Q.1

The distance, of the point $(7, -2, 11)$ from the line $\frac{x-6}{1} = \frac{y-4}{0} = \frac{z-8}{3}$ along the line $\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-5}{6}$, is :

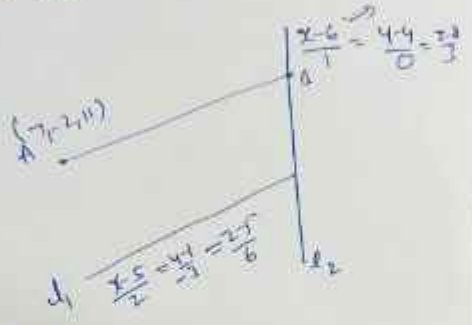
Options

1. 18
2. 12
3. 21
4. 14

Solve :-

distance between AB

$AB \parallel l_1$



The equation of AB

$$\frac{x-7}{2} = \frac{y+2}{-3} = \frac{z-11}{6} = \lambda$$

$x = 2\lambda + 7$, $y = -3\lambda - 2$, $z = 6\lambda + 11$

Point B $(x_2, y_2, z_2) = (2\lambda+7, -3\lambda-2, 6\lambda+11)$

Put (x_2, y_2, z_2) value in line l_2

final Point B

$$\frac{2\lambda+7-6}{1} = \frac{-3\lambda-2-4}{0} = \frac{6\lambda+11-8}{3}$$

$$(2\lambda+7-6) \times 0 = (-3\lambda-6)$$

$$-3\lambda = 6$$

$$\lambda = -2$$

Point B $= (2\lambda+7, -3\lambda-2, 6\lambda+11)$

$= (3, 4, -1)$

distance $|AB| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

$= \sqrt{(3-7)^2 + (4+2)^2 + (11+1)^2}$

$= \sqrt{16 + 36 + 144}$

$= \sqrt{196}$

$= 14$

Q.2

Let $x = x(t)$ and $y = y(t)$ be solutions of the differential equations $\frac{dx}{dt} + ax = 0$ and $\frac{dy}{dt} + by = 0$ respectively, a, b ∈ R. Given that $x(0) = 2$; $y(0) = 1$ and $3y(1) = 2x(1)$, the value of t, for which $x(t) = y(t)$, is :

Options

1. $\log_3 4$
2. $\log_{\frac{4}{3}} 2$
3. $\log_4 3$
4. $\log_{\frac{2}{3}} 2$

Answer → $x = x(t)$ & $y = y(t)$

$\frac{dx}{dt} = -ax$

$\frac{dx}{x} = -a\,dt$

$\int \frac{dx}{x} = \int -a\,dt$

$\log|x| = -at + c$

$x(0) = 2$

$\ln 2 = c$

$\ln|x| = -at + \ln 2$

$\ln \frac{x}{2} = -at$

$\frac{x}{2} = e^{-at}$

$x = 2e^{-at}$

$\frac{dy}{dt} = -by$

$\frac{dy}{y} = -b\,dt$

$\int \frac{dy}{y} = \int -b\,dt$

$\ln|y| = -bt + k$

$y(0) = 1$

$\ln 1 = k$

$k = 0$

$\ln y = -bt$

$y = e^{-bt}$

$3y(1) = 2x(1)$

$3e^{-b} = 2(2e^{-a})$

$3e^{-b} = 4e^{-a}$

$e^{a-b} = \frac{4}{3}$

$a - b = \ln \frac{4}{3}$

$x(t) = y(t)$

$2e^{-at} = e^{-bt}$

$e^{(a-b)t} = 2$

$e^{a-b} \cdot e^{t} = 2$

$\frac{4}{3} e^{t} = 2$

$\ln \frac{4}{3} \cdot t = \ln 2$

$t = \frac{\ln 2}{\ln \frac{4}{3}}$

Q.3

$^{n-1}C_r = (k^2-8)\ ^nC_{r+1}$ if and only if :

Options

1. $2\sqrt{3} < k < 3\sqrt{3}$
2. $2\sqrt{2} < k \leq 3$
3. $2\sqrt{3} < k \leq 3\sqrt{2}$
4. $2\sqrt{2} < k < 2\sqrt{3}$

Answer

$$^{n-1}C_r = (k^2-8)\ ^nC_{r+1}$$

$$^nC_r = \frac{n}{r}\ ^{n-1}C_{r-1}$$

$$^{n-1}C_r = (k^2-8)\frac{n}{r+1}\ ^{n-1}C_r$$

$$k^2-8 = \frac{r+1}{n}$$

$$\therefore \frac{r+1}{n} \leq 1$$

$$k^2-8 \leq 1$$

$$k^2 \leq 9$$

$$|k| \leq 3 \quad \text{— (i)}$$

Since $k^2-8 > 0$

$$k^2 > 8$$

$$|k| > 2\sqrt{2} \quad \text{— (ii)}$$

from (i) & (ii)

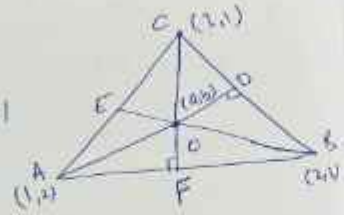
$$2\sqrt{2} < k \leq 3$$

Q.4 If (a, b) be the orthocentre of the triangle whose vertices are (1, 2), (2, 3) and (3, 1), and $I_1 = \int_a^b x \sin(4x - x^2)\,dx$, $I_2 = \int_a^b \sin(4x - x^2)\,dx$, then $36\,\frac{I_1}{I_2}$ is equal to :

Options

1. 66
2. 80
3. 88
4. 72

Answer Q.4



$AD \perp AC$

$(\text{Slope of } AD)(\text{Slope of } BC) = -1$

$\left(\frac{a-1}{b-2}\right)\left(\frac{2-3}{3-1}\right) = -1$

$\frac{a-1}{b-2} \times \frac{-1}{2} = -1$

$\frac{a-1}{b-2} = 2$

$a - 1 = 2b - 4$

$a - 2b = -3 \quad \text{---(i)}$

$(\text{Slope of } BE)(\text{Slope of } AC) = -1$

$\left(\frac{a-2}{b-3}\right) \times \left(\frac{1-3}{2-1}\right) = -1$

$\frac{a-2}{b-3} \times \frac{-2}{1} = -1$

$a - 2 = \frac{1}{2}(b-3)$

$2a - 4 = b - 3$

$2a - b = 1 \quad \text{---(ii)}$

from eqn (i) & (ii)

$a - 2b = -3$

$2a - b = 1$

$b = 7/3$

$a = 5/3$

$a + b = 4$

by King's rule

$I_1 = \int_a^b x \sin(4x - x^2)\,dx$

$I_1 = \int_a^b (a+b-x) \sin(4(a+b-x) - (a+b-x)^2)\,dx$

$I_1 = \int_a^b (4-x) \sin(4-x+x)\,dx$

$I_1 = \int_a^b (4-x) \sin(4-x)\,dx$

$I_1 = \int_a^b 4 \sin(4x - x^2)\,dx - \int_a^b x \sin(4x - x^2)\,dx$

$I_1 = 4I_2 - I_1$

$2I_1 = 4I_2$

$\frac{I_1}{I_2} = 2$

$\therefore \quad 36\,\frac{I_1}{I_2} = 36 \times 2 = 72$

Q.5

If the shortest distance between the lines $\frac{x-4}{1}=\frac{y+1}{2}=\frac{z}{-3}$ and $\frac{x-\lambda}{2}=\frac{y+1}{4}=\frac{z-2}{-5}$ is $\frac{6}{\sqrt{5}}$, then the sum of all possible values of λ is :

Options

1. 7
2. 8
3. 10
4. 5

Shortest distance of two lines $= \left|\frac{(\vec{a_2}-\vec{a_1})\cdot(\vec{b_1}\times\vec{b_2})}{|\vec{b_1}\times\vec{b_2}|}\right|$

$\vec{a_1} = (4,-1,0) \quad \vec{a_2} = (\lambda,-1,2)$

$\vec{b_1} = (1,2,-3) \quad \vec{b_2} = (2,4,-5)$

$\vec{a_2}-\vec{a_1} = (\lambda-4, 0, 2)$

$$S.D. = \left|\frac{\begin{vmatrix}\lambda-4 & 0 & 2\\ 1 & 2 & -3\\ 2 & 4 & -5\end{vmatrix}}{\begin{vmatrix}i & j & k\\ 1 & 2 & -3\\ 2 & 4 & -5\end{vmatrix}}\right|$$

$$SD = \left|\frac{(\lambda-4)2 - 0 + 2(0)}{2\hat{i}-1\hat{j}+0\hat{k}}\right|$$

$$= \left|\frac{2(\lambda-4)}{\sqrt{5}}\right|$$

$$\frac{36}{\sqrt{5}} = \left|\frac{2(\lambda-4)}{\sqrt{5}}\right|$$

$|\lambda-4| = 3$

$\lambda-4 = \pm 3$

$\lambda_1 = 7, \lambda_2 = 1$

Sum of Value λ

$\lambda = 7+1 = 8$

Q.6 The number of common terms in the progressions 4, 9, 14, 19, ... , up to 25^{th} term and 3, 6, 9, 12, ... , up to 37^{th} term is :

Options

1. 8
2. 5
3. 9
4. 7

Ans.

$4, 9, 14 \cdots$

$T_{25} = 4 + (25-1)5$

$= 124$

$3, 6, 9 \cdots$

$T_{37} = 3 + (37-1)3$

$= 111$

Common diff $LCM(5,3) = 15$

AP $= 9, 24, \cdots$

$T_n = 9 + (n-1)15$

clearly $15n - 6 \leq 111$

$15n \leq 117$

$n \leq 7.8$

$\boxed{n = 7}$

Q.7

If $S=\{z \in C : |z-i|=|z+i|=|z-1|\}$, then, n(S) is :

Options

1. 1
2. 3
3. 2
4. 0

Answer

$z \in C$

Sol

$|z-i| = |z-1| = |z+i|$

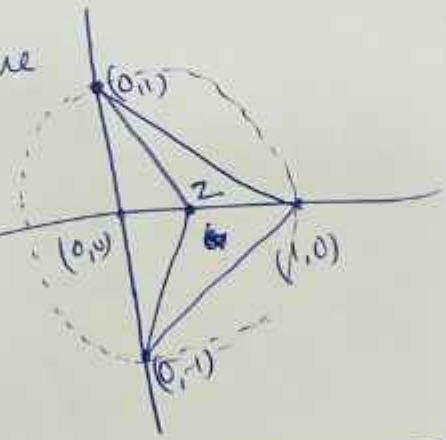
$|z-i|$ = distance between two point z and i

$z \Rightarrow x+iy$, $i \Rightarrow (0,1)$

$|z-1|$ = distance b/w (x,y) and $(1,0)$

$|z+i|$ = distance b/w (x,y) and $(0,-1)$

z in a circumcentre of the triangle formed by points $(0,1)$, $(1,0)$ and $(0,-1)$



So the number of points z satisfying the eqⁿ = 1

Q.8

If $a = \lim\limits_{x\to 0} \dfrac{\sqrt{1+\sqrt{1+x^4}}-\sqrt{2}}{x^4}$ and $b = \lim\limits_{x\to 0} \dfrac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$, then the value of ab^3 is :

Options

1. 25
2. 36
3. 30
4. 32

Answer

$$a = \lim_{x\to 0} \frac{\sqrt{1+\sqrt{1+x^4}}-\sqrt{2}}{x^4}$$

Use binomial theorem

$(1+x^4)^{1/2} = 1+\frac{1}{2}x^4$ Higher power are deleted if x is so small

$(1+x)^n = 1+nx \quad x\to 0$

$$\Rightarrow a = \lim_{x\to 0} \frac{\sqrt{1+1+\frac{1}{2}x^4}-\sqrt{2}}{x^4}$$

$$a = \lim_{x\to 0} \frac{\left(2+\frac{1}{2}x^4\right)^{1/2}-\sqrt{2}}{x^4}$$

$$a = \lim_{x\to 0} \frac{\sqrt{2}\left(2+\frac{1}{4}x^4\right)^{1/2}-\sqrt{2}}{x^4}$$

$$a = \lim_{x\to 0} \frac{\sqrt{2}\left(1+\frac{1}{8}x^4\right)-\sqrt{2}}{x^4}$$

$$a = \lim_{x\to 0} \frac{\sqrt{2}+\frac{\sqrt{2}}{8}x^4-\sqrt{2}}{x^4}$$

$$a = \frac{1}{4\sqrt{2}}$$

$$b = \lim_{x\to 0} \frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$$

$$= \lim_{x\to 0} \frac{\sin^2 x\left(\sqrt{2}+\sqrt{1+\cos x}\right)}{2-(1+\cos x)}$$

$$= \lim_{x\to 0} \frac{\sin^2 x\ 2\sqrt{2}}{1-\cos x}$$

$$= \lim_{x\to 0} \frac{(1-\cos^2 x)2\sqrt{2}}{1-\cos x}$$

$$= \lim_{x\to 0} (1+\cos x)2\sqrt{2}$$

$$b = 4\sqrt{2}$$

Value $= ab^3$

$$= \frac{1}{4\sqrt{2}}(4\sqrt{2})^3$$

$$= (4\sqrt{2})^2$$

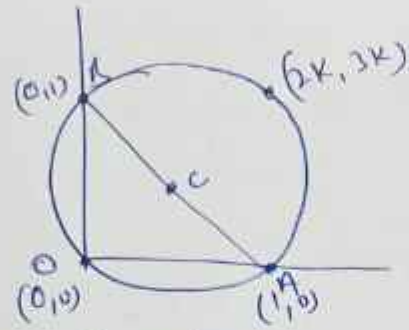
$$\boxed{ab^3 = 32}$$

Q.9 Four distinct points (2k, 3k), (1, 0), (0, 1) and (0, 0) lie on a circle for k equal to :

Options

1. $\frac{2}{13}$
2. $\frac{1}{13}$
3. $\frac{5}{13}$
4. $\frac{3}{13}$

Answer



diameter of circle eqn

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

$$(x-0)(x-1)+(y-1)(y-0)=0$$

$$x^2-x+y^2-y=0$$

$(2k,3k)$ satisfying the eqn of circle.

$$(2k)^2-2k+(3k)^2-3k=0$$

$$13k^2-5k=0$$

$$k(13k-5)=0$$

$$13k-5=0$$

$$\boxed{k=\frac{5}{13}} \qquad k\neq 0$$

Q.10 The function $f: \mathbf{N} - \{1\} \to \mathbf{N}$; defined by $f(n)$ = the highest prime factor of n, is :

Options

1. onto only
2. neither one-one nor onto
3. one-one only
4. both one-one and onto

Q

$f: N - \{1\} \to N$ is defined by

$f(n)$ = the highest prime factor of n.

