SMART ACHIEVERS
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JEE Main - 2024 Session - 2

Answers & Solutions

(Physics, Chemistry and Mathematics)

06 - April - 2024 - Shift - 1

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PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. For a given single electron atom, ratio of shortest wavelengths in Balmer and Lyman series is

(1) 4 : 1 (2) 1 : 4

(3) 1 : 2 (4) 2 : 1

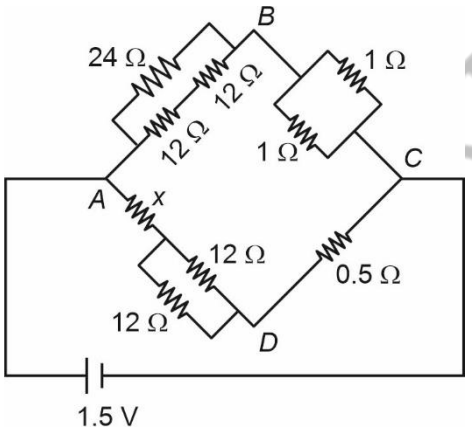
Answer (1)

Sol. $\frac{1}{\lambda_L} = RZ^2\left\{1-\frac{1}{\infty}\right\}$

$\frac{1}{\lambda_B} = RZ^2\left[\frac{1}{4}-\frac{1}{\infty}\right]$

$\frac{\lambda_B}{\lambda_L} = 4$

2. The value of unknown resistance x for which potential difference between point B and D is zero is



(1) 12 Ω (2) 6 Ω

(3) 3 Ω (4) 2 Ω

Answer (2)

Sol. $V_D - V_B = 0$, *i.e.*, it is condition of Wheatstone bridge.

$\frac{12}{6+x} = \frac{0.5}{0.5}$

$x = 6\ \Omega$

3. Which of the following does not depend on the wave nature of light?

A. Reflection

B. Diffraction

C. Photoelectric effect

D. Polarization

E. Interference

(1) C only (2) A, B

(3) A, B, C (4) D, E

Answer (1)

Sol. Theoretical

4. Four particles A, B, C, D have masses $\frac{m}{2}$, m, $2m$ and $4m$. They have equal momentum. The particle that has highest kinetic energy is

(1) A

(2) B

(3) C

(4) D

Answer (1)

Sol. $KE = \frac{p^2}{2m}$

$\Rightarrow KE \propto \frac{1}{m}$

5. Which of the following is not a semiconductor?

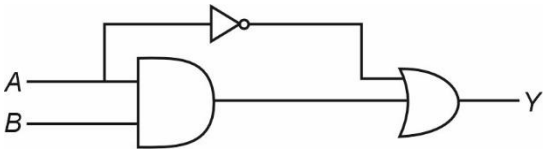
(1) Silicon (2) Germanium

(3) Copper oxide (4) Graphite

Answer (4)

Sol. Theoretical.

6. Find the truth table for the following circuit.



(1)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(2)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	1

(3)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

(4)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

Answer (2)

Sol. $Y = AB + A'$

$= A' + B$

7. A bullet of mass 50 gm enters a metal sheet with speed of 100 m/s and emerges with speed of 40 m/s. The loss in kinetic energy of bullet is

(1) 105 J

(2) 42 J

(3) 210 J

(4) 140 J

Answer (3)

Sol. $|\Delta K| = \frac{1}{2} \times \frac{50}{100}\{100^2 - 40^2\} = \frac{50}{2000} \times 140 \times 60$

= 210 J

8. A ball of mass m and density ρ made to free fall into viscous liquid of density ρ_0. The viscous force on ball is

(1) $mg\left(1 - \frac{\rho}{\rho_0}\right)$ (2) $mg\left(1 - \frac{\rho_0}{\rho}\right)$

(3) $\frac{mg}{1 - \frac{\rho}{\rho_0}}$ (4) $\frac{mg}{1 - \frac{\rho_0}{\rho}}$

Answer (2)

Sol. $\vec{W} + \vec{B} + \vec{F}_{\text{viscous}} = \vec{O}$

$\vec{F}_V = W + B$

$= mg - \rho_0 vg$

$= mg - \rho_0 \frac{m}{\rho} g$

$= mg\left(1 - \frac{\rho_0}{\rho}\right)$

9. For a spring block system, the error in time period calculation is 2% and the error in mass calculation is 1%. Find the percentage error in spring constant k.

(1) 2%

(2) 4%

(3) 5%

(4) 10%

Answer (3)

Sol. $k = 4\pi^2 \cdot \frac{m}{T^2}$

$\frac{dk}{k} \times 100 = \pm\left(\frac{dm}{m} \times 100 + 2 \cdot \frac{dT}{T} \times 100\right)$

$\boxed{\frac{dk}{k} \times 100 = 1 + 2 \times 2 = 5}$

10. Match the dimensions:

a.	Torque	i.	M^0L^2AT
b.	Magnetic moment	ii.	$ML^2T^{-2}A^0$
c.	Magnetic field	iii.	$MLT^{-3}A^{-2}$
d.	Permeability	iv.	$ML^0T^{-2}A^{-1}$

(1) a-i, b-ii, c-iii, d-iv (2) a-ii, b-iv, c-i, d-iii

(3) a-ii, b-i, c-iv, d-iii (4) a-ii, b-iii, c-i, d-iv

Answer (3)

Sol. $\tau = F\ell \equiv ML^2T^{-2}$

$\mu = iA \equiv M^0L^2AT$

$$B \equiv \frac{F}{qV} \equiv \frac{MLT^{-2}}{ATLT^{-1}} \equiv MT^{-2}A^{-1}$$

$$\mu_0 \equiv \frac{Br}{i} \equiv \frac{MT^{-2}A^{-1}\cdot L}{AT} \equiv MLT^{-3}A^{-2}$$

11. Kinetic energy to move a body of mass m from surface of earth to infinite distance form the earth is (g is acceleration due to gravity on surface of earth & R is radius of earth)

