1}$

52. **(c)** The reaction (c) is Hoffmann elimination

53. **(d)** For ideal solution vapour pressure of solution
$= P_A^o X_A + P_B^o X_B$

$= 80 \times \dfrac{2}{5} + 100 \times \dfrac{3}{5} = 92$ torr

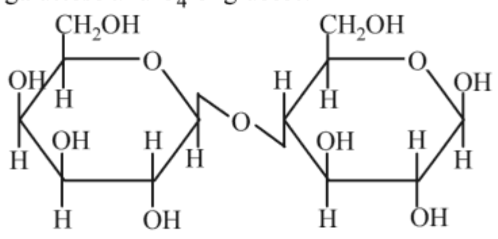
Since observed vapour pressure of solution < ideal vapour pressure, the solution shows negative deviation.

54. **(d)** $\Lambda_m = \dfrac{\kappa \times 1000}{M} = \dfrac{0.0110 \times 1000}{0.05}$
$= 220 \text{ S cm}^2 \text{ mol}^{-1}$

55. **(c)** Gold sol. have negative charge. So $Al_2(SO_4)_3$ is most effective for coagulation.

56. **(d)** $8MnO_4^- + 3S_2O_3^{2-} + H_2O \xrightarrow[\text{alk. solution}]{\text{neutral or}}$
$8MnO_2 + 6SO_4^{2-} + 2OH^-$

57. **(a)** Lactose contains β-glycosidic linkage between C_1 of galactose and C_4 of glucose.



β -D-Galactose β -D-Glucose

58. **(b)** Closed shell (Ne), half filled (P) and completely filled configuration (Mg) are the cause of higher value of I.E.

59. **(d)** de Broglie wavelength $\lambda = \dfrac{h}{mv}$

$\dfrac{\lambda_1}{\lambda_2} = \dfrac{m_2 v_2}{m_1 v_1};\ \dfrac{1}{4} = \dfrac{1}{9} \times \dfrac{v_2}{v_1}$

$\dfrac{v_2}{v_1} = \dfrac{9}{4};\ \dfrac{v_1}{v_2} = \dfrac{4}{9}$

$$KE = \frac{1}{2}mv^2$$

$$\frac{KE_1}{KE_2} = \frac{m_1}{m_2} \times \frac{v_1^2}{v_2^2} = \frac{9}{1} \times \left(\frac{4}{9}\right)^2 = \frac{16}{9}$$

60. (c) When $Q = K_C$, $E = 0$

61. (c) For I[st] order reaction

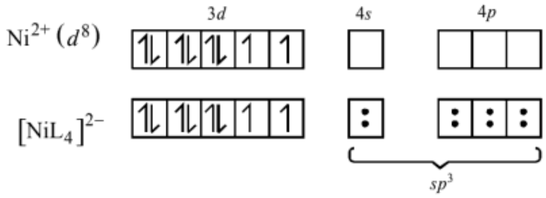
$$k = \frac{2.303}{t}\log\frac{a_o}{0.2a_o}$$

also $t_{1/2} = \frac{0.693}{k}$

$$k = \frac{0.693}{200} \Rightarrow \frac{0.693}{200} = \frac{2.303}{t}\log\frac{1}{0.2}$$

$$t = \frac{2.303}{0.693} \times 200\log\frac{1}{0.2} = 466.675 \approx 467 \text{ sec}$$

62. (a) $[NiL_4]^{2-}$

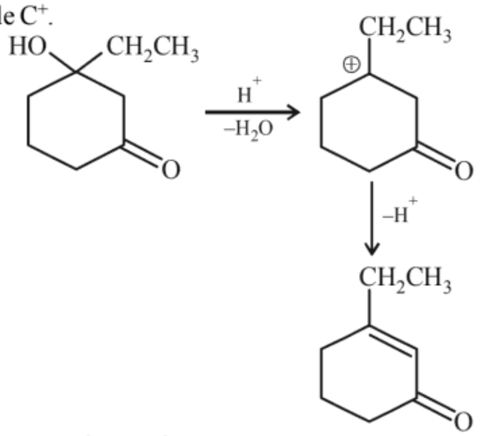


i.e, no. of unpaired electron = 2
hybridization – sp^3.

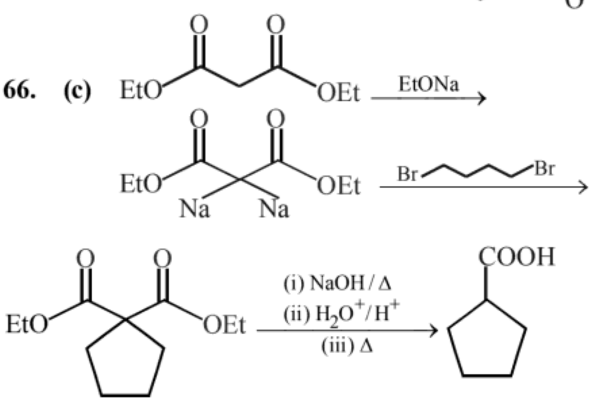
63. (a) Melamine plastic crockery is a copolymer of HCHO and Melamine.

