# JEE Main -2024 Session - 2 Answers \& Solutions 

# (Physics, Chemistry and Mathematics) 

## 5-April 2024 Shift - 1

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## PHYSICS

## SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

## Choose the correct answer:

1. The ratio of radius of gyration of uniform hollow sphere and uniform solid sphere about its diameter is $\qquad$ . (Both having same radius)
(1) $\sqrt{\frac{4}{3}}$
(2) $\sqrt{\frac{5}{3}}$
(3) $\sqrt{\frac{3}{2}}$
(4) $\sqrt{\frac{3}{5}}$

## Answer (2)

Sol. For hollow sphere, $l_{1}=\frac{2}{3} m R^{2}=m k_{1}^{2}, k_{1}=\sqrt{\frac{2}{3}} R$
For solid sphere, $I_{2}=\frac{2}{5} m R^{2}=m k_{1}^{2}, k_{2}=\sqrt{\frac{2}{5}} R$
$\frac{k_{1}}{k_{2}}=\sqrt{\frac{2}{3} \times \frac{5}{2}}=\sqrt{\frac{5}{3}}$.
2. In closed rigid chamber, collision frequency of molecules of ideal gas at $27^{\circ} \mathrm{C}$ is $v$. The collision frequency of gas at temperature $127^{\circ} \mathrm{C}$ becomes
(1) $\frac{\sqrt{3}}{2} v$
(2) $\sqrt{\frac{127}{27}} v$
(3) $\frac{2}{\sqrt{3}} v$
(4) $\frac{27}{127} v$

## Answer (3)

Sol. $\tau=\frac{\lambda}{v_{\text {th }}}=\frac{1 \mathrm{~V}}{\sqrt{2} \pi d^{2} N \mathrm{v}_{\text {th }}}$
$v \propto \frac{v_{\text {th }}}{V}$
$v \propto \frac{\sqrt{T}}{V}$
$\Rightarrow \frac{v_{1}}{v_{2}}=\frac{\sqrt{300}}{\sqrt{400}} \quad$ (at constant volume)

$$
v_{l}=\frac{2}{\sqrt{3}} v
$$

3. If the time period of a pendulum at height $R$ (where $R$ is radius of earth) from surface of earth is $T_{1}$ and at height $2 R$ it is $T_{2}$, then
(1) $3 T_{1}=2 T_{2}$
(2) $2 T_{1}=3 T_{2}$
(3) $T_{1}=3 T_{2}$
(4) $3 T_{1}=4 T_{2}$

## Answer (1)

Sol. $T=2 \pi \sqrt{\frac{l}{g}}$
at $R, g^{\prime}=\frac{G M}{(2 R)^{2}}=\frac{g}{4}$
$2 R, g^{\prime}=\frac{G M}{(3 R)^{2}}=\frac{g}{9}$
$\therefore \quad T_{1}=2\left(2 \pi \sqrt{\frac{l}{g}}\right)$
$T_{2}=3\left(2 \pi \sqrt{\frac{1}{g}}\right)$
$3 T_{1}=2 T_{2}$
4. A point source of light is placed at focus of convex lens, then what is the shape of wavefront after passing through the lens?
(1) Planar
(2) Cylindrical
(3) Spherical
(4) Elliptical

Answer (1)

Sol.

$\therefore$ As the source is at focus, rays get parallel after passing through the lens, hence planar wavefront.
5. A block of mass $m=50 \mathrm{~kg}$ is lifted from ground to a height 20 m in two different ways as shown in figure (1) and (2). The ratio of work done in these two will be

(2)
(1) $1: 1$
(2) $1: 2$
(3) $2: 1$
(4) $1: 5$

Answer (1)
Sol. For I $\rightarrow W_{1}=m g h$
For II $\rightarrow W_{2}=m g h$
6. Two concentric conducting coplanar rings of radius $a$ and $b$ are placed as shown in diagram $(a \ll b)$. Find coefficient of mutual inductance of rings.

(1) $\frac{\mu_{0} \pi b^{2}}{a}$
(2) $\frac{\mu_{0} \pi a^{2}}{2 b}$
(3) $\frac{\mu_{0} a^{2}}{2 b}$
(4) $\frac{\mu_{0} a^{3}}{2 \pi b^{2}}$

Answer (2)
Sol. $\phi_{a b}=\frac{\mu_{0} i}{2 b} \times \pi a^{2}$

$$
M=\frac{\mu_{0} \pi a^{2}}{2 b}
$$

7. Find current $l$ in the given circuit

(1) $\frac{12}{13} \mathrm{~A}$
(2) $\frac{6}{7} \mathrm{~A}$
(3) $\frac{5}{6} \mathrm{~A}$
(4) $\frac{7}{8} \mathrm{~A}$

Answer (2)
Sol. $R_{\text {eq. }}=12+2=14 \Omega$

$$
I=\frac{12}{14}=\frac{6}{7} \mathrm{~A}
$$

8. Find dimension of $\sqrt{G . \mu}$, where $G$ is universal gravitational constant and $\mu$ is energy gradient.
(1) $\left[\mathrm{LT}^{-2}\right]$
(2) $\left[\mathrm{L}^{2} \mathrm{~T}^{-2}\right]$
(3) $\left[\mathrm{LT}^{3}\right]$
(4) $\left[\mathrm{LT}^{-1}\right]$

Answer (2)
Sol. Dimension of $\sqrt{G . \mu}=\sqrt{M^{1} L^{3} T^{-2} \frac{M^{1} L^{2} T^{-2}}{L^{1}}}$

$$
\begin{aligned}
& =\sqrt{M^{0} L^{4} T^{-4}} \\
& =\left[\mathrm{L}^{2} \mathrm{~T}^{-2}\right]
\end{aligned}
$$

9. The correct relation between kinetic energy (K.E) and total energy (T.E) of a satellite orbiting around the planet is
(1) $K \cdot E=|T \cdot E|$
(2) $K \cdot E=2|T \cdot E|$
(3) $K \cdot E=\frac{|T . E|}{2}$
(4) $|T \cdot E|=3 K \cdot E$

Answer (1)
Sol. $T . E=-\frac{G m m}{2 R}$
$K . E=\frac{G m m}{2 R}$
10. Given two circuits. Find the ratio of energy stored in capacitor system. $E_{1}: E_{2}$

(1) $2: 1$
(2) $4: 1$
(3) $9: 1$
(4) $1: 1$

## Answer (3)

Sol. $E_{1}: C_{\text {eq }}=3 C$
$E_{1}=\frac{1}{2} \times 3 C \times V^{2}=\frac{3}{2} C V^{2}$
$E_{2}: C_{\text {eq }}=\frac{C}{3}$

$$
E_{2}=\frac{1}{2} \times\left(\frac{C}{3}\right) \times V^{2}=\frac{C V^{2}}{6}
$$

$E_{1}: E_{2}=9: 1$
11. Match the column :
[Given : Mass of sun $=M_{s}$

$$
\begin{aligned}
& \text { Mass of earch }=M_{e} \\
& \text { Radius of earth }=R
\end{aligned}
$$

Distance between sun and earth =a]

| (a) | Kinetic energy of <br> earth | (i) | $-\frac{G M_{s} M_{e}}{a}$ |
| :--- | :--- | :--- | :--- |
| (b) | Potential energy of <br> earth and sun | (ii) | $\frac{G M_{s} M_{e}}{2 a}$ |
| (c) | Total energy of <br> earth and sun | (iii) | $\frac{G M_{e}}{R}$ |
| (d) | Escape energy <br> from surface of <br> earth per unit mass | (iv) | $-\frac{G M_{s} M_{e}}{2 a}$ |

