09 - 04 - 2024
SHIFT - 1

SMART ACHIEVERS
JEE | NEET | FOUNDATION

JEE Main - 2024 Session - 2
Answers & Solutions

(Physics, Chemistry and Mathematics)

9-APRIL SHIFT - 1

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PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. The dimension of latent heat is

(1) $[M^0L^2T^{-1}]$ (2) $[M^0L^2T^{-2}]$

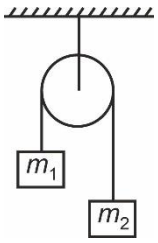
(3) $[M^0LT^{-2}]$ (4) $[M^{-1}L^2T^{-2}]$

Answer (2)

Sol. $[L] = \frac{Q}{M} = \frac{ML^2T^{-2}}{M}$

$= \left[L^2T^{-2}\right]$

2. In the pulley-block system shown, the pulley and the block are ideal. If the acceleration of the blocks is $\frac{g}{8}$, find $m_1 : m_2$ (Given $m_2 > m_1$)



(1) 7 : 9 (2) 5 : 7

(3) 3 : 4 (4) 9 : 11

Answer (1)

Sol. $a = \frac{(m_2 - m_1)g}{(m_1 + m_2)} = \frac{g}{8}$

$\frac{m_1}{m_2} = \frac{7}{9}$

3. Velocity of a particle of mass m as a function of displacement x is given by $v = \alpha\sqrt{x}$.

Work done to move it from $x = 0$ to $x = d$ is

(1) $\frac{m\alpha^2}{2}.d$ (2) $m\alpha^2.d$

(3) $\frac{3m\alpha^2 d}{2}$ (4) $2m\alpha^2 d$

Answer (1)

Sol. $W = \Delta KE$

$W = \frac{1}{2}m\left[\left(\alpha\sqrt{d}\right)^2 - \left(\alpha\sqrt{0}\right)^2\right]$

$W = \frac{m\alpha^2 d}{2}$

4. Two persons are pulling a rope towards themselves with force of 200 N each. If Young's modulus is 2×10^{11} N/m^2 and area of cross is 2 cm^2 for the rope. The elongation in the rope is, if distance of their holding the rope is 2 m.

(1) 10 μm (2) 20 μm

(3) 5 μm (4) 40 μm

Answer (1)

Sol. $\frac{Fl}{YA} = \Delta l$

$\frac{200 \times 2}{2 \times 10^{11} \times 2 \times 10^{-4}} = \Delta l$

$100 \times 10^{-7} = \Delta l$

10×10^{-6} m $= \Delta l$

5. A galvanometer having resistance of 200 Ω shows full deflection at 20 μA. If the galvanometer has to measure current up to 200 *m*A, the shunt resistance required is

(1) $\frac{200}{99}$ Ω (2) $\frac{200}{999}$ Ω

(3) $\frac{20}{99}$ Ω (4) 200 × 999 Ω

Answer (2)

Sol. $20\,\mu A = \left(\frac{R_S}{R_S + 200}\right) 20\,mA$

$R_S + 200 = 1000\ R_S$

$R_S = \frac{200}{999}\ \Omega$

6. A particle oscillating simple harmonic motion such that its speed and acceleration at distance 2 m from mean position are 4 m/s and 16 m/s^2 respectively.

(1) $\sqrt{10}$ m (2) $\sqrt{6}$ m

(3) $\sqrt{8}$ m (4) $\sqrt{3}$ m

Answer (2)

Sol. $4 = \omega\sqrt{A^2 - 4}$ …(i)

$16 = 2\omega^2$ …(ii)

from (i) and (ii)

$A = \sqrt{6}$ m

7. **Assertion :** Object at radius of curvature of biconvex lens forms image at same distance an other side of lens.

Reason : Image of a real object formed by concave lens is always virtual and erect.

(1) Assertion and reason are correct and reason is correct explanation of assertion.

(2) Assertion and reason are correct but reason is not correct explanation of assertion.

(3) Assertion is correct but Reason is incorrect

(4) Assertion is incorrect but Reason is correct

Answer (2)

Sol. Theoretical $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ & $|v| = |u| = 2f$

8. The equivalent energy of 1 gm mass is equal to

(1) 8.3×10^{26} MeV (2) 5.6×10^{26} MeV

(3) 8.3×10^{12} MeV (4) 5.6×10^{12} MeV

Answer (2)

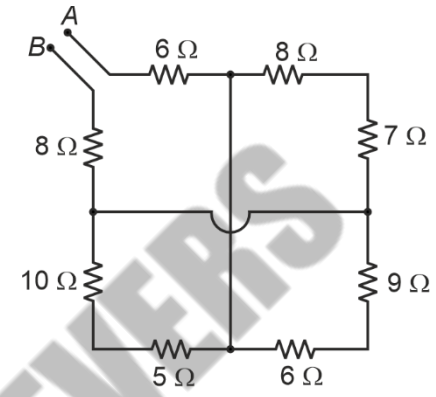
Sol. $E = mc^2$

$= \frac{1 \times 10^{-3} \times 9 \times 10^{16}}{1.6 \times 10^{-19}}$ eV

$= 5.625 \times 10^{32}$ eV

$= 5.6 \times 10^{26}$ MeV

9. Find equivalent resistance between terminal is A and B for the given network.

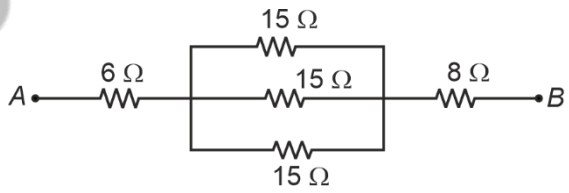


(1) 16 Ω (2) 20 Ω

(3) 15 Ω (4) 19 Ω

Answer (4)

Sol. Simplifying the circuit, we get



$R_{AB} = 14 + \frac{15}{3} = 19\ \Omega$

10. A person covers first half of the distance with 6 m/s and rest half of the distance is covered with 9 m/s and 15 m/s in two equal time intervals. Find average speed of the journey.

(1) 12 m/s (2) 9 m/s

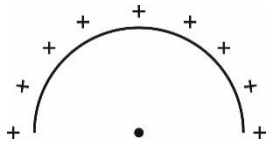
(3) 10 m/s (4) 8 m/s

Answer (4)

Sol. Average speed in second half distance $= \frac{9+15}{2} = 12$ m/s

Average speed of journey $= \frac{2d}{\frac{d}{6}+\frac{d}{12}} = 8$ m/s

11. Find the potential at the centre of a uniformly charged semi-circular wire with uniform linear charge density of 4 nC/m.



(1) 36π volts (2) 29π volts

(3) 9π volts (4) Zero volts

Answer (1)

Sol. $v = \frac{kQ}{R}$

$= \frac{9\times10^{9}(\pi R)\times4\times10^{-9}}{R}$

$= 36\pi$ volt

12. An astronaut takes a body of mass m from the surface of the earth to an altitude $\frac{R}{20}$ (R is the radius of the earth, g is acceleration due to gravity at surface of earth)

Find change in gravitational potential energy of the body.

(1) $\frac{mgR}{20}$ (2) $\frac{mgR}{21}$

(3) $\frac{mgR}{15}$ (4) $\frac{mgR}{25}$

Answer (2)

Sol. $\Delta U = U_f - U_i = \frac{-GMm}{\left(\frac{R+R}{20}\right)} - \left(\frac{-GMm}{R}\right)$

$\Delta U = \frac{GMm}{21R} = m\left(\frac{GM}{R^2}\right)\cdot\frac{R}{21} = \frac{mgR}{21}$

13. Energy associated with a photon is 1.42 eV. Find wavelength of photon. (Take hc = 1240 nmeV)

(1) 628.26 nm (2) 873.24 nm

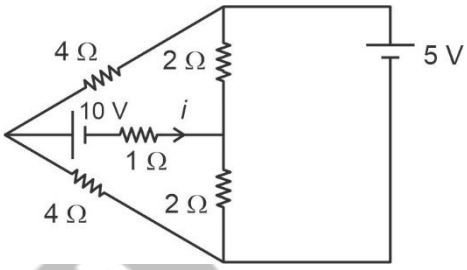
(3) 625.22 nm (4) 820.23 nm

Answer (2)

Sol. $E = \frac{hc}{\lambda}$

$\lambda = \frac{1240n \text{ MeV}}{1.42 \text{ eV}}$

$= 873.24$ nm

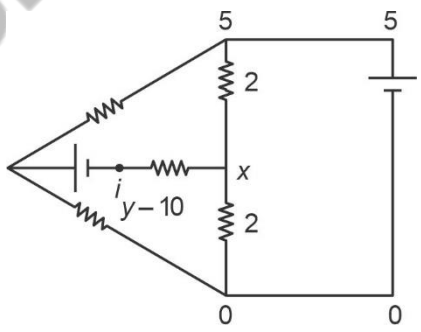
14. 