64. (d) Unstable C^+ has general tendency to rearrange to more stable C^+.

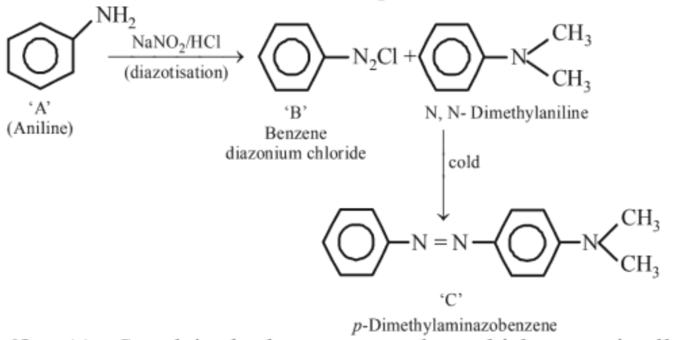
65. (a)



66. (c)



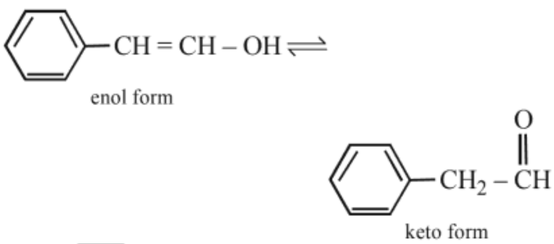
67. (d) The reaction can be completed as follows:



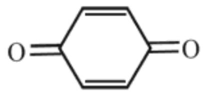
68. (a) Starch is also known as amylum which occurs in all green plants. A molecule of starch $(C_6H_{10}O_5)_n$ is built of a large number of α-glucose rings joined through oxygen atom.

69. (a)

(a)

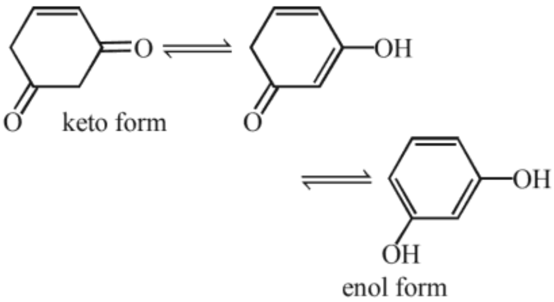


(b)

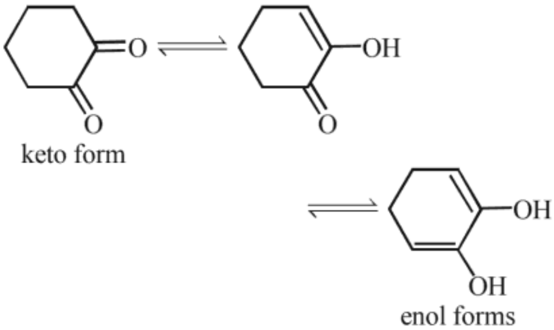


cannot tautomerise

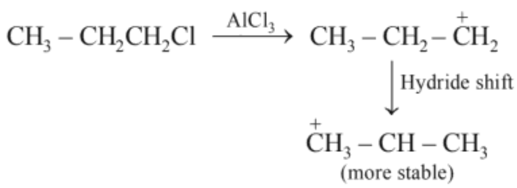
(c)



(d)



70. (c) $CH_3 - \overset{\oplus}{C}H - CH_3$ is formed in the above reaction



PART - III (MATHEMATICS)

71. **(b)** $\underset{(at\,x=0)}{\text{L.H.L}} = \lim_{x\to 0^-} f(x)$

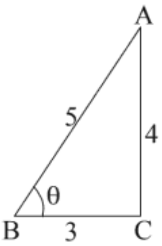
$= \lim_{h\to 0} \dfrac{\sin\{(p+1)(-h)\} - \sin h}{-h}$

$= p+1+1 = p+2$

$\underset{(at\,x=0)}{\text{R.H.L}} = \lim_{x\to 0^+} f(x) = \dfrac{1}{1+1} = \dfrac{1}{2}$

$f(0) = q \Rightarrow p = -\dfrac{3}{2}, q = \dfrac{1}{2}$

72. **(c)** Let $\tan^{-1}\dfrac{4}{3} = \theta \Rightarrow \tan\theta = \dfrac{4}{3}$



$\Rightarrow \cos^{-1}\left(\dfrac{3}{10}\cos\theta + \dfrac{2}{5}\sin\theta\right)$

$= \cos^{-1}\left(\dfrac{3}{10}\times\dfrac{3}{5} + \dfrac{2}{5}.\dfrac{4}{5}\right)$

