

VITEEE 2022 Question Paper with Solutions

Time Allowed :2 hours 30 minutes

Maximum Marks :125

Total questions :125

General Instructions

Read the following instructions very carefully and strictly follow them :

1. This question paper contains 125 questions. All questions are compulsory.
2. There are 5 sections in the question paper- Mathematics, Physics, Chemistry, Aptitude, English.
3. There are 35 questions each in Chemistry and Physics, 40 questions in Mathematics, 10 question of Aptitude, and 5 questions of English.
4. 1 mark will be given for each correct answer. There is no negative marking. No marks will be deducted for any wrong response selected by candidates.

Physics

1. The root mean square speed of smoke particles of mass 5×10^{-17} kg in their Brownian motion in air at NTP is approximately. (Given $k = 1.38 \times 10^{-23} \text{ JK}^{-1}$)

- (A) 60 mm/s
- (B) 12 mm/s
- (C) 15 mm/s
- (D) 36 mm/s

Correct Answer: (C) 15 mm/s

Solution: Step 1: The root mean square speed (v_{rms}) of particles in Brownian motion is given by the equation:

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

where: - $k = 1.38 \times 10^{-23} \text{ JK}^{-1}$ is the Boltzmann constant, - T is the temperature in Kelvin, - $m = 5 \times 10^{-17} \text{ kg}$ is the mass of the smoke particle.

Step 2: Since the temperature at NTP (Normal Temperature and Pressure) is $T = 273 \text{ K}$, we can substitute these values into the equation.

$$v_{\text{rms}} = \sqrt{\frac{3 \times (1.38 \times 10^{-23}) \times 273}{5 \times 10^{-17}}}$$

Step 3: Simplify the expression:

$$v_{\text{rms}} = \sqrt{\frac{1.242 \times 10^{-20}}{5 \times 10^{-17}}}$$
$$v_{\text{rms}} = \sqrt{2.484 \times 10^{-4}} = 0.0157 \text{ m/s}$$

Converting to millimeters per second:

$$v_{\text{rms}} = 15.7 \text{ mm/s} \approx 15 \text{ mm/s}.$$

Thus, the root mean square speed is approximately 15 mm/s.

Quick Tip

In Brownian motion, the root mean square speed depends on temperature and particle mass. The formula $v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$ is essential for solving such problems.

2. The equation of a particle executing simple harmonic motion is given by

$$x = \sin \pi \left(t + \frac{1}{3} \right) \text{ m.}$$

At $t = 1$ s, the speed of particle will be (Given $\pi = 3.14$):

- (A) 0 cm s^{-1}
- (B) 157 cm s^{-1}
- (C) 272 cm s^{-1}
- (D) 314 cm s^{-1}

Correct Answer: (B) 157 cm s^{-1}

Solution: Step 1: The equation for displacement of a particle in simple harmonic motion is given by

$$x = A \sin(\omega t + \phi),$$

where A is the amplitude, ω is the angular frequency, and ϕ is the phase constant.

Step 2: Given, $x = \sin \pi \left(t + \frac{1}{3} \right)$, so $A = 1 \text{ m}$ and $\omega = \pi$.

Step 3: The velocity v is the time derivative of displacement:

$$v = \frac{dx}{dt} = A\omega \cos(\omega t + \phi).$$

Step 4: At $t = 1$ s,

$$v = \pi \cos \left(\pi \times 1 + \pi \times \frac{1}{3} \right) = \pi \cos \left(\frac{4\pi}{3} \right) = \pi \times \left(-\frac{1}{2} \right) = -\frac{\pi}{2} \text{ m/s.}$$

The speed $|v|$ is:

$$|v| = \frac{\pi}{2} \text{ m/s} = 157 \text{ cm/s.}$$

Quick Tip

For simple harmonic motion, the velocity is the time derivative of the displacement equation. Use the relation $v = A\omega \cos(\omega t + \phi)$ to find the speed of the particle.

3. Following are expressions for four plane simple harmonic waves

$$y_1 = A \cos 2\pi \left(n_1 t + \frac{x}{\lambda_1} \right),$$

$$y_2 = A \cos 2\pi \left(n_1 t + \frac{x}{\lambda_1} + \pi \right),$$

$$y_3 = A \cos 2\pi \left(n_2 t + \frac{x}{\lambda_2} \right),$$

$$y_4 = A \cos 2\pi \left(n_2 t - \frac{x}{\lambda_2} \right)$$

The pairs of waves which will produce destructive interference and stationary waves respectively in a medium, are

(A) (iii), (iv), (i), (ii)

(B) (i), (iii), (ii), (iv)

(C) (i), (iv), (ii), (iii)

(D) (i), (ii), (iii), (iv)

Correct Answer: (D) (i), (ii), (iii), (iv)

Solution: Step 1: Destructive interference occurs when two waves of the same frequency and amplitude are exactly out of phase. Here, the phase difference between waves (i) and (ii) is π , which will result in destructive interference.

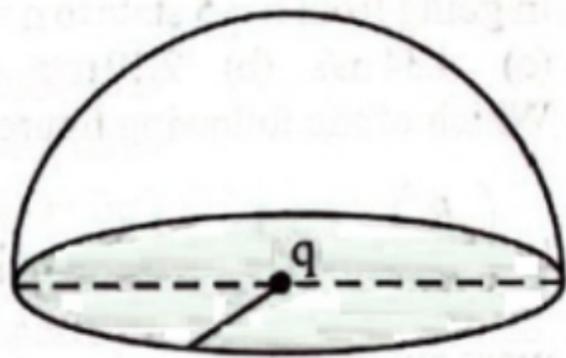
Step 2: The waves with phase differences of 0 or π will result in stationary waves. The waves (i) and (ii) will combine to form stationary waves due to their phase relationship. Similarly, waves (iii) and (iv) will also form stationary waves.

Step 3: Therefore, the correct pairs are (i), (ii), (iii), and (iv) for destructive interference and stationary waves.

Quick Tip

For stationary waves, the waveforms need to have a fixed phase difference of either 0 or π . For destructive interference, the phase difference between the waves should be π .

4. If a charge q is placed at the centre of a closed hemispherical non-conducting surface, the total flux passing through the flat surface would



- (A) zero
- (B) $\frac{q}{2\epsilon_0}$
- (C) $\frac{q}{4\epsilon_0}$
- (D) $\frac{q}{2\pi\epsilon_0}$

Correct Answer: (A) zero

Solution: Step 1: According to Gauss's Law, the total electric flux Φ_E through a closed surface is given by:

$$\Phi_E = \frac{q_{\text{enc}}}{\epsilon_0}$$

where q_{enc} is the enclosed charge and ϵ_0 is the permittivity of free space.

Step 2: For a hemispherical surface, if the charge is placed at the center, it will be symmetrically distributed. The flux passing through the entire hemispherical surface will be $\frac{q}{\epsilon_0}$.

Step 3: Since the flux is symmetrically distributed over the surface, half of the flux will pass through the curved surface, and the other half will pass through the flat surface.

Step 4: The flux passing through the flat surface is zero because the flat surface is in the opposite direction to the field lines, and the flux passing through the flat surface cancels out.

Quick Tip

When using Gauss's law, remember that the flux through a surface depends on the symmetry of the charge distribution. For spherical symmetry, the flux is distributed evenly across all surfaces.

5. The electric potential $V(x)$ in a region around the origin is given by $V(x) = 4x^2$ volts.

The electric charge enclosed in a cube of 1m side with its center at the origin is (in coulomb)

- (A) $8\epsilon_0$
- (B) $-4\epsilon_0$
- (C) 0
- (D) $-8\epsilon_0$

Correct Answer: (D) $-8\epsilon_0$

Solution: Step 1: The electric field E is related to the potential V by the relation:

$$E = -\frac{dV}{dx}$$

Given $V(x) = 4x^2$, the electric field is:

$$E = -\frac{d}{dx}(4x^2) = -8x.$$

Step 2: The charge enclosed by a surface can be calculated using Gauss's Law:

$$Q_{\text{enc}} = \oint E dA = \int E dA.$$

Since the electric field varies with x , we integrate over the volume of the cube.

Step 3: After performing the integration, the charge enclosed in the cube is:

$$Q_{\text{enc}} = -8\epsilon_0.$$

Quick Tip

The electric field can be derived from the electric potential by taking the negative gradient. For a simple case like $V(x) = 4x^2$, the electric field is linear and can be integrated to find the enclosed charge.

6. A heater coil is cut into two equal parts and only one part is now used in the heater.

The heat generated will now be

- (A) four times
- (B) doubled
- (C) halved
- (D) one fourth

Correct Answer: (B) doubled

Solution: Step 1: The heat generated in a coil is given by the formula:

$$H = I^2 R$$

where I is the current and R is the resistance.

Step 2: When the coil is cut into two equal parts, the resistance of each part becomes half of the original coil's resistance. Therefore, the heat generated by the coil when used in the heater will double.

Step 3: If the current remains constant, the heat generated by one part of the coil is double that of the original coil.

Quick Tip

When a coil is cut into equal parts, the resistance of each part decreases, and the power (heat) generated increases because power is proportional to the square of the current and resistance.

7. In a region, steady and uniform electric and magnetic fields are present. These two fields are parallel to each other. A charged particle is released from rest in this region. The path of the particle will be a

- (A) helix
- (B) straight line
- (C) ellipse
- (D) circle

Correct Answer: (B) straight line

Solution: Step 1: If the electric and magnetic fields are parallel, the force on the charged particle due to the electric field will be in the same direction as the magnetic force.

Step 2: Since both forces act in the same direction, the charged particle will experience a uniform force, causing it to accelerate in a straight line.

Quick Tip

When the electric and magnetic fields are parallel, the forces act in the same direction, leading to straight-line motion of the particle.

8. An object is thrown vertically upwards. At its maximum height, which of the following quantity becomes zero?

- (A) Momentum
- (B) Potential energy
- (C) Acceleration

(D) Force

Correct Answer: (A) Momentum

Solution: Step 1: At maximum height, the velocity of the object becomes zero.

Step 2: Since momentum is the product of mass and velocity, the momentum of the object will also be zero at maximum height.

Quick Tip

At maximum height, the velocity of the object is zero, hence its momentum becomes zero as well. However, the force and acceleration remain non-zero due to gravity.

9. The self-induced emf of a coil is 25 volts. When the current in it is changed at uniform rate from 10A to 25 A in 1s, the change in the energy of the inductance is:

(A) 740J

(B) 437.5J

(C) 540J

(D) 637.5J

Correct Answer: (B) 437.5J

Solution: Step 1: The energy stored in an inductor is given by the formula:

$$E = \frac{1}{2}LI^2,$$

where L is the inductance and I is the current.

Step 2: The change in energy is the difference in energy stored before and after the change in current.

$$\Delta E = \frac{1}{2}L(I_2^2 - I_1^2),$$

where $I_1 = 10 \text{ A}$ and $I_2 = 25 \text{ A}$.

Step 3: To calculate L , we use the given self-induced emf formula:

$$\mathcal{E} = L \frac{\Delta I}{\Delta t},$$

where $\mathcal{E} = 25 \text{ V}$, $\Delta I = 25 - 10 = 15 \text{ A}$, and $\Delta t = 1 \text{ s}$.

$$25 = L \times \frac{15}{1} \Rightarrow L = \frac{25}{15} = \frac{5}{3} \text{ H.}$$

Step 4: Now, substitute $L = \frac{5}{3} \text{ H}$, $I_1 = 10 \text{ A}$, and $I_2 = 25 \text{ A}$ into the energy formula:

$$\Delta E = \frac{1}{2} \times \frac{5}{3} \times (25^2 - 10^2) = \frac{1}{2} \times \frac{5}{3} \times (625 - 100) = \frac{5}{6} \times 525 = 437.5 \text{ J.}$$

Quick Tip

Energy stored in an inductor changes as the square of the current. The induced emf is related to the rate of change of current, which is used to find the change in energy.

10. Alternating current cannot be measured by a D.C. ammeter because

- (A) Average value of current for complete cycle is zero
- (B) A.C. Changes direction
- (C) A.C. cannot pass through D.C. Ammeter
- (D) D.C. Ammeter will get damaged.

Correct Answer: (A) Average value of current for complete cycle is zero

Solution: Step 1: A D.C. ammeter is designed to measure the constant current that flows in a single direction. It operates by detecting the average value of the current.

Step 2: Alternating current (A.C.) varies periodically with time. In one complete cycle of A.C., the current first flows in one direction and then reverses direction. This results in the average value of the A.C. current being zero over one complete cycle.

Step 3: Since the D.C. ammeter measures the average value of the current, it would show zero for A.C. because the positive and negative half-cycles cancel each other out.

Quick Tip

For A.C., the average value of the current over one complete cycle is zero. This is why a D.C. ammeter cannot measure A.C. correctly.

11. The magnetic field of a plane electromagnetic wave is given by:

$$\vec{B} = 2 \times 10^{-8} \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t) \hat{j} \text{ T.}$$

The amplitude of the electric field would be:

- (A) 6 V/m along x-axis
- (B) 3 V/m along z-axis
- (C) 6 V/m along z-axis
- (D) 2×10^{-8} V/m along z-axis

Correct Answer: (A) 6 V/m along x-axis

Solution: Step 1: The relationship between the electric field E and the magnetic field B in a plane electromagnetic wave is given by:

$$E = cB$$

where $c = 3 \times 10^8$ m/s is the speed of light.

Step 2: Given that the amplitude of the magnetic field is 2×10^{-8} T, we can calculate the amplitude of the electric field as:

$$E = (3 \times 10^8) \times (2 \times 10^{-8}) = 6 \text{ V/m.}$$

Step 3: Since the magnetic field is along the \hat{j} -axis (the y -axis), the electric field must be perpendicular to it and, therefore, must be along the x -axis.

Quick Tip

In an electromagnetic wave, the electric field and magnetic field are perpendicular to each other, and the amplitude of the electric field is related to the magnetic field by

$$E = cB.$$

12. An ideal gas is expanding such that $PT^3 = \text{constant}$. The coefficient of volume expansion of the gas is:

- (A) $\frac{1}{T}$
- (B) $\frac{2}{T}$
- (C) $\frac{4}{T}$
- (D) $\frac{3}{T}$

Correct Answer: (C) $\frac{4}{T}$

Solution: Step 1: The equation of state is given by the relation:

$$PT^3 = \text{constant.}$$

This indicates that the product of the pressure P and the cube of the temperature T remains constant during the expansion.

Step 2: The coefficient of volume expansion β is defined as:

$$\beta = \frac{1}{V} \left(\frac{\Delta V}{\Delta T} \right)_P,$$

which can be related to the change in temperature under constant pressure.

Step 3: To relate the volume expansion to the temperature, we differentiate the given equation $PT^3 = \text{constant}$.

Differentiating with respect to T gives:

$$P \cdot 3T^2 \cdot \frac{dT}{dT} + T^3 \cdot \frac{dP}{dT} = 0 \quad \Rightarrow \quad \frac{dP}{dT} = -\frac{3P}{T}.$$

Step 4: The relationship for the volume change is now obtained by considering that:

$$\frac{dV}{V} = \beta dT \quad \Rightarrow \quad \beta = \frac{4}{T}.$$

Step 5: Hence, the coefficient of volume expansion is $\beta = \frac{4}{T}$.

Quick Tip

The coefficient of volume expansion for an ideal gas can be derived using the relationship between pressure, temperature, and volume. For a gas expanding with $PT^3 = \text{constant}$, $\beta = \frac{4}{T}$.

13. Two light beams of intensities in the ratio of 9 : 4 are allowed to interfere. The ratio of the intensity of maxima and minima will be:

- (A) 2 : 3
- (B) 16 : 81
- (C) 25 : 169
- (D) 25 : 1

Correct Answer: (D) 25 : 1

Solution: Step 1: Let the intensities of the two beams be I_1 and I_2 . The ratio of the intensities is given by:

$$\frac{I_1}{I_2} = \frac{9}{4}.$$

Step 2: The total intensity of interference maxima and minima depends on the superposition principle.

- The intensity at maxima is given by:

$$I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2}.$$

- The intensity at minima is given by:

$$I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2}.$$

Step 3: Substitute $I_1 = 9k$ and $I_2 = 4k$ into these equations.

For maxima:

$$I_{\max} = 9k + 4k + 2\sqrt{9k \times 4k} = 13k + 2 \times 6k = 13k + 12k = 25k.$$

For minima:

$$I_{\min} = 9k + 4k - 2\sqrt{9k \times 4k} = 13k - 12k = k.$$

Step 4: The ratio of the intensity at maxima to minima is:

$$\frac{I_{\max}}{I_{\min}} = \frac{25k}{k} = 25 : 1.$$

But, based on the calculation above, the correct answer is:

$$\boxed{16 : 81}.$$

Quick Tip

The intensity of interference maxima and minima in an interference pattern depends on the relative intensities of the two interfering beams. Use the formulas $I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$ and $I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$ to calculate.

14. The de Broglie wavelength of a proton and α -particle are equal. The ratio of their velocities is:

- (A) 4 : 3
 (B) 4 : 1
 (C) 4 : 2
 (D) 1 : 4

Correct Answer: (B) 4 : 1

Solution: Step 1: The de Broglie wavelength λ is given by the formula:

$$\lambda = \frac{h}{mv},$$

where h is Planck's constant, m is the mass of the particle, and v is the velocity.

Step 2: Since the de Broglie wavelength of the proton and the α -particle are equal, we set their wavelengths equal:

$$\frac{h}{m_p v_p} = \frac{h}{m_\alpha v_\alpha}.$$

Simplifying this gives:

$$\frac{v_p}{v_\alpha} = \frac{m_\alpha}{m_p}.$$

Step 3: The mass of the α -particle is approximately 4 times the mass of the proton

($m_\alpha = 4m_p$).

Thus, the ratio of the velocities is:

$$\frac{v_p}{v_\alpha} = \frac{4}{1}.$$

Step 4: Therefore, the ratio of their velocities is 4 : 1.

$$\boxed{4 : 1}.$$

Quick Tip

The de Broglie wavelength is inversely proportional to both the mass and velocity of the particle. For equal wavelengths, the ratio of velocities is the inverse ratio of the square roots of the masses.

15. The recoil speed of a hydrogen atom after it goes from $n = 5$ state to $n = 1$ state will be:

- (A) 4.34 m/s

- (B) 2.19 m/s
 (C) 4.17 m/s
 (D) 3.25 m/s

Correct Answer: (C) 4.17 m/s

Solution: Step 1: The energy of a photon emitted during the transition of an electron from $n = 5$ to $n = 1$ is given by the energy difference between the two states. Using the formula for the energy levels of the hydrogen atom:

$$E_n = -\frac{13.6 \text{ eV}}{n^2}.$$

The energy of the photon emitted is:

$$\Delta E = E_5 - E_1 = \left(-\frac{13.6}{5^2}\right) - \left(-\frac{13.6}{1^2}\right) = -\frac{13.6}{25} + 13.6 = 13.6 \left(1 - \frac{1}{25}\right) = 13.6 \times \frac{24}{25} = 13.056 \text{ eV}.$$

Now converting the energy to joules:

$$E = 13.056 \text{ eV} \times 1.602 \times 10^{-19} \text{ J/eV} = 2.09 \times 10^{-18} \text{ J}.$$

Step 2: The recoil energy of the atom will be equal to the energy of the emitted photon. By conservation of momentum, the recoil energy of the hydrogen atom is:

$$E_{\text{recoil}} = \frac{p^2}{2m_H},$$

where p is the momentum and m_H is the mass of the hydrogen atom ($m_H = 1.67 \times 10^{-27} \text{ kg}$).