$\therefore f(6)$ = the highest prime factor of $6 = 3$

$f(12)$ = the highest prime factor of $12 = 3$

Now 6 and 12 are associated to the same element.

$\therefore f$ is not one-to-one Also range of f consist of prime numbers only.

range of $f \neq N$

$\therefore f$ is not onto function.

Range of f is the set of all prime numbers

$f(n)$ is neither one-one nor onto

Q.11 Let $a_1, a_2, \ldots a_{10}$ be 10 observations such that $\sum_{k=1}^{10} a_k = 50$ and $\sum_{\forall k<j} a_k . a_j = 1100$. Then the standard deviation of $a_1, a_2, \ldots, a_{10}$ is equal to :

Options

1. 5
2. $\sqrt{5}$
3. 10
4. $\sqrt{115}$

Answer

$$\sum_{k=1}^{10} a_k = 50$$

$$\Rightarrow a_1 + a_2 + a_3 + \cdots + a_{10} = 50$$

$$\sum_{\forall k<j} a_k a_j = 1100 \Rightarrow [a_1a_2 + a_2a_3 + \cdots + a_9a_{10}] = 1100$$

Standard deviation $= \sqrt{\frac{\sum a_i^2}{n} - \left(\frac{\sum a_i}{n}\right)^2}$

$$a_1 + a_2 + a_3 + \cdots + a_{10} = 50$$

$$(a_1 + a_2 + a_3 + \cdots + a_{10})^2 = 2500$$

$$a_1^2 + a_2^2 + a_3^2 + \cdots + a_{10}^2 + 2\sum_{\forall k<j} a_k a_j = 2500$$

$$\sum_{i=1}^{10} a_i^2 + 2(1100) = 2500$$

$$\sum_{i=1}^{10} a_i^2 = 300$$

$$SD = \sqrt{\frac{300}{10} - \left(\frac{50}{10}\right)^2}$$

$$SD = \sqrt{30 - 25} = \sqrt{5}$$

Q.12

The length of the chord of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, whose mid point is $\left(1,\frac{2}{5}\right)$, is equal to :

Options

1. $\frac{\sqrt{1691}}{5}$
2. $\frac{\sqrt{2009}}{5}$
3. $\frac{\sqrt{1741}}{5}$
4. $\frac{\sqrt{1541}}{5}$

Answer (2)

Equation of chord with given middle point

$T = S_1$

$\frac{x}{25}+\frac{y}{40}=\frac{1}{25}+\frac{1}{100}$

$\frac{8x+5y}{200}=\frac{8+2}{200}$

$y=\frac{10-8x}{5}$

$\frac{x^2}{25}+\frac{(10-8x)^2}{400}=1 \quad \left(\text{Ref } \frac{x^2}{25}+\frac{y^2}{16}=1\right)$

$\frac{16x^2+100+64x^2-160x}{400}=1$

$4x^2-8x-15=0$

$x=\frac{8\pm\sqrt{304}}{8}$

$x_1=\frac{8+\sqrt{304}}{8}$; $x_2=\frac{8-\sqrt{304}}{8}$

Now $y=\frac{10-(8\pm\sqrt{304})}{5}$

$=\frac{2\mp\sqrt{304}}{5}$

$y_1=\frac{2-\sqrt{304}}{5}$, $y_2=\frac{2+\sqrt{304}}{5}$

Distance $=\sqrt{(x_2-x_1)^2+(y_1-y_2)^2}$

$=\sqrt{\frac{4\times304}{64}+\frac{4\times304}{25}}$

$=\frac{\sqrt{1691}}{5}$

Q.13 Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 3(\hat{i} - \hat{j} + \hat{k})$. Let $\vec{c}$ be the vector such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$. Then $\vec{a} \cdot \left((\vec{c} \times \vec{b}) - \vec{b} - \vec{c}\right)$ is equal to :

Options

1. 24
2. 20
3. 32
4. 36

Answer:

$\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$

$\vec{b} = 3(\hat{i} - \hat{j} + \hat{k})$

$\vec{a} \cdot \vec{b} = 3 - 6 + 3 = 0$

$\Rightarrow \vec{a} \cdot \left((\vec{c} \times \vec{b}) - \vec{b} - \vec{c}\right)$

$\Rightarrow \vec{a} \cdot (\vec{c} \times \vec{b}) - \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c}$

$\Rightarrow (\vec{a} \times \vec{c}) \cdot \vec{b} - 0 - 3$

$\Rightarrow (\vec{b}) \cdot \vec{b} - 0 - 3$

$\Rightarrow |\vec{b}|^2 - 3$

$\Rightarrow \left(\sqrt{3^2 + 3^2 + 3^2}\right)^2 - 3$

$\Rightarrow 27 - 3$

$\Rightarrow 24$

Q.14 Let $S = \{1, 2, 3, \ldots, 10\}$. Suppose M is the set of all the subsets of S, then the relation $R = \{(A, B) : A \cap B \neq \phi; A, B \in M\}$ is :

Options

1. symmetric only
2. symmetric and reflexive only
3. reflexive only
4. symmetric and transitive only

X-4

Answer Let $S = \{1,2,3 \cdots 10\}$

$R = \{(A,B) : A \cap B \neq \phi : A,B \in M\}$

for Reflexive

M is subset of 'S'

So $\phi \in M$

for $\phi \cap \phi = \phi$

but Relation $A \cap B \neq \phi$

So it is not reflexive

for symmetric

$ARB \cdot A \cap B \neq \phi$

$B \cap A \neq \phi$

So it is symmetric

for transitive

If $A = \{(1,2), (2,3)\}$

$B = \{(2,3), (3,4)\}$

$C = \{(3,4), (5,4)\}$

$A \cap B \neq \phi \quad B \cap C \neq \phi$

$A \cap C = \phi$

So it not transitive

Only Symmetric

Q.15 If the shortest distance of the parabola $y^2 = 4x$ from the centre of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$ is d, then d^2 is equal to :

Options

1. 36
2. 20
3. 16
4. 24

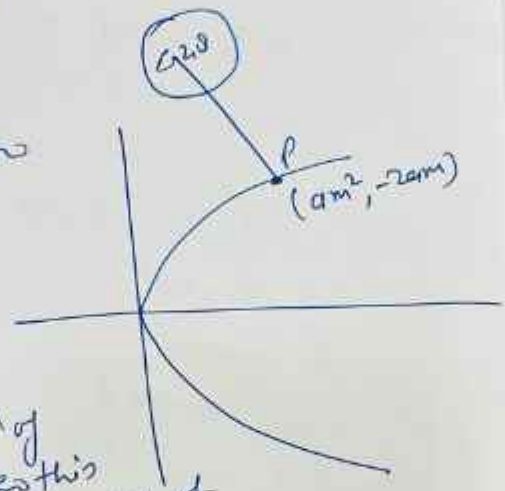
Answer –

$x^2 + y^2 - 4x - 16y + 64 = 0$

$x^2 + y^2 + 2gx + 2fy + c = 0$

Centre $(-g, -f)$

Centre $(2, 8)$



eqn of common normal to parabola $y^2 = 4x$

$a = 1$

$y = mx - 2m - m^3$

this is also the eqn of normal of circle so this line passing through the centre of circle $(2, 8)$

$8 = 2m - 2m - m^3$

$m = -2$

Point $P(4, -4)$

[CP] distance $d = \sqrt{(4-2)^2 + (-4-8)^2}$

$d = \sqrt{4 + 144}$

$d^2 = 20$

Q.16

Consider the matrix $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Given below are two statements :

Statement I : $f(-x)$ is the inverse of the matrix $f(x)$.

Statement II : $f(x)\, f(y) = f(x+y)$.

In the light of the above statements, choose the **correct** answer from the options given below

Options

1. **Statement I** is true but **Statement II** is false
2. **Statement I** is false but **Statement II** is true
3. Both **Statement I** and **Statement II** are true
4. Both **Statement I** and **Statement II** are false

Answer:

matrix $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

[We know Matrix A, $A \cdot A^{-1} = I$]

$f(x) \cdot f(-x) = I$

$$\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-x) & -\sin(-x) & 0 \\ \sin(-x) & \cos(-x) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos x & +\sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 x + \sin^2 x + 0 & 0 & 0 \\ 0 & \sin^2 x + \cos^2 x + 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Statement is true.

$$f(x) \cdot f(y) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos x \cos y - \sin x \sin y & -\cos x \sin y - \sin x \cos y & 0 \\ \sin x \cos y + \cos x \sin y & -\sin x \sin y + \cos x \cos y & 0 \\ 0 + 0 + 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= f(x+y)$$

Q.17 If $\int_0^1 \frac{1}{\sqrt{3+x}+\sqrt{1+x}}\,dx = a + b\sqrt{2} + c\sqrt{3}$, where a, b, c are rational numbers, then $2a + 3b - 4c$ is equal to :

Options

1. 4
2. 10
3. 7
4. 8

Answer -

$$\int_0^1 \frac{1}{\sqrt{3+x}+\sqrt{1+x}}\,dx = a+b\sqrt{2}+c\sqrt{3}$$

$$= \int_0^1 \frac{1}{\sqrt{3+x}+\sqrt{1+x}} \times \frac{\sqrt{3+x}-\sqrt{1+x}}{\sqrt{3+x}-\sqrt{1+x}}\,dx$$

$$= \int_0^1 \frac{\sqrt{3+x}-\sqrt{1+x}}{3+x-1-x}\,dx$$

$$= \int_0^1 \frac{\sqrt{3+x}-\sqrt{1+x}}{2}\,dx$$

$$= \frac{1}{2}\int_0^1 \sqrt{3+x}\,dx - \frac{1}{2}\int \sqrt{1+x}\,dx$$

$$= \frac{1}{3}\left[(4)^{3/2} - (3)^{3/2}\right] - \frac{1}{3}\left[(2)^{3/2} - 1\right]$$

$$= \frac{8}{3} - \frac{2}{3}\sqrt{2} - \sqrt{3} + \frac{1}{3}$$

$$= 3 - \frac{2}{3}\sqrt{2} - \sqrt{3}$$

$$a + b\sqrt{2} + c\sqrt{3}$$

$a = 3,\ b = -\frac{2}{3},\ c = -1$

Value $= 2a + 3b - 4c$

$= 6 - 2 + 4$

$= 8$

Q.18 Consider the function

$$f(x)=\begin{cases}\dfrac{a(7x-12-x^2)}{b|x^2-7x+12|}, & x<3\\ 2^{\frac{\sin(x-3)}{x-[x]}}, & x>3\\ b, & x=3,\end{cases}$$

where $[x]$ denotes the greatest integer less than or equal to x. If S denotes the set of all ordered pairs (a, b) such that $f(x)$ is continuous at $x=3$, then the number of elements in S is :

Options

1. 1
2. 2
3. 4
4. Infinitely many

Answer -

$$f(3^-)=\frac{a}{b}\frac{(7x-12-x^2)}{|x^2-7x+12|}\quad(\text{for } f(x) \text{ to be cont.})$$

$$f(3^-)=-\frac{a}{b}\frac{(x-3)(x-4)}{(x-3)(x-4)}\quad x<3$$

$$f(3^-)=-\frac{a}{b}$$

Then $f(3^+)=2^{\lim_{x\to3^+}\left(\frac{\sin(x-3)}{x-3}\right)}=2$

$$f(3)=b$$

Hence Continuous at $x=3$

$$f(3)=f(3^+)=f(3^-)$$

$$b=2=-\frac{a}{b}$$

$b=2$; $a=-4$

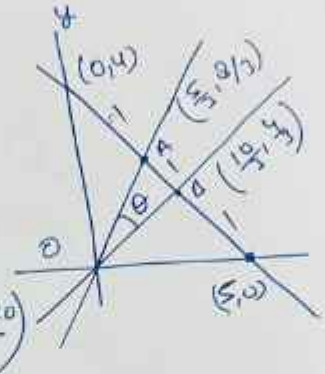
Hence only 1 ordered pair $(-4,2)$.

Q.19 The portion of the line $4x + 5y = 20$ in the first quadrant is trisected by the lines L_1 and L_2 passing through the origin. The tangent of an angle between the lines L_1 and L_2 is :

Options

1. $\frac{8}{5}$
2. $\frac{25}{41}$
3. $\frac{2}{5}$
4. $\frac{30}{41}$

Answer



Point A $= \left(\frac{2\times0+1\cdot5}{1+2}, \frac{2\times4+1\times0}{2+1}\right)$

$= \left(\frac{5}{3}, \frac{8}{3}\right)$

Point B $= \left(\frac{1\times0+2\times5}{1+2}, \frac{1\times4+2\times0}{1+2}\right)$

Coord B $= \left(\frac{10}{3}, \frac{4}{3}\right)$

Slope of OA $= \frac{y_2-y_1}{x_2-x_1} = \frac{\frac{8}{3}-0}{\frac{5}{3}-0}$

$= \frac{8}{5}$

Slope of OB $= \frac{2}{5}$

$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$\tan\theta = \frac{\frac{6}{5}}{1+\frac{16}{25}} = \frac{30}{41}$

$\tan\theta = \frac{30}{41}$

Q.20 If A denotes the sum of all the coefficients in the expansion of $(1-3x+10x^2)^n$ and B denotes the sum of all the coefficients in the expansion of $(1+x^2)^n$, then :

Options

1. $A=3B$
2. $B=A^3$
3. $A=B^3$
4. $3A=B$

Answer

Sum of coefficients in the expansion of $(1-3x+10x^2)^n = A$

$A = (1-3+10)^n = 8^n$

Sum of coefficient in expansion of $(1+x^2)^n = B$

$B = (1+1)^n = 2^n$

$\boxed{A = B^3}$

Q.21 Let the set of all $a \in R$ such that the equation $\cos 2x + a \sin x = 2a - 7$ has a solution be $[p, q]$ and $r = \tan 9^\circ - \tan 27^\circ - \frac{1}{\cot 63^\circ} + \tan 81^\circ$, then pqr is equal to _____.