$= \cos^{-1}\left(\dfrac{9}{50} + \dfrac{8}{25}\right) = \cos^{-1}\left(\dfrac{25}{50}\right) = \cos^{-1}\left(\dfrac{1}{2}\right) = \dfrac{\pi}{3}$

73. **(d)** $\underset{x\to 2}{\text{Lim}} \dfrac{2^x + 2^{3-x} - 6}{\sqrt{2^{-x}} - 2^{1-x}} = \underset{x\to 2}{\text{Lim}} \dfrac{(2^x)^2 - 6.2^x + 2^3}{\sqrt{2^x} - 2}$

[Multiplying N^r and D^r by 2^x]

$\underset{x\to 2}{\text{Lim}} \dfrac{(2^x - 4)(2^x - 2)(\sqrt{2^x} + 2)}{(\sqrt{2^x} - 2)(\sqrt{2^x} + 2)}$

$= \underset{x\to 2}{\text{Lim}} \dfrac{(2^x - 4)(2^x - 2)(\sqrt{2^x} + 2)}{(2^x - 4)}$

$= \underset{x\to 2}{\text{Lim}} (2^x - 2)(\sqrt{2^x} + 2) = (2^2 - 2)(2+2) = 8.$

74. **(d)** Circles

$S_1 : x^2 + y^2 - 2y - 3 = 0$ and

$S_2 : x^2 + y^2 - 8x - 18y + 93 = 0$

$C_1 \equiv (0, 1)$ and $r_1 = 2$

$C_2 \equiv (4, 9)$ and $r_2 = 2$

Since circles have same radius, centre of the smallest circle C_3 is collinear with C_1 and C_2 and mid point of C_1C_2.

$\therefore\ C_3 \equiv (2, 5)$

75. **(b)** Given

$P(E_1) = \dfrac{2+3P}{6}, P(E_2) = \dfrac{2-P}{8}\ \&\ P(E_3) = \dfrac{1-P}{2}.$

According to question,

$P(E_1) + P(E_2) + P(E_3) \le 1$

$\dfrac{2+3P}{6} + \dfrac{2-P}{8} + \dfrac{1-P}{2} \le 1$

$26 - 3P \le 24 \Rightarrow 2 \le 3P \Rightarrow P \ge \dfrac{2}{3}$

So, $\dfrac{2}{3} \le P \le 1$. Then, $P_1 = 1$ and $P_2 = \dfrac{2}{3}$.

$P_1 + P_2 = \dfrac{5}{3}$

76. **(b)** Sum of elements in $A \cap B$

$= \underbrace{(2+4+6+...+200)}_{\text{Multiple of 2}} - \underbrace{(6+12+...+198}_{\text{Multiple of 2 \& 3 i.e. 6}}$

$- \underbrace{(10+20+...+200)}_{\text{Multiple of 5 \& 2 i.e. 10}} + \underbrace{(30+60+...+180}_{\text{Multiple of 2,5 \& 3 i.e. 30}} = 5264$

77. **(c)** Given, $f(x) = |x^2 - 2x - 3|.e^{|9x^2 - 12x + 1|}$

$f(x) = |(x-3)(x+1)|.e^{(3x-2)^2}$

$$f(x) = \begin{cases} (x-3)(x+1).e^{(3x-2)^2} & ;\ x \in (3, \infty) \\ -(x-3)(x+1).e^{(3x-2)^2} & ;\ x \in [-1, 3] \\ (x-3).(x+1).e^{(3x-2)^2} & ;\ x \in (-\infty, -1) \end{cases}$$

Hence at x = –1, 3 f(x) = 0

Clearly, non-differentiable at x = –1 & x = 3.

78. **(a)** We have $f(x)$

$= \int e^{\log_e x^2} \log_{e^x} x dx = \int x^2.\dfrac{\log_e x}{x}.dx = \int x.\log_e x.dx$

$= \dfrac{x^2}{2}\log x - \int \dfrac{x^2}{2}.\dfrac{1}{x}.dx = \dfrac{x^2}{2}\log x - \dfrac{x^2}{4} + C$

Since, $f(1) = -\dfrac{1}{4}$, we get C = 0

$\therefore\ f(e) = \dfrac{e^2}{2} - \dfrac{e^2}{4} = \dfrac{e^2}{4}$

79. **(d)** For unique solution $\Delta \ne 0$

$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \ne 0 \Rightarrow 1(k+2) - 1(2k+3) + 1(4-3) \ne 0$

$\Rightarrow -k + 2 - 3 + 1 \ne 0 \Rightarrow k \ne 0 \therefore S = R - \{0\}$

80. **(b)** Table of values of the objective function:

Corner Point	**Value of Z = 3x – 4y**
(0, 0)	$3\times 0 - 4\times 0 = 0$
(5, 0)	$3\times 5 - 4\times 0 = 15$ (Maximum)
(6, 5)	$3\times 6 - 4\times 5 = -2$
(6, 8)	$3\times 6 - 4\times 8 = -14$
(4, 10)	$3\times 4 - 4\times 10 = -28$
(0, 8)	$3\times 0 - 4\times 8 = -32$ (Minimum)