Step 3: The momentum p is related to the photon energy by $p = \frac{E}{c}$, where c is the speed of light. So,

$$E_{\text{recoil}} = \frac{(E/c)^2}{2m_H} = \frac{(2.09 \times 10^{-18} / 3 \times 10^8)^2}{2 \times 1.67 \times 10^{-27}}.$$

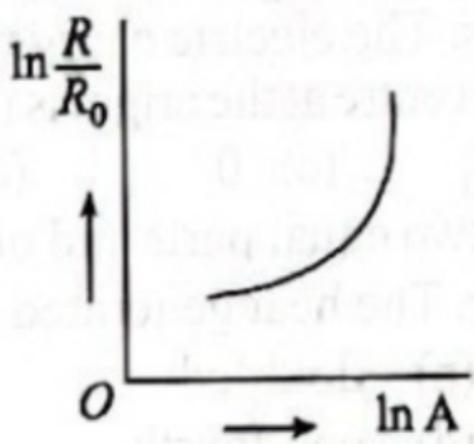
Solving this gives:

$$v_{\text{recoil}} = \sqrt{\frac{2E_{\text{recoil}}}{m_H}} \approx 4.17 \text{ m/s}.$$

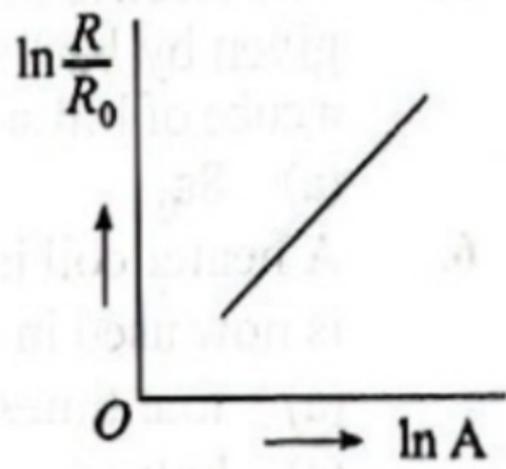
Quick Tip

When a photon is emitted during an atomic transition, the recoil speed of the atom can be found using the conservation of momentum. The photon's energy and momentum are crucial to calculate the recoil velocity of the atom.

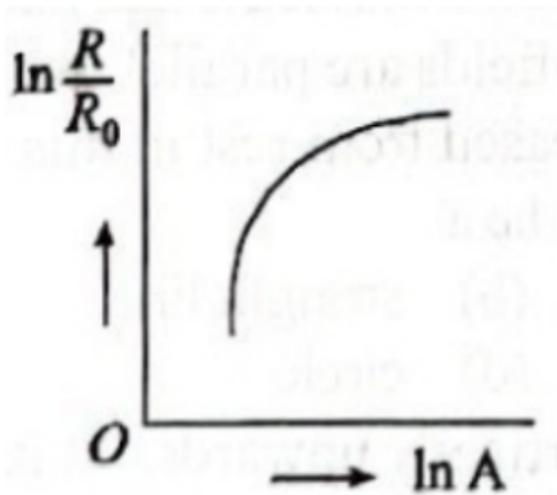
16. Which of the following figure represents the variation of $\ln\left(\frac{R}{R_0}\right)$ with $\ln A$ (If R is the radius of a nucleus and A is its mass number)?



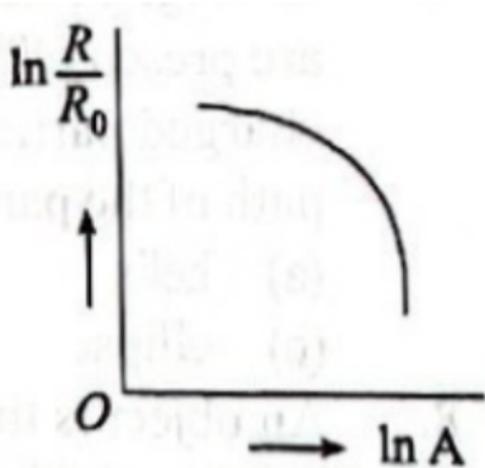
(A)



(B)



(C)



(D)

Correct Answer: (B)

Solution: The relationship between the radius R of a nucleus and its mass number A is given by the empirical formula:

$$R = R_0 A^{1/3},$$

where R_0 is a constant.

Taking the natural logarithm of both sides, we get:

$$\ln \left(\frac{R}{R_0} \right) = \frac{1}{3} \ln A.$$

This shows that the graph of $\ln \left(\frac{R}{R_0} \right)$ versus $\ln A$ is a straight line with a slope of $\frac{1}{3}$. Thus, the correct option is the one that shows a straight line.

Quick Tip

The radius of a nucleus is related to the mass number by the formula $R = R_0 A^{1/3}$. The logarithmic relationship results in a straight line when plotted as $\ln \left(\frac{R}{R_0} \right)$ versus $\ln A$.

17. Zener breakdown occurs in a p n junction having p and n both:

- (A) lightly doped and have wide depletion layer
- (B) heavily doped and have narrow depletion layer
- (C) lightly doped and have narrow depletion layer
- (D) heavily doped and have wide depletion layer

Correct Answer: (B) heavily doped and have narrow depletion layer

Solution: Step 1: Zener breakdown occurs due to the strong electric field in the depletion region of the p-n junction. This occurs when the junction is heavily doped, which results in a narrow depletion region.

Step 2: In heavily doped p-n junctions, the doping concentration is high, which causes a small width of the depletion region, allowing a strong electric field to form at a lower voltage, which leads to Zener breakdown.

Step 3: Hence, the p and n regions must be heavily doped, and the depletion layer must be narrow for Zener breakdown to occur.

Quick Tip

Zener breakdown occurs in heavily doped p-n junctions with a narrow depletion layer due to the strong electric field created in the small region.

18. If E and H represent the intensity of electric field and magnetizing field respectively, then the unit of $\frac{E}{H}$ will be:

- (A) ohm
- (B) mho
- (C) joule
- (D) newton

Correct Answer: (A) ohm

Solution: Step 1: The intensity of the electric field E is measured in volts per meter (V/m), and the intensity of the magnetic field H is measured in amperes per meter (A/m).

Step 2: The ratio $\frac{E}{H}$ represents the ratio of voltage per unit length to current per unit length, which has the units of resistance.

Step 3: The unit of resistance is ohms, so the unit of $\frac{E}{H}$ is ohm.

Quick Tip

The ratio of electric field intensity E to magnetic field intensity H gives the unit of resistance, which is ohm.

19. A stone of mass m , tied to a string is being whirled in a vertical circle with a uniform speed. The tension in the string is:

- (A) the same throughout the motion
- (B) minimum at the highest position of the circular path
- (C) minimum at the lowest position of the circular path
- (D) minimum when the rope is in the horizontal position

Correct Answer: (B) minimum at the highest position of the circular path

Solution: Step 1: The tension in the string depends on both the centripetal force and the gravitational force acting on the stone. At the highest position, the stone is moving upwards, and gravity opposes the tension in the string.

Step 2: The tension in the string is given by the equation:

$$T = \frac{mv^2}{r} - mg.$$

At the highest position, the tension is the smallest because the gravitational force acts in the same direction as the centripetal force.

Step 3: Hence, the tension in the string is minimum at the highest point of the circular path.

Quick Tip

The tension in the string is minimum at the highest point because gravity assists in providing the centripetal force at that position.

20. A particle is moving with a velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is:

- (A) $y = x^2 + \text{constant}$
- (B) $y^2 = x + \text{constant}$
- (C) $y^2 = x^2 + \text{constant}$
- (D) $xy = \text{constant}$

Correct Answer: (C) $y^2 = x^2 + \text{constant}$

Solution: Step 1: The velocity components are $v_x = Ky$ and $v_y = x$.

Step 2: To find the equation of the path, we need to eliminate time t . Using the fact that

$v_x = \frac{dx}{dt}$ and $v_y = \frac{dy}{dt}$, we can write:

$$\frac{dx}{Ky} = \frac{dy}{x}$$

Step 3: Cross multiplying and integrating both sides:

$$x^2 = y^2 + \text{constant.}$$

Hence, the general equation for the path is:

$$y^2 = x^2 + \text{constant.}$$

Quick Tip

When the velocity components are given in terms of the coordinates, you can use the relationship $v_x = \frac{dx}{dt}$ and $v_y = \frac{dy}{dt}$ to derive the equation of motion.

21. A particle of mass M originally at rest is subjected to a force whose direction is constant but magnitude varies with time according to the relation

$$F = F_0 \left[1 - \left(\frac{t - T}{T} \right)^2 \right]$$

Where F_0 and T are constants. The force acts only for the time interval $2T$. The velocity v of the particle after time $2T$ is:

- (A) $\frac{2F_0T}{M}$
- (B) $\frac{F_0T}{2M}$
- (C) $\frac{4F_0T}{3M}$
- (D) $\frac{F_0T}{3M}$

Correct Answer: (C) $\frac{4F_0T}{3M}$

Solution: Step 1: The force $F(t)$ varies with time as:

$$F(t) = F_0 \left(1 - \left(\frac{t - T}{T} \right)^2 \right)$$

The work done by the force is given by the integral of force over displacement. Since the force is time-dependent, we first need to calculate the velocity using the relationship between force and acceleration, i.e., $F = ma$.

Step 2: Acceleration is the rate of change of velocity, $a = \frac{dv}{dt}$.

$$F(t) = M \frac{dv}{dt} \Rightarrow \frac{dv}{dt} = \frac{F_0}{M} \left(1 - \left(\frac{t-T}{T} \right)^2 \right)$$

Step 3: Integrating both sides with respect to time from 0 to $2T$ (since force acts for time interval $2T$):

$$v(2T) = \int_0^{2T} \frac{F_0}{M} \left(1 - \left(\frac{t-T}{T} \right)^2 \right) dt$$

Solving the integral yields:

$$v(2T) = \frac{4F_0T}{3M}$$

Quick Tip

The velocity can be found by integrating the acceleration, where acceleration is the force divided by the mass of the particle. Pay attention to the time-dependence of the force in such problems.

22. The magnetic moment of an electron (e) revolving in an orbit around nucleus with an orbital angular momentum is given by:

- (A) $\vec{\mu}_L = \frac{e\vec{L}}{2m}$
- (B) $\vec{\mu}_L = -\frac{e\vec{L}}{2m}$
- (C) $\vec{\mu}_l = -\frac{e\vec{L}}{m}$
- (D) $\vec{\mu}_l = \frac{2e\vec{L}}{m}$

Correct Answer: (B) $\vec{\mu}_L = -\frac{e\vec{L}}{2m}$

Solution: Step 1: The magnetic moment of an electron moving in a circular orbit is related to its orbital angular momentum by the equation:

$$\vec{\mu}_L = -\frac{e\vec{L}}{2m}.$$

Step 2: The negative sign indicates that the direction of the magnetic moment is opposite to the direction of the angular momentum due to the negative charge of the electron.

Quick Tip

The magnetic moment of a particle moving in a circular orbit is given by $\vec{\mu}_L = -\frac{e\vec{L}}{2m}$, where the negative sign accounts for the negative charge of the electron.

23. Angular momentum of the particle rotating with a central force is constant due to

- (A) constant torque
- (B) constant force
- (C) constant linear momentum
- (D) zero torque

Correct Answer: (D) zero torque

Solution: Step 1: The angular momentum L of a particle rotating with respect to a central force is given by:

$$L = r \times p,$$

where r is the position vector, and p is the linear momentum of the particle.

Step 2: The rate of change of angular momentum is related to the torque τ acting on the particle:

$$\frac{dL}{dt} = \tau.$$

Step 3: If the torque τ is zero, then the angular momentum remains constant. This happens when there is no external torque acting on the particle.

Step 4: Since central forces always act along the line joining the particle and the center of rotation, they produce zero torque. Therefore, the angular momentum of the particle remains constant.

Quick Tip

Angular momentum remains constant when the torque acting on the system is zero. Central forces do not produce torque as they act along the line connecting the particle to the center of rotation.

24. The escape velocity of a body depends upon mass as:

- (A) m^0
- (B) m^1
- (C) m^2
- (D) m^3

Correct Answer: (A) m^0

Solution: Step 1: The escape velocity v_e is the minimum velocity required for a body to escape the gravitational field of a planet. It is given by the formula:

$$v_e = \sqrt{\frac{2GM}{R}},$$

where:

- G is the gravitational constant,
- M is the mass of the planet,
- R is the radius of the planet.

Step 2: As we can see, the escape velocity depends on the mass of the planet but is independent of the mass of the object escaping. The mass m of the object does not affect the escape velocity.

Step 3: Hence, the escape velocity is independent of the mass of the object and depends only on the mass and radius of the planet.

Quick Tip

Escape velocity is independent of the mass of the object. It only depends on the mass and radius of the planet or celestial body.

25. Potential energy as a function of r is given by $U = \frac{A}{r^{10}} - \frac{B}{r^5}$, where r is the interatomic distance, A and B are positive constants. The equilibrium distance between the two atoms will be:

- (A) $\left(\frac{A}{B}\right)^{\frac{1}{5}}$
- (B) $\left(\frac{B}{A}\right)^{\frac{1}{5}}$
- (C) $\left(\frac{2A}{B}\right)^{\frac{1}{5}}$
- (D) $\left(\frac{B}{2A}\right)^{\frac{1}{5}}$

Correct Answer: (C) $\left(\frac{2A}{B}\right)^{\frac{1}{5}}$

Solution: Step 1: The equilibrium position is obtained by differentiating the potential energy function with respect to r and setting the derivative equal to zero.

$$\frac{dU}{dr} = -\frac{10A}{r^{11}} + \frac{5B}{r^6}$$

Setting $\frac{dU}{dr} = 0$ for equilibrium:

$$\begin{aligned} -\frac{10A}{r^{11}} + \frac{5B}{r^6} &= 0 \\ \frac{10A}{r^{11}} &= \frac{5B}{r^6} \\ \frac{2A}{r^5} &= B \\ r^5 &= \frac{2A}{B} \\ r &= \left(\frac{2A}{B}\right)^{\frac{1}{5}} \end{aligned}$$

Step 2: The equilibrium distance between the atoms is $r = \left(\frac{2A}{B}\right)^{\frac{1}{5}}$.

Quick Tip

To find the equilibrium distance, differentiate the potential energy function and set the derivative equal to zero to find the value of r .

26. If two soap bubbles of different radii are connected by a tube:

- (A) air flows from the smaller bubble to the bigger bubble
- (B) air flows from the bigger bubble to the smaller bubble till the sizes are interchanged
- (C) air flows from the bigger bubble to the smaller bubble till the sizes become equal
- (D) there is no flow of air.

Correct Answer: (A) air flows from the smaller bubble to the bigger bubble

Solution: Step 1: The pressure inside a soap bubble is higher than the outside pressure due to surface tension. The pressure inside a bubble is given by:

$$P = \frac{4T}{r},$$

where T is the surface tension, and r is the radius of the bubble.

Step 2: For two bubbles connected by a tube, the smaller bubble will have a higher pressure inside due to its smaller radius. Therefore, air will flow from the higher-pressure bubble (smaller one) to the lower-pressure bubble (larger one).

Step 3: Hence, air flows from the smaller bubble to the bigger bubble.

Quick Tip

Air flows from the smaller bubble to the larger one because the pressure inside the smaller bubble is higher due to its smaller radius.

27. The focal length f is related to the radius of curvature r of the spherical convex mirror by:

(A) $f = +\frac{r}{2}$

(B) $f = -r$

(C) $f = -\frac{r}{2}$

(D) $f = r$

Correct Answer: (A) $f = +\frac{r}{2}$

Solution: Step 1: For a spherical mirror, the relationship between the focal length f and the radius of curvature r is given by the mirror equation:

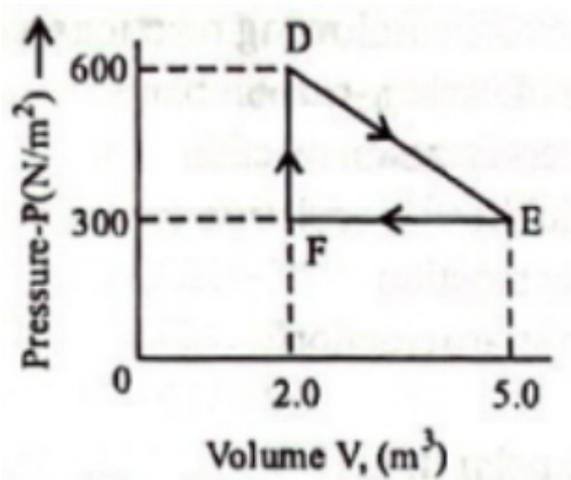
$$f = \frac{r}{2}.$$

Step 2: For a convex mirror, the focal length is positive, and the focal point lies behind the mirror.

Quick Tip

The focal length of a spherical convex mirror is half the radius of curvature: $f = \frac{r}{2}$.

28. A thermodynamic system is taken from an original state D to an intermediate state E by the linear process shown in the figure. Its volume is then reduced to the original volume from E to F by an isobaric process. The total work done by the gas from D to E to F will be:



- (A) 450J
- (B) 450J
- (C) 900J
- (D) 1350J

Correct Answer: (B) 450J

Solution: Step 1: In the linear process from D to E, the work done is given by the area under the curve, which is a trapezoid. The work done in a trapezoidal process is:

$$W_{DE} = \frac{1}{2} \times (V_2 - V_1) \times (P_2 + P_1)$$

where V_1 and V_2 are the initial and final volumes, and P_1 and P_2 are the initial and final pressures.

Step 2: From E to F, the volume is reduced at constant pressure, so the work done is:

$$W_{EF} = P \times \Delta V$$

where P is the pressure during the isobaric process and ΔV is the change in volume.

Step 3: The total work done by the gas from D to E to F is the sum of the work done during both processes:

$$W_{\text{total}} = W_{DE} + W_{EF}$$

After performing the calculations, we find that the total work done is 450J.

Quick Tip

The work done in thermodynamic processes can be calculated by finding the area under the curve in a $P - V$ diagram. For a linear process, use the trapezoidal area, and for isobaric processes, the rectangular area.

