09 - 04 - 2024
SHIFT - 2

SMART ACHIEVERS
JEE | NEET | FOUNDATION

JEE Main - 2024 Session - 2
Answers & Solutions

(Physics, Chemistry and Mathematics)

9 - April - Shift - 2

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PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Dimensional formula of Planck's constant is

(1) $[M^2L^2T^{-1}]$

(2) $[M^1L^2T^{-1}]$

(3) $[M^2L^2T^{-2}]$

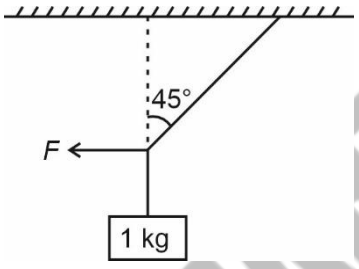
(4) $[ML^2T^{-3}]$

Answer (2)

Sol. $E = h\nu$

$$[h] = \frac{ML^2T^{-2}}{T^{-1}}$$

2. Find the magnitude of force F, if the given system is in equilibrium



(1) 10 N

(2) $\frac{10}{\sqrt{2}}$ N

(3) 0 N

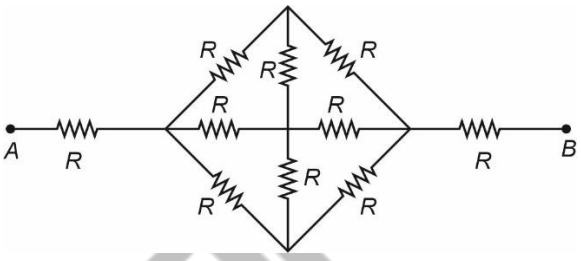
(4) $\frac{1}{10\sqrt{2}}$ N

Answer (2)

Sol. T = 10 N; $T\sin 45 = F$

$$F = \frac{10}{\sqrt{2}} \text{ N}$$

3. The equivalent resistance between terminal A and B in the network shown.

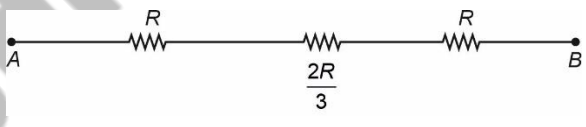


(1) $\frac{4R}{3}$ (2) $\frac{8R}{3}$

(3) $3R$ (4) $\frac{5R}{2}$

Answer (2)

Sol.



$$R_{AB} = \frac{8R}{3}$$

4. A nuclei at rest breaks into two parts with mass ratio 1 : 2. The ratio of their velocities and direction is

(1) Opposite direction 2 : 1

(2) Same direction 1 : 2

(3) Opposite direction 1 : 1

(4) Same direction 1 : 1

Answer (1)

Sol. By conservation of momentum

$m_1v_1 = m_2v_2$

$$\frac{v_1}{v_2} = \frac{m_2}{m_1} = \frac{2}{1}$$

5. Two cars *A* and *B* are moving towards each other with speed 20 m/s each. When 300 m apart, they both apply breaks which causes deceleration of 2 m/s^2. The distance between them when they stop will be :

(1) 100 m

(2) 50 m

(3) 150 m

(4) 200 m

Answer (1)

Sol. $\vec{v}_{AB} = 40\hat{i}$ m/s

$\vec{a}_{AB} = -4\hat{i}$ m/s

$\Rightarrow \quad v^2 = u^2 + 2as$

$0 = 1600 - 8s$

$\Rightarrow \quad s = 200$ m

Distance between them = 300 – 200 = 100 m

6. For a wire, original resistance was 50 Ω at initial temperature was 27°C. When temperature is increased, its resistance becomes 62 Ω. If the thermal coefficient of resistivity of wire is 2.4×10^{-2} K^{-1}, find final temperature.

(1) 45°C

(2) 32°C

(3) 37°C

(4) 48°C

Answer (3)

Sol. $R = R_0(1 + \alpha\Delta T)$

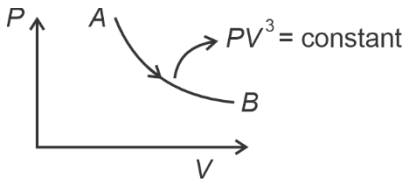
$62 = 50(1 + 2.4 \times 10^{-2}\,\Delta T)$

$1.24 = 1 + 2.4 \times 10^{-2}\,\Delta T$

$\Delta T = 10$

$T = 37°C$

7. Find work done by monatomic gas from *A* to *B*. Here temperature of gas (1 mole) changes from 330 K to 300 K.



(1) 125 J

(2) 250 J

(3) 500 J

(4) 625 J

Answer (1)

Sol. $w = \dfrac{\mu R\Delta T}{1-\alpha} = \dfrac{25}{3} \times \dfrac{30}{2} = 125$ J

8. Two bubbles having radii r_A and r_B are having excess pressure P_A and P_B in them. If $P_A = 3P_B$, find $\dfrac{r_A}{r_B}$.

(1) 9 : 1

(2) 1 : 9

(3) 1 : 3

(4) 3 : 1

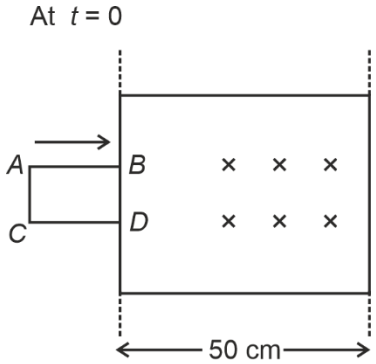
Answer (3)

Sol. $\Delta P = \dfrac{4T}{r}$

$\dfrac{P_A}{P_B} = \dfrac{r_B}{r_A}$

$\dfrac{r_A}{r_B} = \dfrac{1}{3}$

9. Find the induced emf in the square loop of side 15 cm moving with 2 cm/s after 10 seconds.



(1) 0 (2) 0.3 mV

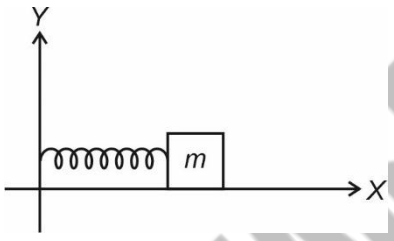
(3) 3 V (4) 9 V

Answer (1)

Sol. At t = 10 seconds, ϕ = Constant

$$\Rightarrow \frac{d\phi}{dt} = 0$$

10. A spring exerts force on block $\vec{F} = -50x^{-b}$ where x is change in length of spring. Find time period of oscillations. (m = 0.5 kg)



(1) 0.63 sec.

