SMART ACHIEVERS
JEE | NEET | FOUNDATION

JEE Main - 2024 Session-2

Answers & Solutions

(Physics, Chemistry and Mathematics)

04 - April - 2024 - Shift - 2

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PHYSICS

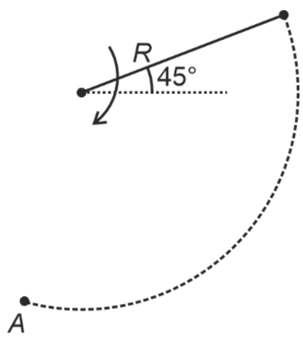
SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. A massless rod has a point mass attached at one end while the other end is hinged. The rod is released from the position shown. The speed of the mass at bottom-most point is

   (R = 14 m, g = 10 m/s²)



(1) $\sqrt{560}$ m/s (2) $\sqrt{280\left(1+\frac{1}{\sqrt{2}}\right)}$ m/s

(3) $\sqrt{280}$ m/s (4) $\sqrt{280\left(1+\frac{1}{\sqrt{3}}\right)}$ m/s

Answer (2)

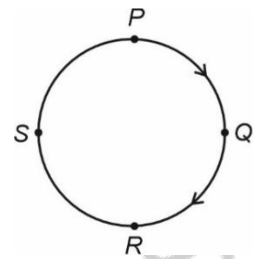
Sol. Conserving energy,

$$v=\sqrt{2g\left[R+R\sin 45^\circ\right]}$$

$$=\sqrt{20\times 14\left(1+\frac{1}{\sqrt{2}}\right)}$$

$$=\sqrt{280\left(1+\frac{1}{\sqrt{2}}\right)}\text{ m/s}$$

2. P, Q, R, S are 4 symmetric points on a horizontal circle of radius 4 km. What is displacement when a car moves from P to R along the given circular path?



(1) $4\sqrt{2}$ km (2) 4π km

(3) 8 km (4) 4 km

Answer (3)

Sol. $PR = 2r$

3. One mole of an monoatomic ideal gas compressed adiabatically from volume 2 V to V. If initial temperature of gas was T then magnitude work done in this process is

(1) $\frac{3}{2}RT\left(2^{\frac{1}{2}}-1\right)$ (2) $\frac{3}{2}RT\left(2^{\frac{2}{3}}-1\right)$

(3) $\frac{2}{3}RT\left(2^{\frac{2}{3}}-1\right)$ (4) $\frac{2}{3}RT\left(\sqrt{2}-1\right)$

Answer (2)

Sol. $W=-\dfrac{nR\Delta T}{P-1}$

$\Rightarrow\ T_i=T$

$T_f=T(2)^{\frac{2}{3}}$

$\Delta T=T\left(2^{\frac{2}{3}}-1\right)$

$$W=\frac{RT\left(2^{\frac{2}{3}}-1\right)}{\frac{5}{3}-1}$$

$$=\frac{3}{2}RT\left(2^{\frac{2}{3}}-1\right)$$

4. A 2 kg brick is placed on an inclined plane of inclination 45°. The brick is at rest. The minimum co-efficient of static friction is

(1) 0.5 (2) $\sqrt{3}$

(3) 1 (4) $\frac{1}{\sqrt{3}}$

Answer (3)

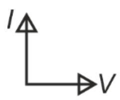
Sol. $N = mg\cos45°$

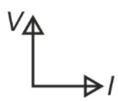
$f_s = mg\sin45°$

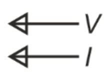
$\Rightarrow \quad mg\sin45° \le \mu mg\cos45°$

$\Rightarrow \quad \mu \ge 1.$

5. Correct match for phasors of voltage and current for given elements is

(a) Inductive (p) 

(b) Capacitive (q) 

(c) Resistive (r) 

(1) (a) $\rightarrow$ (p), (b) $\rightarrow$ (q), (c) $\rightarrow$ (r)

(2) (a) $\rightarrow$ (q), (b) $\rightarrow$ (p), (c) $\rightarrow$ (r)

(3) (a) $\rightarrow$ (p), (b) $\rightarrow$ (p), (c) $\rightarrow$ (r)

(4) (a) $\rightarrow$ (q), (b) $\rightarrow$ (q), (c) $\rightarrow$ (r)

Answer (2)

6. With regard to gravitation parameters, the dimensions of T^2 are same as that of

(1) $\frac{r^3}{GM}$ (2) $\frac{GM}{r^3}$

(3) $\frac{r^{3/2}}{GM}$ (4) $\frac{r^2}{GM}$

Answer (1)

Sol. $T^2 = \frac{4\pi^2}{GM} r^3$

7. A point charge q is kept at the centre of the one of the surface of a cube. Flux linked with cube is

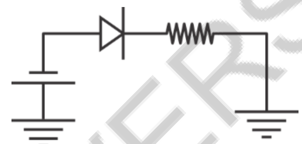
(1) $\frac{q}{\varepsilon_0}$ (2) $\frac{q}{8\varepsilon_0}$

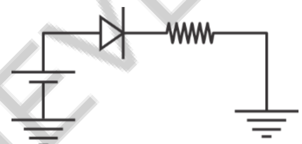
(3) $\frac{q}{2\varepsilon_0}$ (4) $\frac{q}{4\varepsilon_0}$

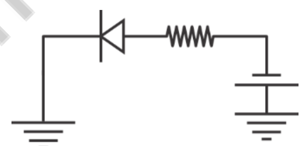
Answer (3)

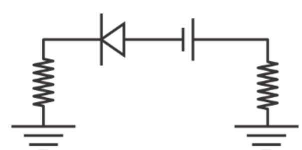
Sol. $\phi = \frac{1}{2}\frac{Q_{in}}{\varepsilon_0} = \frac{q}{2\varepsilon_0}$

8. Which of the following circuits would have the diode in conducting state?

(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. For conducting state :

$V_p > V_n$.

9. A heater of rating of 50 W – 200 V is connected with source voltage of 100 V. Power consumed by heater is

(1) 100 W (2) 25 W

(3) 50 W (4) 12.5 W

Answer (4)

Sol. $R = \dfrac{V_r^2}{P_r} = \dfrac{200 \times 200}{50} = 800\ \Omega$

$P = \dfrac{V^2}{R} = \dfrac{100 \times 100}{800} = 12.5\ \text{W}$

10. Wavelengths assigned to gamma rays, infra-red rays, UV rays and microwaves are λ_1, λ_2, λ_3 & λ_4 respectively. Then :

(1) $\lambda_1 < \lambda_2 < \lambda_3 < \lambda_4$ (2) $\lambda_1 < \lambda_3 < \lambda_2 < \lambda_4$

(3) $\lambda_1 > \lambda_2 > \lambda_3 > \lambda_4$ (4) $\lambda_2 < \lambda_3 < \lambda_1 < \lambda_4$

Answer (2)

Sol. ← Gamma Ray UV Infra Micro
Increasing Energy

11. The width of the one slit in YDSE is four times the other slit. Then ratio of maximum to the minimum intensity at screen is

(1) 9 : 1 (2) 16 : 1

(3) 4 : 1 (4) 1 : 1

Answer (1)

Sol. $I_1 = I_0$

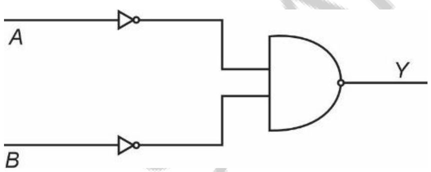
$I_2 = 4I_0$

$I_{max} = \left[\sqrt{I_0} + \sqrt{4I_0}\right]^2$

$= 9I_0$

$I_{min} = I_0$

12. The circuit diagram shown is equivalent to



(1) OR (2) NOR

(3) AND (4) NAND

Answer (1)

Sol. $Y = \overline{\overline{A} \cdot \overline{B}} = A + B$

13. **Statement 1 :** In photoelectric effect, number of photoelectrons emitted are proportional to frequency of incident light.