(1) (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)
(2) (a)-(i), (b)-(ii), (c)-(iv), (d)-(iii)
(3) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)
(4) (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)

Answer (3)
Sol. $U=-\frac{G M_{s} M_{e}}{a}$
$K: U: E=1:-2:-1$
Escape energy from earth surface per unit mass = $\frac{G M_{e}}{R}$
12. Three helium atoms form carbon at high temperature due to fusion. Masses of helium and carbon nuclei in a.m.u. are 4.0002 and 12 respectively. Find energy released in the process.
(1) 0.18 MeV
(2) 0.56 MeV
(3) 0.10 MeV
(4) 21.3 KeV

Answer (2)
Sol. $3 \mathrm{He} \rightarrow \mathrm{C}$
$\Delta m=0.0006$
$\Delta E=0.0006 \times 931$
$=0.56 \mathrm{MeV}$
13. For the graph $\left(v_{0}-v\right)$ for photoelectric effect given below for two metal, $M_{1}$ and $M_{2}$
Stopping potential ( $v_{0}$ )


Statement-I : For the incident light of same frequency, kinetic energy of ejected electron from metal $M_{1}$ will be more than that of metal $M_{2}$.

Statement-II: Slope of the graph is equal to $\frac{h}{e}$, where $h$ is planck's constant and $e$ is electronic charge.
(1) Both statements are correct
(2) Statement-I is correct while statement-II is incorrect
(3) Statement-II is correct while statement-I is incorrect
(4) Both are incorrect

Answer (1)

Sol. $h v=e v_{0}+\phi$
$e v_{0}=h \nu-\phi$
$v_{0}=\frac{-\phi}{e}+\frac{h}{e} v$

Slope of curve $=\frac{h}{e}$
$K \cdot E=E-\phi$
$(K \cdot E)_{1}>(K \cdot E)_{2}$ for same light.
14. Figure shows two long co-axial cylindrical cables, carrying same current along their wall in opposite directions. The magnetic field will be zero at

(1) None of the points
(2) $A$ and $B$
(3) $A$ and $C$
(4) B and C

## Answer (3)

Sol. By applying ampere's-circuital law magnetic field is zero at $A$ and $C$.
15. Find the normal force between the table and 5 kg block as shown in the diagram (take $g=10 \mathrm{~m} / \mathrm{s}^{2}$ )

(1) 306 N
(2) 303 N
(3) 296 N
(4) 297 N

## Answer (4)

Sol.


$$
\begin{gathered}
m g-\mathrm{N}=m a \\
\mathrm{~N}=m(g-a) \\
=30 \times 9.9 \\
=297 \mathrm{~N}
\end{gathered}
$$

16. Find net heat exchanged in the given cyclic process,

(1) 81.5 J
(2) 61.5 J
(3) 100.2 J
(4) 40.2 J

## Answer (2)

Sol. In cyclic process $\Delta Q=\Delta \mathrm{W}$ as $\Delta U=0$
$\Rightarrow \pi \times \frac{280}{2} \times \frac{280}{2} \times 10^{-6} \times 10^{3}$
17. Find kinetic energy of $\mathrm{O}_{2}$ at temperature 300 K ?
(1) $10.35 \times 10^{-21} \mathrm{~J} /$ molecule
(2) $9.35 \times 10^{-22} \mathrm{~J} /$ molecule
(3) $20.70 \times 10^{-21} \mathrm{~J} /$ molecule
(4) $10.70 \times 10^{-21} \mathrm{~J} /$ molecule

## Answer (1)

Sol. Total KE of $\mathrm{O}_{2}$ molecule $=$ Translational $\mathrm{KE}+$ Rotational KE

$$
\begin{aligned}
& =\frac{3}{2} K T+\frac{2}{2} K T \\
& =\frac{5}{2} K T \\
& =\frac{5}{2} \times 1.38 \times 10^{-23} \times 300 \\
& =10.35 \times 10^{-21} \mathrm{~J} / \text { molecule }
\end{aligned}
$$

18. The speed of electron in the first orbit of hydrogen atom is
(1) $1.2 \times 10^{5} \mathrm{~m} / \mathrm{s}$
(2) $2.2 \times 10^{5} \mathrm{~m} / \mathrm{s}$
(3) $2.8 \times 10^{6} \mathrm{~m} / \mathrm{s}$
(4) $2.2 \times 10^{6} \mathrm{~m} / \mathrm{s}$

## Answer (4)

Sol. For $n^{\text {th }}$ orbit,

$$
\begin{aligned}
V_{n} & =\frac{2.186 \times 10^{6}}{n} \mathrm{~m} / \mathrm{s} \\
& \approx 2.2 \times 10^{6} \mathrm{~m} / \mathrm{s} \text { for } n=1
\end{aligned}
$$

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19. An electron and a proton are placed at a certain distance apart. Then the ratio of coulombic force and gravitational force between them is of order
(1) $10^{32}$
(2) $10^{39}$
(3) $10^{36}$
(4) $10^{42}$

Answer (2)
Sol. $\frac{F_{c}}{F_{g}}=\frac{k q_{1} q_{2}}{G m_{1} m_{2}}$

$$
\begin{aligned}
& =\frac{9 \times 10^{9} \times(1.6)^{2} \times 10^{-38}}{6.67 \times 10^{-11} \times 9.1 \times 10^{-31} \times 1.67 \times 10^{-27}} \\
& =2.27 \times 10^{39}
\end{aligned}
$$

20. 

## SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.
21. In YDSE; for wavelength $\lambda=5000 \AA$, slit distance $d=3 \mathrm{~mm}$ and screen distance of 2 m . The intensity at a point which is 3 cm away from central maxima (Assume intensity of light for each source is $I_{0}$ ) is $x \%_{0}$ then $x$ is $\qquad$ .

## Answer (4)

Sol. $\Delta x=\frac{d y}{D}=\frac{3 \times 10^{-3} \times 3 \times 10^{-2}}{2}$

$$
\begin{aligned}
& \Delta d=\frac{2 \pi}{5000 \times 10^{-10}} \times \frac{9 \times 10^{-5}}{2} \\
& =\frac{2 \pi}{10^{-6}} \times 9 \times 10^{-5} \\
& \Delta d=180 \pi
\end{aligned}
$$

$$
\Rightarrow \text { maxima }
$$

$\therefore$ Intensity $=410$
$x=4$
22. What is angle (in degrees) between resultant of $2 \vec{q}-2 \vec{p}$ and $2 \vec{q}+2 \vec{p}$ with $\vec{q}$ ?

