Solve:-

$\log_e a, \log_e b, \log_e c$ in A.P.

$$b^2 = ac \quad ---(i)$$

Also

$\log \frac{a}{2b}, \log_e c\left(\frac{2b}{3c}\right), \log\left(\frac{3c}{a}\right)$ in A P

$$\left(\frac{2b}{3c}\right)^2 = \frac{a}{2b} \times \frac{3c}{a}$$

$$\frac{b}{c} = \frac{3}{2}$$

from (i)

$$b^2 = a \times \frac{2b}{3}$$

$$\frac{a}{b} = \frac{3}{2}$$

$$\boxed{a:b:c = 9:6:4}$$

Q-2

Solve -

$$\overline{X} = \frac{24}{5}, \quad \sigma^2 = \frac{194}{5}$$

Let first four observation be x_1, x_2, x_3, x_4

Here, $\frac{x_1+x_2+x_3+x_4+x_5}{5} = \frac{24}{5}$ —(i)

$$\frac{x_1+x_2+x_3+x_4}{4} = \frac{7}{2}$$

$$x_1+x_2+x_3+x_4 = 14$$

Now from (i)

$$x_5 = 10$$

$$\sigma^2 = \frac{195}{24}$$

$$\frac{\Sigma x_i^2}{5} - \left(\frac{\Sigma x_i}{5}\right)^2 = \frac{195}{24}$$

$$\frac{x_1^2+x_2^2+x_3^2+x_4^2+x_5^2}{5} - \frac{576}{25} = \frac{194}{25}$$

$$x_1^2+x_2^2+x_3^2+x_4^2 = 54$$

Variand of 4 observation.

$$\boxed{\sigma^2 = \frac{\sum_{i=1}^{4} x_i^2}{4} - \left(\frac{\sum_{i=1}^{4} x_i}{4}\right)^2 = \frac{54}{4} - \frac{49}{4} = \frac{5}{4}}$$

Q)

Solve:-

Assume $\sin^{-1} \cancel{x} = \theta$

$$\cos 2\theta = \frac{1}{9}$$

$$1 - 2\sin^2\theta = \frac{1}{9}$$

$$\sin\theta = \pm\frac{2}{3} = x$$

as m & n are co-prime of Natural number

$$x = \frac{2}{3}$$

$$m = 2, \ n = 3$$

Quadratic equation $2x^2 - 3x + 1 = 0$

$$\alpha = 1, \ \beta = \frac{1}{2}$$

$\boxed{\left(1, \frac{1}{2}\right) \text{ lies of } 5x + 8y = 9}$

Q54

Solve Differential equation

$$x\cos\frac{y}{x}\frac{dy}{dx} = y\cos\frac{y}{x} + x$$

$$\cos\frac{y}{x}\left[x\frac{dy}{dx} - y\right] = x$$

divide by x^2

$$\cos\frac{y}{x}\left(\frac{x\frac{dy}{dx} - y}{x^2}\right) = \frac{1}{x}$$

$$\frac{y}{x} = t$$

$$\frac{x\frac{dy}{dx} - y}{x^2} = \frac{dt}{dx}$$

$$\cos t\left(\frac{dt}{dx}\right) = \frac{1}{x}$$

$$\int \cos t\,dt = \int \frac{1}{x}\,dx$$

$$\sin t = \ln|x| + c$$

$$\sin\frac{y}{x} = \ln|x| + c$$

$$y(1) = \pi/3$$

$$\frac{\sqrt{3}}{2} = c$$

so $\alpha = \sqrt{3}$

$$\boxed{\alpha^2 = 3}$$

⑤

<u>Solve :-</u>

$L_1 (3x+2y=14)$

and $L_2 (5x-y=6)$

$A(2,4)$

$4x+3y=9$ and $6x+y=5$

Solve $B(\frac{1}{2},2)$

Finding Eqn of AB: $4x-3y+4=0$

Calculate distance PM

$$= \left| \frac{4(5)-3(-2)+4}{5} \right|$$

$$= 6$$

Q.6

Solve

$$\sqrt{3}x - y + 1 = 0$$

Parallel line eq^n

$$\sqrt{3}x - y + c = 0$$

line passes thru $(2,3)$ must satisfies

$$2\sqrt{3} - 3 + c = 0$$

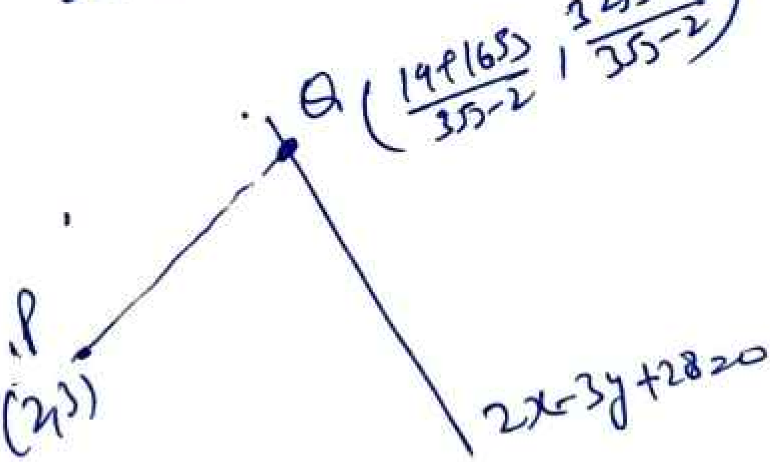
$$c = 3 - 2\sqrt{3}$$

line $\sqrt{3}x - y + 3 - 2\sqrt{3} = 0$ —(i)

$2x - 3y + 28 = 0$ —(ii)

Solve eq^n (i) & (ii)

$$x = \frac{19+6\sqrt{3}}{3\sqrt{3}-2}, \quad y = \frac{32\sqrt{3}-6}{3\sqrt{3}-2}$$



distance $|PQ|$

$$= \sqrt{\left(\frac{19+6\sqrt{3}}{3\sqrt{3}-2} - 2\right)^2 + \left(\frac{32\sqrt{3}-6}{3\sqrt{3}-2} - 3\right)^2}$$

$$\frac{1}{3\sqrt{3}-2}\sqrt{(23)^2 + (23\sqrt{3})^2}$$

$$\frac{\cancel{46}^{\,2} \times (3\sqrt{3}+2)}{\cancel{23}}$$

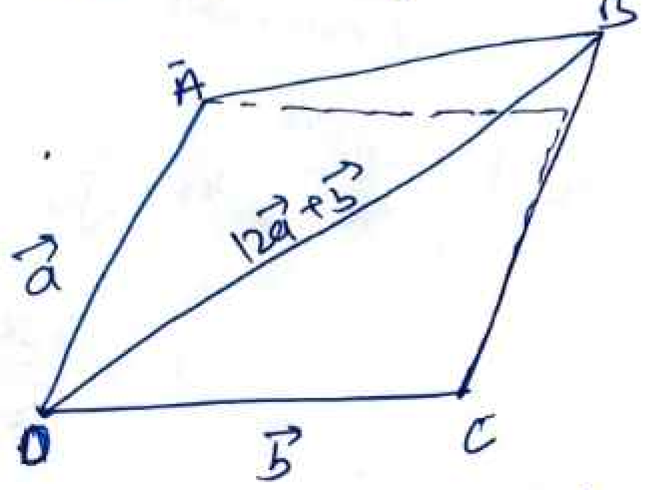
$$\boxed{PQ = 4 + 6\sqrt{3}}$$

Q7

Solve :-

Area of parallelogram

$S = |\vec{a} \times \vec{b}|$



Area of quadrilateral = Area($\triangle OAB$) + Area($\triangle OBC$)