Find current i in the given DC circuit.

(1) 1.25 A (2) 5 A

(3) 2.5 A (4) 1 A

Answer (3)

Sol. 

$\frac{x-5}{2}+\frac{x}{2}+\frac{x-y+10}{1}=0$

$x-5+x+2x-2y+20=0$

$4x-2y=-15$ …(i)

$\frac{y-5}{4}+\frac{y-0}{4}+\frac{y-10-x}{1}=0$

$y - 5 + y - 0 - 4y - 40 - 4x = 0$

$6y - 4x = 45$ …(ii)

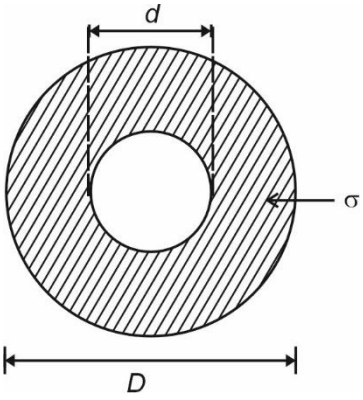
$i = 2.5$ A

$4y = 30$

$y = \frac{15}{2}$

And $x = 0$

15. An object of diameter D has cavity of diameter d as shown. Relative density of material of object is σ. Find $\frac{D}{d}$ such that the object just completely submerge in water.



(1) $\left(\frac{\sigma-1}{\sigma}\right)^{\frac{1}{3}}$ (2) $\left(\frac{\sigma}{\sigma-1}\right)^{\frac{1}{3}}$

(3) $\left(\frac{\sigma^2-1}{\sigma^2}\right)^{\frac{1}{3}}$ (4) $\left(\frac{2\sigma-1}{2\sigma}\right)^{\frac{1}{3}}$

Answer (2)

Sol. $V = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3$

$V_C = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$

$\frac{4}{3}\pi\left(\frac{D}{2}\right)^3 = \frac{4}{3}\pi\left\{\left(\frac{D}{2}\right)^3 - \left(\frac{d}{2}\right)^3\right\}\sigma$

$D^3 = (D^3 - d^3)\sigma$

$1 = \left\{1 - \left(\frac{d}{D}\right)^3\right\}\sigma$

$1 - \frac{1}{\sigma} = \left(\frac{d}{D}\right)^3$

$\left(\frac{\sigma}{\sigma-1}\right)^{\frac{1}{3}} = \frac{D}{d}$

16. A capacitor is connected with a bulb to an ac source. After some time dielectric is introduced between the plates of the capacitor. The brightness of the bulb will

(1) Increase

(2) Decrease

(3) No change

(4) First increase then decrease

Answer (1)

Sol. $c' = kc$

$X_c = \frac{1}{\omega c}$

$X_c' = \frac{1}{kc\omega}$

z decreases

$\therefore$ i will increase.

17. Two vector $\vec{A}$ and $\vec{B}$ having magnitude A and B respectively are inclined at angle $\theta = \cos^{-1}\left(\frac{5}{9}\right)$. If they satisfied the relation $\left|\vec{A}+\vec{B}\right| = \sqrt{2}\left|\vec{A}-\vec{B}\right|$ and given that $\left|\vec{A}\right| = n\left|\vec{B}\right|$, then value of n may be

(1) $\sqrt{3}$ (2) 3

(3) $\frac{2}{\sqrt{3}}$ (4) $\frac{1}{\sqrt{3}}$

Answer (2)

Sol. $\sqrt{A^2+B^2+2AB\cos\theta}=\sqrt{2}\sqrt{A^2+B^2-2AB\cos\theta}$

$$(nB)^2+B^2+2(nB)B\times\frac{5}{9}=2\left[(nB)^2+B^2-2(nB)B\times\frac{5}{9}\right]$$

$$n^2+1-\frac{10}{3}n=0$$

$3n^2 + 3 - 10n = 0$

$3n^2 - 10n + 3 = 0$

$3n^2 - 9n - n + 3 = 0$

$3n(n - 3) - 1(n - 3) = 0$

$n = 3$

$$n=\frac{1}{3}$$

18. In an adiabatic process $\left(\gamma=\frac{3}{2}\right)$, a gas expands to double of its initial volume.

Find the work done by gas if initial temperature is T. (Number of moles of gas = 1)

(1) $RT\left(2-\sqrt{2}\right)$ (2) $RT\left(2+\sqrt{2}\right)$

(3) $\frac{RT}{2+\sqrt{2}}$ (4) $\frac{RT}{2-\sqrt{2}}$

Answer (1)

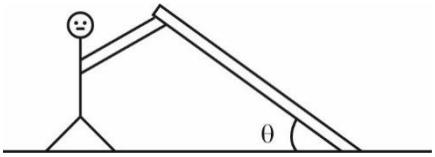
Sol. $TV^{\gamma-1}=T_1(2V)^{\gamma-1}$

$$\Rightarrow\quad T_1=\frac{T}{\sqrt{2}}$$

$$W=\frac{nR\left(T-\frac{T}{\sqrt{2}}\right)}{\frac{3}{2}-1}=\sqrt{2}RT\left(\sqrt{2}-1\right)$$

$$=RT\left(2-\sqrt{2}\right)$$

19. A man is holding a rod as shown in figure from one end while the other end is making an angle θ with the ground. Find the contact force between rod and man if weight of rod is W.



(1) $\frac{W}{2}\sin\theta$

(2) $\frac{W}{3}\cos\theta$

(3) $\frac{W}{3}\sin\theta$

(4) $\frac{W}{2}\cos\theta$

Answer (4)

Sol. $mg\cos\theta\frac{l}{2}=N\times l$

$$N=\frac{mg}{2}\cos\theta=\frac{W\cos\theta}{2}$$

20. If the energy of α-particle, proton and an electron are same and the simplest ratio of their de–Broglie wavelength is

(1) 2 : 1 : 1244

(2) 1836 : 4 : 1

(3) 1 : 4 : 7340

(4) 1 : 4 : 1836

Answer (3)

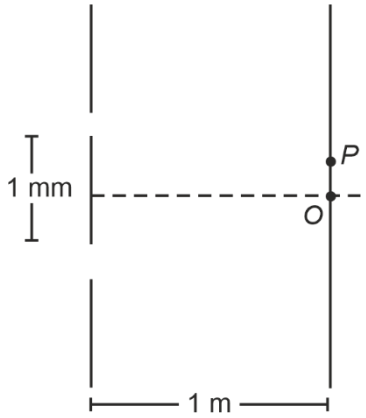
Sol. $\lambda=\frac{h}{\sqrt{2mE}}$

$$\lambda\propto\frac{1}{\sqrt{m}}$$

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. In the *YDSE* set up shown below, point *P* is having $\left(\frac{1}{4}\right)^{th}$ the maximum intensity. If the minimum distance of *P* from *O* is $N \times 10^{-4}$ m, find *N*. (Given wavelength of light = 600 nm)



Answer (2)

Sol. $\frac{I_0}{4} = I_0 \cos^2 \frac{\phi}{2}$

$\phi = 120°$

$\Delta x = \frac{\lambda}{2\pi} \times \frac{2\pi}{3} = \frac{\lambda}{3}$

$\Delta x = \frac{dy}{D} = \frac{\lambda}{3}$

$y = \frac{\lambda D}{3d} = \frac{600 \times 10^{-9} \times 1}{3 \times 10^{-3}}$

$= 2 \times 10^{-4}$ m

22. A choke coil draws 4 A current from 20 V DC and $\frac{4}{5}$ A current from 20 V AC (of frequency of 50 Hz). The inductance of the coil is $\frac{x}{10}$ H. Find nearest integer *x*.