## Answer (0)

Sol. $\vec{r}=(2 \vec{q}-2 \vec{p})+(2 \vec{q}+2 \vec{p})$

$$
\vec{r}=2 \times 2 \vec{q}=4 \vec{q}
$$

Angle of $\vec{r}$ with $\vec{q}=$ angle between $4 \vec{q}$ and $\vec{q}=0^{\circ}$
23. In a LCR series AC circuit as given, the voltage across the capacitor is $25 \sqrt{x}$ volts then $x$ is


## Answer (8)

Sol. $Z=\sqrt{\left(L \omega-\frac{1}{\omega C}\right)^{2}+R^{2}}$

$$
\begin{aligned}
& =\sqrt{\left(100-\frac{1}{100 \times 20 \times 10^{-6}}\right)^{2}+300^{2}} \\
& =\sqrt{(400)^{2}+300^{2}}
\end{aligned}
$$

$$
Z=500 \Omega
$$

$$
i=\frac{50 \sqrt{2}}{500} \mathrm{~A}
$$

$$
\chi_{C}=\frac{1}{100 \times 20 \times 10^{-6}}=\frac{1000}{2}=500 \Omega
$$

$$
V_{C}=50 \sqrt{2}=25 \sqrt{8}
$$

24. The magnetic field at centre $O$ is $k \times 10^{-7} \mathrm{~T}$. Radius given. Find $k$.


## Answer (3)

Sol. $B$ due to section $B C$ and $D A$ of wire $=0$
$B_{n e t}=B_{A B}+B_{C D}$
$B_{\text {net }}=\frac{\mu_{0} i \pi}{4 \pi \cdot 4 \pi}+\frac{\mu_{0} i \pi}{4 \pi \cdot 2 \pi}=\frac{\mu_{0} i}{\pi}\left(\frac{1}{16}+\frac{1}{8}\right)$
$B_{n e t}=\frac{3 \mu_{0} i}{\pi \times 16}=\frac{\mu_{0}}{4 \pi} \times \frac{3 i}{4}=10^{-7} \times \frac{3 \times 4}{4}$

$$
B_{n e t}=3 \times 10^{-7} \mathrm{~T} \otimes
$$

25. A particle is moving in straight line with constant acceleration with initial velocity of zero. The ratio of distance travelled by particle in $(n-1)^{\text {th }}$ second to that in $n^{\text {th }}$ second, where $n=10$ is $\frac{A}{B}$ then $(A+B)$ is $\qquad$ ( $A$ and $B$ are co-prime number)

## Answer (36)

Sol. $S_{n}=u+\frac{a}{2}(2 n-1) \Rightarrow \frac{S_{9}}{S_{10}}=\frac{17}{19}$
26. Find the tension ( $T$ ) in the given string (in $N$ ).


Answer (192)
Sol. $T=(6+10)(g+a)$

$$
\begin{aligned}
& =16 \times 12 \\
& =192 \mathrm{~N}
\end{aligned}
$$

27. An alternating voltage applied to a series $\angle R C$ circuit as shown in figure.


The current the circuit is $\qquad$ Ampere

## Answer (20)

Sol. $z=\sqrt{R^{2}+\left(x_{C}-x_{L}\right)^{2}}=R$

$$
i=\frac{200}{z}=20 \text { Ampere }
$$

28. 
29. 
30. 

## CHEMISTRY

## SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

## Choose the correct answer :

1. Which metal shows highest and maximum number of oxidation states?
(1) Mn
(2) Fe
(3) Co
(4) Cr

Answer (1)
Sol. Mn shows highest oxidation state of +7 . Oxidation number of Mn ranges from +2 to +7 .
2. Consider the reaction:

$$
\mathrm{Fe}_{2} \mathrm{O}_{3}(\mathrm{~s})+3 \mathrm{CO}(\mathrm{~g}) \rightleftharpoons 2 \mathrm{Fe}(\mathrm{~s})+3 \mathrm{CO}_{2}(\mathrm{~g})
$$

Which of the following will not affect the equilibrium state?
(I) Addition of $\mathrm{Fe}_{2} \mathrm{O}_{3}$
(II) Addition of $\mathrm{CO}_{2}$
(III) Decreasing mass of $\mathrm{Fe}_{2} \mathrm{O}_{3}$
(IV) Removal of CO
(1) (II) and (IV)
(2) (I) and (IV)
(3) (I) and (III)
(4) All will affect equilibrium state

Answer (3)
Sol. As conc. of solid species is constant (active mass $=1$ ) at a given temperature, for solid addition or decreasing mass do not affect equilibrium state.
3. Find out work done on the gas for following process

(1) 61.5 Joule
(2) -61.5 Joule
(3) +246 Joule
(4) -246 Joule

Answer (1)
Sol. $W=\pi r^{2}$

$$
\begin{aligned}
= & (3.14)\left(140 \times 10^{3}\right)\left(140 \times 10^{-6}\right) \mathrm{J} \\
& =61.544 \text { Joule }
\end{aligned}
$$

4. Identify the correct Lewis dot structure of $\mathrm{NO}_{2}^{-}$
(1) $[: O ̈=N=O ̈:]^{\ominus}$
(2) $[: \ddot{̣}-\ddot{N}=\ddot{O}]^{\ominus}$
(3) $[: \ddot{O}-N-\ddot{O}:]^{\ominus}$
(4) $[: O ̈=\ddot{N}=\ddot{O}:]^{\circ}$

Answer (2)
Sol. The correct Lewis structure of $\mathrm{NO}_{2}^{-}$is
$[: \ddot{O}-\ddot{\mathrm{N}}=\underset{O}{\mathrm{O}}]^{\ominus}$
5. What is correct order of strength of following ligands according to spectrochemical series
$\mathrm{Cl}^{-}, \mathrm{F}^{-}, \mathrm{I}^{-}, \mathrm{CN}^{-}, \mathrm{OH}^{-}$
(1) $\mathrm{CN}^{-}<\mathrm{OH}^{-}<\mathrm{F}^{-}<\mathrm{Cl}^{-}<\mathrm{I}^{-}$
(2) $\mathrm{I}^{-}<\mathrm{Cl}^{-}<\mathrm{F}^{-}<\mathrm{OH}^{-}<\mathrm{CN}^{-}$
(3) $\mathrm{I}^{-}<\mathrm{Cl}^{-}<\mathrm{F}^{-}<\mathrm{CN}^{-}<\mathrm{OH}^{-}$
(4) $\mathrm{F}^{-}<\mathrm{Cl}^{-}<\mathrm{I}^{-}<\mathrm{OH}^{-}<\mathrm{CN}^{-}$

Answer (2)
Sol. Spectrochemical series : (NCERT)

$$
\begin{aligned}
\mathrm{I}^{-} & <\mathrm{Br}^{-}<\mathrm{SCN}^{-}<\mathrm{Cl}^{-}<\mathrm{S}^{2-}<\mathrm{F}^{-}<\mathrm{OH}^{-}<\mathrm{C}_{2} \mathrm{O}_{4}^{2-}<\mathrm{NCS}^{-}<\mathrm{EDT}^{4-}<\mathrm{NH}_{3}<\mathrm{en}<\mathrm{CN}^{-}<\mathrm{CO}^{2}
\end{aligned}
$$

6. 



Product will be :
(1) Alcohol
(2) Alkane
(3) Alkene
(4) Ester

Answer (2)

Sol.

7. If the number of neutrons in the most abundant isotope of boron is ' $x$ ' and its highest oxidation state in unsaturated compound is ' $y$ ', then find the value of $(x+y)$
(1) 6
(2) 4
(3) 3
(4) 9

## Answer (4)

Sol. The most abundant isotope of boron is ${ }_{5} \mathrm{~B}^{11}$. Number of neutrons in it $=x=6$.