Answer

$$\cos 2x + a \sin x = 2a - 7$$
$$a \sin x - 2a = -7 - \cos 2x$$
$$a(\sin x - 2) = -7 - (1 - \sin^2 x)$$
$$a(\sin x - 2) = -7 - 1 + 2\sin^2 x$$
$$a(\sin x - 2) = 2\sin^2 x - 8$$
$$a(\sin x - 2) = 2(\sin x - 2)(\sin x + 2)$$

$$\sin x = 2$$
$$a = 2(\sin x + 2) = 2(2+2) = 6$$

$$p = 2, \quad q = 6$$
$$r = \tan 9^\circ - \tan 27^\circ - \frac{1}{\cot 63^\circ} + \tan(90^\circ - 9^\circ)$$
$$r = \tan 9^\circ + \cot 9^\circ - \tan 27^\circ - \frac{1}{\cot 63^\circ}$$
$$= \tan 9^\circ + \cot 9^\circ - \tan 27^\circ - \cot 27^\circ$$
$$= \frac{1}{\sin 9^\circ \cos 9^\circ} - \frac{1}{\sin 27^\circ \cdot \cos 27^\circ}$$
$$= \frac{2}{\sin 18^\circ} - \frac{2}{\sin 54^\circ}$$
$$= 2\left[\frac{4}{\sqrt{5}-1} - \frac{4}{\sqrt{5}+1}\right]$$

$$r = 4$$
$$p \cdot q \cdot r = 2 \times 6 \times 4 = 48$$

Q.22

Let $A=\begin{bmatrix}2 & 0 & 1\\ 1 & 1 & 0\\ 1 & 0 & 1\end{bmatrix}$, $B=[B_1, B_2, B_3]$, where B_1, B_2, B_3 are column matrics, and

$AB_1=\begin{bmatrix}1\\0\\0\end{bmatrix}$, $AB_2=\begin{bmatrix}2\\3\\0\end{bmatrix}$, $AB_3=\begin{bmatrix}3\\2\\1\end{bmatrix}$

If $\alpha=|B|$ and β is the sum of all the diagonal elements of B, then $\alpha^3+\beta^3$ is equal to _____.

Answer

$$A=\begin{bmatrix}2 & 0 & 1\\ 1 & 1 & 0\\ 1 & 0 & 1\end{bmatrix} \qquad B=[B_1, B_2, B_3]$$

$$B_1=\begin{bmatrix}x_1\\y_1\\z_1\end{bmatrix},\quad B_2=\begin{bmatrix}x_2\\y_2\\z_2\end{bmatrix},\quad B_3=\begin{bmatrix}x_3\\y_3\\z_3\end{bmatrix}$$

$$AB_1=\begin{bmatrix}2 & 0 & 1\\ 1 & 1 & 0\\ 1 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\y_1\\z_1\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$$

$$x_1=1,\ y_1=-1,\ z_1=-1$$

$$AB_2=\begin{bmatrix}2 & 0 & 1\\ 1 & 1 & 0\\ 1 & 0 & 1\end{bmatrix}\begin{bmatrix}x_2\\y_2\\z_2\end{bmatrix}=\begin{bmatrix}2\\3\\0\end{bmatrix}$$

$$x_2=2,\ y_2=1,\ z_2=-2$$

$$AB_3=\begin{bmatrix}2 & 0 & 1\\ 1 & 1 & 0\\ 1 & 0 & 1\end{bmatrix}\begin{bmatrix}x_3\\y_3\\z_3\end{bmatrix}=\begin{bmatrix}3\\2\\1\end{bmatrix}$$

$$x_3=2,\ y_3=0,\ z_3=-1$$

$$B=\begin{bmatrix}1 & 2 & 2\\ -1 & 1 & 0\\ -1 & -2 & -1\end{bmatrix}$$

$$\alpha=|B|=3$$

$$\beta=1$$

$$\boxed{\alpha^3+\beta^3=27+1=28}$$

Q.23 Let for a differentiable function $f : (0, \infty) \to \mathbf{R}$, $f(x) - f(y) \geq \log_e\left(\frac{x}{y}\right) + x - y$, $\forall\, x, y \in (0, \infty)$.

Then $\sum_{n=1}^{20} f'\left(\frac{1}{n^2}\right)$ is equal to _____.

$$f(x) - f(y) \geq \ln x - \ln y + x - y$$

$$\frac{f(x) - f(y)}{x - y} \geqslant \frac{\ln x - \ln y}{x - y} + 1$$

if $x > y$

taking $\lim_{y \to x} f(x) \geq \ln x + 1$

$$\lim_{y \to x} f'(x) \leq \frac{1}{x} + 1$$

let $x < y$

$$\lim_{y \to x} f'(x^+) \leq \frac{1}{x} + 1$$

$$f'(x^-) = f'(x^+)$$

$$f'(x) = \frac{1}{x} + 1$$

$$f'\left(\frac{1}{x^2}\right) = x^2 + 1$$

$$\sum_{x=1}^{20} (x^2 + 1) = \sum_{x=1}^{20} x^2 + 20$$

$$= \frac{x(x+1)(2x+1)}{6} + 20$$

$$= \frac{20(20+1)(2\times 20+1)}{6} + 20$$

$$= \frac{20 \times 21 \times 41}{6} + 20$$

$$\boxed{\sum_{n=1}^{20} f'\left(\frac{1}{n^2}\right) = 2890}$$

Q.24 A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required and let $a = P(X=3)$, $b = P(X \geq 3)$ and $c = P(X \geq 6 | X > 3)$. Then $\frac{b+c}{a}$ is equal to _____.

Answer

$$a = P(X=3) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \left(\frac{5}{6}\right)^2 \times \frac{1}{6} = \frac{25}{216}$$

$$b = P(X \geq 3) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^3 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^4 \cdot \frac{1}{6} + \cdots \infty$$

$$b = \frac{\left(\frac{5}{6}\right)^2 \cdot \frac{1}{6}}{1 - \frac{5}{6}} = \frac{25}{36}$$

$$P\left(\frac{X \geq 6}{X > 3}\right) = \frac{P(X \geq 6) \cap P(X > 3)}{P(X > 3)} = \frac{P(X \geq 6)}{P(X > 3)}$$

$$\Rightarrow P(X \geq 6) = \left(\frac{5}{6}\right)^5 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^6 \cdot \frac{1}{6} + \cdots \infty$$

$$= \frac{\left(\frac{5}{6}\right)^5 \cdot \frac{1}{6}}{1 - \frac{5}{6}} = \left(\frac{5}{6}\right)^5$$

$$c = \frac{\left(\frac{5}{6}\right)^5}{\left(\frac{5}{6}\right)^3} = \frac{25}{36}$$

$$\frac{b+c}{a} = \frac{\left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^2}{\left(\frac{5}{6}\right)^2 \cdot \frac{1}{6}} = 12$$

Q.25 If the solution of the differential equation $(2x+3y-2)\,dx+(4x+6y-7)\,dy=0$, $y(0)=3$, is $\alpha x+\beta y+3\log_e|2x+3y-\gamma|=6$, then $\alpha+2\beta+3\gamma$ is equal to _____.

Ans

$$(2x+3y-2)dx+(4x+6y-7)dy=0$$

$$\frac{dy}{dx}=-\frac{2x+3y-2}{4x+6y-7}$$

$$=-\frac{2x+3y-2}{2(2x+3y-2)-3}$$

Let $2x+3y-2=t$

$$2+3\frac{dy}{dx}=\frac{dt}{dx}$$

$$\frac{dy}{dx}=\frac{1}{3}\left(\frac{dt}{dx}-2\right)$$

$$\frac{1}{3}\left(\frac{dt}{dx}-2\right)=\frac{-t}{2t-3}$$

$$\frac{dt}{dx}-2=\frac{-3t}{2t-3}$$

$$\frac{dt}{dx}=\frac{-3t}{2t-3}+2$$

$$=\frac{-3t+4t-6}{2t-3}$$

$$=\frac{t-6}{2t-3}$$

$$\frac{2t-3}{t-6}\,dt=dx$$

$$\frac{2(t-6)+9}{t-6}\,dt=dx$$

$$2\int dt+9\int\frac{1}{t-6}\,dt=\int dx$$

$$2t+9\ln|t-6|=x+c$$

$$2(2x+3y-2)+9\ln|2x+3y-8|=x+c$$

$x=0,\ y=3$

$$2(7)+9\ln|1|=0+c$$

$$c=14$$

$$4x+6y-4+9\ln|2x+3y-8|=x+14$$

$$3x+6y+9\ln|2x+3y-8|=18$$

$$x+2y+3\ln|2x+3y-8|=6$$

$\alpha=1,\ \beta=2,\ \gamma=8$

$$\alpha+2\beta+3\gamma$$

$$1+4+24=29$$

Q.26 Let $f(x)=x^3+x^2f'(1)+xf''(2)+f'''(3)$, $x \in \mathbf{R}$. Then $f'(10)$ is equal to _____.

Given **202**

Answer

$$f(x) = x^3 + x^2 \cdot f'(1) + x \cdot f''(2) + f'''(3)$$

$$f'(x) = 3x^2 + 2x f'(1) + f''(2)$$

$$f''(x) = 6x + 2f'(1)$$

$$f'''(x) = 6$$

$$f'''(3) = 6$$

$$f''(2) = 12 + 2f'(1)$$

$$f'(1) = 3 + 2f'(1) + 12 + 2f'(1)$$

$$f'(1) = 15 + 4f'(1)$$

$$3f'(1) = -15$$

$$f'(1) = -5$$

$$f''(2) = 2$$

$$f(x) = x^3 + x^2 \cdot (-5) + x \cdot 2 + 6$$

$$f'(x) = 3x^2 - 10x + 2$$

$$f'(10) = 300 - 100 + 2$$

$$= 202$$

Q.27 The least positive integral value of α, for which the angle between the vectors $\alpha\hat{i} - 2\hat{j} + 2\hat{k}$ and $\alpha\hat{i} + 2\alpha\hat{j} - 2\hat{k}$ is acute, is _____.

Given **5**

Answer

$$\cos\theta = \frac{(\alpha i - 2j + 2k)\cdot(\alpha i + 2\alpha j - 2k)}{\sqrt{\alpha^2+4+4}\ \sqrt{\alpha^2+4\alpha^2+4}}$$

$$\cos\theta = \frac{\alpha^2 - 4\alpha - 4}{\sqrt{\alpha^2+8}\ \sqrt{5\alpha^2+4}}$$

$$\Rightarrow \quad \alpha^2 - 4\alpha - 4 > 0$$

$$(\alpha-2)^2 > 8$$

$$|\alpha - 2| > 2\sqrt{2}$$

$$\alpha - 2 > 2\sqrt{2} \quad \text{or} \quad \alpha - 2 < -2\sqrt{2}$$

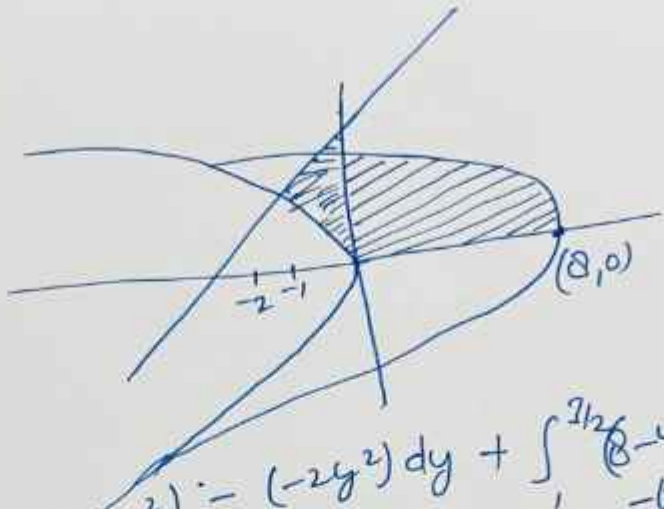
$$\alpha \in (-\infty, 2-2\sqrt{2}) \cup (2\sqrt{2}+2, \infty)$$

$$\alpha \in (-\infty, -0.82) \cup (4.82, \infty)$$

Least positive integral value of $\alpha = 5$

Q.28 Let the area of the region $\{(x, y) : x-2y+4\geq 0,\ x+2y^2\geq 0,\ x+4y^2\leq 8,\ y\geq 0\}$ be $\frac{m}{n}$, where m and n are coprime numbers. Then $m+n$ is equal to _____.

Answer



$$A = \int_0^1 \left[(8-4y^2) - (-2y^2)\right] dy + \int_1^{3/2} \left[(8-4y^2) - (-2y-4)\right] dy$$

$$= \left[8y - \frac{2y^3}{3}\right]_0^1 + \left[12y - y^2 - \frac{4y^3}{3}\right]_1^{3/2}$$

$$\frac{m}{n} = \frac{107}{12}$$

$$m+n = 119$$

Q.29 If $8 = 3 + \frac{1}{4}(3 + p) + \frac{1}{4^2}(3 + 2p) + \frac{1}{4^3}(3 + 3p) + \cdots\cdots \infty$, then the value of p is _____.

Given **12**

Answer

$$8 = 3 + \frac{1}{4}(3+P) + \frac{1}{4^2}(3+2P) + \cdots \infty$$

Sum of infinite term of AGP

$$8 = \frac{a}{1-r} + \frac{d \cdot r}{(1-r)^2}$$

$$8 = \frac{3}{1-\frac{1}{4}} + \frac{P \cdot \frac{1}{4}}{\left(1-\frac{1}{4}\right)^2}$$

$$8 = 4 + \frac{4P}{9}$$

$$4 = \frac{4P}{9}$$

$$\boxed{P = 9}$$

Q.30 If α satisfies the equation $x^2+x+1=0$ and $(1+\alpha)^7=A+B\alpha+C\alpha^2$, A, B, $C \geq 0$, then $5(3A-2B-C)$ is equal to _____.