Answer (4)

Sol. $R = 5\ \Omega$

$\sqrt{5^2 + x_L^2} = \frac{20}{\left(\frac{4}{5}\right)} = 25$

$25 + x_L^2 = 625$

$x_L = \sqrt{600}$

$L = \frac{10\sqrt{6}}{20\pi \times 50} = \frac{\sqrt{6}}{2\pi}$ H

$= 0.4$ H

23. String wrapped on a circular disc of radius $r = 20$ cm. The moment of inertia of disc is 0.4 kgm^2. The string is pulled with a constant force of 40 N. The angular velocity of the disc at time $t = 2$ sec is *K* rad/s.

Find the value of *K*. (Initially disc is at rest)

Answer (40)

Sol. $\tau = I\alpha$

$40 \times 0.2 = 0.4 \cdot \alpha \Rightarrow \alpha = 20$ rad/s^2

$\therefore \omega = \omega_i + \alpha t$

$\boxed{\omega = 40 \text{ rad/s}}$

24.

25.

26.

27.

28.

29.

30.

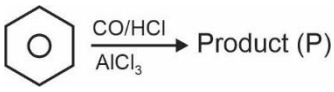
CHEMISTRY

SECTION - A

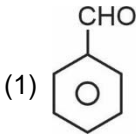
Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

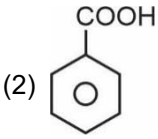
Choose the correct answer :

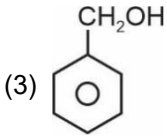
1. For the reaction:

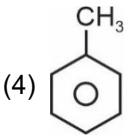


Product (P) is

(1) 

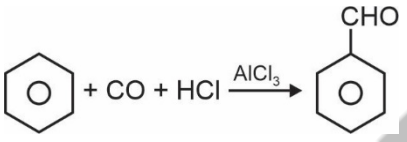
(2) 

(3) 

(4) 

Answer (1)

Sol. This is Gattermann – Koch reaction



2. Chemical formula of compound present in tooth enamel?

(1) $Ca_{10}(PO_4)_6(OH)_2$

(2) $Ca_8(PO_4)_4(OH)_2$

(3) $Ca_6(PO_4)_3(OH)_2$

(4) $Ca_8(PO_4)_6(OH)_2$

Answer (1)

Sol. The chemical formula of compound present in tooth enamel is $[3Ca_3(PO_4)_2 \cdot Ca(OH)_2]$.

3. Which of the following has sp^2 hybridisation?

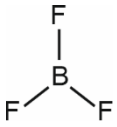
(1) BF_3

(2) H_2SO_4

(3) NH_4^+

(4) NH_3

Answer (1)

Sol. 

Steric number = 3

Hybridisation = sp^2

4. Which of the following orbitals has the highest energy?

(1) n = 6 l = 0

(2) n = 5 l = 2

(3) n = 4 l = 2

(4) n = 3 l = 1

Answer (2)

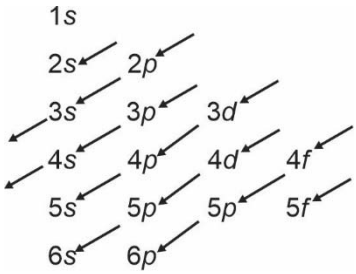
Sol. (i) n = 6 l = 0 $\Rightarrow$ 6*s*

(ii) n = 5 l = 2 $\Rightarrow$ 5*d*

(iii) n = 4 l = 2 $\Rightarrow$ 4*d*

(iv) n = 3 l = 1 $\Rightarrow$ 3*p*

Energy of $3p < 4d < 6s < 5d$



5. Equal volume of 1 M HCl and 1 M H_2SO_4 neutralised by dil. NaOH and heat released is x and y KCal mol^{-1} respectively which is correct.

(1) x = y

(2) x = 0.5y

(3) x = 0.4y

(4) x = 2y

Answer (2)

Sol. When 1 mol of H^+ is neutralised by 1 mol of OH^- then 13.7 KCal mol^{-1} energy is released energy during neutralisation of 1M HCl is equal to x, then energy released during complete neutralisation of H_2SO_4 by dil. NaOH = 2x

y = 2x

x = 0.5y

6. Consider the following electronic configuration :

$Cu^{2+} = [Ar]\ 3d^9 4s^0$

$Cu^{+} = [Ar]\ 3d^{10} 4s^0$

Which option is correct?

(1) Cu^{2+} is more stable in aqueous solution

(2) Cu^{+} is more stable in aqueous solution

(3) Cu^{+} and Cu^{2+} are equally stable in aqueous solution

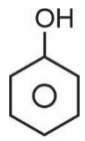
(4) Depends upon copper salt

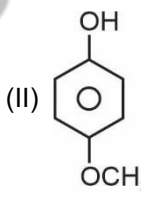
Answer (1)

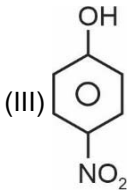
Sol. $Cu^{+} \longrightarrow Cu^{2+} + Cu$

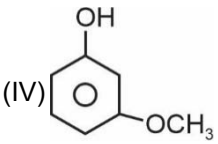
Cu^{+} undergoes disproportionation due to high hydration energy of Cu^{2+}.

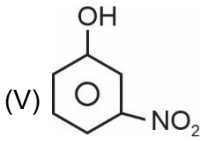
7. Arrange the following in increasing order of acidity:

(I) 

(II) 

(III) 

(IV) 

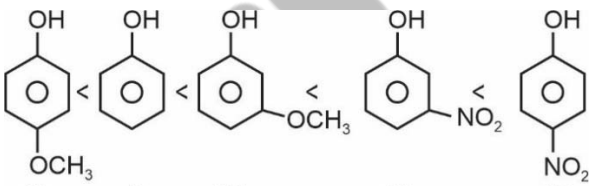
(V) 

(1) I < II < III < IV < V (2) II < I < IV < V < III

(3) III < V < IV < I < II (4) II < IV < III < I < V

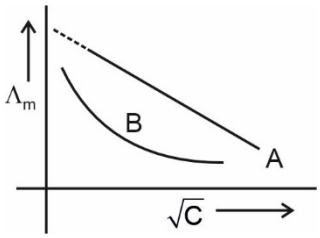
Answer (2)

Sol. The acidic strength of:



(II) (I) (IV) (V) (III)

8. Molar conductance vs $\sqrt{\text{concentration}}$ curve for two electrolytes 'A' and 'B' are shown. Identify the nature of both electrolytes-



(1) A → Strong Electrolyte
B → Strong Electrolyte

(2) A → Weak Electrolyte
B → Strong Electrolyte

(3) A → Strong Electrolyte
B → Weak Electrolyte

(4) A → Weak electrolyte
B → Weak Electrolyte

Answer (3)

Sol. Λ_m of strong electrolyte varies linearly with $\sqrt{C}$ according to Debye-Huckel – Onsager equation

$$\Lambda_m = \Lambda_m^0 - A\sqrt{C}$$

While Λ_m of weak electrolyte varies non-linearly with $\sqrt{C}$ due to change in percentage ionization with concentration.

9. Consider the following reactions:

(I) $CH_3 - CH = CH_2 \xrightarrow[(ii)\,H_2O_2/OH^-]{(i)\,B_2H_6/THF} (A)$

(II) $CH_3 - CH = CH_2 \xrightarrow{H^+/H_2O} (B)$

Identify the major product(s) (A) and (B) formed in the above reactions.