$= \frac{1}{2}\{|\vec{a} \times (12\vec{a} + 4\vec{b})| + |\vec{b} \times (12\vec{a} + 4\vec{b})|\}$

$= \frac{1}{2}\{0 + 4\vec{a}\times\vec{b} + 0 + 12\vec{a}\times\vec{b}$.

$= 8|(\vec{a}\times\vec{b})|$

$$\boxed{\text{Ratio} = \frac{8\cancel{|(\vec{a}\times\vec{b})|}}{\cancel{|\vec{a}\times\vec{b}|}} = 8}$$

Q-8

Solve :-

$$\int \frac{\sin^{\frac{3}{2}} x}{\sqrt{\sin^3 x \cos^3 x\,(\sin(x-\theta))}}\,dx + \int \frac{\cos^{3/2} x}{\sin^3 x \cos^3 x\,(\sin(x-\theta))}\,dx$$

$$= \int \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x \cos^2 x \sqrt{\tan x \cos\theta - \sin\theta}}\,dx$$

$$+ \int \frac{\cos^{3/2} x}{\sin^2 x \cos^{3/2} x \sqrt{\cos\theta - \sin\theta \cot x}}\,dx$$

$$= \int \frac{\sec^2 x}{\sqrt{\tan x \cos\theta - \sin\theta}} + \int \frac{\operatorname{cosec}^2 x}{\sqrt{\cos\theta - \cot x \sin\theta}}\,dx$$

$$\tan x \cos\theta - \sin\theta = t^2$$

$$\sec^2 x\,dx = \frac{2t\,dt}{\cos\theta}$$

Similarly $\quad \operatorname{cosec}^2 x\,dx = \dfrac{2k\,dk}{\sin\theta}$

$$= \int \frac{2t\,dt}{\cos\theta\, t} + \int \frac{2k\,dk}{\sin\theta \cdot k}$$

$$= \frac{1}{\cos\theta} \cdot 2t + \frac{1}{\sin\theta} \cdot 2k$$

$$= 2\sec\theta \sqrt{\tan x \cos\theta - \sin\theta} + 2\operatorname{cosec}\theta \sqrt{\cos\theta - \cot x \sin\theta}$$

comparing

$$\boxed{AB = 8\operatorname{cosec} 2\theta}$$

Q-9

Solve:-

Set $\{1,2,3,4\}$

Minimum order pair are for equivalance.

$\{(1,2)(1,3),(1,1)(2,2)(3,3)(4,4),$
$(2,1),(3,1),(2,3),(3,2)\}$

$\boxed{\text{Thus number of elements} = 10}$

Q-10

Solve

$$f(x) = \frac{x}{x^2-6x-16}$$

$$f'(x) = \frac{-(x^2+16)}{(x^2-6x-16)^2} < 0$$

$$f'(x) < 0$$

Thus $f(x)$ is decreasing in

$\boxed{(-\infty,-2) \cup (-2,8) \cup (8,\infty)}$

Q-11

Set $= \{1, 2, 3 \cdots 50\}$

$P(A) =$ Probability that number in multiple of 4

$P(B) =$ Probability that number in multiple of 6

$P(C) =$ Probability " " " " of 7

Now

$$P(A) = \frac{12}{50}$$

$$P(B) = \frac{8}{50}$$

$$P(C) = \frac{7}{50}$$

$$P(A \cap B) = \frac{4}{50}, \quad P(B \cap C) = \frac{1}{50}, \quad P(A \cap C) = \frac{1}{50}$$

$$P(A \cap B \cap C) = 0$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

$$= \frac{12}{50} + \frac{8}{50} + \frac{7}{50} - \frac{4}{50} - \frac{1}{50} - \frac{1}{50}$$

$$= \frac{21}{50}$$

13

Solve:

$$\frac{3\cos 2x+\cos^3 2x}{\cos^6 x-\sin^6 x}=x^3-x^2+6$$

$$\frac{\cancel{\cos 2x}(3+\cos^2 2x)}{\cancel{(\cos^2 x-\sin^2 x)}(\cos^4 x+\sin^4 x+\sin^2 x\cos^2 x)}=x^3-x^2+6$$

$$\frac{3+\cos^2 2x}{(\cos^2 x+\sin^2 x)^2-2\sin^2 x\cos^2 x+\sin^2 x\cos^2 x}=x^3-x^2+6$$

$$\frac{3+\cos^2 2x}{1-\frac{4\sin^2 x\cos^2 x}{4}}=x^3-x^2+6$$

$$\frac{4(3+\cos^2 2x)}{4-\sin^2 2x}=x^3-x^2+6$$

$$\frac{4\cancel{(3+\cos^2 2x)}}{\cancel{(3+\cos^2 2x)}}=x^3-x^2+6$$

$$x^3-x^2+2=0$$

$$(x+1)(x^2-2x+2)=0$$

so, sum of real solution $=-1$

12

Solve-

Direction of $\vec{PR} = (4, 1, -1)$

Direction of $\vec{PQ} = (1, 4, -1)$

Now, $\cos\alpha = \left| \frac{4+4+1}{\sqrt{18}\,\sqrt{18}} \right|$

$$\boxed{\alpha = \pi/3}$$

Q-17

Solve:-

$$z = 2 - i\left(2\tan\frac{5\pi}{8}\right)$$

$$z = r(\cos\theta + i\sin\theta)$$

$$r\cos\theta = 2$$

$$r\sin\theta = -2\tan\frac{5\pi}{8}$$

$$r = \sqrt{2^2 + \left(2\tan\frac{5\pi}{8}\right)^2}$$

$$r = \left|2\sec\frac{5\pi}{8}\right|$$

$$= \left|2\sec\left(\pi - \frac{5\pi}{8}\right)\right|$$

$$= 2\sec\frac{3\pi}{8}$$

$$\theta = \tan^{-1}\cancel{\tan}\left(\frac{-2\tan\frac{5\pi}{8}}{2}\right)$$

$$= \tan^{-1}\left(\tan\left(\pi - \frac{5\pi}{8}\right)\right)$$

$$\theta = \frac{3\pi}{8}$$

$$\boxed{(r, \theta) = \left(2\sec\frac{3\pi}{8}, \frac{3\pi}{8}\right)}$$

Q-18

Solve

$$|P^{-1}AP - 2I|$$

$$\Rightarrow |P^{-1}AP - 2P^{-1}P|$$

$$\Rightarrow |P^{-1}\cancel{P}(A-2I)P|$$

$$\Rightarrow |P^{-1}|\,|A-2I|\,|P|$$

$$\Rightarrow |A-2I|$$

$$= \begin{bmatrix} 0 & 1 & 2 \\ 6 & 0 & 11 \\ 3 & 3 & 0 \end{bmatrix}$$

$$= 33+36$$

$$= 69$$

Primefactor

3 & 23

$$\boxed{\text{Sum} = 26}$$

Q-16

Answer

Case-I 3 shelf empty : (8,0,0,0) → 1 way

Case-II 2 shelf empty

(7,1,0,0), (6,2,0,0), (5,3,0,0) (4,4,0,0)