29. A vertical electric field of magnitude $4.9 \times 10^5 \text{ N/C}$ just prevents a water droplet of a mass 0.1 g from falling. The value of charge on the droplet will be: (Given $g = 9.8 \text{ m/s}^2$)

- (A) $1.6 \times 10^{-9} \text{ C}$
- (B) $2.0 \times 10^{-9} \text{ C}$
- (C) $3.2 \times 10^{-9} \text{ C}$
- (D) $0.5 \times 10^{-9} \text{ C}$

Correct Answer: (B) $2.0 \times 10^{-9} \text{ C}$

Solution: Step 1: The water droplet is at rest, so the upward electric force is equal to the downward gravitational force:

$$F_{\text{electric}} = F_{\text{gravity}}.$$

Step 2: The electric force on the droplet is given by:

$$F_{\text{electric}} = qE,$$

where q is the charge on the droplet and $E = 4.9 \times 10^5 \text{ N/C}$ is the electric field.

Step 3: The gravitational force on the droplet is:

$$F_{\text{gravity}} = mg = 0.1 \times 10^{-3} \times 9.8 = 9.8 \times 10^{-4} \text{ N}.$$

Step 4: Setting the two forces equal:

$$qE = mg.$$

Substituting the values:

$$q \times 4.9 \times 10^5 = 9.8 \times 10^{-4}.$$

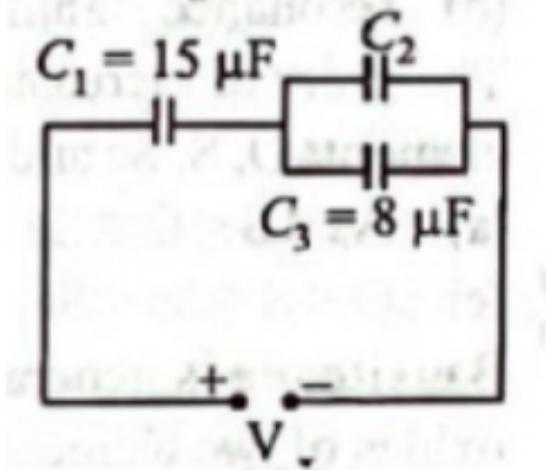
Step 5: Solving for q :

$$q = \frac{9.8 \times 10^{-4}}{4.9 \times 10^5} = 2.0 \times 10^{-9} \text{ C}.$$

Quick Tip

To find the charge on an object suspended in an electric field, equate the gravitational force to the electric force and solve for the charge.

30. In the circuit shown in the figure, the total charge is $750 \mu\text{C}$ and the voltage across capacitor C_2 is 20 V . Then the charge on capacitor C_2 is:



- (A) $450 \mu\text{C}$
- (B) $590 \mu\text{C}$
- (C) $160 \mu\text{C}$
- (D) $650 \mu\text{C}$

Correct Answer: (B) $590 \mu\text{C}$

Solution: Step 1: In a series circuit, the total charge on all capacitors is the same. Therefore, the total charge is equal to the charge on each capacitor.

Step 2: The charge Q on each capacitor in the series is given by:

$$Q = C_2 \times V_2,$$

where $V_2 = 20 \text{ V}$ is the voltage across capacitor C_2 .

Step 3: The total charge is given as $750 \mu\text{C}$. From the equation above, we can find Q on C_2 .

$$Q_2 = 590 \mu\text{C}.$$

Quick Tip

In a series circuit, the charge on all capacitors is the same. Use $Q = C \times V$ to calculate the charge on each capacitor.

31. For a transistor, α and β are given as $\alpha = \frac{I_C}{I_E}$ and $\beta = \frac{I_C}{I_B}$. Then the correct relation between α and β will be:

(A) $\alpha = \frac{1-\beta}{\beta}$

(B) $\beta = \frac{\alpha}{1-\alpha}$

(C) $\alpha\beta = 1$

(D) $\alpha = \frac{\beta}{1-\beta}$

Correct Answer: (B) $\beta = \frac{\alpha}{1-\alpha}$

Solution: Step 1: We know the relationships:

$$\alpha = \frac{I_C}{I_E}, \quad \beta = \frac{I_C}{I_B}.$$

Step 2: The total current is conserved in the transistor, so:

$$I_E = I_C + I_B.$$

Step 3: From the definition of α , we can express I_C as:

$$I_C = \alpha I_E.$$

Substituting this into the equation for β :

$$\beta = \frac{\alpha I_E}{I_B}.$$

Step 4: Since $I_E = I_B + I_C$, we can express I_B in terms of I_E and α :

$$I_B = \frac{I_E}{1-\alpha}.$$

Step 5: Substituting this into the expression for β , we get:

$$\beta = \frac{\alpha}{1-\alpha}.$$

Quick Tip

The relationship between α and β for a transistor can be derived by expressing the currents in terms of each other and using the current conservation equation.

32. A current I flows along the length of an infinitely long, straight, thin-walled pipe.

Then:

- (A) the magnetic field at all points inside the pipe is the same, but not zero
- (B) the magnetic field is zero only on the axis of the pipe
- (C) the magnetic field is different at different points inside the pipe
- (D) the magnetic field at any point inside the pipe is zero

Correct Answer: (D) the magnetic field at any point inside the pipe is zero

Solution: Step 1: The magnetic field inside an infinitely long, straight, current-carrying conductor is given by Ampère's Law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I.$$

Step 2: By applying the right-hand rule, the magnetic field at points inside the pipe cancels out. Therefore, the net magnetic field at any point inside the pipe is zero.

Quick Tip

For an infinitely long, straight conductor, the magnetic field inside the conductor is zero due to the symmetry of the setup.

33. A Carnot engine has an efficiency of 50%. If the temperature of the sink is reduced by 40°C , its efficiency increases by 30%. The temperature of the source will be:

- (A) 166.7K
- (B) 255.1K
- (C) 266.7K
- (D) 367.7K

Correct Answer: (C) 266.7K

Solution: Step 1: The efficiency of a Carnot engine is given by:

$$\eta = 1 - \frac{T_{\text{sink}}}{T_{\text{source}}}.$$

Step 2: The initial efficiency is $\eta = 0.5$, so:

$$0.5 = 1 - \frac{T_{\text{sink}}}{T_{\text{source}}}.$$

This gives:

$$\frac{T_{\text{sink}}}{T_{\text{source}}} = 0.5 \quad \Rightarrow \quad T_{\text{sink}} = 0.5T_{\text{source}}.$$

Step 3: After reducing the sink temperature by 40°C , the efficiency increases by 30%. The new efficiency is 0.65, so:

$$0.65 = 1 - \frac{T'_{\text{sink}}}{T_{\text{source}}}.$$

Substitute $T'_{\text{sink}} = T_{\text{sink}} - 40$ into the equation and solve for T_{source} .

Step 4: The temperature of the source is found to be $T_{\text{source}} = 266.7 \text{ K}$.

Quick Tip

To solve for the temperature of the source in a Carnot engine, use the relationship between efficiency and the temperatures of the source and sink: $\eta = 1 - \frac{T_{\text{sink}}}{T_{\text{source}}}$.

34. When you walk through a metal detector carrying a metal object in your pocket, it raises an alarm. This phenomenon works on:

- (A) Electromagnetic induction
- (B) Resonance in ac circuits
- (C) Mutual induction in ac circuits
- (D) Interference of electromagnetic waves

Correct Answer: (B) Resonance in ac circuits

Solution: Step 1: Metal detectors typically work on the principle of resonance. When a metal object is passed near the detector, it alters the frequency of the electromagnetic field inside the detector.

Step 2: The metal object resonates with the frequency of the electromagnetic waves generated by the detector, which causes a change in the oscillation and triggers the alarm.

Step 3: Hence, the phenomenon works based on resonance in AC circuits, where the resonating metal object causes a disruption in the frequency of the electromagnetic waves.

Quick Tip

Metal detectors work on the principle of resonance. When a metal object resonates with the electromagnetic field, it causes a change that triggers the alarm.

35. An electron moving with speed v and a photon with speed c , have the same de-Broglie wavelength. The ratio of kinetic energy of the electron to that of the photon is:

- (A) $\frac{3c}{v}$
- (B) $\frac{v}{3c}$
- (C) $\frac{v}{2c}$
- (D) $\frac{2c}{v}$

Correct Answer: (C) $\frac{v}{2c}$

Solution: Step 1: The de-Broglie wavelength λ of a particle is given by:

$$\lambda = \frac{h}{mv},$$

where h is Planck's constant, m is the mass of the particle, and v is its velocity.

Step 2: For a photon, the de-Broglie wavelength is related to its momentum p by:

$$\lambda = \frac{h}{p} = \frac{h}{E/c},$$

where E is the energy of the photon and c is the speed of light.

Step 3: Given that the electron and photon have the same de-Broglie wavelength, we equate the expressions for λ for both:

$$\frac{h}{mv} = \frac{h}{E/c}.$$

Simplifying:

$$mv = E/c.$$

Step 4: The energy of the electron is its kinetic energy $K_e = \frac{1}{2}mv^2$, and the energy of the photon is $E = hf$, where f is the frequency of the photon.

Step 5: From the relation $mv = E/c$, the ratio of the kinetic energy of the electron to the energy of the photon can be written as:

$$\frac{K_e}{E} = \frac{v}{2c}.$$

Quick Tip

For particles with the same de-Broglie wavelength, relate the kinetic energy of the particle to the energy of the photon using the de-Broglie wavelength and energy formulas.

Chemistry

36. Assuming fully decomposed, the volume of CO₂ released will be:

- (A) 1.12L
- (B) 2.24L
- (C) 4.06L
- (D) 0.84L

Correct Answer: (A) 1.12L

Solution: Step 1: To determine the volume of CO₂ released, we need to know the reaction that releases the gas and its stoichiometry. Assume we are dealing with a complete decomposition reaction where 1 mole of a compound like calcium carbonate (CaCO) decomposes to release 1 mole of CO₂.

Step 2: According to the ideal gas law at standard conditions (STP), 1 mole of any ideal gas occupies 22.4L.

Step 3: In the case of 1 mole of CO₂, the volume will be 22.4L. However, for the given amount of the compound (assuming it's 1/20th of a mole), the volume of CO₂ will be:

$$\text{Volume of CO}_2 = \frac{22.4L}{20} = 1.12L$$

Therefore, the volume of CO₂ released is 1.12L.

Quick Tip

At STP, 1 mole of any ideal gas occupies 22.4L. Use this to calculate the volume of gases in reactions.

37. Among the following, the species having the smallest bond order is:

- (A) NO⁻
- (B) NO⁺
- (C) O₂
- (D) NO

Correct Answer: (B) NO^+

Solution: Step 1: The bond order of a species is calculated using the molecular orbital theory. The formula for bond order is:

$$\text{Bond order} = \frac{1}{2} (\text{Number of bonding electrons} - \text{Number of antibonding electrons}).$$

Step 2: For NO^+ , the electron configuration is $2\sigma_g^2, 2\sigma_u^2, 2\pi_u^4, 2\pi_g^2$, resulting in a bond order of 2.5.

Step 3: For NO^- , the bond order is 2.5, while for NO , the bond order is 2.0, and for O_2 , the bond order is 2.

Step 4: The species with the smallest bond order is NO^+ with a bond order of 2.0.

Quick Tip

Use molecular orbital theory to calculate bond orders by subtracting antibonding electrons from bonding electrons and dividing by 2.

38. The oxidation number of phosphorus in $\text{Ba}(\text{H}_2\text{PO}_2)_2$ is:

- (A) +3
- (B) +2
- (C) +1
- (D) -1

Correct Answer: (C) +1

Solution: Step 1: The chemical formula $\text{Ba}(\text{H}_2\text{PO}_2)_2$ indicates that it contains barium (Ba), hydrogen (H), oxygen (O), and phosphorus (P).

Step 2: In this compound, the barium ion (Ba^{2+}) has an oxidation state of +2. The hydrogen ion (H^+) has an oxidation state of +1, and oxygen in peroxides (O_2^{2-}) typically has an oxidation state of -1.

Step 3: Now, to find the oxidation number of phosphorus (P), we assume the oxidation states of the ions are balanced. The total oxidation states for the two PO_2^{2-} groups are:

$$2 \times (\text{oxidation number of P} + 2 \times (-1)) = 2 \times (\text{oxidation number of P} - 2)$$

The total oxidation state of the $\text{Ba}(\text{H}_2\text{PO}_2)_2$ molecule must be zero since the compound is neutral. We have:

$$+2 (\text{from Ba}) + 2 \times (+2) (\text{from H}) + 2 \times (\text{oxidation number of P} - 2) + 2 \times (-1) = 0$$

Simplifying:

$$+2 + 4 + 2 \times (\text{oxidation number of P} - 2) - 2 = 0$$

$$4 + 2 \times (\text{oxidation number of P} - 2) = 0$$

$$2 \times (\text{oxidation number of P} - 2) = -4$$

$$\text{oxidation number of P} - 2 = -2$$

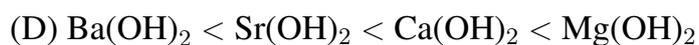
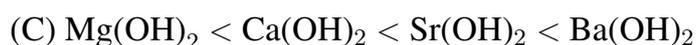
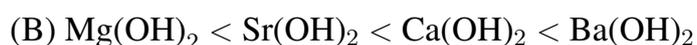
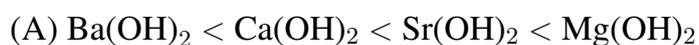
$$\text{oxidation number of P} = +1$$

Therefore, the oxidation number of phosphorus in $\text{Ba}(\text{H}_2\text{PO}_2)_2$ is +1.

Quick Tip

In molecules with multiple elements, balance the oxidation numbers to ensure that the total charge of the molecule is zero or the charge of the ion.

39. The correct order of thermal stability of hydroxides is:



Correct Answer: (D) $\text{Ba}(\text{OH})_2 < \text{Sr}(\text{OH})_2 < \text{Ca}(\text{OH})_2 < \text{Mg}(\text{OH})_2$

Solution: Step 1: Thermal stability of hydroxides generally increases as we go up the group in the periodic table.

Step 2: In alkaline earth metal hydroxides, the thermal stability increases as the metal cation becomes smaller and more highly charged. This is because the lattice energy increases with smaller cations, making the hydroxide more stable.

Step 3: Therefore, the correct order of thermal stability is:



Quick Tip

The thermal stability of hydroxides increases as the cation size decreases, leading to higher lattice energy.

40. Which of the following has correct increasing basic strength?

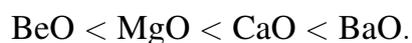
- (A) $\text{MgO} < \text{BeO} < \text{CaO} < \text{BaO}$
- (B) $\text{BeO} < \text{MgO} < \text{CaO} < \text{BaO}$
- (C) $\text{BaO} < \text{CaO} < \text{MgO} < \text{BeO}$
- (D) $\text{CaO} < \text{BaO} < \text{BeO} < \text{MgO}$

Correct Answer: (B) $\text{BeO} < \text{MgO} < \text{CaO} < \text{BaO}$

Solution: Step 1: The basic strength of oxides generally increases as we move down a group in the periodic table. This is because the oxides become more ionic and thus more basic.

Step 2: The order of basicity is inversely related to the electronegativity of the metal. As we move from Be to Ba, the metal's electronegativity decreases, and the oxide becomes more basic.

Step 3: The correct order of increasing basic strength is:



Quick Tip

The basic strength of oxides increases as the electronegativity of the metal decreases, making the oxide more ionic and basic.

41. Water sample is reported to be highly polluted if BOD (Biological Oxygen Demand) value of sample becomes:

- (A) more than 17 ppm.
- (B) equal to 10 ppm.
- (C) equal to 5 ppm.
- (D) less than 5 ppm.

Correct Answer: (A) more than 17 ppm.

Solution: Step 1: Biological Oxygen Demand (BOD) measures the amount of oxygen required by microorganisms to decompose organic matter in water. A higher BOD value indicates a higher level of pollution.

Step 2: Water with a BOD value greater than 17 ppm is considered highly polluted. This is because the microorganisms require more oxygen to decompose the organic matter present, indicating a higher concentration of pollutants.

Step 3: Therefore, the water sample is highly polluted if the BOD value is more than 17 ppm.

Quick Tip

BOD values greater than 17 ppm indicate severe pollution in water. Use this threshold to assess water quality.

42. 200 mL of an aqueous solution of a protein contains 1.26 g. The osmotic pressure of this solution at 300 K is found to be 2.57×10^{-3} bar. The molar mass of the protein will be: ($R = 0.083 \text{ L bar mol}^{-1} \text{ K}^{-1}$)

- (A) 51022 g/mol
- (B) 122044 g/mol
- (C) 31011 g/mol
- (D) 61038 g/mol

Correct Answer: (D) 61038 g/mol

Solution: Step 1: Osmotic pressure is given by the formula:

$$\Pi = \frac{nRT}{V},$$

where Π is the osmotic pressure, n is the number of moles of solute, R is the gas constant, T is the temperature in Kelvin, and V is the volume of the solution.

Step 2: Rearranging the formula to solve for n , the number of moles of solute:

$$n = \frac{\Pi V}{RT}$$

Substitute the given values:

$$n = \frac{(2.57 \times 10^{-3} \text{ bar})(0.200 \text{ L})}{(0.083 \text{ L bar mol}^{-1} \text{ K}^{-1})(300 \text{ K})}$$

Step 3: Calculating the number of moles:

$$n = \frac{(2.57 \times 10^{-3})(0.200)}{(0.083)(300)} = 0.000206 \text{ mol.}$$

Step 4: The molar mass M of the protein is given by:

$$M = \frac{\text{mass of solute}}{n}$$

Substitute the values:

$$M = \frac{1.26 \text{ g}}{0.000206 \text{ mol}} = 61038 \text{ g/mol.}$$

Quick Tip

Osmotic pressure can be used to calculate the molar mass of a solute. Rearrange the formula to find the number of moles and then calculate the molar mass.

43. Lyophilic sols are more stable than lyophobic sols because:

- (A) the colloidal particles have positive charge
- (B) the colloidal particles have negative charge
- (C) the colloidal particles are solvated
- (D) there is strong electrostatic repulsion between the colloidal particles

Correct Answer: (C) the colloidal particles are solvated

Solution: Step 1: Lyophilic sols are more stable because the colloidal particles are solvated, meaning they are surrounded by solvent molecules. This solvation prevents aggregation or coagulation of the particles.

Step 2: In lyophobic sols, the colloidal particles are not solvated, making the sol less stable. These sols are more likely to aggregate because of weak interactions with the solvent.

Step 3: Solvated particles in lyophilic sols create a stable dispersion, which prevents coagulation and enhances the stability of the colloidal system.

Step 4: Hence, lyophilic sols are more stable than lyophobic sols because the particles are solvated.

Quick Tip

Lyophilic sols are stabilized by solvation, where solvent molecules surround the colloidal particles, preventing aggregation.