(1) (A): $CH_3 - \underset{\displaystyle OH}{\underset{|}{CH}} - CH_3$

(B): $CH_3 - \underset{\displaystyle OH}{\underset{|}{CH}} - CH_3$

(2) (A): $CH_3 - CH_2 - CH_2 - OH$

(B): $CH_3 - \underset{\displaystyle OH}{\underset{|}{CH}} - CH_3$

(3) (A): $CH_3 - \underset{\displaystyle OH}{\underset{|}{CH}} - CH_3$

(B): $CH_3 - CH_2 - CH_2 - OH$

(4) (A): $CH_3 - CH_2 - CH_2 - OH$

(B): $CH_3 - CH_2 - CH_2 - OH$

Answer (2)

Sol. Hydroboration of unsymmetrical alkene followed by hydrolysis by alkaline H_2O_2 results in the formation of a product as if H_2O has been added to alkene according to anti-Markovnikov rule.

$$3CH_3 - CH = CH_2 + B_2H_6 \xrightarrow{THF} 2(CH_3CH_2CH_2)_3B$$

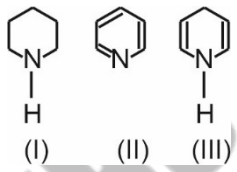
$$(CH_3CH_2CH_2)_3B + 3H_2O_2 \xrightarrow{OH^-} \underset{(A)}{3CH_3CH_2CH_2OH} + H_3BO_3$$

Acid catalysed addition of water to unsymmetrical alkene follows Markovnikov rule with the possibility of rearrangement.

$$CH_3 - CH = CH_2 + H^+ \longrightarrow CH_3 - \underset{\text{(More stable)}}{\overset{+}{C}H} - CH_3$$

$$CH_3 - \overset{+}{C}H - CH_3 + H_2O \longrightarrow (CH_3 - \underset{\displaystyle OH}{\underset{|}{CH}} - CH_3) + H^+$$

10. Consider the following compounds



(I) (II) (III)

Correct order of their basicity is

(1) III > II > I

(2) I > II > III

(3) II > I > III

(4) II > III > I

Answer (2)

Sol. In case of III the lone pair of N is delocalised by resonance hence III is weakest. In case of I and II the substituents of I are electron loosing due to their +I effect hence I is strongest nitrogenous base/Lewis base/base.

11. Which of the following is colourless?

(1) Eu^{3+}

(2) Lu^{3+}

(3) Nd^{3+}

(4) Sm^{3+}

Answer (2)

Sol. $Lu^{3+} = [Xe]\ 4f^{14}\ 5d^0\ 6s^0$

Due to completely filled 4*f* subshell or absence, of unpaired electron Lu^{3+} is colourless.

12. Which among the following have single unpaired electron?

N_2, O_2, CN^-, O_2^-, C_2^{2-}, N_2^-

(1) O_2, N_2

(2) CN^-, C_2^{2-}

(3) CN^-, O_2^-

(4) N_2^-,O_2^-

Answer (4)

Sol. $N_2^- \ \sigma_{1s}^2 \ \sigma^{*2}_{1s} \ \sigma_{2s}^2 \ \sigma^{*2}_{2s} \ \pi_{2p_x}^2 = \pi_{2p_y}^2 \ \sigma_{2p_z}^2 \ \pi^{*1}_{2p_x}$

$O_2^- \ \sigma_{1s}^2 \ \sigma^{*2}_{1s} \ \sigma_{2s}^2 \ \sigma^{*2}_{2s} \ \sigma_{2p_z}^2 \ \pi_{2p_x}^2 = \pi_{2p_y}^2 \ \pi^{*2}_{2p_x} = \pi^{*1}_{2p_y}$

N_2^- and O_2^- have 1 unpaired electron.

13. **Statement-I:** Sulphur exist as S_8 while oxygen exist as O_2.

Statement-II: In oxygen $p\pi - p\pi$ bonding occurs while it is not effective in sulphur.

(1) Statement-I and Statement-II are true

(2) Statement-I is true and Statement-II is false

(3) Statement-I is false and Statement-II is true

(4) Both Statement-I and Statement-II are false

Answer (1)

Sol. Due to bigger size of sulphur atom, it doesn't have effective $p\pi - p\pi$ bonding.

Therefore, in molecular form, oxygen exist as O_2 while sulphur exist as S_8.

14. Identify the correct order of de-Broglie wavelength of proton (λ_p), electron (λ_e) and alpha(λ_a)– particles, moving with same kinetic energy.

(1) $\lambda_p < \lambda_a < \lambda_e$

(2) $\lambda_p < \lambda_e < \lambda_a$

(3) $\lambda_a < \lambda_p < \lambda_e$

(4) $\lambda_a < \lambda_e < \lambda_p$

Answer (3)

Sol. For moving particles, $\lambda = \dfrac{h}{\sqrt{2m(KE)}}$

As, KE $\rightarrow$ same and $m_e < m_p < m_a$

Therefore, $\lambda_a < \lambda_p < \lambda_e$

15. Consider the reaction between PbS and HNO_3 :

$PbS + HNO_3 \longrightarrow$

Which of the following is not formed?

(1) NO (2) NO_2

(3) S (4) $Pb(NO_3)_2$

Answer (2)

Sol. $3PbS + 8HNO_3 \longrightarrow 3Pb^{2+} + 6NO_3^- + 3S\downarrow + 2NO + 4H_2O$

Hence, NO_2 is not formed.

16. Which of the following statement is incorrect?

(1) $KMnO_4$ and NaOH can be used as secondary standard

(2) Primary standard should not undergo change in air

(3) Reaction of primary standard with another substance should not be instantaneous

(4) Primary standard should be soluble in H_2O

Answer (3)

Sol. Reaction of primary standard with another substance should be instantaneous and stoichiometric.

17. Purification method of organic compound does not depends on

(1) Name of compound

(2) Shape of compound

(3) Density of compound

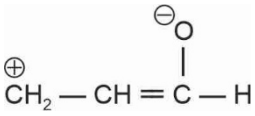
(4) Solubility of compound

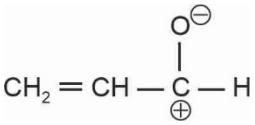
Answer (2)

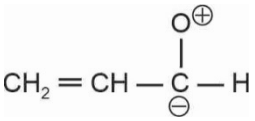
Sol. Organic compounds are purified based on their nature and impurity present in it, the solubility and density of impurity are different from organic compound and on the basis of these difference compounds are purified.

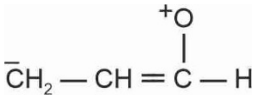
Shape of compound does not play role in purification.

18. Arrange the following in increasing order of their stability

(a) 

(b) 

(c) 

(d) 

(1) (a) < (b) < (c) < (d)

(2) (a) < (d) < (c) < (b)

(3) (a) < (c) < (d) < (b)

(4) (d) < (c) < (a) < (b)

Answer (4)

Sol. Stability of the given ionic species will be decided by the fact that +ve charge on more electronegative element will be less stable and as the distance between opposite charges increases, stability decreases.

Therefore, the correct stability order is

(d) < (c) < (a) < (b)

19. Consider the given complex: $[Co(en)_2)Cl_2]^+$

Statement-I: The number of stereoisomers for the above compound is 3.

Statement-II: Geometry of the above complex is octahedral.

(1) Statement-I is correct but statement-II is incorrect

(2) Both statements-I and II are correct

(3) Statements-I is incorrect but statement-II is correct

(4) Both statements-I and II are incorrect

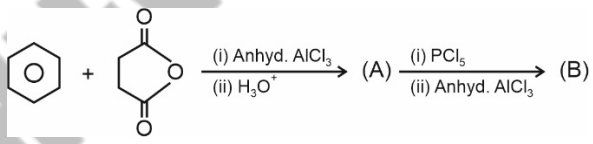
Answer (2)

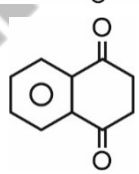
Sol. cis-2

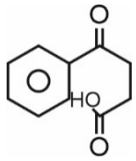
trans-1

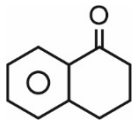
Hybridisation is $sp^3d^2 \Rightarrow$ Geometry = octahedral

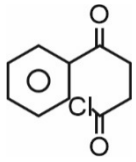
20. Identify the final product (B) obtained in the given reaction sequence:



(1) 

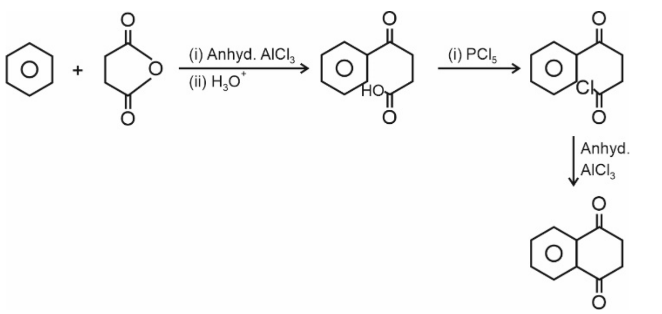
(2) 

(3) 

(4) 

Answer (1)

Sol.



SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Number of ambident nucleophile among following is

$CN^{\ominus}$; $SCN^{\ominus}$; $NO_2^{\ominus}$; $CH_3-COO^{\ominus}$; $C_2O_4^{2\ominus}$

$NH_2^{\ominus}$; SO_4^{2-}

Answer (3)

Sol. Nucleophiles which have more than one different points of linkage (i.e. electron rich centres) are called as ambident nucleophiles.

Examples are $CN^{\ominus}$; $SCN^{\ominus}$; $NO_2^{\ominus}$

(1) $CN^{\ominus} \rightarrow$ C – or N – atom can donate $e^{\ominus}$ pair

(2) $SCN^{\ominus} \rightarrow$ S – or N – atom can donate $e^{\ominus}$ pair

(3) $NO_2^{\ominus} \rightarrow$ N – or O – atom can donate $e^{\ominus}$ pair

22. Total number of essential amino acids are ______.

Answer (10)

Sol.

Valine
Leucine
Isoleucine
Arginine
Lysine
Threonine
Methionine
Phenylalanine
Tryptophan
Histidine

→ Total 10 amino acids are essential which is not synthesised in our body and need to be included in our diet.

23. Heat of solution of $CuSO_4$ and $CuSO_4\cdot5H_2O$ is given as (–70 kJ/mole) and (+12 kJ/mole) respectively. Then $\Delta H_{hydration}$ for converting $CuSO_4$ to $CuSO_4\cdot5H_2O$ is (–x kJ/mole). Find out x (Nearest integer)

Answer (82)

Sol. $CuSO_4 \xrightarrow{-x} CuSO_4\cdot5H_2O \xrightarrow{+12} CuSO_4\cdot tH_2O$

(overall: –70)

$-x + 12 = -70$

$x = 82$ kJ/mole

24. Calculate the molality of 500 mL solution of $CuSO_4$ having density of 1.25 g mL^{-1} and having molarity 2×10^{-1} M at 32°C. What is the value of x if the molality calculated is $x \times 10^{-2}$ molal?

Answer (16)

Sol. Molarity = 0.2 $= \dfrac{n_{CuSO_4}}{V_{sol^n}}$

$\Rightarrow \quad n_{CuSO_4} = 0.2 \times \dfrac{500}{1000} = 0.1$

mass of solution = 1.25 × 500 = 625 g

mass of solvent = 625 – 0.1 × (159.5)

= 609.05 g

$$\text{Molality} = \frac{n_{CuSO_4}}{m_{solvent}\,(\text{In kg})}$$

$$= \frac{0.1 \times 1000}{609.05}$$

$$= 0.164$$

$$= 16.4 \times 10^{-2}$$

So, $x \simeq 16$

25. Given $E^{\circ}_{MnO_4^-/Mn^{2+}} = 1.33\,V$

How many of the given species can be oxidised by MnO_4^- in aqueous solution among the following?

(a) Cu [Given $E^{\circ}_{Cu^{2+}/Cu} = 0.34\,V$]

(b) Cr [Given $E^{\circ}_{Cr^{3+}/Cr} = -0.74\,V$]

(c) Ag [Given $E^{\circ}_{Ag^{+}/Ag} = 0.80\,V$]

(d) Au [Given $E^{\circ}_{Au^{+}/Au} = 1.68\,V$]

Answer (3)

Sol. If the standard cell potential is positive, then the reaction is feasible.

(a) $E^{\circ}_{cell} = (-0.34 + 1.33)\,V = 0.99\,V$

(b) $E^{\circ}_{cell} = (0.74 + 1.33)\,V = 2.07\,V$

(c) $E^{\circ}_{cell} = (-0.80 + 1.33)\,V = 0.53\,V$

(d) $E^{\circ}_{cell} = (-1.68 + 1.33)\,V = -0.35\,V$

26. Rate of a reaction is given as : rate = $k[A]^2[B]$. If concentration of both reactants is doubled then rate becomes x times the previous and the overall order of reaction is y, then what is the value of (x + y)?

Answer (11)

Sol. rate = $k[A]^2[B]$

order of reaction = 2 + 1 = 3

So, y = 3

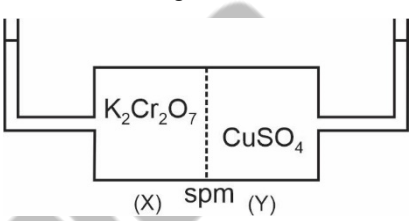
on doubling the concentration

rate becomes $2^2 \cdot 2 = 8$ times

So, x = 8

x + y = 8 + 3 = 11

27. Equimolar solution of $K_2Cr_2O_7$ and $CuSO_4$ are separated by a semi-permeable membrane. How many of the following statements are incorrect?



(i) Green colour of $CuCr_2O_7$ is observed as side (Y)

(ii) Green colour of $CuCr_2O_7$ is observed as side (X)

(iii) Molarity of $K_2Cr_2O_7$ will decrease

(iv) Molarity of $CuSO_4$ will decrease

Answer (3)

Sol. $K_2Cr_2O_7 \rightarrow 2K^+ + Cr_2O_7^{2-}$

$CuSO_4 \rightarrow Cu^{2+} + SO_4^{2-}$

van't Hoff factor for $K_2Cr_2O_7 = 3$

van't Hoff factor for $CuSO_4 = 2$

Therefore, solvent molecules will migrate from (Y) to (X) resulting in decrease of molarity of $K_2Cr_2O_7$.

$\therefore$ (i), (ii) and (iv) are incorrect.

28.

29.

30.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

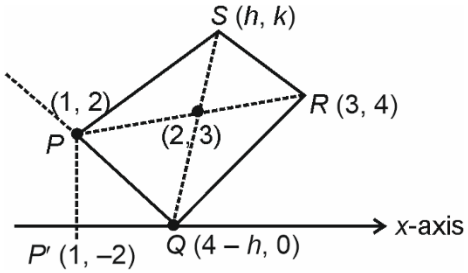
1. A ray of light passing through (1, 2) after reflecting on x-axis at point Q passes through $R(3, 4)$. If $S(h, k)$ is such that $PQRS$ is a parallelogram, then find the value of hk^2.

(1) 90 (2) 84

(3) 96 (4) 108

Answer (2)

Sol.



$\therefore$ $k = 6$ (using diagram)

P' lies on RQ

$$\frac{4}{h-1}=\frac{6}{2}$$

$\Rightarrow$ $4 = 3h - 3$

$\Rightarrow$ $3h = 7$

$$\Rightarrow \quad h=\frac{7}{3}$$

$$hk^2=\frac{7}{3}\times 36=84$$

2. Tetrahedral dice having outcomes (1, 2, 3, 4) has 3 outcomes a, b, c (which are visible). Probability that $ax^2 + bx + c = 0$ has real roots is $\frac{m}{n}$. (m, n are coprime). Then $m + n$ =?

(1) 4 (2) 5

(3) 6 (4) 7

Answer (2)

Sol. $a, b, c \in \{1, 2, 3, 4\}$

And a, b, c are distinct

For real roots of $ax^2 + bx + c = 0$

$b^2 \geq 4ac$

So, we get (a, b, c) as (1, 3, 2),

(2, 3, 1), (1, 4, 2), (2, 4, 1), (1, 4, 3) and (3, 4, 1)

So, total 6 values of (a, b, c) are possible for required condition.