Case-III 4-ways

→ 1-shelf empty

(6,1,1,0) (5,2,1,0) (4,3,1,0)

(3,3,2,0) (4,2,2,0)

Case-IV 5-ways

→ 0-shelf empty

(1,2,3,2)

(2,2,2,2)

(3,3,1,1)

(4,2,1,1)

(5,1,1,1)

5-ways

$\boxed{\text{Total} = 15 \text{ ways}}$

Q-17

Solve :-

$$y = \log\left(\frac{1-x^2}{1+x^2}\right)$$

$$y' = \frac{dy}{dx} = \frac{-4x}{1-x^4}$$

$$y'' = \frac{-4(1+3x^4)}{(1-x^4)^2}$$

$$y' - y'' = \frac{-4x}{1-x^4} + \frac{4(1+3x^4)}{(1-x^4)^2}$$

at $x = \frac{1}{2}$

$$y' - y'' = \frac{736}{225}$$

$$\boxed{\text{Thus } 225(y' - y'') = 736}$$

Q-18

Solve :-

$$f(x) = 2x + 3(x)^{2/3}, \quad x \in R$$

$$f'(x) = 2 + 3 \times \frac{2}{3}(x)^{-1/3}$$

$$0 = 2\left(1 + \frac{1}{x^{1/3}}\right)$$

$$= 2\left(\frac{x^{1/3} + 1}{x^{1/3}}\right)$$

$$x = -1,\ 0$$

$+ \quad -1 \quad - \quad 0 \quad +$

at $x = -1$ exactly one point of local maxima

$x = 0$ exactly one point of local minima

x has exactly one point of local maxima and one point of local minima

Q-19

Solve-

Unit vector $\hat{u} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\vec{w_1} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}$$

$$\vec{w_2} = \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$$

$$\vec{w_3} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$$

angle between $\hat{u} \cdot \vec{w_1}$ in $\pi/2$

$$\vec{u} \cdot \vec{w_1} = |\hat{u}||\vec{w_1}| \cos \pi/2$$

$$x + z = 0 \quad \text{— (i)}$$

$$\hat{u} \cdot \vec{w_2} = |\vec{u}||\vec{w_2}| \cos \frac{\pi}{3}$$

$$\frac{y}{\sqrt{2}} + \frac{z}{\sqrt{2}} = 1 \cdot 1 \times \frac{1}{2}$$

$$y + z = \frac{1}{\sqrt{2}} \quad \text{— (ii)}$$

$$\vec{u} \cdot \vec{w_3} = |\hat{u}||\vec{w_3}| \cos \frac{2\pi}{3}$$

$$x + y = -\frac{1}{\sqrt{2}} \quad \text{— (iii)}$$

Solve (i), (ii) & (iii)

$$x = -\frac{1}{\sqrt{2}}, \quad y = 0, \quad z = \frac{1}{\sqrt{2}}$$

$$\hat{u} - \vec{v} = -\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k} - \frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j} - \frac{1}{\sqrt{2}}\hat{k}$$

$$\boxed{|\hat{u} - \vec{v}|^2 = \left(\sqrt{\frac{4}{2} + \frac{1}{2}}\right)^2 = \frac{5}{2}}$$

Q-22

Sol^n :- Parabola $x^2 = 8y$

Chord with mid-point (x_1, y_1) is $T = S_1$

$\therefore x x_1 - 4(y + y_1) = x_1^2 - 8y_1$

$\therefore (x_1, y_1) = \left(1, \frac{5}{4}\right)$

$x - 4\left(y + \frac{5}{4}\right) = 1 - 8 \times \frac{5}{4} = -9$

$\therefore x - 4y + 4 = 0$ --- (i)

(α, β) lies on (i) & also on $y^2 = 4x$

$\therefore \alpha - 4\beta + 4 = 0$ --- (ii)

& $\beta^2 = 4\alpha$ --- (iii)

Solving (ii) & (iii)

$\beta^2 = 4(4\beta - 4) \Rightarrow \beta^2 - 16\beta + 16 = 0$

$\therefore \beta = 8 \pm 4\sqrt{3}$

$\alpha = 4\beta - 4 = 28 \pm 16\sqrt{3}$

$\therefore (\alpha, \beta) = (28 + 16\sqrt{3}, 8 + 4\sqrt{3})$ & $(28 - 16\sqrt{3}, 8 - 4\sqrt{3})$

$\therefore |(\alpha - 28)\cdot(\beta - 8)| = |16\sqrt{3} \times 4\sqrt{3}| = 192$

Q.2)

Solve:-

$\because$ a, b, c are in AP

$2b = a + c$

$a - 2b + c = 0$

$ax + by + c = 0$

Point P $(1, -2)$

$$\begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 2 & 5 & \alpha & : & \beta \\ 1 & 2 & 3 & : & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & -2 \\ 0 & 3 & \alpha-2 & : & \beta-12 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & -2 \\ 0 & 0 & \alpha-8 & : & \beta-6 \end{bmatrix}$$

for infinity solution

RANK [A] = Ran [A|B]

& Rank [A] < No. of unknown

$\alpha - 8 = 0$ & $\beta - 6 = 0$

$\alpha = 8$ $\quad \beta = 6$

distance $(PQ)^2 = \left(\sqrt{(8-1)^2 + (6+2)^2}\right)^2$

$\boxed{(PQ)^2 = 49 + 64 = 113}$

$$\int e^{-v} v\,dv = \int dx$$

$$e^{-v}(-1-v) = x+c$$

$$e^{v}(x+c)+1+v=0$$

$$f(1)=1$$

$$e^{y/x}(x+c)+1+\frac{y}{x}=1$$

$$c=-1-\frac{2}{e}$$

$$e^{y/x}\left(-1-\frac{2}{e}+x\right)+1+\frac{y}{x}=0$$

$$y=f(a)=1$$

$$\cancel{-1}-\frac{2}{e}+a\cancel{+1}=0$$

$$a=\frac{2}{e}$$

$$\boxed{ae=2}$$

Q-29

Solve

$$f(x) = \sqrt{\lim_{r \to x} \frac{2r^2\left[(f(r))^2 - f(x)f(r)\right]}{r^2 - x^2} - r^3 e^{\frac{f(r)}{r}}}$$