(1) $2mgR$ (2) $\frac{1}{2}mgR$

(3) mgR (4) $\frac{1}{4}mgR$

Answer (3)

Sol. $K + U = 0$

$$\Rightarrow K = \frac{GMm}{R} = mgR$$

12. Find the ratio of root mean square speed of oxygen and helium molecules at same temperature.

(1) $\frac{2\sqrt{2}}{1}$ (2) $\frac{1}{2\sqrt{2}}$

(3) $\frac{1}{4}$ (4) $\frac{1}{32}$

Answer (2)

Sol. $V_{rms} = \sqrt{\frac{3RT}{M}}$

$$\frac{(V_{rms})_{O_2}}{(V_{rms})_{He}} = \frac{M\mu_e}{M_{O_2}} = \sqrt{\frac{4}{32}}$$

$$= \frac{1}{2\sqrt{2}}$$

13. The specific heat capacity for a gas following the relation $PV^2 = RT$ is (C_V is heat capacity at constant volume and R is gas constant)

(1) C_V (2) $C_V + R$

(3) $\frac{R}{3} + C_V$ (4) R

Answer (1)

Sol. $\frac{PV^2}{PV} = C \Rightarrow V = \text{constant}$

$\Rightarrow$ C_V

14. A screw gauge has circular scale 100 divisions with pitch 1 mm. Upon keeping a wire between studs, main scale reading is 1 mm and circular scale divisions 42th coincide with reference line.

Find the diameter of circular cross-section wire in mm.

(1) 1.42 (2) 1.40

(3) 1.38 (4) 0.39

Answer (1)

Sol. Diameter = Main scale reeding + circular scale reading × least count

d = 1 mm + (42 × 0.01) mm

d = 1.42 mm

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Ratio of angle of prism and minimum deviation is one for a prism whose refractive index is $\sqrt{3}$ then angle of prism (in degrees) is _____.

Answer (60)

Sol. $A = \delta$

$$\frac{\sin\left(\frac{A+\delta}{2}\right)}{\sin\frac{A}{2}} = \frac{\sin A}{\sin\frac{A}{2}} = \sqrt{3}$$

$$2\cos\frac{A}{2} = \sqrt{3}$$

$\Rightarrow$ $A = 60°$

22. Time period of a simple harmonic motion is 3.14 seconds, with amplitude 0.06 m. The maximum velocity of particle is $k \times 10^{-2}$ m/s. Find the value of k.

Answer (12)

Sol. $V_{max} = A\omega = A.\frac{2\pi}{T}$

$$V_{max} = 0.06 \times \frac{2\pi}{3.14}$$

$$\boxed{V_{max} = 0.12 \text{ m/s}}$$

23. A body uniformly accelerates [starting from rest] to speed of 80 km/hr in time t and then maintains this speed for time interval of $3t$. Average speed for whole motion is _____ km/hr.

Answer (70)

Sol. $< v >= \frac{40 \times t + 80 \times 3t}{4t} = \frac{40 + 240}{4} = 70$

24. Radiation of energy 3.5 eV is incident on a metal. The stopping potential required is 0.5 V. The work function of the metal is _____ eV.

Answer (3)

Sol. $nf - \phi = eV$

$\Rightarrow$ $\phi = 3.5 - 0.5 = 3$ eV

25. If the radius of earth is reduced to $\frac{3}{4}$th of its original radius, then the time period of earth's rotation becomes K hours 30 minutes. Find the value of K.

Answer (13)

Sol. $\vec{\tau}_{ext} = 0 \Rightarrow$ Angular momentum is conserved

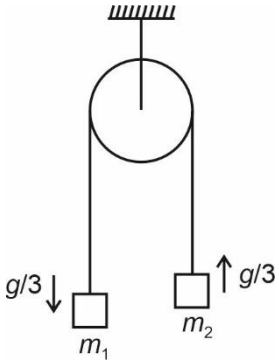
$$\frac{2}{5}mR^2 \cdot \omega = \frac{2}{5}m\left(\frac{3R}{4}\right)^2 .\omega_1$$

$$\omega_1 = \frac{16\omega}{9}$$

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{\omega} \cdot \frac{9}{16} = 24 \times \frac{9}{16} \text{ hours}$$

T_1 = 13 hours 30 minutes

26. Two masses m_1 and m_2 are attached through a thin string passing over frictionless and massless pulley. The acceleration of masses is as shown. Then $\frac{m_1}{m_2}$ is _____



Answer (2)

Sol. $a = \frac{m_1 - m_2}{m_1 + m_2}g = \frac{g}{3}$

$$\Rightarrow \quad \frac{m_1}{m_2} = 2$$

27.

28.

29.

30.

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Among the given molecules, identify the one which undergoes nucleophilic addition reaction at fastest rate?

   (1) HCHO

   (2) CH_3CHO

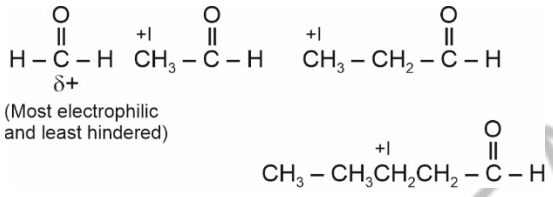
   (3) CH_3CH_2CHO

   (4) $CH_3CH_2CH_2CHO$

Answer (1)

Sol. Rate of N.A.R. α electrophilicity of C-atom

$$\alpha \frac{1}{\text{Steric Hinderance}}$$



2. Which compound will absorb light of highest frequency?

   (1) $[Cr[H_2O]_6]^{3+}$ (2) $[CrCl_6]^{3-}$

   (3) $[Cr(CN)_6]^{3-}$ (4) $[CrCl_3(H_2O)_3]$

Answer (3)

Sol. More the crystal field splitting energy more will be the frequency of absorbed light.