Statement 2 : Maximum kinetic energy of photoelectrons is proportional to frequency of incident light.

(1) Statement 1 is true and Statement 2 is true and correct explanation of 1

(2) Statement 1 is true and Statement 2 is true and not correct explanation of 1

(3) Statement 1 is true and Statement 2 is false

(4) Statement 1 is false and Statement 2 is true

Answer (4)

Sol. $h\nu = h\nu_0 + \text{KE}$

$\nu \uparrow = \text{KE} \uparrow$

14. A metallic rod of length 4 m is rotating about perpendicular bisector of the rod with angular velocity of 2 rad/s in presence of transverse magnetic field of 0.5 T. Potential difference developed across ends of rod is

(1) 16 V

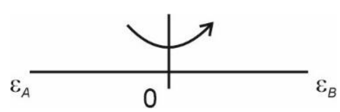
(2) 8 V

(3) 0 V

(4) 32 V

Answer (3)

Sol.



$\varepsilon_A = \varepsilon_B$

$\Delta V_{AB} = 0$

15. **Assertion (A) :** The contact angle depends on material of solid and liquid.

Reason (R) : Height of the liquid in a capillary tube is independent of the radius of the tube.

(1) Both (A) and (R) are true and (R) is the correct explanation of (A)

(2) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(3) (A) is true but (R) is false

(4) (A) is false but (R) is true

Answer (3)

Sol. Contact angle is dependent on materials.

Also, $h = \frac{2s\cos\theta}{\rho g r}$

$\Rightarrow$ h depends on r.

16. A ray of light is incident (just close to) at critical angle on slab of thickness $\frac{4}{\sqrt{3}}$ cm. Refractive index of slab is $\sqrt{12}$. The lateral displacement of ray when it emerges from air is

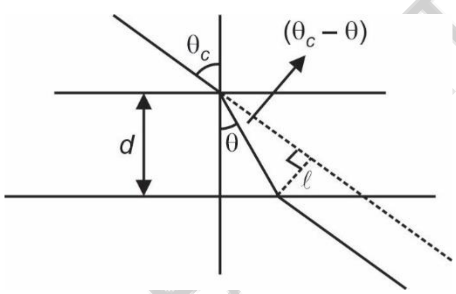
(1) $2\left(1+\frac{\sqrt{11}}{\sqrt{143}}\right)$ cm

(2) $2\left(1-\frac{\sqrt{11}}{\sqrt{143}}\right)$ cm

(3) $\left(1+\frac{\sqrt{11}}{\sqrt{143}}\right)$ cm

(4) $4\left(1-\frac{\sqrt{11}}{\sqrt{143}}\right)$ cm

Answer (2)



Sol. $\Rightarrow$ $\ell = (d\sec\theta)\sin(\theta_c - \theta)$

$\ell = (d\sec\theta) = \sin\theta_c\cos\theta - d\sec\theta\cos\theta_c\sin\theta$

$= d\sin\theta_c - d\tan\theta\cos\theta_c$

$\Rightarrow$ $\sin\theta_c = \mu\sin\theta$

$$\frac{1}{\sqrt{12}} = \sqrt{12}\sin\theta$$

$$\sin\theta = \frac{1}{12} \quad \cos\theta = \frac{\sqrt{143}}{12}$$

$$\text{and } \sin\theta_c = \frac{1}{\sqrt{12}}$$

$$\cos\theta_c = \frac{\sqrt{11}}{\sqrt{12}}$$

$$\ell = 4\sqrt{3}\times\frac{1}{\sqrt{12}} - 4\sqrt{3}\times\frac{1}{\sqrt{143}}\frac{\sqrt{11}}{\sqrt{12}}$$

$$= 2 - \frac{2\sqrt{11}}{\sqrt{143}}$$

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Two point mass m and $2m$ are on straight line. If mass m moves toward centre of mass by distance 2 cm, then the distance must mass $2m$ should move so that centre of mass does not change _______ cm.

Answer (1)

Sol. $m(2m) = 2m(x)$

$x = 1$ cm

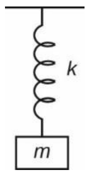
22. A body of mass 4 kg is at a height of R (radius of earth) from surface of earth. The weight of the body is _______ N.

Answer (10)

Sol. $g' = \frac{g}{4} = \frac{5}{2}$ m/s^2

$\Rightarrow$ Weight = mg' = 10 N

23. A mass m is in equilibrium (which is connected with a light spring as shown) and energy associated is E. Instead, if these had been mass of $2m$ then in equilibrium energy associated is E', then $\frac{E'}{E}$ is __________ .



Answer (4)

Sol. $\frac{1}{2}kx^2 - m\varepsilon x = E$

$x = \frac{m\varepsilon}{k}$

$\frac{1}{2}k\frac{m^2\varepsilon^2}{k^2} - m\varepsilon\frac{m\varepsilon}{k} = -\frac{m^2\varepsilon^2}{2k} = \varepsilon$

$\varepsilon \propto m^2$

24. A bar magnet of magnetic moment $M = 0.5\text{A m}^2$ is under the influence of a magnetic field 8 T. Find the work done (J) to move the magnet from stable to unstable equilibrium position.

Answer (8)

Sol. $W = \Delta U$

$\Rightarrow \quad W = 2 \times M \times B$

$= 8$ J

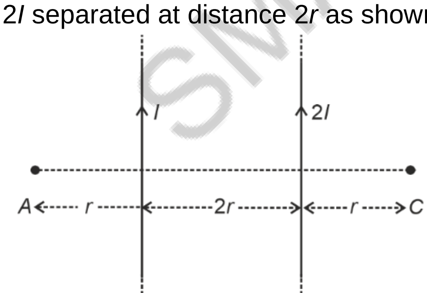
25. For methane, translation degrees of freedom is f_1 while rotational degrees of freedom is f_2. Find $f_1 + f_2$.

Answer (6)

Sol. $f_1 = 3$

$f_2 = 3$ [$\because$ Non-linear]

26. Two infinite straight conductor currying current I and $2I$ separated at distance $2r$ as shown in figure.



The ratio of magnetic field at point A to that of point C is $\frac{x}{7}$, then find x.

Answer (5)

Sol. $B_A = \frac{\mu_0 I}{2\pi r} + \frac{\mu_0(2I)}{2\pi(3r)} = \frac{\mu_0 I}{2\pi r} \times \frac{5}{3}$

$B_C = \frac{\mu_0(2I)}{2\pi r} + \frac{\mu_0 I}{2\pi(3r)} = \frac{\mu_0 I}{2\pi r} \times \frac{7}{3}$

$\frac{B_A}{B_C} = \frac{5}{7}$

27. The position of particle oscillation on x-axis is given as $x = 10\sin\left(\omega t + \frac{\pi}{3}\right)$. If time period of oscillation is 3.14 second, then displacement of particle at $t = 0$ is given as $n\sqrt{3}$ metre, then n is ______

Answer (5)

Sol. $T = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi}{T} = 2$ rad/sec.

then $x = 10\sin\left(2t + \frac{\pi}{3}\right)$

at $t = 0$

$x = 10\sin\left(\frac{\pi}{3}\right)$

$= 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}$

28. Two wires A and B of same length and same material ae having radius of cross sections of 2 mm and 4 mm respectively. If resistance of wire B is 2 Ω then resistance of wire A is ______ Ω.

Answer (8)

Sol. $R = \rho\frac{l}{A} = \frac{C}{r^2}$

$\frac{R_A}{R_B} = \frac{4^2}{2^2}$ $R_A = 4 \times 2 = 8$ Ω.