So, required probability = $\frac{6}{4!}=\frac{6}{24}$

$$=\frac{1}{4}=\frac{m}{n}$$

$m + n = 5$

3. A circle passes through (0, 0) and (1, 0) and touches the circle $x^2 + y^2 = 9$. Then the locus of the centre of the circle is

(1) Circle

(2) Parabola

(3) Hyperbola

(4) Straight line

Answer (1)

Sol. Circle will touch internally.

$C_1C_2 = |r_1 - r_2|$

$\sqrt{h^2+k^2} = 3 - \sqrt{h^2+k^2}$

$\Rightarrow 2\sqrt{h^2+k^2} = 3$

$\Rightarrow x^2 + y^2 = \frac{9}{4}$

4. $\vec{A}, \vec{B}$ and $\vec{C}$ are given as

$\vec{A} = \alpha\hat{i} + 4\hat{j} + 5\hat{k}$

$\vec{B} = 2\hat{i} + 5\hat{j} + 6\hat{k}$

$\vec{C} = \vec{A} + \vec{B}$

$|\vec{C}| = |\vec{A} - \vec{B}|$

The value of α and $|\vec{C}|^2$ is equal to

(1) 25, 731 (2) –25, 669

(3) –25, 731 (4) 25, 669

Answer (3)

Sol. $|\vec{C}| = |\vec{A} + \vec{B}|$

$|\vec{C}| = |\vec{A} - \vec{B}|$

$\Rightarrow \vec{A}\cdot\vec{B} = 0$

$\Rightarrow 2\alpha + 20 + 30 = 0$

$\Rightarrow \alpha = -25$

$|\vec{C}|^2 = |\vec{A}|^2 + |\vec{B}|^2 + 2\vec{A}\cdot\vec{B}$

$= \alpha^2 + 16 + 25 + 4 + 25 + 36$

$= 731$

5. If set $A = \{Z : |Z - 1| \le 1\}$ and set $B = \{Z : |Z - 5i| \le |Z - 5|\}$, If $Z = a + ib$, (where $a, b \in I$) then sum of modulus squares of $A \cap B$ is

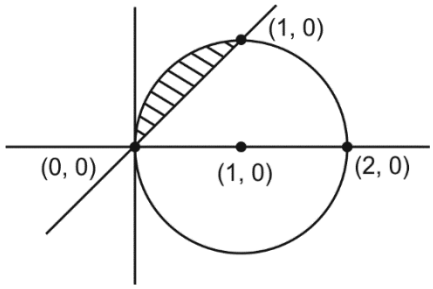
(1) 0 (2) 2

(3) 4 (4) 5

Answer (2)

Sol. $|Z - 1| \le 1$

$|Z - 5i| \le |Z - 5|$



$\Rightarrow Z = a + ib, a, b \in I$

$\Rightarrow (a, b) = \{(0, 0), (1, 1)\}$

$\Rightarrow |Z| = \sqrt{a^2+b^2} \Rightarrow \sqrt{a^2+b^2} \leftarrow \{0, \sqrt{2}\}$

Sum of squares of modulus = $0^2 + (\sqrt{2})^2 = 2$

6. If $\frac{1}{(1+d)(1+2d)} + \frac{1}{(1+2d)(1+3d)} + \ldots + \frac{1}{(1+9d)(1+10d)} = 1.$

The value of $50d$ is ($d > 0$)

(1) 50 (2) 60

(3) 25 (4) 30

Answer (3)

Sol. $\frac{1}{(1+d)(1+2d)} + \frac{1}{(1+2d)(1+3d)} + \frac{1}{(1+3d)(1+4d)} + \ldots + \frac{1}{(1+9d)(1+10d)} = 1.$

$\frac{1}{d}\left[\left(\frac{1}{1+d} - \frac{1}{1+2d}\right) + \left(\frac{1}{1+2d} - \frac{1}{1+3d}\right) + \ldots + \left(\frac{1}{1+9d} - \frac{1}{1+10d}\right)\right] = 1$

$\frac{1}{d}\left[\frac{1}{1+d} - \frac{1}{1+10d}\right] = 1$

$\frac{9d}{d\cdot(1+d)(1+10d)} = 1 \Rightarrow 9 = (10d+1)(d+1)$

$\therefore 10d^2 + 11d - 8 = 0$

$\Rightarrow d = \frac{1}{2}$

$\therefore 50d = \frac{1}{2}$

7. If $f(x) = \begin{cases} \left(\frac{8}{7}\right)^{\frac{\tan 8x}{\tan 7x}} & , \ x \in \left(0, \frac{\pi}{2}\right) \\ a-8 & , \ x = \frac{\pi}{2} \\ (1+|\cot x|)^{\frac{b}{a}|\tan x|} & , \ x \in \left(\frac{\pi}{2}, \pi\right) \end{cases}$

If the function $f(x)$ is continuous at $x = \frac{\pi}{2}$ then $a^2 + b^2$ is equal to

(1) 97 (2) 85

(3) 81 (4) 100

Answer (4)

Sol. $\lim\limits_{x \to \frac{\pi^-}{2}} f(x) = f\left(\frac{\pi}{2}\right) = \lim\limits_{x \to \frac{\pi^+}{2}} f(x)$ for continuity at $x = \frac{\pi}{2}$

$\Rightarrow \lim\limits_{x \to \frac{\pi^-}{2}} \left(\frac{8}{7}\right)^{\left(\frac{\tan 8x}{\tan 7x}\right)}$ Let $x = \frac{\pi}{2} - h$

$\Rightarrow \lim\limits_{h \to 0} \left(\frac{8}{7}\right)^{\frac{\tan(4\pi - 8h)}{\tan\left(3\pi + \frac{\pi}{2} h\right)}} = \lim\limits_{h \to 0} \left(\frac{8}{7}\right)^{\frac{\tan(-8h)}{\cot(h)}} = \left(\frac{8}{7}\right)^0 = 1$

$\Rightarrow a - 8 = 1 \Rightarrow a = 9$

$\lim\limits_{x \to \frac{\pi^+}{2}} (1+|\cot x|)^{\frac{b}{a}|\tan x|}$, $x = \frac{\pi}{2} + h$

$\lim\limits_{h \to 0} (1-\tan h)^{-\frac{b}{9}\cot h} = \lim\limits_{h \to 0} (1-\tan h)^{-\frac{b}{9}\cot h}$

$= \lim\limits_{h \to 0} (1-\tan h)^{\left(\frac{-1}{\tan h}\right)\cdot(-\tan h)\cdot\left(\frac{-b}{9}\cot h\right)}$

$= e^{\frac{b}{9}} = 1 \qquad \Rightarrow b = 0$

$\Rightarrow a^2 + b^2 = 81 + 0 = 81$

8. If $\cos\theta\cos(60-\theta)\cos(60+\theta) \le \frac{1}{8}$. Find the sum of values of θ for which $\cos 3\theta$ is maximum

(1) 6π (2) 4π

(3) 3π (4) 7π

Answer (1)

Sol. $\cos\theta\cos(60-\theta)\cos(60+\theta) \le \frac{1}{8}$

$\Rightarrow \frac{1}{4}\cos 3\theta \le \frac{1}{8}$

$\cos 3\theta \le \frac{1}{2}$

$\because \cos 3\theta \le$ maximum

$\Rightarrow \cos 3\theta = \frac{1}{2}$, $3\theta = 2n\pi \pm \frac{\pi}{3}$, $n \in I$

$\therefore \ \theta = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9}$

$= \frac{54\pi}{9} = 6\pi$

9. If $(x^2 + y^2)dy = 5xy\,dx$. Find the general solution of DE.

(1) $\frac{5}{8}\ln\left|\frac{y-2x}{y+2x}\right| = \frac{1}{4}\ln\left|\frac{y}{x}\right| - \ln|x| + C$

(2) $\frac{5}{8}\ln\left|\frac{y+2x}{y-2x}\right| = \frac{1}{4}\ln\left|\frac{y}{x}\right| - 2\ln|x| + C$

(3) $\frac{5}{8}\ln\left|\frac{y-2x}{y+2x}\right| = -\frac{1}{4}\ln\left|\frac{y}{x}\right| + \ln|x^2| + C$