Answer

$x^2+x+1=0$

$x=\omega$ is root of eqn

$1+\omega+\omega^2=0$

Let $\alpha=\omega$

$(1+\omega)^7=(-\omega^2)^7=-\omega^{14}$

$=-\omega^2.\omega^{12}$

$=-\omega^2(\omega^3)^4$

$\boxed{\omega^3=1}$

$=-\omega 2$

$(1+\alpha)^7=1+\omega=A+B\omega+C\omega^2$

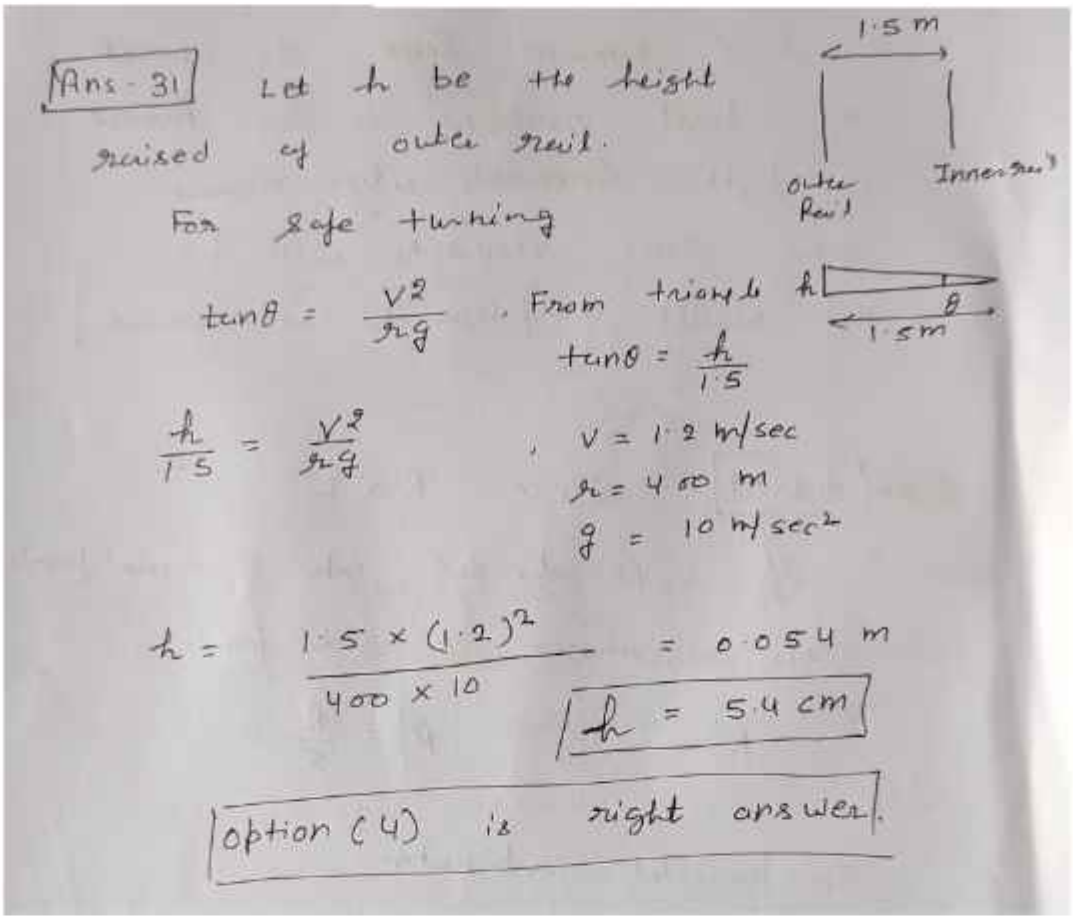
$A=1, B=1, C=0$

$5(3A-2B-C)=5(3-2-0)=5$

Q.31 A train is moving with a speed of 12 m/s on rails which are 1.5 m apart. To negotiate a curve of radius 400 m, the height by which the outer rail should be raised with respect to the inner rail is (Given, $g = 10\ m/s^2$) :

Options

1. 6.0 cm
2. 4.8 cm
3. 4.2 cm
4. 5.4 cm



Q.32 Given below are two statements :

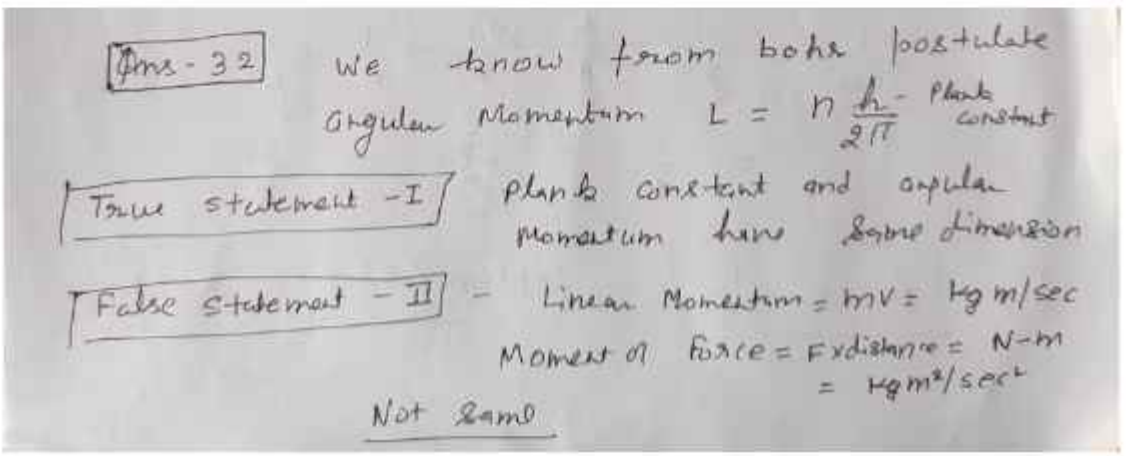
Statement (I) : Planck's constant and angular momentum have same dimensions.

Statement (II) : Linear momentum and moment of force have same dimensions.

In the light of the above statements, choose the **correct** answer from the options given below :

Options

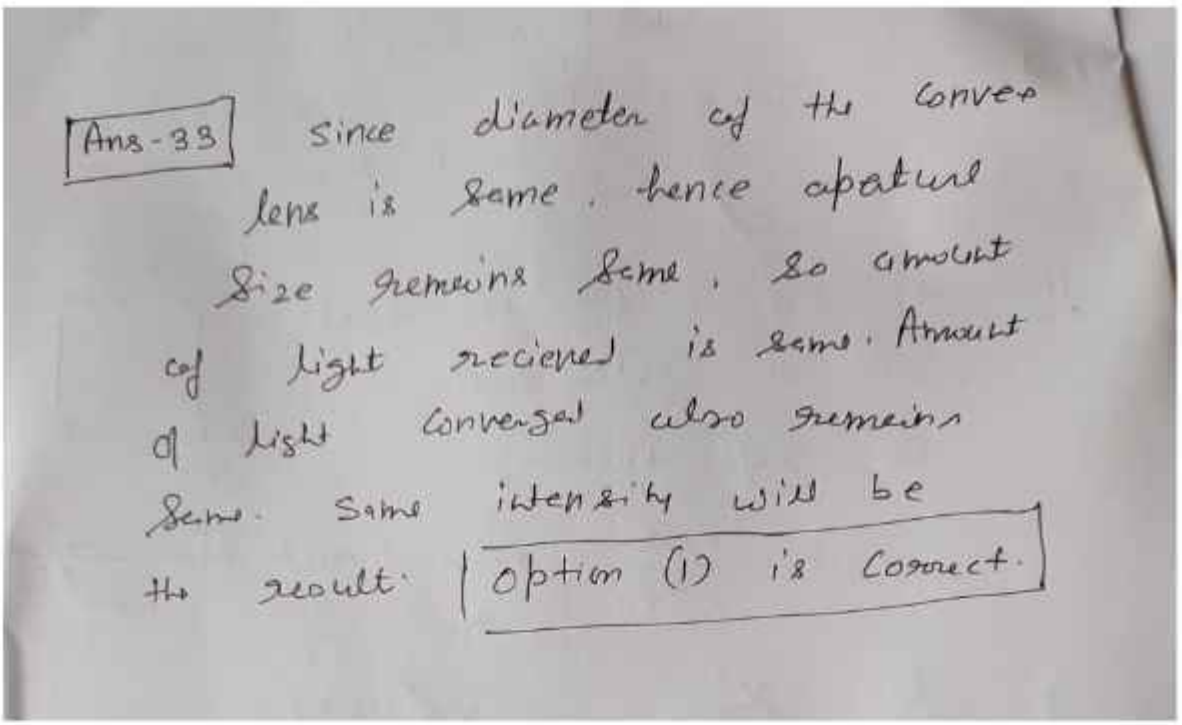
1. Both **Statement I** and **Statement II** are false
2. Both **Statement I** and **Statement II** are true
3. **Statement I** is true but **Statement II** is false
4. **Statement I** is false but **Statement II** is true



Q.33 A convex lens of focal length 40 cm forms an image of an extended source of light on a photo-electric cell. A current I is produced. The lens is replaced by another convex lens having the same diameter but focal length 20 cm. The photoelectric current now is :

Options

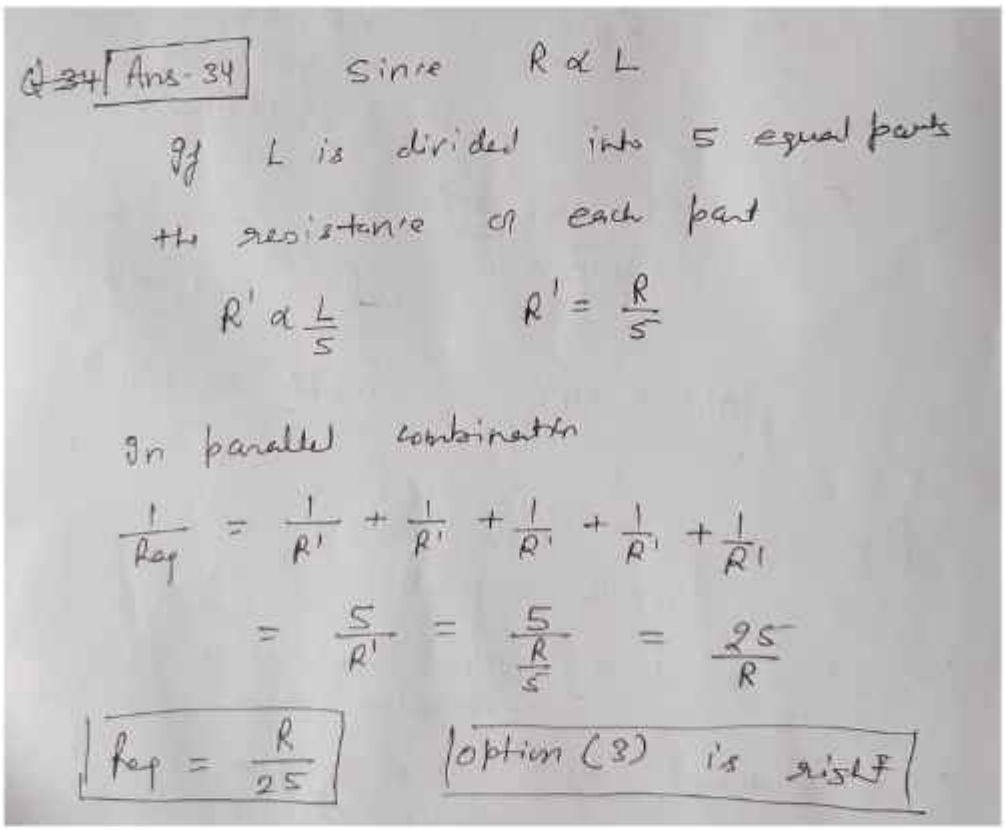
1. I
2. 4 I
3. 2 I
4. $\frac{I}{2}$



Q.34 A wire of resistance R and length L is cut into 5 equal parts. If these parts are joined parallely, then resultant resistance will be :

Options

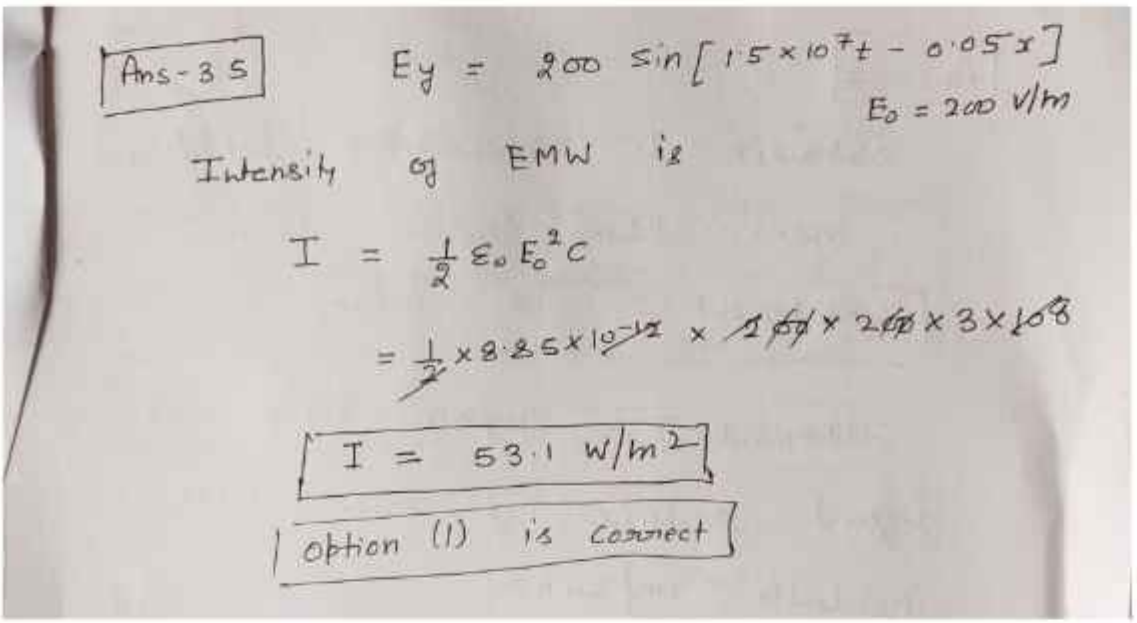
1. 25 R
2. $\frac{1}{5}$ R
3. $\frac{1}{25}$ R
4. 5 R



Q.35 A plane electromagnetic wave propagating in x-direction is described by $E_y = (200\ Vm^{-1})\ \sin[1.5\times10^7 t - 0.05\ x]$; The intensity of the wave is :
(Use $\epsilon_0 = 8.85\times10^{-12}\ C^2N^{-1}m^{-2}$)

Options

1. $53.1\ Wm^{-2}$
2. $35.4\ Wm^{-2}$
3. $26.6\ Wm^{-2}$
4. $106.2\ Wm^{-2}$



Q.36 Identify the physical quantity that cannot be measured using spherometer :

Options

1. Thickness of thin plates
2. Radius of curvature of concave surface
3. Specific rotation of liquids
4. Radius of curvature of convex surface

Ans. 36 Spherometer is a instrument used to measure small thickness and radius of curvature of objects. It can not measure specific rotation of liquids.