The highest oxidation state of boron in its unsaturated compound like $(\mathrm{BN})_{x}$ (Inorganic graphite) is +3
$\therefore y=3$

$$
x+y=6+3=9
$$

8. Consider the following statements:

Statement-1: Stability of +1 oxidation state increases down the group in Group-13
Statement-2: Atomic radius of Ga is greater than Al
(1) Statement-1 is correct, statement-2 is correct and statement-2 is correct explanation of statement-1
(2) Statement-1 is correct, statement-2 is correct but statement-2 is not correct explanation of statement-1
(3) Statement- 1 is correct, statement- 2 is incorrect
(4) Statement-1 is incorrect, statement-2 is correct

## Answer (3)

Sol. S-1: Stability of +1 oxidation state increases down the group due to inert pair effect $\Rightarrow S-1$ is true

S-2: Radius of Ga is smaller than AI due to d-block contraction $\Rightarrow S-2$ is false
9. Which of the following given complexes has least magnetic moment?
(1) $\left[\mathrm{Fe}\left(\mathrm{H}_{2} \mathrm{O}\right)_{6}\right]^{2+}$
(2) $\left[\mathrm{Co}\left(\mathrm{H}_{2} \mathrm{O}\right)_{6}\right]^{2+}$
(3) $\left[\mathrm{Ti}\left(\mathrm{H}_{2} \mathrm{O}\right)_{6}\right]^{3+}$
(4) $\left[\mathrm{Cr}\left(\mathrm{H}_{2} \mathrm{O}\right)_{6}\right]^{2+}$

## Answer (3)

Sol. $\left[\mathrm{Fe}\left(\mathrm{H}_{2} \mathrm{O}\right)_{6}\right]^{2+} \Rightarrow \mathrm{Fe}^{2+}=3 d^{6} \Rightarrow \mathrm{n}=4$
$\left[\mathrm{Co}\left(\mathrm{H}_{2} \mathrm{O}\right)_{6}\right]^{2+} \Rightarrow \mathrm{Co}^{2+}=3 d^{7} \Rightarrow \mathrm{n}=3$
$\left[\mathrm{Ti}\left(\mathrm{H}_{2} \mathrm{O}\right)_{6}\right]^{3+} \Rightarrow \mathrm{Ti}^{3+}=3 d^{1} \quad \Rightarrow \mathrm{n}=1$
$\left[\mathrm{Cr}\left(\mathrm{H}_{2} \mathrm{O}\right)_{6}\right]^{2+} \Rightarrow \mathrm{Cr}^{2+}=3 d^{4} \quad \Rightarrow \mathrm{n}=4$
$\left[\mathrm{Ti}\left(\mathrm{H}_{2} \mathrm{O}\right)_{6}\right]^{3+}$ has only 1 unpaired electron hence least magnetic moment.
10. Assertion : Trans But-2-ene is less polar than cis But-2-ene

Reason: Trans But-2-ene has zero dipole moment
(1) Both Assertion and Reason are true and Reason is a correct explanation of Assertion
(2) Both Assertion and Reason are true but Reason is not a correct expiation of Assertion.
(3) Assertion is true and Reason is false
(4) Assertion is false and Reason is true.

## Answer (1)

Sol.


Trans but-2-ene
$\mu=0$ for trans but-2-ene, whereas for cis but-2-ene
$\mu \neq 0$

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11. Arrange the following carbocations in increasing order of their stability
(I) $\left(\mathrm{CH}_{3}\right)_{3}$ - $^{\oplus}$
(II)

(III) $\stackrel{\oplus}{\mathrm{C}} \mathrm{H}_{3}$
(IV) $\mathrm{CH}_{3}-\stackrel{\oplus}{\mathrm{C}} \mathrm{H}_{2}$
(1) I $<$ II $<$ IV $<$ III
(2) II $<$ III $<$ IV $<$ I
(3) II $<$ I $<$ III $<$ IV
(4) III $<$ IV $<$ II $<$ I

Answer (4)
Sol. Stability of carbocation $\propto$-number of $\alpha$-hydrogen atoms


So, stability III < IV < II < I
12. Which postulates of Dalton's theory is wrong?
(1) Matter consists of indivisible atoms
(2) All atoms of a given element have identical properties but different masses
(3) Compounds are formed when atoms of different elements combine in a fixed ratio
(4) Chemical reaction involve reorganisation of atoms

## Answer (2)

Sol. All atoms of a given elements have identical properties including identical mass.
13. In which of the following compound Mn will have highest oxidation state?
(1) $\mathrm{MnO}_{4}^{-}$
(2) $\mathrm{MnO}_{2}$
(3) $\mathrm{MnO}_{4}^{2-}$
(4) $\mathrm{Mn}_{2} \mathrm{O}_{3}$

Answer (1)
Sol. $\mathrm{MnO}_{4}^{-}$possess +7 oxidation state of Mn which is highest among the given compounds.
14. Which of the following cation will give green colour in Borax Bead Test?
(1) Iron
(2) Cobalt
(3) Manganese
(4) Nickel

## Answer (1)

Sol. Iron
Cobalt
Manganese
Nickel

## Green

Blue
Colourless/Violet
Grey
15. Identify the change occurring in oxidation state of Mn in cell reaction of dry cell in clock during its use
(1) $+3 \rightarrow+4$
(2) $+2 \rightarrow+7$
(3) $+4 \rightarrow+3$
(4) $+7 \rightarrow+2$

## Answer (3)

Sol. Discharging cell reaction of dry cell
$\underset{\text { Anode }}{\mathrm{Zn}(\mathrm{s})}+\underset{\text { Cathode }}{2 \mathrm{Mnn}_{2}(\mathrm{~s})}+2 \mathrm{NH}_{4} \mathrm{Cl}(\mathrm{s}) \longrightarrow$

$$
\mathrm{ZnCl}_{2}(\mathrm{~s})+2 \mathrm{Mn}^{+3} \mathrm{O}(\mathrm{OH})(\mathrm{s})+2 \mathrm{NH}_{3} \uparrow
$$

16. Identify the correct relationship between potential energy and total energy of an electron in Bohr's model for single electron species.
(1) $P E=2 T E$
(2) $\mathrm{PE}=-2 \mathrm{TE}$
(3) $\mathrm{PE}=\mathrm{TE}$
(4) $P E=-T E$

Answer (1)

Sol. In Bohr's model for H-like atoms/ions,

$$
T E\left(E_{n}\right)=\frac{-13.6 Z^{2}}{n^{2}} e V
$$

andPE $\left(U_{n}\right)=2 T E\left(E_{n}\right)$
and $K E\left(K_{n}\right)=-T E\left(E_{n}\right)$
17. If value of mass of electron is $9.1 \times 10^{-31} \mathrm{~kg}$. Find out velocity of electron in first orbit of H -atom.
(1) $2.18 \times 10^{5} \mathrm{~m} / \mathrm{s}$
(2) $2.18 \times 10^{6} \mathrm{~m} / \mathrm{s}$
(3) $2.18 \times 10^{7} \mathrm{~m} / \mathrm{s}$
(4) $2.18 \times 10^{4} \mathrm{~m} / \mathrm{s}$

## Answer (2)

Sol. $13.6 \times 1.6 \times 10^{-19}=\frac{1}{2} \times\left(9.1 \times 10^{-31}\right) \times v^{2}$
$v=2.186 \times 10^{6} \mathrm{~m} / \mathrm{s}$
18. If $\mathrm{C}=42.1 \%, \mathrm{H}=6.4 \%, \mathrm{O}=52.5 \%$ and molar mass is $342 \mathrm{~g} / \mathrm{mol}$, then molecular formula of compound.
(1) $\mathrm{C}_{11} \mathrm{H}_{22} \mathrm{O}_{12}$
(2) $\mathrm{C}_{12} \mathrm{H}_{22} \mathrm{O}_{11}$
(3) $\mathrm{C}_{10} \mathrm{H}_{20} \mathrm{O}_{11}$
(4) $\mathrm{C}_{11} \mathrm{H}_{22} \mathrm{O}_{11}$

