SMART ACHIEVERS
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JEE Main - 2024 Session - 2
Answers & Solutions

(Physics, Chemistry and Mathematics)

08 - April - 2024 - Shift - 1

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PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. The correct expression for Bernoulli's theorem is (symbols have their usual meaning)

(1) $P+\rho gh+\frac{1}{2}\rho v^2=\text{constant}$

(2) $P+\frac{1}{2}\rho gh+\frac{1}{2}\rho v^2=\text{constant}$

(3) $P+\rho gh+\rho v^2=\text{constant}$

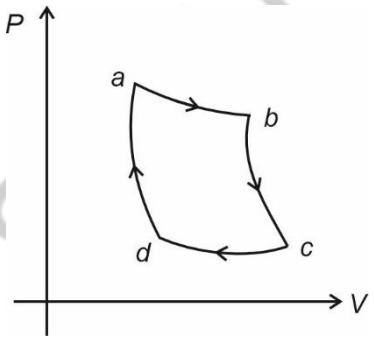
(4) $P+2\rho gh+\rho v^2=\text{constant}$

Answer (1)

Sol. According to Bernoulli's theorem

$$P+\rho gh+\frac{1}{2}\rho v^2=\text{constant}$$

2. The PV curve shown in diagram consists of two isothermal & two adiabatic curves. Then



(1) $\frac{V_a}{V_d}=\frac{V_b}{V_c}$ (2) $\frac{V_a}{V_d}=\left(\frac{V_b}{V_c}\right)^{-1}$

(3) $\frac{V_a}{V_d}=\left(\frac{V_c}{V_b}\right)^2$ (4) $\frac{V_a}{V_d}=\frac{V_c}{V_b}$

Answer (1)

Sol. $T_a = T_b$, $T_c = T_b$

$T_bV_b^{\gamma-1} = T_cV_c^{\gamma-1}$

$T_aV_a^{\gamma-1} = T_dV_d^{\gamma-1}$

$\frac{V_b}{V_a}=\frac{V_c}{V_d}$

$\therefore \quad \frac{V_a}{V_d}=\frac{V_b}{V_c}$

3. In a series LCR circuit, the value of resistance as well as $(X_L - X_C)$ is halved, then the new current amplitude (I_2) will satisfy (I_1 is old current amplitude)

(1) $I_2 = 2I_1$

(2) $I_2 = 0$

(3) $I_2=\frac{I_1}{2}$

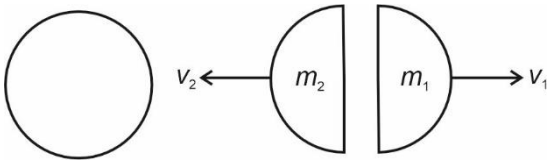
(4) $I_2 = I_1$

Answer (1)

Sol. $I_1=\frac{V_0}{\sqrt{R^2+(X_L-X_C)^2}}$,

$$I_2=\frac{V_0}{\sqrt{\left(\frac{R}{2}\right)^2+\left(\frac{X_L-X_C}{2}\right)^2}}=2I_1$$

4. A ball initially at rest breaks in two masses m_1 and m_2 that move with speed v_1 and v_2 respectively as shown in figure.



The ratio of kinetic energy of right mass to the left mass is

(1) $\frac{m_1}{m_2}$ (2) $\frac{m_2}{m_1}$

(3) $\frac{m_1^2}{m_2^2}$ (4) $\frac{m_2^2}{m_1^2}$

Answer (2)

Sol. From momentum conservation :

$|m_1v_1| = |m_2v_2| = p$

$$K_1 = \frac{p^2}{2m_1}$$

$$K_2 = \frac{p^2}{2m_2}$$

$$\frac{K_1}{K_2} = \frac{m_2}{m_1}$$

5. Critical angle for a pair of medium is given to be 45°. Find the ratio of the refractive index of rarer medium to denser.

(1) $1:\sqrt{3}$

(2) $1:\sqrt{2}$

(3) 1 : 2

(4) 2 : 1

Answer (2)

Sol. $\sin C = \frac{\mu_{Rarer}}{\mu_{Denser}}$

$\Rightarrow \sin 45° = \frac{\mu_{Rarer}}{\mu_{Denser}}$

$\Rightarrow \mu_R : \mu_D = 1:\sqrt{2}$

6. A ball of mass 150 g moving with speed 20 m/s is catched in 0.1 sec. Find the average force exerted by the hands.

(1) 40 N (2) 60 N

(3) 20 N (4) 30 N

Answer (4)

Sol. $F_{avg} = \frac{mv}{t} = \frac{0.15 \times 20}{0.1} = 30\ N$

7. For a number given as ($a \times 10^b$) the order of number is

(1) b if $a \geq 5$

(2) b if $a \leq 5$

(3) b if $5 < a < 10$

(4) a when $b \geq 5$

Answer (2)

Sol. Rules for scientific notation

8. An electron and a proton are having same kinetic energy. Find ratio of their linear momentum. (mass of electron = 9.1×10^{-31} kg, mass of proton = 1.67×10^{-27} kg)

(1) 1.67×10^{-3} kg.m/s

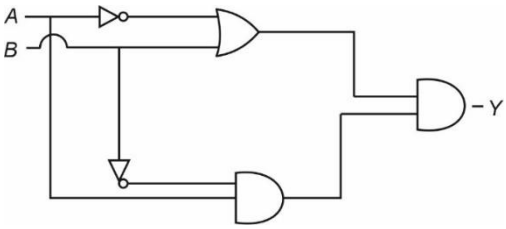
(2) 1.33×10^{-2} kg.m/s

(3) 1.23×10^{-2} kg.m/s

(4) 2.33×10^{-2} kg.m/s

Answer (4)

Sol. $\frac{P_e^2}{2m_e} = \frac{P_p^2}{2m_p} \Rightarrow \frac{P_e}{P_p} = \sqrt{\frac{m_e}{m_p}} = 2.33 \times 10^{-2}$

9. 

For given logic circuit, correct relation between input (x, y) and output (Y) is

(1) $y = 0$ (2) $y = A \cdot \overline{B}$

(3) $y = A + \overline{B}$ (4) $y = \overline{A} \cdot B$

Answer (1)

Sol. $\left(\overline{A} + B\right) \cdot \left(A \cdot \overline{B}\right)$

$\overline{A} \cdot \left(A\overline{B}\right) + B \cdot \left(A \cdot \overline{B}\right)$

= zero

10. Two organ pipes having same length, one is open while other is closed. Find ratio of 7th overtone of these organ pipes.

(1) $\frac{15}{16}$ (2) $\frac{16}{15}$

(3) $\frac{14}{15}$ (4) $\frac{13}{14}$

Answer (2)

Sol. $n_o = (n+1)\frac{V}{2l}$, $n_c = (2n+1)\frac{V}{4l} = \frac{15V}{4l}$

$= \frac{4V}{l}$

11. Two planets of mass m_1 and m_2 are revolving around their sun in radius of their orbits r_1 and r_2 respectively. Angular momentum of planets are in ratio of 3 then $\frac{T_1}{T_2}$ is (T_1 and T_2 are periods of revolutions)

(1) $27\left(\frac{m_2}{m_1}\right)^3$ (2) $\frac{1}{27}\left(\frac{m_2}{m_1}\right)^3$

(3) $\left(\frac{r_1}{r_2}\right)^3$ (4) $\left(\frac{r_2}{r_1}\right)^{3/2}$

Answer (2)