(2) 3.14 sec.

(3) 1.57 sec.

(4) 0.31 sec.

Answer (1)

Sol. $T = 2\pi\sqrt{\frac{0.5}{50}} = \frac{2\pi}{10} = \frac{\pi}{5}$ sec.

11. A proton and deuteron, having same kinetic energy, enters a transverse uniform magnetic field. Radius of circular paths for proton and deuteron are in ratio of

(1) $\sqrt{2}$

(2) $\frac{1}{2\sqrt{2}}$

(3) $\frac{1}{\sqrt{2}}$

(4) $2\sqrt{2}$

Answer (3)

Sol. $r = \frac{\sqrt{2mk}}{qB}$

$$\frac{r_p}{r_d} = \sqrt{\frac{m_p}{m_d}}$$

$$\frac{q_d}{q_p} = \frac{1}{\sqrt{2}}.$$

12. A satellite of mass 10^3 kg is orbiting in an orbit of radius $2r$ from centre of the planet of radius r. If satellite is given energy $E = \frac{GM}{6r}$, then find new radius of orbit in which satellite will revolve.

(M = mass of planet)

(1) $14r$

(2) $6r$

(3) $8r$

(4) $12r$

Answer (2)

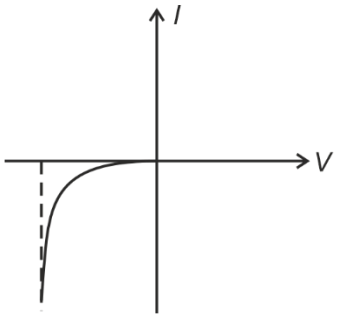
Sol. $E_1 = -\frac{GMm}{4r}$

$$E_f = -\frac{GMm}{4r} + \frac{GMm}{6r} = -\frac{GMm}{2x}$$

$$-\frac{1}{12r} = -\frac{1}{2x}$$

$x = 6r$

13. For which of the following is the *I-V* characteristics shown below is possible?



(1) Transistor

(2) Zener diode

(3) Solar cell

(4) Diode used as rectifier

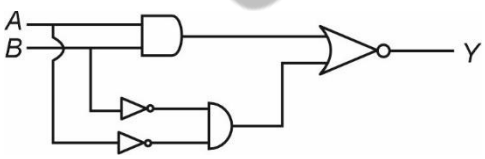
Answer (2)

Sol. As Zener diode operates in reverse bias, it is for Zener diode.

14. For the circuit shown, the truth table

A	B	Y
0	0	0
0	1	x
1	0	y
1	1	0

. Find 'x' and 'y'.



(1) 0, 0

(2) 0, 1

(3) 1, 0

(4) 1, 1

Answer (4)

Sol. $Y = \overline{AB + \bar{A}\bar{B}}$

15. A ball of radius 10^{-4} m and density 10^5 kg/m^3 is dropped from a height h into water (viscosity = 9.8×10^{-6} *Pa–s*) such that after falling into liquid, its speed does not change. Find the approximate value of h.

(1) 2200 m

(2) 2350 m

(3) 2470 m

(4) 2520 m

Answer (3)

Sol. Velocity just before entering water = Terminal velocity

$$\sqrt{2 \times g \times h} = \frac{2}{9} r^2 g \frac{(\rho - \sigma)}{\eta}$$

$$\sqrt{2g \times h} = \frac{2}{9} \times 10^{-8} \times g \times \frac{(10^5 - 10^3)}{9.8 \times 10^{-6}}$$

$\boxed{h \approx 2470 \text{ m}}$

16.

17.

18.

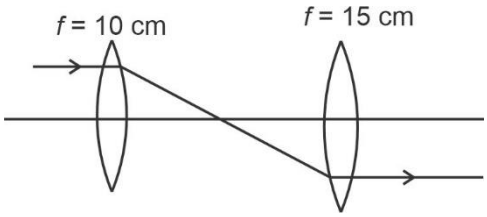
19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. In given ray diagram, find distance u (in cm) between two convex lenses.



Answer (25)

Sol. $f_1 + f_2 = L = 25$ cm

22. Find the work done (in J) by force $F = 3x^2 + 2x - 5$ in moving a particle from $x = 2$ to $x = 4$.

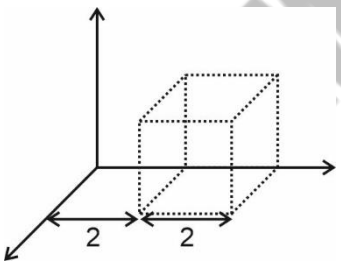
Answer (58)

Sol. $W = \int_2^4 F \cdot dx$

$= \left[x^3 + x^2 - 5x\right]_2^4$

$= 58$ J

23. There is an imaginary cube of side 2 m where edges are along axes. The electrostatic field varies as $\vec{E}(x) = 2x\hat{i}$, then flux through cube in Nm^2/C is _____ .



Answer (16)

Sol. $E_1 = 4$

$E_2 = 8$

$\Rightarrow \Delta\phi = (8 - 4)\, 2^2 = 16$

24. If work function of a metal is 2.13 eV and energy per photon of incident light is 3.13 eV, then maximum kinetic energy of photoelectrons (in eV) will be_____.

Answer (1)

Sol. $KE_{max} = hr - \phi_0$

$= (3.13 - 2.13)$ eV

$= 1$ eV

25. A photon of energy of 10.2 eV is incident on hydrogen atom in ground state. Thereafter number of emitted lines will be

Answer (1)

Sol. $\Delta E = 10.2$ eV

e^- will be excited to $n = 2$

26.

27.

28.

29.

30.