## Answer (2)

Sol. Molar mass matches with $\mathrm{C}_{12} \mathrm{H}_{22} \mathrm{O}_{11}(342 \mathrm{~g} / \mathrm{mol})$
\% of C in $\mathrm{C}_{12} \mathrm{H}_{22} \mathrm{O}_{11}=\frac{12 \times 12 \times 100}{342}=42.1 \%$
19. Which one of the following compounds will give positive ninhydrin test?
(1) Starch
(2) Cellulose
(3) Egg Albumin
(4) Lipid

## Answer (3)

Sol. Ninhydrin test is given by all $\alpha$-amino acids. Egg albumin is a protein which on hydrolysis gives number of $\alpha$-amino acids. So, Egg albumin will give positive ninhydrin test.
20. Consider the following compounds
(a)

(b)

(c)

(d)


Select the option with correct order of their boiling points
(1) $d>a>b>c$
(2) $a>b>d>c$
(3) a $>$ d $>$ c $>d$
(4) a $>d>b>c$

## Answer (2)

Sol. B.P. $\mathrm{C}_{2} \mathrm{H}_{5}-\mathrm{O}-\mathrm{C}_{2} \mathrm{H}_{5} \simeq 307 \mathrm{~K}$


## SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.


Sum of total number of oxygen atoms in $A$ and $B$ is

## Answer (14)

Sol.

(B)

This is preparation method of picric acid.
Total number of oxygen atoms in

$$
\begin{gathered}
A=7 \\
B=7 \\
\text { Sum }=7+7=14
\end{gathered}
$$

22. Sum of number of $\sigma$ and $\pi$ bonds in ethylene molecule is $x$. Then value of $x$ is:

## Answer (6)

Sol. Ethylene :

$\sigma$ bond $=5$
$\pi$ bond $=1$
$x=5+1=6$
23. What is the number of valence electrons on $N$ in $\mathrm{NO}_{2}^{-}$?

## Answer (8)

Sol. $\mathrm{NO}_{2}^{-}$:


Total valence electrons of N in $\mathrm{NO}_{2}^{-}=8$
24. How many of the following can be prepared by Sandmeyer's reaction:






Answer (2)

Sol.




25. Find out overall order of reaction

|  | $[\mathrm{A}]$ | $[\mathrm{B}]$ | Rate of <br> reaction |
| :--- | :--- | :--- | :--- |
| Exp-1 | 0.1 | 0.1 | $6 \times 10^{-1}$ |
| Exp-2 | 0.3 | 0.2 | $72 \times 10^{-1}$ |
| Exp-3 | 0.3 | 0.4 | $288 \times 10^{-1}$ |
| Exp-4 | 0.4 | 0.1 | $24 \times 10^{-1}$ |

Answer (3)

Sol. Exp-1 and Exp-4
$r \alpha[A]^{1}$
Exp-2 and Exp-3
Rate $\propto[B]^{2}$
$r \propto[A]^{1}[B]^{2}$
Overall order $=1+2$

$$
=3
$$

26. 



If mass of aniline taken is 9.3 g and produced mass of product is 26.4 g . Then calculate the percentage yield of reaction.
[Atomic mass of $\mathrm{Br}: 80 \mathrm{~g} \mathrm{~mol}^{-1}$ ]

## Answer (80)

Sol. Moles of aniline taken $=\frac{9.3}{93}=0.1$ as molar mass of aniline is $93 \mathrm{~g} \mathrm{~mol}^{-1}$


Molar mass $=330 \mathrm{~g} \mathrm{~mol}^{-1}$

$$
\begin{aligned}
\text { Yield } & =\frac{\text { Produced mass }(\text { real })}{\text { Produced mass (ideal) }} \times 100 \\
& =\frac{26.4 \times 100}{330 \times 0.1}=80
\end{aligned}
$$

27. The limiting molar conductivities of a divalent cation $\left(\mathrm{M}^{2+}\right)$ and a monovalent anion ( $\mathrm{A}^{-}$) are $57 \mathrm{~S} \mathrm{~cm}^{2}$ $\mathrm{mol}^{-1}$ and $73 \mathrm{~S} \mathrm{~cm}^{2} \mathrm{~mol}^{-1}$ respectively. Then find the limiting molar conductivity shown by their compound in $\mathrm{S} \mathrm{cm}^{2} \mathrm{~mol}^{-1}$ is $\qquad$ -.

## Answer (203)

Sol. Compound of $\mathrm{M}^{2+}$ and $\mathrm{A}^{-}$is $\mathrm{MA}_{2}$.
Limiting Molar conductivity of $\mathrm{M}^{2+}=57 \mathrm{~S} \mathrm{~cm}^{2} \mathrm{~mol}^{-1}$
Limiting Molar conductivity of $\mathrm{A}^{-}=73 \mathrm{~S} \mathrm{~cm}^{2} \mathrm{~mol}^{-1}$
Limiting Molar conductivity of $\mathrm{MA}_{2}=57+2 \times 73$

$$
=203 \mathrm{~S} \mathrm{~cm}^{2} \mathrm{~mol}^{-1}
$$

28. One litre solution of 0.2 M glucose is separated with its pure solvent with semi-permeable membrane.
0.1 moles of NaCl is added to the solution. The change in osmotic pressure of solution will be at 300 K (nearest integer)
(Take $\mathrm{R}=0.083$ )

## Answer (5)

Sol. $\pi_{\text {initial }}=\mathrm{iCRT}=1 \times 0.2 \times 0.083 \times 300$

$$
\begin{aligned}
\pi_{\text {final }} & =(0.2+0.2) 0.083 \times 300 \\
& =0.4 \times 0.083 \times 300 \\
\Delta \pi & =(0.4-0.2) \times 0.083 \times 300 \\
& =4.98 \\
& \simeq 5
\end{aligned}
$$

29. 
30. 

## MATHEMATICS

## SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

## Choose the correct answer :

1. The value of $\int_{0}^{\frac{\pi}{4}} \frac{d x}{1+\tan x}$ equals to
(1) $\frac{\pi}{8}+\ln 2$
(2) $\frac{\pi}{4}+\ln 2$
(3) $\frac{\pi}{8}+\frac{1}{2} \ln 2$
(4) $\frac{\pi}{8}+\frac{1}{4} \ln 2$

Answer (4)
Sol. $I=\int_{0}^{\frac{\pi}{4}} \frac{d x}{1+\tan x}$

$$
I=\int_{0}^{\frac{\pi}{4}} \frac{d x}{1+\tan \left(\frac{\pi}{4}-x\right)}
$$

$$
=\int_{0}^{\frac{\pi}{4}} \frac{d x}{1+\frac{1-\tan x}{1+\tan x}}
$$

$$
=\int_{0}^{\frac{\pi}{4}}\left(\frac{1+\tan x}{2}\right) d x
$$

$$
\left.=\frac{1}{2} x+\frac{1}{2} \ln |\sec x|\right]_{0}^{\frac{1}{4}}
$$

$$
=\frac{\pi}{8}+\frac{1}{2} \ln \sqrt{2}
$$

$$
=\frac{\pi}{8}+\frac{1}{4} \ln 2
$$

2. $\int_{-\pi}^{\pi} \frac{2 x(1+\sin x) d x}{\left(1+\cos ^{2} x\right)}$ is equal to
(1) $\pi^{2}$
(2) $2 \pi$
(3) $\frac{3 \pi}{2}$
(4) $\frac{\pi^{2}}{2}$