29.

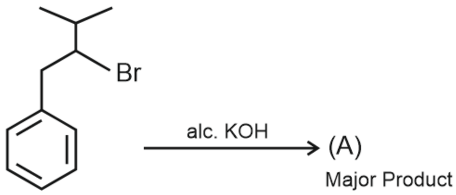
30.

CHEMISTRY

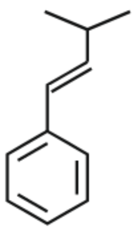
SECTION - A

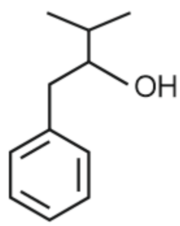
Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

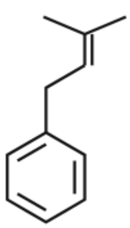
Choose the correct answer :

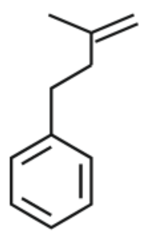
1. 

. A is

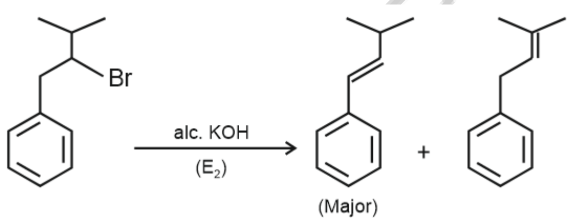
(1) 

(2) 

(3) 

(4) 

Answer (1)

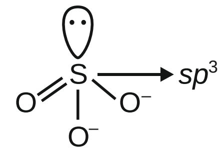
Sol. 

2. Which one of the following has pyramidal shape?

(1) $S_2O_3^{2-}$ (2) SO_4^{2-}

(3) SO_3^{2-} (4) $S_2O_7^{2-}$

Answer (3)

Sol. 

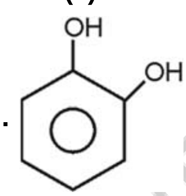
Pyramidal shape

3. IUPAC name of Catechol is

(1) Benzene, 1,2-diol (2) Benzene, 1,3-diol

(3) Benzene, 1,4-diol (4) 3-Hydroxyphenol

Answer (1)

Sol. 

Benzene, 1,2-diol

4. Which one of the following has the most negative (highest –ve) electron gain enthalpy?

(1) Li (2) Na

(3) F (4) Cl

Answer (4)

Sol. Chlorine = –349 kJ mol^{-1}

Sodium = –53 kJ mol^{-1}

Fluorine = –328 kJ mol^{-1}

Lithium = –60 kJ mol^{-1}

5. Consider the following statements.

Statement I: The number of emitted photoelectrons increases with increase in frequency of incident light.

Statement II: Kinetic energy of emitted photoelectrons increases with increase in frequency of incident light.

(1) Statement I is true but Statement II is false

(2) Statement I is false but Statement II is true

(3) Both statement I and Statement II are true

(4) Both statement I and Statement II are false

Answer (2)

Sol. The number of emitted photoelectrons independent of the frequency of incident light but kinetic energy of emitted photoelectrons increases with increase in frequency of incident light.

6. Arrange the following in increasing order of first ionization enthalpy: Al, Ga, In, Tl, B

(1) Tl < In < Ga < Al < B (2) In < Al < Ga < Tl < B

(3) In < Ga < Al < B < Tl (4) B < Al < Ga < In < Tl

Answer (2)

Sol. Due to poor shielding by electrons in *d*-subshell of Ga and *f*-subshell of Tl, their ionization energy increases than the expected value. So, correct order of IE-

In < Al < Ga < Tl < B

7. Find out number of unpaired electrons in *d*-subshell for $[Co(H_2O)_6]^{3+}$.

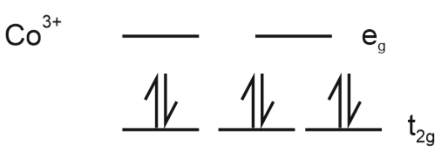
(1) 3 (2) 4

(3) 0 (4) 2

Answer (3)

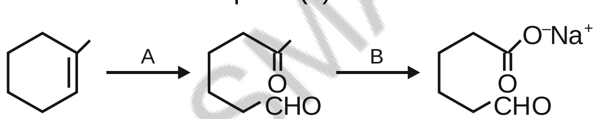
Sol. Co^{3+} : $3d^6 4s^0$

Co^{3+} in excited state will undergo pairing with H_2O.



$\boxed{n = 0}$

Correct answer is option (3).

8. 

What are A and B respectively?

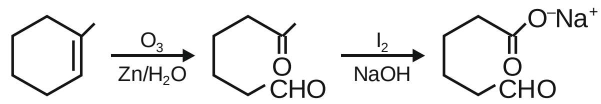
(1) (A) O_3/Zn-H_2O; (B) H_3O^+

(2) (A) O_3/H_2O; (B) I_2/NaOH

(3) (A) O_2/Zn-H_2O; (B) I_2/NaOH

(4) (A) $KMnO_4/H^+$, Δ; (B) $LiAlH_4$

Answer (3)

Sol. 

9. Which of the following statement is INCORRECT

(1) In homogeneous mixture, Composition is uniform

(2) Compounds are formed when atoms of different elements combine together in any ratio

(3) Atoms of same element have identical atomic mass and properties

(4) In heterogeneous mixture, Composition is not uniform

Answer (2)

Sol. Compounds are formed when atoms of different elements combine together in fixed ratio

10. Match the column I and column II.

	Column I		Column II
(i)	α-Glucose and α-Galactose	(a)	Homologues
(ii)	α-Glucose and α-Fructose	(b)	Epimer
(iii)	α-Glucose and β-Glucose	(c)	Anomer
(iv)	α-Ribose and α-Glucose	(d)	Functional isomers

Select the option with correct match.

(1) (i) → (b); (ii) → (d); (iii) → (a), (iv) → (c)

(2) (i) → (b); (ii) → (d); (iii) → (c), (iv) → (a)

(3) (i) → (d); (ii) → (b); (iii) → (c), (iv) → (a)

(4) (i) → (a); (ii) → (c); (iii) → (d), (iv) → (b)

Answer (2)

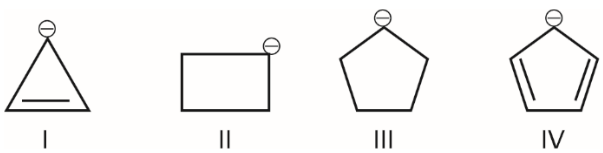
Sol. α-Glucose and α-Galactose are C–4 epimers

α-Glucose is Aldohexose and α-Fructose is ketohexose hence functional isomers

α-Glucose and β-Glucose are different in configuration at C–1 i.e. Anomeric carbon hence are anomers

α-Ribose is pentose while α-Glucose is hexose hence homologues

11. Arrange the following anions in the decreasing order of their stability.



(1) I > II > III > IV

(2) IV > III > II > I

(3) III > II > I > IV

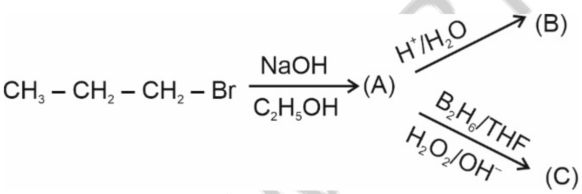
(4) II > IV > III > I

Answer (2)

Sol. Cyclopentadienyl anion (IV) is most stable and cyclopropenyl anion (I) is least stable as (IV) is aromatic and (I) is antiaromatic. Anion (II) is less stable than (III) due to higher angle strain.

