06-04-2024
Shift - 2

SMART ACHIEVERS
JEE | NEET | FOUNDATION

JEE Main - 2024 Session - 2
Answers & Solutions

(Physics, Chemistry and Mathematics)

06 - April - 2024 - Shift - 1

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PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. There are two fixed charged spheres *P* and *Q* repelling each other with force of 16 N. A third neutral sphere is placed between the charged spheres. The new force between spheres is (assuming all three spheres are insulating spheres)

   (1) 8 N

   (2) 32 N

   (3) 16 N

   (4) 4 N

Answer (3)

Sol. Electric force between two charges doesn't depend on intervening medium.

2. A tree branch holds a weight of 200 N by a uniform chain of mass 10 kg. The force applied by branch to hold this weight is (take $g = 10\ m/s^2$)

   (1) 150 N

   (2) 100 N

   (3) 200 N

   (4) 300 N

Answer (4)

Sol. $F = 200 + 100 = 300$ N

3. If kinetic energy of a block of mass *m* increases 36 times. By what percentage will the momentum increase?

   (1) 6%

   (2) 600%

   (3) 60%

   (4) 500%

Answer (4)

Sol. $k = \frac{P^2}{2m}$

$36k = \frac{P_1^2}{2m} \Rightarrow \boxed{P_1 = 6P}$ (Increased by 500%)

4. A ball is projected vertically upward from a building. Time taken to reach ground is T_1. Another ball is projected downward from the same building with same speed. Time taken to reach ground is T_2. If a third ball is released from the building, time taken to reach ground is

   (1) $\sqrt{T_1T_2}$ (2) $\sqrt{T_1^2 + T_2^2}$

   (3) $\sqrt{T_1^2 - T_2^2}$ (4) $2\sqrt{T_1T_2}$

Answer (1)

Sol. $-H = V_0T_1 - \frac{1}{2}gT_1^2$

$-H = -V_0T_2 - \frac{1}{2}gT_2^2$

$H = \frac{1}{2}g\left(\frac{T_1+T_2}{2}\right)^2 - \frac{1}{2}g\left(\frac{T_1-T_2}{2}\right)^2$

$T = \sqrt{\frac{2H}{g}}$

$\Rightarrow \quad T = \sqrt{T_1T_2}$

5. The weight of an object measured on the surface of earth is 300 N. What will be weight of the same object at depth $\frac{R}{4}$ inside the earth?

(R = radius of earth)

(1) 220 N

(2) 225 N

(3) 200 N

(4) 210 N

Answer (2)

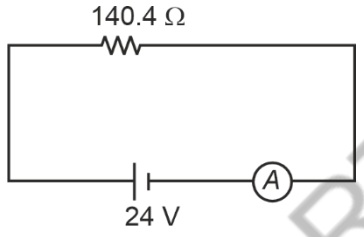
Sol. $W_1 = mg = 300$ N

$$W_2 = mg' = mg\left(1-\frac{d}{R}\right)$$

$$= mg\left(\frac{3}{4}\right)$$

= 225 N

6. An ammeter consists of 240 Ω galvanometer and 10 Ω shunt resistance is connected in circuit as shown. Reading of ammeter is



(1) 0.18 A (2) 0.16 A

(3) 0.32 A (4) 3.2 A

Answer (2)

Sol. $R_A = \frac{10\times 240}{250} = 9.6\ \Omega$

$$R = 140.4 + 9.6 = 150\ \Omega$$

$$i = \frac{24}{150} = 0.16$$

7. An isolated system contains one mole of helium, given a heat of 48 J. If the temperature of system changes by 2°C, then find work done. (take R = 8.35/mole-K)

(1) 32.20 J

(2) 37.34 J

(3) 40.74 J

(4) 41.74 J

Answer (4)

Sol. Q = 48 J

$$\Delta Q = \Delta V + \Delta W$$

$$48 = 1\times\frac{3R}{2}(2) + W$$

$$W = 48 - \frac{3}{4}(8.35)$$

= 41.74

8. Find the longest wavelength of Paschen series for hydrogen atom. (Rydberg constant = 10^7/m)

(1) 2.06 μm

(2) 20.6 μm

(3) 4.86 μm

(4) 48.6 μm

Answer (1)

Sol. $\frac{1}{\lambda} = R\left(\frac{1}{3^2}-\frac{1}{4^2}\right)$

$$\lambda = \frac{144}{7\times 10^7} = 20.57\times 10^{-7}$$

9. Find net kinetic energy (maximum possible) associated with 20 diatomic molecules (Here k_B is Boltzmann constant and T is absolute temperature of diatomic gas).

(1) 35 k_BT

(2) 70 k_BT

(3) 60 k_BT

(4) 30 k_BT

Answer (2)

Sol. $E_i = (5+2)\frac{1}{2}kT$

$= \frac{7}{2}kT$

$E_T = 20 \times \frac{7}{2}kT$

$= 70\ k_BT$

10. **Statement-I :** Dimensions of specific heat capacity is $[L^2T^{-2}K^{-1}]$

Statement-II : Dimensions of universal gas constant is $[ML^2T^{-1}K^{-1}]$

(1) Both statements are incorrect

(2) Both statements are correct

(3) Statement-I is correct but statement-II is incorrect

(4) Statement-I is incorrect but statement-II is correct

Answer (3)

Sol. $S = \frac{Q}{m\Delta T} = \frac{ML^2T^{-2}}{mK} = [L^2T^{-2}K^{-1}]$

$R = \frac{ML^2T^{-2}}{K} = [ML^2T^{-2}K^{-1}]$

11. The displacement (x) of a particle vary as $x^2 = 1 + t^2$ and acceleration is given function of x as x^{-n}, then find n.