Answer (1)
Sol.
$I=\int_{-\pi}^{\pi} \frac{(2 x(1+\sin x)) d x}{\left(1+\cos ^{2} x\right)}=\int_{0}^{\pi} \frac{2 x(1+\sin x)+(-2 x)(1+\sin (-x)) d x}{\left(1+\cos ^{2} x\right)}$

$$
I=4 \int_{0}^{\pi} \frac{x \sin x d x}{\left(1+\cos ^{2} x\right)}
$$

$$
I=4 \int_{0}^{\pi} \frac{(\pi-x) \sin (\pi-x)}{1+(\cos (\pi-x))^{2}} d x
$$

$$
I=4 \int_{0}^{\pi} \frac{\pi \sin x d x}{1+\cos ^{2} x}-4 \int_{0}^{\pi} \frac{x \sin x d x}{\left(1+\cos ^{2} x\right)}
$$

$\Rightarrow 2 I=4 \pi \int_{0}^{\pi} \frac{\sin x d x}{1+\cos ^{2} x}$
$\Rightarrow I=2 \pi \int_{0}^{\pi} \frac{\sin x}{1+\cos ^{2} x} d x=2 \pi \cdot \frac{\pi}{2}=\pi^{2}$
3. If $\frac{d y}{d x}+2 y=\sin 2 x \& y(0)=\frac{3}{4}$, then $y\left(\frac{\pi}{8}\right)$
(1) $e^{\frac{\pi}{8}}$
(2) $e^{\frac{\pi}{6}}$
(3) $e^{\frac{-\pi}{4}}$
(4) None

Answer (3)
Sol. I.F. $=e \int 2 d x$
$y e^{2 x}=\int e^{2 x} \sin 2 x d x$
Let $2 x=t$.
$d x=\frac{d t}{2}$
$y e^{2 x}=\frac{1}{2} \int e^{t} \sin t d t$
$=\frac{1}{4}\left[e^{t}[\sin t-\cos t]\right]+c$
$y e^{2 x}=\frac{e^{2 x}}{4}(\sin 2 x-\cos 2 x)+c$
Now $y(0)=\frac{3}{4}$
$\Rightarrow \frac{3}{4}=-\frac{1}{4}+c$
$\Rightarrow c=1$
$\therefore y=\frac{\sin 2 x-\cos 2 x}{4}+e^{-2 x}$
$\therefore y\left(\frac{\pi}{8}\right)=e^{-\frac{\pi}{4}}$
4. Consider the equation $a x^{2}+b x+c=0$ then find the probability if $a, b, c \in A$, where $A=\{1,2,3,4 \ldots 8\}$ that the equation has equal roots.
(1) $\frac{1}{512}$
(2) $\frac{1}{64}$
(3) $\frac{1}{8}$
(4) $\frac{1}{4}$

## Answer (2)

Sol. For equal roots : $b^{2}-4 a c=0$
$\Rightarrow b^{2}=4 a c$
$\Rightarrow a c$ must be perfect square
$\Rightarrow \quad(a, c)=\{(1,1),(1,4),(2,2),(2,8),(3,3),(4,1)$,

$$
(4,4),(5,5),(6,6),(7,7),(8,2),(8,8)\}
$$

Correspond $b$ must lie in set $A$
$\Rightarrow \quad(a, b, c) \in\{(1,2,1),(1,4,4),(2,4,2),(2,8,8)$,
$(3,6,3),(4,4,1),(4,8,4),(8,8,2)\}$
$\Rightarrow$ probability $=\frac{8}{8^{3}}=\frac{1}{64}$
5. Let $f(x)=x^{2}-5 x$ and $g(x)=7 x-x^{2}$. Then the area between curves equals to
(1) 36
(2) 70
(3) 72
(4) 50

Answer (3)
Sol. Given $f(x)=x^{2}-5 x$,
$g(x)=7 x-x^{2}$


Intersection of (1) and (2)
$x^{2}-5 x=7 x-x^{2}$
$2 x^{2}-12 x=0$
$2\left(x^{2}-6 x\right)=0$
$2 x(x-6)=0$
$x=0, x=6$
$\therefore \quad A=\int_{0}^{6}(g(x)-f(x)) d x$
$=\int_{0}^{6}\left(7 x-x^{2}-\left(x^{2}-5 x\right)\right) d x$
$=\int_{0}^{6}\left(7 x-x^{2}-x^{2}+5 x\right) d x$
$=\int_{0}^{6}\left(12 x-2 x^{2}\right) d x$
$=12\left(\frac{6^{2}}{2}\right)-\frac{2}{3}(6)^{3}$
$=216-144$
$=72$ sq. units

JEE Main Session -2 (05-04-2024)-Shift - 1
6. When 4 dice are rolled, then find the probability of 16 as a sum.
(1) $\frac{5^{4}}{6^{4}}$
(2) $\frac{5^{3}}{6^{4}}$
(3) $\frac{5^{2}}{6^{4}}$
(4) $\frac{5}{6^{4}}$

## Answer (2)

Sol. Total ways to distribute 16 as a sum of 4 numbers are:

| 6 | 6 | 2 | 2 | $\rightarrow 4!/ 2!2!$ | $=$ | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 6 | 6 | 3 | 1 | $\rightarrow 4!/ 2!$ | $=$ | 12 |
| 6 | 5 | 4 | 1 | $\rightarrow 4!$ | $=$ | 24 |
| 6 | 5 | 3 | 2 | $\rightarrow 4!$ | $=$ | 24 |
| 6 | 4 | 4 | 2 | $\rightarrow 4!/ 2!$ | $=$ | 12 |
| 6 | 4 | 3 | 3 | $\rightarrow 4!/ 2!$ | $=$ | 12 |
| 5 | 5 | 4 | 2 | $\rightarrow 4!/ 2!$ | $=12$ |  |
| 5 | 5 | 3 | 3 | $\rightarrow 4!/ 2!\cdot 2!$ | $=$ | 6 |
| 5 | 5 | 1 | 5 | $\rightarrow 4!/ 3!$ | $=$ | 4 |
| 5 | 4 | 4 | 3 | $\rightarrow 4!/ 2!$ | $=12$ |  |
| 4 | 4 | 4 | 4 | $\rightarrow 1$ |  |  |

Required Probability $=\frac{125}{6^{4}}$
7. A rectangle $A B C D$ with $A B=2$ and $B C=4$ is inscribed in rectangle PQRS such that vertices of ABCD lie on sides of PQRS then maximum possible area of rectangle PQRS is (sq. unit)
(1) 9
(2) 20
(3) 18
(4) 12

Answer (3)

Sol.