$\therefore$ Correct stability order is

IV > III > II > I

12. 

What are B and C respectively?

(1) Propan-1-ol and propan-2-ol

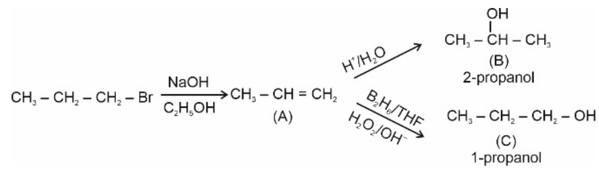
(2) Propan-2-ol and propan-1-ol

(3) Both are propan-1-ol

(4) Both are propan-2-ol

Answer (2)

Sol.



13. Spin only magnetic moment of V_2O_5 (in BM)

(1) 0

(2) 1

(3) 2

(4) 3

Answer (1)

Sol. $V_2O_5 \Rightarrow V^{+5}$

$V^{+5} = [Ar]3d^0 4s^0$

There is no unpaired electron in V^{+5}

So spin only magnetic moment is zero.

14. $KMnO_4$ + conc. H_2SO_4 $\xrightarrow{\text{Salt (X)}}$ Greenish yellow gas is produced

Salt (X) contains

(1) F^-

(2) Cl^-

(3) Br^-

(4) I^-

Answer (2)

Sol. The reaction/oxidation of F^- is not possible by the chemical reagent $KMnO_4/H_2SO_4$.

The oxidation of other halides produces dihalogen.

Cl_2 : Greenish yellow

Br_2 : Red

I_2 : Violet

Hence that salt contains Cl^-.

15. Which of the following represents correct unit of slope of graph between molar conductivity ($\wedge_m$) and (conc)$^{1/2}$:

(1) S cm$^{1/2}$ mol$^{-1/2}$

(2) S cm$^{3/2}$ mol^{-2}

(3) S cm$^{7/2}$ mol$^{-3/2}$

(4) S cm$^{5/2}$ mol$^{-3/2}$

Answer (3)

Sol. Debye-Hückel-Onsager equation

$$\wedge_m = \wedge_m^o - A\sqrt{C}$$

Slope of $\wedge_m$ vs $\sqrt{C}$ = –A

Unit of slope = $\dfrac{\text{Unit of } \wedge_m}{\text{Unit of } \sqrt{C}} = \dfrac{\text{S cm}^2\text{mol}^{-1}}{(\text{mol cm}^{-3})^{1/2}}$

S $cm^{7/2}$ $mol^{-3/2}$

16. Which of the following is used as adsorbent in adsorption chromatography?

(1) Silica gel

(2) Alumina

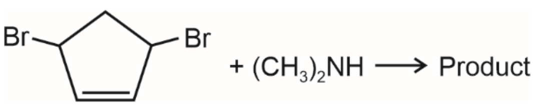
(3) Benzene

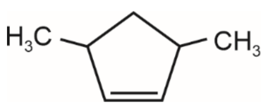
(4) Both (1) and (2)

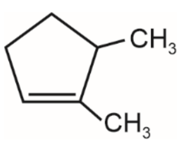
Answer (4)

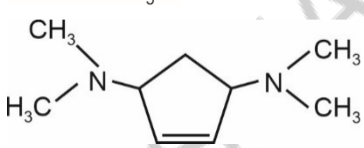
Sol. Commonly used adsorbents are silica gel and alumina.

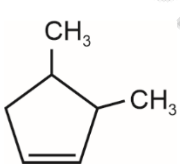
17. Identify the correct product formed in the following reaction.



(1) 

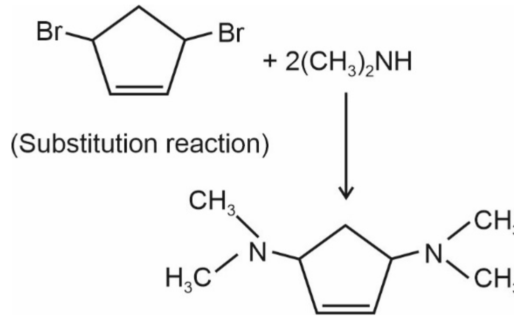
(2) 

(3) 

(4) 

Answer (3)

Sol.



18.

19.

20.

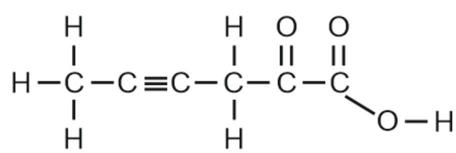
SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. What is the sum of number of σ and π bonds present in 2-oxo-hex-4-yn-oic acid?

Answer (18)

Sol.



Number of σ bonds : 14

Number of π bonds = 4

Total $\sigma + \pi$ bonds = 14 + 4 = 18

22. Find out magnitude of heat (q) for an isothermal irreversible expansion against external pressure of 8 bar if volume increases by 10 L (in joule).

Answer (8000)

Sol. $W = -P_{ext}(\Delta V)$

$= -8 \times 10^5\ N/m^2 \times (10 \times 10^{-3}\ m^3)$

$= -8 \times 10^5 \times 10^{-2}$ joule

$= -8 \times 10^3$

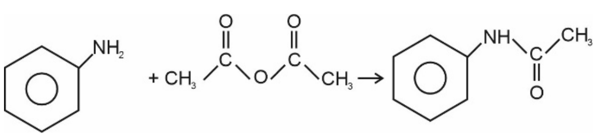
$= -8000$ J

$q + W = \Delta E$

$q + W = 0 \Rightarrow q = -W = +8000$ J

23. What is the maximum amount of acetanilide formed when acetic anhydride in excess is treated with 18 gm of aniline. (nearest integer)

Answer (26)

Sol. 

18 gm.

moles of aniline $= \frac{18}{93}$

mass of acetanilide formed $= \frac{18}{93} \times 135$

$= 26.129$

$= 26$

24. We have a complex of Fe^{3+} ion having electronic configuration according to crystal field theory is $t_{2g}^5 e_g^{\circ}$. If complex is $[Fe(NH_3)_x(CN)_y]$, then value of (x + y) is ____

Answer (6)

Sol. Given electronic configuration of Fe^{3+} ion in complex = $t_{2g}^5 e_g^{\circ}$ then complex should be $[Fe(NH_3)_3(CN)_3]$

x = 3, y = 3

x + y = 6

25. Consider the following reaction at equilibrium at a certain temperature T Kelvin whose $K_c = 3 \times 10^{-13}$

$$SO_2(g) + \frac{1}{2}O_2(g) \rightleftharpoons SO_3(g)$$

The value of K_c' for the following reaction is a × 10^{+b} (Scientific notation). Find the value of (a + b).

$$2SO_3(g) \rightleftharpoons 2SO_2(g) + O_2(g)$$

Answer (26)

Sol. $SO_2(g) + \frac{1}{2}O_2(g) \rightleftharpoons SO_3(g) \quad K_c = 3 \times 10^{-13}$

The equilibrium constant (K'_c) for the following reaction

$$2SO_3(g) \rightleftharpoons 2SO_2(g) + O_2(g) \qquad K'_c = \left(\frac{1}{K_c}\right)^2$$

$$K'_c = \left(\frac{1}{3 \times 10^{-13}}\right)^2 = 1.11 \times 10^{25}$$

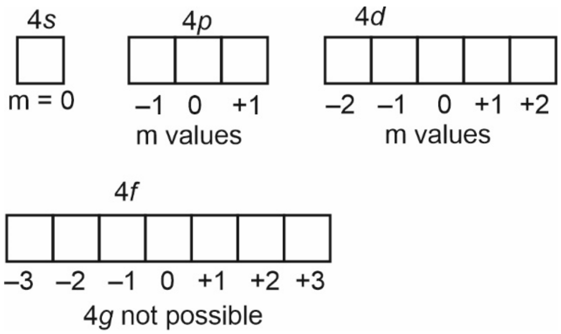
a = 1.11 and b = 25

a + b = 26.11

26. Maximum number of orbitals possible when n = 4 and m = 0?

Answer (4)

Sol.