Crystal field splitting energy depends on ligand strength here

Order of ligand strength

$Cl^- < H_2O < CN^-$; So splitting energy will also follow same order

3. Find the ratio of shortest wavelengths in Lyman and Balmer series of H-atom.

   (1) $\frac{1}{4}$ (2) $\frac{4}{1}$

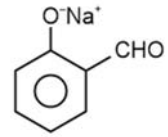
   (3) $\frac{1}{2}$ (4) $\frac{2}{1}$

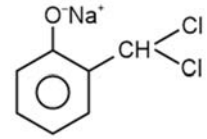
Answer (1)

Sol. $(\lambda_{\text{shortest}})_{\text{Lyman}} = \frac{1}{R}$; $(\lambda_{\text{shortest}})_{\text{Balmer}} = \frac{4}{R}$

$$\frac{\lambda_{\text{Lyman}}}{\lambda_{\text{Balmer}}} = \frac{\frac{1}{R}}{\frac{4}{R}} = \frac{1}{4}$$

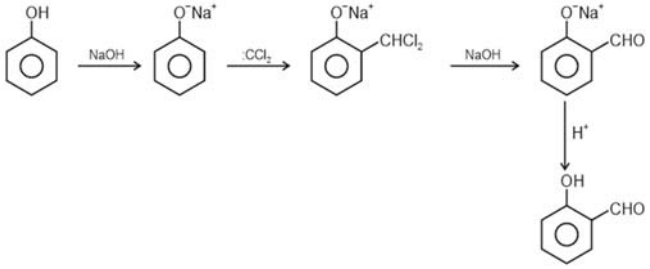
4. Which of the following is not the intermediate observed in Reimer Tiemann Reaction?

   (1)  (2) $:CCl_2$

   (3)  (4) $CHCl_3$

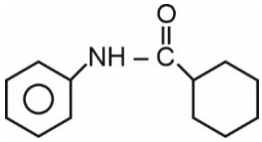
Answer (4)

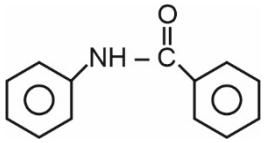
Sol. $CHCl_3 + NaOH \longrightarrow :CCl_2$

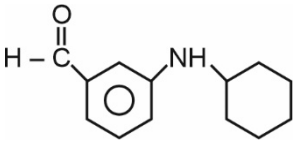


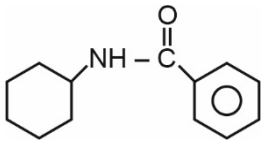
$CHCl_3$ is the reagent.

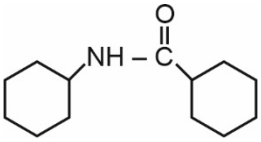
5. Correct metamer of the following compound is



(1) 

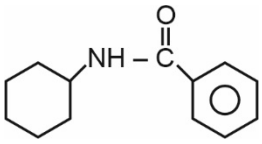
(2) 

(3) 

(4) 

Answer (3)

Sol. Metamers have the same functional group but differ in the alkyl/aryl groups attached to it keeping the molecular formula unchanged. Therefore, metamer of the given compound is



6. How many of the following do not belong to Lanthanoids?

Eu, Er, Lu, Cm, Yb, Tb

(1) 5

(2) 4

(3) 3

(4) 1

Answer (4)

Sol. $_{63}Eu$, $_{65}Tb$, $_{68}Er$, $_{70}Yb$ and $_{71}Lu$ belong to Lanthanoids.

$_{96}Cm$ belongs to actinoids.

7. Density of x M solution of NaOH is 1.12 g/mL and molality is 3 m, then the value of x is

(1) 3

(2) 2.8

(3) 3.8

(4) 3.5

Answer (1)

Sol. Given molality of NaOH = 3 m

It means 3 moles of NaOH present in 1000 g of solvent.

Mass of solute (NaOH) = 3 × 40 = 120 g

Mass of solution = 1000 + 120 = 1120 g

$$\text{Density of solution} = 1.12 = \frac{1120}{\text{volume}}$$

$$\text{Volume of solution} = \frac{1120}{1.12} = 1000 \text{ mL}$$

$$\text{Molarity of solution} = \frac{3}{1000} \times 1000$$

$$= 3 \text{ M}$$

8. Which of the following is not a semiconductor?

(1) Si

(2) Graphite

(3) CuO

(4) Ge

Answer (2)

Sol. Graphite is not a semiconductor, it is an allotrope of carbon and good conductor of electricity.

CuO is a p-type semiconductor.

Si and Ge are also semiconductors.

9. Match List-I with List-II and choose the correct option.

	List-I (Reagent)		**List-II (Radical)**
(i)	dil. HCl	(A)	Pb^{2+}
(ii)	$NH_4Cl + NH_4OH + (NH_4)_2CO_3$	(B)	Al^{3+}
(iii)	$NH_4Cl + NH_4OH + H_2S$	(C)	Mn^{2+}
(iv)	$NH_4Cl + NH_4OH$	(D)	Sr^{2+}

(1) (i)-(A), (ii)-(D), (iii)-(C), (iv)-(B)
(2) (i)-(D), (ii)-(A), (iii)-(C), (iv)-(B)
(3) (i)-(A), (ii)-(D), (iii)-(B), (iv)-(C)
(4) (i)-(B), (ii)-(C), (iii)-(D), (iv)-(A)

Answer (1)

Sol. (i) dil. HCl
Group-I: Pb^{2+}
(ii) $NH_4Cl + NH_4OH + (NH_4)_2CO_3$
Group-V: Ba^{2+}, Sr^{2+}, Ca^{2+}
(iii) $NH_4Cl + NH_4OH + H_2S$
Group-IV: Co^{2+}, Ni^{2+}, Mn^{2+}, Zn^{2+}
(iv) $NH_4Cl + NH_4OH$
Group-III: Al^{3+}, Fe^{3+}

10. Choose the correct option based on matching.

	Hybridization		Shape
A	sp^3	I	Octahedral
B	sp^3d	II	Tetrahedral
C	sp^2	III	Trigonal bipyramidal
D	sp^3d^2	IV	Trigonal planar

(1) A(I); B(II); C(III); D(IV)
(2) A(II); B(III); C(IV); D(I)
(3) A(II); B(III); C(I); D(IV)
(4) A(III); B(II); C(IV); D(I)

Answer (2)

Sol.