Minimum of Z = – 32 at (0, 8)

81. **(c)** $\sim(p\vee(q\wedge r))\equiv\sim p\wedge\sim(q\wedge r)\equiv\sim p\wedge(\sim q\vee\sim r)$ [By De Morgan's Law]

$\equiv(\sim p\wedge\sim q)\vee(\sim p\wedge\sim r)$ [By Distributive Law]

82. **(d)** $f(x)=e^{(x^3-3x+2)}$

Let $g(x)=x^3-3x+2$;

$g'(x)=3x^2-3=3(x^2-1)\geq 0$ for $x\in(-\infty,-1]$

Therefore, $f(x)$ is increasing function.

Hence, $f(x)$ is one-one

Now, the range of $f(x)$ is $(0, e^4]$.

But co-domain is $(0, e^5]$.

Hence, $f(x)$ is an into function.

83. **(a)** Given, $BA=AB^2\Rightarrow A^{-1}(BA)=B^2$

$\Rightarrow(A^{-1}BA)^m=B^{2m}\Rightarrow A^{-1}B^mA=B^{2m}$

$\Rightarrow B^mA=AB^{2m}$

Also, $BA=AB^2\Rightarrow A=B^{-1}AB^2$

$\Rightarrow A^n=B^{-1}A^nB^{2n}$

84. **(a)** Let, $y=\dfrac{x^2+2x+c}{x^2+4x+3c}$

or $(y-1)x^2+(4y-2)x+3cy-c=0$

Now, x is real. Hence,

$D=(4y-2)^2-4(y-1)(3cy-c)\geq 0,\forall y\in\mathbb{R}$

or $(2y-1)^2-(y-1)(3cy-c)\geq 0,\forall y\in\mathbb{R}$

or $(4-3c)y^2+(-4+c+3c)y+1-c\geq 0,\forall y\in\mathbb{R}$

or $4-3c>0$ and $(4c-4)^2-4(4-3c)(1-c)\leq 0$

or $c<\dfrac{4}{3}$ and $4(c-1)^2-(4-3c)(1-c)\leq 0$

or $c<\dfrac{4}{3}$ and $(c-1)\times(4c-4+4-3c)\leq 0$

or $c<\dfrac{4}{3}$ and $(c-1)(c)\leq 0$

or $c<\dfrac{4}{3}$ and $0\leq c\leq 1$

or $0\leq c\leq 1$

85. **(c)** Here, $f(x)=\dfrac{x}{\sqrt{a^2+x^2}}-\dfrac{d-x}{\sqrt{b^2+(d-x)^2}}$

$\Rightarrow f'(x)=\dfrac{a^2}{(a^2+x^2)^{3/2}}+\dfrac{b^2}{(b^2+(d-x)^2)^{3/2}}>0$

$\forall x\in\boldsymbol{R}$

$\therefore f(x)$ is an increasing function of x.

86. **(b)** $f(0)=\sin 0=0,\ f(0^+)\to 0^+$

$f(0^-)=\lim\limits_{x\to 0^-}\sin(x^2-3x)=\lim\limits_{h\to 0}\sin(h^2+3h)\to 0^+$

Thus, $f(0^+)>f(0)$ and $f(0^-)>f(0)$.

Hence, $x=0$ is a point of minima.

87. **(d)** $I_1=\displaystyle\int_{-\frac{1}{2}}^{1}2xf(2x(1-2x))\,dx$

$\Rightarrow 2x=t\Rightarrow 2dx=dt\Rightarrow I_1=\dfrac{1}{2}\displaystyle\int_{-1}^{2}tf(t(1-t))\,dt$

$\Rightarrow 2I_1=\displaystyle\int_{-1}^{2}(1-t)f[(1-t)(1-(1-t))]\,dt$

$\Rightarrow 2I_1=\displaystyle\int_{-1}^{2}f(t(1-t))\,dt-\int_{-1}^{2}tf(t(1-t))\,dt$

$\Rightarrow 2I_1=I_2-2I_1\Rightarrow 4I_1=I_2\Rightarrow\dfrac{I_2}{I_1}=4.$

88. **(d)** The given differential equation is

$$(1+e^{\frac{x}{y}})\,dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0$$

$$\frac{dx}{dy}=\frac{e^{\frac{x}{y}}\left(\frac{x}{y}-1\right)}{\left(e^{x/y}+1\right)}\quad\text{...(i)}$$

$=g\left(\dfrac{x}{y}\right)\ \because\ \dfrac{dx}{dy}=g\left(\dfrac{x}{y}\right)$

$\therefore$ eq. (i) is the homogeneous differential equation so, put

$\dfrac{x}{y}=v$ i.e., $x=vy\Rightarrow\dfrac{dx}{dy}=v+y\dfrac{dv}{dy}$

Then, eq. (i) becomes

$v+y\dfrac{dv}{dy}=\dfrac{e^v(v-1)}{e^v+1}\Rightarrow y\dfrac{dv}{dy}=\dfrac{e^v(v-1)}{e^v+1}-v$