27. How many of the given statements are true for fuel cell?

   (a) It is a type of Galvanic cell

   (b) It is used for providing electrical power in space programme.

   (c) Hydrogen and oxygen are bubbled through porous carbon electrodes into concentrated NaOH solution

   (d) It produces electricity with an efficiency of 40%

   (e) It is pollution free cell

Answer (4)

Sol. Fuel cell produces electricity with an efficiency of 70%

28. An element of *d*-block (Z) of 4th period has spin only magnetic moment of its Z^{3+} form is 3.9 BM, then find minimum atomic number of element (Z).

Answer (24)

Sol. $\mu = 3.9$ BM

It means there must be 3 unpaired electrons in Z^{3+} ion

$Cr^{+3} \Rightarrow [Ar]\ 3d^{3}45^{0}$

29. 3 g of acetic acid is dissolved in 500 g of water. Depression in freezing point of solution is x × 10^{-1}K. Value of x to the nearest integer.

   Given : K_a of $CH_3COOH = 1.8 \times 10^{-5}$ and

   K_f of water = 1.86 K/molal

   Density of water = 1 g/mL

Answer (2)

Sol. $CH_3COOH \rightleftharpoons CH_3COO^- + H^+$

(Assuming α << 1)

$$\alpha = \sqrt{\frac{K_a}{C}} = \sqrt{\frac{1.8 \times 10^{-5}}{10^{-1}}} = \sqrt{1.8 \times 10^{-4}}$$

$$= 1.3 \times 10^{-2}$$

$$= 0.013$$

So, i = 1 + (2 – 1)(0.013)

= 1.013

$$\Delta T_f = 1.013 \times 1.86 \times \frac{3 \times 1000}{60 \times 500}$$

= 0.188

= 1.88×10^{-1}

$x \simeq 2$

30.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. If *a*, *b*, *c* are in A.P. and *a* + 1, *b*, *c* + 3 are in G.P., arithmetic mean of *a*, *b*, *c* is 8, then the value of cube of geometric mean of *a*, *b*, *c* is

(1) 312 (2) 314
(3) 318 (4) 128

Answer (1)

Sol. $a, b, c, \rightarrow$ A.P.

$a+1, b, c+3 \rightarrow$ G.P.

$\frac{a+b+c}{3} = 8 \quad \Rightarrow \quad a + c = 16$

$2b = a + c \quad\quad c = 16 - a$

$\Rightarrow \quad b = 8 \quad\quad \ldots(1)$

$64 = (a + 1)(c + 3)$

$64 = ac + 3a + c + 3$

$64 = a(16 - a) + 3a + 16 - a + 3$

$64 = 16a - a^2 + 2a + 19$

$a^2 - 18a + 45 = 0$

$(a - 15)(a - 3) = 0$

$\Rightarrow \quad a = 15 \quad \text{or} \quad a = 3$

$\Rightarrow \quad c = 1 \quad \text{or} \quad c = 13$

$\left((abc)^{\frac{1}{3}}\right)^3 = 1 \times 15 \times 8 \quad \text{or} \quad 13 \times 3 \times 8$

$= 120 \quad\quad \text{or} \quad 312$

2. If $\int_{-1}^{1} \frac{\cos \alpha x}{1+3^x} = \frac{2}{\pi}$ then α is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{2}$
(3) $\frac{\pi}{3}$ (4) π

Answer (2)

Sol. $I = \int_{-1}^{1} \frac{\cos \alpha x}{1+3^x}$

$\Rightarrow \quad I = \int_0^1 \frac{\cos \alpha x}{1+3^x} + \frac{\cos \alpha x}{1+3^{-x}} dx$

$= \int_0^1 \cos \alpha x \, dx$

$= \left.\frac{\sin \alpha x}{\alpha}\right]_0^1 = \frac{\sin \alpha}{\alpha} = \frac{2}{\pi}$

$\Rightarrow \quad \alpha = \frac{\pi}{2}$

3. If coefficient of x^4, x^5, x^6 of $(1 + x)^n$ are in A.P., then maximum value of *n* is equal to

(1) 28 (2) 21
(3) 14 (4) 7

Answer (3)

Sol. $(1+x)^n = {}^nC_0 1 + {}^nC_1 x^1 + {}^nC_2 x^2 + {}^nC_3 x^3 + {}^nC_4 x^4 + {}^nC_5 x^5 + {}^nC_6 x^6 + \cdots$

${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in A.P.

${}^nC_5 - {}^nC_4 = {}^nC_6 - {}^nC_5$

$\frac{n!}{5!(n-5)!} - \frac{n!}{4!(n-4)!} = \frac{n!}{6!(n-6)!} - \frac{n!}{(n-5)!5!}$

$\frac{1}{5!(n-5)!} - \frac{1}{4!(n-4)!} = \frac{1}{6!(n-6)!} - \frac{1}{5!(n-5)!}$

$\frac{1}{4!(n-5)!}\left[\frac{1}{5} - \frac{1}{n-4}\right] = \frac{1}{5!(n-6)!}\left[\frac{1}{6} - \frac{1}{n-5}\right]$

$30(n-9)(n-6) = 5(n-4)(n-11)$

$30[n^2 - 6n - 9n + 54] = 5[n^2 - 11n - 4n + 44]$

$30n^2 - 450n + 1620 = 5n^2$

$$\frac{1}{(n-5)}\left[\frac{1}{5}-\frac{1}{n-4}\right]=\frac{1}{5}\left[\frac{1}{6}-\frac{1}{n-5}\right]$$

$$\frac{1}{n-5}\left[\frac{n-4-5}{5(n-4)}\right]=\frac{1}{5}\left[\frac{n-5-6}{6(n-5)}\right]$$

$$\frac{n-9}{5(n-4)}=\frac{1}{5}\left[\frac{n-11}{6}\right]$$

$6(n-9) = (n-11)(n-4)$

$\Rightarrow\ 6n - 54 = n^2 - 15n + 44$

$n^2 - 21n + 98 = 0$

$n_{max.} = 14$

4. Let relation defined as $(x_1, y_1)\ R\ (x_2, y_2)$

$x_1 \le x_2$, $y_1 \le y_2$ and given that

(a) R is reflexive but not symmetric.

(b) R is transitive.

then

(1) (a) is true, (b) is false

(2) (a) is false, (b) is true

(3) Both are true

(4) Both are false

Answer (3)

Sol. $(x_1, y_1)\ R\ (x_2, y_2)$

When $x_1 \le x_2$, $y_1 \le y_2$

For reflexive

$(x_1, y_1)\ R\ (x_1, y_1)$

$\Rightarrow\ x_1 \le x_1\ \&\ y_1 \le y_1$

so, R is reflexive

For symmetric,

When $(x_1, y_1)\ R\ (x_2, y_2)$

$\Rightarrow\ x_1 \le x_2\ \&\ y_1 \le y_2$

for $(x_2, y_2)\ R\ (x_1, y_1)$

$x_2 \le x_1\ \&\ y_2 \le y_1$

That is not necessarily true so R is not symmetric

For transitive,

If $(x_1, y_1)\ R\ (x_2, y_2) \Rightarrow x_1 \le x_2\ \&\ y_1 \le y_2$

& $(x_2, y_2)\ R\ (x_3, y_3) \Rightarrow x_2 \le x_3\ \&\ y_2 \le y_3$

For $(x_1, y_1)\ R\ (x_3, y_3) \Rightarrow x_1 \le x_3\ \&\ y_1 \le y_3$

So, R is transitive

Both (a) & (b) are true.