Sol. $m_1 v_1 r_1 = L \quad \Rightarrow \quad v_1 = \frac{L}{m_1 r_1}$

$m_1 v_2 r_2 = 3L \quad \Rightarrow \quad v_2 = \frac{3L}{m_2 r_2}$

$$\frac{T_1}{T_2} = \frac{\frac{2\pi r_1}{v_1}}{\frac{2\pi r_2}{v_2}} = \frac{r_1 v_2}{r_2 v_1} = \frac{r_1}{r_2}\frac{m_1 r_1}{L}\frac{3L}{m_2 r_2}$$

$$= \frac{3m_1}{m_2}\cdot\frac{r_1^2}{r_2^2}$$

Also, $\frac{T_1}{T_2} = \frac{r_1^{3/2}}{r_2^{3/2}}$

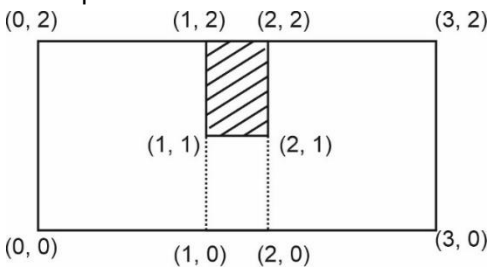
$$\frac{T_1^{4/3}}{T_2^{4/3}} = \frac{r_1^2}{r_2^2}$$

$$\frac{T_1}{T_2} = \frac{3m_1}{m_2}\cdot\frac{(T_1)^{4/3}}{(T_2)^{4/3}}$$

$$\frac{m_2}{3m_1} = \left(\frac{T_1}{T_2}\right)^{1/3}$$

$$\frac{T_1}{T_2} = \frac{1}{27}\left(\frac{m_2}{m_1}\right)^3$$

12. From a rectangular sheet, the shaded portion is removed. Find the co-ordinates of centre of mass after the portion has been removed.



(1) (1.5, 0.9) (2) (2.5, 1.5)

(3) (1, 1) (4) (2, 2)

Answer (1)

Sol. $x_{com} = \frac{6\sigma \times 1.5 + (-\sigma \times 1.5)}{5\sigma} = 1.5$

$y_{com} = \frac{6\sigma \times 1 + (-\sigma \times 1.5)}{5\sigma} = 0.9$

13. An $\bar{e}$ passing through cross magnetic and electric field undergoes zero deviation. If the kinetic energy of electron is 5 μeV and magnetic field is B_0, electric field will be

(1) 800 B_0 (2) 2320 B_0

(3) 1320 B_0 (4) 2000 B_0

Answer (3)

Sol. $E = Bv$

$$\frac{1}{2}mv^2 = 5\times10^{-6}\times1.6\times10^{-19}$$

$$\therefore\ v = 1.32\times10^3 \text{ m/s}$$

E = 1320 B_0.

14. Which of the following is incorrect for paramagnetic materials?

(1) They are strongly attracted by magnetic field

(2) Magnetic susceptibility is slightly more than zero

(3) They align in direction of magnetic field

(4) None of the above

Answer (1)

Sol. Theory based.

15. If radius of earth reduced by one fourth of it's present value, then duration of days will be

(1) 13 hours and 30 min (2) 13 hours and 20 min

(3) 18 hours and 20 min (4) 16 hours and 10 min

Answer (1)

Sol. From conservation of angular momentum

$$\frac{2}{5}mR^2\omega_0 = \frac{2}{5}m\times\frac{9R^2}{16}\omega$$

$$\frac{\omega_0}{\omega} = \frac{9}{16}$$

$$T = T_0\left(\frac{9}{16}\right)$$

= 13 hours, 30 min

16. An electromagnetic radiation of intensity 360 W/cm^2 is incident normally on a non-reflecting surface having area A. Average force on the surface is found to be 2.4 × 10^{-4} N. Find the value of A.

(1) 0.02 m^2 (2) 0.2 m^2

(3) 2 m^2 (4) 20 m^2

Answer (1)

Sol. $F = \frac{IA}{C} \Rightarrow A$= 0.02 m^2

17. A solenoid of 10 turns, cross section 36 cm^2 and of resistance 10 Ω is placed in magnetic field which is varying at constant rate of 0.5 T / sec. Find power of heat dissipation.

(1) 1.8 W (2) 3.8 W

(3) 2.34 W (4) 7.6 W

Answer (3)

Sol. $V = \varepsilon = \frac{d\phi}{dt} = \frac{dB}{dt}$ NA

$$= 0.5\times10\times36\times10^{-4}$$

$$P = \frac{V^2}{R} = \frac{(0.5)^2\times10^2\times(36)^2\times10^{-8}}{10\times10\times10^{-6}}$$

$$= \frac{1}{4}\times36\times36\times10^{-2}$$

= 2.34 W

18. The diameter of a sphere having mass 8.635 gm is measured by a vernier scale. 10 divisions of vernier scale coincides with 9 divisions of main scale and one main scale division is 1 mm. The reading of main scale is 2 cm and 2 divisions of vernier coincide with a main scale division, the density of the sphere is

(1) 2.2 g/cm^3 (2) 2 g/cm^3

(3) 2.5 g/cc (4) 1.75 g/cm^3

Answer (2)

Sol. LC = 0.1 mm

d = 2 cm + 2 × 0.1 mm

= 20.2 mm

= 2.02 cm

$$\rho = \frac{m}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3} = \frac{8.635}{\frac{4}{3}\pi(1.01)^3} = 2 \text{ g/cm}^3$$

19. Three particles having different masses have same momentum.

Find the ratio of their kinetic energy.

(m_1 = 400 gm, m_2 = 1.2 kg, m_3 = 1.6 kg)

(1) 1 : 2 : 3

(2) 3 : 2 : 1

(3) 2.5 : 0.8 : 0.6

(4) 2.8 : 0.6 : 0.8

Answer (3)

Sol. $KE = \frac{P^2}{2m}$

$$KE_1 : KE_2 : KE_3 = \frac{1}{0.4} : \frac{1}{1.2} : \frac{1}{1.6}$$

$$= 2.50 : 0.83 : 0.63$$

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. In a clock, second hand and minute hand are of 75 cm and 60 cm respectively. After 30 minutes, ratio of distance travelled by tip of second hand to that of minute hand is *x*. Find *x*.

Answer (75)

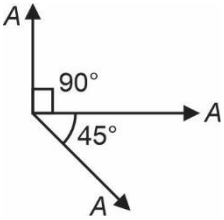
Sol. 30 minutes $\Rightarrow$ 30 revol. by second hand

$\frac{1}{2}$ revol. by minute hand

$$\text{Ratio} = \frac{2\pi r_s \times 30}{2\pi r_m \times \frac{1}{2}}$$

$$= \frac{75}{60} \times 60 = 75$$

22. If the resultant of the vectors shown is $A\sqrt{x}$, find *x*.



Answer (3)

Sol. $R = \sqrt{\left(A\sqrt{2}\right)^2 + (A)^2} = A\sqrt{3}$

23. In a diffraction pattern of a monochromatic light of wavelength 6000 pm, slit width is 3 mm. If the angular position of 2nd minima is $N \times 10^{-6}$ radians, find *N*.

Answer (4)

Sol. $\theta = \frac{2\lambda}{a} = \frac{2 \times 6000 \times 10^{-12}}{3 \times 10^{-3}} = 4 \times 10^{-6}$ rad

24. The ratio of specific heat at constant volume of one mole monoatomic gas to the one mole diatomic gas is given as $\frac{a}{b}$ where *a* and *b* are co-prime number, then find (*a* + *b*).

Answer (8)

Sol. $(C_V)_1 = \frac{3}{2}R$

$$(C_V)_2 = \frac{5}{2}R$$

$$\frac{(C_V)_1}{(C_V)_2} = \frac{3}{2} \times \frac{2}{5} = \frac{3}{5}$$

25.

26.

27.

28.

29.

30.

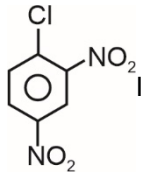
CHEMISTRY

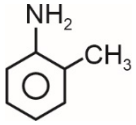
SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Consider following statements:

Statement-I:  IUPAC name is 4-chloro-1, 3-dinitrobenzene.

Statement-II:  IUPAC name is 2-methylaniline.