$\Rightarrow y\dfrac{dv}{dy}=\dfrac{ve^v-e^v-ve^v-v}{e^v+1}-v$

$\Rightarrow\dfrac{e^v+1}{e^v+v}dv=-\dfrac{1}{y}dy$

On integrating both sides, we get

$\displaystyle\int\frac{e^v+1}{e^v+v}dv=-\int\frac{1}{y}dy$ Put $e^v+v=t$

$\Rightarrow e^v+1=\dfrac{dt}{dv}\Rightarrow dv=\dfrac{dt}{e^v+1}$

$\therefore\displaystyle\int\frac{e^v+1}{t}\frac{dt}{e^v+1}-\log|y|+\log C$

$\Rightarrow\log|t|+\log|y|=\log C$

$\Rightarrow\log|e^v+v|+\log|y|=\log C$ $(\because t=e^v+v)$

$\Rightarrow\log|(e^v+v)y|=C\Rightarrow|(e^v+v)y|=C$

$\Rightarrow(e^v+v)y=C$. So, put $v=\dfrac{x}{y}$, we get

$$\left(e^{x/y}+\frac{x}{y}\right)y=C\Rightarrow ye^{x/y}+x=C$$

This is the required solution of the given differential equation.

89. **(a)** $||A^{-1}|(AB)^{T}| = ||A^{-1}|\cdot(A\cdot \text{adj}\, A)^{T}|$

$$= \left|\frac{1}{|A|}\cdot(|A|\cdot I)^{T}\right| = \left|\frac{1}{|A|}\cdot|A|\cdot I^{T}\right| = \left|I^{T}\right| = 1$$

90. **(a)** $f(x) = \dfrac{\sqrt{6x^2+5x-6}}{\sqrt{4-x}-\sqrt{x+4}}$

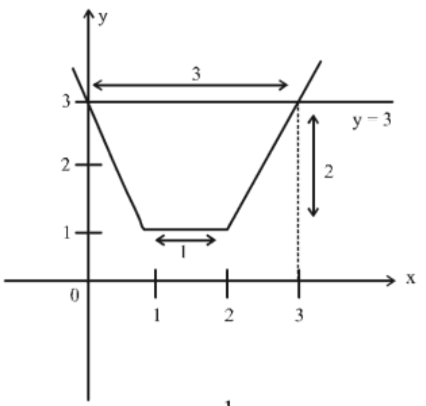
$6x^2 + 5x - 6 \geq 0 \Rightarrow x \in \left(-\infty, -\frac{3}{2}\right] \cup \left[\frac{2}{3}, \infty\right)$

for $\sqrt{4-x}-\sqrt{x+4} \Rightarrow x \in [-4, 4]$

$\Rightarrow x \in \left[-4, \frac{-3}{2}\right] \cup \left[\frac{2}{3}, 4\right]$

91. **(c)** $|z| = \dfrac{|1+i\sqrt{3}|\;|\cos\theta + i\sin\theta|}{2|1-i|\;|\cos\theta - i\sin\theta|} = \dfrac{2}{2\sqrt{2}} = \dfrac{1}{\sqrt{2}}$

92. **(b)** Given $y = |x-1| + |x-2|$ and $y = 3$
Since, graph of the given curves are



$\therefore$ Required area $= \frac{1}{2}(1+3)\times 2 = 4$

93. **(d)** In the expansion of $\left(\dfrac{x\sqrt{y}}{3} - \dfrac{3}{y\sqrt{x}}\right)^{12}$, n = 12 (even)

then middle term is $\frac{12}{2}+1 = 7^{\text{th}}$ term.

$(r+1)^{\text{th}}$ term; $T_{r+1} = {}^{12}C_r\left[\dfrac{x\sqrt{y}}{3}\right]^{12-r}\cdot\left(-\dfrac{3}{y\sqrt{x}}\right)^{r}$

$\therefore\ T_7 = T_{6+1} = {}^{12}C_6\left(\dfrac{x\sqrt{y}}{3}\right)^{6}\left(-\dfrac{3}{y\sqrt{x}}\right)^{6}$

$= {}^{12}C_6\,\dfrac{x^6y^3}{y^6x^3} = {}^{12}C_6\, x^3y^{-3} = C(12, 6)x^3y^{-3}$

94. **(d)** Given : $5^{1+x}+5^{1-x}, \frac{a}{2}, 5^{2x}+5^{-2x}$ are in A.P.