5. The value of

$$\frac{1\times 2^2 + 2\times 3^2 + ... + 100\times(101)^2}{1^2\times 2 + 2^2\times 3 + ... + 100^2\times 101}$$

(1) $\frac{305}{301}$ (2) $\frac{301}{305}$

(3) $\frac{350}{310}$ (4) $\frac{310}{350}$

Answer (1)

Sol. The given problem can be written as

$$\frac{\sum_{n=1}^{100} n(n+1)^2}{\sum_{n=1}^{100} n^2(n+1)}$$

$$\Rightarrow \frac{\sum_{n=1}^{100} n^3 + 2n^2 + n}{\sum_{n=1}^{100} n^3 + n^2}$$

$$=\frac{\left(\frac{100(101)}{2}\right)^2 + \frac{2.100(101)(201)}{6} + \frac{100(101)}{2}}{\left(\frac{100(101)}{2}\right)^2 + \frac{100(101)(201)}{6}}$$

$$=\frac{\frac{(100)(101)}{4} + \frac{2(201)}{6} + \frac{1}{2}}{\frac{100(101)}{2} + \frac{201}{6}}$$

$$=\frac{300(101) + 4(201) + 6}{300(101) + 2(201)} = \frac{15555}{15351} = \frac{5185}{5117} = \frac{305}{301}$$

6. A parabola $y^2 = 12x$ has a chord PQ with mid-point (4, 1) then equation of PQ passes through

(1) $\left(\frac{1}{2},-20\right)$ (2) $\left(\frac{1}{2},-10\right)$

(3) $\left(10,\frac{1}{2}\right)$ (4) $\left(-10,\frac{-1}{2}\right)$

Answer (1)

Sol. Chord with the given middle point is given by $\Rightarrow$ $T = S_1$

$\Rightarrow \quad yy_1 - 6(x + x_1) = y_1^2 - 12x_1 \quad ((x_1, y_1) \equiv (4, 1))$

$y - 6(x + 4) = 1 - 48$

$\Rightarrow \quad y - 6x + 23 = 0$

$\left(\frac{1}{2},-20\right)$ is correct answer.

7. Let $\vec{a} = 2\hat{i} + \lambda\hat{j} - 3\hat{k}$

$\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$

If $\vec{a}+\vec{b}$ is perpendicular to $\vec{a}-\vec{b}$, then λ is

(1) $\sqrt{17}$ (2) 17

(3) 5 (4) $\sqrt{5}$

Answer (1)

Sol. $\vec{c} = \vec{a} + \vec{b} = 5\hat{i} + (\lambda - 2)\hat{j} - 2\hat{k}$

$\vec{d} = \vec{a} - \vec{b} = -\hat{i} + (\lambda + 2)\hat{j} - 4\hat{k}$

Now, $\vec{c}\cdot\vec{d} = 0$

$(5\hat{i} + (\lambda - 2)\hat{j} - 2\hat{k})\cdot(-\hat{i} + (\lambda + 2)\hat{j} - 4\hat{k}) = 0$

$-5 + \lambda^2 - 4 - 8 = 0$

$\lambda = 17$

8. If $\begin{matrix}\cos^{-1}x - \sin^{-1}y = \alpha \\ x, y \in (-1, 1)\end{matrix}$ if $\alpha \in \left[\frac{-\pi}{2}, \pi\right]$

Then minimum value of $x^2 + y^2 + 2xy\sin\alpha$ is

(1) $\frac{-1}{2}$ (2) -1

(3) $\frac{1}{2}$ (4) 0

Answer (4)

Sol. $\cos^{-1}x - \frac{\pi}{2} + \cos^{-1}y = \alpha$

$\cos^{-1}x + \cos^{-1}y = \frac{\pi}{2} + \alpha$

$\because \quad \alpha \in \left(-\frac{\pi}{2}, \pi\right)$

then $\frac{\pi}{2} \in \left(0, \frac{3\pi}{2}\right)$

$\cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right) = \frac{\pi}{2} + \alpha$

$xy - \sqrt{1-x^2}\sqrt{1-y^2} = -\sin\alpha$

$xy + \sin\alpha = \sqrt{1-x^2}\sqrt{1-y^2}$

$\frac{x^2}{y^2} + \sin^2\alpha + 2xy\sin\alpha = 1 - x^2 - y^2 + x^2y^2$

$\underbrace{x^2 + y^2 + 2xy\sin\alpha}_{E} = \cos^2\alpha$

Now min value of E is 0

9. Team A plays 10 matches, probability of winning is $\frac{1}{3}$ and losing is $\frac{2}{3}$. They win x matches and lose y matches. Probability such that $|x - y| \le 2$ is P then find 3^9P.

(1) 8288 (2) 8381

(3) 8461 (4) 8911

Answer (1)

Sol. Probability of winning matches $= \frac{1}{3}$ and losing matches $= \frac{2}{3}$

We need to find $|x - y| \le 2$

x = Number of winning matches

Y = Number of losing matches.

As we know $x + y = 10$

$|x - y| \le 2$

So, Case I, $x = 4$, $y = 6$

$${}^{10}C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^6 = \frac{210.2^6}{3^{10}}$$

Case II, $x = 5$, $y = 5$

$${}^{10}C_5\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^5 = \frac{252.2^5}{3^{10}}$$

Case III, $x = 6$, $y = 4$

$${}^{10}C_6\left(\frac{1}{3}\right)^6\left(\frac{2}{3}\right)^4 = \frac{210.2^4}{3^{10}}$$

So required probability $= \frac{2^4}{3^{10}}[2^2.210 + 2.252 + 210]$

$$= \frac{1554.2^4}{3^{10}} = \frac{518.2^4}{3^9},$$

Now, $3^9P = 8288$

10. $f(x) = \begin{cases} \frac{(72)^x - 9^x - 8^x + 1}{\sqrt{2} - \sqrt{1+\cos x}}; x \neq 0 \\ a\log 2 \log 3 \quad ; x = 0 \end{cases}$

is continuous at $x = 0$. Then a^2 equals to

(1) 1152 (2) 572

(3) 1225 (4) 1005

Answer (1)

Sol. $\lim_{x \to 0} \frac{(9^x - 1)(8^x - 1)}{(1 - \cos x)}\left(\sqrt{2} + \sqrt{1+\cos x}\right)$

$$\lim_{x \to 0} \frac{\left(\frac{9^x - 1}{x}\right)\left(\frac{8^x - 1}{x}\right)}{\left(\frac{1 - \cos x}{x^2}\right)} \times 2\sqrt{2}$$

$= 4\sqrt{2}\ln 9 \times \ln 8$

$= 24\sqrt{2}\log 2 \log 2$

$\Rightarrow a = 24\sqrt{2}$

$a^2 = 1152$

Option (1) is correct

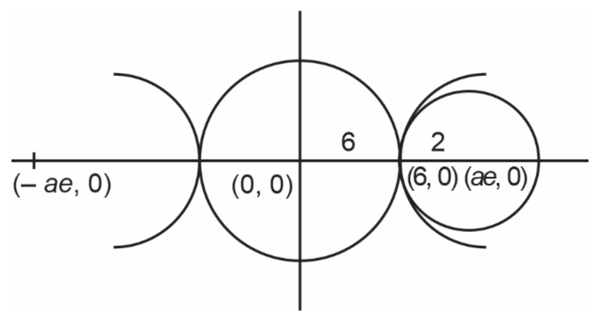
11. For a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, C_1 is a circle touching hyperbola having centre at origin and C_2 is circle centred at four and touching hyperbola at vertices, if area of C_1 = 36π and area of C_2 = 4π. Find $a^2 + b^2$ = ?