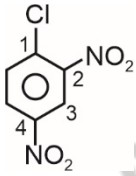
(1) Both statement-I and statement-II are correct

(2) Statement-I is correct, statement-II is incorrect

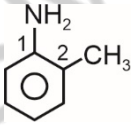
(3) Statement-I is incorrect, statement-II is correct

(4) Both statement-I and statement-II are incorrect

Answer (3)

Sol. Statement-I  1-chloro-2,4-dinitrobenzene

$\Rightarrow$ Statement-I is incorrect

Statement-II  2-methyl aniline

$\Rightarrow$ Statement-II is correct

2. We have two complexes $[Fe(H_2O)_6]^{2+}$ and $[Cu(H_2O)_6]^{2+}$, the magnetic properties respectively are

(1) Diamagnetic and Diamagnetic

(2) Paramagnetic and Paramagnetic

(3) Diamagnetic and Paramagnetic

(4) Paramagnetic and Diamagnetic

Answer (2)

Sol. $[Fe(H_2O)_6]^{2+} \Rightarrow Fe^{2+} \Rightarrow 3d^6 \Rightarrow t_{2g}^4 eg^2 \Rightarrow n = 4$

Paramagnetic

$[Cu(H_2O)_6]^{2+} \Rightarrow Cu^{+2} \Rightarrow 3d^9 \Rightarrow t_{2g}^6 eg^3 \Rightarrow n = 1$

paramagnetic

3. Match the following

	Column-I (Molecule)		**Column-II (Shape)**
(i)	NH_3	(p)	Trigonal bipyramidal
(ii)	BrF_5	(q)	Tetrahedral
(iii)	PCl_5	(r)	Pyramidal
(iv)	CCl_4	(s)	Square pyramidal

(1) (i)-(q), (ii)-(p), (iii)-(s), (iv)-(r)

(2) (i)-(s), (ii)-(r), (iii)-(q), (iv)-(p)

(3) (i)-(r), (ii)-(s), (iii)-(p), (iv)-(q)

(4) (i)-(r), (ii)-(s), (iii)-(q), (iv)-(p)

Answer (3)

Sol. $NH_3 \rightarrow$ Pyramidal (sp^3)

$BrF_5 \rightarrow$ Square pyramidal (sp^3d^2)

$PCl_5 \rightarrow$ Trigonal bipyramidal (sp^3d)

$CCl_4 \rightarrow$ Tetrahedral (sp^3)

4. Statement-I : Stability of +1 oxidation state increases as Ga< In < Tl

Statement-II : Stability of +1 oxidation state increases down the group due to inert pair effect.

(1) Statement-I and Statement-II both are correct

(2) Statement-I and Statement-II both are incorrect

(3) Statement-I is correct and Statement-II is incorrect

(4) Statement-I is incorrect and Statement-II is correct

Answer (1)

Sol. +1 oxidation state for group 13 elements increases down the group due to inert pair effect.

5. $CoCl_3.xNH_3$ on reaction with excess $AgNO_3$(aq.) gives two mole of AgCl as precipitate. Summation of oxidation state of Co in $CoCl_3.xNH_3$ and x is :

(1) 7 (2) 8

(3) 9 (4) 10

Answer (2)

Sol. $CoCl_3.xNH_3 \xrightarrow{AgNO_3} 2AgCl\downarrow$

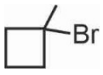
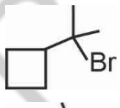
So, one Cl-atom is inside co-ordination sphere.

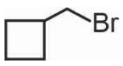
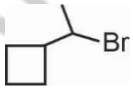
$\left[\overset{+3}{Co}(NH_3)_5Cl\right]Cl_2$

$\Rightarrow$ x = 5

So, (O.N. + x) = 5 + 3 = 8

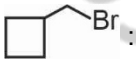
6. The molecule which will undergo S_N2 reaction with the fastest rate?

(1)  (2) 

(3)  (4) 

Answer (3)

Sol. Rate of S_N2 increases with decrease in steric hinderance near the leaving group.

 : 1° halide : least hindered

7. $x \rightleftharpoons y$; $k_1 = 1$

$y \rightleftharpoons z$; $k_2 = 2$

$z \rightleftharpoons w$; $k_3 = 4$

Find k_{eq} for $x \rightleftharpoons w$

(1) 12 (2) 8

(3) 2 (4) 4

Answer (2)

Sol. $x \rightleftharpoons y$; $k_1 = 1$ …(i)

$y \rightleftharpoons z$; $k_2 = 2$ …(ii)

$z \rightleftharpoons w$; $k_3 = 4$ …(iii)

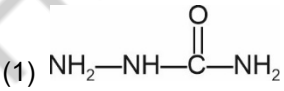
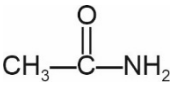
On adding equation (i), (ii) and (iii)

$x \rightleftharpoons w$

$k_{eq} = k_1 \times k_2 \times k_3$

$= 1 \times 2 \times 4 = 8$

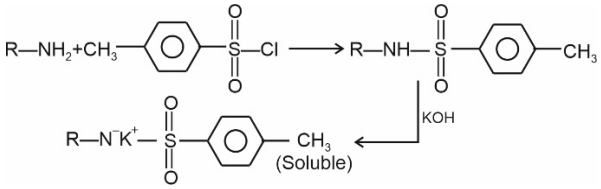
8. Which of the following compounds will not give Hinsberg's Test?

(1)  (2) 

(3) $CH_3—CH_2—NH_2$ (4) $CH_3—NH—CH_3$

Answer (2)

Sol. Hinsberg's Test is



9. Electron and proton have same de-Broglie wavelength. What is the ratio of their kinetic energy $\left(\text{i.e. } \frac{KE_e}{KE_P} = ?\right)$ $\left(\text{Given } \frac{M_e}{M_P} = \frac{1}{1836}\right)$

(1) 1836 (2) $\sqrt{1836}$

(3) $\frac{1}{1836}$ (4) $\frac{1}{\sqrt{1836}}$

Answer (1)

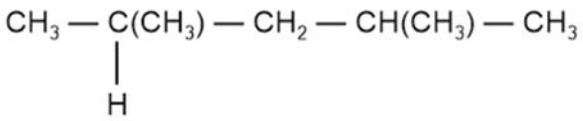
Sol. $\lambda_e = \lambda_P$

$$\Rightarrow \frac{h}{\sqrt{2M_e KE_e}} = \frac{h}{\sqrt{2M_P KE_P}}$$

$$\Rightarrow (M_e \times KE_e) = (M_P \times KE_P)$$

$$\frac{KE_e}{KE_P} = \frac{M_P}{M_e} = 1836$$

10. Total number of secondary carbon atom present in given compound is

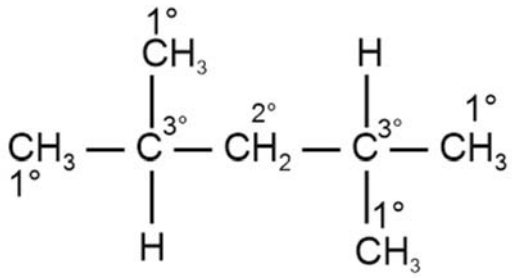


(1) 1 (2) 2

(3) 3 (4) 4

Answer (1)

Sol.



11. Which one of the following statements regarding D-Glucose is incorrect?

(1) It does not give Schiff's test.

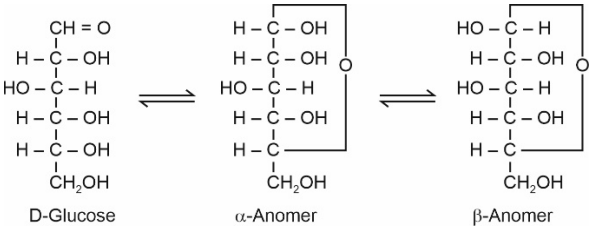
(2) It has asymmetrical C-atoms.