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Correct order of bond angle of following compounds is BF_3, PF_3, ClF_3

   (1) $BF_3 > PF_3 > ClF_3$ (2) $PF_3 > ClF_3 > BF_3$

   (3) $ClF_3 > PF_3 > BF_3$ (4) $BF_3 > ClF_3 > PF_3$

Answer (1)

Sol. $BF_3 \Rightarrow sp^2 \Rightarrow$ Bond angle = 120°

$PF_3 \Rightarrow sp^3 \Rightarrow$ Bond angle $\approx 109°28'$

$ClF_3 \Rightarrow sp^3d \Rightarrow$ Bond angle $\approx 90°$

2. Identify the correct electronic configuration of Einstenium is

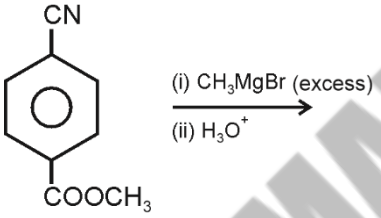
   (1) [Rn] $5f^{14}6d^{1}7s^{2}$ (2) [Rn] $5f^{11}7s^{2}$

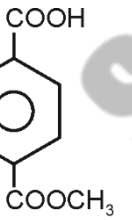
   (3) [Rn] $5f^{10}6d^{1}7s^{2}$ (4) [Rn] $5f^{11}6d^{1}7s^{1}$

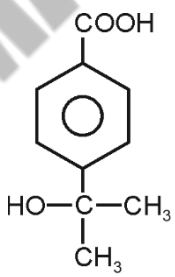
Answer (2)

Sol. Es (Z = 99) $\Rightarrow$ [Rn] $5f^{11}7s^{2}$

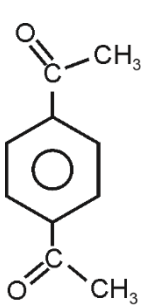
3. The product obtained in the following reaction is:



(1)


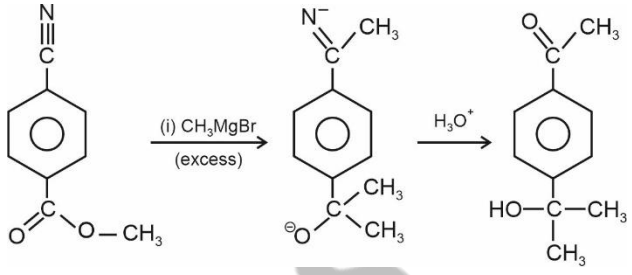
(2)


(3)


(4)


Answer (3)

Sol.



4. Ca^{2+} makes which type of complex with EDTA?

   (1) Trigonal bipyramidal

   (2) Square planar

   (3) Tetrahedral

   (4) Octahedral

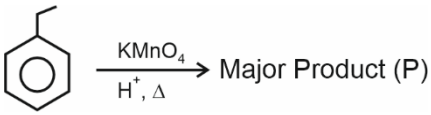
Answer (4)

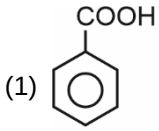
Sol. Co-ordination number of Ca^{2+} with EDTA is 6

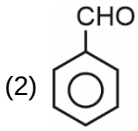
Hybridisation = sp^3d^2

Shape = Octahedral

5. Consider the following reaction and identify the major product P.



(1)


(2)


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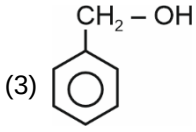
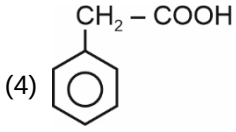
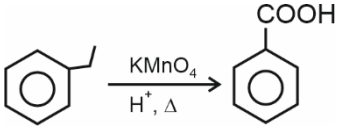


(4)


Answer (1)

Sol. The reaction is benzylic oxidation reaction



6. Match the complexes given in List-I with the hybridisation of central metal atom/ion given in List-II and choose the correct option.

	List-I (Complexes)		**List-II (Hybridisation)**
(A)	$K_2[Ni(CN)_4]$	(I)	sp^3
(B)	$[Ni(CO)_4]$	(II)	sp^3d^2
(C)	$[Co(NH_3)_6]Cl_3$	(III)	dsp^2
(D)	$Na_3[CoF_6]$	(IV)	d^2sp^3

(1) (A)-(I); (B)-(II) (C)-(III); (D)-(IV)

(2) (A)-(III); (B)-(I) (C)-(IV); (D)-(II)

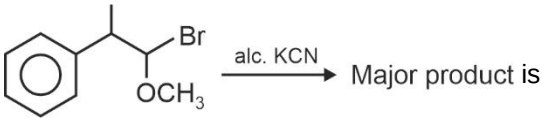
(3) (A)-(IV); (B)-(III) (C)-(II); (D)-(I)

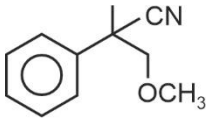
(4) (A)-(I); (B)-(II) (C)-(IV); (D)-(III)

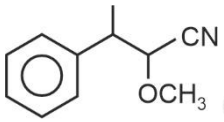
Answer (2)

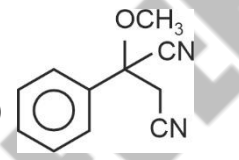
Sol.

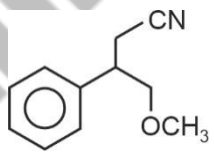
(A)	$K_2[Ni(CN)_4]$ $Ni^{2+}: 3d^8$	;	dsp^2 hybridisation as CN^- is strong field ligand
(B)	$[Ni(CO)_4]$ $Ni^0: 3d^8 4s^2$	;	sp^3 hybridisation as CO is strong field ligand
(C)	$[Co(NH_3)_6]Cl_3$ $Co^{3+}: 3d^6$	;	d^2sp^3 hybridisation as NH_3 is strong field ligand
(D)	$Na_3[CoF_6]$ $[CoF_6]^{3-}; Co^{3+}: 3d^6$	;	sp^3d^2 hybridisation as F^- ion is a weak field ligand

7. 

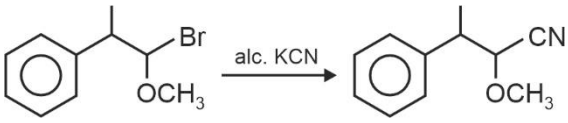
(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. 