$$f^2(x) = \lim_{r \to x} 2r^2 \frac{\left[f^2(r) - f(x)f(r)\right]}{r^2 - x^2} - r^3 e^{\frac{f(r)}{r}}$$

$$= \lim_{r \to x} \left(\frac{2r^2 f(r)}{r+x} \frac{f(r) - f(x)}{r - x} - r^3 e^{\frac{f(r)}{r}} \right.$$

$$f^2(x) = \frac{2x^2 f(x)}{2x} f'(x) - x^3 e^{\frac{f(x)}{x}}$$

$$y = f(x)$$

$$y^2 = xy \frac{dy}{dx} - x^3 e^{\frac{y}{x}}$$

$$\frac{y}{x} = \frac{dy}{dx} - \frac{x^2}{y} e^{y/x}$$

Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v = v + x\frac{dv}{dx} - \frac{x}{v} e^{v}$$

$$\frac{dv}{dx} = \frac{e^v}{v} \Rightarrow e^{-v} v \cdot dv = dx$$

(25)

Solve :-

$$\int_{\pi/6}^{\pi/3} \sqrt{1-\sin 2x}\, dx = \alpha + \beta\sqrt{2} + \gamma\sqrt{3}$$

$$= \int_{\pi/6}^{\pi/3} |\sin x - \cos x|\, dx$$

$$= \int_{\pi/6}^{\pi/4} (\cos x - \sin x)\, dx + \int_{\pi/4}^{\pi/3} (\sin x - \cos x)\, dx$$

$$= -1 + 2\sqrt{2} - \sqrt{3}$$

$$= \alpha + \beta\sqrt{2} + \gamma\sqrt{3}$$

$\alpha = -1$, $\beta = 2$, $\gamma = -1$

$$\boxed{3\alpha + 4\beta - \gamma = 6}$$

(26)

Solve :-
$$x^2 - \sqrt{6}x + 6 = 0$$

$$x = \frac{\sqrt{6} \pm i\sqrt{6}}{2} = \frac{\sqrt{6}}{2}(1 \pm i)$$

$$\alpha = \sqrt{3}\left(e^{i\pi/4}\right), \quad \beta = \sqrt{3}\left(e^{-i\pi/4}\right)$$

$$\therefore \frac{\alpha^{99}}{\beta} + \alpha^{98} = \alpha^{98}\left(\frac{\alpha}{\beta} + 1\right)$$

$$\alpha^{98}\left(\frac{\alpha+\beta}{\beta}\right) = 3^{49}\left(e^{i99\pi/4}\right) \times \sqrt{2}$$

$$= 3^{49}(-1+i)$$

$$3^n(a+ib) = 3^{49}(-1+i)$$

$$n = 49, \quad a = -1, \quad b = 1$$

$$\boxed{n+a+b = 49 - 1 + 1 = 49}$$

Q-28

Solve :-

$64^{32^{32}}$

Let $n = 32^{32}$

from $(1+63)^n = 1 + n \cdot 63 + {}^nC_2 \cdot 63^2 + \cdots + 63^n$

$= 1 + 9\left(7n + {}^nC_2\, 9 \cdot 7^2 + {}^nC_3\, 9^2 \cdot 7^3 + \cdots + 7^n \cdot 9^{n-1}\right)$

$(1+63)^n = 1 + 9k$

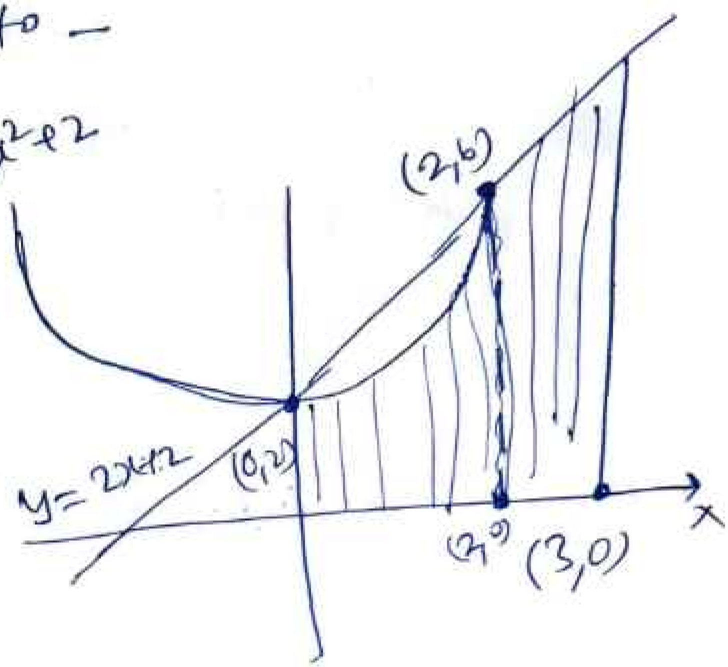
divible by 9

$\boxed{\text{Remainder} = 1}$

Q-27

Solve $\{(x,y): 0 \le x \le 3,\ 0 \le y \le \min(x^2+2, 2x+2)\}$ be A

Then 12A is equal to –



$$A=\int_0^2 (x^2+2)\,dx$$

$$+\int_2^3 (2x+2)\,dx$$

$$A=\frac{41}{3}$$

$$\boxed{12A = 41\times4 = 164}$$

Solve (3) & (4)

$\lambda = -1, \mu = 1$

$\therefore M(1, 3, 2)$

$\therefore N(4, 3, -2)$

$$\boxed{\therefore \vec{OM} \cdot \vec{ON} = 4 + 9 - 4 = 9}$$

(29)

Solve:- $L_1 : \frac{x-5}{4} = \frac{y-4}{1} = \frac{z-5}{3} = \lambda$

direction cosine $b_1 = (4,1,3)$

$M(4\lambda+5, \lambda+4, 3\lambda+5)$

$L_2 \Rightarrow \frac{x+8}{12} = \frac{y+2}{5} = \frac{z+11}{9} = \mu$

$N(12\mu-8, 5\mu-2, 9\mu-11)$

$\overrightarrow{MN} = (4\lambda-12\mu+13, \lambda-5\mu+6, 3\lambda-9\mu+16)$ --- (1)

$b_2(12, 5, 9)$ ---

Now

$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 1 & 3 \\ 12 & 5 & 9 \end{vmatrix} = -6\hat{i} + 8\hat{k}$ --- (2)

equation (1) and (2

$\frac{4\lambda-12\mu+13}{-6} = \frac{\lambda-5\mu+6}{0} = \frac{3\lambda-9\mu+16}{8}$

(i) and (ii) $\lambda - 5\mu + 6 = 0$ --- (3)

(i) and (iii) $\lambda - 3\mu + 4 = 0$ --- (4)

(30)