(3) It forms a dicarboxylic acid on reaction with Br_2 water

(4) In aqueous solution it exists as an equilibrium mixture of two anomeric forms.

Answer (3)

Sol. D-Glucose is an aldohexose which mainly exists in two cyclic anomeric forms. Since aldehyde group is not free, it does not give Schiff's test.



It has asymmetrical C-atom and is dextrorotatory. Br_2 water oxidises glucose to monocarboxylic acid called gluconic acid. In aqueous solution it exists as an equilibrium mixture of α- and β-anomers.

12. One mole of monoatomic gas and one mole of diatomic gas is present in a mixture. Find out ratio of heat capacities at constant volume and constant pressure $\left(i.e.\ \frac{C_v}{C_P}\right)$

(1) $\frac{2}{3}$ (2) $\frac{7}{5}$

(3) $\frac{5}{7}$ (4) $\frac{3}{5}$

Answer (1)

Sol. $C_v = \frac{1\left(\frac{3R}{2}\right)+1\left(\frac{5R}{2}\right)}{2}$

$$= \frac{8R}{4} = 2R$$

$$C_p = \frac{1\left(\frac{5R}{2}\right)+1\left(\frac{7R}{2}\right)}{2}$$

$$= \frac{12R}{4} = 3R$$

$$\frac{C_v}{C_P} = \frac{2R}{3R}$$

$$= \frac{2}{3}$$

13. Which of the following has all paired electrons in t_{2g}?

(1) $[Cr(H_2O)_6]^{3+}$ (2) $[Co(H_2O)_6]^{2+}$

(3) $[Co(H_2O)_6]^{3+}$ (4) $[Fe(H_2O)_6]^{2+}$

Answer (3)

Sol. $[Cr(H_2O)_6]^{3+} \Rightarrow Cr^{+3} \Rightarrow 3d^3 \Rightarrow d^2sp^3 \Rightarrow t_{2g}^3 e_g^0$

$[Co(H_2O)_6]^{2+} \Rightarrow Co^{+2} \Rightarrow 3d^7 \Rightarrow sp^3d^2 \Rightarrow t_{2g}^5 e_g^2$

$[Co(H_2O)_6]^{3+} \Rightarrow Co^{3+} \Rightarrow 3d^6 \Rightarrow d^2sp^3 \Rightarrow t_{2g}^6 e_g^0$

$[Fe(H_2O)_6]^{2+} \Rightarrow Fe^{2+} \Rightarrow 3d^6 \Rightarrow sp^3d^2 \Rightarrow t_{2g}^4 e_g^2$

In $[Co(H_2O)_6]^{3+}$ all electron are present in t_{2g} set t_{2g} set have all paired electrons.

14. Which of the following will undergo disproportionation reaction in aqueous alkaline medium?

(1) I_2, Cl_2 only (2) F_2, Cl_2 only

(3) I_2, Br_2 only (4) Cl_2, Br_2, I_2 only

Answer (4)

Sol. $\overset{\circ}{X}_2(aq) + \underset{\text{(Cold and diluted)}}{OH^{\ominus}} \longrightarrow \overset{-1}{X^{\ominus}}(aq) + \overset{+1}{XO^{\ominus}}(aq)$

$\overset{\circ}{X}_2 + \underset{\text{(Hot and Conc.)}}{OH^{\ominus}} \longrightarrow \overset{-1}{X^{\ominus}}(aq) + \overset{+5}{XO_3^{\ominus}}(aq)$

[X = Cl, Br or I]

15. Match the List-I (Complexes) with List-II (Colour) and choose the correct option.

	List-I (Complex)		List-II (Colour)
(i)	$Fe_4[Fe(CN)_6]_3$	(A)	Red
(ii)	$[Fe(SCN)]^{2+}$	(B)	Green
(iii)	$FeSO_4 \cdot 7H_2O$	(C)	Prussian blue

(1) (i)-(C), (ii)-(A), (iii)-(B)

(2) (i)-(B), (ii)-(A), (iii)-(C)

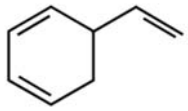
(3) (i)-(A), (ii)-(B), (iii)-(C)

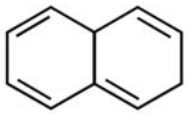
(4) (i)-(C), (ii)-(B), (iii)-(A)

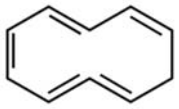
Answer (1)

Sol. (i)-(C), (ii)-(A), (iii)-(B)

16. Which of the following molecules is aromatic?

(1) 

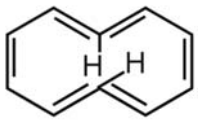
(2) 

(3) 

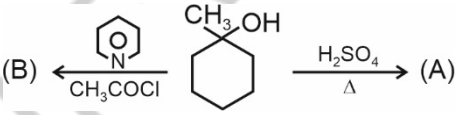
(4) 

Answer (4)

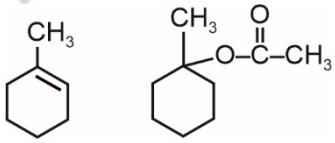
Sol. In (1) & (2) there is no cyclic delocalisation in (3) the two ring changes its plane due to hinderance of the two H-atoms:

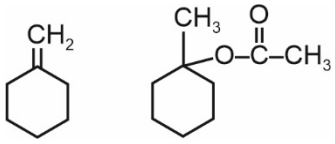


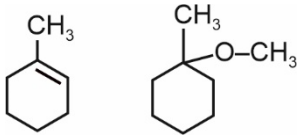
In (4) all conditions are present for aromaticity.

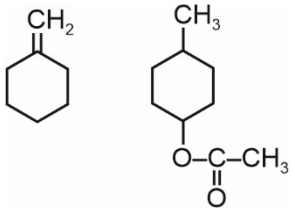
17. 

Product (A) and (B) are respectively:

(1) 

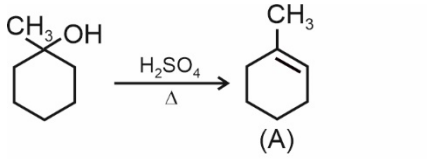
(2) 

(3) 

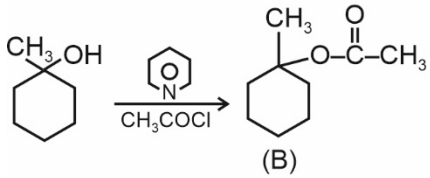
(4) 

Answer (1)

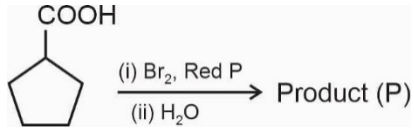
Sol.



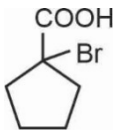
(A)

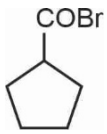


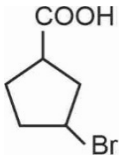
(B)

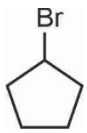
18. 

The product (P) is :

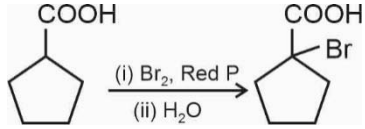
(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. 