$P D=4 \cos \theta \quad \Rightarrow P S=4 \cos \theta+2 \sin \theta$
$D S=2 \sin \theta$
$A P=4 \sin \theta$
$Q A=2 \cos \theta \quad \Rightarrow P Q=2 \cos \theta+4 \sin \theta$
$\Rightarrow$ Area of $P Q R S$
$=P Q R S=4(2 \cos \theta+\sin \theta)(\cos \theta+2 \sin \theta)$
$=4\left[2 \cos ^{2} \theta+2 \sin ^{2} \theta+5 \sin \theta \cos \theta\right]$
$=8+10 \sin 2 \theta$
Area is maximum when $\sin 2 \theta=1 \Rightarrow \theta=45^{\circ}$
$\Rightarrow$ max area $=8+10=18$
8. Two lines passing through $(2,3)$ parallel to coordinate axes. A circle of unit radius touches both the lines and lie on the origin side.
Then the shortest distance of point $(5,5)$ from the circle is
(1) 2
(2) 3
(3) 4
(4) $\sqrt{13}$

Answer (3)

Sol.

$\Rightarrow$ Centre $\equiv(1,2)$
$\Rightarrow$ Equation of circle is $(x-1)^{2}+(y-2)^{2}=1$

$P C=5$
$C B=1$
$\Rightarrow P B=$ shortest distance $=4$
9. If $f(1)=1 \lim _{t \rightarrow x} \frac{t^{2} f(x)-x^{2} f(t)}{t-x}=1$ then $2 f(2)+3 f(3)$ equals to
(1) 1
(2) 7
(3) 9
(4) 13

## Answer (4)

Sol. $\lim _{t \rightarrow x} \frac{t^{2} f(x)-x^{2} f(t)}{t-x}=1$
$\lim _{t \rightarrow x} \frac{2 t f(x)-x^{2} f^{\prime}(t)}{1}=1$
$2 x f(x)-x^{2} f^{\prime}(x)=1$
$\Rightarrow \frac{d y}{d x}-\frac{2}{x} y=-1$
I.F. $=e^{\int-\frac{2}{x} d x}=\frac{1}{x^{2}}$
$\therefore \frac{y}{x^{2}}=\int-\frac{1}{x^{2}} d x$
$\frac{y}{x^{2}}=\frac{1}{x}+c$
$\therefore y=x+x^{2} c$
Now $f(1)=1$
$\Rightarrow c=0$
$\therefore y=x$
Now $2 f(2)+3 f(3)=2 \times 2+3 \times 3$
$=4+9=13$
Option (D) is correct.
10. $\int_{0}^{\pi / 4} \frac{39 \sin x d x}{2 \sin x+3 \cos x}$ equals to
(1) $\frac{3 \pi}{2}+9 \ln \left(\frac{5}{3 \sqrt{2}}\right)$
(2) $\frac{3 \pi}{2}-9 \ln \left(\frac{5}{3 \sqrt{2}}\right)$
(3) $\frac{3 \pi}{2}+3 \ln \left(\frac{5}{3 \sqrt{2}}\right)$
(4) $\frac{3 \pi}{2}-3 \ln \left(\frac{5}{3 \sqrt{2}}\right)$

## Answer (2)

Sol. $\sin x=A(2 \sin x+3 \cos x)+B(2 \cos x-3 \sin x)$

$$
\left.\left.\begin{array}{ll} 
& 2 A-3 B=1 \\
\Rightarrow & 3 A+2 B=0
\end{array}\right\} B=\frac{-3}{13} \begin{array}{l}
13 \\
B
\end{array}\right] \begin{aligned}
& \therefore \quad I=39 \int \frac{2}{\frac{2}{13}(2 \sin x+3 \cos x)-\frac{3}{13}(2 \cos x-3 \sin x)} \\
& (2 \sin x+3 \cos x)
\end{aligned} x
$$

$$
=39\left[\frac{2}{13} x-\frac{3}{13} \ln |2 \sin x+3 \cos x|\right]_{0}^{\frac{\pi}{4}}
$$

$$
=[6 x-9 \ln |2 \sin x+3 \cos x|]_{0}^{\frac{\pi}{4}}
$$

$$
=\frac{6 \pi}{4}-9 \ln \left(\frac{5}{\sqrt{2}}\right)+9 \ln (3)
$$

$$
=\frac{3 \pi}{2}-9 \ln \left(\frac{5}{3 \sqrt{2}}\right)
$$

11. If $f(x)=\frac{\sin 3 x+\alpha \sin x+\beta \cos 3 x}{x^{3}}$ is
continuous at $x=0$ then $f(0)$ is
(1) -4
(2) 4
(3) 9
(4) 3

Answer (1)
Sol. $\operatorname{limit}_{x \rightarrow 0} \frac{\sin 3 x+\alpha \sin x+\beta \cos 3 x}{x^{3}}$

$$
\operatorname{limint}_{x \rightarrow 0} \frac{\left(3 x-\frac{27 x^{3}}{3!}+\ldots\right)+\alpha\left[x-\frac{x^{3}}{3!}+\ldots\right]+\beta\left[1-\frac{9 x^{2}}{2!}+\frac{81 x^{4}}{4!}+\ldots\right]}{x^{3}}
$$

$\therefore \quad \beta=0 \& \alpha=-3$,
$\therefore f(0)=\operatorname{limit}_{x \rightarrow 0} \frac{\left[-\frac{27}{3!}+\frac{3}{3!}\right] x^{3}+\ldots}{x^{3}}$
$=\frac{-27+3}{3!}=\frac{-24}{3!}=-4$

JEE Main Session -2 (05-04-2024)-Shift - 1
12. If intersection point of lines $\frac{x-3}{3}=\frac{y}{2}=\frac{z+1}{2}$ and $\frac{x-7}{4}=\frac{y}{1}=\frac{z+1}{1}$ is $P$. Then the distance of $P$ from $\left(\frac{-2}{5}, \frac{-3}{5}, \frac{2}{5}\right)$ is equal to
(1) $\sqrt{13}$
(2) $\sqrt{19}$
(3) $\sqrt{11}$
(4) $\sqrt{17}$

## Answer (3)

Sol. $\frac{x-3}{3}=\frac{y}{2}=\frac{z+1}{2}=\lambda$ (say)
$x=3 \lambda+3$
$y=2 \lambda$
$z=2 \lambda-1$
And $\frac{x-7}{4}=\frac{y}{1}=\frac{z+1}{1}=\mu($ say $)$
$x=4 \mu+7$
$y=\mu$
$z=\mu-1$
As these lines intersect, so

$$
3 \lambda+3=4 \mu+7, \quad 2 \lambda=\mu
$$

We get

$$
\begin{aligned}
& 3 \lambda+3=4(2 \lambda)+7 \\
& 5 \lambda=-4 \\
& \lambda=\frac{-4}{5} \Rightarrow \mu=\frac{-8}{5}
\end{aligned}
$$

So we get points as $\left(\frac{3}{5}, \frac{-8}{5}, \frac{-13}{5}\right)=P$
Now, distance of $P$ from $\left(\frac{-2}{5}, \frac{-3}{5}, \frac{2}{5}\right)$

$$
\begin{aligned}
& =\sqrt{\left(\frac{3}{5}+\frac{2}{5}\right)^{2}+\left(\frac{-8}{5}+\frac{3}{5}\right)^{2}+\left(\frac{-13}{5}-\frac{2}{5}\right)^{2}} \\
& =\sqrt{1+1+9}=\sqrt{11}
\end{aligned}
$$

13. 10 items out of which 3 are defective. If $x$ denotes number of defective items if 5 items are drawn and $\operatorname{Var}(X)=\sigma^{2}$ then $96 \sigma^{2}$ is equal to
(1) 96
(2) 56
(3) 21
(4) 28