Solve :-

$$x^2 - 2^y = 2023$$

let put $y = 1 \in N$

$$x^2 = 2023 + 2 = 2025$$

$$x^2 = (45)^2$$

$$x = 45$$

$$\boxed{\sum (x+y) = 45 + 1 = 46}$$

JEE Main - 2024 Solution

Physics, 29/01/24 Shift - II

Ans - 31

$$Q = \frac{a^4 b^3}{c^2}$$

$$\frac{\Delta Q}{Q} \times 100 = \left(4\frac{\Delta a}{a} + 3\frac{\Delta b}{b} + 2\frac{\Delta c}{c}\right) \times 100$$

Percentage error in d = $4 \times 3\% + 3 \times 4\% + 2 \times 5\%$

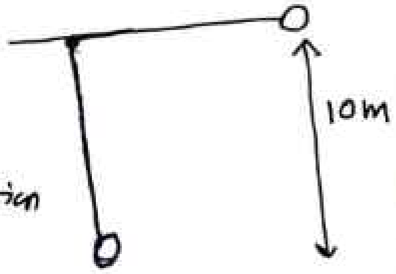
$= (12 + 12 + 10)$

$= 34\%$

$\boxed{\text{Ans} = 34\%}$

$\boxed{\text{Option (3) is Correct}}$

Ans - 32



Let speed at lowest point v. Now from conservation of energy

$$\frac{1}{2}\cancel{m}v^2 = \frac{90}{100} \times \cancel{m}gh$$

$$v^2 = 2 \times 0.9 \times 10 \times 10 = 180$$

$$v = \sqrt{180} = 6\sqrt{5} \text{ m/sec}$$

$\boxed{\text{Ans} = 6\sqrt{5} \text{ m/s}}$ $\boxed{\text{Option (1) is Correct}}$

$\boxed{\text{Ans-33}}$

Power = 200 W

$E_1 = \frac{hc}{\lambda_1}$, $E_2 = \frac{hc}{\lambda_2}$

$$n_1 = \frac{P}{\frac{hc}{\lambda_1}} = \frac{P\lambda_1}{hc}$$

$$n_2 = \frac{P\lambda_2}{hc}$$

$$\frac{n_1}{n_2} = \frac{\frac{P\lambda_1}{hc}}{\frac{P\lambda_2}{hc}} = \frac{\lambda_1}{\lambda_2} = \frac{300}{500} = \frac{3}{5}$$

$\boxed{\text{Ans} = 3:5}$

$\boxed{\text{option (4) is correct}}$

$\boxed{\text{Ans - 34}}$

$\vec{E} = (6\hat{i} + 5\hat{j} + 3\hat{k})$ N/C

$\vec{A} = 30\hat{i}$ m²

$$\phi = \vec{E}\cdot\vec{A} = (6\hat{i} + 5\hat{j} + 3\hat{k})\cdot(30\hat{i})$$

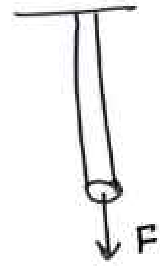
$$= 6 \times 30 = 180$$

$\boxed{\text{Ans} = 180}$

$\boxed{\text{option (3) is correct}}$

$\boxed{\text{Ans- 35}}$

$$\frac{\Delta l}{l} = \frac{\frac{F}{A}}{Y}$$



$$\frac{l}{L} = \frac{F}{AY} \quad -②$$

$$\frac{l'}{L} = \frac{\frac{F}{2}}{\frac{A}{\cancel{4}_2} Y}$$

(radius is half $A' = \pi \left(\frac{r}{2}\right)^2 = \frac{A}{4}$)

$$\frac{l'}{L} = \frac{2F}{AY} \quad -①$$

$$l' = 2l$$

(on comparing ① and ②)

$\boxed{\text{Ans = 2 times}}$

$\boxed{\text{Option (2) is Correct}}$

$\boxed{\text{Ans-36}}$

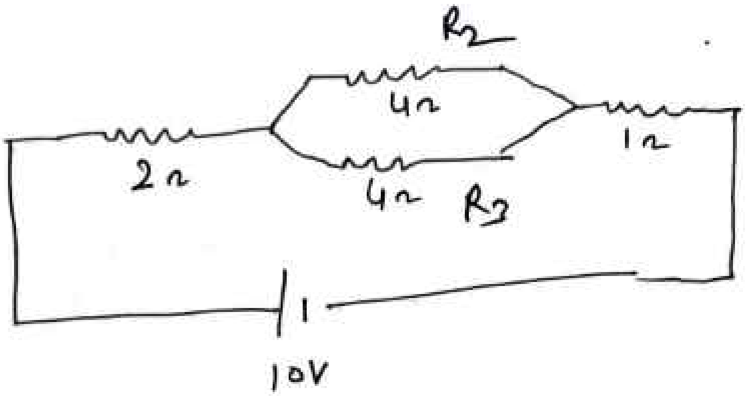
statement I is correct as per the rutherford's model.

Statement II is Wrong, not as per the rutherford's model

statement I is correct Statement II is wrong

$\boxed{\text{option (1) is Correct}}$

Ans - 37



$R_{eq.}$

$$= 2+1+\frac{4\times4}{4+4}$$

$$= 2+1+2 = 5\Omega$$

$$I = \frac{V}{R} = \frac{10}{5} = 2A$$

current in R_3 = 1 A [both R_2 and and R_3 same equal current flow]

$\boxed{\text{Ans} = 1\text{ A}}$

option (2) is correct

Ans - 38 for mixture

$$C_V = \frac{N_1 C_{V_1} + N_2 C_{V_2}}{N_1 + N_2}$$

$$5 = \frac{6N + 2\times3}{N+2}$$

$$5N + 10 = 6N + 6$$

$\boxed{N = 4}$ $\boxed{\text{Ans} = 4}$

option (2) is correct

$\boxed{\text{Ans-39}}$ Virtual, magnified image is only formed by concave mirror.

$u = -x$

$m = 2$

$V = 15 - x$

$-\frac{V}{u} = 2$, $\frac{15-x}{x} = 2$

$x = 5cm$

$u = -5$, $V = 10$

$\frac{1}{f} = \frac{1}{V} + \frac{1}{u} = \frac{1}{10} - \frac{1}{5} = -\frac{1}{10}$

$$\boxed{f = -10 \text{ cm}}$$

$\boxed{\text{option 1 is correct}}$

$\boxed{\text{Ans-40}}$

$$T^2 \propto r^3$$

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$$

$$\left(\frac{200}{T_2}\right)^2 = \left(\frac{r}{\frac{r}{4}}\right)^3$$

$$T_2^2 = \frac{200 \times 200}{4 \times 4 \times 4} = \frac{50 \times 50}{4}$$

$$\boxed{T_2 = \frac{50}{2} = 25}$$

$\boxed{\text{option (4) is correct}}$

Ans- 41 $\frac{4}{3}\pi R^3 = 27 \times \frac{4}{3}\pi r^3$

$$\boxed{r = \frac{R}{3}}$$

Work done = change in surface energy

$= T\left(27 \times 4\pi\left(\frac{R}{3}\right)^2 - 4\pi R^2\right)$

$= T \times 8\pi R^2$

$$\boxed{\text{Work done} = 8\pi R^2 T}$$

option 1 is correct

Ans- 42 $V = 100 \sin(100t) V$

$I = 100 \sin\left(100t + \frac{\pi}{3}\right) mA$

$V_0 = 100V \quad I_0 = 100 mA = 100 \times 10^{-3} A$
$= \frac{1}{10} A$

$P = \frac{V_0 I_0}{2} \cos\phi = \frac{100 \times 10}{20} \times \cos\frac{\pi}{3}$