This is HVZ reaction

19. Which of the following reaction(s) is/are correct?

(a) $Fe^{3+} + I^- \longrightarrow I_2 + Fe^{2+}$

(b) $Fe^{3+} + I^- \longrightarrow FeI_3$

(c) $Fe^{2+} + S_2O_8^{2-} \longrightarrow Fe + 2SO_4^{2-}$

(d) $Fe^{2+} + S_2O_8^{2-} \longrightarrow Fe^{3+} + 2SO_4^{2-}$

(1) (a) only (2) (b) and (c) only

(3) (a) and (d) only (4) (b) and (d) only

Answer (3)

Sol. Fe^{3+} ions oxidises I^- ions to I_2 and itself gets reduced to Fe^{2+} ions

$2Fe^{3+} + 2I^- \longrightarrow I_2 + 2Fe^{2+}$

Fe^{2+} ions are oxidised by $S_2O_8^{2-}$ to Fe^{3+} ions and Itself gets reduced to SO_4^{2-} ions

$2Fe^{2+} + S_2O_8^{2-} \longrightarrow 2Fe^{3+} + 2SO_4^{2-}$

20. Match the following

Column-I	Column-II
(i) Borax bead test	(a) $MCO_3 \xrightarrow[\Delta]{HCl} MCl_2 + CO_2 + H_2O$
(ii) Cobalt nitrate test	(b) $Na_2B_4O_7 + Co^{2+} \rightarrow Co(BO_2)_2$
(iii) Flame test	(c) $HgO + C \rightarrow Hg + CO$
(iv) Charcoal cavity test	(d) $CoO + ZnO \rightarrow CoO.ZnO$

(1) i-(d), ii-(c), iii-(b), iv-(a)

(2) i-(b), ii-(d), iii-(a), iv-(c)

(3) i-(a), ii-(b), iii-(c), iv-(d)

(4) i-(d), ii-(b), iii-(a), iv-(c)

Answer (2)

Sol. Borox bead test :

$Na_2B_4O_7 + Co^{2+} \xrightarrow{\Delta} \underset{\text{Blue}}{Co(BO_2)_2}$

Cobalt nitrate test :

$CoO + ZnO \longrightarrow \underset{\text{Green}}{CoO.ZnO}$

Flame test :

$MCO_3 \xrightarrow[\Delta]{HCl} MCl_2 + CO_2 + H_2O$: Flame test

$\underset{\text{Brick red}}{M^{2+}} : Ca^{2+}, \quad \underset{\text{Crimson red}}{Sr^{2+}}, \quad \underset{\text{Apple green}}{Ba^{2+}}$

Charcoal cavity test :

$\underset{\text{Red}}{HgO(s)} + \text{Charcoal} \longrightarrow \underset{\text{Silver grey}}{Hg} + CO$

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Find the spin only magnetic moment (nearest integer) of M in MO_4^{2-}, M being the atom having least atomic radii among Sc, Ti, V, Cr, Mn, Zn

Answer (0)

Sol. Radii $\rightarrow$ Sc > Ti > Mn $\simeq$ Zn > V > Cr

So, M is Cr.

$\overset{+6}{Cr}O_4^{2-}$: $\overset{+6}{Cr} \rightarrow [Ar]4s^0 3d^0 \Rightarrow$ zero unpaired electron

$\mu_{spin} = 0$

22. A solution contains 100 g water and 10 g of AB_2. The boiling point of solution was found to be 100.52°C. The degree of dissociation of $AB_2(\alpha)$ = _____$\times 10^{-1}$

$$\left[\text{MW of } AB_2 = \frac{200\text{ g}}{\text{mol}}; K_b = 0.52\frac{K\cdot kg}{\text{mole}}\right]$$

Answer (5)

Sol. ΔT_b = (i) (.52) (m)

$$0.52 = (i)\,(0.52)\left(\frac{10(10)}{(200)(1)}\right)$$

i = 2

$2 = 1 + 2\alpha$

$1 = 2\alpha$

$\alpha = 0.5$

23. Find the mass (in g) of O_2 required for the complete combustion of 900 g glucose.

Answer (960)

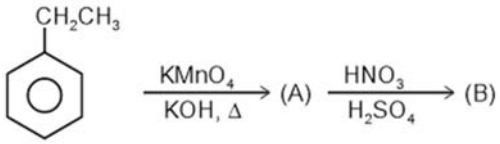
Sol. Glucose has molecular formula = $C_6(H_2O)_6$ or $C_6H_{12}O_6$

$C_6H_{12}O_6 + 6O_2 \longrightarrow 6CO_2 + 6H_2O$

$$\text{Moles of glucose} = \frac{900}{180} = 5$$

Hence moles of O_2 required = 5 × 6 = 30

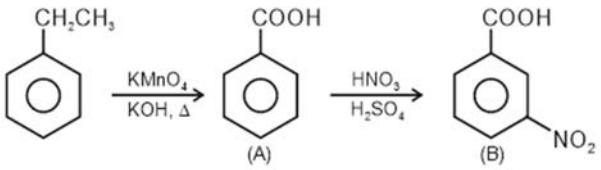
Mass (in g) of O_2 required = 30 × 32 = 960

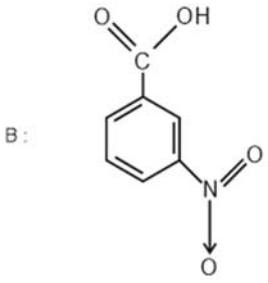
24. 

The number of π–bonds in product (B) is

Answer (5)

Sol.





No. of π-bonds = 5

25. Find out magnitude of work done on the gas at 18°C when 1 mole of an ideal gas undergoes compression from 9 litre to 1 litre through a reversible isothermal process (in joule) (Nearest integer). (Take log3 = 0.48)

Answer (5349)

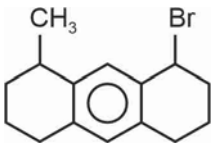
Sol. W = 2.303 × (1) × (8.314) × (291) log9

= (2.303) (8.314) (291) (0.48) (2)

= 4981.2 joule

$\approx$ 5349 J

26. Find the number of optical isomers of the following compound.



Answer (4)

Sol. The given structure has two chiral centres without possibility of symmetry hence optical isomers $2^n = 2^2 = 4$

27. Consider the reaction.

$$A + B \rightarrow C$$

Time taken by A to become $\frac{1}{4^{th}}$ of initial concentration is twice the time taken by it to become $\frac{1}{2}$ of its same concentration. Rate of change of [B] with time gives an equation, whose slope is negative and intercept is positive. The overall order of reaction is

Answer (1)

Sol. For I order kinetics, $t_{75\%} = 2 \times t_{50\%}$

Therefore, order w.r.t. [A] = 1

For zero order kinetics,

$$[R]_t = [R]_0 - kt$$

Negative slope and positive intercept

Therefore, order w.r.t. [B] = 0

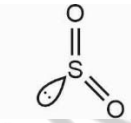
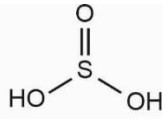
Overall order = 0 + 1 = 1

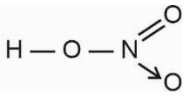
28. How many of the given compounds follow(s) octet rule?

H_2SO_4, CO_2, SO_2, SO_3, H_2SO_3, NO_2, HNO_3

Answer (2)

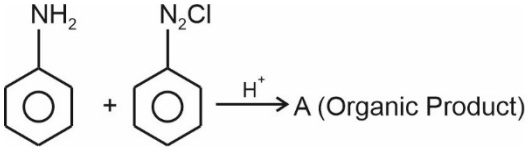
Sol. H_2SO_4 : HO—S(=O)(=O)—OH , CO_2 : O=C=O

SO_2 :  , H_2SO_3 : 

NO_2 : O←N=O , HNO_3 : 

Only CO_2 and HNO_3 follow octet rule.

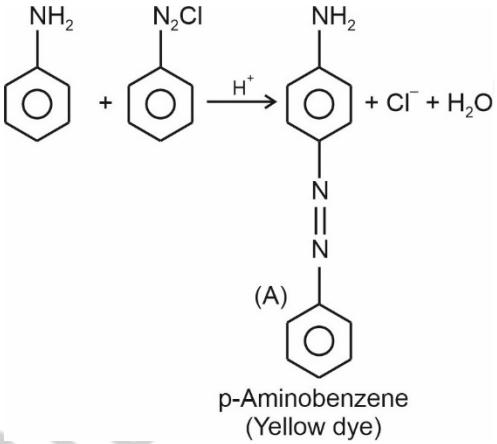
29. Consider the following reaction



What is the mass of nitrogen (in g) in one mole A?