(1) 40 (2) 43

(3) 64 (4) 56

Answer (3)

Sol.



Radius of C_1 = 6

Radius of C_2 = 2

$2ae = 16$

$ae = 8$

$b^2 = a^2e^2 - a^2$

$\Rightarrow b^2 + a^2 = 64$

12. $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ and $X = I + \text{adj}(A) + (\text{adj } A)^2 + \ldots$

adj $(A)^{10}$

then sum of elements of X is

(1) 88 (2) –88

(3) 124 (4) 0

Answer (2)

Sol. $(\text{adj } A) = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

$(\text{adj } A)^2 = \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix}$

$(\text{adj } A)^3 = \begin{bmatrix} 1 & -6 \\ 0 & 1 \end{bmatrix}$

$(\text{adj } A)^4 = \begin{bmatrix} 1 & -8 \\ 0 & 1 \end{bmatrix}$

$(\text{adj } A)^r = \begin{bmatrix} 1 & (-2r) \\ 0 & 1 \end{bmatrix}$

$$X = \sum_{r=0}^{10} (\text{adj } A)^r = \begin{bmatrix} \sum_{r=0}^{10} 1 & \sum_{r=0}^{10} (-2r) \\ \sum_{r=0}^{10} (0) & \sum_{r=0}^{10} (1) \end{bmatrix}$$

$$X = \begin{bmatrix} 11 & -110 \\ 0 & 11 \end{bmatrix}$$

$\Rightarrow$ Sum of elements = –110 + 22 = –88

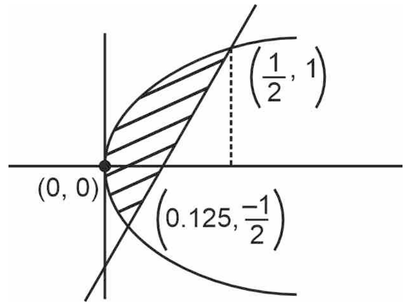
13. Find area bounded by $y^2 \le 2x$ and $y \ge 4x - 1$

(1) $\frac{9}{32}$ (2) $\frac{11}{32}$

(3) $\frac{11}{8}$ (4) $\frac{11}{3}$

Answer (1)

Sol. Given region is



$$\text{Area} = \int_{-1/2}^{1} \left(\frac{y+1}{4} - \frac{y^2}{2}\right) dy$$

$$= \left[\frac{y^2}{8} + \frac{y}{4} - \frac{y^3}{6}\right]_{-1/2}^{1}$$

$$= \left(\frac{1}{8} + \frac{1}{4} - \frac{1}{6}\right) - \left(\frac{1}{32} - \frac{1}{8} + \frac{1}{48}\right)$$

$$= \frac{5}{24} - \left(\frac{3 - 12 + 2}{96}\right)$$

$$= \frac{5}{24} + \frac{7}{96}$$

$$= \frac{27}{96} = \frac{9}{32}$$

14. $(x^2 + 1)^2 dy + (y(2x^3 + x) - 2)dx = 0$, $y(0) = 0$, then $y(2)$ is equal to

(1) $\frac{2}{5}\tan^{-1}2$ (2) $\frac{3}{5}\tan^{-1}2$

(3) $\frac{2}{5}\tan^{-1}3$ (4) $\frac{3}{5}\tan^{-1}3$

Answer (1)

Sol. $\frac{dy}{dx} = \frac{2 - y\left(2x^3 + x\right)}{\left(x^2 + 1\right)^2}$

$$\frac{dy}{dx} + \frac{2x^3 + x}{\left(x^2 + 1\right)^2} y = \frac{2}{\left(x^2 + 1\right)^2}$$

I.F. $= e^{\int \frac{2x^3+x}{(x^2+1)^2}dx}$

$= x^2 + 1$

$$y\cdot(x^2+1) = \int \frac{2}{(x^2+1)}dx + c$$

$y(x^2 + 1) = 2\tan^{-1}x + c$

$y(0) = 0 \quad \Rightarrow c = 0$

$$\Rightarrow \quad y = \frac{2\tan^{-1}x}{x^2+1}$$

$$y(2) = \frac{2\tan^{-1}2}{5}$$

15. If $f(x) = \int_0^x (t + \sin(1-e^t))dt$ then

$\lim_{x\to 0}\frac{f(x)}{x^3}$ is equal to

(1) $\frac{1}{6}$ (2) $\frac{1}{24}$

(3) $\frac{-1}{6}$ (4) $\frac{1}{2}$

Answer (3)

Sol. $\lim_{x\to 0}\left(\frac{f(x)}{x^3}\right)$, $\lim_{x\to 0} f(x) = 0$ $\left(\frac{0}{0}\text{form}\right)$

$\Rightarrow$ Using L' Hopital rule

$$\Rightarrow \quad \lim_{x\to 0}\left(\frac{f'(x)}{3x^2}\right), f'(x) = x + \sin(1-e^x)$$

$$= \lim_{x\to 0}\left(\frac{x+\sin(1-e^x)}{3x^2}\right) = \lim_{x\to 0}\frac{1+\cos(1-e^x)(-e^x)}{6x}$$

$$= \lim_{x\to 0}\frac{(e^x)\sin(-e^x)(-e^x)+\cos(1-e^x)(-e^x)}{6}$$

$$= \frac{-1}{6}$$

16. If a, b, c are in increasing A.P. and $a + 1$, b, $c + 3$ are in G.P. If A.M. of a, b, c is 8. Find cube of G.M. of a, b, c.

(1) 123 (2) 312

(3) 415 (4) 213

Answer (2)

Sol. $2b = a + c$ … (1)

$b^2 = (a + 1)(c + 3)$ … (2)

$\frac{a+b+c}{3} = 8$ … (3)

$\Rightarrow \frac{3b}{3} = 8$

$b = 8$

$\Rightarrow \quad ac + 3a + c + 3 = 64$

$3a + c + ac = 61$ … (4)

$a + c = 16$

$c = 16 - a$

from equation (4)

$3a + 16 - a + a(16 - a) = 61$

$2a + 16 + 16a - a^2 = 61$

$a^2 - 18a + 45 = 0$

$(a - 15)(a - 3) = 0$

$a = 15$, $b = 8$, $c = 1 \rightarrow$ rejected

$a = 3$, $b = 8$, $c = 13$

$\left((a\cdot b\cdot c)^{1/3}\right)^3 = 3 \times 8 \times 13 = 312$

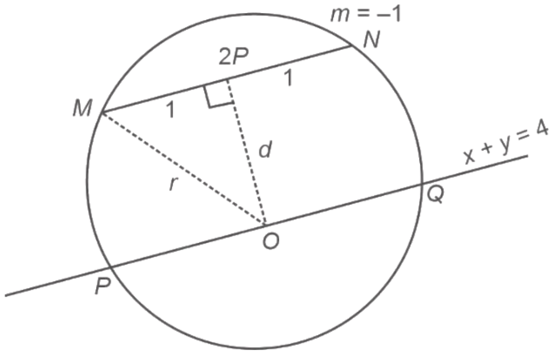
17. The radius of a circle is $\sqrt{10}$. $x + y = 4$ is the line intersecting the circle at P & Q. A chord MN is of length 2 m having slope –1. Find perpendicular distance between the two chords PQ and MN.

(1) 2 (2) 3

(3) 4 (4) 5

Answer (2)

Sol. Radius of circle = $\sqrt{10}$



In ΔMPO,

$\sqrt{1^2 + d^2} = r$

$1 + d^2 = 10$ [$\because$ $r = \sqrt{10}$]

$d^2 = 9$

$d = \pm 3$

Since, distance is positive, distance between chord is 3.