## Answer 2)

Sol.

| $X$ | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: |
| $P(X)$ | $\frac{{ }^{7} C_{5}}{{ }^{10} C_{5}}$ | $\frac{{ }^{7} C_{4} \cdot{ }^{3} C_{1}}{{ }^{10} C_{5}}$ | $\frac{{ }^{7} C_{3} \cdot{ }^{3} C_{3}}{{ }^{10} C_{5}}$ | $\frac{{ }^{7} C_{2} \cdot{ }^{3} C_{3}}{{ }^{10} C_{5}}$ | $\Rightarrow$


| $x$ | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: |
| $P(X)$ | $\frac{1}{12}$ | $\frac{5}{12}$ | $\frac{5}{12}$ | $\frac{1}{12}$ |

$\sigma^{2}=\sum(x-u)^{2} P(x)$
$\mu=\sum x P(x)=0+\frac{5}{12}+\frac{10}{12}+\frac{3}{12}=\frac{18}{12}$

$$
=\frac{3}{2}
$$

$\sigma^{2}=\sum\left(x-\frac{3}{2}\right)^{2} P(x)=\frac{9}{4} \times \frac{1}{12}+\frac{1}{4} \times \frac{5}{12}+\frac{1}{4} \times \frac{5}{12}+\frac{9}{4} \times \frac{1}{12}$
$=\frac{1}{48}(9+5+5+9)=\frac{28}{48}=\frac{7}{12}$
$\Rightarrow 96 \sigma^{2}=56$
14.
15.
16.
17.
18.
19.
20.

SECTION - B
Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.
21. If $f(x)=x^{5}+2 x^{3}+3 x+1$ and $g(f(x))=x$, then $\frac{g(1)}{g^{\prime}(1)}$ is equal to

## Answer (0)

Sol. $g(f(x))=x$
$g^{\prime}(f(x)) \times f^{\prime}(x)=1$
$g^{\prime}(f(x))=\frac{1}{f^{\prime}(x)}$
$f(x)=1$
$\Rightarrow x^{5}+2 x^{3}+3 x+1=1$
$x^{5}+2 x^{3}+3 x=0$
$\Rightarrow x=0$ is the only solution because $f(x)$ is increasing.
$\Rightarrow g(1)=0(f(x)=1 a+x=0)$
$\therefore \frac{g(1)}{g^{\prime}(1)}=0$
22. If $\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\ldots+\frac{l}{\sqrt{99}+\sqrt{100}}=m$
$\frac{1}{1 \times 2}+\frac{1}{2 \times 3}+\ldots+\frac{1}{99 \times 100}=n$.
Then the value of $\frac{m}{n}$ equals to

## Answer (100)

Sol. $\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\ldots+\frac{1}{\sqrt{99}+\sqrt{100}}$

$$
=\begin{array}{r}
\frac{1}{(\sqrt{1}+\sqrt{2})} \frac{\sqrt{2}-\sqrt{1}}{(\sqrt{2}-\sqrt{1})}+\frac{1}{(\sqrt{2}+\sqrt{3})} \frac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}-\sqrt{2})}+\ldots+ \\
\\
\frac{1}{\sqrt{99}+\sqrt{100}} \times \frac{\sqrt{100}-\sqrt{99}}{\sqrt{100}+\sqrt{99}}
\end{array}
$$

$\sqrt{2}-\sqrt{1}+\sqrt{3}-\sqrt{2}+\ldots \sqrt{100}-\sqrt{99}$
$=\sqrt{100}-\sqrt{1}$
$\Rightarrow 99=m$
$\frac{1}{1 \times 2}+\frac{1}{2 \times 3}+\ldots+\frac{1}{99 \times 100}=n$
$\frac{2-1}{1 \times 2}+\frac{3-2}{2 \times 3}+\ldots+\frac{100-99}{99 \times 100}$
$\Rightarrow \frac{1}{1}-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\ldots+\frac{1}{99}-\frac{1}{100}$
$\Rightarrow \quad 1-\frac{1}{100}=\frac{99}{100}=n$
$\frac{m}{n}=\frac{99}{99}=100$
100
23. If $a \in R$ and $|2 a-1| \leq 3[a]+2\{a\}$, Here [ $x]$ represents greatest integer value of $x$ and $\{x\}$ represents fractional part value of $x$.

Then the value of $72 a_{\text {min }}$ equals to

## Answer (18)

Sol. $|2 a-1| \leq 3[a]+2\{a\}$
$|2 a-1| \leq[a]+2 a$
For $2 a-1<0$ i.e., $a<\frac{1}{2}-2 a+1 \leq[a]+2 a$
$4 a+[a]-1 \geq 0 \Rightarrow[a] \geq 1-4 a \Rightarrow a \geq 0.25$ and for
$a>\frac{1}{2}$
$2 a-1 \leq[a]+2 a$
$\Rightarrow[a] \geq-1 \Rightarrow a \geq-1$
So $a_{\text {min. }}=0.25$
$\Rightarrow 72 .(0.25)=\frac{72}{4}=18$
24. The number of distinct real roots of
$|x||x+2|-5|x+1|-1=0$ equals to

## Answer (3)

Sol.

| 1 | 1 | 1 |
| :--- | :--- | :--- |
| -2 | -1 | 0 |

(I) $x \geq 0$

$$
\begin{aligned}
& x^{2}+2 x-5 x-5-1=0 \\
& x^{2}-3 x-6=0 \rightarrow 1 \text { root }+\mathrm{ve}
\end{aligned}
$$

(II) $-1 \leq x<0$

$$
-x^{2}-2 x-5 x-5-1=0
$$

$x^{2}+7 x+6=0 \rightarrow x=-1$ is a root
(III) $-1<x \leq-2$
$x^{2}-2 x+5 x+5-1=0$
$x^{2}-3 x-4=0 \rightarrow$ No root in given range
(IV) $x<-2$

$$
\begin{aligned}
& x^{2}+2 x+5 x+5-1=0 \\
& x^{2}+7 x+4=0 \rightarrow \text { One root less than }-2
\end{aligned}
$$

$\therefore$ The number of distinct roots are 3
25. Let $2 x+3 y-k=0$ is a curve which intersects axis at points $A$ and $B$. A circle is drawn through $A$ and $B$ as diameter has equation $x^{2}+y^{2}-3 x-2 y=0$.
If the latus rectum of ellipse $x^{2}+9 y^{2}=k^{2}$ is $/$ then $3 /$ is equal to

Answer (04.00)
Sol.


Equation of circle with $A B$ as diameter

$$
\begin{aligned}
& \left(x-\frac{k}{2}\right) x+y\left(y-\frac{k}{3}\right)=0 \\
& \Rightarrow \quad x^{2}+y^{2}-\frac{k x}{2}-\frac{k y}{3}=0
\end{aligned}
$$

Comparing, $k=6$
Latus rectum of ellipse

$$
\begin{aligned}
& x^{2}+9 y^{2}=k^{2}=6^{2} \\
& \Rightarrow \frac{x^{2}}{6^{2}}+\frac{y^{2}}{2^{2}}=1 \quad \Rightarrow \text { L.R. }=\frac{2 b^{2}}{a}=\frac{2(4)}{6}
\end{aligned}
$$

$$
=\frac{4}{3}
$$

26. 
27. 
28. 
29. 
30. 