(1) 1 (2) 3

(3) 4 (2) 2

Answer (2)

Sol. $x = \sqrt{1+t^2}$

$v = \frac{1}{2}(1+t^2)^{-\frac{1}{2}}(2t)$

$= \frac{t}{\sqrt{a+1+t^2}}$

$a = \frac{1}{x^3} = x^{-3}$

12.

13.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. A convex lens has a focal length of f = 20 cm, R_1 = 15 cm, R_2 = 30 cm. The refractive index of the lens is $\frac{x}{2}$. The value of x is _____ .

Answer (3)

Sol. $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$\Rightarrow \quad \frac{1}{20}=(\mu-1)\left(\frac{1}{15}+\frac{1}{30}\right)=(\mu-1)\frac{3}{30}$

$\Rightarrow \quad \mu-1=\frac{1}{2} \Rightarrow \mu=1.5$

22. For a device, power consumed = 110 W and voltage supplied is 220 V. The number of electrons that flow in 1 s is $\frac{x}{4}\times10^{17}$. Find x.

Answer (125)

Sol. $P = VI$

$\Rightarrow \quad I = 0.5$ A

$\Rightarrow$ Number of electrons

$= \frac{0.5\times1}{1.6\times10^{-19}}$

$= \frac{1000}{32}\times10^{17}$

$= 31.25 \times 10^{17}$

23. In a photoelectric setup, work function of the material is 2.13 eV, wavelength used in 300 nm. If hc = 1240 eV.nm, stopping potential for the set-up is _____ V.

Answer (2)

Sol. $\frac{hc}{\lambda}-\phi=eV_s$

$\Rightarrow$ (4.13 – 2.13) eV = eV_s

$\Rightarrow \quad V_s = 2$ Volts

24. A car of mass 800 kg is moving in a circular path of radius 300 m on a banked road with angle 30°. Coefficient of friction between the car and road is 0.2. Find the maximum safe speed (to the nearest integer in m/s) with which the car can travel.

$\left(\text{Take } \sqrt{3}=1.7\right)$

Answer (52)

Sol. $V_{max}=\sqrt{\frac{rg(\mu+\tan\theta)}{1-\mu\tan\theta}}=\sqrt{\frac{300\times10(0.2+\tan30)}{1-0.2\tan30}}$

$V_{max}=\sqrt{2680}=51.76$ m/s

25. Two sources produce, individually, intensities of I and $4I$ at a location. If they are coherent, then difference between I_{max} and I_{min} is nI. Find n.

Answer (8)

Sol. $I_{max}=\left(\sqrt{I_1}+\sqrt{I_2}\right)^2=9I$

$I_{min}=\left(\sqrt{I_1}-\sqrt{I_2}\right)^2=I$

$\Rightarrow \quad n = 8$

26. An object of mass 30 kg and relative density 5 is immersed inside water. The weight of the object inside water is 10x N. Find the value of x.

Answer (24)

Sol. $W = mg - V_{object} \times \rho_{water} \cdot g$

$W=300-\frac{m_{object}}{\rho_{object}}\cdot\rho_{water}\cdot g$

$W=300-\frac{30}{5}\times1\times10$

$\boxed{W=240\ N}$

27.

28.

29.

30.

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The molarity of NaCl solution is 3 M. Calculate the molality of the solution. [Given density of the solution = 1.25 g/mL]

   (1) 2.9 (2) 2.79

   (3) 1.85 (4) 3.85

Answer (2)

Sol. Given molarity of solution = 3 M means 3 moles of NaCl is present in 1000 mL of solution.

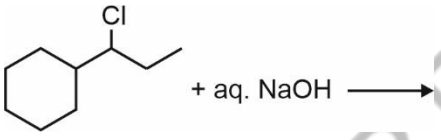
Mass of solution = d × v

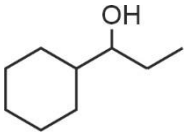
= 1.25 × 1000

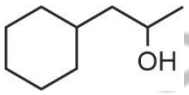
= 1250 g

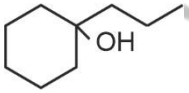
Mass of solute = 3 × 58.5 = 175.5 g

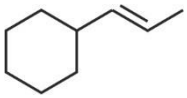
2. Identify the major product formed in the following reaction.



(1) 

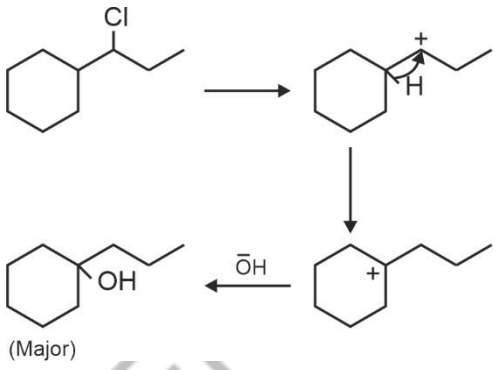
(2) 

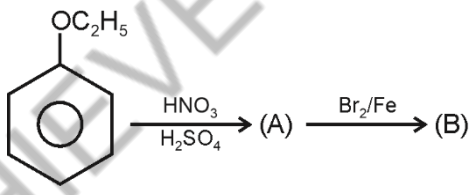
(3) 

(4) 

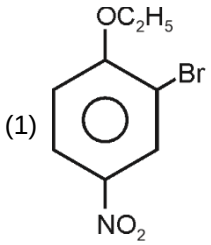
Answer (3)

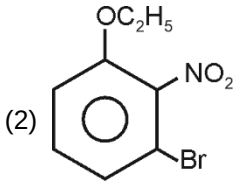
Sol.