Option (3) is correct

Q.37 Given below are two statements :

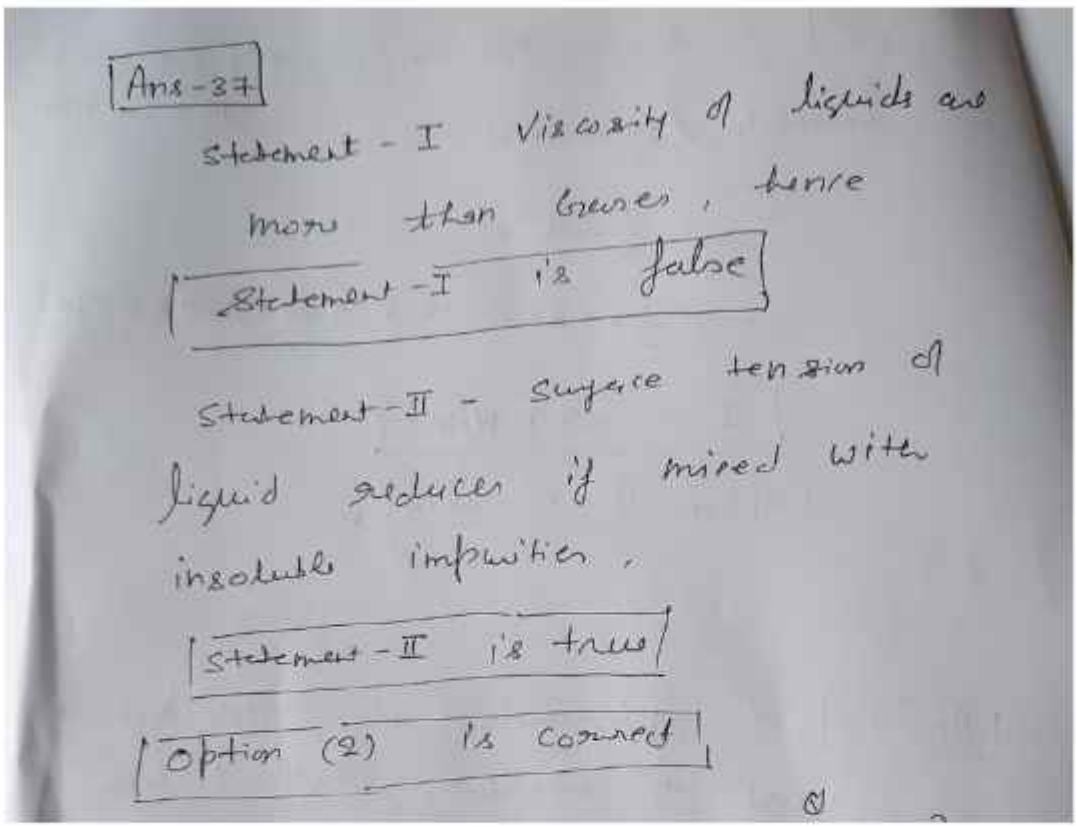
Statement (I) : Viscosity of gases is greater than that of liquids.

Statement (II) : Surface tension of a liquid decreases due to the presence of insoluble impurities.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

Options

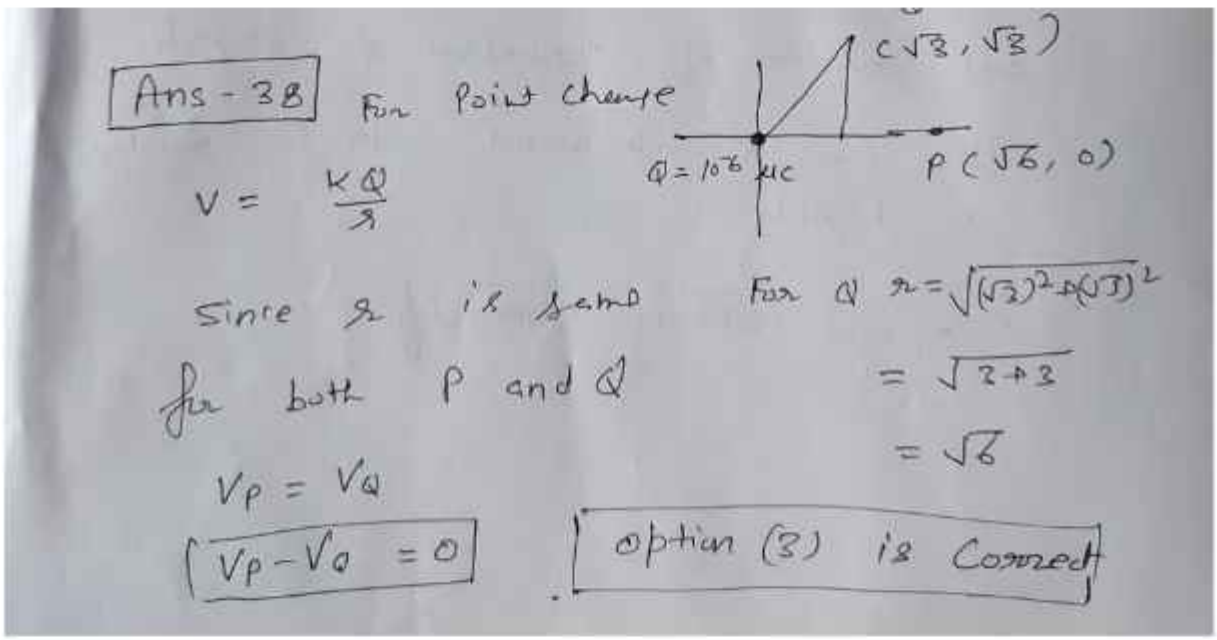
1. Both **Statement I** and **Statement II** are correct
2. **Statement I** is incorrect but **Statement II** is correct
3. **Statement I** is correct but **Statement II** is incorrect
4. Both **Statement I** and **Statement II** are incorrect



Q.38 An electric charge 10^{-6} μC is placed at origin (0, 0)m of X – Y co-ordinate system. Two points P and Q are situated at $(\sqrt{3}, \sqrt{3})$ m and $(\sqrt{6}, 0)$ m respectively. The potential difference between the points P and Q will be :

Options

1. 3 V
2. $\sqrt{3}$ V
3. 0 V
4. $\sqrt{6}$ V



Q.39 A proton moving with a constant velocity passes through a region of space without any change in its velocity. If $\vec{E}$ and $\vec{B}$ represent the electric and magnetic fields respectively, then the region of space may have :

(A) $E=0, B=0$

(B) $E=0, B \neq 0$

(C) $E \neq 0, B=0$

(D) $E \neq 0, B \neq 0$

Choose the **most appropriate** answer from the options given below :

Options

1. (A), (B) and (D) only
2. (A), (C) and (D) only
3. (A), (B) and (C) only
4. (B), (C) and (D) only

Ans-39 Since partical comes out with constant velocity it means magnitude and direction both remain same. Hence $\vec{E}=0$. If $\vec{E} \neq 0$ then $\vec{B} \neq 0$, case of velocity selector

Hence option (1) is correct

Q.40 The average kinetic energy of a monoatomic molecule is 0.414 eV at temperature :
(Use $K_B = 1.38 \times 10^{-23}$ J/mol-K)

Options

1. 1600 K
2. 3200 K
3. 1500 K
4. 3000 K

Ans. 40

Average K.E of molecule

$= \frac{3}{2} K_B T$

$0.414 \times 1.6 \times 10^{-19}$ J

$= \frac{3}{2} \times 1.38 \times 10^{-23} T$

$T = \frac{0.414 \times 2 \times 1.6 \times 10^{-19}}{3 \times 1.38 \times 10^{-23}}$

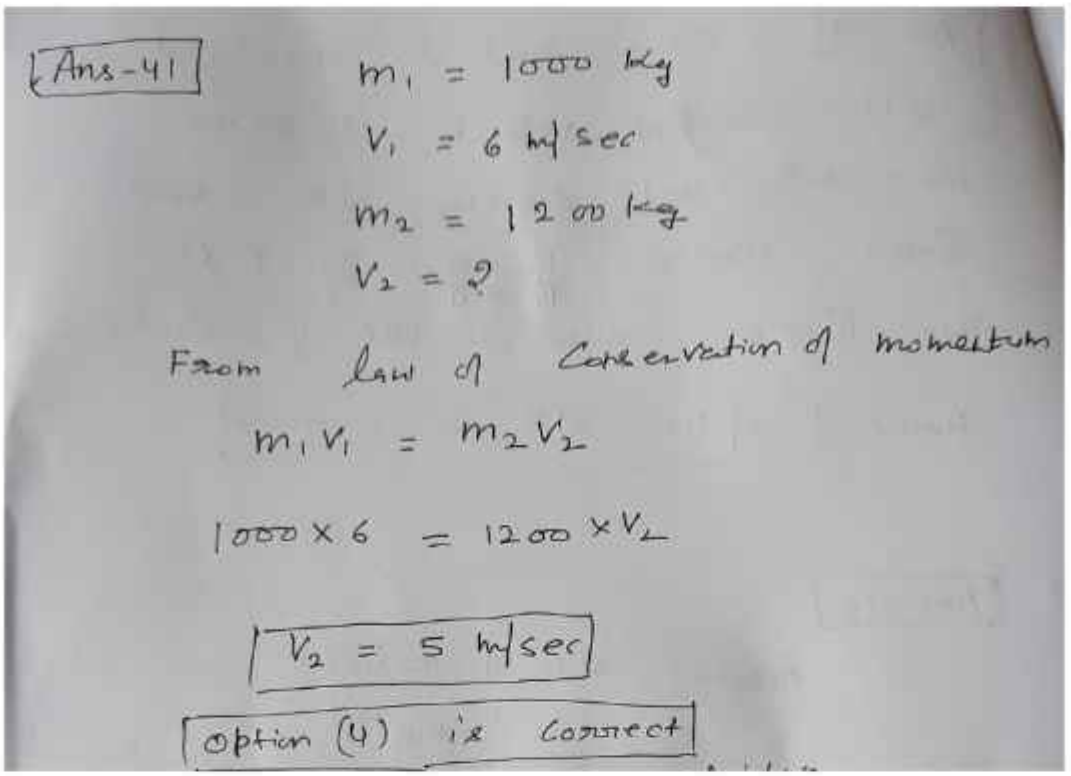
$T = 3200$ K

Option (2) is correct

Q.41 A body of mass 1000 kg is moving horizontally with a velocity 6 m/s. If 200 kg extra mass is added, the final velocity (in m/s) is :

Options

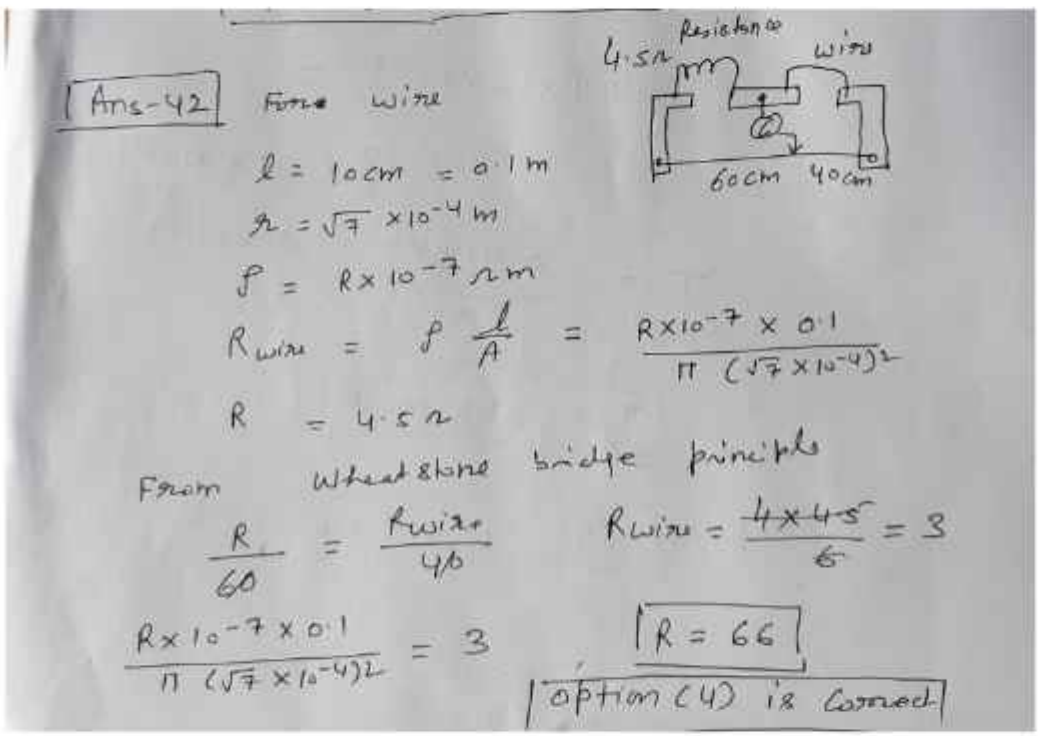
1. 3
2. 2
3. 6
4. 5



Q.42 A wire of length 10 cm and radius $\sqrt{7} \times 10^{-4}$ m is connected across the right gap of a meter bridge. When a resistance of 4.5 Ω is connected on the left gap by using a resistance box, the balance length is found to be at 60 cm from the left end. If the resistivity of the wire is $R \times 10^{-7}$ Ωm, then value of R is :

Options

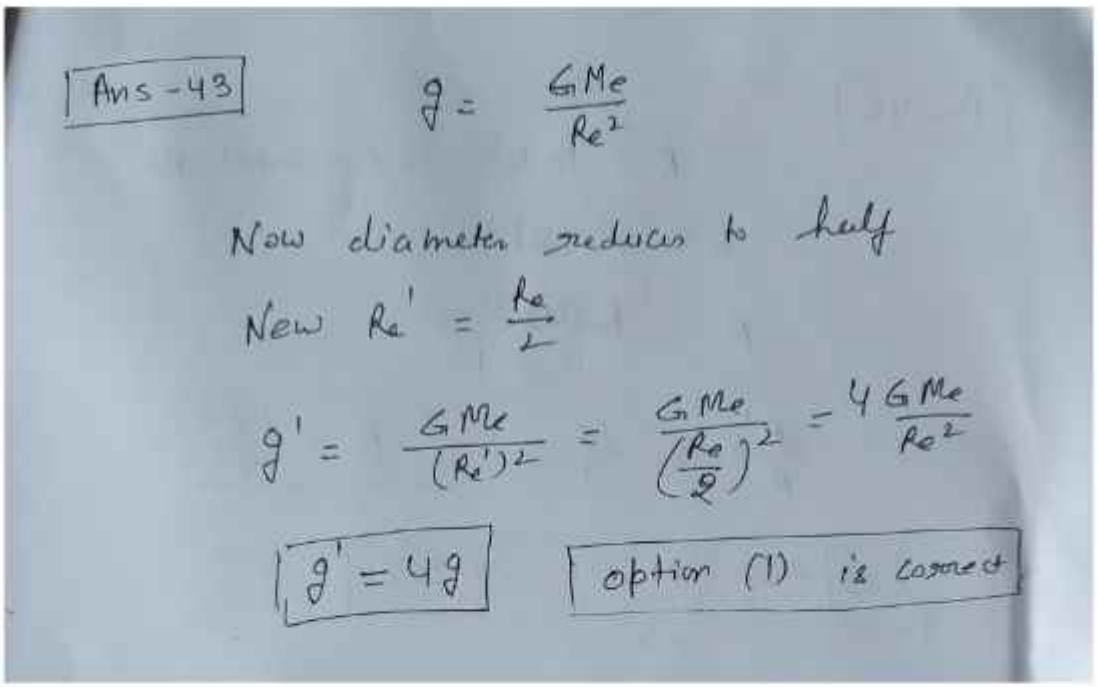
1. 63
2. 35
3. 70
4. 66



Q.43 The acceleration due to gravity on the surface of earth is g. If the diameter of earth reduces to half of its original value and mass remains constant, then acceleration due to gravity on the surface of earth would be :

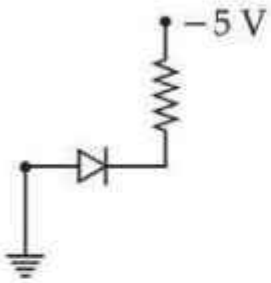
Options

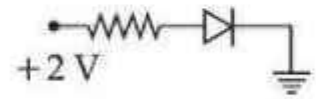
1. $4\,g$
2. $2\,g$
3. $\frac{g}{2}$
4. $\frac{g}{4}$

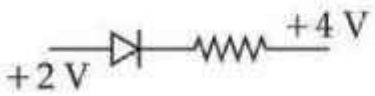


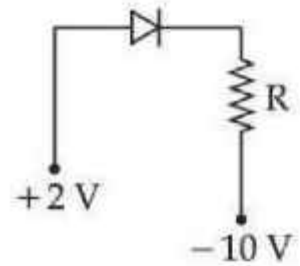
Q.44 Which of the following circuits is reverse - biased ?