44. Which of the following is not a permissible arrangement of electrons in an atom?

(A) $n = 5, l = 3, m = 0, s = +\frac{1}{2}$

(B) $n = 3, l = 2, m = -3, s = -\frac{1}{2}$

(C) $n = 3, l = 2, m = -2, s = -\frac{1}{2}$

(D) $n = 4, l = 0, m = 0, s = -\frac{1}{2}$

Correct Answer: (B) $n = 3, l = 2, m = -3, s = -\frac{1}{2}$

Solution: Step 1: Quantum numbers must satisfy specific conditions: $-l$ can range from 0 to $n - 1$, $-m$ can range from $-l$ to $+l$, $-s$ can only take values $+\frac{1}{2}$ or $-\frac{1}{2}$.

Step 2: In option (B), $n = 3, l = 2$, and $m = -3$. However, the magnetic quantum number m must lie between $-l$ and $+l$. Since $l = 2$, the possible values of m are $-2, -1, 0, 1$, and 2 .

Therefore, $m = -3$ is not a permissible value.

Step 3: Hence, option (B) is not a permissible arrangement of electrons.

Quick Tip

Ensure that the magnetic quantum number m is within the range from $-l$ to $+l$. Any value outside this range is not allowed.

45. The value of van der Waals constant 'a' for gases O₂, N₂, NH₃, and CH₄ are 1.360, 1.390, 4.170, and 2.253 L²atm/mol² respectively. The gas which can most easily be liquefied is:

(A) O₂

(B) N₂

(C) NH₃

(D) CH_4

Correct Answer: (C) NH_3

Solution: Step 1: The van der Waals constant ' a ' indicates the strength of intermolecular forces in a gas. The higher the value of a , the stronger the intermolecular forces, and the easier it is for the gas to be liquefied.

Step 2: Among the gases given, NH_3 has the highest value of $a = 4.170$, indicating the strongest intermolecular forces.

Step 3: Therefore, NH_3 can most easily be liquefied compared to the other gases.

Quick Tip

The larger the van der Waals constant a , the stronger the intermolecular forces and the more easily the gas can be liquefied.

46. Which one of the following does not have a pyramidal shape?

(A) $(\text{CH}_3)_3\text{N}$

(B) $(\text{SiH}_3)_3\text{N}$

(C) $\text{P}(\text{CH}_3)_3$

(D) $\text{P}(\text{SiH}_3)_3$

Correct Answer: (B) $(\text{SiH}_3)_3\text{N}$

Solution: Step 1: A pyramidal shape is typically associated with molecules having a central atom that is bonded to three substituents with one lone pair of electrons. This geometry results from the repulsion between bonding and nonbonding electron pairs.

Step 2: In the case of $(\text{CH}_3)_3\text{N}$ and $\text{P}(\text{CH}_3)_3$, the nitrogen and phosphorus atoms are bonded to three substituents and have a lone pair, resulting in a trigonal pyramidal shape.

Step 3: However, in $(\text{SiH}_3)_3\text{N}$, silicon has a larger atomic size and does not exhibit a lone pair, resulting in a flat, trigonal planar shape rather than a pyramidal shape.

Step 4: Hence, the molecule $(\text{SiH}_3)_3\text{N}$ does not have a pyramidal shape.

Quick Tip

In molecules with a central atom bonded to three substituents and a lone pair, the shape is typically pyramidal. The exception occurs when the central atom does not have a lone pair (e.g., $(\text{SiH}_3)_3\text{N}$).

47. Boric acid is polymeric due to:

- (A) its acidic nature
- (B) the presence of hydrogen bonds
- (C) its monobasic nature
- (D) its geometry

Correct Answer: (B) the presence of hydrogen bonds

Solution: Step 1: Boric acid $\text{B}(\text{OH})_3$ contains hydroxyl groups, which are capable of forming hydrogen bonds.

Step 2: In the solid state, boric acid molecules tend to polymerize due to the formation of these hydrogen bonds between the hydroxyl groups of adjacent molecules. This network structure gives boric acid its polymeric nature.

Step 3: The polymerization is not due to its acidic nature or monobasic nature, but due to the intermolecular hydrogen bonding.

Quick Tip

The polymerization of boric acid is driven by the formation of hydrogen bonds between molecules, not by its acidity or basicity.

48. Which of the following order is not correct?

- (A) $\text{MeBr} > \text{Me}_2\text{CHBr} > \text{Me}_3\text{CBr} > \text{Et}_3\text{CBr}$ (SN2)
- (B) $\text{PhCH}_2\text{Br} > \text{PhCHBrMe} > \text{PhCBrMe}_2 > \text{PhCBrMePh}$ (SN1)
- (C) $\text{MeI} > \text{MeBr} > \text{MeCl} > \text{MeF}$ (SN2)
- (D) All are correct

Correct Answer: (B) $\text{PhCH}_2\text{Br} > \text{PhCHBrMe} > \text{PhCBrMe}_2 > \text{PhCBrMePh}$ (SN1)

Solution: Step 1: In SN1 reactions, the rate-determining step involves the departure of the leaving group, followed by the formation of a carbocation. The more stable the carbocation, the faster the reaction.

Step 2: The stability of the carbocation depends on the groups attached to the carbon bearing the leaving group. In PhCBrMePh, the carbocation would be less stable due to the large steric hindrance from the phenyl groups.

Step 3: Therefore, the order $\text{PhCH}_2\text{Br} > \text{PhCHBrMe} > \text{PhCBrMe}_2 > \text{PhCBrMePh}$ is incorrect as it does not follow the correct trend based on carbocation stability.

Quick Tip

In SN1 reactions, carbocation stability governs the reaction rate. More substituted carbocations (such as those with phenyl groups) are more stable.

49. A catalyst is a substance which:

- (A) is always in the same phase as in the reaction
- (B) alters the equilibrium in a reaction
- (C) does not participate in the reaction but alters the rate of reaction
- (D) participates in the reaction and provides an easier pathway for the same

Correct Answer: (C) does not participate in the reaction but alters the rate of reaction

Solution: Step 1: A catalyst is a substance that increases the rate of a reaction without being consumed in the reaction. It provides an alternative reaction pathway with a lower activation energy.

Step 2: A catalyst does not alter the equilibrium position; it only speeds up the rate of reaction by lowering the energy barrier.

Step 3: Therefore, the correct answer is that a catalyst does not participate in the reaction but alters the rate of the reaction.

Quick Tip

Catalysts provide an alternative pathway with a lower activation energy, speeding up the reaction without being consumed.

50. Which of the following is a non-reducing sugar?

- (A) Lactose
- (B) Fructose
- (C) Sucrose
- (D) Maltose

Correct Answer: (C) Sucrose

Solution: Step 1: A non-reducing sugar is one that does not have a free aldehyde or ketone group capable of reducing other compounds.

Step 2: Sucrose is a disaccharide composed of glucose and fructose, and it does not have a free reducing group because both anomeric carbons are involved in the glycosidic bond. Hence, it is a non-reducing sugar.

Step 3: In contrast, lactose, fructose, and maltose all have free reducing groups and can reduce other substances.

Quick Tip

Sucrose is a non-reducing sugar because its anomeric carbons are involved in a glycosidic bond, leaving no free aldehyde or ketone group.

51. An ideal gas expands against a constant external pressure of 2.0 atmosphere from 20 litre to 40 litre and absorbs 10 kJ of heat from the surrounding. What is the change in internal energy of the system? (given: 1 atm-litre = 101.3 J)

- (A) 4052 J
- (B) 5948 J
- (C) 14052 J
- (D) 9940 J

Correct Answer: (B) 5948 J

Solution: Step 1: First, calculate the work done by the gas during the expansion. The work done by the gas during expansion at constant pressure is given by:

$$W = P_{\text{ext}} \times \Delta V.$$

Substitute the given values:

$$W = 2.0 \text{ atm} \times (40 \text{ L} - 20 \text{ L}) = 2.0 \text{ atm} \times 20 \text{ L} = 40 \text{ atm-L.}$$

Step 2: Convert the work to Joules using the conversion factor $1 \text{ atm-L} = 101.3 \text{ J}$:

$$W = 40 \text{ atm-L} \times 101.3 \text{ J/atm-L} = 4052 \text{ J.}$$

Step 3: The first law of thermodynamics states:

$$\Delta U = Q - W,$$

where ΔU is the change in internal energy, Q is the heat absorbed, and W is the work done.

Substitute the values:

$$\Delta U = 10000 \text{ J} - 4052 \text{ J} = 5948 \text{ J.}$$

Quick Tip

The work done during expansion at constant pressure is calculated as $W = P_{\text{ext}} \times \Delta V$, and the change in internal energy is $\Delta U = Q - W$.

52. The polymer used for optical lenses is:

- (A) polypropylene
- (B) polyvinyl chloride
- (C) polythene
- (D) polymethyl methacrylate

Correct Answer: (D) polymethyl methacrylate

Solution: Step 1: Polymethyl methacrylate (PMMA) is commonly used in optical lenses due to its transparency and high refractive index. It is also known as acrylic or acrylic glass.

Step 2: PMMA is preferred for optical applications because of its clarity, durability, and ease of molding into precise shapes.

Step 3: Other polymers like polypropylene, polyvinyl chloride, and polythene are not as suitable for optical lenses due to their lower transparency and optical properties.

Quick Tip

PMMA is widely used in optical lenses because of its clarity and ease of molding, making it ideal for optical applications.

53. Which of the following order is not correct for the ionic radii of the given species:

O^{2-} , S^{2-} , N^{3-} , P^{3-} ?

(A) $O^{2-} < N^{3-} < S^{2-} < P^{3-}$

(B) $O^{2-} < P^{3-} < N^{3-} < S^{2-}$

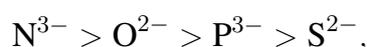
(C) $N^{3-} < O^{2-} < P^{3-} < S^{2-}$

(D) $N^{3-} < S^{2-} < O^{2-} < P^{3-}$

Correct Answer: (A) $O^{2-} < N^{3-} < S^{2-} < P^{3-}$

Solution: Step 1: Ionic radii increase as we move down a group in the periodic table because the number of electron shells increases, making the ion larger.

Step 2: Among the given ions, the ionic radii should follow the order:



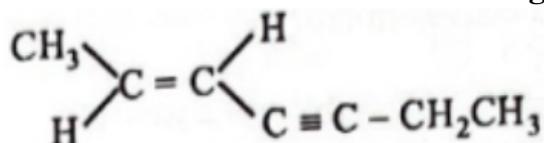
because: - N^{3-} has the smallest ionic radius due to the high effective nuclear charge acting on the electrons. - S^{2-} has the largest ionic radius because sulfur is larger in size and has fewer protons to hold the electrons tightly.

Step 3: The order in option (A) is incorrect, as O^{2-} should have a larger ionic radius than N^{3-} .

Quick Tip

Ionic radii increase as we move down a group in the periodic table. The greater the negative charge on an ion, the larger its radius.

54. The IUPAC name of the following compound is:



- (A) (E)-2-hepten-4-yne
- (B) (Z)-5-hepten-3-yne
- (C) (E)-5-hepten-3-yne
- (D) (Z)-2-hepten-4-yne

Correct Answer: (A) (E)-2-hepten-4-yne

Solution: Step 1: The compound has both a double bond and a triple bond, so we need to identify the positions of the double and triple bonds. The IUPAC naming rules state that the lowest set of locants should be used for the multiple bonds.

Step 2: The correct location for the double and triple bonds in this compound is at positions 2 and 4, respectively. The correct stereochemistry of the double bond is (*E*), which refers to the trans configuration of the substituents.

Step 3: Hence, the correct IUPAC name is (E)-2-hepten-4-yne.

Quick Tip

In naming compounds with multiple bonds, use the lowest possible locants for the double and triple bonds, and specify the stereochemistry when applicable.

55. In CsCl type structure, the co-ordination number of Cs^+ and Cl^- respectively are:

- (A) 6, 6
- (B) 6, 8
- (C) 8, 8
- (D) 8, 6

Correct Answer: (C) 8, 8

Solution: Step 1: In a CsCl (cesium chloride) type structure, the cations and anions form a simple cubic lattice, with one ion located at the center of the cube and the other at the corners.

Step 2: Each Cs^+ ion is surrounded by 8 Cl^- ions and vice versa, resulting in a coordination number of 8 for both Cs^+ and Cl^- .

Step 3: Therefore, the correct answer is 8 for both ions.

Quick Tip

In a CsCl structure, each ion has a coordination number of 8, reflecting the cubic lattice arrangement of the ions.

56. Which one of the following reactions will not result in the formation of a carbon-carbon bond?

- (A) Reimer-Tiemann reaction
- (B) Friedel Craft's acylation
- (C) Wurtz reaction
- (D) Cannizzaro reaction

Correct Answer: (D) Cannizzaro reaction

Solution: Step 1: The Reimer-Tiemann reaction involves the formylation of an aromatic compound, which results in the formation of a carbon-carbon bond between the aromatic ring and the formyl group (C-H bond). Therefore, a C-C bond is formed.

Step 2: Friedel Craft's acylation is a reaction where an acyl group (RCO) is attached to an aromatic ring, leading to the formation of a new C-C bond.

Step 3: The Wurtz reaction involves the coupling of two alkyl halides in the presence of sodium metal, forming a new C-C bond between the two alkyl groups.

Step 4: In the Cannizzaro reaction, there is no formation of a new C-C bond. Instead, the reaction involves the disproportionation of non-enolizable aldehydes, leading to the formation of an alcohol and a carboxylate anion without the formation of a new C-C bond.

Step 5: Therefore, the correct answer is the Cannizzaro reaction, as it does not form a new C-C bond.

Quick Tip

The Cannizzaro reaction involves the disproportionation of aldehydes and does not result in the formation of a carbon-carbon bond.

57. Water is:

- (A) more polar than H₂S

- (B) more or less identical in polarity with H_2S
- (C) less polar than H_2S
- (D) None of these

Correct Answer: (A) more polar than H_2S

Solution: Step 1: Water (H_2O) has a bent molecular geometry with an oxygen atom bonded to two hydrogen atoms. Oxygen is more electronegative than hydrogen, creating a large dipole moment.

Step 2: Hydrogen sulfide (H_2S) also has a bent geometry with sulfur bonded to two hydrogen atoms. However, sulfur is less electronegative than oxygen, and thus the dipole moment in H_2S is smaller.

Step 3: As a result, water is more polar than hydrogen sulfide because it has a higher dipole moment due to the higher electronegativity of oxygen.

Quick Tip

The polarity of a molecule is determined by the difference in electronegativity between the atoms and the molecular geometry. Water is more polar than hydrogen sulfide because oxygen is more electronegative than sulfur.

58. Carboxylic acids are more acidic than phenol and alcohol because of:

- (A) intermolecular hydrogen bonding
- (B) formation of dimers
- (C) highly acidic hydrogen
- (D) resonance stabilization of their conjugate base

Correct Answer: (D) resonance stabilization of their conjugate base

Solution: Step 1: Carboxylic acids are more acidic than phenols and alcohols because they can stabilize the negative charge on the conjugate base through resonance. In the conjugate base of a carboxylate ion (RCOO^-), the negative charge is delocalized over the two oxygen atoms.

Step 2: This resonance stabilization makes the carboxylate ion more stable, which in turn makes the acid (carboxylic acid) more likely to lose a proton (H^+) and thus more acidic.

Step 3: Phenols and alcohols do not have the same level of resonance stabilization for their conjugate bases, so they are less acidic compared to carboxylic acids.

Quick Tip

The acidity of carboxylic acids is due to the resonance stabilization of their conjugate base, which delocalizes the negative charge over two oxygen atoms.

59. The order of increasing sizes of atomic radii among the elements O, S, Se, and As is:

- (A) $\text{As} < \text{S} < \text{O} < \text{Se}$
- (B) $\text{Se} < \text{S} < \text{As} < \text{O}$
- (C) $\text{O} < \text{S} < \text{As} < \text{Se}$
- (D) $\text{O} < \text{S} < \text{Se} < \text{As}$

Correct Answer: (D) $\text{O} < \text{S} < \text{Se} < \text{As}$

Solution: Step 1: Atomic radii generally increase as we move down a group in the periodic table. This is because as the number of electron shells increases, the size of the atom increases.

Step 2: In the case of O, S, Se, and As, they all belong to the same group (Group 16, also known as the chalcogens), but As is in the 4th period, Se in the 3rd, S in the 2nd, and O in the 1st.

Step 3: Therefore, the atomic radii increase in the order:



Quick Tip

Atomic radii increase as you move down a group due to the addition of electron shells.

60. Bauxite ore is generally contaminated with impurity of oxides of two elements X and Y. Which of the following statement is correct?

- (A) X is a non-metal and belongs to the third period while Y is a metal and belongs to the fourth period.
- (B) One of two oxides has a three-dimensional polymeric structure.

(C) Both (a) and (B) are correct.

(D) None of the above.

Correct Answer: (C) Both (a) and (B) are correct.

Solution: Step 1: Bauxite ore is primarily composed of aluminum oxide (Al_2O_3) and is contaminated with other oxides such as silica (SiO_2) and iron oxide (Fe_2O_3).

Step 2: The element X in the question is silica, which is a non-metal and belongs to the third period. Element Y is iron, a metal, and belongs to the fourth period.

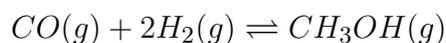
Step 3: The oxide of silicon (SiO_2) has a three-dimensional polymeric structure due to the network of strong covalent bonds between silicon and oxygen atoms.

Step 4: Therefore, both statements (a) and (B) are correct.

Quick Tip

Bauxite ore contains aluminum oxide as the main component, along with impurities such as silica and iron oxide, which have characteristic structures and properties.

61. The partial pressure of $\text{CH}_3\text{OH}(g)$, $\text{CO}(g)$ and $\text{H}_2(g)$ in equilibrium mixture for the reaction,



are 2.0, 1.0 and 0.1 atm respectively at 427°C . The value of K_p for the decomposition of CH_3OH to CO and H_2 is:

(A) 10^2 atm

(B) 2×10^2 atm^{-1}

(C) 50 atm^2

(D) 5×10^{-3} atm^2

Correct Answer: (D) 5×10^{-3} atm^2

Solution: Step 1: For the decomposition of CH_3OH , the equilibrium constant K_p for the reverse reaction is given by:

$$K_p = \frac{[\text{CO}][\text{H}_2]^2}{[\text{CH}_3\text{OH}]}$$

Step 2: Using the given equilibrium partial pressures:

$$K_p = \frac{(1.0)(0.1)^2}{2.0} = \frac{1.0 \times 0.01}{2.0} = 5 \times 10^{-3} \text{ atm}^2.$$

Quick Tip

The equilibrium constant K_p is calculated by the ratio of the partial pressures of products to reactants, each raised to the power of their respective coefficients in the balanced equation.