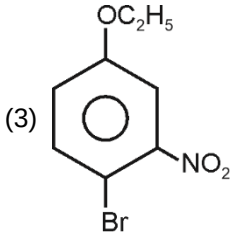


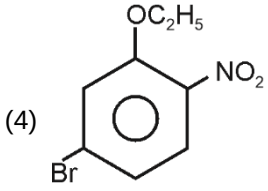
3. 

Product B is:

(1) 

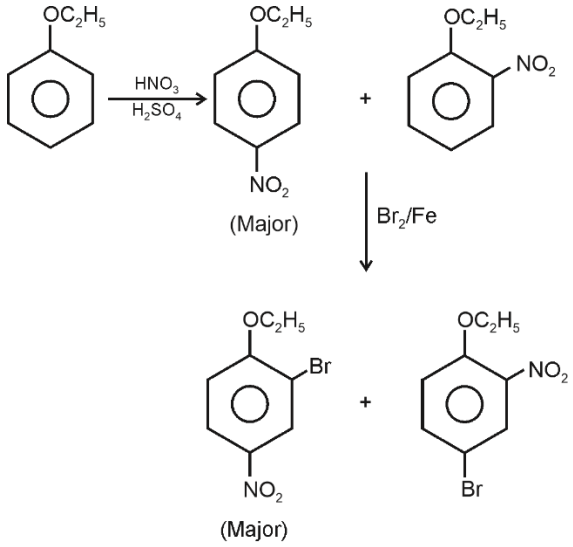
(2) 

(3) 

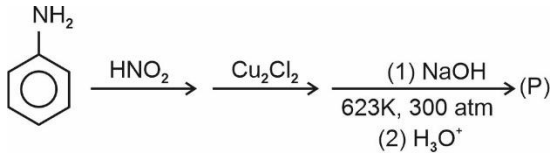
(4) 

Answer (A)

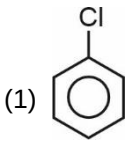
Sol.

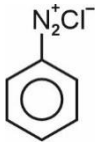


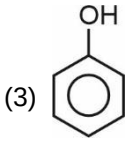
4.

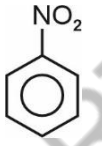


Product P is :

(1) 

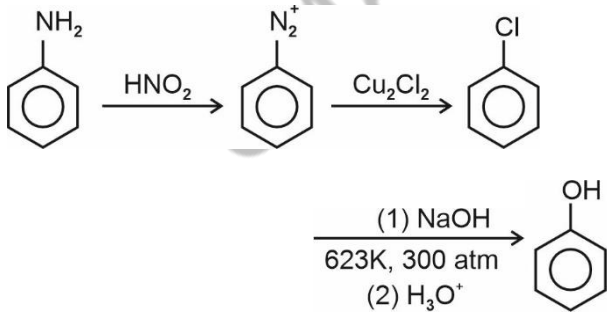
(2) 

(3) 

(4) 

Answer (3)

Sol.



5. Which of the following *d*-block elements has maximum unpaired electron in ground state electronic configuration?

(1) Ti(22) (2) V(23)

(3) Mn(25) (4) Cr(24)

Answer (4)

Sol. Ti(22) = [Ar] $3d^2 4s^2$ n = 2

V(23) = [Ar] $3d^3 4s^2$ n = 3

Mn(25) = [Ar] $3d^5 4s^2$ n = 5

Cr(24) = [Ar] $3d^5 4s^1$ n = 6

Cr(24) has maximum unpaired electron i.e., 6.

6. Find out shortest wavelength of Paschen series for H-atom.

(1) $\frac{9}{R}$ (2) $\frac{16}{R}$

(3) $\frac{144}{7R}$ (4) $\frac{7R}{144}$

Answer (1)

Sol. $\frac{1}{\lambda} = R(1)^2\left(\frac{1}{9} - \frac{1}{\infty}\right)$

$\frac{1}{\lambda} = R\left(\frac{1}{9}\right)$

$\boxed{\lambda = \frac{9}{R}}$

7. Match the column.

	Column-I (Compounds)		**Column-II (Configurations)**
A.	$TiCl_4$	(1)	$e^3t_2^3$
B.	FeO_4^{2-}	(2)	$e^2t_2^0$
C.	$FeCl_4^{2-}$	(3)	$e^2t_2^3$
D.	$MnCl_4^{2-}$	(4)	$e^0t_2^0$

(1) A(4), B(2), C(1), D(3)

(2) A(4), B(3), C(2), D(1)

(3) A(1), B(2), C(3), D(4)

(4) A(2), B(4), C(3), D(1)

Answer (1)

Sol. $TiCl_4 \Rightarrow Ti^{4+} \Rightarrow e^0t_2^0$

$FeO_4^{2-} \Rightarrow Fe^{6+} \Rightarrow e^2t_2^0$

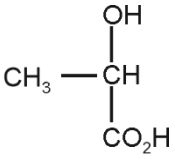
$FeCl_4^{2-} \Rightarrow Fe^{2+} \Rightarrow e^3t_2^3$

$MnCl_4^{2-} \Rightarrow Mn^{2+} \Rightarrow e^2t_2^3$

8. $CH_3COOH \xrightarrow{LiAlH_4} A \xrightarrow{PCC} B \xrightarrow[(ii)\ H^+]{(i)\ CN^-} C \xrightarrow{Hydrolysis} D$