18. If $f(x) = \dfrac{2x}{\sqrt{1-9x^2}}$ and $(fofo \ldots f(x)) \ldots$ 10 times

$= \dfrac{2x}{\sqrt{1-9\alpha x^2}}$ then $\sqrt{3\alpha + 1}$ is equal to

(1) 1024 (2) 512

(3) 240 (4) 108

Answer (1)

Sol. $f(x) = \dfrac{2x}{\sqrt{1-9x^2}}$

$$f(f(x)) = \frac{\frac{2x}{\sqrt{1-9x^2}}}{\sqrt{1-9\left(\frac{4x^2}{1-9x^2}\right)}} = \frac{2x}{\sqrt{1-9x^2-36x^2}} = \frac{2x}{\sqrt{1-45x^2}}$$

$$f(f(f(x))) = \frac{\frac{2x}{\sqrt{1-9x^2}}}{\sqrt{\frac{1-45(4x^2)}{(1-9x^2)}}} = \frac{2x}{\sqrt{1-189x^2}}$$

$\Rightarrow$ $f_K(x) = \dfrac{2x}{\sqrt{1-a_K x^2}}$

$a_1 = 9$

$a_2 = 9 \times 4 + 9 = 4a_1 + 9$

$a_3 = 4a_2 + 9 \ldots$ so on

$\Rightarrow$ $a_n = 4a_{n-1} + 9$

$\Rightarrow$ $a_K = 9, 9 \times 5, 9 \times 21, 9 \times 85 \ldots$

$b_K = \dfrac{a_K}{9} = 1, 5, 21, 85 \ldots$

Notice that b_K is first difference G.P.

$(S - 1) = 4, (21 - 5) = 16, (85 - 21) = 64$

$S_K = 1 + 5 + 21 + 85 + \ldots + b_{10}$

$b_K = 1 + 5 + 21 + \ldots + b_9 + b_{10}$

$O = 1 + 4 + 16 + 64 + \ldots + \quad \ldots b_{10}$

$\Rightarrow$ $b_{10} = 1 + 4 + 16 + 64 + \ldots$

$$b_{10} = 1 \cdot \left(\frac{4^{10}-1}{4-1}\right) = \left(\frac{4^{10}-1}{3}\right)$$

$$a_{10} = \frac{9\left(4^{10}-1\right)}{3} = 3\left(4^{10}-1\right)$$

$$f_{10}(x) = \frac{2x}{\sqrt{1-3\left(4^{10}-1\right)x^2}}$$

$\Rightarrow$ $9\alpha = 3(4^{10} - 1)$

$\Rightarrow$ $3\alpha = 4^{10} - 1$ $\Rightarrow$ $(3\alpha + 1) = 4^{10}$

$\Rightarrow$ $\sqrt{3\alpha + 1} = 4^5 = 2^{10}$

$= 1024$

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. In a group *A* there are 4 men and 5 women and in group *B* there are 5 men and 4 women, if 4 people are selected from each group. Find number of ways to select 4 men and 4 women.

Answer (5626)

Sol.

Total	4M 5W	5 M 4 W
	A	B
	0M, 4W	4M, 0W $\to {}^5C_4 \times {}^5C_4$
	1M, 3W	3M, 1W $\to {}^4C_1 \times {}^5C_3 \times {}^5C_3 \times {}^4C_1$
	2M, 2W	2M, 2 W $\to ({}^4C_2)^2 \times ({}^5C_2)^2$
	3M, 1W	1M, 3W $\to {}^4C_3 \times {}^5C_1 \times {}^5C_1 \times {}^4C_3$
	4M, 0W	0M, 4W $\to {}^4C_4 \times {}^4C_4$

$({}^4C_0)^2\,({}^5C_4)^2 + ({}^4C_1)^2 \times ({}^5C_3)^2 + ({}^4C_2)^2 \times ({}^5C_2)^2 + ({}^4C_3)^2 \times ({}^5C_1)^2 + ({}^4C_4)^2 \times ({}^5C_0)^2$

= 5626

22. If $f(x) = 3\sqrt{x-2} + \sqrt{4-x}$

If minimum value = α

Maximum value = β

find $\alpha^2 + \beta^2$

Answer (22)

Sol. $3\sqrt{x-2} + \sqrt{4-x}$

Let $x = 2\sin^2\theta + 4\cos^2\theta$

$= 3\sqrt{2\sin^2\theta + 4\cos^2\theta - 2} + \sqrt{4 - 2\sin^2\theta - 4\cos^2\theta}$

$= 3\sqrt{2\cos^2\theta} + \sqrt{2\sin^2\theta}$

$= 3\sqrt{2}|\cos\theta| + \sqrt{2}|\sin\theta|$

$= 3\sqrt{2}|\cos\theta| + \sqrt{2}|\sin\theta|$

$= 3\sqrt{2}\cos\theta + \sqrt{2}\sin\theta \le \sqrt{18+2}$

$3\sqrt{2}\cos\theta + \sqrt{2}\sin\theta \le \sqrt{20}$

Minimum value exist when $\theta = \frac{\pi}{2}$

So minimum value = $\sqrt{2}$

$\Rightarrow \alpha^2 + \beta^2 = 20 + 2 = 22$

23. $\int(\text{cosec}x)^5\,dx = -\alpha\,\text{cosec}x\cot x\left(\text{cosec}^2 x + \frac{3}{2}\right) + \frac{\beta}{2}\log\left|\tan\frac{x}{2}\right| + C$

Find α + β.

Answer (1)

Sol. Take,

$I = \int(\text{cosec}x)^5\,dx$

$\Rightarrow \int \text{cosec}^3 x.\,\text{cosec}^2 x\,dx$

$\Rightarrow \text{cosec}^3 x.\int \text{cosec}^2 x\,dx -$

$\int(3\,\text{cosec}^2 x(-\text{cosec}x\cot x)\int \text{cosec}^2 x\,dx$

$\Rightarrow \text{cosec}^3 x\,(-\cot x) -$

$\int 3\text{cosec}^2 x(-\text{cosec}x\cot x)(-\cot x)\,dx$

$I = -\text{cosec}^3 x\cot x - 3\int \text{cosec}^3 x.\cot^2 x\,dx$

$= -\text{cosec}^3 x\cot x - 3\int \text{cosec}^3 x\,(\text{cosec}^2 x - 1)dx$

$= -\text{cosec}^3 x\cot x - 3\int \text{cosec}^5 dx + 3\int \text{cosec}^3 x\,dx$

$\Rightarrow I = -\text{cosec}^3 x\cot x - 3I + 3\int \text{cosec}^3 x\,dx$

$4I = -\text{cosec}^3 x\cot x + 3\int \text{cosec}^3 x\,dx$

$\therefore 4I = -\text{cosec}^3 x\cot x$

$$+3\left\{-\frac{1}{2}\operatorname{cosec}x\cot x-\frac{1}{2}\log\left|\tan\frac{x}{2}\right|+C\right\}$$

$$I=-\frac{1}{4}\operatorname{cosec}^3 x\cot x-\frac{3}{8}\operatorname{cosec}x\cot x$$

$$+\frac{3}{8}\log\left|\tan\frac{x}{2}\right|+C$$

$$\Rightarrow\quad -\frac{1}{4}\operatorname{cosec}x\cot x\left[\operatorname{cosec}^2 x+\frac{3}{2}\right]$$

$$+\frac{1}{2}\left(\frac{3}{4}\right)\log\left|\tan\frac{x}{2}\right|$$

$$\alpha=\frac{1}{4},\ \beta=\frac{3}{4}$$

$$\alpha+\beta=\frac{1}{4}+\frac{3}{4}=1$$

24.

25.

26.

27.

28.

29.

30.