Options

1. 

2. 

3. 

4. 

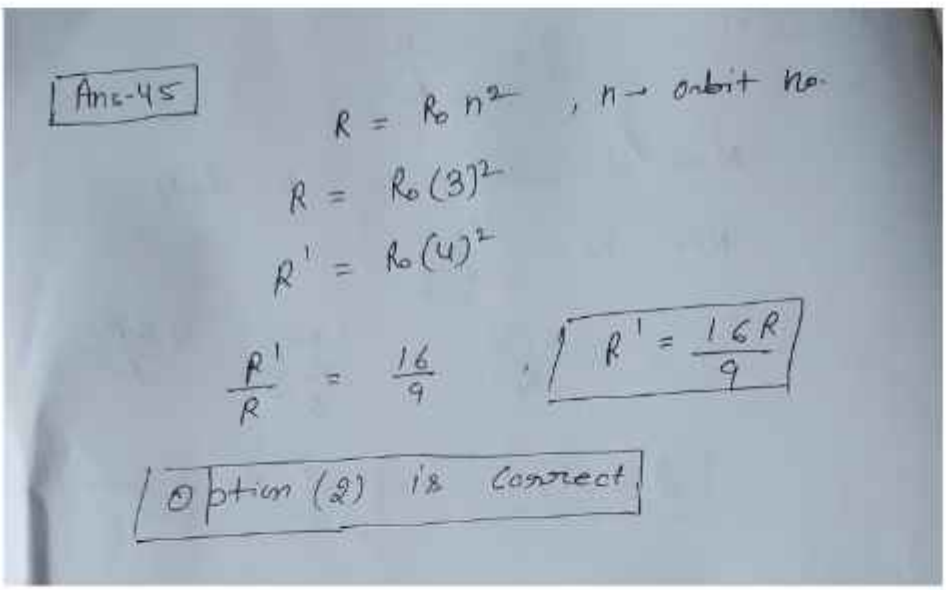
Ans-44 In reverse biased +ve terminal of diode should be at lower potential than negative terminal.

Option (3) is Correct

Q.45 The radius of third stationary orbit of electron for Bohr's atom is R. The radius of fourth stationary orbit will be :

Options

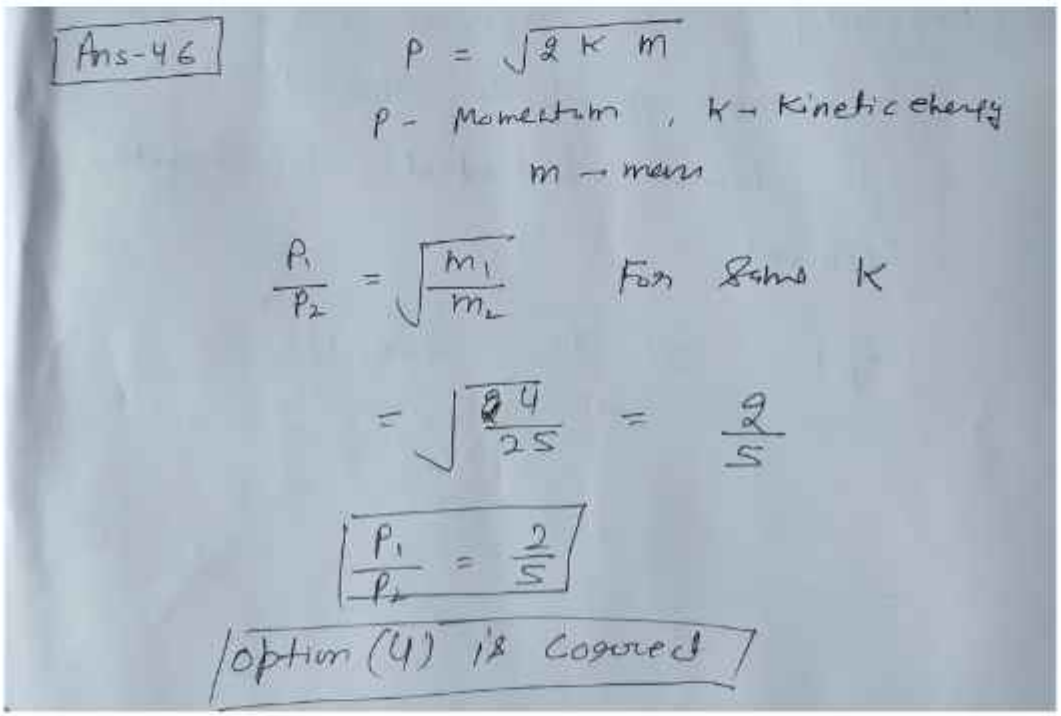
1. $\frac{3}{4}R$
2. $\frac{16}{9}R$
3. $\frac{4}{3}R$
4. $\frac{9}{16}R$



Q.46 Two bodies of mass 4 g and 25 g are moving with equal kinetic energies. The ratio of magnitude of their linear momentum is :

Options

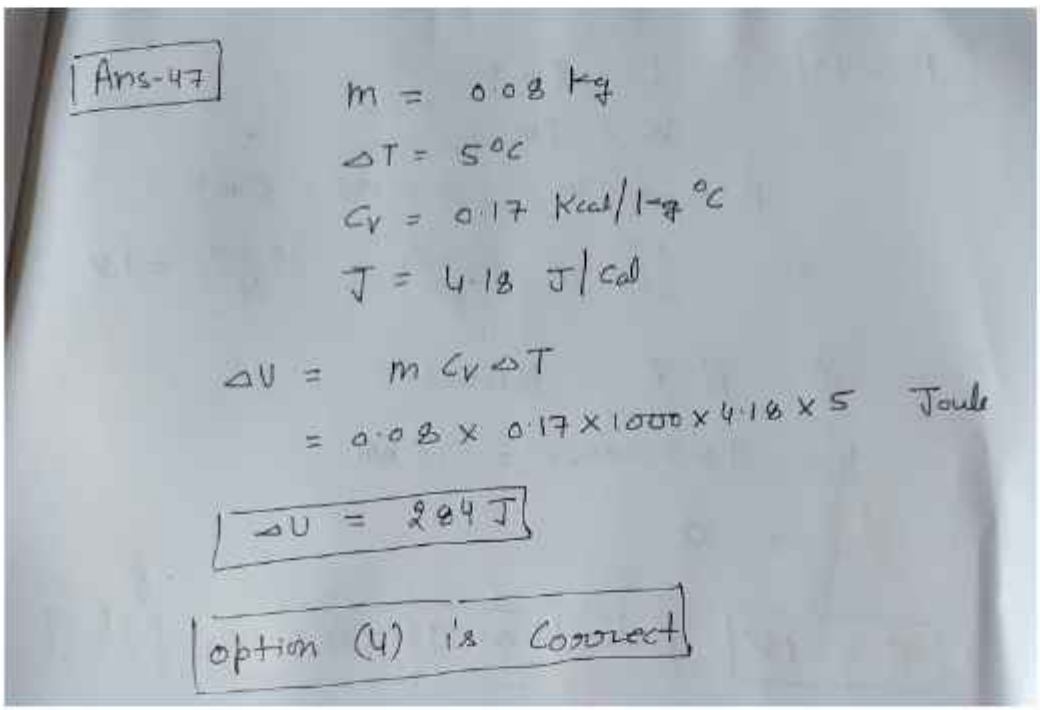
1. 3 : 5
2. 4 : 5
3. 5 : 4
4. 2 : 5



Q.47 0.08 kg air is heated at constant volume through 5°C. The specific heat of air at constant volume is 0.17 kcal/kg°C and J = 4.18 joule/cal. The change in its internal energy is approximately.

Options

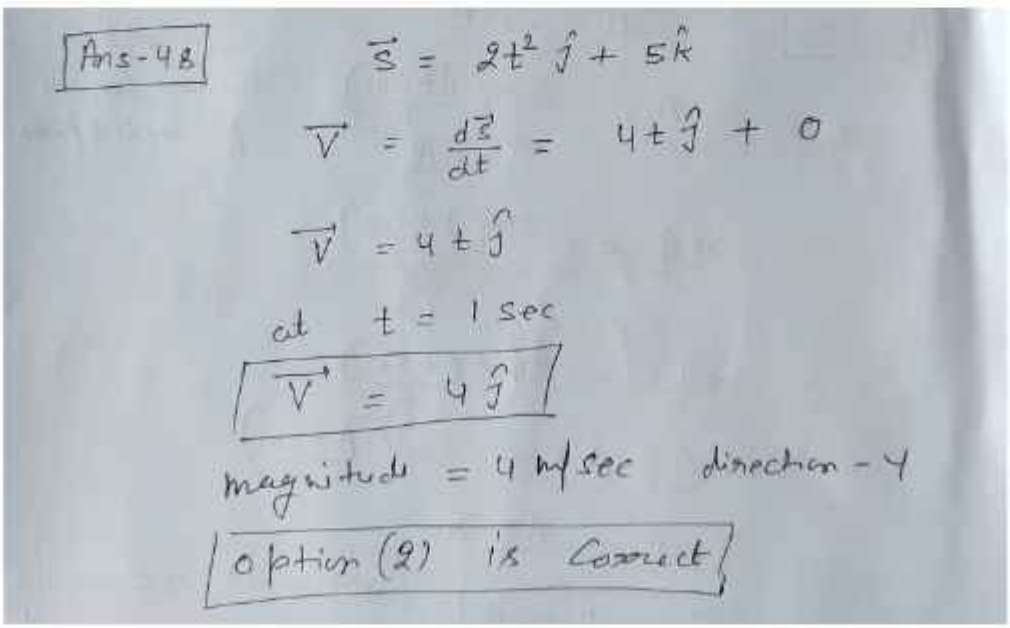
1. 142 J
2. 318 J
3. 298 J
4. 284 J



Q.48 Position of an ant (S in metres) moving in Y-Z plane is given by $S = 2t^2\hat{j} + 5\hat{k}$ (where t is in second). The magnitude and direction of velocity of the ant at t = 1 s will be :

Options

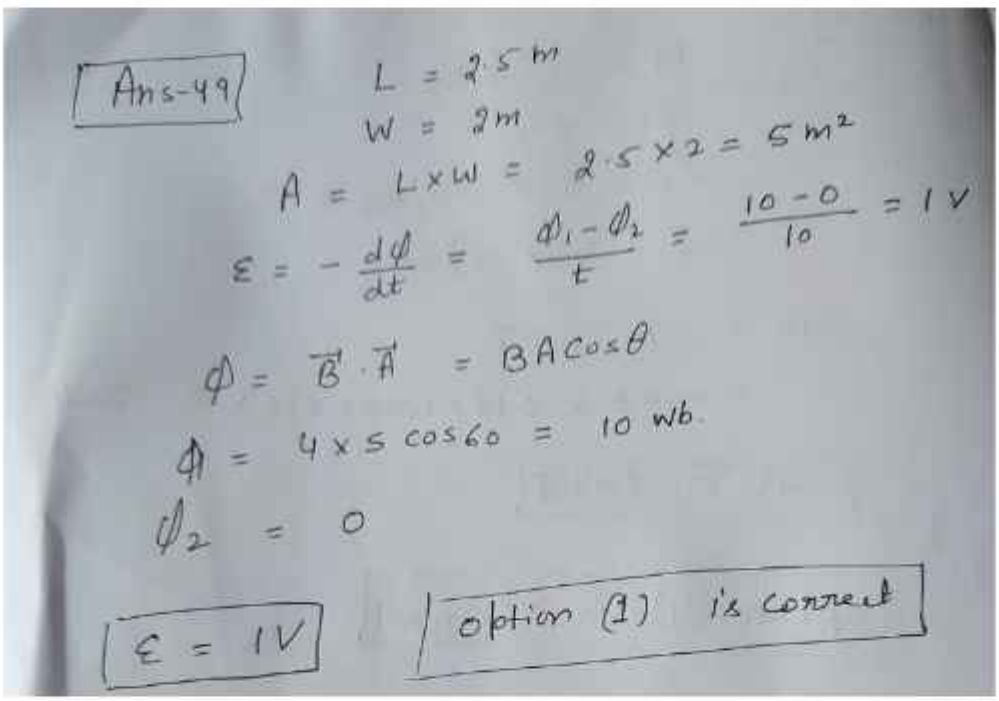
1. 9 m/s in z-direction
2. 4 m/s in y-direction
3. 16 m/s in y-direction
4. 4 m/s in x-direction



Q.49 A rectangular loop of length 2.5 m and width 2 m is placed at 60° to a magnetic field of 4 T. The loop is removed from the field in 10 s. The average emf induced in the loop during this time is :