Select the correct option

(1) A is CH_3CHO

(2) D is 

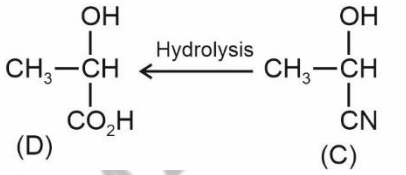
(3) B is CH_3CH_2OH

(4) C is $CH_3CH_2CO_2H$

Answer (2)

Sol. $CH_3COOH \xrightarrow{LiAlH_4} \underset{(A)}{CH_3CH_2OH} \xrightarrow{PCC} \underset{(B)}{CH_3CHO}$

$\xrightarrow[(ii)\ H^+]{(i)\ CN^-}$



9. Which of the following statement is incorrect-

(1) Enzymes are biocatalyst

(2) Enzymes are not specific

(3) Enzymes are globular protein

(4) Oxidase enzymes catalyse the oxidation of C-N and C-O bonds

Answer (2)

Sol. (1) Enzymes are biocatalyst that catalyse numerous biological process.

(2) They are very specific in nature

(3) Enzymes are mainly globular proteins

(4) Oxidase are enzymes specific to oxidation – reduction reactions involving oxidation of C-N and C-O bonds

10. Find relation between K_P and K_C for given reaction :

$$C(s) + \frac{1}{2}O_2(g) \longrightarrow CO(g)$$

(1) $K_P = K_C(RT)^1$

(2) $K_P = K_C(RT)^{-1}$

(3) $K_P = K_C(RT)^{1/2}$

(4) $K_P = K_C(RT)^{-1/2}$

Answer (3)

Sol. $K_P = K_C(RT)^{\Delta n_g}$

$$\Delta n_g = \frac{1}{2}$$

$K_P = K_C(RT)^{1/2}$

11. Given for two first order reactions $\frac{t^1_{1/2}}{t^2_{1/2}} = \frac{2}{5}$.

Then $\frac{t^1_{2/3}}{t^2_{4/5}} = ?$

(1) 0.273

(2) 0.468

(3) 0.318

(4) 2.55

Answer (1)

Sol. $\frac{K_2}{K_1} = \frac{2}{5}$

$$t^1_{2/3} = \frac{2.303}{K_1}\log 3$$

$$t^2_{4/5} = \frac{2.303}{K_2}\log 5$$

$$\frac{t^1_{2/3}}{t^2_{4/5}} = \frac{K_2}{K_1}\frac{\log 3}{\log 5}$$

$$= \frac{2}{5} \times \frac{0.477}{0.699}$$

$$= 0.273$$

12. Among the following anions, identify the anion which gives pale yellow precipitate with aq. $AgNO_3$. The precipitate is partially soluble in aq. NH_4OH solution.

(1) I^- (2) Cl^-

(3) Br^- (4) NO_2^-

Answer (3)

Sol. $I^- + Ag^+ \longrightarrow AgI$ (Yellow ppt.)

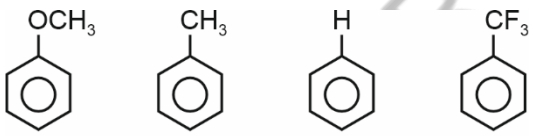
$Cl^- + Ag^+ \longrightarrow AgCl$ (White ppt.)

$Br^- + Ag^+ \longrightarrow AgBr$ (Pale yellow ppt.)

$NO_2^- + Ag^+ \longrightarrow AgNO_2$ (White ppt.)

AgBr is partially soluble in aq. NH_4OH solution whereas AgI is insoluble in aq. NH_4OH solution.

13. Arrange the following compounds in increasing order of electrophilic aromatic substitution.



(i) (ii) (iii) (iv)

(1) (iv) < (iii) < (ii) < (i)

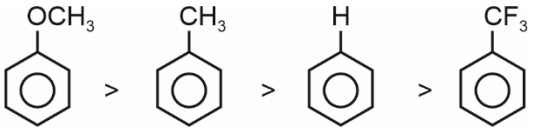
(2) (ii) < (iii) < (iv) < (i)

(3) (iv) < (ii) < (iii) < (i)

(4) (i) < (ii) < (iii) < (iv)

Answer (1)

Sol. Rate of EAS is



(i) is activated due to resonance and (ii) due to hyperconjugation, (iv) is deactivated due to reverse hyperconjugation.

14. IUPAC name of complex compound $[Pt(Br)_2(PPh_3)_2]$.

(1) Dibromido di(triphenyl phosphine) platinum(II)

(2) Dibromido bis(triphenyl phosphine) platinum(II)

(3) bis(triphenyl phosphine) dibromide platinum(II)

(4) bis(triphenyl phosphine) dibromide platinate(II)

Answer (2)

Sol. Dibromido bis(triphenyl phosphine) platinum(II) is the correct IUPAC name of given complex compound.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. For a certain reaction. Δ_rH is 400 kJ/mol and ΔS = 0.2 kJ/mol K. Above what minimum temperature in kelvin, the reaction become spontaneous

Answer (2000)