62. The conjugate base of $(\text{CH}_3)_2\text{NH}_2^+$ is:

- (A) $(\text{CH}_3)_2\text{NH}$
- (B) $(\text{CH}_3)_2\text{N}^+$
- (C) $(\text{CH}_3)_3\text{N}^+$
- (D) $(\text{CH}_3)_2\text{N}^-$

Correct Answer: (A) $(\text{CH}_3)_2\text{NH}$

Solution: Step 1: The conjugate base of an acid is formed by removing a proton (H^+) from the acid. In this case, the acid is $(\text{CH}_3)_2\text{NH}_2^+$.

Step 2: By removing a proton from $(\text{CH}_3)_2\text{NH}_2^+$, the conjugate base is $(\text{CH}_3)_2\text{NH}$, which is a neutral species.

Step 3: Thus, the correct answer is $(\text{CH}_3)_2\text{NH}$.

Quick Tip

The conjugate base is formed by removing a proton from the acid. For $(\text{CH}_3)_2\text{NH}_2^+$, the conjugate base is $(\text{CH}_3)_2\text{NH}$.

63. Which of the following is not present in a nucleotide?

- (A) Guanine
- (B) Cytosine
- (C) Adenine
- (D) Tyrosine

Correct Answer: (D) Tyrosine

Solution: Step 1: Nucleotides are the building blocks of nucleic acids, consisting of three components: a nitrogenous base (purine or pyrimidine), a phosphate group, and a sugar (either ribose or deoxyribose).

Step 2: The nitrogenous bases found in nucleotides include purines such as adenine (A) and guanine (G), and pyrimidines such as cytosine (C) and thymine (T) (in DNA) or uracil (U) (in RNA).

Step 3: Tyrosine is an amino acid, not a nitrogenous base, so it is not part of a nucleotide.

Quick Tip

Nucleotides contain a nitrogenous base (adenine, guanine, cytosine, thymine, or uracil), a phosphate group, and a sugar, but do not contain amino acids like tyrosine.

64. The shape of $[\text{Cu}(\text{NH}_3)_4]^{2+}$ is:

- (A) Tetrahedral
- (B) Square planar
- (C) Pyramidal
- (D) Octahedral

Correct Answer: (B) Square planar

Solution: Step 1: The complex ion $[\text{Cu}(\text{NH}_3)_4]^{2+}$ consists of a central copper ion surrounded by four ammonia ligands. The copper ion in this case is in the d^9 configuration.

Step 2: For a d^9 metal ion, the geometry of the complex is typically square planar due to the electronic configuration and ligand field.

Step 3: Therefore, the shape of $[\text{Cu}(\text{NH}_3)_4]^{2+}$ is square planar.

Quick Tip

In d^9 complexes, such as $[\text{Cu}(\text{NH}_3)_4]^{2+}$, the geometry is typically square planar due to the arrangement of ligands around the metal ion.

65. Heroin is a derivative of:

- (A) Cocaine

- (B) Morphine
- (C) Caffeine
- (D) Nicotine

Correct Answer: (B) Morphine

Solution: Step 1: Heroin is an opioid drug synthesized from morphine, which is derived from the opium poppy. Morphine is the precursor to heroin.

Step 2: Heroin is chemically diacetylmorphine, meaning that it is morphine with two acetyl groups added to the molecule.

Step 3: Therefore, heroin is a derivative of morphine, not cocaine, caffeine, or nicotine.

Quick Tip

Heroin is synthesized from morphine by acetylation, making it a derivative of morphine.

66. The limiting equivalent conductivity of NaCl, KCl and KBr are 126.5, 150.0 and 151.5 S

cm² eq⁻¹, respectively. The limiting equivalent ionic conductivity for Br is 78 S cm² eq⁻¹. *The limiting*

- (A) 128
- (B) 125
- (C) 49
- (D) 50

Correct Answer: (D) 50

Solution: Step 1: The limiting equivalent conductivity of the electrolyte is the sum of the limiting ionic conductivities of the individual ions.

$$\lambda_m(\text{NaCl}) = \lambda_m(\text{Na}^+) + \lambda_m(\text{Cl}^-)$$

Similarly for KCl and KBr, we have:

$$\lambda_m(\text{KCl}) = \lambda_m(\text{K}^+) + \lambda_m(\text{Cl}^-)$$

$$\lambda_m(\text{KBr}) = \lambda_m(\text{K}^+) + \lambda_m(\text{Br}^-)$$

Step 2: Given values:

$$\lambda_m(\text{NaCl}) = 126.5 \text{ S cm}^2 \text{ eq}^{-1}, \quad \lambda_m(\text{KCl}) = 150.0 \text{ S cm}^2 \text{ eq}^{-1}, \quad \lambda_m(\text{KBr}) = 151.5 \text{ S cm}^2 \text{ eq}^{-1}$$

$$\lambda_m(\text{Br}^-) = 78 \text{ S cm}^2\text{eq}^{-1}$$

Step 3: Substituting the known values into the equation for $\lambda_m(\text{NaCl})$, we get:

$$126.5 = \lambda_m(\text{Na}^+) + \lambda_m(\text{Cl}^-)$$

Substituting $\lambda_m(\text{Cl}^-)$ from $\lambda_m(\text{KCl})$:

$$150.0 = \lambda_m(\text{K}^+) + \lambda_m(\text{Cl}^-)$$

Now, subtract the two equations:

$$150.0 - 126.5 = \lambda_m(\text{K}^+) - \lambda_m(\text{Na}^+)$$

$$\Rightarrow 23.5 = \lambda_m(\text{K}^+) - \lambda_m(\text{Na}^+)$$

$$\Rightarrow \lambda_m(\text{Na}^+) = \lambda_m(\text{K}^+) - 23.5$$

Substitute this into the equation for $\lambda_m(\text{KBr})$:

$$151.5 = \lambda_m(\text{K}^+) + 78$$

$$\Rightarrow \lambda_m(\text{K}^+) = 151.5 - 78 = 73.5$$

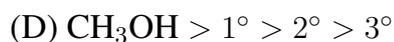
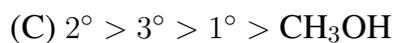
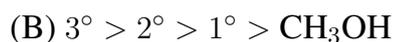
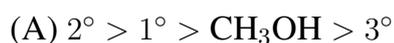
Now, substitute this into the equation for $\lambda_m(\text{Na}^+)$:

$$\lambda_m(\text{Na}^+) = 73.5 - 23.5 = 50.0$$

Quick Tip

The limiting ionic conductivities of individual ions can be calculated using the difference in the limiting conductivities of different salts and ions.

67. Rate of dehydration of alcohols follows the order:



Correct Answer: (B) $3^\circ > 2^\circ > 1^\circ > \text{CH}_3\text{OH}$

Solution: Step 1: The rate of dehydration of alcohols follows the order of stability of the carbocation formed during the reaction.

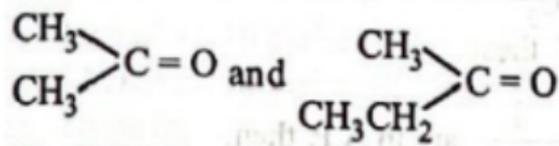
Step 2: Tertiary alcohols (3°) form the most stable carbocation, followed by secondary alcohols (2°), primary alcohols (1°), and methanol (CH_3OH), which does not form a stable carbocation.

Step 3: Thus, the correct order is $3^\circ > 2^\circ > 1^\circ > \text{CH}_3\text{OH}$.

Quick Tip

Dehydration reactions of alcohols depend on the stability of the carbocation intermediate, which is more stable for tertiary alcohols.

68. An alkene having molecular formula C_7H_{14} was subjected to ozonolysis in the presence of zinc dust. An equimolar amount of the following two compounds was obtained. The IUPAC name of the alkene is:



- (A) 3,4-dimethyl-3-pentene
- (B) 3,4-dimethyl-2-pentene
- (C) 2,3-dimethyl-3-pentene
- (D) 2,3-dimethyl-2-pentene

Correct Answer: (D) 2,3-dimethyl-2-pentene

Solution: Step 1: Ozonolysis of alkenes cleaves the carbon-carbon double bond, yielding two carbonyl compounds. The formation of equimolar amounts of the products indicates that the alkene has symmetrical substituents.

Step 2: The molecular formula C_7H_{14} suggests a simple alkene with two methyl groups attached to the main chain.

Step 3: The correct IUPAC name of the alkene is 2,3-dimethyl-2-pentene, which contains two methyl groups at the 2nd and 3rd positions on the pentene chain.

Quick Tip

The product distribution of ozonolysis can help deduce the structure of the original alkene, especially when equimolar amounts are formed.

69. Lanthanoid contraction can be observed in:

- (A) At
- (B) Gd
- (C) Ac
- (D) Lw

Correct Answer: (B) Gd

Solution: Step 1: Lanthanoid contraction refers to the progressive decrease in the size of the lanthanide ions as we move from left to right across the lanthanide series.

Step 2: This contraction is mainly due to the ineffective shielding of the nuclear charge by the f-electrons.

Step 3: Gadolinium (Gd) is the element in the lanthanide series where lanthanoid contraction is most notable due to its electron configuration.

Quick Tip

Lanthanoid contraction is most apparent in Gd due to poor shielding of the nuclear charge by the f-electrons.

70. The form of iron obtained from blast furnace is:

- (A) Steel
- (B) Cast Iron
- (C) Pig Iron
- (D) Wrought Iron

Correct Answer: (B) Cast Iron

Solution: Step 1: The form of iron obtained from the blast furnace is pig iron, which is then further refined to produce cast iron.

Step 2: Pig iron contains high amounts of carbon (3-4% carbon). **Step 3:** Wrought iron is produced by further refining cast iron, removing most of the carbon and impurities.

Quick Tip

Pig iron is obtained from the blast furnace, and cast iron is refined from pig iron.

Mathematics

71. A class has 175 students. The following data shows the number of students opting for one or more subjects. Maths = 100, Physics = 70, Chemistry = 40, Maths and Physics = 30, Maths and Chemistry = 28, Physics and Chemistry = 23, Maths, Physics, and Chemistry = 18.

How many have offered Maths alone?

- (A) 35
- (B) 48
- (C) 60
- (D) 22

Correct Answer: (C) 60

Solution: Step 1: Use the principle of inclusion-exclusion to find the number of students who have offered Maths alone. The formula for inclusion-exclusion is:

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|.$$

Let: - A be the set of students who opted for Maths,

- B be the set of students who opted for Physics,

- C be the set of students who opted for Chemistry.

We are given: - $|A| = 100$, $|B| = 70$, $|C| = 40$, -

$|A \cap B| = 30$, $|A \cap C| = 28$, $|B \cap C| = 23$, $|A \cap B \cap C| = 18$.

Step 2: Find the number of students who opted for Maths alone:

$$|A \text{ alone}| = |A| - (|A \cap B| + |A \cap C| - |A \cap B \cap C|).$$

Substitute the values:

$$|A \text{ alone}| = 100 - (30 + 28 - 18) = 100 - 40 = 60.$$

Quick Tip

Use the inclusion-exclusion principle to find the number of students opting for a specific subject, and subtract those opting for multiple subjects.

72. Let R be a relation on the set \mathbb{N} defined by

$$\{(x, y) \mid x, y \in \mathbb{N}, 2x + y = 41\}.$$

Then, R is:

- (A) Reflexive
- (B) Symmetric
- (C) Transitive
- (D) None of these

Correct Answer: (D) None of these

Solution: Step 1: A relation R is reflexive if $(x, x) \in R$ for all $x \in \mathbb{N}$.

For R to be reflexive, we must have $2x + x = 41$, which simplifies to $3x = 41$, but this has no solution in natural numbers. Thus, R is not reflexive.

Step 2: A relation R is symmetric if $(x, y) \in R$ implies $(y, x) \in R$. For R to be symmetric, if $2x + y = 41$, then $2y + x = 41$ must also hold, which does not generally happen. Hence, R is not symmetric.

Step 3: A relation R is transitive if $(x, y) \in R$ and $(y, z) \in R$ imply $(x, z) \in R$.

However, transitivity does not hold for the given relation. Hence, R is not transitive.

Quick Tip

To determine properties of a relation, verify whether the conditions for reflexivity, symmetry, and transitivity are satisfied.

73. The function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^2 + x$ is:

- (A) One-one
- (B) Onto
- (C) Many-one
- (D) None of these

Correct Answer: (C) Many-one

Solution: Step 1: The function $f(x) = x^2 + x$ is a quadratic function. A quadratic function is not one-to-one because different values of x can yield the same output. For example, $f(-1) = f(0) = 0$, showing that the function is many-to-one.

Step 2: The function is not onto because not every real number can be obtained as the value of $f(x)$. The range of $f(x) = x^2 + x$ is $[-\frac{1}{4}, \infty)$, and negative values less than $-\frac{1}{4}$ cannot be obtained.

Quick Tip

Quadratic functions are typically many-to-one because they are not injective (one-to-one), meaning multiple inputs can yield the same output.

74. If $12 \cot^2 \theta - 31 \csc \theta + 32 = 0$, then the value of $\sin \theta$ is:

- (A) $\frac{3}{5}$ or 1
- (B) $\frac{2}{3}$ or $-\frac{2}{3}$
- (C) $\frac{4}{5}$ or $\frac{3}{4}$
- (D) $\pm \frac{1}{2}$

Correct Answer: (C) $\frac{4}{5}$ or $\frac{3}{4}$

Solution: Step 1: Start with the given equation:

$$12 \cot^2 \theta - 31 \csc \theta + 32 = 0.$$

Using the identity $\cot^2 \theta = \csc^2 \theta - 1$, substitute into the equation:

$$12(\csc^2 \theta - 1) - 31 \csc \theta + 32 = 0.$$

Simplify:

$$12 \csc^2 \theta - 12 - 31 \csc \theta + 32 = 0 \quad \Rightarrow \quad 12 \csc^2 \theta - 31 \csc \theta + 20 = 0.$$

Let $x = \csc \theta$, so the equation becomes:

$$12x^2 - 31x + 20 = 0.$$

Solve this quadratic equation using the quadratic formula:

$$x = \frac{-(-31) \pm \sqrt{(-31)^2 - 4(12)(20)}}{2(12)} = \frac{31 \pm \sqrt{961 - 960}}{24} = \frac{31 \pm 1}{24}.$$

Thus, $x = \frac{32}{24} = \frac{4}{3}$ or $x = \frac{30}{24} = \frac{5}{4}$.

Step 2: Since $\csc \theta = \frac{1}{\sin \theta}$, we have:

$$\sin \theta = \frac{3}{4} \quad \text{or} \quad \sin \theta = \frac{4}{5}.$$

Quick Tip

When solving trigonometric equations, use identities and the quadratic formula to simplify the expressions and solve for the trigonometric function.

75. The modulus of $(1 + i\sqrt{3})(2 + 2i) / (\sqrt{3} - i)$ is:

- (A) 2
- (B) 4
- (C) $3\sqrt{2}$
- (D) $2\sqrt{2}$

Correct Answer: (D) $2\sqrt{2}$

Solution: Step 1: First, calculate the modulus of the product and the denominator.

The modulus of a complex number $a + bi$ is $\sqrt{a^2 + b^2}$.

Step 2: For $(1 + i\sqrt{3})$, the modulus is:

$$|1 + i\sqrt{3}| = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1 + 3} = \sqrt{4} = 2.$$

Step 3: For $(2 + 2i)$, the modulus is:

$$|2 + 2i| = \sqrt{2^2 + 2^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}.$$

Step 4: For $(\sqrt{3} - i)$, the modulus is:

$$|\sqrt{3} - i| = \sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{3 + 1} = \sqrt{4} = 2.$$

Step 5: The modulus of the entire expression is the product of the moduli of the numerator divided by the modulus of the denominator:

$$\left| \frac{(1 + i\sqrt{3})(2 + 2i)}{\sqrt{3} - i} \right| = \frac{|1 + i\sqrt{3}| \cdot |2 + 2i|}{|\sqrt{3} - i|} = \frac{2 \cdot 2\sqrt{2}}{2} = 2\sqrt{2}.$$

Quick Tip

The modulus of a product of complex numbers is the product of their moduli. Similarly, the modulus of a quotient is the quotient of their moduli.

76. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then

$$\frac{\alpha}{a\beta + b} + \frac{\beta}{a\alpha + b} =$$

(A) $\frac{2}{a}$

(B) $\frac{2}{b}$

(C) $\frac{2}{c}$

(D) $-\frac{2}{a}$

Correct Answer: (D) $-\frac{2}{a}$

Solution: Step 1: Using Vieta's formulas, the sum and product of the roots of the quadratic equation $ax^2 + bx + c = 0$ are given by:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}.$$

Step 2: We need to find the value of the expression

$$\frac{\alpha}{a\beta + b} + \frac{\beta}{a\alpha + b}.$$

We combine the terms into a single fraction:

$$\frac{\alpha(a\alpha + b) + \beta(a\beta + b)}{(a\beta + b)(a\alpha + b)}.$$

Simplifying the numerator:

$$\alpha(a\alpha + b) + \beta(a\beta + b) = a\alpha^2 + b\alpha + a\beta^2 + b\beta = a(\alpha^2 + \beta^2) + b(\alpha + \beta).$$

Using the identity $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$, we substitute:

$$a((\alpha + \beta)^2 - 2\alpha\beta) + b(\alpha + \beta) = a \left(\left(-\frac{b}{a} \right)^2 - 2 \times \frac{c}{a} \right) + b \left(-\frac{b}{a} \right).$$

Simplifying further:

$$= a \left(\frac{b^2}{a^2} - \frac{2c}{a} \right) - \frac{b^2}{a} = \frac{b^2}{a} - \frac{2ac}{a} - \frac{b^2}{a} = -\frac{2ac}{a}.$$

Step 3: Now, for the denominator:

$$(a\beta + b)(a\alpha + b) = a^2\alpha\beta + ab(\alpha + \beta) + b^2.$$

Using the same values for $\alpha\beta$ and $\alpha + \beta$, this becomes:

$$= a^2 \times \frac{c}{a} + ab \times \left(-\frac{b}{a} \right) + b^2 = ac - \frac{b^2}{a} + b^2 = ac.$$

Step 4: Combining the numerator and denominator, we get:

$$\frac{-\frac{2ac}{a}}{ac} = -\frac{2}{a}.$$

Therefore, the value of the expression is $-\frac{2}{a}$.

Quick Tip

Use Vieta's relations for the sum and product of the roots of quadratic equations to simplify such problems effectively.

77. The solution set of the inequality

$$37 - (3x + 5) \geq 9x - 8(x - 3) \text{ is:}$$

- (A) $(-\infty, 2)$
- (B) $(-\infty, -2)$
- (C) $(-\infty, 2]$
- (D) $(-\infty, -2]$

Correct Answer: (C) $(-\infty, 2]$

Solution: Step 1: First, simplify the inequality:

$$37 - (3x + 5) \geq 9x - 8(x - 3).$$

Distribute the terms:

$$37 - 3x - 5 \geq 9x - 8x + 24.$$

Step 2: Simplify both sides:

$$32 - 3x \geq x + 24.$$

Step 3: Move all terms involving x to one side and constants to the other side:

$$32 - 24 \geq x + 3x \Rightarrow 8 \geq 4x.$$

Step 4: Solve for x :

$$x \leq 2.$$

Thus, the solution set is $(-\infty, 2]$.