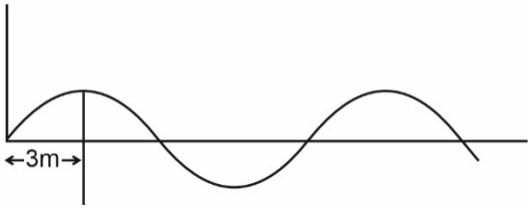
Answer (42)

Sol.



One mole A has three mole nitrogen atoms hence mass of nitrogen in 1 mole A = 14 × 3 = 42 g

30. Frequency of following electromagnetic wave is given by _____ × 10^6 Hz.



Answer (25)

Sol. $\lambda = 12$ m

$$\nu = \frac{c}{\lambda} = \frac{3\times10^8}{12}$$

$$= \frac{1}{4}\times10^8$$

$$= 0.25 \times 10^8$$

$$= 25 \times 10^6 \text{ Hz}$$

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. For $8^{2x} - 168^{x} + 48 = 0$, the sum of values of x is equal to

(1) $1+\log_6 8$ (2) $1+\log_8 6$

(3) $\log_8 6$ (4) 16

Answer (2)

Sol. Let $8^x = t$

$t^2 - 16t + 48 = 0$

$\Rightarrow (t-12)(t-4) = 0$

$t = 12$ or $t = 4$

$\Rightarrow 8^x = 12, 4$

$\Rightarrow x_1 = \log_8 12$ and $x_2 = \log_8 4$

$x_1 + x_2 = \log_8 12 + \log_8 4$

$= \log_8 48$

$= 1+\log_8 6$

2. If $I(x) = \int \frac{6dx}{\sin^2 x(1+\cot x)^2}$ and $I(0) = 3$ then $I\left(\frac{\pi}{12}\right)$ is equal to

(1) $\frac{21-9\sqrt{3}}{3-\sqrt{3}}$ (2) $\frac{21+9\sqrt{3}}{3-\sqrt{3}}$

(3) $\frac{21}{3-\sqrt{3}}$ (4) $\frac{3+\sqrt{3}}{3-\sqrt{3}}$

Answer (1)

Sol. $I(x) = \int \frac{6dx}{\sin^2 x(1+\cot x)^2}$

$I(x) = \int \frac{6dx}{(\sin x+\cos x)^2}$

$I(x) = \int \frac{6dx}{\sin^2 x+\cos^2 x+2\sin x\cos x}$

$I(x) = \int \frac{6\sec^2 x dx}{\tan^2 x+1+2\tan x}$

$I(x) = \int \frac{6\sec^2 x\,dx}{(1+\tan x)^2}$, $t = \tan x$

$\Rightarrow dt = \sec^2 x dx$

$= \int \frac{6dt}{(1+t)^2} = \frac{-6}{(1+t)} + c$

$\Rightarrow I(x) = \frac{-6}{1+\tan x} + c$

$I(0) = \frac{-6}{1+0} + c = 3 \Rightarrow c = 9$

$I(x) = 9 - \frac{6}{1+\tan x} = 9 - \frac{6}{1+\tan\left(\frac{\pi}{12}\right)}$

$I\left(\frac{\pi}{12}\right) = 9 - \frac{6}{1+(2-\sqrt{3})} = \frac{9(3-\sqrt{3})-6}{(3-\sqrt{3})}$

$= \left(\frac{21-9\sqrt{3}}{3-\sqrt{3}}\right)$

3. Let $f(x) = \cos x - x + 1$, $x \in [0, \pi]$, then

(1) $f(x)$ is increasing in $(0, \pi)$

(2) $f(x)$ is decreasing in $(0, \pi)$

(3) $f(x)$ is increasing in $(0, \pi/2)$ and decreasing in $(\pi/2, \pi)$

(4) $f(x)$ is decreasing in $(0, \pi/2)$ and increasing in $(\pi/2, \pi)$

Answer (2)

Sol. $f'(x) = -\sin x - 1 < 0 \ \forall \ x \in (0, \pi)$

$\therefore$ $f(x)$ is decreasing $\forall \ x \in (0, \pi)$

4. If $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$, if sum of diagonal elements of A^{13} is 3^n then n is equal to

(1) 5 (2) 7

(3) 9 (4) 13

Answer (2)

Sol. Trace = Sum of eigen values

tr (A^{13}) = Sum of eigen values

$$\Rightarrow \begin{vmatrix} 2-\lambda & -1 \\ 1 & 1-\lambda \end{vmatrix} = 0$$

$(2 - \lambda)(1 - \lambda) + 1 = 0 \Rightarrow \lambda^2 - 3\lambda + 3 = 0 \nearrow \lambda_1 \ \searrow \lambda_2$

$\Rightarrow \ \lambda_1 + \lambda_2 = 3, \qquad \lambda_1\lambda_2 = 3$

To get $(A^{13})'$ trace $\Rightarrow \lambda_1^{13} + \lambda_2^{13}$

$$\lambda_1^2 + \lambda_2^2 = (\lambda_1 + \lambda_2)^2 - 2\lambda_1\lambda_2$$

$= 9 - 6 = 3$

$$\lambda_1^3 + \lambda_2^3 = (\lambda_1 + \lambda_2)(\lambda_1^2 + \lambda_2^2 - \lambda_1\lambda_2) = (3)(3-3) = 0$$

$$\lambda_1^3 = -\lambda_2^3 \Rightarrow \lambda_1^3\lambda_2^3 = 27 \Rightarrow -\lambda_1^6 = 27$$

$$\Rightarrow \ \lambda_1^6 = \lambda_2^6 \Rightarrow \lambda_1^{12} = \lambda_2^{12} \Rightarrow \lambda_1^{12} = 27^2$$

$$\lambda_1\lambda_1^{12} + \lambda_2\lambda_2^{12} = \lambda_1^{12}(\lambda_1 + \lambda_2)$$

$= (27)^2 \cdot 3 = 3^6 \cdot 3^1 = 3^7$

5. 3 blue balls and 4 yellow balls are in a box. 3 balls are drawn on random. Let variance and mean be x and y respectively then value of $3x + 4y$ is

(1) 5.21 (2) 8.39

(3) 7.34 (4) 6.54

Answer (3)

Sol. Let z denote the number of blue balls in sample of 3 balls drawn from a box containing 3 blue balls and 4 yellow balls.

So z can be 0, 1, 2, 3

$P(z = 0) = P$(no blue ball)

$$= \frac{4}{7} \times \frac{4}{7} \times \frac{4}{7} = \frac{64}{343}$$

$$P(z = 1) = 3\left(\frac{4}{7} \times \frac{4}{7} \times \frac{3}{7}\right) = \frac{144}{343}$$

$$P(z = 2) = 3\left(\frac{4}{7} \times \frac{3}{7} \times \frac{3}{7}\right) = \frac{108}{343}$$

$$P(z = 3) = \frac{3}{7} \times \frac{3}{7} \times \frac{3}{7} = \frac{27}{243}$$

z	0	1	2	3
$P(z)$	$\frac{64}{343}$	$\frac{144}{343}$	$\frac{108}{343}$	$\frac{27}{343}$

Mean = $\Sigma z \cdot P(z) = 0 \cdot \frac{64}{343} + \frac{144}{343} + 2 \cdot \frac{108}{343} + 3 \cdot \frac{27}{343}$

$$y = \frac{441}{343}$$

Variance = $\sigma_x^2 = \sum x^2 \cdot [P(x)] - (\text{mean})^2$

$$x = \left[\frac{144}{343} + 4 \cdot \frac{108}{343} + 9 \cdot \frac{27}{343}\right] - \left(\frac{441}{343}\right)^2$$

$$3x + 4y = 3\left[\frac{819}{343} - \left(\frac{441}{343}\right)^2\right] + 4 \cdot \frac{441}{343}$$

$$= \frac{2457}{343} - 4.96 + 5.14$$

= 7.16 + 0.18

= 7.34

6. If $C_1 = (x-\alpha)^2 + (y-\beta)^2 = r_1^2$,

$C_2 : (x-6)^2 + \left(y - \frac{15}{2}\right)^2 = r_2^2$ touches each other at (6, 6). If line joining centres of C_1 and C_2 is divided by (6, 6) in 2 : 1 internally, then $(\alpha + \beta) + 4\left(r_1^2 + r_2^2\right)$ is equal to

(1) 54

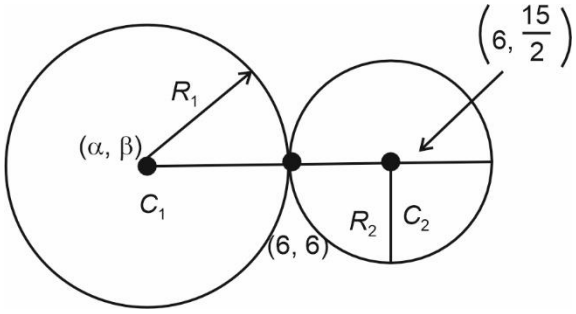
(2) 36

(3) 18

(4) 27

Answer (1)

Sol.