8. Which of the following is correct for strong electrolyte (B > 0)

(1) $\lambda_m - \lambda_m^0 - B\sqrt{C} = 0$

(2) $\lambda_m + \lambda_m^0 - B\sqrt{C} = 0$

(3) $\lambda_m - \lambda_m^0 + B\sqrt{C} = 0$

(4) $\lambda_m + \lambda_m^0 + B\sqrt{C} = 0$

Answer (3)

Sol. $\lambda_m = \lambda_m^0 - B\sqrt{C}$

$\lambda_m - \lambda_m^0 + B\sqrt{C} = 0$

9. Which one of the following statements regarding glucose is incorrect?

(1) Glucose is one of the monosaccharides of sucrose

(2) Glucose dissolves in water because it has aldehyde group.

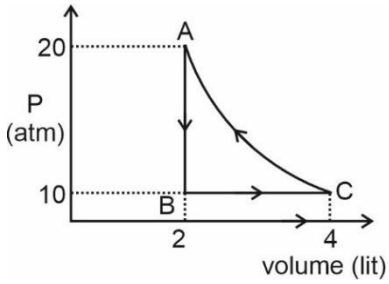
(3) Glucose has six carbon atoms in its structure

(4) Glucose is an aldose

Answer (2)

Sol. Glucose is an aldohexose having molecular formula $C_6H_{12}O_6$. It is soluble in water due to number of hydroxyl groups which can form H-bonds with water. α(D) Glucose condenses with β(D) fructose to form sucrose.

10.



What is the work done on the gas in cyclic process ABCA

(1) +773.7 J

(2) –773.7 J

(3) +4762.3 J

(4) –4762.3 J

Answer (1)

Sol. $W_{AB} = 0$

$W_{BC} = -10\ (4 - 2)$

$= -20$ atm. Lit

$W_{CA} = 2.303(40) \log 2$

$= 27.636$ atm. Lit

$W_{total} = 7.636$ atm. Lit

$= 773.7$ Joule

11. Which of the following compounds does not give Tollen's test?

(1) Formaldehyde

(2) Formic acid

(3) Benzaldehyde

(4) Acetone

Answer (4)

Sol. Aldehyde and Formic acid can give Tollen's test with ammoniacal silver nitrate solution.

12. Which of the following will give positive Iodoform test?

(1) $CH_3 - CH_2 - CH_2 - CHO$

(2) $CH_3 - \underset{\underset{OH}{|}}{CH} - CH_3$

(3) $CH_3 - CH_2 - \overset{\overset{O}{||}}{C} - CH_2 - CH_3$

(4) $CH_3 - CH_2 - CH_2 - CH_2 - OH$

Answer (2)

Sol. Molecules having

Groups as $-\overset{\overset{O}{||}}{C} - CH_3$ or $-\underset{\underset{OH}{|}}{CH} - CH_3$ gives positive iodoform test.

13. Match the List and choose correct option.

	List-I		List-II
(i)	Ni-Cd cell	(a)	Rechargeable
(ii)	Fuel cell	(b)	Anode ($Zn \rightarrow Zn^{2+} + 2e^-$)
(iii)	Mercury cell	(c)	Used in hearing aid
(iv)	Leclanche cell	(d)	Combustion energy in to electrical energy

(1) (i)-(a); (ii)-(d); (iii)-(c), (iv)-(b)

(2) (i)-(b); (ii)-(a); (iii)-(c), (iv)-(d)

(3) (i)-(d); (ii)-(a); (iii)-(c), (iv)-(b)

(4) (i)-(a); (ii)-(b); (iii)-(c), (iv)-(d)

Answer (1)

Sol. Ni-Cd cell is secondary cell and are rechargeable mercury cell is used in hearing aid.

14. What is the correct order of C – C bond length of ethane, ethene and ethyne?

(1) Ethane > Ethene > Ethyne

(2) Ethene > Ethane > Ethyne

(3) Ethyne > Ethene > Ethane

(4) Ethyne > Ethane > Ethene

Answer (1)

Sol. Correct order of C – C bond length is

Ethane > Ethene > Ethyne

C – C > C = C > C ≡ C

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Fuming sulphuric acid has how many oxygen atoms?

Answer (7)

Sol. Fuming sulphuric acid is oleum ($H_2S_2O_7$)

$\therefore$ 7 O-atoms are present in fuming sulphuric acid.

22. Total sum of number of electrons in π^* orbitals of O_2. O_2^+ and O_2^- is

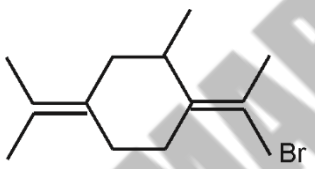
Answer (6)

Sol. O_2 $(16e^-)$:

$$\sigma_{1s}^2\ \sigma_{1s}^{*2}\ \sigma_{2s}^2\ \sigma_{2s}^{*2}\ \sigma_{2p_z}^2 \begin{pmatrix} \pi_{2p_x}^2 \\ \pi_{2p_y}^2 \end{pmatrix} \begin{pmatrix} \pi_{2p_x}^{*1} \\ \pi_{2p_x}^{*1} \end{pmatrix} \sigma_{2p_z}^*$$

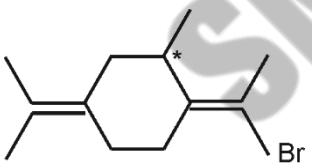
Total number of e^- in π^* orbitals of O_2, O_2^+, O_2^- = 2 + 1 + 3 = 6

23. How many total number of stereoisomers are possible for the following structure



Answer (4)

Sol.



The structure has two stereogenic centres, one geometrical centre and one optical centre. Hence it has total 4 stereoisomers.

$2^2 = 4$

24. Among the elements – Sc, Ti, V, Cr, Mn find magnetic moment of element which have highest ionization enthalpy in +2 oxidation state. [Nearest integer]

Answer (6)

Sol. Sc^{+2} Ti^{+2} V^{+2} Cr^{+2} Mn^{+2}

Mn^{+2} will have highest I.E. due to its stable half filled configuration.

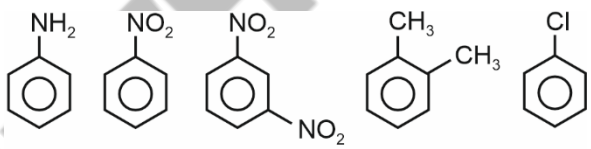
$Mn \rightarrow [Ar]\ 4s^2\ 3d^5 \rightarrow 5$ unpaired $e^{\ominus}$

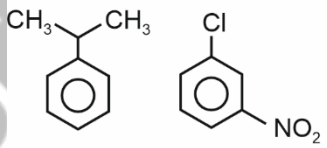
$\mu_{spin} = \sqrt{5(5+2)}$ BM

$= \sqrt{35}$

$\simeq 6$

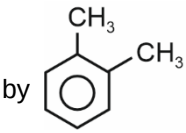
25. How many of the following compounds will give Friedel Craft's reaction?