Options

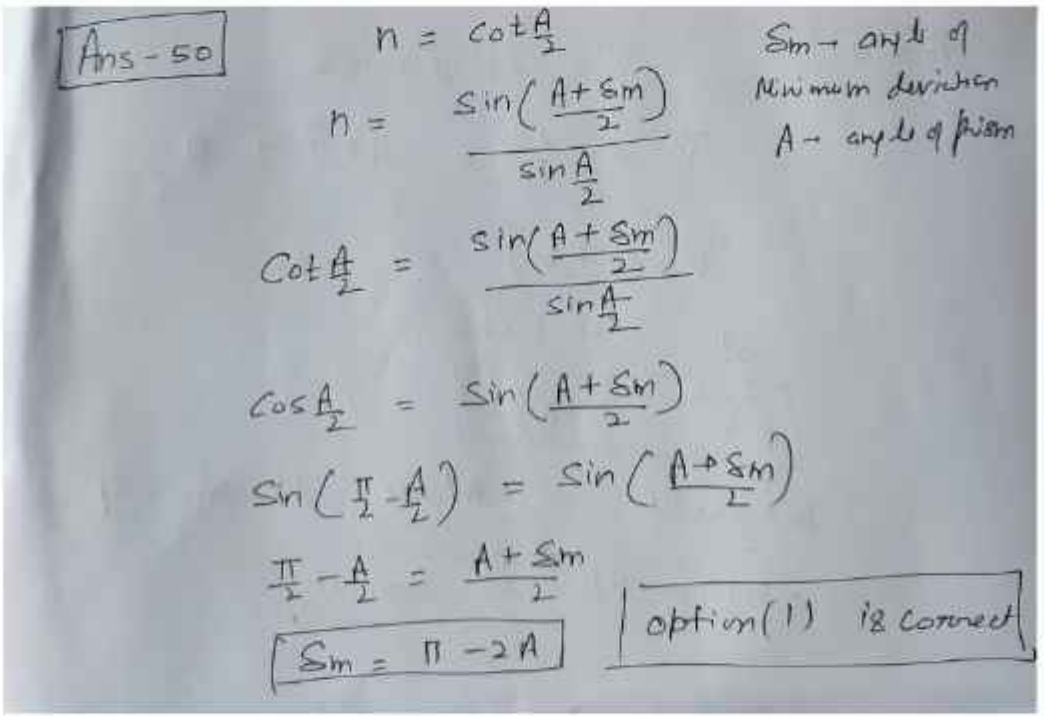
1. $+1\ \text{V}$
2. $-2\ \text{V}$
3. $-1\ \text{V}$
4. $+2\ \text{V}$



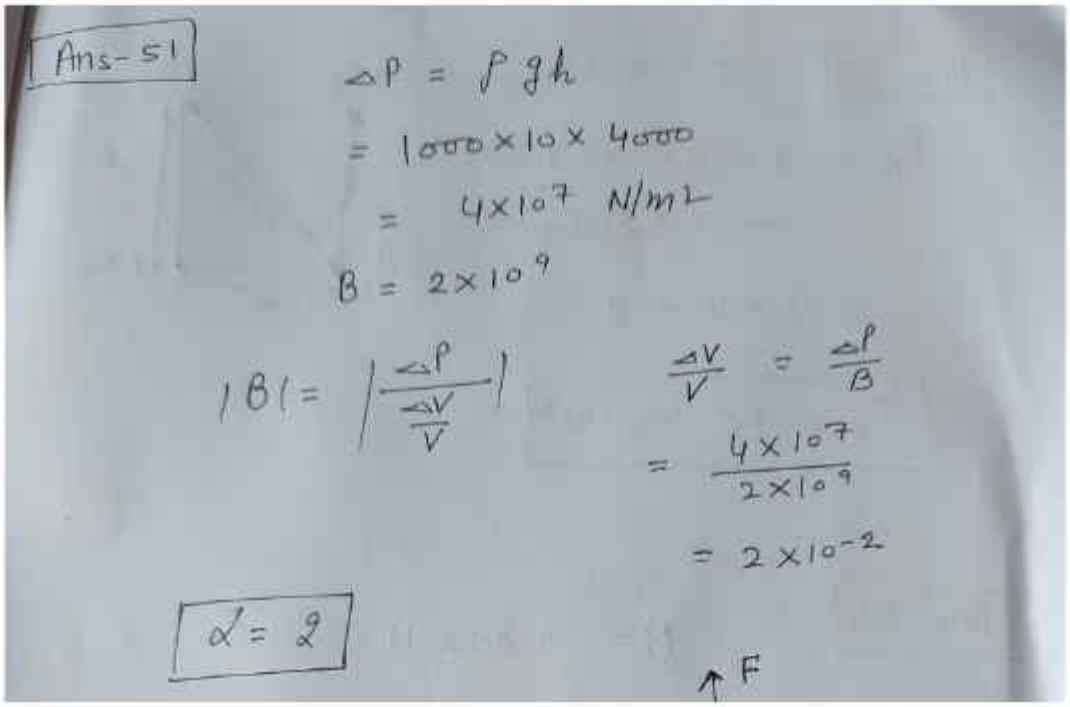
Q.50 If the refractive index of the material of a prism is $\cot\left(\frac{A}{2}\right)$, where A is the angle of prism then the angle of minimum deviation will be :

Options

1. $\pi - 2A$
2. $\frac{\pi}{2} - A$
3. $\frac{\pi}{2} - 2A$
4. $\pi - A$

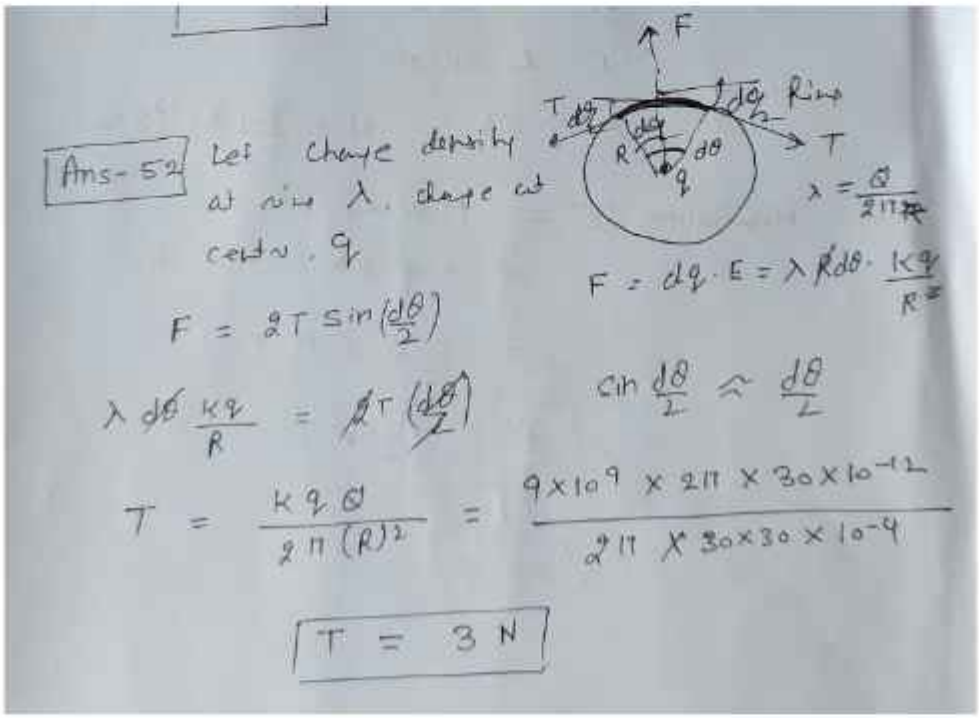


Q.51 If average depth of an ocean is 4000 m and the bulk modulus of water is $2\times10^{9}\ \text{Nm}^{-2}$, then fractional compression $\frac{\Delta V}{V}$ of water at the bottom of ocean is $\alpha\times10^{-2}$. The value of α is _____. (Given, $g=10\ \text{ms}^{-2}$, $\rho=1000\ \text{kg m}^{-3}$)

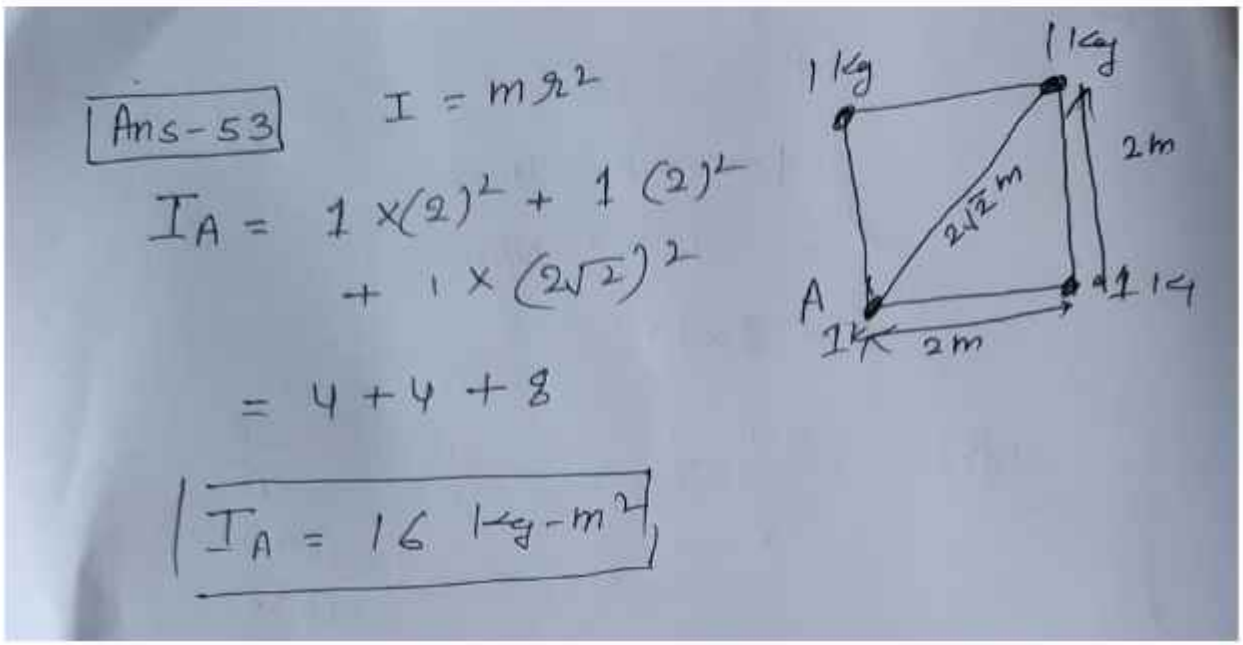


Q.52 A thin metallic wire having cross sectional area of 10^{-4} m^2 is used to make a ring of radius 30 cm. A positive charge of 2π C is uniformly distributed over the ring, while another positive charge of 30 pC is kept at the centre of the ring. The tension in the ring is _____ N; provided that the ring does not get deformed (neglect the influence of gravity).

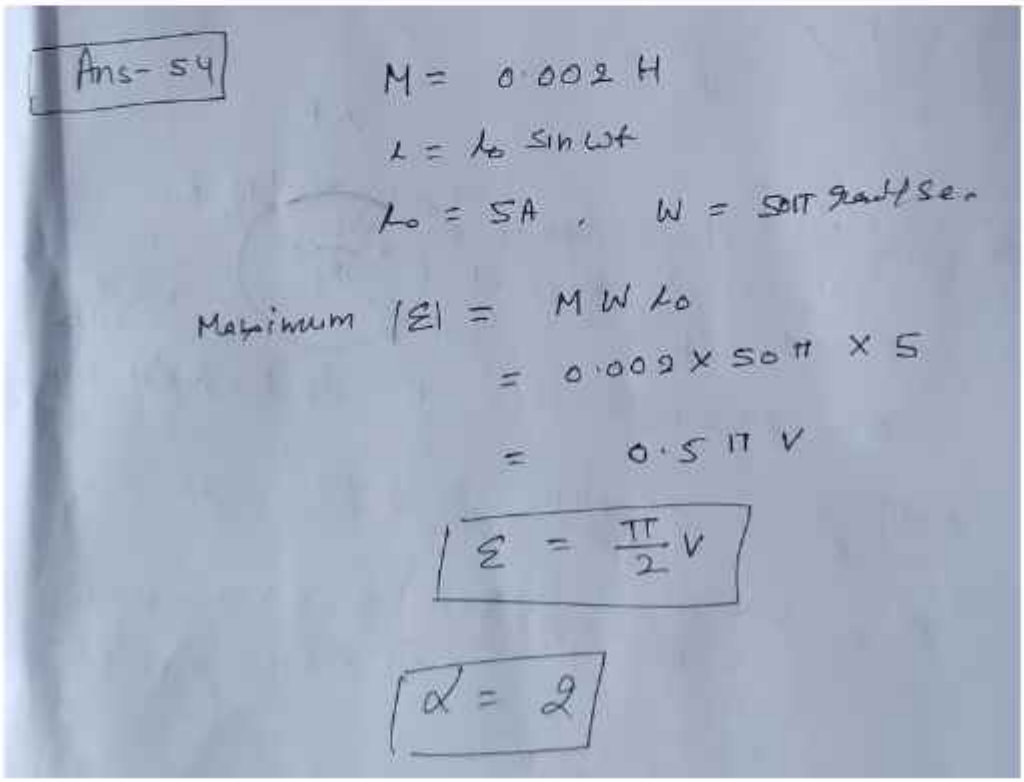
$\left(\text{given, } \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ SI units}\right)$



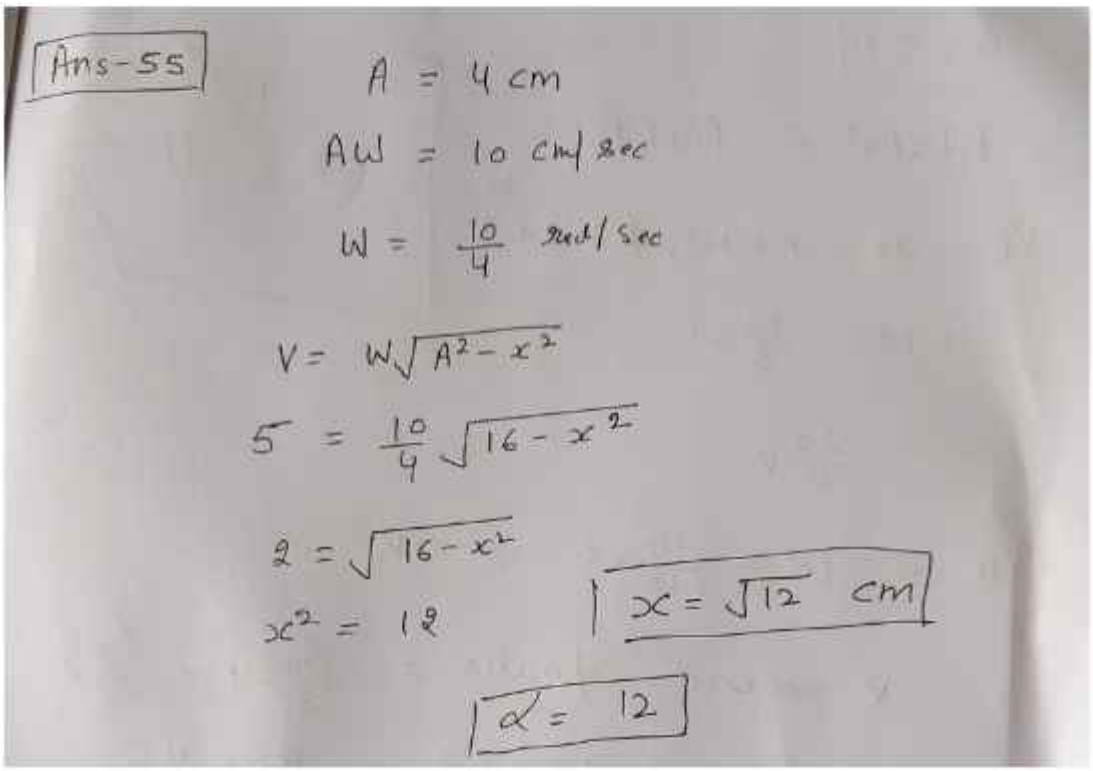
Q.53 Four particles each of mass 1 kg are placed at four corners of a square of side 2 m. Moment of inertia of system about an axis perpendicular to its plane and passing through one of its vertex is _____ kgm^2.