Hybridization	Shape
sp^3	Tetrahedral
sp^3d	Trigonal bipyramidal
sp^2	Trigonal planar
sp^3d^2	Octahedral

11. Which of the following will have positive electron gain enthalpy?

(A) $Na + e^{\ominus} \rightarrow Na^{\ominus}$
(B) $O + 2e^{\ominus} \rightarrow O^{2\ominus}$
(C) $Be + e^{\ominus} \rightarrow Be^{\ominus}$
(D) $F + e^{\ominus} \rightarrow F^{\ominus}$
(E) $N + e^{\ominus} \rightarrow N^{\ominus}$

(1) (B, C, E) (2) (A, B, E)
(3) (A, C, D) (4) (A, B, C)

Answer (1)

Sol. Be & N have stable fully filled & half-filled electronic configuration respectively

$O + e \rightarrow O^- +$ $\quad \Delta H_{eg1} = -141$ kJ/mol

$O^- + e \rightarrow O^{2-}$ $\quad \Delta H_{eg2} = +780$ kJ/mol

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$O + 2e \rightarrow O^{2-}$ $\quad \Delta H = +639$ kJ/mol

12. Consider the given reaction :

$H_2 + I_2 \rightleftharpoons 2HI$

If equal number of molecules of H_2, I_2 and HI are present at equilibrium. Then $K_p = t \times 10^{-1}$

Find out t

(1) 10 (2) 0.01
(3) 0.1 (4) 1

Answer (1)

Sol. $H_2 + I_2 \quad 2HI$

$$K_p = \frac{(P_{HI})^2}{(P_{H_2})(P_{I_2})} = 1$$

$K_p = t \times 10^{-1}$

$\boxed{t = 10}$

13. **Statement-I:** Gallium has low melting point, so it is used in thermometers.

Statement-II: A substance having 253 K can be measured by Ga thermometer.

(1) Statement-I and Statement-II both correct

(2) Statement-I and Statement-II both incorrect

(3) Statement-I correct and Statement-II incorrect

(4) Statement-II correct and Statement-I incorrect

Answer (3)

Sol. Melting point of Gallium is nearly 302 K so, it can't measure temperature of 253 K.

14. Choose the correct option regarding the following statements:

Statement-I: 2, 4, 6-Trinitrotoluene is picric acid.

Statement-II: Reaction of 4-hydroxybenzene-1, 3-disulphonic acid with conc. HNO_3 gives picric acid.

(1) Both statement-I and statement-II are true

(2) Both statement-I and statement-II are false

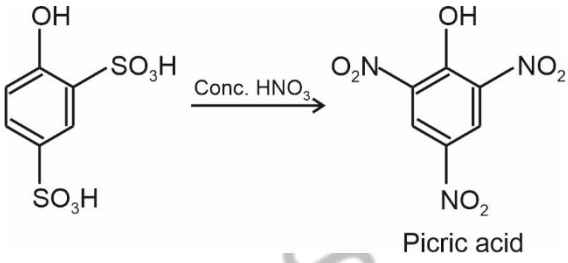
(3) Statement-I is true but statement-II is false

(4) Statement-I is false but statement-II is true

Answer (4)

Sol. 2, 4, 6-Trinitrotoluene is an explosive and not picric acid. Therefore statement-I is false

2, 4, 6-Trinitrotophenol is called picric acid. It is synthesised by the reaction of 4-hydroxybenzene-1,3-disulphonic acid with conc. HNO_3.



Picric acid

$\therefore$ Statement-II is true.

15. Among the following which is not a base of DNA?

(1) Adenine

(2) Uracil

(3) Guanine

(4) Cytosine

Answer (2)

Sol. Uracil is present in RNA. Instead of uracil, thymine is present in DNA.

16. Identify the correct match among the given species and respective shape of molecule

Species	Shape
(A) NH_4^+	See-Saw
(B) SF_4	Tetrahedral
(C) ClF_3	T-shaped
(D) XeF_6	Square planar

(1) A (2) B

(3) C (4) D

Answer (3)

Sol. NH_4^+ : Tetrahedral ClF_3 : T-shaped

SF_4 : See-Saw XeF_6 : Distorted Octahedral

17. Which of the following statement is incorrect?

(1) Glycerol is purified by vacuum distillation

(2) Aniline is purified by steam distillation

(3) Chloroform and aniline can be separated by distillation

(4) Ethanol and water are azeotropic mixture can be separated by distillation

Answer (4)

Sol. Ethanol and water are azeotropic mixture and can't be separated by distillation

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Find out ratio of t_{99} and t_{90} for first order

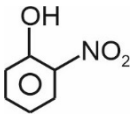
Answer (2)

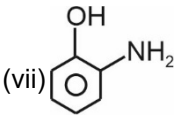
Sol. $t_{99} = \frac{2.303}{k}\log\frac{100}{1}$

$t_{90} = \frac{2.303}{k}\log\frac{100}{10}$

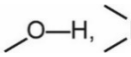
22. How many of the following show(s) H-bonding.

(i)  (ii) H_2O (iii) CH_3OH

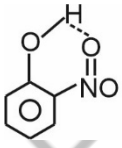
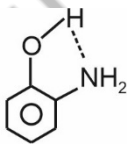
(iv) HF (v) NH_3 (vi) 

(vii)  (viii) C_2H_6

Answer (6)

Sol. The system having suitable electropositive pole H and suitable electronegative pole can show H-bonding. Suitable positive pole H: F—H,  , suitable negative pole: Electronegative element F, O, N with sufficient negative charge.