$\frac{\alpha + 2.6}{3} = 6, \quad \frac{\beta + \frac{2.15}{2}}{3} = 6$

$\Rightarrow \alpha = 6, \beta = 3$

$\Rightarrow$ Also, $\frac{R_1}{R_2} = \frac{2}{1}$

$R_2 = \sqrt{(6-6)^2 + \left(6 - \frac{15}{2}\right)^2} = \frac{3}{2}$

$\Rightarrow R_1 = 2R_2 = 3$

$\Rightarrow \alpha + \beta + 4\left(r_1^2 + r_2^2\right) = 6 + 3 + 4\left(3^2 + \frac{9}{4}\right)$

$= 54$

7. $R = (a, b) : a + 5b = 42$ and $a, b \in N$ has m elements and $\sum_{n=1}^{m}(1 + i^{n!}) = x + iy$. (Where $i = \sqrt{-1}$) find $x + y + m$

(1) 20 (2) 12

(3) 8 (4) 13

Answer (1)

Sol. $R = (a, b) : a + 5b = 42$

Then $R = \{(2, 8), (7, 7), (12, 6), (17, 5), (22, 4), (27, 3), (32, 2), (37, 1)\}$

$m = 8$

and $\sum_{n=1}^{m}(1 + i^{n!}) = x + iy$

$\therefore \quad \sum_{n=1}^{8}(1 + i^{n!}) = 8 + (i + i^2 + i^6 + 1 + 1 + 1 + 1 + 1)$

$= 11 + i$

$\therefore \quad x = 11, y = 1$

$\therefore \quad x + y + m = 20$

8. If $y = \int \frac{e^{\tan x}}{\cos^2 x(1 + e^{2\tan x})} dx$ and $y(0) = 6$, then $y\left(\frac{\pi}{4}\right)$ is equal to

(1) $\tan^{-1}(e) - \frac{\pi}{4}$ (2) $\tan^{-1}(e) + 6 - \frac{\pi}{4}$

(3) $\tan^{-1}(e) - 6 + \frac{\pi}{4}$ (4) $\tan^{-1}\left(\frac{1}{e}\right) + \frac{\pi}{4} - 6$

Answer (2)

Sol. Put $e^{\tan x} = t$

$\Rightarrow e^{\tan x}.\sec^2 x dx = dt$

$y = \int \frac{dt}{1 + t^2} \Rightarrow \tan^{-1}(e^{\tan x}) + c = y$

$\therefore \quad y = \tan^{-1}(e^{\tan x}) + c$

$y(0) = 6$

$6 = \tan^{-1}(1) + c \Rightarrow c = 6 - \frac{\pi}{4}$

$\therefore\ y = \tan^{-1}\left(e^{\tan x}\right) + 6 - \frac{\pi}{4}$

$y\left(\frac{\pi}{4}\right) = \tan^{-1}(e) + 6 - \frac{\pi}{4}$

9. The area bounded by $y = \min\{\sin x, \cos x\}$ and x-axis in $-\pi \le x \le \pi$ interval is equal to (in sq. units)

(1) 4 (2) 8

(3) $2-\sqrt{2}$ (4) $4-2\sqrt{2}$

Answer (1)

Sol. $f(x) = \min\{\sin x, \cos x\}$

$$\Rightarrow f(x) = \begin{cases} \cos x & , \ x \in \left(-\pi, -\frac{3\pi}{4}\right) \\ \sin x & , \ x \in \left(-\frac{3\pi}{4}, \frac{\pi}{4}\right) \\ \cos x & , \ x \in \left(\frac{\pi}{4}, \pi\right) \end{cases}$$

Area bounded by $f(x)$ and x-axis

$$= \int_{-\pi}^{\pi} |f(x)dx = \int_{-\pi}^{-\frac{3\pi}{4}} |\cos x|\, dx + \int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} |\sin x\, dx| + \int_{\frac{\pi}{4}}^{\pi} |\cos x)\, dx$$

$$= \frac{1}{\sqrt{2}} + 2 + 2 - \frac{1}{\sqrt{2}} = 4 \text{ sq. unit}$$

10. Let $f(\theta) = \frac{\sin^4\theta + 3\cos^2\theta}{\sin^4\theta + \cos^2\theta}$, then range of $f(\theta) \in [a, b]$. The sum of infinite G.P., where first term is 64 and common ratio is $\frac{a}{b}$ is equal to

(1) 32 (2) 64

(3) 96 (4) 108

Answer (3)

Sol. $f(\theta) = 1 + \frac{2\cos^2\theta}{\sin^4\theta + \cos^2\theta}$

$$= 1 + \frac{2\cos^2\theta}{\left(1-\cos^2\theta\right)^2 + \cos^2\theta}$$

$$= 1 + \frac{2\cos^2\theta}{1 + \cos^4\theta - 2\cos^2\theta + \cos^2\theta}$$

$$= 1 + \frac{2\cos^2\theta}{1 + \cos^4\theta - \cos^2\theta}$$

$$= 1 + \frac{2}{\cos^2\theta + \frac{1}{\cos^2\theta} - 1}$$

$f(\theta)_{max} = 1 + \frac{2}{2-1} = 3$

$f(\theta)_{min} = 1 + 0 = 1$

$\therefore\ a = 1, b = 3$

$\therefore\ r = \frac{1}{3}, a = 64$

$\therefore\ S_\infty = \frac{64}{1-\frac{1}{3}} = \frac{64\times 3}{2} = 96$

11.

1
2 3
4 5 6
7 8 9 10

If this pattern continue then which row number, the number 5437 lies

(1) 103 (2) 104

(3) 102 (4) 105

Answer (2)

Sol. Number of term :

{1}, {2}, {3} ….

To find term number 5437 will lie

Let r^{th} term have it.

$1+2+...+r=\frac{r(r+1)}{2}$

$\Rightarrow \frac{(r-1)r}{2} \le 5437 \le \frac{r(r+1)}{2}$

$\Rightarrow r(r+1) \ge 2.5437$

If $r = 104$, $104 \times 105 > 2.5437$

12. If $A=\begin{bmatrix}2 & a & 1\\ 1 & 3 & 1\\ 0 & 5 & b\end{bmatrix}$ and $A^3 = 4A^2 - A - 21I$ then $(2a + 3b)$ is equal to

(1) 33 (2) 23

(3) 13 (4) 7

Answer (3)

Sol. $A=\begin{bmatrix}2 & a & 1\\ 1 & 3 & 1\\ 0 & 5 & b\end{bmatrix}$

A satisfy characteristic equation

$\Rightarrow \begin{vmatrix}2-\lambda & a & 1\\ 1 & 3-\lambda & 1\\ 0 & 5 & b-\lambda\end{vmatrix}=0$

$(2-\lambda)\left[\lambda^2-(b+3)\lambda+3b-5\right]-(a-3+\lambda)=0$

$-\lambda^3+(b+3+2)\lambda^2+\lambda(-2b-6-3b+4)$

$+6b-10-a+3=0$

$\lambda^3-(b+5)\lambda^2+\lambda(5b+2)+(a+7-6b)=0$

$A^3-(b+5)A^2+\lambda(5b+2)+a+7-6b=0$

$\Rightarrow b + 5 = 4$

$5b + 2 = 1 \Rightarrow b = -1$

$a + 7 - 6b = 21 \Rightarrow a = 8$

$2a + 3b = 16 - 3 = 13$

13. If sum of two positive numbers is 24 then the probability of product of numbers is not less than $\frac{3}{4}$ times the maximum possible product of a and b then probability of such event is $\frac{m}{n}$ (m, n are O prime) then $n - m$ is

(1) 1 (2) 3

(3) 5 (4) 7

Answer (1)

Sol. Take two numbers as a and b

$a + b = 24$

$a = 24 - b$

Now product of these numbers.