Sol. For reaction to be spontaneous,

$\Delta G < O$

$\Delta H - T\Delta S < O$

$$T > \frac{\Delta H}{\Delta S} = \frac{400}{0.2} = 2000\ K$$

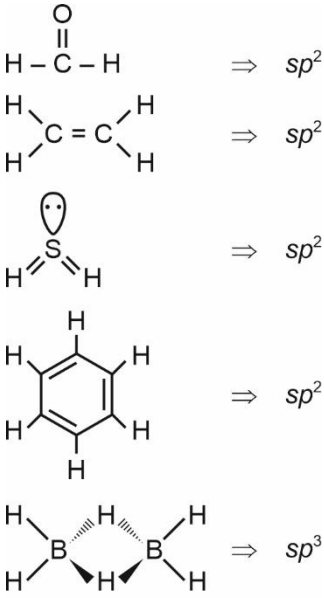
Minimum temperature for spontaneity = 2000 K

22. The number of compounds having central atom is sp^2 hybridised

HCHO, C_2H_4, SO_2, C_6H_6, B_2H_6, BF_3, $SiO_2(s)$, CH_2Cl_2

Answer (5)

Sol.



If steric number is 3, then hybridisation is sp^2.

23. Among the following, how many metal ions act as oxidising agents?

Sn^{2+}, Sn^{4+}, Pb^{4+}, Pb^{2+}, Tl^{+}, Tl^{3+}

Answer (2)

Sol. Due to inert pair effect, Pb^{2+} is more stable than Pb^{4+} and Tl^{+} is more stable than Tl^{3+}. Therefore, Pb^{4+} and Tl^{3+} only will act as oxidising agents

24. Calculate the magnetic moment in B.M. of the one from $VO_2^{\oplus}$, $MnO_4^{\ominus}$ and $Cr_2O_7^{2-}$ which is having least oxidizing property

Answer (0)

Sol. For 3-d transition series,

Oxidizing power $V^{+5} < Cr^{+6} < Mn^{-7}$

μ_{spin} of V^{+5} :

$V^{+5} \rightarrow [Ar]\ 4s^0 3d^0$

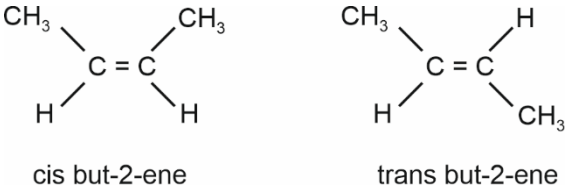
Number of unpaired $e^- = 0$

$\mu_{spin} = 0$

25. How many geometrical isomers are there in but-2-ene?

Answer (2)

Sol. But-2-ene has one stereogenic centre and it has two geometrical isomer as given below.



cis but-2-ene trans but-2-ene

26.

27.

28.

29.

30.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. If $\int \frac{dx}{a^2\sin^2 x + b^2\cos^2 x} = \frac{1}{12}\tan^{-1}(3\tan x) + c$

then the maximum value of $a\sin x + b\cos x$ is

(1) $\sqrt{10}$ (2) $\sqrt{20}$

(3) $2\sqrt{10}$ (4) $2\sqrt{5}$

Answer (3)

Sol. $I = \int \frac{\sec^2 x dx}{b^2 + a^2\tan^2 x}$

$\tan x = t \Rightarrow \sec^2 x dx = dt$

$I = \int \frac{dt}{b^2 + a^2t^2} = \frac{1}{ba}\tan^{-1}\left(\frac{at}{b}\right)$

$\Rightarrow I = \frac{1}{ab}\tan^{-1}\left(\frac{a}{b}\tan x\right) + c$

$\Rightarrow ab = 12 \Rightarrow a^2 = 36$

$\frac{a}{b} = 3 \Rightarrow b^2 = 4$

$\Rightarrow$ Maximum value of $a\sin x + b\cos x$ is

$\sqrt{a^2 + b^2} = \sqrt{40} = 2\sqrt{10}$

2. Range of $\frac{1}{7 - \sin 5x}$ equals to

(1) $\left[\frac{1}{7}, \frac{1}{5}\right]$ (2) $\left[\frac{1}{7}, \frac{1}{6}\right]$

(3) $\left[\frac{1}{8}, \frac{1}{5}\right]$ (4) $\left[\frac{1}{8}, \frac{1}{6}\right]$

Answer (4)

Sol. We know that,

$-1 \le \sin 5x \le 1$

$-1 \le -\sin 5x \le 1$

$6 \le 7 - \sin 5x \le 8$

$\frac{1}{8} \le \frac{1}{7 - \sin 5x} \le \frac{1}{6}$

$\therefore$ Range is $\left[\frac{1}{8}, \frac{1}{6}\right]$

3. There are letters to be delivered to 5 different location, then find the probability that letter is delivered to exactly 2 correct address assuming each letter is delivered to unique address.

(1) $\frac{1}{6}$ (2) $\frac{1}{5}$

(3) $\frac{1}{12}$ (4) $\frac{1}{4}$

Answer (3)

Sol. $\Rightarrow$ Select any two correct address

$\Rightarrow \left({}^5C_2\right)$

Remaining 3 have to be dearranged

$\Rightarrow 3!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!}\right)$

$= 6\left(1 - 1 + \frac{1}{2} - \frac{1}{6}\right) = 3 - 1 = 2$ ways

$\Rightarrow {}^5C_2 \cdot 2 = 10$

Probability $= \frac{10}{5!} = \frac{1}{12}$

4. The 315th word in dictionary arranged in order for the word 'NAGPUR' is

(1) NRAGPU (2) NRPGUA

(3) NPRGUA (4) NRAPGU

Answer (4)

Sol. Letters N,A,G,P,U,R.