Hence H_2O, CH_3OH, HF and NH_3 can show intermolecular H bonding and

 and  Can show interamolecular H bonding.

23. $KMnO_4 \xrightarrow{H^+}$ X(product having Mn)

What is the difference in spin only magnetic moment (in B.M.) between the given reactant and product. (Nearest integer)

Answer (6)

Sol. In acidic medium $KMnO_4$ goes to Mn^{2+} e.g. $MnSO_4$.

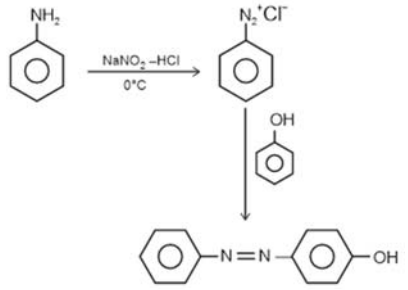
The $KMnO_4$ has zero unpaired electrons (hence it is diamagnetic) because it has Mn at +7 oxidation state (electronic configuration $(n-1)d^0\ ns^0$).

Mn^{2+} has 5 unpaired electrons as it has $(n-1)d^5\ ns^0$ electronic configuration so it has 5.92 B.M. magnetic moment so difference will be 5.92 B.M. Nearest integer = 6.

24. 9.3 g of aniline was treated with $NaNO_2$ and HCl at 0°C to get product A which reacts with phenol to form a product B. Assuming 100% yield in each step, what is the weight of product B obtained?

Answer (20)

Sol.



9.3 g

$\frac{9.3}{93} = 0.1$ mole.

Final product formed = 0.1 mol.

= 0.1 × 198

= 19.8 gm

= 20

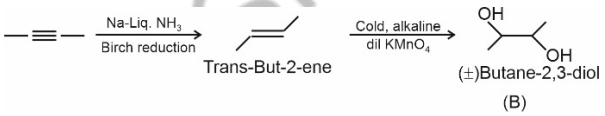
25. Consider the following sequence of reactions

$$But-2-yne \xrightarrow{Na-Liq.\ NH_3} A \xrightarrow{Cold,\ alkaline,\ dil.\ KMnO_4} B$$

Find the number of oxygen in B (1 molecule)

Answer (2)

Sol.



Hence the number of O atom in one molecule of B is 2.

26. According to the reaction,

$$HX_{(aq)} \rightleftharpoons H^+_{(aq)} + X^-_{(aq)} \quad K_a = (1.2 \times 10^{-5})$$

Find out osmotic pressure of 0.03 M HX solution at 300 K (in atm).

Answer (1)

Sol. $K_a = \frac{C\alpha^2}{1-\alpha}$

$1.2 \times 10^{-5} = (3 \times 10^{-2})\ (\alpha^2)$

$\alpha^2 = 0.4 \times 10^{-3}$

$\alpha^2 = 4 \times 10^{-4}$

$\alpha = 0.02$

$\pi = iCRT$

$i = 1 + \alpha = 1.02$

$\pi = (1.02)\ (0.03)\ (0.0821)\ (300)$

= 0.7536 atm

≈ 1

27.

28.

29.

30.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. If $A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2}+\alpha \\ 2r & 2 & n^2-\beta \\ 3r-1 & 3 & \frac{n}{2}(3n-1) \end{vmatrix}$ then the value of $2A_{10} - A_8$ is equal to

(1) $4\alpha + 2\beta$ (2) $2n$

(3) 0 (4) $2\alpha + 4\beta$

Answer (1)

Sol. $A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2}+\alpha \\ 2r & 2 & n^2-\beta \\ 3r-2 & 3 & \frac{3n^2}{2}-\frac{n}{2} \end{vmatrix}$

$A_r = \begin{vmatrix} 0 & 1 & \frac{n^2}{2}+\alpha \\ 0 & 2 & n^2-\beta \\ -2 & 3 & \frac{3n^2}{2}-\frac{n}{2} \end{vmatrix}$

$A_r = -2\left[(n^2-\beta) - 2\left(\frac{n^2}{2}+\alpha\right)\right] = -2\left[n^2-\beta-n^2-2\alpha\right]$

$\Rightarrow \quad A_r = (\beta+2\alpha)\cdot 2$

$\Rightarrow \quad 2A_{10} - A_8 = 4(\beta + 2\alpha) - 2(\beta + 2\alpha)$

$= 2(\beta + 2\alpha) = 4\alpha + 2\beta$

2. The value of $\int_0^{\pi/4} \frac{\cos^2 x \sin^2 x}{(\cos^3 x + \sin^3 x)^2} dx$ is equal to

(1) $\frac{1}{6}$ (2) $\frac{1}{3}$

(3) $\frac{1}{2}$ (4) 1

Answer (1)

Sol. $\int_0^{\pi/4} \frac{\tan^2 x \sec^2 x}{(1+\tan^3 x)^2} dx$

Put $1 + \tan^3 x = t$

$\Rightarrow 3\tan^2 x \sec^2 x dx = dt$

$\frac{1}{3}\int_1^2 \frac{dt}{t^2}$

$\frac{1}{3}\left[\frac{t^{-1}}{-1}\right]_1^2$

$= \frac{1}{6}$

3. Let α, β be the distinct roots of the quadratic equation $x^2 - (t^2 - 5t + 6)x + 1 = 0$, $a_n = \alpha^n + \beta^n$, then the minimum value of $\frac{a_{2023} + a_{2025}}{a_{2024}}$ is

(1) $-\frac{1}{4}$ (2) $\frac{1}{4}$

(3) $-\frac{1}{2}$ (4) $\frac{1}{2}$

Answer (1)