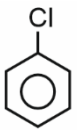




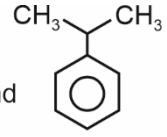
Answer (3)

Sol. Friedel Craft's reaction is not given by those aromatic compounds which have strong deactivating groups like $-NO_2$ group. Even aniline does not give Friedel Crafts reaction because the Lewis acid $AlCl_3$ will from co-coordinate bond with $-NH_2$ group thus converting it into strongly deactivating group, Friedel Crafts reaction is given by





and



26.

27.

28.

29.

30.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. If $\frac{z-2i}{z+2i}$ is purely imaginary, then maximum value of $|z + 8 + 6i|$ is equal to

(1) 6 (2) 8

(3) 10 (4) 12

Answer (4)

Sol. $\left(\frac{x+(y-2)i}{x+(y+2)i}\right)\frac{(x-(y+2)i)}{(x-(y+2)i)}$ = purely imaginary.

$\Rightarrow x^2 + (y-2)(y+2) = 0$

$\Rightarrow x^2 + y^2 = 4$

Maximum value = 10 + 2 = 12

2. $\int_{\frac{1}{4}}^{\frac{3}{4}} \cos\left(2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right)dx =$

(1) $\frac{-1}{4}$ (2) $\frac{3}{2}$

(3) $\frac{1}{16}$ (4) $\frac{-4}{3}$

Answer (1)

Sol. $\int_{\frac{1}{4}}^{\frac{3}{4}} \cos\left(2\cot^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)\right)dx$

$x = \cos 2\theta \quad \Rightarrow dx = (-\sin 2\theta d\theta)2$

$-2\int_{\alpha}^{\beta} \cos\left(2\cot^{-1}|\tan\theta|\right)\sin 2\theta d\theta$

$= -2\int_{\alpha}^{\beta} -\cos 2\theta \cdot \sin 2\theta d\theta$

$= \int_{\alpha}^{\beta} \sin 4\theta d\theta$

$= \frac{-\cos 4\theta}{4}\Big|_{\alpha}^{\beta}$

$= \frac{-1}{4}\left(2\cos^2 2\theta - 1\right)\Big|_{\alpha}^{\beta}$

$= \frac{-1}{4}\left(2\cdot\left(x^2\right)-1\right)\Big|_{\frac{1}{4}}^{\frac{3}{4}}$

$= \frac{-1}{4}\left(2x^2-1\right)\Big|_{\frac{1}{4}}^{\frac{3}{4}}$

$= \frac{-1}{4}\left(2\cdot\left(\frac{9}{16}\right)-1\right)-2\left(\frac{1}{16}\right)+1\Bigg)$

$= \frac{-1}{4}\left(\frac{18}{16}-1-\frac{2}{16}+1\right)$

$= \frac{-1}{4}(1) = \frac{-1}{4}$

3. $\lim_{x\to 0}\frac{e-(1+2x)^{\frac{1}{2x}}}{x}$

(1) e (2) $\frac{e}{4}$

(3) $\frac{e}{8}$ (4) $\frac{11}{24}e$

Answer (1)

Sol. Using expansion

$\lim_{x\to 0}\frac{e-e\left[1-\frac{2x}{2}+\frac{11\times 4x^2}{24}+\cdots\right]}{x}$

$\lim_{x\to 0} e-\frac{11x}{24}e\ldots$

$= e$

4. In the given data

x_f	f_i
C	2
$2C$	1
$3C$	1
$4C$	1
$5C$	1
$6C$	1

If $\sigma^2 = 160$. Find the value of $|C|$.

(1) 7 (2) 5

(3) 6 (4) 4

Answer (1)

Sol.

x_i	$f(x_i)$	$xf(x)$	$x^2f(x)$
C	2	$2C$	$2C^2$
$2C$	1	$2C$	$4C^2$
$3C$	1	$3C$	$9C^2$
$4C$	1	$4C$	$16C^2$
$5C$	1	$5C$	$25C^2$
$6C$	1	$6C$	$36C^2$

$\sigma^2 = E(x^2) - [E(x)]^2$, $\Sigma f(x_i) = 7$

$E(x) = \Sigma xf(x) = 22C$

$E(x^2) = \Sigma x^2 f(x) = 92C^2$

$$\sigma^2 = 160 = \frac{92C^2}{7} - \left(\frac{22C}{7}\right)^2$$

$\Rightarrow C = \pm 7$

5. $\int_{-1}^{2} \log(x + \sqrt{x^2+1})dx$

(1) $\log[(2+\sqrt{5})^2(\sqrt{2}-1)] - \sqrt{5} + \sqrt{2}$

(2) $\log[(2+\sqrt{5})^2(\sqrt{2}-1)] + \sqrt{5} - \sqrt{2}$

(3) $\log[(2+\sqrt{5})^2(\sqrt{2}-1)] + \sqrt{5} + \sqrt{2}$

(4) $\log(2+\sqrt{5})^2 + \sqrt{5} + \sqrt{2}$

Answer (1)

Sol. $\int_{-1}^{2} \underset{II}{1} \cdot \underset{I}{\log(x + \sqrt{x^2+1})}dx$

$$= x\log(x+\sqrt{x^2+1}) - \int_{-1}^{2}\left(\frac{1+\frac{x}{\sqrt{x^2+1}}}{x+\sqrt{x^2+1}}\right)x\,dx$$

$$= x\log(x+\sqrt{x^2+1}) - \int_{-1}^{2}\frac{x}{\sqrt{x^2+1}}dx$$

$$= x\log(x+\sqrt{x^2+1}) - \sqrt{x^2+1}\Big]_{-1}^{2}$$

$$= [2\log(2+\sqrt{5}) - \sqrt{5}] - [-\log(\sqrt{2}-1) - \sqrt{2}]$$

$$= \log[(2+\sqrt{5})^2(\sqrt{2}-1)] - \sqrt{5} + \sqrt{2}$$

6. The sum of coefficients of $x^{\frac{2}{3}}$ and $x^{\frac{2}{5}}$ in the binomial expansion of $\left(x^{\frac{2}{3}} + \frac{x^{\frac{-2}{5}}}{2}\right)^9$ is