(4) $\frac{5}{4}\ln\left|\frac{y-2x}{y+2x}\right| = \frac{1}{4}\ln\left|\frac{y}{x}\right| + 2\ln|x| + C$

Answer (1)

Sol. $\frac{dy}{dx} = \frac{5xy}{x^2+y^2}$

let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{5x \cdot vx}{x^2 + v^2x^2} = \frac{5v}{1+v^2}$

$\Rightarrow x\frac{dv}{dx} = \frac{5v}{1+v^2} - v \Rightarrow \frac{4v - v^3}{1+v^2} = \frac{v\left(4-v^2\right)}{1+v^2}$

$$\int\frac{1+v^2}{v(2-v)(2+v)}dv=\int\frac{dv}{x}\Rightarrow\int\frac{1+v^2}{v(v-1)(v+2)}=-\int\frac{dx}{x}$$

Let $\frac{1+v^2}{v(v-2)(v+2)}=\frac{A}{v}+\frac{B}{v-2}+\frac{C}{v+2}$

$B=\frac{5}{8},\quad A=\frac{1}{-4},C=\frac{5}{8}$

$\therefore\ \int\frac{1+v^2}{v(2-v)(2+v)}\,dv=-\frac{dx}{x}$

$=\int\left(-\frac{1}{4v}+\frac{5}{8(v-2)}+\frac{5}{8(v+2)}\right)dv=-\int\frac{dx}{x}$

$=-\frac{1}{4}\ln|v|+\frac{5}{8}\ln\left|\frac{(v-2)}{(v+2)}\right|=-\ln|x|+C$

$\Rightarrow-\frac{1}{4}\ln\left|\frac{y}{x}\right|+\frac{5}{8}\ln\left|\frac{y-2x}{y+2x}\right|=-\ln|x|+C$

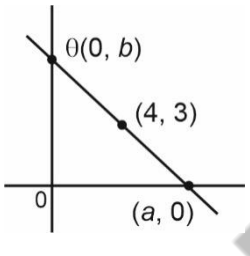
10. Find the maximum area (in sq. units) of a line pass through (4, 3)

(1) 30 (2) 31

(3) 24 (4) 32

Answer (3)

Sol.



$\frac{x}{a}+\frac{y}{b}=1$

As line passes through (4, 3)

$\frac{4}{a}+\frac{3}{b}=1$

$4b + 3a = ab$

$3a - ab = -4b$

$a=\frac{-4b}{3-b}=\frac{4b}{b-3}$

Now area of triangle = $\left|\frac{1}{2}\times a\times b\right|$

$=\left|\frac{1}{2}\times\frac{4b}{b-3}\times b\right|$

$\Rightarrow\ f(b)=\frac{2b^2}{b-3}$

$\Rightarrow\ f'(b)=\frac{2b(b-6)}{(b-3)^2}=0$

[At $b = 6$ we get maximum area]

i.e. $\frac{1}{2}\times\frac{4\times6}{3}\times6=24$ sq. units

11. $\int\frac{2+\tan x}{3+\tan x}dx=\frac{1}{2}\lambda x-\frac{1}{10}\log|a\sin x+b\cos x|$.

Then the value of $\lambda+\frac{a}{b}$ is equal to

(1) $\frac{13}{5}$ (2) $\frac{15}{26}$

(3) $\frac{26}{15}$ (4) $\frac{5}{13}$

Answer (3)

Sol. $\int\frac{2+\tan x}{3+\tan x}dx$

$=\int\left(1-\frac{1}{3+\tan x}\right)dx$

$=x-\int\frac{\cos x}{3\cos x+\sin x}dx$

Consider $I'=\int\frac{\cos x}{3\cos x+\sin x}dx$

$\cos x=A(3\cos x+\sin x)+B(-3\sin x+\cos x)$

$A - 3B = 0$

$3A + B = 1$

$\Rightarrow \quad B = \frac{1}{10}, A = \frac{3}{10}$

$I' = \frac{3}{10}x + \frac{1}{10}\log|3\cos x + \sin x|$

$I = \frac{7x}{10} - \frac{1}{10}\log|3\cos x + \sin x|$

$= \frac{1}{2}\left(\frac{7x}{5}\right) - \frac{1}{10}\log|3\cos x + \sin x|$

$a = 1, b = 3, \ \lambda = \frac{7}{5}$

$\therefore \quad \lambda + \frac{a}{b}$

$= \frac{7}{5} + \frac{1}{3} = \frac{26}{15}$

12. If $y^2 = 4x$ and $x^2 + y^2 = 5$, then the area of smaller part of the circle cut by parabola is

(1) $\frac{2}{3} + \frac{5\pi}{2} + \sin^{-1}\frac{1}{\sqrt{5}}$

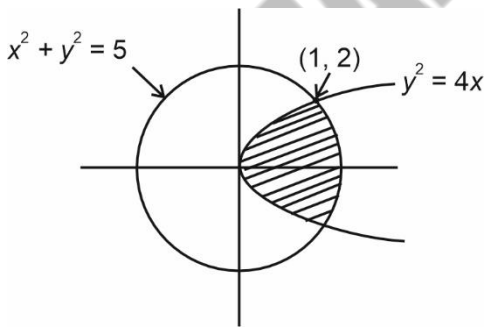
(2) $\frac{2}{3} + \frac{5\pi}{2} - 5\sin^{-1}\frac{1}{\sqrt{5}}$

(3) $\frac{2}{3} + \frac{5\pi}{4} + \sin^{-1}\frac{1}{\sqrt{5}}$

(4) $\frac{2}{3} + \frac{5\pi}{4} - 5\sin^{-1}\frac{1}{\sqrt{5}}$

Answer (2)

Sol.



Area = $2\left[\int_0^1 2\sqrt{x}dx + \int_1^{\sqrt{5}}\sqrt{5-x^2}dx\right]$

$= 2\left[\left(\frac{4}{3}x^{\frac{3}{2}}\right)_0^1 + \left(\frac{x}{2}\sqrt{5-x^2} + \frac{5}{2}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)\right)_1^{\sqrt{5}}\right]$

$= 2\left[\frac{4}{3} + \frac{5\pi}{4} - 1 - \frac{5}{2}\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)\right]$

$= \left(\frac{2}{3} + \frac{5\pi}{2} - 5\sin^{-1}\frac{1}{\sqrt{5}}\right)$ sq. unit

13. If $\frac{ydy}{dx} + 3 = \frac{2dy}{dx}$ is a parabola passing through (1, 0). Then, the vertices of the parabola satisfy the equation

(1) $3x + 2y = 6$ (2) $3x + 2y = -6$

(3) $3x + 2y = 9$ (4) $3x + 2y = -9$

Answer (3)

Sol. $ydy + 3dx = 2dy$

$\Rightarrow \quad \frac{y^2}{2} + 3x = 2y + c\Big|_{(1,0)}$

$\Rightarrow \quad c = 3$

$\therefore \quad (y-2)^2 = -6\left(x - \frac{5}{3}\right)$

$\therefore \quad \text{Vertex} = \left(\frac{5}{3}, 2\right)$

14. If α, β are the roots of the equation $x^2 - 2\sqrt{2}x + 1 = 0$ then equation whose roots are $\alpha^4 + \beta^4$ and $\frac{\alpha^6 + \beta^6}{6}$ is

(1) $x^2 - 66x + 1110 = 0$

(2) $x^2 - 33x + 1122 = 0$

(3) $x^2 - 34x + 1122 = 0$

(4) $x^2 - 67x + 1122 = 0$

Answer (4)

Sol. $x^2 - 2\sqrt{2}x + 1 = 0 \begin{matrix}\nearrow \alpha \\ \searrow \beta\end{matrix}$

$\alpha^4 + \beta^4, \dfrac{\alpha^6 + \beta^6}{6}$

$\Rightarrow \alpha + \beta = 2\sqrt{2}$, $\alpha\beta = 1$

$\Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = 8 \Rightarrow \alpha^2 + \beta^2 = 6$

$\Rightarrow \alpha^4 + \beta^4 + 2\alpha^2\beta^2 = 36 \Rightarrow \alpha^4 + \beta^4 = 34$