Sol. Given equation

$x^2 - (t^2 - 5t + 6)x + 1 = 0$

$\therefore a_{2025} - (t^2 - 5t + 6)\, a_{2024} + a_{2023} = 0$

$$\Rightarrow \frac{a_{2025}+a_{2023}}{a_{2024}} = \left(t^2-5t+6\right)$$

$$= t^2-5t+\frac{25}{4}+6-\frac{25}{4}$$

$$= \underbrace{\left(t+\frac{5}{2}\right)^2}_{\geq 0}+\left(-\frac{1}{4}\right)$$

$\therefore$ Minimum value is $-\frac{1}{4}$

4. The shortest distance between two lines

$\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}$ and $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$ is

(1) $4\sqrt{3}$

(2) $8\sqrt{3}$

(3) $6\sqrt{3}$

(4) $2\sqrt{3}$

Answer (1)

Sol. $L_1 : \frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}$

$L_2 : \frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$

Shortest distance $= \frac{\begin{vmatrix}-4 & 16 & 0\\ 2 & -7 & 5\\ 2 & 1 & -3\end{vmatrix}}{\sqrt{256+256+256}}$

$= \frac{192}{16\sqrt{3}}$

$= \frac{12}{\sqrt{3}}$

$= 4\sqrt{3}$

5. R is defined on set $X = \{1, 2, \ldots, 20\}$

$R_1 = \{(x, y) : 2x - 3y = 2\}$

$R_2 = \{(x, y) : 5x - 4y = 0\}$

If M, N represent the number of elements to be added to make R_1 and R_2 symmetric respectively. Then value of $M + N$ equals to

(1) 10 (2) 8

(3) 12 (4) 11

Answer (1)

Sol. R is defined on $X = \{1, 2, 3, \ldots., 20\}$

$R_1 = \{(x, y) : 2x - 3y = 2\}$

$R_2 = \{(x, y) : 5x + 4y = 0\}$

As $2x - 3y = 2$

So $2x$ and $3y$ both has to be even or odd simultaneously and $2x$ can't be odd so $2x$ and $3y$ both will be even.

So $R_1 = \{(4, 3), (7, 4), (10, 6), (13, 8), (16, 10), (19, 12)\}$

for symmetric we need to add 6 elements here as (3, 4), (4, 7) and (6, 10), (8, 13), (10, 16), (12, 19)

So $M = 6$

For R_2 $5x - 4y = 0$

So $5x$ and $4y$ has to be equal. $4y$ is always even so $5x$ will also be even

$R_2 = \{(4, 5), (8, 10), (12, 15), (16, 20)\}$

So 4 elements (5, 4), (10, 8) (15, 12), (20, 16) need to be added

$N = 4$

$M + N = 10$

6. If $\frac{dy}{dx}+\frac{y}{x\ell nx}=\frac{1}{x^2\ell nx}$ and $y(e^{-1}) = 0$. Then $y(e)$ equals to

(1) $\frac{e^2+1}{e}$ (2) $\frac{e^2-1}{e}$

(3) $\frac{e^2+2}{e}$ (4) $\frac{e^2-2}{e}$

Answer (2)

Sol. I.F = $e^{\int \frac{1}{x\ell nx}dx}$

Put $\ell nx = t$

$\frac{1}{x}dx = dt.$

I.F = $e^{\int \frac{1}{t}dt}$

= $e^{\ell nt}$

= t

= ℓnx.

$\therefore \quad y.\ell nx = \int \ell nx \cdot \left(\frac{1}{x^2 \ell nx}\right)dx$

$y.\ell nx = \int \frac{1}{x^2}dx$

$y\ell nx = \frac{-1}{x} + c \qquad \ldots(2)$

Given y (e^{-1}) = 0

$0 = -e + c$

$c = e$

$\therefore$ From (2)

We get,

$y\ell nx = \frac{-1}{x} + e$

$\therefore$ Put $x = e$

$y = \frac{-1}{e} + e$

$y = \frac{e^2 - 1}{e}$

Option (b) is correct

7. Interval in which x^x is strictly increasing is

(1) $(0, \infty)$ (2) $\left(0, \frac{1}{e}\right]$

(3) $\left[\frac{1}{C^2}, \infty\right)$ (4) $\left[\frac{1}{e}, \infty\right)$

Answer (4)

Sol. Let $f(x) = x^x$

$\Rightarrow \quad f'(x) = x^x(1 + \ln x)$

For strictly increasing, $f'(x) > 0$

$\Rightarrow \quad 1 + \ln x > 0$

$\Rightarrow \quad x > \frac{1}{e}$

8. If $\frac{dy}{dx} + \frac{y}{1+x^2} = e^{-\tan^{-1}x}$, then which of the following is true

(1) $ye^{\tan^{-1}x} = \frac{x^2}{2} + c$ (2) $ye^{\tan^{-1}x} = \frac{1}{x} + c$

(3) $ye^{\tan^{-1}x} = x + c$ (4) $ye^{\tan^{-1}x} = -x + c$

Answer (3)

Sol. $\frac{dy}{dx} + \frac{y}{1+x^2} = e^{-\tan^{-1}x}$

I.F = $e^{\int \frac{1}{1+x^2}dx}$

= $e^{\tan^{-1}x}$

Now,

$y\, e^{\tan^{-1}x} = \int e^{\tan^{-1}x} \cdot e^{-\tan^{-1}x}dx$

$\Rightarrow ye^{\tan^{-1}x} = \int dx$

$\Rightarrow ye^{\tan^{-1}x} = x + c$

9. A company produces automobiles. It has two factories factory '*A*' produces 60% of the automobiles and rest is produced by factory *B*. 80% of the automobiles produced by '*A*' is upto the standards and 90% of the automobiles by '*B*' is upto the standards. If an automobile is selected we found it as standard, the probability it came from *B* is *P*. Then 126 *P* equals to

(1) 54 (2) 52

(3) 48 (4) 27

Answer (1)

Sol. P (standard automobile from A) $= \frac{6}{10} \times \frac{8}{10} = \frac{12}{25}$

P (standard automobile from B) $= \frac{4}{10} \times \frac{9}{10} = \frac{9}{25}$