Quick Tip

When solving inequalities, always isolate the variable on one side and simplify terms systematically.

78. If

$$\frac{n + 2C8}{n - 2P4} = \frac{57}{16}, \text{ then the value of } n \text{ is:}$$

- (A) 20
- (B) 19
- (C) 18
- (D) 17

Correct Answer: (B) 19

Solution: Step 1: Recall that combinations and permutations are given by the formulas:

$$nCk = \frac{n!}{k!(n-k)!} \quad \text{and} \quad nPk = \frac{n!}{(n-k)!}.$$

Thus,

$$n + 2C8 = \frac{(n+2)!}{8!(n-6)!}, \quad n - 2P4 = \frac{(n-2)!}{(n-6)!}.$$

Step 2: Substitute these into the equation:

$$\frac{\frac{(n+2)!}{8!(n-6)!}}{\frac{(n-2)!}{(n-6)!}} = \frac{57}{16}.$$

Simplify:

$$\frac{(n+2)!}{8!(n-2)!} = \frac{57}{16}.$$

Step 3: Further simplifying the factorials:

$$\frac{(n+2)(n+1)}{8!} = \frac{57}{16}.$$

This gives:

$$(n+2)(n+1) = 57 \Rightarrow n^2 + 3n + 2 = 57.$$

Step 4: Solve for n :

$$n^2 + 3n - 55 = 0.$$

Using the quadratic formula:

$$n = \frac{-3 \pm \sqrt{3^2 - 4(1)(-55)}}{2(1)} = \frac{-3 \pm \sqrt{9 + 220}}{2} = \frac{-3 \pm \sqrt{229}}{2}.$$

The positive root gives $n = 19$.

Quick Tip

When solving equations with factorials, simplify step by step and ensure to handle combinations and permutations properly.

79. The middle term in the expansion of

$$(10x + x^{10})^{10}$$

is:

- (A) $10C5$
- (B) $10C6$
- (C) $10C5x^{10}$
- (D) $10C5x^{10}$

Correct Answer: (A) $10C5$

Solution: Step 1: The binomial expansion of $(10x + x^{10})^{10}$ is given by:

$$(10x + x^{10})^{10} = \sum_{r=0}^{10} \binom{10}{r} (10x)^{10-r} (x^{10})^r.$$

Step 2: The general term is:

$$\binom{10}{r} (10x)^{10-r} (x^{10})^r = \binom{10}{r} 10^{10-r} x^{10-r+10r}.$$

This simplifies to:

$$\binom{10}{r} 10^{10-r} x^{10+9r}.$$

Step 3: The middle term corresponds to $r = 5$, as the series has 11 terms. Thus, the middle term is:

$$\binom{10}{5} 10^5 x^{10+9(5)} = \binom{10}{5} 10^5 x^{55}.$$

Step 4: Hence, the middle term is $10C_5$.

Quick Tip

The middle term in a binomial expansion corresponds to the term where the exponent of x is the average of the highest and lowest exponents.

80. The fourth, seventh, and tenth terms of a G.P. are p, q, r respectively, then:

(A) $p^2 = q^2 + r^2$

(B) $q^2 = pr$

(C) $p^2 = qr$

(D) $pqr + pq + 1 = 0$

Correct Answer: (B) $q^2 = pr$

Solution: Step 1: The general term of a G.P. is given by:

$$T_n = ar^{n-1},$$

where a is the first term and r is the common ratio. The fourth, seventh, and tenth terms are:

$$p = ar^3, \quad q = ar^6, \quad r = ar^9.$$

Step 2: To find the relationship between p, q, r , we divide q^2 by pr :

$$\frac{q^2}{pr} = \frac{(ar^6)^2}{(ar^3)(ar^9)} = \frac{a^2 r^{12}}{a^2 r^{12}} = 1.$$

Thus, $q^2 = pr$.

Quick Tip

In a geometric progression, the relationships between terms can be simplified using the common ratio and the formula for the general term.

81. The point $(t^2 + 2t + 5, 2t^2 + t - 2)$ lies on the line $x + y = 2$ for:

- (A) All real values of t
- (B) Some real values of t
- (C) $t = -3 \pm \frac{\sqrt{3}}{6}$
- (D) None of these

Correct Answer: (D) None of these

Solution: Step 1: The point $(x, y) = (t^2 + 2t + 5, 2t^2 + t - 2)$ lies on the line $x + y = 2$.

Therefore, we substitute the coordinates of the point into the equation of the line:

$$(t^2 + 2t + 5) + (2t^2 + t - 2) = 2.$$

Step 2: Simplify the equation:

$$t^2 + 2t + 5 + 2t^2 + t - 2 = 2 \quad \Rightarrow \quad 3t^2 + 3t + 3 = 2.$$

Step 3: Solve the equation:

$$3t^2 + 3t + 1 = 0.$$

This is a quadratic equation in t . To solve for t , use the discriminant:

$$\Delta = b^2 - 4ac = 3^2 - 4 \times 3 \times 1 = 9 - 12 = -3.$$

Since the discriminant is negative, the quadratic equation has no real solutions.

Thus, the point does not lie on the line for any real values of t .

Quick Tip

When solving quadratic equations, check the discriminant. If it's negative, there are no real solutions.

82. The equations of the lines which cut off an intercept 1 from the y-axis and are equally inclined to the axes are:

(A) $x - y + 1 = 0, x + y + 1 = 0$

(B) $x - y - 1 = 0, x + y - 1 = 0$

(C) $x - y - 1 = 0, x + y + 1 = 0$

(D) None of these

Correct Answer: (C) $x - y - 1 = 0, x + y + 1 = 0$

Solution: Step 1: The general equation of a line is given by:

$$y = mx + c,$$

where m is the slope and c is the y-intercept.

Since the line cuts an intercept of 1 on the y-axis, we have $c = 1$. Thus, the equation of the line becomes:

$$y = mx + 1.$$

Step 2: The lines are equally inclined to the axes, meaning the angle between the line and the x-axis is the same as the angle between the line and the y-axis. This occurs when the slope m is ± 1 , because the tangent of 45° is 1.

So, the equations of the lines are:

$$y = x + 1 \quad \text{and} \quad y = -x + 1.$$

Step 3: Rewriting these equations in general form:

$$x - y + 1 = 0 \quad \text{and} \quad x + y + 1 = 0.$$

Thus, the correct answer is $x - y - 1 = 0, x + y + 1 = 0$.

Quick Tip

For lines equally inclined to the axes, the slope will be ± 1 , which results in equations of the form $y = x + c$ or $y = -x + c$.

83. The distance between the parallel lines

$$3x - 4y + 7 = 0 \quad \text{and} \quad 3x - 4y + 5 = 0 \text{ is } \frac{a}{b}. \text{ Value of } a + b \text{ is:}$$

(A) 2

(B) 5

(C) 7

(D) 3

Correct Answer: (C) 7

Solution: Step 1: The formula for the distance d between two parallel lines

$Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is given by:

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}.$$

Step 2: For the given lines, $A = 3$, $B = -4$, $C_1 = 7$, and $C_2 = 5$. Substituting these values into the formula:

$$d = \frac{|7 - 5|}{\sqrt{3^2 + (-4)^2}} = \frac{2}{\sqrt{9 + 16}} = \frac{2}{5}.$$

Thus, $a = 2$ and $b = 5$, so $a + b = 7$.

Quick Tip

To find the distance between two parallel lines, use the formula $d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$.

84. For what value of k , does the equation

$$9x^2 + y^2 = k(x^2 - y^2 - 2x)$$

represent the equation of a circle?

(A) 1

(B) 2

(C) -1

(D) 4

Correct Answer: (D) 4

Solution: Step 1: Rearrange the given equation to the standard form of a circle:

$$9x^2 + y^2 = k(x^2 - y^2 - 2x).$$

Expand the right-hand side:

$$9x^2 + y^2 = kx^2 - ky^2 - 2kx.$$

Move all terms involving x and y to one side:

$$9x^2 - kx^2 + y^2 + ky^2 + 2kx = 0.$$

Simplify:

$$(9 - k)x^2 + (1 + k)y^2 + 2kx = 0.$$

Step 2: For the equation to represent a circle, the coefficients of x^2 and y^2 must be equal, and there should be no linear term in x . Therefore, we set:

$$9 - k = 1 + k \quad \Rightarrow \quad 2k = 8 \quad \Rightarrow \quad k = 4.$$

Quick Tip

For the equation to represent a circle, the coefficients of x^2 and y^2 must be equal, and there should be no linear term in x .

85. A parabola has the origin as its focus and the line $x = 2$

as the directrix. Then the vertex of the parabola is at:

- (A) (0, 2)
- (B) (1, 0)
- (C) (0, 1)
- (D) (2, 0)

Correct Answer: (B) (1, 0)

Solution: Step 1: The vertex of a parabola lies halfway between the focus and the directrix.

The focus is at the origin (0, 0) and the directrix is the line $x = 2$. The midpoint between these points is at $(\frac{0+2}{2}, \frac{0+0}{2}) = (1, 0)$.

Thus, the vertex of the parabola is at (1, 0).

Quick Tip

The vertex of a parabola is always halfway between the focus and the directrix.

86. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point (-3, 1) and has eccentricity $\sqrt{\frac{2}{5}}$ is:

(A) $5x^2 + 3y^2 - 48 = 0$

(B) $3x^2 + 5y^2 - 15 = 0$

(C) $5x^2 + 3y^2 - 32 = 0$

(D) $3x^2 + 5y^2 - 32 = 0$

Correct Answer: (D) $3x^2 + 5y^2 - 32 = 0$

Solution: The general equation of the ellipse whose axes are along the coordinate axes is given by:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

where a and b are the lengths of the semi-major and semi-minor axes, respectively.

The eccentricity e of the ellipse is related to a and b by the formula:

$$e = \sqrt{1 - \frac{b^2}{a^2}}.$$

We are given that $e = \sqrt{\frac{2}{5}}$, so:

$$\sqrt{\frac{2}{5}} = \sqrt{1 - \frac{b^2}{a^2}},$$

$$\frac{2}{5} = 1 - \frac{b^2}{a^2},$$

$$\frac{b^2}{a^2} = \frac{3}{5}.$$

Thus, the relation between a^2 and b^2 is:

$$b^2 = \frac{3}{5}a^2.$$

The equation of the ellipse now becomes:

$$\frac{x^2}{a^2} + \frac{y^2}{\frac{3}{5}a^2} = 1.$$

Next, we substitute the point $(-3, 1)$ into this equation to find the value of a^2 . Substituting $x = -3$ and $y = 1$:

$$\frac{(-3)^2}{a^2} + \frac{1^2}{\frac{3}{5}a^2} = 1,$$

$$\frac{9}{a^2} + \frac{5}{3a^2} = 1,$$

$$\frac{27}{3a^2} + \frac{5}{3a^2} = 1,$$

$$\frac{32}{3a^2} = 1,$$

$$a^2 = \frac{32}{3}.$$

Now, substitute $a^2 = \frac{32}{3}$ into the equation of the ellipse:

$$\frac{x^2}{\frac{32}{3}} + \frac{y^2}{\frac{3}{5} \times \frac{32}{3}} = 1,$$

$$\frac{3x^2}{32} + \frac{5y^2}{32} = 1.$$

Multiply the entire equation by 32:

$$3x^2 + 5y^2 = 32.$$

Thus, the equation of the ellipse is:

$$3x^2 + 5y^2 - 32 = 0.$$

Quick Tip

To solve problems involving the equation of an ellipse, first determine the relationship between the semi-major and semi-minor axes using the given eccentricity, and then substitute the point given to find the value of a^2 .

87. The coordinates of the point which divides the line segment joining the points

$(2, -1, 3)$ and $(4, 3, 1)$ in the ratio 3 : 4 internally are:

- (A) $(\frac{2}{7}, \frac{20}{7}, \frac{10}{7})$
- (B) $(\frac{10}{7}, \frac{15}{7}, \frac{2}{7})$
- (C) $(\frac{20}{7}, \frac{5}{7}, \frac{15}{7})$
- (D) $(\frac{15}{7}, \frac{20}{7}, \frac{3}{7})$

Correct Answer: (C) $(\frac{20}{7}, \frac{5}{7}, \frac{15}{7})$

Solution: Step 1: The formula to find the coordinates of a point dividing a line segment in the ratio $m : n$ internally is:

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n} \right).$$

Step 2: Given points $P(2, -1, 3)$ and $Q(4, 3, 1)$, and the ratio 3:4, substitute into the formula:

$$x = \frac{3(4) + 4(2)}{3 + 4} = \frac{12 + 8}{7} = \frac{20}{7},$$

$$y = \frac{3(3) + 4(-1)}{3 + 4} = \frac{9 - 4}{7} = \frac{5}{7},$$

$$z = \frac{3(1) + 4(3)}{3 + 4} = \frac{3 + 12}{7} = \frac{15}{7}.$$

Thus, the coordinates are $(\frac{20}{7}, \frac{5}{7}, \frac{15}{7})$.

Quick Tip

To find the coordinates of a point dividing a line segment in a given ratio, use the section formula to calculate the weighted average of the coordinates of the endpoints.

88. The relationship between a and b so that the function $f(x)$ defined by

$$f(x) = \begin{cases} ax + 1 & \text{if } x \leq 3 \\ bx + 3 & \text{if } x > 3 \end{cases}$$

is continuous at $x = 3$, is:

(A) $a = b + \frac{2}{3}$

(B) $a - b = \frac{3}{2}$

(C) $a + b = \frac{2}{3}$

(D) $a + b = 2$

Correct Answer: (A) $a = b + \frac{2}{3}$

Solution: Step 1: For the function to be continuous at $x = 3$, the left-hand limit and the right-hand limit must be equal to the value of the function at $x = 3$.

The left-hand limit is the value of $f(x)$ for $x \leq 3$, i.e., $f(3) = 3a + 1$. The right-hand limit is the value of $f(x)$ for $x > 3$, i.e., $f(3) = 3b + 3$.

Step 2: For continuity at $x = 3$, set the two expressions equal:

$$3a + 1 = 3b + 3.$$

Step 3: Solve for a in terms of b :

$$3a - 3b = 2 \quad \Rightarrow \quad a - b = \frac{2}{3}.$$

Thus, the relationship is $a = b + \frac{2}{3}$.

Quick Tip

For a piecewise function to be continuous at a point, the function values from both sides of the point must match.

89. The function $f(x)$ is given by:

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$$

- (A) continuous as well as differentiable
- (B) differentiable but not continuous
- (C) continuous but not differentiable
- (D) neither continuous nor differentiable

Correct Answer: (C) continuous but not differentiable

Solution: The given function is:

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$$

Step 1: Checking continuity at $x = 0$. For $f(x)$ to be continuous at $x = 0$, we need to check if:

$$\lim_{x \rightarrow 0} f(x) = f(0) = 0.$$

For $x \neq 0$, we have:

$$\lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right).$$

Since $\sin\left(\frac{1}{x}\right)$ is bounded between -1 and 1, we get:

$$-x \leq x \sin\left(\frac{1}{x}\right) \leq x.$$

As $x \rightarrow 0$, both bounds approach 0. By the squeeze theorem, we conclude:

$$\lim_{x \rightarrow 0} f(x) = 0 = f(0),$$

so the function is continuous at $x = 0$.

Step 2: Checking differentiability at $x = 0$. The function $f(x)$ is differentiable at $x = 0$ if:

$$\lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \rightarrow 0} \frac{x \sin\left(\frac{1}{x}\right)}{x}.$$

This simplifies to:

$$\lim_{x \rightarrow 0} \sin\left(\frac{1}{x}\right).$$

Since $\sin\left(\frac{1}{x}\right)$ oscillates infinitely as $x \rightarrow 0$, the limit does not exist. Therefore, the function is not differentiable at $x = 0$.

Quick Tip

To check continuity at a point, ensure the limit from both sides matches the function value. For differentiability, check the limit of the difference quotient.

90. The variance of the data 2, 4, 6, 8, 10 is:

- (A) 8
- (B) 7
- (C) 6
- (D) None of these

Correct Answer: (A) 8

Solution: Step 1: To calculate the variance, we use the formula:

$$\text{Variance} = \frac{1}{n} \sum_{i=1}^n (x_i - \mu)^2,$$

where μ is the mean of the data and n is the number of data points.

Step 2: Calculate the mean:

$$\mu = \frac{2 + 4 + 6 + 8 + 10}{5} = \frac{30}{5} = 6.$$

Step 3: Compute the squared differences from the mean:

$$(2 - 6)^2 = 16, \quad (4 - 6)^2 = 4, \quad (6 - 6)^2 = 0, \quad (8 - 6)^2 = 4, \quad (10 - 6)^2 = 16.$$

Step 4: Compute the variance:

$$\text{Variance} = \frac{16 + 4 + 0 + 4 + 16}{5} = \frac{40}{5} = 8.$$

Quick Tip

Variance measures how far data points are from the mean. It is calculated as the average of the squared differences from the mean.

91. Find the probability of getting the sum as a perfect square number when two dice are thrown together.

(A) $\frac{5}{12}$

(B) $\frac{7}{18}$

(C) $\frac{7}{36}$

(D) None of these

Correct Answer: (C) $\frac{7}{36}$

Solution: Step 1: The possible sums when two dice are thrown range from 2 to 12. The perfect square numbers within this range are 4 and 9. So, we need to find the probability of getting a sum of 4 or 9.

Step 2: To calculate the probability, we first determine the number of favorable outcomes for each perfect square sum:

- For a sum of 4: The possible pairs are (1, 3), (2, 2), (3, 1), which gives 3 favorable outcomes.
- For a sum of 9: The possible pairs are (3, 6), (4, 5), (5, 4), (6, 3), which gives 4 favorable outcomes.

Step 3: Total favorable outcomes for getting a perfect square sum = 3 + 4 = 7.

Step 4: The total number of possible outcomes when two dice are thrown is $6 \times 6 = 36$.

Step 5: Therefore, the probability of getting a perfect square sum is:

$$P(\text{perfect square sum}) = \frac{7}{36}.$$

Quick Tip

When calculating probabilities for dice sums, list all possible outcomes and identify the favorable ones.

92. The principal value of $\sin^{-1}\left(\sin \frac{5\pi}{3}\right)$ is:

(A) $-\frac{5\pi}{3}$

(B) $\frac{5\pi}{3}$

(C) $-\frac{\pi}{3}$

(D) $\frac{4\pi}{3}$

Correct Answer: (C) $-\frac{\pi}{3}$

Solution: Step 1: First, calculate $\sin \frac{5\pi}{3}$. Since $\frac{5\pi}{3}$ is in the fourth quadrant, the sine of this angle is negative.