Total words start with letter A

5! = 120

Words start with G

5! = 120

Words with N at first place and A at 2nd place

4! = 24

Words with N at first and G at 2nd place

4! = 24

Words with N at first and P at 2nd place

4! = 24

So total words 120 + 120 + (24)3

= 312

313th word = NRAGPU

314th word = NRAGUP

315th word = NRAPGU

So, 315th word = NRAPGU

5. Let $A = [1, 2, 3, 4, 5]$, m be the number of relation such as $4x \le 5y$ XRY and n be the minimum number of elements to be added from $A \times A$ to make symmetric relation. Then the value of $n + m$.

(1) 26 (2) 25

(3) 24 (4) 23

Answer (2)

Sol. $A = [1, 2, 3, 4, 5]$

XRY when $4x \le 5y$

So $R = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 2), (2, 3), (2, 4), (2, 5), (3, 3), (3, 4), (3, 5), (4, 4), (4, 5), (5, 4), (5, 5)\}$

$m = 16$

As $(1, 2) \in R$ then $(2, 1)$ is to be added

$(1, 3) \in R$ So $(3, 1)$ will be added

$(1, 4) \in R$ So $(4, 1) \in R$

$(1, 5) \in R$ So $(5, 1) \in R$

$(2, 3) \in R$ So $(3, 2) \in R$

$(2, 4) \in R$ So $(4, 2) \in R$

$(2, 5) \in R$ So $(5, 2) \in R$

$(3, 4) \in R$ So $(4, 3) \in R$

$(3, 5) \in R$ So $(5, 3) \in R$

to make R symmetric

So $n = 9$

$m + n = 25$

6. If the area bounded by the region (x, y) such that $\left\{(x, y)\left|\frac{a}{x^2} < y < \frac{1}{x}\right.\right.$ such that $1 < x < 2, 0 < a < 1\}$ is $\left(\ln 2 - \frac{2}{7}\right)$ sq. units then $(7a - 3)$ is equal to

(1) 0 (2) 1

(3) 2 (4) 4

Answer (2)

Sol. $\Rightarrow \int_1^2 \left(\frac{1}{x} - \frac{a}{x^2}\right) dx = \left(\ln|x| + \frac{a}{x}\right)_1^2$

$$\left(\ln 2 + \frac{a}{2}\right) - (\ln 1 + a) = \ln 2 - \frac{a}{2}$$

$$= \ln 2 - \frac{2}{7} = \ln 2 - \frac{a}{2}$$

$\Rightarrow a = \frac{4}{7}$

$\Rightarrow 7a - 3 = 1$

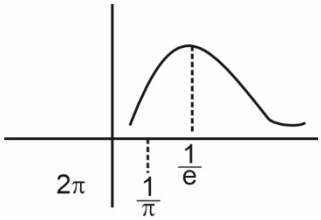
7. If the function $f(x) = \left(\frac{1}{x}\right)^{2x}$ $x > 0$, attains the maximum value of $x = \frac{1}{e}$, then

(1) $e^{\pi} < \pi^e$ (2) $e^{2\pi} < (2\pi)^e$

(3) $(2e)^{\pi} > (\pi)^{2e}$ (4) $e^{\pi} > \pi^e$

Answer (4)

Sol. $f\left(\frac{1}{\pi}\right) < f\left(\frac{1}{e}\right)$



$(\pi)^{\frac{2}{\pi}} < e^{\frac{2}{e}} \Rightarrow \pi^{2e} < e^{2\pi}$

$e^{\pi} > \pi^{e}$

8. If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \left(\left(\vec{a} \times \left(\hat{i} + \hat{j}\right) \times \hat{i}\right) \times \hat{i}\right)$ then the square of projection of $\vec{a}$ on $\vec{b}$ is

(1) $\frac{1}{3}$ (2) 2

(3) $\frac{1}{2}$ (4) $\frac{2}{5}$

Answer (4)

Sol. $\vec{a} \times \left(\hat{i} + \hat{j}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$

$= -\hat{i} + \hat{j} + 3\hat{k}$

$\left(\left(\vec{a} \times \left(\hat{i} + \hat{j}\right)\right) \times \hat{i}\right) = -\hat{k} + 3\hat{j}$

$\left(\left(\left(\vec{a} \times \left(\hat{i} + \hat{j}\right)\right) \times \hat{i}\right) \times \hat{i}\right) = -\hat{j} - 3\hat{k}\left(\vec{b}\right)$

$\therefore$ Projection of $\vec{a}$ on $\vec{b}$ $= \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

$= \frac{-2}{\sqrt{10}}$

Square of projection $= \frac{4}{10} = \frac{2}{5}$

9. $\lim\limits_{n\to\infty} \frac{\sum(n^4 - 2n^3 + n^2)}{\sum((3n)^4 + n^3 - n^2)}$ is equal to

(1) $\frac{1}{81}$ (2) $\frac{1}{72}$

(3) $\frac{1}{57}$ (4) $\frac{1}{93}$

Answer (1)

Sol. $\lim\limits_{n\to\infty} \frac{\sum(n^4 - 2n^3 + n^2)}{\sum((3n)^4 + n^3 - n^2)} = \lim\limits_{n\to\infty} \frac{\sum n^4}{\sum(3n)^4}$

(As Σn^2 will dominate and has highest powers of n)

$= \lim\limits_{n\to\infty} \frac{1}{34} \frac{\sum n^4}{\sum n^4}$

$= \frac{1}{81}$

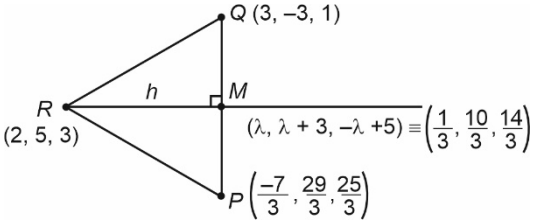
10. If (α,β,γ) is the mirror image of Q (3, –3, 1) in the line $\frac{x-0}{1} = \frac{y-3}{1} = \frac{z-5}{-1}$ and R (2, 5, 3). If the area of ΔPQR is λ, then $\frac{\lambda^2}{546}$ equals to

(1) $\frac{125}{81}$ (2) $\frac{25}{81}$

(3) $\frac{1}{81}$ (4) $\frac{5}{81}$

Answer (1)

Sol.