(1) $\frac{{}^9C_4}{2^5}$ (2) $\frac{{}^9C_6}{2^4}$

(3) 0 (4) $\frac{63}{8}$

Answer (1)

Sol. $T_{r+1} = {}^9C_r \left(\frac{x^{\frac{-2}{5}}}{2}\right)^r \left(x^{\frac{2}{3}}\right)^{9-r}$

$$= {}^9C_r \frac{1}{2^r} x^{\frac{2}{3}(9-r)+\left(\frac{-2r}{5}\right)}$$

$$= {}^9C_r \frac{1}{2^r} x^{6-\frac{16r}{15}}$$

Coefficient of $x^{\frac{2}{3}}$ $\Rightarrow 6 - \frac{16r}{15} = \frac{2}{3}$

$\Rightarrow 90 - 16r = 10$

$\Rightarrow r = 5$

Coefficient of $x^{\frac{2}{5}}$ $\Rightarrow$ $6-\frac{16r}{15}=\frac{2}{5}$

$\Rightarrow$ $90 - 16r = 6$

$\Rightarrow$ $r=\frac{84}{16}\notin I$

$\Rightarrow$ Sum $={}^{9}C_5\frac{1}{2^5}+0$

$\Rightarrow$ $\frac{63}{16}$

7. Dice is thrown 3 times, then find the probability that $x_1 < x_2 < x_3$. (here $x_1, x_2, x_3 \in [1, 6]$)
(where x_1, x_2, x_3 are outcomes on dice)

(1) $\frac{7}{54}$ (2) $\frac{5}{54}$

(3) $\frac{11}{54}$ (4) $\frac{17}{54}$

Answer (2)

Sol. Given condition is $x_1 < x_2 < x_3$

So, $n(E) = {}^{6}C_3$

$n(s) = 6^3 = 216$

Then required probability = $\frac{{}^{6}C_3}{216}$

= $\frac{20}{216}=\frac{5}{54}$

8. If $f'(x) = 3f(x) + x$ and $f(0) = 1$, then $f(x)$ is

(1) $\frac{-x}{3}+\frac{10}{9}e^{-3x}$ (2) $\frac{-x}{3}-\frac{1}{9}+\frac{10}{9}e^{3x}$

(3) $\frac{-x}{3}-\frac{10}{9}e^{-3x}$ (4) $\frac{-x}{2}-\frac{1}{9}+\frac{10}{9}e^{2x}$

Answer (2)

Sol. $\frac{dy}{dx}=3y+x$

$\Rightarrow$ $\frac{dy}{dx}-3y=x$

IF $=e^{\int -3dx}=e^{-3x}$

$y\cdot e^{-3x}=\int e^{-3x}\cdot x+c$

$\Rightarrow$ $y\cdot e^{-3x}=\frac{x\cdot e^{-3x}}{-3}+\frac{1}{3}\int e^{-3x}dx+c$

$y\cdot e^{-3x}=-\frac{1}{3}xe^{-3x}-\frac{1}{9}e^{-3x}+c$

$\Rightarrow$ $y=\frac{-1}{3}x-\frac{1}{9}+c\cdot e^{3x}$

$y(0)=1$

$1=\frac{-1}{9}+c$ $\Rightarrow$ $c=\frac{10}{9}$

$y=\frac{-x}{3}-\frac{1}{9}+\frac{10}{9}e^{3x}$

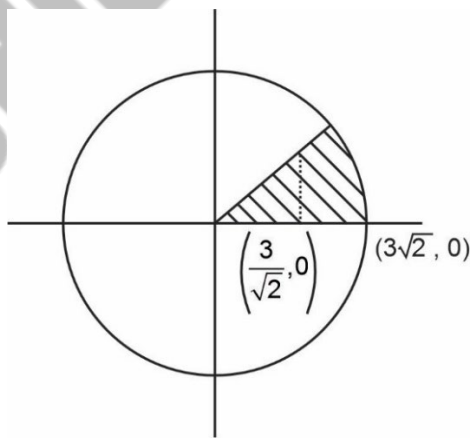
9. Find the area bounded by ellipse $x^2 + 3y^2 = 18$ below the line $y = x$ is (in first quadrant)

(1) $3\pi + 1$ (2) $\sqrt{3}\pi$

(3) $3\pi-\frac{3}{4}$ (4) $3\pi+\frac{1}{4}$

Answer (2)

Sol.



Area = $\int_0^{\frac{3}{\sqrt{2}}} x\,dx+\int_{\frac{3}{\sqrt{2}}}^{3\sqrt{2}}\sqrt{\frac{18-x^2}{3}}dx$

$=\frac{1}{2}\left(x^2\right)_0^{\frac{3}{\sqrt{2}}}+\frac{1}{\sqrt{3}}\left[\frac{x}{2}\sqrt{18-x^2}+9\sin^{-1}\left(\frac{x}{3\sqrt{2}}\right)\right]_{\frac{3}{\sqrt{2}}}^{3\sqrt{2}}$

$$= \frac{1}{2}\left(\frac{9}{2}\right) + \frac{1}{\sqrt{3}}\left[9\sin^{-1}(1) - \frac{3}{2\sqrt{2}}\frac{3\sqrt{3}}{\sqrt{2}} - 9\sin^{-1}\left(\frac{1}{2}\right)\right]$$

$$= \frac{9}{4} + \frac{1}{\sqrt{3}}\left(\frac{9\pi}{2} - \frac{9\sqrt{3}}{4} - \frac{9\pi}{6}\right)$$

$$= \sqrt{3}\pi$$

10. $x^2 - \sqrt{2}x - \sqrt{3} = 0$ and $P_n = \alpha^n + \beta^n$. The value of $P_{12} - \sqrt{2}P_{11} - \sqrt{3}P_{10} + P_{11} - \sqrt{2}P_{10}$ is

(1) $\sqrt{3}P_9$ (2) $\left(2+\sqrt{5}\right)P_9$

(3) $\sqrt{5}P_9$ (4) $\left(3+\sqrt{5}\right)P_9$

Answer (1)