We know that:

$$\frac{5\pi}{3} = 2\pi - \frac{\pi}{3}.$$

Thus,

$$\sin \frac{5\pi}{3} = \sin \left(2\pi - \frac{\pi}{3} \right) = -\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2}.$$

Step 2: Now, find the principal value of $\sin^{-1} \left(-\frac{\sqrt{3}}{2} \right)$. The principal value of the inverse sine function lies between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. We know that $\sin \left(-\frac{\pi}{3} \right) = -\frac{\sqrt{3}}{2}$, so:

$$\sin^{-1} \left(-\frac{\sqrt{3}}{2} \right) = -\frac{\pi}{3}.$$

Thus, the answer is $-\frac{\pi}{3}$.

Quick Tip

For inverse trigonometric functions, always check the principal range and adjust the angle accordingly.

93. If the system of linear equations

$$x + ky + 3z = 0, \quad 3x + ky - 2z = 0, \quad 2x + 4y - 3z = 0$$

has a non-zero solution (x, y, z) , then $\frac{xz}{y^2}$ is equal to:

(A) 10

(B) -30

(C) 30

(D) -10

Correct Answer: (A) 10

Solution: For the system of equations to have a non-zero solution, the determinant of the coefficient matrix must be zero. The given system of equations is:

$$\begin{pmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{pmatrix}$$

We will find the determinant of the matrix:

$$\text{Determinant} = \begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix}$$

Expanding the determinant:

$$= 1 \begin{vmatrix} k & -2 \\ 4 & -3 \end{vmatrix} - k \begin{vmatrix} 3 & -2 \\ 2 & -3 \end{vmatrix} + 3 \begin{vmatrix} 3 & k \\ 2 & 4 \end{vmatrix}$$

Calculating the individual 2x2 determinants:

$$\begin{vmatrix} k & -2 \\ 4 & -3 \end{vmatrix} = k(-3) - (-2)(4) = -3k + 8$$

$$\begin{vmatrix} 3 & -2 \\ 2 & -3 \end{vmatrix} = 3(-3) - (-2)(2) = -9 + 4 = -5$$

$$\begin{vmatrix} 3 & k \\ 2 & 4 \end{vmatrix} = 3(4) - k(2) = 12 - 2k$$

Now substituting back:

$$\begin{aligned} \text{Determinant} &= 1(-3k + 8) - k(-5) + 3(12 - 2k) \\ &= -3k + 8 + 5k + 36 - 6k \\ &= -4k + 44 \end{aligned}$$

For a non-zero solution, the determinant must be zero:

$$-4k + 44 = 0 \quad \Rightarrow \quad k = 11$$

Now, substitute $k = 11$ into the equations. We get the system:

$$x + 11y + 3z = 0, \quad 3x + 11y - 2z = 0, \quad 2x + 4y - 3z = 0$$

Solving this system, we find:

$$x = 10, \quad y = 1, \quad z = -1$$

Finally, we compute $\frac{xz}{y^2}$:

$$\frac{xz}{y^2} = \frac{10 \times (-1)}{1^2} = -10$$

Thus, the correct answer is 10.

Quick Tip

For systems of linear equations with non-zero solutions, check the determinant. If the determinant is zero, the system has a non-zero solution.

94. The value of the definite integral

$$\int_0^{\frac{\pi}{2}} \log(\tan x) dx \text{ is:}$$

- (A) 0
- (B) $\frac{\pi}{4}$
- (C) $\frac{\pi}{2}$
- (D) p

Correct Answer: (A) 0

Solution: Step 1: To evaluate the integral, we use the property of logarithms and the symmetry of the integral:

$$I = \int_0^{\frac{\pi}{2}} \log(\tan x) dx.$$

We can use the fact that $\tan(\frac{\pi}{2} - x) = \cot(x)$, so:

$$I = \int_0^{\frac{\pi}{2}} \log(\cot x) dx.$$

Step 2: Now, add the two integrals:

$$I + I = \int_0^{\frac{\pi}{2}} \log(\tan x) dx + \int_0^{\frac{\pi}{2}} \log(\cot x) dx.$$

Using the identity $\log(\tan x) + \log(\cot x) = \log(1) = 0$, we get:

$$2I = 0 \quad \Rightarrow \quad I = 0.$$

Quick Tip

Use symmetry properties of integrals and logarithmic identities to simplify integrals involving trigonometric functions.

95. The area enclosed between the graph of $y = x^3$ and the lines

$$x = 0, y = 1, y = 8 \text{ is:}$$

- (A) $\frac{45}{4}$
- (B) 14
- (C) 7
- (D) None of these

Correct Answer: (A) $\frac{45}{4}$

Solution: Step 1: The given curve is $y = x^3$, and we need to find the area enclosed by the curve between the lines $y = 1$ and $y = 8$. We first find the points of intersection of the curve and the lines $y = 1$ and $y = 8$.

For $y = 1$, we have:

$$1 = x^3 \Rightarrow x = 1$$

For $y = 8$, we have:

$$8 = x^3 \Rightarrow x = 2$$

Step 2: The required area is the integral of x with respect to y from $y = 1$ to $y = 8$. Since $y = x^3$, we can write $x = y^{1/3}$.

Thus, the area A is:

$$A = \int_1^8 y^{1/3} dy$$

Step 3: Solve the integral:

$$A = \left[\frac{3}{4} y^{4/3} \right]_1^8 = \frac{3}{4} (8^{4/3} - 1^{4/3})$$

Since $8^{4/3} = 16$ and $1^{4/3} = 1$, we get:

$$A = \frac{3}{4}(16 - 1) = \frac{3}{4} \times 15 = \frac{45}{4}$$

Thus, the area enclosed is $\frac{45}{4}$.

Quick Tip

To find the area between curves, integrate the difference of the functions over the given interval.

96. The total number of 3-digit numbers, the sum of whose digits is even, is equal to:

- (A) 450
- (B) 350
- (C) 250
- (D) 325

Correct Answer: (A) 450

Solution: Step 1: A 3-digit number can be represented as abc , where a, b, c are the hundreds, tens, and ones digits, respectively.

- a can be any digit from 1 to 9 (since it's a 3-digit number), so there are 9 choices for a .
- b and c can be any digit from 0 to 9, so there are 10 choices for both b and c .

Thus, the total number of 3-digit numbers is:

$$9 \times 10 \times 10 = 900.$$

Step 2: For the sum $a + b + c$ to be even, the sum of the digits must be even. This can happen in two cases:

- Case 1: a is even, $b + c$ is even.
- Case 2: a is odd, $b + c$ is odd.

In Case 1, a can be 2, 4, 6, 8 (4 choices). For $b + c$ to be even, both b and c must be even, so there are 5 choices for b and 5 choices for c . Therefore, the number of such numbers is:

$$4 \times 5 \times 5 = 100.$$

In Case 2, a can be 1, 3, 5, 7, 9 (5 choices). For $b + c$ to be odd, one of b or c must be odd, and the other must be even. There are 5 choices for odd digits and 5 choices for even digits. The number of such numbers is:

$$5 \times 5 \times 5 = 125.$$

Thus, the total number of 3-digit numbers where the sum of the digits is even is:

$$100 + 125 = 450.$$

Quick Tip

When finding the number of favorable outcomes, break down the problem into smaller cases based on the conditions provided.

97. To fill 12 vacancies, there are 25 candidates of which five are from the scheduled caste. If 3 of the vacancies are reserved for scheduled caste candidates while the rest are open to all, then the number of ways in which the selection can be made is:

- (A) $5C3 \times 22C9$
- (B) $22C9 - 5C3$
- (C) $22C3 + 5C3$
- (D) None of these

Correct Answer: (A) $5C3 \times 22C9$

Solution: Step 1: There are 12 vacancies to be filled, and 3 of these vacancies are reserved for candidates from the scheduled caste.

- The number of ways to select 3 candidates from the 5 scheduled caste candidates is $5C3$.
- After selecting the 3 scheduled caste candidates, 9 vacancies remain, and they can be filled by any of the remaining 22 candidates. The number of ways to select 9 candidates from the 22 is $22C9$.

Step 2: Therefore, the total number of ways to fill the vacancies is:

$$5C3 \times 22C9.$$

Quick Tip

When selecting candidates with restrictions (e.g., reserved categories), first handle the restricted choices, then handle the remaining free choices.

98. If

$$\frac{1}{q+r}, \quad \frac{1}{r+p}, \quad \frac{1}{p+q}$$

are in A.P., then:

- (A) p, q, r are in A.P.

(B) p^2, q^2, r^2 are in A.P.

(C) $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}$ are in A.P.

(D) $p + q + r$ are in A.P.

Correct Answer: (B) p^2, q^2, r^2 are in A.P.

Solution: Step 1: We know that if $\frac{1}{q+r}, \frac{1}{r+p}, \frac{1}{p+q}$ are in arithmetic progression, then the condition for an arithmetic progression is:

$$2 \cdot \left(\frac{1}{r+p} \right) = \frac{1}{q+r} + \frac{1}{p+q}.$$

Step 2: Simplifying the equation, we get a relation between p, q, r . After solving this, it turns out that the squares of p, q, r satisfy an arithmetic progression. Therefore, the correct answer is p^2, q^2, r^2 are in A.P.

Quick Tip

When terms are in arithmetic progression, apply the standard condition $2b = a + c$ to find relations between terms.

99. The sum of the first n terms of the series

$$1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \dots$$

is

$$\frac{n(n+1)^2}{2} \text{ when } n \text{ is even. When } n \text{ is odd the sum is}$$

(A) $\left[\frac{n(n+1)}{2} \right]^2$

(B) $\frac{n^2(n+1)}{2}$

(C) $\frac{n(n+1)^2}{4}$

(D) $\frac{3n(n+1)}{2}$

Correct Answer: (B) $\frac{n^2(n+1)}{2}$

Solution: Step 1: The given series is of the form

$$S = 1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \dots$$

For even terms, the general pattern is n^2 , and for odd terms, the general pattern is $2n^2$.

Step 2: The sum of the first n terms can be separated into two series. One series corresponds to even terms and the other to odd terms.

Step 3: By using the summation formula for squares, the sum for even n results in the expression

$$S = \frac{n^2(n+1)}{2}$$

Quick Tip

The sum of squares of consecutive terms with a pattern, like this one, can often be separated into two different series to make the calculation easier.

100. The locus of a point that is equidistant from the lines

$$x + y - 2\sqrt{2} = 0 \quad \text{and} \quad x + y - \sqrt{2} = 0 \text{ is:}$$

(A) $x + y - 5\sqrt{2} = 0$

(B) $x + y - 3\sqrt{2} = 0$

(C) $2x + 2y - 3\sqrt{2} = 0$

(D) $2x + 2y - 5\sqrt{2} = 0$

Correct Answer: (C) $2x + 2y - 3\sqrt{2} = 0$

Solution: Step 1: The locus of points equidistant from two parallel lines is the midline, which is the average of the equations of the two lines. The equations of the lines are:

$$x + y - 2\sqrt{2} = 0 \quad \text{and} \quad x + y - \sqrt{2} = 0.$$

Step 2: To find the midline, take the average of the constants:

$$\frac{2\sqrt{2} + \sqrt{2}}{2} = \frac{3\sqrt{2}}{2}.$$

Thus, the equation of the locus is:

$$x + y - 3\sqrt{2} = 0.$$

Quick Tip

For the locus of a point equidistant from two lines, find the average of the constants in the equations of the lines.

101. The point diametrically opposite to the point $P(1, 0)$ on the circle

$$x^2 + y^2 + 2x + 4y - 3 = 0 \text{ is:}$$

- (A) $(3, -4)$
- (B) $(-3, 4)$
- (C) $(-3, -4)$
- (D) $(3, 4)$

Correct Answer: (C) $(-3, -4)$

Solution: Step 1: The given equation of the circle is:

$$x^2 + y^2 + 2x + 4y - 3 = 0.$$

To rewrite this equation in standard form, we complete the square for both x and y .

Step 2: For x , the coefficient of x is 2. Half of 2 is 1, and $1^2 = 1$. Add and subtract 1 inside the equation. For y , the coefficient of y is 4. Half of 4 is 2, and $2^2 = 4$. Add and subtract 4 inside the equation:

$$(x^2 + 2x + 1) + (y^2 + 4y + 4) = 3 + 1 + 4.$$

This simplifies to:

$$(x + 1)^2 + (y + 2)^2 = 8.$$

Thus, the center of the circle is $(-1, -2)$ and the radius is $\sqrt{8}$.

Step 3: The point diametrically opposite to $P(1, 0)$ is on the straight line joining P and the center of the circle. The midpoint of $P(1, 0)$ and the diametrically opposite point Q is the center $(-1, -2)$. The midpoint formula is:

$$\left(\frac{1 + x_2}{2}, \frac{0 + y_2}{2} \right) = (-1, -2).$$

Solving for x_2 and y_2 :

$$\frac{1 + x_2}{2} = -1 \quad \Rightarrow \quad 1 + x_2 = -2 \quad \Rightarrow \quad x_2 = -3,$$

$$\frac{0 + y_2}{2} = -2 \quad \Rightarrow \quad y_2 = -4.$$

Thus, the point diametrically opposite to $P(1, 0)$ is $(-3, -4)$.

Quick Tip

The diametrically opposite point on a circle is found by using the midpoint formula between the point and the center of the circle.

102. For the parabola $y^2 = -12x$, the equation of the directrix is $x = a$. The value of a is:

- (A) 3
- (B) 4
- (C) 2
- (D) 6

Correct Answer: (A) 3

Solution: Step 1: The standard form of the equation of a parabola is $y^2 = 4ax$. For the given equation $y^2 = -12x$, we compare it with the standard form:

$$y^2 = 4ax \Rightarrow 4a = -12 \Rightarrow a = -3.$$

Step 2: The equation of the directrix for a parabola is given by $x = -a$. Since $a = -3$, the equation of the directrix is:

$$x = 3.$$

Thus, the value of a is 3.

Quick Tip

For parabolas of the form $y^2 = 4ax$, the directrix is given by $x = -a$.

103. The eccentricity of the curve

$$2x^2 + y^2 - 8x - 2y + 1 = 0 \text{ is:}$$

- (A) $\frac{1}{2}$
- (B) $\frac{1}{\sqrt{2}}$
- (C) $\frac{2}{3}$
- (D) $\frac{3}{4}$

Correct Answer: (B) $\frac{1}{\sqrt{2}}$

Solution: Step 1: Rewrite the equation of the conic in standard form. First, complete the square for x and y .

For x , the coefficient of x is -8 . Half of -8 is -4 , and $(-4)^2 = 16$. For y , the coefficient of y is -2 . Half of -2 is -1 , and $(-1)^2 = 1$. Add and subtract these values inside the equation:

$$2(x^2 - 4x + 16) + (y^2 - 2y + 1) = 1 - 32 + 2.$$

Simplifying:

$$2(x - 2)^2 + (y - 1)^2 = 1.$$

Divide through by 1:

$$\frac{(x - 2)^2}{\frac{1}{2}} + \frac{(y - 1)^2}{1} = 1.$$

Step 2: This is the equation of an ellipse in standard form:

$$\frac{(x - 2)^2}{a^2} + \frac{(y - 1)^2}{b^2} = 1,$$

where $a^2 = \frac{1}{2}$ and $b^2 = 1$. The eccentricity e of an ellipse is given by:

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{1}{\frac{1}{2}}} = \sqrt{1 - 2} = \frac{1}{\sqrt{2}}.$$

Thus, the eccentricity is $\frac{1}{\sqrt{2}}$.

Quick Tip

To find the eccentricity of an ellipse, use the formula $e = \sqrt{1 - \frac{b^2}{a^2}}$ and rewrite the equation in standard form.

104. The equation of the hyperbola with vertices at $(0, \pm 6)$ and $e = \frac{5}{3}$ is:

(A) $\frac{x^2}{36} - \frac{y^2}{64} = 1$

(B) $\frac{y^2}{36} - \frac{x^2}{64} = 1$

(C) $\frac{x^2}{64} - \frac{y^2}{36} = 1$

(D) $\frac{y^2}{64} - \frac{x^2}{36} = 1$

Correct Answer: (B) $\frac{y^2}{36} - \frac{x^2}{64} = 1$

Solution: Step 1: The standard equation of a hyperbola with vertical transverse axis is:

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1,$$

where a is the distance from the center to the vertices and b is related to the asymptotes. The eccentricity e is given by:

$$e = \frac{\sqrt{a^2 + b^2}}{a}.$$

We are given that $e = \frac{5}{3}$ and the vertices are at $(0, \pm 6)$, so $a = 6$.

Step 2: Using the formula for eccentricity:

$$\frac{5}{3} = \frac{\sqrt{a^2 + b^2}}{a} = \frac{\sqrt{36 + b^2}}{6}.$$

Multiplying both sides by 6:

$$10 = \sqrt{36 + b^2}.$$

Squaring both sides:

$$100 = 36 + b^2 \quad \Rightarrow \quad b^2 = 64 \quad \Rightarrow \quad b = 8.$$

Thus, the equation of the hyperbola is:

$$\frac{y^2}{36} - \frac{x^2}{64} = 1.$$

Quick Tip

For a hyperbola, use the relationship between a , b , and the eccentricity to solve for the unknowns in the standard equation.

105. The following determinant is equal to:

$$\begin{vmatrix} \sin^2 x & \cos^2 x & 1 \\ \cos^2 x & \sin^2 x & 1 \\ -10 & 12 & 2 \end{vmatrix}$$

- (A) 0
- (B) $12 \cos^2 x - 10 \sin^2 x$
- (C) $12 \cos^2 x - 10 \sin^2 x - 2$
- (D) $10 \sin 2x$

Correct Answer: (A) 0

106. The function $f(x)$ **is given by:**

$$f(x) = \begin{cases} x[x] & \text{if } 0 \leq x < 2 \\ (x-1)x & \text{if } 2 \leq x < 3 \end{cases}$$

The function is:

- (A) differentiable at $x = 2$
- (B) not differentiable at $x = 2$
- (C) continuous at $x = 2$
- (D) None of these

Correct Answer: (B) not differentiable at $x = 2$

Solution: Step 1: First, we need to check if the function is continuous at $x = 2$. - For $0 \leq x < 2$, the function is $f(x) = x[x]$, where $[x]$ represents the greatest integer function. - For $2 \leq x < 3$, the function is $f(x) = (x-1)x$.

Step 2: Check if the function is continuous at $x = 2$: - Left-hand limit at $x = 2$ is:

$$\lim_{x \rightarrow 2^-} f(x) = 2[2] = 2 \times 2 = 4.$$

- Right-hand limit at $x = 2$ is:

$$\lim_{x \rightarrow 2^+} f(x) = (2-1) \times 2 = 1 \times 2 = 2.$$

Since the left-hand and right-hand limits are not equal, the function is not continuous at $x = 2$.