$(\lambda - 3) + 1(\lambda + 6) - 1(-\lambda + 4) = 0$

$3\lambda - 1 = 0 \Rightarrow \lambda = \frac{1}{3}$

Area of $\Delta PQR = 2 \times \frac{1}{2}\ (QM \cdot MR)$

$$(QM)(MR) = \left(\sqrt{\left(3-\frac{1}{3}\right)^2 + \left(-3\frac{-10}{3}\right)^2 + \left(1-\frac{14}{3}\right)^2}\right)$$

$$\left(\sqrt{\left(2-\frac{1}{3}\right)^2 + \left(5-\frac{10}{3}\right)^2 + \left(3-\frac{14}{3}\right)^2}\right)$$

$$(QM)(MR) = \frac{1}{9}\left(\sqrt{64+19^2+11^2}\right)\left(\sqrt{25+25+25}\right)$$

$$= \frac{\sqrt{546\times125}}{9}$$

$$= \frac{5}{9}\sqrt{546\times5}$$

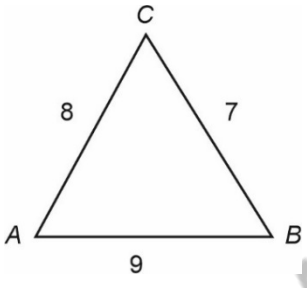
11. Sides of a triangle are $AB = 9$, $BC = 7$, $AC = 8$. Then $\cos 3C$ equals to

(1) $\frac{-262}{343}$ (2) $\frac{181}{247}$

(3) $\frac{81}{93}$ (4) $\frac{-283}{285}$

Answer (1)

Sol.



$$\cos C = \frac{8^2+7^2-9^2}{2\times8\times7} = \frac{32}{2\times8\times7} = \frac{2}{7}$$

$\cos 3C = 4\cos^3 C - 3\cos C$

$$= 4\times\frac{8}{343} - \frac{6}{7} = \frac{32-6\times49}{343}$$

$$= \frac{-262}{343}$$

12. The locus of P such that the ratio of distance P from $A(3, 1)$ and $B(1, 2)$ is 5 : 4 is

(1) $81x^2 - 92x + 81y^2 - 180y = 35$

(2) $81x^2 + 92x + 81y^2 - 19y = 35$

(3) $81x^2 - 48x + 81y^2 + 20y = 35$

(4) $81x^2 - 90x + 81y^2 - 180y = 35$

Answer (4)

Sol. Take point $P(x, y)$

$$\underset{(3,\ 1)}{5} \quad \underset{P}{\bullet} \quad \underset{(1,\ 2)}{4}$$

$$x = \frac{5+12}{9},\ y = \frac{10+4}{9}$$

$$P = \left(\frac{17}{9}, \frac{14}{9}\right) \text{ (internally)}$$

for externally division.

$$x = -\frac{7}{9},\ y = \frac{6}{9}$$

$$P' = \left(\frac{-7}{9}, \frac{6}{9}\right)$$

Locus of P is the circle whose diameter is PP'

$$\left(x-\frac{-17}{9}\right)\left(x+\frac{7}{9}\right) + \left(y-\frac{14}{9}\right)\left(y-\frac{6}{9}\right) = 0$$

$(9x - 17)(9x + 7) + (9y - 14)(9y - 6) = 0$

So $81x^2 - 90x + 81y^2 - 180y = 35$

13. If $\left|\frac{z_1 - 2z_2}{\frac{1}{2} - \bar{z}_1 z_2}\right| = 2$ then

(1) z_1 lie on circle with radius 1 and z_2 lie on circle with radius 2

(2) z_1 lie on circle with radius 1 and z_2 lie on circle with radius 1

(3) z_1 lie on circle with radius $\frac{1}{2}$ and z_2 lie on circle with radius 1

(4) z_1 lie on circle with radius 1 and z_2 lie on circle with radius $\frac{1}{2}$

Answer (4)

Sol. $|z_1 - 2z_2| = |1 - 2\bar{z}_1 z_2|$

$\Rightarrow (z_1 - 2z_2)(\bar{z}_1 - 2\bar{z}_2) = (1 - 2\bar{z}_1 z_2)(1 - 2z_1\bar{z}_2)$