$\therefore\ 2.\frac{a}{2} = 5^{1+x}+5^{1-x}+5^{2x}+5^{-2x}$

$\Rightarrow a = 5.5^{x}+5(5^{x})^{-1}+(5^{x})^{2}+(5^{x})^{-2}$

Let $5^{x} = t \therefore\ a = 5t+\frac{5}{t}+t^2+\frac{1}{t^2}$

$\Rightarrow a = t^2+\frac{1}{t^2}+5\left(t+\frac{1}{t}\right)$

$\Rightarrow a = \left(t+\frac{1}{t}\right)^2 - 2 + 5\left(t+\frac{1}{t}\right)$

Put $t+\frac{1}{t} = A$

$\therefore a = A^2+5A-2$ [add & subtract $\left(\frac{b}{2a}\right)^2$]

$\Rightarrow a = \left[A^2+5A-\left(\frac{5}{2}\right)^2\right]+\left(\frac{5}{2}\right)^2-2$

$\Rightarrow a = \left(A-\frac{5}{2}\right)^2+\frac{17}{4} \Rightarrow a \geq \frac{17}{4}.$

95. **(a)** Equation of parabola is $y^2 = ax = 4\cdot\frac{a}{4}x$ ...(i)

$\therefore$ Focus of parabola is $\left(\frac{a}{4}, 0\right)$.

Focal chord of the parabola is $2x - y - 8 = 0$

$\therefore 2\cdot\frac{a}{4} - 0 - 8 = 0 \Rightarrow a = 16$

Putting the value of a in (i), we get

$y^2 = 16x \Rightarrow y^2 = 4\cdot 4x$

So, directrix is $x + 4 = 0$.

96. **(c)** $ab\sin x + b\sqrt{(1-a^2)}\cos x$

Now, $\sqrt{(ab)^2+(b\sqrt{(1-a^2)})^2}$

$= \sqrt{a^2b^2+b^2(1-a^2)} = b\sqrt{(a^2+1-a^2)} = b$

$\Rightarrow b\{(a\sin x + \sqrt{(1-a^2)}\cos x)\}$

Let, $a = \cos\alpha$,

$\therefore \sqrt{(1-a^2)} = \sin\alpha \Rightarrow b\sin(x+a)$

$\because -1 \leq \sin(x+\alpha) \leq 1$

$\therefore c-b \leq b\sin(x+\alpha)+c \leq b+c$

$\therefore b\sin(x+\alpha)+c \in [c-b, c+b]$

97. **(c)** The equation of a line perpendicular to $3x+2y+5=0$ is $2x-3y+\lambda=0$ ...(i)
This passes through the point (3, 4).
$\therefore 3\times 2 - 3\times 4 + \lambda = 0 \Rightarrow \lambda = 6$
Putting $\lambda = 6$ in (i), we get $2x - 3y + 6 = 0$, which is the required equation.

98. **(d)** Let E be the event that the randomly drawn card is a spade. Let F_i be the event that i spades nre missing from the 52-card, for i can take values 0, 1, 2

$$P(E) = P\left(\frac{E}{F_0}\right)P(F_0) + P\left(\frac{E}{F_1}\right)P(F_1) + P\left(\frac{E}{F_2}\right)P(F_2)$$

$$= \frac{13}{50}\frac{\binom{13}{0}\binom{39}{2}}{\binom{52}{2}} + \frac{12}{50}\frac{\binom{13}{1}\binom{39}{1}}{\binom{52}{2}} + \frac{11}{50}\frac{\binom{13}{2}\binom{39}{0}}{\binom{52}{2}} = \frac{1}{4}$$

99. (b) We know that $\sum P_i = 1$

$\therefore \quad a+4a+3a+7a+8a+10a+6a+9a=1$

$\Rightarrow \quad 48a = 1 \Rightarrow a = \dfrac{1}{48}$

Now, $P(X<3) = P(0)+P(1)+P(2)$

$= a+4a+3a = 8a = 8.\dfrac{1}{48} = \dfrac{1}{6}$

$P(X \geq 4) = P(4)+P(5)+P(6)+P(7)$

$= 8a+10a+6a+9a = 33a = \dfrac{33}{48}$

and $P(0<X<5) = P(1)+P(2)+P(3)+P(4)$

$= 4a+3a+7a+8a = 22a = \dfrac{22}{48} = \dfrac{11}{24}$

100. (c) Since, $|\vec{a}| = \sqrt{11}, |\vec{c}| = \sqrt{22}$

Now, $|\vec{a}| = |\vec{b}\times\vec{c}| = |\vec{b}||\vec{c}|\sin\theta \Rightarrow \sqrt{11} = \sqrt{50}\sqrt{22}\sin\theta$

$\Rightarrow \sin\theta = \dfrac{1}{10}$

$\because |\vec{b}+\vec{c}|^2 = |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b}\cdot\vec{c}$

$= |\vec{b}|^2 + |\vec{c}|^2 + 2|\vec{b}||\vec{c}|\cos\theta$

$= 50+22+2\times\sqrt{50}\times\sqrt{22}\times\dfrac{\sqrt{99}}{10} \quad = 72+66$

$\Rightarrow \left|72 - |\vec{b}+\vec{c}|^2\right| = 66$

101. (b)