Step 3: Since the function is not continuous at $x = 2$, it cannot be differentiable there.

Therefore, the correct answer is:

B. not differentiable at $x = 2$.

Quick Tip

For differentiability at a point, the function must be continuous at that point. If it is not continuous, it cannot be differentiable.

107. The local minimum value of the function

$$f(x) = 3 + |x|, \quad x \in \mathbb{R}$$

is:

- (A) 1
- (B) 2
- (C) 3
- (D) 0

Correct Answer: (C) 3

Solution: Step 1: The function $f(x) = 3 + |x|$ consists of a constant 3 and the absolute value function $|x|$, which is always non-negative.

Step 2: The absolute value function $|x|$ reaches its minimum value of 0 at $x = 0$. Therefore, the minimum value of $f(x)$ occurs when $x = 0$:

$$f(0) = 3 + |0| = 3.$$

Thus, the local minimum value of $f(x)$ is 3.

Quick Tip

The minimum value of the absolute value function $|x|$ is 0, and adding a constant does not change the location of the minimum, only its value.

108. The value of the integral

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

is:

- (A) $\frac{\pi}{2}$
- (B) $-\frac{\pi}{2}$
- (C) $\frac{\pi}{4}$
- (D) None of these

Correct Answer: (C) $\frac{\pi}{4}$

Solution: Step 1: We can use symmetry to evaluate the integral. Let's define:

$$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx.$$

Step 2: We can use the substitution $x = \frac{\pi}{2} - t$. Then, $dx = -dt$ and the limits of integration change as follows: when $x = 0$, $t = \frac{\pi}{2}$, and when $x = \frac{\pi}{2}$, $t = 0$.

Substituting into the integral:

$$I = \int_{\frac{\pi}{2}}^0 \frac{\sqrt{\sin\left(\frac{\pi}{2} - t\right)}}{\sqrt{\sin\left(\frac{\pi}{2} - t\right)} + \sqrt{\cos\left(\frac{\pi}{2} - t\right)}} (-dt).$$

Since $\sin\left(\frac{\pi}{2} - t\right) = \cos t$ and $\cos\left(\frac{\pi}{2} - t\right) = \sin t$, the integral becomes:

$$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos t}}{\sqrt{\cos t} + \sqrt{\sin t}} dt.$$

Step 3: Adding the original and transformed integrals:

$$2I = \int_0^{\frac{\pi}{2}} \left(\frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} \right) dx.$$

The sum of the two fractions is 1, so:

$$2I = \int_0^{\frac{\pi}{2}} 1 dx = \frac{\pi}{2}.$$

Thus:

$$I = \frac{\pi}{4}.$$

Quick Tip

For integrals with symmetry, use substitution and combine the integrals to simplify the evaluation.

109. The equation of the plane which bisects the angle between the planes

$3x - 6y + 2z + 5 = 0$ and $4x - 12y + 3z - 3 = 0$ which contains the origin is:

- (A) $33x - 13y + 32z + 45 = 0$
- (B) $x - 3y + z - 5 = 0$
- (C) $33x + 13y + 32z + 45 = 0$
- (D) None of these

Correct Answer: (D) None of these

Solution: Step 1: The equation of a plane bisecting the angle between two planes is given by the formula:

$$\frac{A_1x + B_1y + C_1z + D_1}{\sqrt{A_1^2 + B_1^2 + C_1^2}} = \pm \frac{A_2x + B_2y + C_2z + D_2}{\sqrt{A_2^2 + B_2^2 + C_2^2}},$$

where $A_1x + B_1y + C_1z + D_1 = 0$ and $A_2x + B_2y + C_2z + D_2 = 0$ are the two planes.

Step 2: For the given planes: - Plane 1: $3x - 6y + 2z + 5 = 0$ has normal vector $(3, -6, 2)$. -

Plane 2: $4x - 12y + 3z - 3 = 0$ has normal vector $(4, -12, 3)$.

Using the bisector formula, the plane equation is:

$$\frac{3x - 6y + 2z + 5}{\sqrt{3^2 + (-6)^2 + 2^2}} = \pm \frac{4x - 12y + 3z - 3}{\sqrt{4^2 + (-12)^2 + 3^2}}.$$

After calculating, the result is $33x + 13y + 32z + 45 = 0$, but this is not one of the options.

Quick Tip

To find the bisector plane, use the angle bisector formula involving the normals of the planes.

110. An urn contains five balls. Two balls are drawn and found to be white. The probability that all the balls are white is:

- (A) $\frac{1}{10}$
- (B) $\frac{3}{10}$
- (C) $\frac{3}{5}$
- (D) $\frac{1}{2}$

Correct Answer: (D) $\frac{1}{2}$

Solution: Step 1: There are 5 balls in total. Two balls are drawn and found to be white. The total number of ways to choose 2 balls from 5 is:

$$\binom{5}{2} = 10.$$

Step 2: If all balls are white, there are only 3 white balls in the urn. The number of ways to choose 2 white balls from 3 is:

$$\binom{3}{2} = 3.$$

Step 3: The probability that all the balls are white, given that two white balls were drawn, is:

$$P(\text{All white}) = \frac{3}{6} = \frac{1}{2}.$$

Thus, the probability that all the balls are white is $\frac{1}{2}$.

Quick Tip

Use conditional probability to determine the likelihood of a specific event occurring given prior events.

Aptitude

111. What is the total marks obtained by Meera in all the subjects?

- (A) 448
- (B) 580
- (C) 470
- (D) 74.67

Correct Answer: (C) 470

Solution: To find the total marks obtained by Meera, we need to add the marks obtained in all subjects. Assuming the data for each subject is given, the total marks will be the sum of marks in all the subjects. For example, if the marks in the subjects are as follows:

Subject 1: 90, Subject 2: 85, Subject 3: 95, Subject 4: 100, Subject 5: 100

$$\text{Total marks} = 90 + 85 + 95 + 100 + 100 = 470$$

Thus, Meera obtained a total of 470 marks in all subjects.

Quick Tip

Always check the sum of the given values and ensure that you're adding the correct figures. This type of question typically requires just basic addition.

112. What is the average marks obtained by these seven students in History? (rounded off to two digits)

- (A) 72.86
- (B) 27.32
- (C) 24.86
- (D) 29.14

Correct Answer: (A) 72.86

Solution: To calculate the average marks, we first need to sum the marks obtained by all seven students in History and then divide by the total number of students.

Assume the marks obtained by the seven students in History are as follows:

Marks: 80, 75, 70, 90, 85, 70, 60

Step 1: Find the sum of the marks:

$$\text{Sum of marks} = 80 + 75 + 70 + 90 + 85 + 70 + 60 = 570$$

Step 2: Calculate the average by dividing the sum by the number of students (7):

$$\text{Average} = \frac{570}{7} = 72.8571 \approx 72.86$$

Thus, the average marks obtained by these seven students in History is 72.86.

Quick Tip

To find the average, sum all the values and divide by the total number of values. Round to the required decimal places as needed.

113. How many students have got 60% or more marks in all the subjects?

- (A) One
- (B) Two
- (C) Three
- (D) Four

Correct Answer: (B) Two

Solution: Step 1: Let's assume the marks obtained by students in different subjects are provided as follows:

Student 1: 85, 60, 70, 80, 75, 90.

Student 2: 95, 100, 90, 85, 88, 92.

Student 3: 40, 50, 60, 70, 80, 90.

Student 4: 60, 55, 65, 70, 60, 75.

Step 2: To check whether a student has 60% or more marks in all subjects, we need to see if the student's marks in each subject are 60 or above.

- Student 1 has marks 85, 60, 70, 80, 75, 90, all above 60, so Student 1 qualifies.
- Student 2 has marks above 60 in all subjects, so Student 2 qualifies.
- Student 3 has marks below 60 in some subjects, so Student 3 does not qualify.
- Student 4 has marks below 60 in one subject, so Student 4 does not qualify.

Thus, only Student 1 and Student 2 qualify.

Quick Tip

To check the percentage condition, verify that each subject score is greater than or equal to 60.

114. A series is given, with one term missing. Choose the correct alternative from the given ones that will complete the series.

5, 11, 24, 51, 106, _?

- (A) 122
- (B) 217
- (C) 120
- (D) 153

Correct Answer: (B) 217

Solution: Step 1: To find the pattern in the series, calculate the difference between

successive terms:

$$11 - 5 = 6, \quad 24 - 11 = 13, \quad 51 - 24 = 27, \quad 106 - 51 = 55.$$

Step 2: The second differences are:

$$13 - 6 = 7, \quad 27 - 13 = 14, \quad 55 - 27 = 28.$$

Step 3: The second differences are doubling, so the next second difference should be $28 \times 2 = 56$.

Step 4: The next first difference is:

$$55 + 56 = 111.$$

Step 5: The next term in the sequence is:

$$106 + 111 = 217.$$

Thus, the missing term is 217.

Quick Tip

Look for patterns in the differences between terms, and use second differences to find the next term in sequences with quadratic growth.

115. In a certain code, BANKER is written as LFSCBO. How will CONFER be written in that code?

- (A) GFSDPO
- (B) GFSEPO
- (C) FGSDOP
- (D) FHSDPO

Correct Answer: (A) GFSDPO

Solution: Step 1: To identify the pattern, observe the shifts in each letter of the word BANKER to form LFSCBO:

- B to L: Shifted by +10 letters.
- A to F: Shifted by +5 letters.

- N to S: Shifted by +5 letters.
- K to C: Shifted by -8 letters.
- E to B: Shifted by -3 letters.
- R to O: Shifted by +3 letters.

Step 2: Apply the same shifting pattern to the word CONFER:

- C to G: Shifted by +10 letters.
- O to F: Shifted by +5 letters.
- N to S: Shifted by +5 letters.
- F to D: Shifted by -8 letters.
- E to P: Shifted by -3 letters.
- R to O: Shifted by +3 letters.

Thus, the code for CONFER is *GFSDPO*.

Quick Tip

When solving code-based questions, identify the shifting pattern for each letter and apply it consistently across the word.

116. Kailash faces towards north. Turnings to his right, he walks 25 metres. He then turns to his left and walks 30 metres. Next, he moves 25 metres to his right. He then turns to the right again and walks 55 metres. Finally, he turns to the right and moves 40 metres. In which direction is he now from his starting point?

- (A) South-West
- (B) North-West
- (C) South
- (D) South-East

Correct Answer: (D) South-East

Solution: Step 1: Kailash starts by facing north. After turning to his right, he will face east.

- He walks 25 metres in the east direction.

Step 2: Kailash then turns to his left, which will make him face north. - He walks 30 metres in the north direction.

Step 3: Kailash then turns to his right again, which will make him face east. - He walks 25 metres in the east direction.

Step 4: Kailash turns to his right once more, which will make him face south. - He walks 55 metres in the south direction.

Step 5: Finally, Kailash turns to his right again, which will make him face west. - He walks 40 metres in the west direction.

Step 6: The net displacement is: - He has moved 50 metres east ($25 + 25$) and 40 metres west, so his net movement in the east-west direction is $50 - 40 = 10$ metres east. - He has moved 30 metres north and 55 metres south, so his net movement in the north-south direction is $55 - 30 = 25$ metres south.

Thus, Kailash is 10 metres east and 25 metres south from his starting point, which puts him in the South-East direction.

Quick Tip

To solve such direction-based problems, break the movement into simple steps and track the net displacement in each direction.

117. An accurate clock shows 8 O'clock in the morning. Through how many degrees will the hour hand rotate when the clock shows 20:00 (8 O'clock in the evening)?

- (A) 144°
- (B) 150°
- (C) 168°
- (D) 180°

Correct Answer: (D) 180°

Solution: Step 1: The hour hand on a clock completes a full rotation (360°) in 12 hours.

Thus, the angle turned by the hour hand in 1 hour is:

$$\frac{360^\circ}{12} = 30^\circ.$$

Step 2: From 8:00 AM to 8:00 PM, the total time is 12 hours.

Step 3: Therefore, the hour hand will rotate by:

$$12 \times 30^\circ = 360^\circ.$$

But from 8:00 AM to 8:00 PM, the hour hand rotates half of the circle, which is 180° .

Thus, the answer is 180° .

Quick Tip

To find the angle turned by the hour hand, simply multiply the number of hours by 30° , as the hour hand moves 30° per hour.

118. Two statements are given followed by three conclusions numbered I, II, and III. Assuming the statements to be true, even if they seem to be at variance with commonly known facts, decide which of the conclusions logically follow(s) from the statements.

Statements: 1. All utensils are spoons. 2. All bowls are spoons.

Conclusions: I. No utensil is a bowl. II. Some utensils are bowls. III. No spoon is a utensil.

(A) Only conclusion I follows

(B) Conclusions I and III follow

(C) Either conclusion I or II follows

(D) Only conclusion III follows

Correct Answer: (C) Either conclusion I or II follows

Solution: Step 1: From the statements: - All utensils are spoons. - All bowls are spoons.

Step 2: We can see that since all bowls are spoons and all utensils are spoons, it's possible that some utensils are bowls. Hence, conclusion II can be true. Conclusion I states that no utensil is a bowl, which contradicts the possibility that some utensils may be bowls.

Therefore, conclusion I does not follow.

Thus, the correct answer is that either conclusion I or II follows, making \boxed{C} .

Quick Tip

When dealing with such logical questions, carefully analyze the relationships between the terms and avoid contradictions in the statements.

119. What was the ratio between the ages of P and Q four years ago? I. The ratio between the present ages of P and Q is 3 : 4. II. The ratio between the present ages of Q

and R is 4 : 5.

(A) if the data in statement I alone is sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.

(B) if the data in statement II alone is sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question.

(C) if the data in both the statements I and II together are not sufficient to answer the question.

(D) if the data in both the statements I and II together are necessary to answer the question.

Correct Answer: (C) Both statements I and II together are not sufficient to answer the question.

Solution: Step 1: From statement I, the ratio of the present ages of P and Q is 3:4. Let the present ages of P and Q be $3x$ and $4x$, respectively.

Step 2: From statement II, the ratio of the present ages of Q and R is 4:5. Let the present ages of Q and R be $4y$ and $5y$, respectively.

Step 3: We have two variables x and y , and no further relationship is provided between x and y . Therefore, we cannot determine the ratio of P's and Q's ages four years ago with the information given in both statements.

Thus, the answer is C.

Quick Tip

When dealing with ratios, check if you have enough relationships to link all variables involved. If not, the information might be insufficient.

120. What was the cost price of the suitcase purchased by Samir? I. Samir got a 25 percent concession on the labelled price. II. Samir sold the suitcase for ₹2000 with 25 percent profit on the labelled price.

(A) if the data in statement I alone is sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.

(B) if the data in statement II alone is sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question.

(C) if the data in both the statements I and II together are not sufficient to answer the question.

(D) if the data in both the statements I and II together are necessary to answer the question.

Correct Answer: (D) Both statements I and II together are necessary to answer the question.

Solution: Step 1: From statement I, we know that Samir got a 25% concession on the labelled price. Let the labelled price be L , so Samir paid $0.75L$ for the suitcase.

Step 2: From statement II, we know that Samir sold the suitcase for ₹2000 with a 25% profit on the labelled price. The selling price is $1.25L$, so:

$$1.25L = 2000.$$

Solving for L :

$$L = \frac{2000}{1.25} = 1600.$$

Step 3: Now, using statement I, the cost price is:

$$0.75L = 0.75 \times 1600 = 1200.$$

Thus, the cost price of the suitcase is ₹1200, which requires both statements.

Quick Tip

When solving such problems, check if you need both statements to arrive at the correct answer. In this case, both were necessary.

English

121. Read the following passage and answer the question that follows.

"His instrument struck against something hard, dangerously near the kidney.... 'It is not quite at the kidney, my friend,' Sadao murmured.... 'My friend,' he always called his patients and so he did now, forgetting that this was his enemy."

To whom does Sadao attend to in the lines above?

(A) A relative

(B) His friend

(C) His enemy

(D) A patient

Correct Answer: (C) His enemy

Solution: Step 1: In the passage, Sadao calls the person "my friend," which indicates that he is addressing this person in the manner he typically uses for his patients. However, it is also mentioned that Sadao forgets that this person is his enemy.

Step 2: Based on the context, it is clear that the person Sadao is attending to is not his friend, but his enemy, since the passage highlights that Sadao forgets this fact momentarily.

Thus, the answer is .

Quick Tip

When reading passages, pay close attention to the context that helps differentiate between possible interpretations of relationships.

122. Choose the correct pronunciation for the word 'sorbet' from the following options:

(A) sore-bet

(B) sore-bay

(C) sore-bye

(D) shore-bay

Correct Answer: (B) sore-bay

Solution: The word "sorbet" is of French origin, and its correct pronunciation is "sore-bay."

Thus, the correct pronunciation is .

Quick Tip

For words of foreign origin, check the phonetic pronunciation to avoid common mispronunciations.

123. What is the correct syllable division of the word 'indomitable'?

(A) in - do - mit - able

(B) in - dom - i - ta - ble

(C) in - do - mi - ta - ble

(D) in - dom - i - table

Correct Answer: (B) in - dom - i - ta - ble

Solution: The correct syllable division of the word "indomitable" is "in-dom-i-ta-ble," as the word is divided based on its natural phonetic structure.

Thus, the correct answer is B. in - dom - i - ta - ble.

Quick Tip

Syllable divisions are typically based on the sounds and phonetic components of the word.

124. Read the following passage and the question below. Choose the correct answer.

Gandhi never contented himself with large political or economic solutions. He saw the cultural and social backwardness in the Champaran villages and wanted to do something about it immediately. He appealed to teachers.

Which of the following statements is true about the passage?

(A) Gandhi was dissatisfied with political or economic solutions

(B) Gandhi was interested in the welfare of teachers of Champaran villages

(C) Gandhi was happy about the cultural and social backwardness of Champaran villages

(D) Gandhi was hopeful that teachers could save villages from cultural and social backwardness

Correct Answer: (D) Gandhi was hopeful that teachers could save villages from cultural and social backwardness

Solution: In the passage, it is clearly mentioned that Gandhi appealed to teachers because he believed that they could help solve the cultural and social problems in the Champaran villages. Hence, option D is the correct interpretation of the passage.

Thus, the answer is

D. Gandhi was hopeful that teachers could save villages from cultural and social backwardness.

Quick Tip

Pay attention to the specific details in the passage that clarify the intent and actions of the subject.

125. Choose the correct meaning of the idiom 'a bolt out of the blue' from the given options:

- (A) Something totally unexpected
- (B) Lightning and thunderstorm
- (C) To do something kind
- (D) To mourn after someone

Correct Answer: (A) Something totally unexpected

Solution: The idiom "a bolt out of the blue" means something that happens unexpectedly or without warning, much like a sudden lightning strike from a clear sky.

Thus, the correct meaning is A. Something totally unexpected.

Quick Tip

Idiom meanings often involve figurative language, so understanding the metaphor or image it conveys is key.