$= \frac{100}{20} \times \frac{1}{2}$

$= \frac{100}{40} = 2.5W$

$\boxed{P = 2.5 W}$ option (2) is correct

$\boxed{\text{Ans-43}}$

$$R = \frac{mv}{qB}$$

$q_1 = q_2 = q$

B - same

$P = \sqrt{2mK}$

$$R = \frac{\sqrt{2mK}}{qB}$$

For first $\quad R_1 = \frac{\sqrt{2m_1 K}}{qB}$

$$R_2 = \frac{\sqrt{2m_2 K}}{qB}$$

$$\boxed{\frac{m_1}{m_2} = \left(\frac{R_1}{R_2}\right)^2}$$

$\boxed{\text{option (4) is Correct}}$

$\boxed{\text{Ans-44}}$ direction of wave x-direction

$E = 9.6\,\hat{j}$ (y-direction)

$E_0 = 9.6$

$$B_0 = \frac{E_0}{c} = \frac{9.6}{3\times10^8} = 3.2\times10^{-8}\,T$$

direction should be perpendicular to both (means z, $\hat{k}$)

$$\boxed{B = 3.2\times10^{-8}\,\hat{k}\ T}$$

$\boxed{\text{option (3) is correct}}$

Ans-45

$$x = (t^3 - 6t^2 + 20t + 15)\ m$$

$$\frac{dx}{dt} = v = 3t^2 - 12t + 20$$

$$\frac{dv}{dt} = a = 6t - 12$$

$a = 0 \qquad 6t - 12 = 0, \qquad t = 2\ sec$

v at $t = 2$

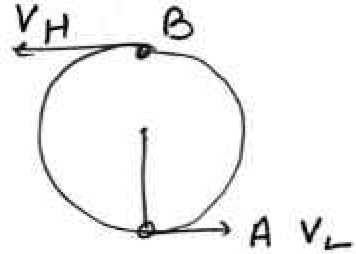
$$v = 3(2)^2 - 12 \times 2 + 20$$

$$= 32 - 24 = 8\ m/sec$$

$$\boxed{v = 8\ m/sec}$$

option (1) is correct

Ans-46



$$\frac{(K.E.)_A}{(K.E.)_B}$$

minimum velocity at A to complete the circle $v = \sqrt{5gL}$

$V_A = \sqrt{5gL} \qquad KE_A = \frac{1}{2}m(5gL)$

$\qquad\qquad\qquad KE_B = \frac{1}{2}m(gL)$

$V_B = \sqrt{gL}$

$$\boxed{\frac{(K.E.)_A}{(K.E.)_B} = \frac{5}{1}}$$

option (2) is correct

Ans-47

Truth table for arrangement

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

option (4) is correct

Ans-48

$$\Delta x = \frac{7\lambda}{4}$$

$$\Delta\phi = \frac{2\pi}{\lambda}\cdot\Delta x = \frac{2\pi}{\lambda}\cdot\frac{7\lambda}{4} = \frac{7\pi}{2}$$

$$I_{eq} = I + I + 2I\cos\frac{7\pi}{2}$$

$$= 2I$$

$$I_{max} = 4I$$

Ratio $\frac{I_{eq}}{I_{max}} = \frac{2I}{4I} = \frac{1}{2}$

option (3) is correct

$\boxed{\text{Ans-49}}$

$PV = nRT$ $\qquad$ $P = 1.38$ atm

$PV = \frac{N}{N_A} RT$ $\qquad$ $= 1.38 \times 10^5$ N/m² or Pa

$PV = N K_B T$ $\qquad$ $V = 1 m^3$

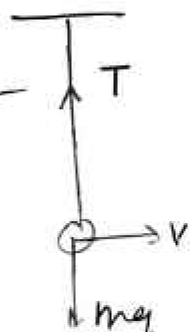
$$\cancel{1.38} \times 10^5 \times 1 = 2 \times 10^{25} \times \cancel{1.38} \times 10^{-23} T$$

$$T = \frac{1000}{2} = 500 K$$

$$\boxed{T = 500 K}$$

$\boxed{\text{option (3) is correct}}$

$\boxed{\text{Ans - 50}}$



$$T - mg = \frac{mv^2}{r} = mr\omega^2$$

$\omega = \frac{2\pi N}{60}$

$$T = mg + mr\omega^2$$

$= \frac{2\pi \times 10}{60}$

$= \frac{\pi}{3}$

$$= mg + mr\left(\frac{\pi}{3}\right)^2$$

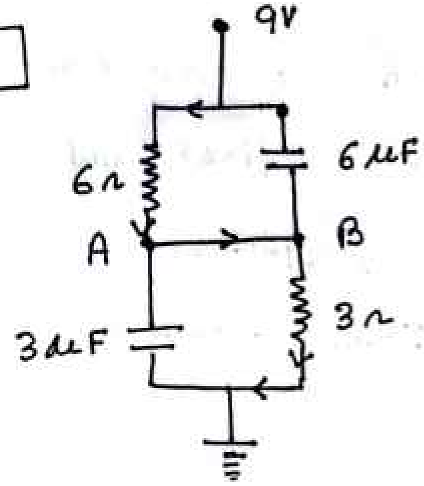
$$= mg + mr\frac{\pi^2}{9} = mg + \frac{mg}{9} \quad \boxed{g = \pi^2}$$

$$= \frac{10 mg}{9} = \frac{10 \times 900 \times 9.8}{1000 \times 9}$$

$$\boxed{T = 9.8 N}$$

$\boxed{\text{option (1) is correct}}$

Ans - 51



Now point A and B are joined.
current will follow path as shown in figure

$$i = \frac{9-0}{9}$$
$$= 1\ A$$

Potential across 6 μF capacitor

$= 9 -$ Potential at A or B

$= [9 - (9 - 6\times1)] = 6V$

charge stored $= 6\times6 = 36\ \mu C$

$\boxed{\text{Ans} = 36}$

Ans- 52

$q = 4\ \mu C = 4\times10^{-6}\ C$

$\vec{V} = 4\times10^{6}\ \hat{J}$ m/sec

$\vec{B} = 2\hat{k}\ T$

$\vec{F} = q(\vec{V}\times\vec{B}) = 4\times10^{-6}(8\times10^{6}(\hat{J}\times\hat{k}))$

$= 32\hat{i}\ N$

$\boxed{\text{Value of } x = 32}$

$\boxed{\text{Ans- 53}}$ $\lambda = 6000\,\mathring{A} = 6\times10^{-7}$ m

distance between first and third minima $= 3\times10^{-3}$ m.