Marks Obtained	Number of students f_i	Mid points x_i	$f_i\ x_i$	$\lvert x_i - \bar{x}\rvert$	$f_i \lvert x_i - \bar{x}\rvert$
10-20	2	15	30	30	60
20-30	3	25	75	20	60
30-40	8	35	280	10	80
40-50	14	45	630	0	0
50-60	8	55	440	10	80
60-70	3	65	195	20	60
70-80	2	75	150	30	60
	40		**1800**		**400**

Here, N = $\sum_{i=1}^{7} f_i = 40, \sum_{i=1}^{7} f_i x_i = 1800$

Therefore, $\bar{x} = \dfrac{1}{N}\sum_{i=1}^{7} f_i x_i = \dfrac{1800}{40} = 45$

Now, $\sum_{i=1}^{7} f_i\,|x_i - \bar{x}| = 400$

$\therefore$ M.D. $(\bar{x}) = \dfrac{1}{N}\sum_{i=1}^{7} f_i\,|x_i-\bar{x}| = \dfrac{1}{40}\times 400 = 10$

102. (c) Centre at (0, 0), $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ at point (4, –1)

$\dfrac{16}{a^2}+\dfrac{1}{b^2}=1 \Rightarrow 16b^2+a^2=a^2b^2$ ...(i)

at point (–2, 2), $\dfrac{4}{a^2}+\dfrac{4}{b^2}=1$

$\Rightarrow \quad 4b^2+4a^2=a^2b^2$ ...(ii)

$\Rightarrow \quad 16b^2+a^2=4a^2+4b^2$

From equations (i) and (ii)

$\Rightarrow \quad 3a^2=12b^2 \Rightarrow a^2=4b^2$

$b^2 = a^2(1-e^2) \Rightarrow e^2 = \dfrac{3}{4} \Rightarrow e = \dfrac{\sqrt{3}}{2}$

103. (a) $S = \log_7 x^2 + \log_7 x^3 + \log_7 x^4 + ...20$ terms

$\because S = 460$

$\Rightarrow \log_7(x^2\cdot x^3\cdot x^4\cdot......x^{21}) = 460$

$\Rightarrow \log_7 x^{(2+3+4......21)} = 460$

$\Rightarrow (2+3+4+.....+21)\log_7 x = 460$

$\Rightarrow \dfrac{20}{2}(2+21)\log_7 x = 460$

$\Rightarrow \log_7 x = \dfrac{460}{230} = 2 \Rightarrow x = 7^2 = 49$

104. (a) Let a, b, c, be the direction ratios of the required line. Then, equation of line passing through $(1, 2, -4)$ and having DR's (a, b, c) is

$\dfrac{x-1}{a} = \dfrac{y-2}{b} = \dfrac{z+4}{c}$

Now, as line (i) is perpendicular to the lines

$\dfrac{x-8}{3} = \dfrac{y+19}{-16} = \dfrac{z-10}{7}$ and

$$\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$$

$\therefore 3a-16b+7c=0$ ...(i)

$3a+8b-5c=0$ ...(ii)

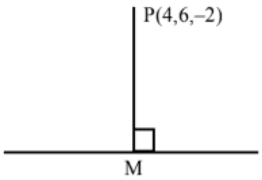
On solving (ii) and (iii), we get

$$\frac{a}{24}=\frac{b}{36}=\frac{c}{72} \; i.e., \frac{a}{2}=\frac{b}{3}=\frac{c}{6}$$

So, the required equation is

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+4}{6}$$

105. (d)



Equation of line is $\frac{x+3}{3}=\frac{y-2}{3}=\frac{z-3}{-1}=\lambda$

$M(3\lambda-3, 3\lambda+2, 3-\lambda)$

D.R of PM $(3\lambda-7, 3\lambda-4, 5-\lambda)$

Since PM is perpendicular to line

$\Rightarrow 3(3\lambda-7)+3(3\lambda-4)-1(5-\lambda)=0$

$\Rightarrow \lambda_2=2$

Put $\lambda=2$ in cordinate M.

$\Rightarrow M(3, 8, 1) \Rightarrow PM = \sqrt{14}$

106. (a) Let Q(x, y, z) be any point which is equidistant from A(0, 2, 3) and B(2, –2, 1), then

QA = QB

Squaring both sides, we get $QA^2 = QB^2$

$$\Rightarrow \sqrt{(x-0)^2+(y-2)^2+(z-3)^2}$$

$$= \sqrt{(x-2)^2+(y+2)^2+(z-1)^2} \text{ (Using distance formula)}$$

$\Rightarrow 4x-8y-42+4=0 \Rightarrow x-2y-2+1=0$

$\Rightarrow x-2y-1=0$

Hence, the required locus is $x-2y-1=0$

107. (d) Here, $\lim\limits_{x\to\frac{\pi}{2}} \frac{\cos x}{(1-\sin x)^{\frac{1}{3}}}=\lim\limits_{t\to 0}\frac{\sin t}{(1-\cos t)^{\frac{1}{3}}}$

$$= \lim_{t\to 0}\frac{2\sin\frac{t}{2}\cos\frac{t}{2}}{\left(2\sin^2\frac{t}{2}\right)^{\frac{1}{3}}}=\lim_{t\to 0} 2^{\frac{2}{3}}\cos\frac{t}{2}\left(\sin\frac{t}{2}\right)^{\frac{2}{3}} = 0$$