Sol. $x^2 - \sqrt{2}x - \sqrt{3} = 0$

$\alpha^2 - \sqrt{2}\alpha - \sqrt{3} = 0$

$\alpha^{n+2} - \sqrt{2}\alpha^{n+1} - \sqrt{3}\alpha^n = 0$ ...(i)

Similarly,

$\beta^{n+2} - \sqrt{2}\beta^{n+1} - \sqrt{3}\beta^n = 0$ ...(ii)

$(\alpha^{n+2} + \beta^{n+2}) - \sqrt{2}(\alpha^{n+1} + \beta^{n+1}) - \sqrt{3}(\alpha^n + \beta^n)$

$P_{n+2} - \sqrt{2}P_{n+1} - \sqrt{3}P_n = 0$ ...(iii)

Put $n = 10$

$P_{12} - \sqrt{2}P_{11} - \sqrt{3}P_{10} = 0$

Now in the expression

$P_{12} - \sqrt{2}P_{11} - \sqrt{3}P_{10} + P_{11} - \sqrt{2}P_{10}$

$= P_{11} - \sqrt{2}P_{10}$

Put $n = 9$ in eq. (iii)

$P_{11} - \sqrt{2}P_{10} - \sqrt{3}P_9 = 0$

$P_{11} - \sqrt{2}P_{10} = \sqrt{3}P_9$

11. If range of function $f(x) = \frac{1}{2 + \sin 3x + \cos 3x}$ is $[a, b]$. If α and β be arithmetic and geometric mean of a, b then $\left(\frac{\alpha}{\beta}\right)$ is equal to

(1) $\frac{1}{\sqrt{2}}$ (2) $\sqrt{2}$

(3) $\frac{1}{2}$ (4) $\sqrt{3}$

Answer (2)

Sol. $f(x) = \frac{1}{2 + \sin(3x) + \cos 3x}$

$\sin(3x) + \cos(3x) \in [-\sqrt{2}, \sqrt{2}]$

$2 + \sin(3x) + \cos(3x) \in [2 - \sqrt{2},\ 2 + \sqrt{2}]$

$\Rightarrow \frac{1}{2 + \sin(3x) + \cos(3x)} \in \left[\frac{1}{2+\sqrt{2}}, \frac{1}{2-\sqrt{2}}\right]$

$\Rightarrow a = \frac{1}{2+\sqrt{2}} = \frac{(2-\sqrt{2})}{2}$

$b = \frac{1}{2-\sqrt{2}} = \frac{2+\sqrt{2}}{2}$

$\alpha = \frac{a+b}{2} = \frac{1}{2}\cdot\frac{1}{2}\cdot 4 = 1$

$\beta = \sqrt{ab} = \sqrt{\frac{1}{4}(2-\sqrt{2})(2+\sqrt{2})} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

then, $\frac{\alpha}{\beta} = \sqrt{2}$

12. If $\int_0^x \sqrt{1-\left(y'(t)\right)^2}\,dt = \int_0^x y'(t)dt$ and $0 \le x \le 3$, $y \ge 0$, $y(0) = 0$, then find $y'' + 1 + y$.

(1) $\frac{x}{\sqrt{2}} - 1$ (2) $\frac{x}{\sqrt{2}} + 1$

(3) $\frac{x}{2} + 1$ (4) $\frac{x}{2} - 1$

Answer (2)

Sol. $\sqrt{1-\left(y'(x)\right)^2}=y'(x)$

$\Rightarrow \quad 1-\left(y'(x)\right)^2=\left(y'(x)\right)^2$

$\Rightarrow \quad 2\left(y'(x)\right)^2=1$

$\Rightarrow \quad y'(x)=\frac{1}{\sqrt{2}},\frac{-1}{\sqrt{2}}$

$y(x)=\frac{1}{\sqrt{2}}x,\frac{-1}{\sqrt{2}}x$

$\therefore \quad y=\frac{1}{\sqrt{2}}x$

13. If $y=e^{3\sin^{-1}x}$, then value of

$(1-x^2)\frac{d^2y}{dx^2}-\frac{xdy}{dx}$ at $x=\frac{1}{2}$ equals to

(1) $9e^{\frac{\pi}{6}}$ (2) $3e^{\frac{\pi}{6}}$

(3) $3e^{\frac{\pi}{2}}$ (4) $e^{\frac{\pi}{6}}$

Answer (3)

Sol. $y=e^{3\sin^{-1}x}$

$\frac{dy}{dx}=e^{3\sin^{-1}x}\cdot\frac{3}{\sqrt{1-x^2}}$

$\sqrt{1-x^2}\frac{dy}{dx}=3y$

Differentiating

$\sqrt{1-x^2}\frac{d^2y}{dx^2}-\frac{2x}{2\sqrt{1-x^2}}\frac{dy}{dx}=\frac{3dy}{dx}$

$(1-x^2)\frac{d^2y}{dx^2}-\frac{xdy}{dx}=3y$

$\therefore$ At $x=\frac{1}{2}$

$\boxed{3y=3e^{\frac{\pi}{2}}}$

Option (3) is correct

14. If A is 2×2 matrix such that $AB^{-1}=A^{-1}$ where $B=\begin{bmatrix}1 & 5\\ 3 & 1\end{bmatrix}$. If $C=BAB^{-1}$ and C satisfy $C^4+\beta C^2+\alpha I=0$ then $(2\beta-\alpha)$ is equal to

(1) 12

(2) 8

(3) 10

(4) 14

Answer (3)

Sol. $AB^{-1}=A^{-1}$ and $C=BAB^{-1}=BA^{-1}\Rightarrow BA^{-1}=A$

$C^4+\beta C^2+\alpha I=0$

$C^2=BA^{-1}\,BA^{-1}=A^2$

$\Rightarrow \quad A^2B^{-1}=I\Rightarrow A^2=B$

$A^2=B\Rightarrow B$ satisfy characteristic eq.