$\Rightarrow |z_1|^2 + 4|z_2|^2 - 2z_1\bar{z}_2 - 2\bar{z}_1 z_2$

$= 1 - 2z_1\bar{z}_2 - 2\bar{z}_1 z_2 + 4|z_1|^2|z_2|^2$

$\Rightarrow |z_1|^2 + 4|z_2|^2 - 4|z_1|^2|z_2|^2 - 1 = 0$

$(|z_1|^2 - 1)(4|z_2|^2 - 1) = 0$

$\Rightarrow |z_1| = 1$ and $|z_2| = \frac{1}{2}$

14. If the orthocentre of triangle formed by (8, 3), (5, 1) and (h, k) is (6, 1), then (h, k) lie on

(1) $x^2 + y^2 = 64$

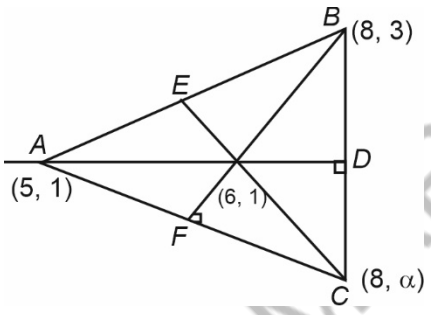
(2) $x^2 + y^2 = 68$

(3) $x^2 + y^2 = 65$

(4) $x^2 + y^2 = 71$

Answer (2)

Sol.

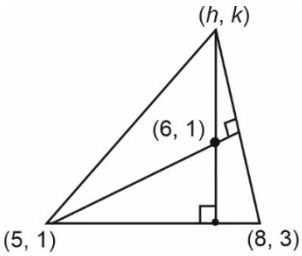


Slope of BF = 1

$\Rightarrow$ Slope of $AC \equiv \left(\frac{\alpha - 1}{8 - 5}\right) = -1$

$\Rightarrow \alpha - 1 = -3$

$\Rightarrow \alpha = -2$



(h, k) lie on $(y - 1) = \frac{-3}{2}(x - 6)$

$2y - 2 + 3x - 18 = 0$

$2y + 3x = 20$ ...(1)

(h, k) lies on circumcircle eg. of circumcircle is $x^2 + y^2 = 68$

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If α, β are the roots of the equation $x^2 - \sqrt{2}x - 8 = 0$ and $A_n = \alpha^n + \beta^n$, $n \in N$, then the value of $\frac{A_{10} - \sqrt{2}A_9}{2A_8}$

Answer (4)

Sol. $x^2 - \sqrt{2}x - 8 = 0$

$A_{10} - \sqrt{2} \cdot A_9 - 8A_8 = 0$

$\Rightarrow \frac{A_{10} - \sqrt{2} \cdot A_9}{A_8} = 8$

$\Rightarrow \frac{A_{10} - \sqrt{2} \cdot A_9}{2 \cdot A_8} = 4$

22. If ${}^{n+1}C_{r+1} : {}^{n}C_{r} : {}^{n-1}C_{r-1} = 55 : 35 : 21$

The value of $n + r$ is

Answer (16)

Sol. $\frac{n+1}{r+1} \times {}^{n}C_{r} : {}^{n}C_{r} : \frac{r}{n} {}^{n}C_{r} = 55 : 35 : 21$

$\Rightarrow \frac{n+1}{r+1} = \frac{55}{35}$ and $\frac{n}{r} = \frac{35}{21}$

$\Rightarrow \frac{n+1}{r+1} = \frac{11}{7}$ and $\frac{n}{r} = \frac{5}{3}$

$\Rightarrow$ $7n + 7 = 11r + 11$

$7n - 11r = 4$ … (1)

$3n - 5r = 0$ … (2)

Solving (1) and (2)

$r = 6$ and $n = 7$

$\Rightarrow$ $n + r = 10 + 6 = 16$

23. If the order of matrix A is 3 and $|A| = 3$ then the value of $\det(adj(-4adj\,(-3adj\,(2A^{-1}))))$ is $2^m . 3^n$. The value of $m + 2n$ =

Answer (44)

Sol. $|adj(-4adj(-3adj(2A^{-1})))|$

$= |-4adj(-3adj(2A^{-1}))|^2$

$=4^6 |-3adj(2A^{-1})|^4$

$=4^6.3^{12} |adj(2A^{-1})|^4$

$=4^6.3^{12} |2A^{-1}|^8$

$=4^6.3^{12}.2^{24} |A^{-1}|^8$

$=4^6.3^{12}.2^{24}. \frac{1}{|A|^8} = 3^{12}.\frac{2^{36}}{3^8}$

$= 3^4 . 2^{36}$

$m = 36$ $n = 4 \Rightarrow m + 2n = 36 + 8 = 44$

24. If $\int_0^3 \left([x^2] + \left[\frac{x^2}{2} \right] \right) dx$

$= a + b\sqrt{2} + c\sqrt{6} - \sqrt{3} - \sqrt{5} - \sqrt{7}$ $(a, b, c \in I)$ then $(a + b + c)$ equals

Answer (23.00)

Sol. $\int_0^3 \left([x^2] + \left[\frac{x^2}{2} \right] \right) dx = \int_0^1 0\,dx + \int_1^{\sqrt{2}} 1\,dx + \int_{\sqrt{2}}^{\sqrt{3}} 3\,dx +$

$\int_{\sqrt{3}}^{2} 4\,dx + \int_2^{\sqrt{5}} 6\,dx + \int_{\sqrt{5}}^{\sqrt{6}} 7\,dx + \int_{\sqrt{6}}^{\sqrt{7}} 9\,dx + \int_{\sqrt{7}}^{\sqrt{8}} 10\,dx + \int_{\sqrt{8}}^{3} 12\,dx$

$= 31 - 6\sqrt{2} - \sqrt{3} - \sqrt{5} - \sqrt{7} - 2\sqrt{6}$

$\Rightarrow$ $a = 31, b = -6, c = -2$

$\Rightarrow$ $a + b + c = 23$

25.

26.

27.

28.

29.

30.