$D = 50$cm $= 0.5$ m

$a = ?$

$$(3-1)\frac{D\lambda}{a} = 3\times10^{-3}$$

$$\frac{2\times0.5\times\cancel{6}^{2}\times10^{-7}}{\cancel{3}\times10^{-3}} = a$$

$$a = 2\times10^{-4}\ \text{m}$$

$\boxed{\text{Ans} = 2}$

$\boxed{\text{Ans - 54}}$ To get the Balmer series line minimum energy needed is to raise the e^- from Ground state to ~~third~~ second excited state $[n = 3]$

$$\text{Energy} = 13.6\left[1-\frac{1}{9}\right] = \frac{13.6\times8}{9}\ \text{eV}$$

$$= \frac{108.8}{9} = 12.1\ \text{eV}$$

$$= \frac{121}{10}\ \text{eV}$$

Potential required $= \frac{121}{10}$ V

$\boxed{\text{Ans} = 121}$

Ans-55

$|a_t| = |a_n|$

$R = 0.5\ m$

$t = 0 \quad V_0 = 4\ m/sec$

$$\frac{dV}{dt} = \frac{V^2}{R}$$

$$\int_4^V \frac{dV}{V^2} = \int_0^t \frac{dt}{R} \qquad \left[-\frac{1}{V}\right]_4^V = \frac{t}{0.5} = 2t$$

$$\frac{1}{4} - \frac{1}{V} = 2t$$

$$V = \frac{4}{1-8t} \quad , \quad \frac{ds}{dt} = \frac{4}{1-8t}$$

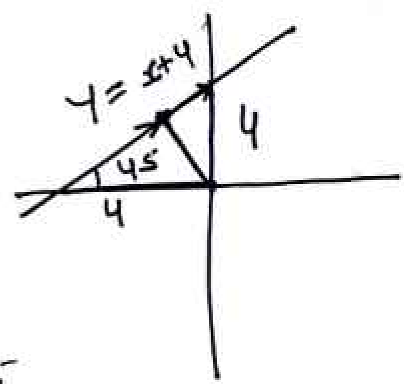
$$\int_0^{t} \frac{dt}{1-8t} = \int_0^{2\pi R} \frac{ds}{4} \qquad 2\pi R = 2\pi \times 0.5 = \pi$$

$$\frac{\ln(1-8t)}{-8} = \frac{\pi}{4} \qquad \ln(1-8t) = -2\pi$$

$$1-8t = e^{-2\pi} \quad , \quad t = \frac{1}{8}\left(1-e^{-2\pi}\right)$$

$$\boxed{\alpha = 8}$$

Ans-56



Angular Momentum

$= mvr$

$= 5 \times 3\sqrt{2} \times 4 \sin 45$

$= 5 \times 3\cancel{\sqrt{2}} \times 4 \times \frac{1}{\cancel{\sqrt{2}}}$

$= 60$

$\boxed{\text{Ans} = 60}$

$\boxed{\text{Ans - 57}}$

Amplitude $= A$, $T = 6\pi$

$$W = \frac{2\pi}{T} = \frac{2\pi}{6\pi} = \frac{1}{3}$$

S.H.M eq^n $y = A \sin \frac{1}{3} t$

time req^d to reach $y = \frac{\sqrt{3}}{2} A$

$$\frac{\sqrt{3} A}{2} = A \sin \frac{t}{3}$$

$$\frac{\sqrt{3}}{2} = \sin \frac{t}{3}$$

$\frac{t}{\cancel{3}} = \frac{2\pi}{\cancel{3}}$, $t = \frac{6\pi}{3}$ to reach $\frac{\sqrt{3} A}{2}$ after A.

$t = 2\pi$

Time to reach A $= \frac{6\pi}{4} = \frac{3\pi}{2}$

Time to travel from A to $\frac{\sqrt{3} A}{2} = \frac{2\pi}{\cancel{8}} - \frac{3\pi}{2}$

$\frac{4\pi - 3\pi}{2}$

$= \frac{\pi}{2}$

$\boxed{x = 2}$

$\boxed{\text{Ans- 58}}$ $V_P = V_Q$ $A_P l_P = A_Q l_Q$

$$\frac{l_P}{l_Q} = \frac{A_Q}{A_P} = \frac{1}{4}$$

$$\frac{\Delta l}{l} = \frac{F}{AY}$$

$$\Delta l = \frac{Fl}{AY}.$$

$$\frac{\Delta l_P}{\Delta l_Q} = \frac{\frac{F_1 l_P}{A_P Y}}{\frac{F_2 l_Q}{A_Q \cdot Y}}$$ for same extension.

$$\boxed{\frac{F_1}{F_2} = 16}$$

$\boxed{\text{Ans-59}}$ $l = 5\,m$

$B = 0.60 \times 10^{-4}\ wb/m^2$

$V = 10\ m/sec$

$$e = BVl = 0.6 \times 10^{-4} \times 10 \times 5$$

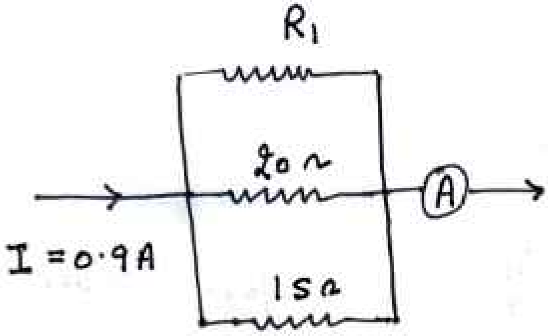
$$= 30 \times 10^{-4}\ V$$

$$\boxed{e = 3 \times 10^{-3} V}$$

$\boxed{\text{Ans} = 3}$

$\boxed{\text{Ans - 60}}$

$I = 0.9\,A$



All resistance are in parallel potential difference will be same

$V = 2\cancel{0} \times 0\cancel{.}3 = 6\,V$

Current in $15\,\Omega = \frac{6}{15} = 0.4\,A$

Current in $R_1 = 0.9 - (0.3 + 0.4)$

$= 0.9 - 0.7 = 0.2\,A$

Potential drop in R_1

$0\cancel{.}2\,R_1 = 60$

$\boxed{R_1 = 30\,\Omega}$

[NTA - SOLUTION - 29TH-JAN.24. SHIFT-2]

A-1 Correct option is-(3) - Ethanol is least acidic and p-nitro Phenol is most acidic

A-2 Correct option is(4) I mean only "A" & "B"

A-3 Correct option is(4). I mean Eu^{2+}

A-4 Correct option is(2) by using AgCN.