$(1-\lambda)(1-\lambda)-15=0\Rightarrow\lambda^2-2\lambda-14=0$

$B^2-2B-14I=0 \qquad \Rightarrow A^4-2A^2-14I=0$

$\Rightarrow C^4-2C^2-14I=0$

$\Rightarrow \beta=-2,\ \alpha=-14$

$\Rightarrow 2\beta-\alpha=-4+14=10$

15. If $\frac{1}{\alpha+1}+\frac{1}{\alpha+2}+\frac{1}{\alpha+3}+\ldots+\frac{1}{\alpha+1012}-\left(\frac{1}{2\times1}+\frac{1}{4\times3}+\frac{1}{6\times5}+\ldots+\frac{1}{2024}\times\frac{1}{2023}\right)=\frac{1}{2024}$, then α is equal to

(1) 2012

(2) 1012

(3) 1011

(4) 506

Answer (3)

Sol. $\sum_{r=1}^{1012}\frac{1}{(2r)(2r-1)}=\sum_{r=1}^{1012}\left(\frac{1}{(2r-1)}-\frac{1}{2r}\right)$

$=\left(1-\frac{1}{2}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\ldots.+\left(\frac{1}{2023}-\frac{1}{2024}\right)$

$$= \left(1+\frac{1}{3}+\frac{1}{5}+\ldots+\frac{1}{2023}\right)$$

$$-\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\ldots+\frac{1}{2024}\right)$$

$$= \left(1+\frac{1}{3}+\frac{1}{5}+\ldots+\frac{1}{2023}\right)-\frac{1}{2}$$

$$\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\ldots\frac{1}{1012}\right)$$

$$= \left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots+\frac{1}{2023}\right)-\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\ldots+\frac{1}{2022}\right)$$

$$-\frac{1}{2}\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}\ldots+\frac{1}{1012}\right)$$

$$= \left(1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{2023}\right)-\frac{1}{2}\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\ldots\frac{1}{1011}\right)$$

$$-\frac{1}{2}\left(1+\frac{1}{2}+\frac{1}{3}\ldots+\frac{1}{1012}\right)$$

$$= \frac{1}{1012}+\frac{1}{1013}+\ldots+\frac{1}{2023}-\frac{1}{2024}$$

$$\Rightarrow \quad \frac{1}{\alpha+1}+\frac{1}{\alpha+2}+\ldots+\frac{1}{\alpha+1012}=\frac{1}{2024}$$

$$+\left(\frac{1}{1012}+\frac{1}{1013}+\ldots\frac{1}{2023}\right)-\frac{1}{2024}$$

$$= \frac{1}{1012}+\ldots+\frac{1}{2023}$$

$\alpha + 1012 = 2023 \Rightarrow \alpha = 1011$

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Sum of infinite terms of a, ar, ar^2... and a^3r^3, a^3r^6, a^3r^9.... is 57 and 9747 respectively, then $a + 18r$ is

Answer (31)

Sol. a, ar, ar^2,...

$$\frac{a}{1-r}=57 \qquad ...(1)$$

a^3r^3, a^3r^6, a^3r^9 ....

$$\frac{a^3}{1-r^3}=9747 \quad ...(2)$$

Equation $\frac{(1)^3}{(2)}$ given,

$$\frac{\frac{a^3}{(1-r)^3}}{\frac{a^3}{1-r^3}}=\frac{57^3}{9747}$$

$$\Rightarrow \frac{1-r^3}{(1-r)^3}=19$$

$$\frac{(1-r)(1+r^2+r)}{(1-r)^3}=19 \quad (r\neq 1)$$

$1 + r^2 + r = 19 + 19r^2 - 38r$

$18r^2 - 39r + 18 = 0$

$\Rightarrow r=\frac{2}{3}$ and $\left(\frac{3}{2}\right)$ rejected

$\therefore r=\frac{2}{3}$ and $a = 19$

Now $a + 18r = 19 + 12 = 31$

22. The number of numbers between 100 to 1000 such that sum of their digits is 14, is

Answer (70.00)

Sol. Number in this range will be 3-digit number.

$N = \overline{abc}$ such that $a + b + c = 14$

Also, $a \geq 1, \quad a, b, c \in \{0, 1, 2, ...9\}$

Case I

All 3-digit same

$\Rightarrow 3a = 14$ not possible

Case II

Exactly 2 digit same:

$\Rightarrow 2a + c = 14$

$(a, c) \in \{(3, 8), (4, 6), (5, 4), (6, 2), (7, 0)\}$

$\Rightarrow \left(\frac{3!}{2!}\right)$ ways $\Rightarrow 5 \times 3 - 1$

$= 15 - 1 = 14$

Case III

All digits are distinct

$a + b + c = 14$

without losing generality $a > b > c$

$$(a, b, c) \in \begin{cases} (9, 5, 0), (9, 4, 1), (9, 3, 2) \\ (8, 6, 0), (8, 5, 1), (8, 4, 2) \\ (7, 6, 1), (7, 5, 2), (7, 4, 3) \\ (6, 5, 3) \end{cases}$$

$\Rightarrow 8 \times 3! + 2(3! - 2!) = 48 + 8 = 56$

$= 0 + 14 + 56 = 70$

23. Find the number of solutions of $3\sin^{-1}x + 2\cos^{-1}x = \frac{2\pi}{5}$.

Answer (0)

Sol. $\sin^{-1}x = \frac{2\pi}{5} - \pi = \frac{-3\pi}{5}$

$\frac{-3\pi}{5} < \frac{-\pi}{2}$

$\therefore$ No real solution

24. If $f(x) = 2(2 - p)x - (p^2 - 6p + 8)\cos 4x + 7$, then for what values of p, does $f(x)$ not have a vertical point?

Answer (4)

Sol. $f'(x) = 2(2 - p) + 4 \cdot \sin 4x(p - 2)(p - 4)$

$= (p - 2)((4\sin 4x)(p - 4) - 2), p \neq 2$

$4\sin 4x(p - 4) - 2 \neq 0$

$\Rightarrow \sin 4x(p - 4) \neq \frac{1}{2}$

$\sin 4x \neq \frac{1}{2(p - 4)}$

$\frac{1}{2(p - 4)} > 1$

$\frac{1}{2(p - 4)} - 1 > 0 \Rightarrow y \in \left(4, \frac{9}{2}\right)$

$\frac{1}{2(p - 4)} < -1 \Rightarrow p \in \left(\frac{7}{2}, 4\right)$

$\therefore \quad p \in \phi$

$\therefore \quad p = 4$ is the only required value

25.

26.

27.

28.

29.

30.