Required probability $= \frac{\frac{9}{25}}{\frac{12}{25} + \frac{9}{25}}$

$P = \frac{9}{21} = \frac{3}{7}$

$126\ P = 54$

10. If $\sigma = 4$(standard deviation) and $\bar{x} = 10$ (mean) of 20 observations. One term was taken wrong *i.e*, instead of 12 they have taken 8. Then the correct standard deviation is

(1) 1.8 (2) $\sqrt{3.96}$

(3) $\sqrt{3.84}$ (4) 1.93

Answer (2)

Sol. Mean = $\bar{x} = 10$

$\sigma = 4,\ n = 20$

Take observations as $x_1, x_2, \ldots x_{20}$

$\frac{x_1 + x_2 + \ldots + x_{20}}{20} = 200$

$x_1 + x_2 + \ldots + x_{20} = 200$

One term, say x_{20} is wrongly written as 8

So, $x_1 + x_2 + \ldots + x_{19} = 200 - 8$

$x_1 + x_2 + \ldots + x_{19} = 192$

Now $(x_{20})_{new} = 12$

So, $\bar{x}_{new} = \frac{192 + 12}{20} = \frac{204}{20} = \frac{102}{10} = 10.2$

$\sigma^2 = \frac{\sum x_1^2}{n} - (\bar{x})^2 = 4$

$\frac{x_1^2 + x_2^2 + \ldots + x_{20}^2}{20} = 4 + 100$

$\Rightarrow \frac{x_1^2 + x_2^2 + \ldots + x_{19}^2 + 64}{20} = 104$

$\Rightarrow x_1^2 + x_2^2 + \ldots + x_{19}^2 + 64 = 2080$

$\Rightarrow x_1^2 + x_2^2 + \ldots + x_{19}^2 = 2016$

Now $\sigma_{new}^2 = \frac{2016 + 144}{20} - (10.2)^2$

$\sigma_{new}^2 = 108 - 104.04$

$\sigma = \sqrt{3.96}$

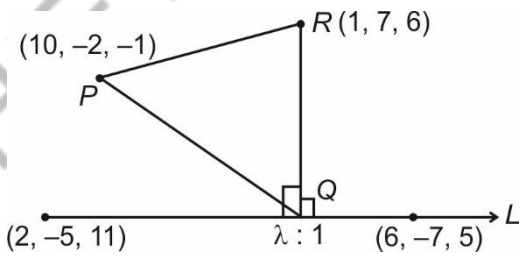
11. A point $P(10, -2, -1)$ and $R(1, 7, 6)$, if Q is a foot of perpendicular from R to the line joining points $(2, -5, 11)$ and $(6, -7, 5)$. Then $(PQ)^2$ is

(1) $\frac{3509}{14}$ (2) $\frac{3600}{7}$

(3) $\frac{3509}{7}$ (4) $\frac{3409}{7}$

Answer (1)

Sol.



$\left(\frac{6\lambda + 2}{\lambda + 1}, \frac{-7\lambda - 5}{\lambda + 1}, \frac{5\lambda + 11}{\lambda + 1}\right)$

$\vec{L} = 4\hat{i} - 2\hat{j} - 6\hat{k}$

$\overrightarrow{RQ} \perp \vec{L} = 4\left(\frac{6\lambda + 2}{\lambda + 1} - 1\right) - 2\left(\frac{-7\lambda - 5}{\lambda + 1} - 7\right)$

$-6\left(\frac{5\lambda + 11}{\lambda + 1} - 6\right)$

$\overrightarrow{RQ} \cdot \vec{L} = \frac{4(5\lambda + 1) - 2(-14\lambda - 12) - 6(-\lambda + 5)}{(\lambda + 1)} = 0$

$= \frac{\lambda(20 + 28 + 6) + (4 + 24 - 30)}{\lambda + 1} = 0$

$= \lambda(54) - 2 = 0 \Rightarrow \lambda = \frac{1}{27}$

$$Q = \left(\frac{\frac{6}{27}+2}{\frac{1}{27}+1}, \frac{\frac{-7}{27}-5}{\frac{1}{27}+1}, \frac{\frac{5}{27}+11}{\frac{1}{27}+1} \right)$$

$$\equiv \left(\frac{60}{28}, \frac{-142}{28}, \frac{302}{28} \right)$$

$$= R' = \left(\frac{15}{7}, \frac{71}{14}, \frac{151}{14} \right)$$

$$\Rightarrow (QP) = \sqrt{\left(10 - \frac{15}{7}\right)^2 + \left(-2 - \frac{71}{14}\right)^2 + \left(-1 - \frac{151}{14}\right)^2}$$

$$= \sqrt{\frac{3509}{14}}$$

12.

13.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If $\cot^{-1}3 + \cot^{-1}4 + \cot^{-1}5 + \cot^{-1} n = \frac{\pi}{4}$, value of n is

Answer (47)

Sol. $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{n}\right) = \frac{\pi}{4}$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{7}{12}}{1-\frac{1}{12}}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{n}\right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{7}{11}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{n}\right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{23}{24}\right) + \tan^{-1}\left(\frac{1}{n}\right) = \frac{\pi}{4}$$

$$\Rightarrow \frac{\frac{23}{24}+\frac{1}{n}}{1-\frac{23}{24.n}} = 1$$

$$\Rightarrow \frac{23n+24}{24n-23} = 1$$

$\Rightarrow n = 47$

22. If $A = [100, 700]$, how many numbers are in A which are neither multiple of 3 nor 4?

Answer (300)

Sol. Let A_1 denotes multiple of 3,

B_1 denotes multiple of 4

$\therefore$ We need to find $\bar{A}_1 \cap \bar{B}_1$

$\because \quad |A_1 \cap B_1| = |\overline{A_1 \cup B_1}|$

$|A_1 \cup B_1| = n(A_1) + n(B_1) - n(A_1 \cap B_1)$

$= 200 + 151 - 50$

$= 301$

$\therefore \quad |\overline{A_1 \cup B_1}| = 601 - 301 = 300$

23. If the second, third, fourth terms of the expression $(x + y)^n$ is 135, 30, $\frac{10}{3}$ respectively, then the value of $9(n^3 + x^2 + y)$ is