$f(b) = b \times (24 - b)$

$= 24b - b^2$

$f'(b) = 24 - 2b$

$\Rightarrow f'(b) = 0$

$\Rightarrow b = 12$

$\Rightarrow f''(12) = -2 < 0$

So, at $b = 12$ product is maximum

$\Rightarrow a = b = 12$

Maximum possible product = 144

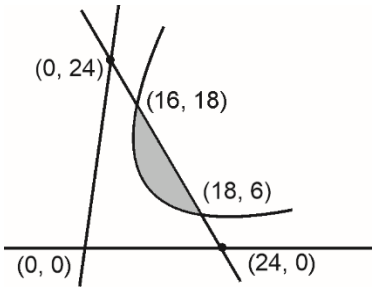
Now $ab \ge \frac{3}{4} \cdot 144 = 36.3 = 108$

So, probability

$=\frac{12\sqrt{2}}{24\sqrt{2}}$

$=\frac{1}{2}=\frac{m}{n}$

$\Rightarrow n - m = 1$



14. Let $\sin\pi = \frac{-3}{5}$, $\pi < x < \frac{3\pi}{2}$, then $80(\tan^2 x - \cos x)$ is equal to

(1) 109 (2) 108

(3) 9 (4) 8

Answer (1)

Sol. $\tan x = \frac{3}{4}$, $\cos x = \frac{-4}{5}$

$\therefore\ 80\left(\frac{9}{16}+\frac{4}{5}\right) = 80\left(\frac{45+64}{80}\right)$

= 109

15. If $|z + 2| = 1$, $\text{Im}\left(\frac{z+1}{z+2}\right) = \frac{1}{5}$ then, $Re(z) < -2$ is equal to

(1) $\frac{24}{25}$ (2) $\frac{2}{5}$

(3) $\frac{12}{5}$ (4) $\frac{3}{5}$

Answer (1)

Sol. $z = x + iy,\ x, y \in \mathbb{R}$

$$\text{Im}\left[\frac{(x+1)+iy}{(x+2)+iy}\right] = \frac{1}{5}$$

$$\Rightarrow \text{Im}\left(\frac{((x+2)-iy)((x+1)+iy)}{(x+2)^2+y^2}\right) = \frac{1}{5}$$

$$= \frac{(x+2)y - y(x+1)}{(x+2)^2+y^2} = \frac{1}{5}$$

$5y = (x + 2)^2 + y^2$

$\Rightarrow (x + 2)^2 + y^2 - 5y = 0$

$(x + 2)^2 + y^2 - 1 = 0 \qquad \Rightarrow y = \frac{1}{5} = 0.2$

$$\Rightarrow (x+2)^2 = \frac{24}{25}$$

16. The value of

$$\lim_{x\to 0}\frac{1-\sqrt{\cos x}\cdot\sqrt[2]{\cos 2x}\cdot\sqrt[3]{\cos 3x}+\ldots\sqrt[n]{\cos nx}}{x^2}$$ is equal to

(1) $\frac{n^2+n+1}{4}$ (2) $\frac{n^2-n-1}{4}$

(3) $\frac{n^2+n-1}{4}$ (4) $\frac{n^2-n+1}{4}$

Answer (3)

Sol. The given form is $\left(\frac{0}{0}\right)$ form. Applying L' Hospital

$$L = \frac{\frac{1}{2}\sqrt{\cos x}(-\sin x)\left(\sqrt[2]{\cos 2x}\cdot\sqrt[3]{\cos 3x}\ldots\right)}{2x}$$

$$\frac{-\frac{1}{2}\sqrt{\cos 2x}(-\sin 2x)\times 2\left(\sqrt{\cos x}\cdot\sqrt[3]{\cos 3x}\ldots\right)}{2x}$$

$$-\frac{1}{3}(\cos 3x)^{2/3}(-\sin 3x)\times 3\left(\sqrt{\cos x}\ldots\right)-\ldots$$

$$\ldots-\frac{1}{n}(-\sin nx)(n)(\cos nx)^{1/n-1}$$

$$= \frac{1}{2}\left(\frac{1}{2}+2+3+\ldots n\right) = \frac{1}{2}\left[\frac{(n)(n+1)}{2}-\frac{1}{2}\right] = \frac{1}{4}\left(n^2+n-1\right)$$

17. If $I(n) = \int_0^1 (1-x^k)^n dx,\ k \in N$, and $147I(20) = 148I(21)$, then k is equal to

(1) 17

(2) 21

(3) 7

(4) 15

Answer (3)

Sol. $I(n) = \int_0^1 (1-x^k)^n dx$

$$I(21)=\int_0^1(1-x^k)^{21}dx$$

$$=\int_0^1(1-x^k)(1-x^k)^{20}dx=\int_0^1(1-x^k)^{20}dx$$

$$-\int_0^1 x^k(1-x^k)^{20}dx$$

$$I(21)=I(20)-\int_0^1 x^k\cdot(1-x^k)^{20}$$

$$=I(20)-\int_0^1 x\cdot\underbrace{x^{k-1}(1-x^k)^{20}}dx$$

$$I(21)=I(20)-\left[\frac{(1-x^k)^{21}}{-21k}x\Big|_0^1-\int_0^1\frac{(1-x^k)^{21}}{-21k}dx\right]$$

$$I(21)=I(20)+\frac{1}{21k}I(21)$$

$\Rightarrow$ $I(21)(21k+1)=21kI(20)$

$\Rightarrow$ $21k=147\Rightarrow k=7$

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Find the number of 3 digit numbers which are not divisible by 3 and made using the digits {2, 3, 5, 7, 4} and repetition is not allowed.

Answer (36.00)

Sol. Possible triplets for which number is divisible by 3

(2, 3, 7), (2, 3, 4), (3, 5, 7), (3, 5, 4)

$\therefore$ Number of required numbers = ${}^5C_3\cdot 3! - 4\times 3!$

$= 3!\times(6)$

$= 6\times 6 = 36$

22. Let $f(x)=(2x-3)^{2/3}(x+2)$. The number of critical points of $f(x)$ is equal to

Answer (2.00)

Sol. $f'(x)=(2x-3)^{2/3}+\frac{(x+2)}{(2x-3)^{1/3}}\times\frac{2}{3}\times 2$

$$=(2x-3)^{2/3}+\frac{4}{3}\frac{(x+2)}{(2x-3)^{1/3}}$$

$$=\frac{(2x-3)+\frac{4}{3}(x+2)}{(2x-3)^{1/3}}$$

$\Rightarrow$ $\frac{6x-9+4x+8}{3(2x-3)^{1/3}}$

$=\frac{10x-1}{(3)(2x-3)^{1/3}}$

$\therefore$ $f'(x)=0$ at $x=\frac{1}{10}$

$f(x)$ is nondifferentiable at $x=\frac{3}{2}$

$\therefore$ 2 critical points are there.

23.

24.

25.

26.

27.

28.

29.

30.