$\Rightarrow \dfrac{\alpha^6 + \beta^6}{6} = \dfrac{(\alpha^2 + \beta^2)}{6}(\alpha^4 + \beta^4 - \alpha^2\beta^2)$

$= \left(\dfrac{6}{6}\right)[34 - 1] = 33$

$\Rightarrow$ roots of the equation are 33, 44

$\Rightarrow$ equation is

$x^2 - (33 + 34)x + 33.34 = 0$

$\Rightarrow x^2 - 67x + 1122 = 0$

15. If $f(x) = x^2 - 8$, $g(x) = \dfrac{x}{x-9}$ and $a = f(g(10))$ and $b = g(f(3))$ and e and l be eccentricity and length of latus rectum of conic $\dfrac{x^2}{|a|} + \dfrac{y^2}{|b|} = 1$, then $(92l^2 + 46e^2)$ is

(1) 48 (2) 46

(3) 45 (4) 92

Answer (2)

Sol. $f(x) = x^2 - 8$

$g(x) = \dfrac{x}{x-9}$

$\Rightarrow f(g(10)) = f(10) = a = 92$

$g(f(3)) = g(1) = b = -\dfrac{1}{8}$

$\Rightarrow$ conic :

$\dfrac{x^2}{92} + \dfrac{y^2}{1/8} = 1$

$l(\text{L}\cdot\text{R}) = \dfrac{2(1/8)}{\sqrt{92}} = \dfrac{1}{4\sqrt{92}} \Rightarrow l^2 = \dfrac{1}{16(92)}$

$e^2 = 1 - \dfrac{1/8}{92} = \dfrac{92\times 8 - 1}{92\times 8}$

$\Rightarrow 92l^2 + 46e^2 = \dfrac{1}{16} + \dfrac{735}{16}$

$= \dfrac{736}{16} = 46$

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. The remainder when $(428)^{2024}$ is divided by 21 is

Answer (1)

Sol. $(428)^{2024} \equiv 8^{2024} \pmod{21}$

$(8^2) \equiv 1 \pmod{21}$

$(8)^{2024} \equiv 1 \pmod{21}$

$\therefore$ remainder = 1.

22. If A is 3×3 matrix,

$\det(3adj(2adjA)) = 2^{-13} \cdot 3^{-10}$ and

$\det'(3adj(2A)) = 2^m \cdot 3^n$, then $2m + 2n$ is equal to

Answer (14.00)

Sol. $P \Rightarrow |3adj(2A)| = 3^3 \cdot |adj2A| = 3^3 \cdot |2A|^2 = 3^3 \cdot 2^6 |A|^2$

$|3adj(2adj(A))| = 3^3 \cdot |adj(2adjA)| = 3^3 \cdot |(2adjA)|^2$

$= 3^3 \cdot (2^3)^2 |adjA|^2$

$= 3^3 \cdot 2^6 (|A|^2) = 3^3 \cdot 2^6 |A|^4$

$= 2^{-13} \cdot 3^{-10}$

$\Rightarrow |A|^4 = 3^{-13} \cdot 2^{-19}$

$\Rightarrow |A|^2 = 3^{-6.5} \cdot 2^{-9.5}$

$\Rightarrow P = 3^3 \cdot 2^6 \cdot 3^{-6.5} \cdot 2^{-9.5} = 3^{-3.5} \cdot 2^{-3.5} = 2^m \cdot 3^n$

$\Rightarrow m = 3.5, n = 3.5$

$\Rightarrow 2m + 2n = 14$

23. If $f(x) = 3ax^3 + bx^2 + cx + 41$ and $f(1) = 41$,

$f'(1) = 2$ and $f''(1) = 4$ then $(a^2 + b^2 + c^2)$ is

Answer (08.00)

Sol. $f(1) = 3a + b + c + 41 = 41 \Rightarrow 3a + b + c = 0$

$f'(x) = 9ax^2 + 2bx + c \Rightarrow f'(1) = 9a + 2b + c = 2$

$f''(x) = 18ax + 2b \Rightarrow f''(1) = 18a + 2b = 4$

$\Rightarrow (4 - 9a) + c = 2$

$3a + c + (2 - 9a) = 0$

$\Rightarrow c - 6a + 2 = 0 \Rightarrow a = 0$

$c - 9a + 2 = 0 \quad c = -2$

$b = 2$

$\Rightarrow a^2 + b^2 + c^2 = 0 + 4 + 4 = 8$

24. If domain of $f(x) = \sin^{-1}\left(\frac{x-1}{2x+3}\right)$ is $R - (\alpha, \beta]$ then $12\alpha\beta$ is equal to

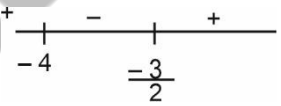
Answer (32.00)

Sol. $f(x) = \sin^{-1}\left(\frac{x-1}{2x+3}\right)$

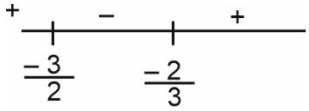
$\Rightarrow 2x + 3 \neq 0$ and $-1 \le \frac{x-1}{2x+3} \le 1$

$\Rightarrow \frac{x-1-2x-3}{2x+3} \le 0 \Rightarrow \frac{x+4}{2x+3} \ge 0$

$\Rightarrow x \in (-\infty, -4] \cup \left(\frac{-3}{2}, \infty\right)$



Also, $0 \le \frac{x-1+2x+3}{2x+3} \Rightarrow \frac{3x+2}{2x+3} \ge 0$



$\Rightarrow x \in \left(-\infty, \frac{-3}{2}\right) \cup \left[\frac{-2}{3}, \infty\right)$

$\Rightarrow$ domain $\Rightarrow x \in (-\infty, -4] \cup \left(\frac{-2}{3}, \infty\right)$

$\Rightarrow R - \left(-4, \frac{-2}{3}\right]$

$\Rightarrow \alpha = -4, \quad \beta = \frac{-2}{3}$

$\Rightarrow 12 \times (-4)\left(\frac{-2}{3}\right) = 32$

25. $A = \{2, 4, 6, 8\}$

$B = \{3, 7, 6, 9\}$

$R : A \times B \to A \times B$ such that

$(a_1, b_1)\ R(a_2, b_2) \Rightarrow a_1 + a_2 = b_1 + b_2$

$(a_1, b_1) \in A$ and $(a_2, b_2) \in B$.

Then the number of elements in the relation is

Answer (9)

Sol. $A = \{2, 4, 6, 8\}$

$B = \{3, 7, 6, 9\}$

$(a_1, b_1)\ R(a_2, b_2)$ when $a_1 + a_2 = b_1 + b_2$

(2, 6) R(7, 3)

(2, 4) R(9, 7)

(4, 8) R(7, 3)

(4, 6) R(9, 7)

(6, 2) R(3, 7)

(6, 4) R(7, 9)

(6, 8) R(9, 7)

(8, 4) R(3, 7)

(8, 6) R(7, 9)

So total 9 elements are in the relation.

26. If $f(m + n) = f(m) + f(n)$, $f(1) = 1$

then $\sum_{k=1}^{2022} f(\lambda + k) \le (2022)^2$, then $\lambda_{max} \in I$ is

Answer (1010)

Sol. $f(m + n) = f(m) + f(n)$

$\Rightarrow f(x) = kx$

$\because f(1) = 1 \Rightarrow k = 1$

$\therefore f(x) = x$

Now, $\sum_{k=1}^{2022} f(\lambda + k) \le (2022)^2$

$\Rightarrow \sum_{k=1}^{2022} (\lambda + k) \le (2022)^2$

$= \underbrace{\lambda + \lambda + ... + \lambda}_{2022} + (1 + 2 + 3 + ... + 2022) \le (2022)^2$

$2022\lambda + \frac{2022 \times 2023}{2} \le (2022)^2$

$\lambda \le \left(2022 - \frac{2023}{2}\right)$

$\Rightarrow \lambda \le \frac{2021}{2}$

$\lambda \le 1010.5$

$\therefore \lambda_{max} = 1010$

27.

28.

29.

30.