Answer (1153)

Sol. $(x + y)^n = {}^nC_0\, x^0 y^n + {}^nC_1 x^1 y^{n-1} + {}^nC_2 x^2 y^{n-2}$

$+ {}^nC_3x^3y^{1-3} + ...$

First term $\Rightarrow {}^nC_1x^1y^{n-1} = 135 \ldots(1)$

Second term $\Rightarrow {}^nC_2x^2y^{n-2} = 30 \ldots(2)$

Third term $\Rightarrow {}^nC_3x^3y^{n-3} = \frac{10}{3} \ldots(3)$

Dividing equation (1) by (2)

$$\frac{2n}{n(n-1)} \times \frac{1}{xy} = \frac{135}{30}$$

$$\frac{2}{(n-1)xy} = \frac{9}{2}$$

$$(n-1)xy = \frac{4}{9} \quad \ldots(4)$$

Dividing equation (2) by (3)

$$\frac{3}{(n-2)xy} = 9$$

$$(n-2)xy = \frac{1}{3} \quad \ldots(5)$$

Dividing equation (4) by (5)

$$\frac{n-1}{n-2} = \frac{4}{3}$$

$\Rightarrow 3n - 3 = 4n - 8$

$\Rightarrow n = 5$

Now equation (1) becomes

$5 \times y^4 = 135 \Rightarrow x = \frac{27}{y^4} \quad \ldots(6)$

And equation (2) becomes

$10x^2y^3 = 30$

$\Rightarrow x^2y^3 = 3 \quad \ldots(7)$

From equation (6) and (7)

$$\Rightarrow \frac{3^6}{y^8}xy^3 = 3$$

$\Rightarrow y^5 = 3^5$

$\Rightarrow y = 3$

$$\Rightarrow x = \frac{1}{3}$$

$\Rightarrow 9(n^3 + x^2 + y)$

$$= 9\left(5^3 + \frac{1}{3^2} + 3\right)$$

$$= 9\left(\frac{1153}{9}\right)$$

$= 1153$

24. Find the number of triangles formed whose vertices are also a vertices of regular octagon but the side of triangle is not common with sides of octagon is

Answer (56)

Sol.

$\Rightarrow$ Total triangles = ${}^8C_3$

Triangles with all 3 sides common with octagon = 0

Triangle with 2 sides common with octagon $\Rightarrow$ Choose vertex

$= \left({}^8C_1\right) \Rightarrow 8$ triangles

Triangle with exactly 1 side common

$= \left({}^8C_1\right)$ ways to choose a side, remaining vertex can be selected in 4 vertices $= {}^4C_1$ ways $= {}^8C_1 \cdot {}^4C_1 = 32$

$\Rightarrow 56 - (8 + 0 + 32) = 16$ triangles

25. The number of real solutions of $x|x + 5| + 2|x + 7| - 2 = 0$ is equal to

Answer (3)

Sol. (I) $x \geq -5$

$x^2 + 5x + 2x + 14 - 2 = 0$

$x^2 + 7x + 12 = 0$

$x = 3, 4$

(II) $-7 < x < -5$

$-x^2 - 5x + 2x + 14 - 2 = 0$

$-x^2 - 3x + 12 = 0$

$x^2 + 3x - 12 = 0$

$x = 2.275, -5.275$, here $x \neq 2.275$

So, $x = -5.275$

(III) $x \leq -7$

$-x^2 - 5x - 2x - 14 - 2 = 0$

$-x^2 - 7x - 16 = 0$

$x^2 + 7x + 16 = 0$

$D < 0 \rightarrow$ no real roots

Only 3 solutions possible

26. The number of points of discontinuities of $f(x) = 2x^2 + [x^2] - [x]$ where $[\cdot]$ is greatest integer function and $x \in [-1, 2]$ is equals to

Answer (4)

Sol. $f(x) = 2x^2 + [x^2] - [x]$, $x \in [-1, 2]$

This function may be discontinuous at $x = -1, 0, 1, \sqrt{2}, \sqrt{3}$ and 2.

For continuity at $x = -1$

$f(-1) = 4$

$$\lim_{h\to 0} f(-1+h) = \lim_{h\to 0} 2(-1+h)^2 + [(-1+h)^2] - [-1+h]$$

$= 3$

$\therefore$ $f(x)$ is discontinuous at $x = -1$

For continuity at $x = 0$, $f(0) = 0$

$f(0^-) = 1$

$\therefore$ $f(x)$ is discontinuous at $x = 0$

Continuity at $x = 1$

$$\text{L.H.L} = \lim_{h\to 0} f(1-h) = 2(1-h)^2 + [(1-h)^2] - [1-h]$$

$= 2$

$f(1) = 2.1^2 + 1 - 1 = 2$

$$\text{R.H.L} = \lim_{h\to 0} f(1+h) = 2(1+h)^2 + [(1+h)^2] - [1+h]$$

$= 2$

$\therefore$ $f(x)$ is continuous at $x = 1$

For continuity, at $x = \sqrt{2}$ and $\sqrt{3}$ similarly it is discontinuous

For continuity at $x = 2$

$f(2) = 2.2^2 + [2^2] - [2] = 10$

$$\text{L.H.L} = \lim_{h\to 0} 2(2-h)^2 + [(2-h)^2] - [2-h]$$

$= 8 + 3 - 1$

$= 10$

$\therefore$ $f(x)$ is continuous at $x = 2$

$f(x)$ is discontinuous at x = $-1, 0, \sqrt{2}$ and $\sqrt{3}$.

No. of points of discontinuity = 4

27.

28.

29.

30.