108. (c) $A=\begin{vmatrix} x^2+2 & 2x+1 & 1 \\ 2x+1 & x+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$

after expanding along R_1

$= (x-1)^2(x-3)$, which is clearly negative for $x < 1$

109. (b) Let $y=\sqrt{\sec\sqrt{x}}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx}=\frac{1}{2\sqrt{\sec\sqrt{x}}}\cdot\sec\sqrt{x}\cdot\tan\sqrt{x}\cdot\frac{1}{2\sqrt{x}}$$

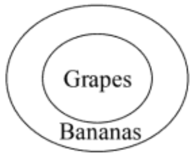
$$=\frac{1}{4\sqrt{x}}(\sec\sqrt{x})^{1/2}\frac{\sin\sqrt{x}}{\cos\sqrt{x}} = \frac{1}{4\sqrt{x}}(\sec\sqrt{x})^{3/2}\cdot\sin\sqrt{x}$$

110. (c) $f\left(x+\frac{1}{x}\right)=x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2$

$\therefore f(x) = x^2-2$

PART - IV (APTITUDE TEST)

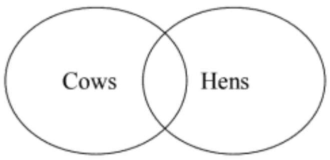
111. (c)



Conclusion I. [√] II. [×]
III. [√] IV. [×]

Hence, only I and III follow.

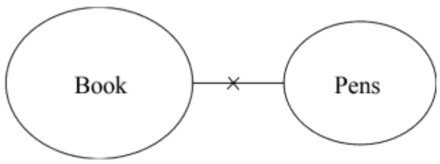
112. (a)



Conclusion I. [√] II. [×]
III. [×] IV. [×]

Hence, only I follow.

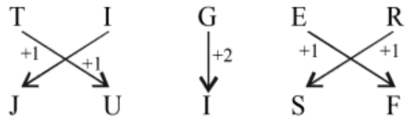
113. (c)



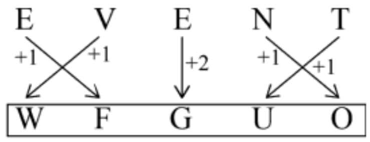
Conclusion I. [√] II. [√]
III. [√] IV. [×]

Hence, only I, II and III follow.

114. (b) As,

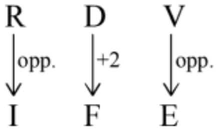


Similarly,

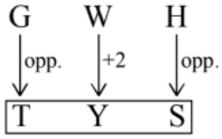


115. (a) 14. [B, C, D, F, G, J, K, L, N, P, Q, R, S and Z]

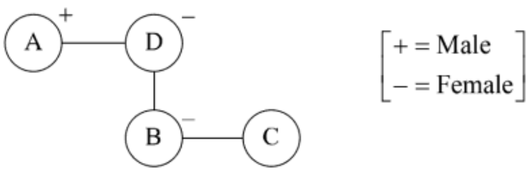
116. (c) As,



Similarly,

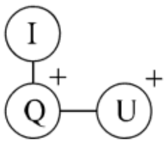


117. (b)



Hence, 'B' is Niece of 'A'.

118. (c) From statement (I) and (II)



Hence, statement I and II together are sufficient to answer the question.

119. (a) The series is as follows

1 4 10 [22] 46 94
(+3, +6, +12, +24, +48)

120. (c) The series is as follows

2 6 12 20 [30] 42 56
(+4, +6, +8, +10, +12, +14)

PART - V (ENGLISH)

121. (c) The phrase suggests overcoming the disparity between countries with advanced AI tools and those without access. The UN is urging international cooperation so that developing nations can benefit equally. Bridging this divide ensures fairness in how AI transforms societies globally.

122. (b) The mention of the expert group highlights the seriousness with which the UN is approaching AI governance. It shows a strategic effort to include multidisciplinary knowledge and global perspectives in setting fair, effective policies.

123. (d) The passage focuses on ensuring that AI is developed and used responsibly, with fairness, inclusivity, and global collaboration at its core. It emphasizes both the potential and the risks of AI and calls for policies that protect human rights while closing access gaps.

124. (a) "Inclusive" in this context means ensuring that everyone is involved or considered, much like "comprehensive," which suggests covering all elements or groups fairly. The other options suggest limiting or partial involvement.

125. (c) "Strategies" imply structured, goal-oriented actions. "Chaos" represents complete disorder or lack of planning, making it a contextual opposite.