A-5 Correct option is(3). I mean NH_3 is the gas that gives only brown ppt with Nessler's Reagent

A-6 Correct option is(4) being unsymmetrical nature of alkene

A-7 Correct option is(1) D.M.G form chelates with Ni^{2+} ions in basic medium.

A-8 Correct option is(2). I mean  . Chlorobenzene

A-9 Correct option is (2) cyclohex-2-en,1-ol

A-10 Correct option is(3). I mean nitrogen

A-11 Correct option is (2) "B" and "D" both are correct for Zn, Hg and Cd

A-12 Correct option is 1 I mean 2-hydroxybenzaldehyde

A-13 Correct option is 1.

A-14 Correct option is 4

Acidic nature ∝ % S-Character

∝ $\frac{1}{\quad}$

$\propto \dfrac{1}{\text{no of alkyl group}}$

A-15 Correct option is 1. small size & high E.N.

A-16 Correct option is 2.

A-17 correct option is 4.

A-18 Correct option is (3)

A-19 Correct option is (1) because Fluorine is being most E N and smaller size have most electron density so it has high electron repulsion and max e^--gain enthalpy

A-20 Correct option is 1. because nucleic acid belongs to nucleotides and Protein belongs to α-amino acid and rest us as usual

A 21 Brown ring is nitroso ferrous Sulphate

$$[Fe(NO)]SO_4$$

$$x + (+1) + (-2) = 0$$

$$x - 1 = 0$$

$$\boxed{x = +1}$$

A-22 Total no. of antibonding M.Os are 4.
One from 2s and 3-from 2p.

A-23
$$N_2 + 3H_2 \rightleftharpoons 2NH_3$$

$$K_c = \frac{[NH_3]^2}{[N_2]^1[H_2]^3} = \frac{(15 \times 10^{-2})^2}{(2\times10^{-2})^1 \times (3\times10^{-2})^3}$$

$$K_c = \frac{\cancel{15}^{5} \times \cancel{15}^{5} \times 10^{-4}}{\cancel{27}_{\cancel{9}_3} \times 2 \times 10^{-8} \times 100} = \frac{25}{6} \times 10^2$$

$$K_c = \frac{25 \times 50}{3} = \frac{1250}{3} = 416.66 \simeq \underline{\underline{417}}$$

A-25

molarity = 0.8 M

mass of solution = 1000 × 1.06
= 1060 gm

mass of H_2SO_4 = 0.8 × 98
= 78.4 gm

mass of solvent = 1060.0 − 78.4 = 981.6 gm

$$\text{molality} = \frac{0.8 \times (10^3)^2}{9816} = \frac{8000 \times 10^3}{9816} = 0.8149 \times 10^3 = \underline{815}$$

A-26

$$w = \frac{E}{F} \times q$$

$$w = \frac{197}{3F} \times q$$

$$1.314 = \frac{197}{3F} \times q$$

$$q = \frac{1.314 \times 3}{197} F = 0.02 F$$

$q = 2 \times 10^{-2}$ F

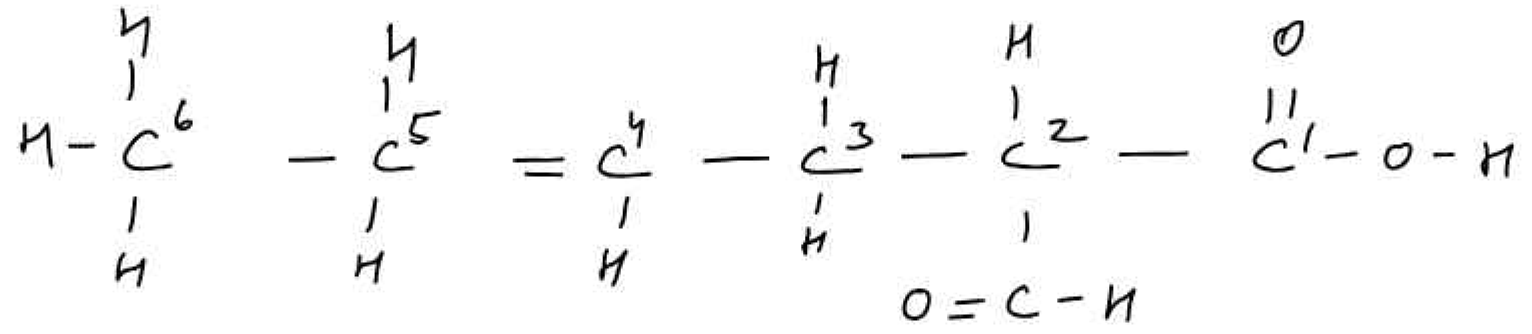
on comparing with $x \times 10^{-2}$ F then, it will be $\boxed{x = 2}$

A-27 Total 3-molecules are CH_4, BF_3 & CO_2 in which net dipole moment is zero.

A-28 Total heat required $= \frac{30.4 \times 284}{154}$

= 56 KJ

A-29



Total $\boxed{\begin{array}{l}\sigma\text{-bonds} = 20 \\ \pi\text{-bonds} = 2\end{array}} = \underline{22}$

A-30 Let the molarity of 25ml NaOH solution is M, then

50ml 0.5 M $H_2C_2O_4 \equiv$ 25ml M_1 NaOH

or

50ml 1N $H_2C_2O_4 \equiv$ 25ml N_1 NaOH [for NaOH, $N = M$]

Law of equivalence,

$$50 \times 1 = 25 \times N_1$$

$$\boxed{N_1 = 2N}$$

For NaOH, $\boxed{N_1 = 2N = 2M}$

To Prepare 50 ml of NaOH solution, NaOH is required,

$$M = \frac{w \times 1000}{m \times V(ml)}$$

$$2 = \frac{w \times 100\cancel{0}}{4\cancel{0} \times 5\cancel{0}} \Rightarrow \cancel{2} = \frac{w \times 10}{4 \times \cancel{5}}\cancel{2}$$

$$\boxed{W_{NaOH} = 4\,gm}$$

A 24

$$K = \frac{0.693}{36}$$

$$K = \frac{0.693}{36} = \frac{2.303}{24}\log\frac{1}{x}$$

$$\Rightarrow \frac{0.\cancel{2303} \times 0.3010}{36} = \frac{\cancel{2.303}}{24}\log\frac{1}{x}$$

$$\Rightarrow \frac{24 \times 0.3010}{36} = \log\frac{1}{x}$$

$$\frac{\quad\quad}{36} = \log\frac{1}{x}$$

$$\Rightarrow \frac{1}{x} = \text{Antilog}\left(\frac{24\times 0.3010}{36}\right)$$

$$\Rightarrow \frac{1}{x} = \text{Antilog}\,(0.2006)$$

$$\Rightarrow \frac{1}{x} = 1.587$$

$$x = \frac{1000}{1.587}$$

$$x = 0\,630119$$

$$\boxed{x = 63.0119 \times 10^{-2}}$$

on comparing with $x = \underline{\quad\quad} \times 10^{-2}$

then, it will be 63