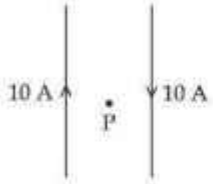
Q.54 Two coils have mutual inductance 0.002 H. The current changes in the first coil according to the relation $i=i_0 \sin\omega t$, where $i_0=5$A and $\omega=50\pi$ rad/s. The maximum value of emf in the second coil is $\frac{\pi}{\alpha}$ V. The value of α is _____.



Q.55 A particle executes simple harmonic motion with an amplitude of 4 cm. At the mean position, velocity of the particle is 10 cm/s. The distance of the particle from the mean position when its speed becomes 5 cm/s is $\sqrt{\alpha}$ cm, where α= _____.



Q.56 Two long, straight wires carry equal currents in opposite directions as shown in figure. The separation between the wires is 5.0 cm. The magnitude of the magnetic field at a point P midway between the wires is _____ μT. (Given : $\mu_0 = 4\pi \times 10^{-7}$ TmA^{-1})



Ans - 56

$B_P = B_1 + B_2$

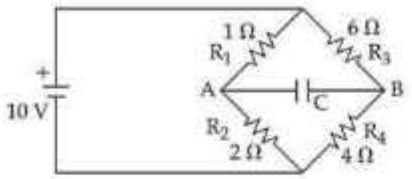
$B_1 = B_2 = \frac{\mu_0 I}{2\pi r}$

$B_P = \frac{2\mu_0 I}{2\pi r}$

$= \frac{2 \times 2 \times 10^{-7} \times 10}{25 \times 10^{-2}}$

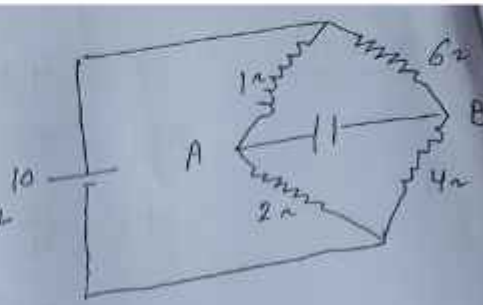
$B_P = 160\,\mu T$

Q.57 The charge accumulated on the capacitor connected in the following circuit is _____ μC.
(Given C = 150 μF)



Ans - 57

Potential of Point A



V_A = 10 − Potential drop in 1Ω

$= 10 - \frac{10}{3} \times 1$

$= \frac{20}{3}$ V

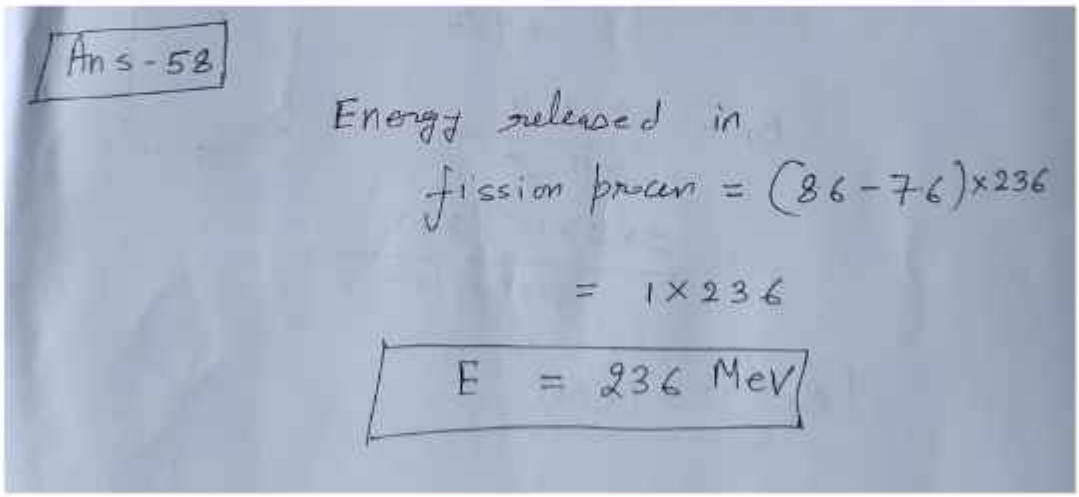
$V_B = 10 - \frac{10}{10} \times 6 = 4$ V

V across Capacitor $= \frac{20}{3} - 4 = \frac{8}{3}$ V

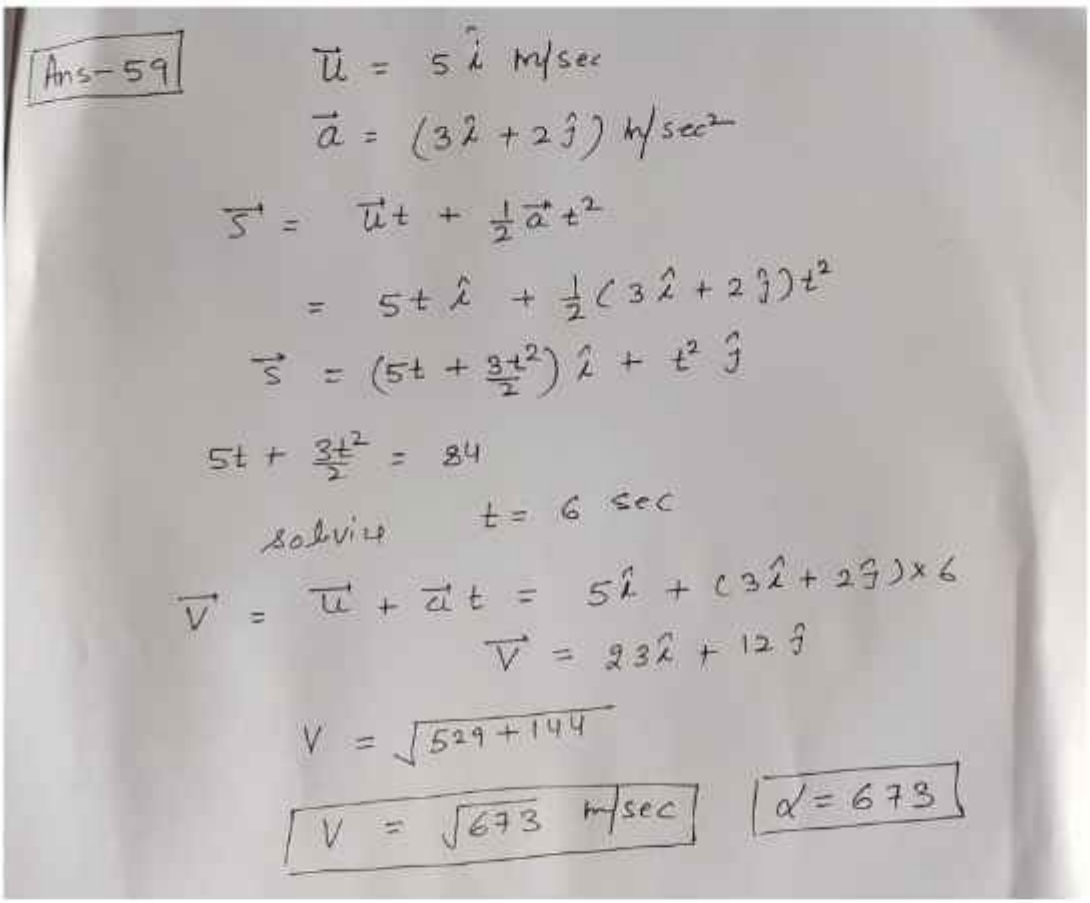
$Q = CV = 150 \times \frac{8}{3} = 400\ \mu C$

$Q = 400\ \mu C$

Q.58 In a nuclear fission process, a high mass nuclide ($A \approx 236$) with binding energy 7.6 MeV/Nucleon dissociated into middle mass nuclides ($A \approx 118$), having binding energy of 8.6 MeV/Nucleon. The energy released in the process would be _____ MeV.

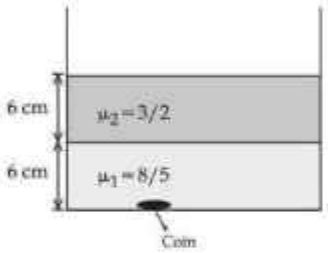


Q.59 A particle starts from origin at t=0 with a velocity $5\hat{i}$ m/s and moves in x-y plane under action of a force which produces a constant acceleration of $\left(3\hat{i}+2\hat{j}\right)$ m/s^2. If the x-coordinate of the particle at that instant is 84 m, then the speed of the particle at this time is $\sqrt{\alpha}$ m/s. The value of α is _____.

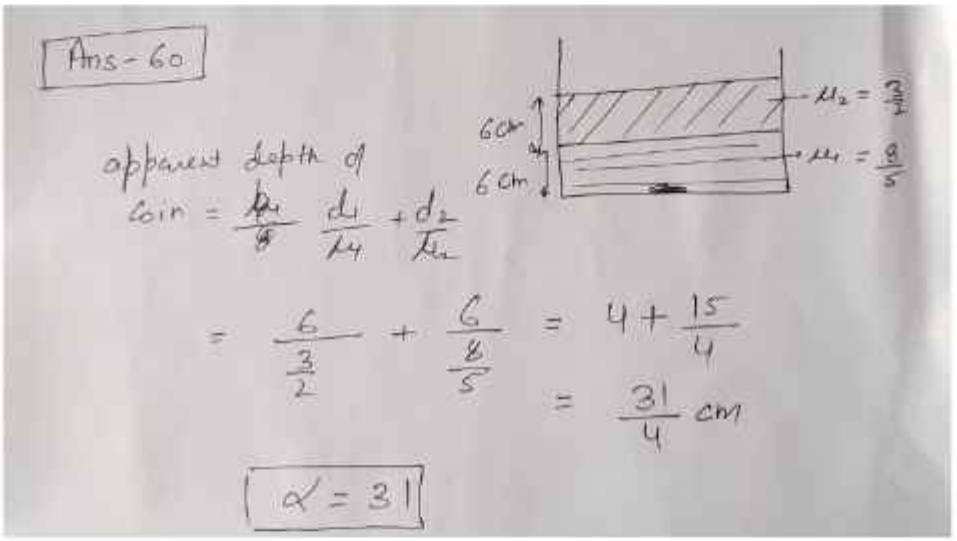


Q.60

Two immiscible liquids of refractive indices $\frac{8}{5}$ and $\frac{3}{2}$ respectively are put in a beaker as shown in the figure. The height of each column is 6 cm. A coin is placed at the bottom of the beaker. For near normal vision, the apparent depth of the coin is $\frac{\alpha}{4}$ cm. The value of α is _____.



Ans - 60



Q.61 Element not showing variable oxidation state is :

Options

1. Bromine
2. Iodine
3. Fluorine
4. Chlorine

Ans.61 Option-3 is correct because Fluorine does not contains vacant d-subshell and orbitals. Hence, it does not show variable O.S & Expansion of octet.

Q.62 The correct statement regarding nucleophilic substitution reaction in a chiral alkyl halide is :

Options

1. Racemisation occurs in S_N1 reaction and retention occurs in S_N2 reaction.
2. Racemisation occurs in both S_N1 and S_N2 reactions.
3. Retention occurs in S_N1 reaction and inversion occurs in S_N2 reaction.
4. Racemisation occurs in S_N1 reaction and inversion occurs in S_N2 reaction.

Ans-62 Correct option is 4.
Chiral alkyl halide may be 2°c and 3°c.
3° carbocation carries Racemisation by SN^1 but 2°-carbocation carries inversion & Racemisation both by SN^1 & SN^2.

Q.63 NaCl reacts with conc. H_2SO_4 and $K_2Cr_2O_7$ to give reddish fumes (B), which react with NaOH to give yellow solution (C). (B) and (C) respectively are :

Options

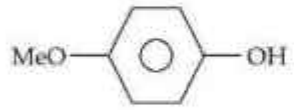
1. CrO_2Cl_2, $KHSO_4$
2. CrO_2Cl_2, $Na_2Cr_2O_7$
3. CrO_2Cl_2, Na_2CrO_4
4. Na_2CrO_4, CrO_2Cl_2

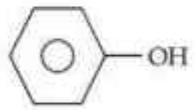
Ans-63. Correct option is (3)
i.e. CrO_2Cl_2 and Na_2CrO_4.

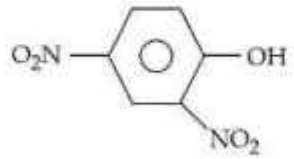
Q.64 The ascending order of acidity of $-OH$ group in the following compounds is :

(A) Bu – OH

(B) 

(C) 

(D) 

(E) 

Choose the **correct** answer from the options given below :

Options

1. (C) < (A) < (D) < (B) < (E)
2. (A) < (D) < (C) < (B) < (E)
3. (A) < (C) < (D) < (B) < (E)
4. (C) < (D) < (B) < (A) < (E)

Ans-64 Correct option is (3). greater is the E.W.G., greater is M-effect and greater will be its acidic nature

Q.65 The electronic configuration for Neodymium is :
[Atomic Number for Neodymium 60]

Options

1. $[Xe]\ 4f^6\ 6s^2$
2. $[Xe]\ 4f^1\ 5d^1\ 6s^2$
3. $[Xe]\ 5f^7\ 7s^2$
4. $[Xe]\ 4f^4\ 6s^2$

Ans 5. Correct option is (4).
$[Xe].4f^4.6s^2$

Q.66 A solution of two miscible liquids showing negative deviation from Raoult's law will have :

Options

1. increased vapour pressure, increased boiling point
2. decreased vapour pressure, decreased boiling point
3. decreased vapour pressure, increased boiling point
4. increased vapour pressure, decreased boiling point

Ans 6. Correct option is (3).
In negative deviation, V.P. of solution is lower than expected V.P. of solvent and
V.P. of solution $\propto \frac{1}{\text{B.P. of solution}}$

Q.67

Cyclohexene  is _____ type of an organic compound.

Options

1. Alicyclic
2. Benzenoid aromatic
3. Acyclic
4. Benzenoid non-aromatic

Ans 67. correct option is (1)
∵ cyclohexane has both aliphatic & cyclic nature

Q.68 Given below are two statements :
Statement (I) : p-nitrophenol is more acidic than m-nitrophenol and o-nitrophenol.
Statement (II) : Ethanol will give immediate turbidity with Lucas reagent.
In the light of the above statements, choose the **correct** answer from the options given below :

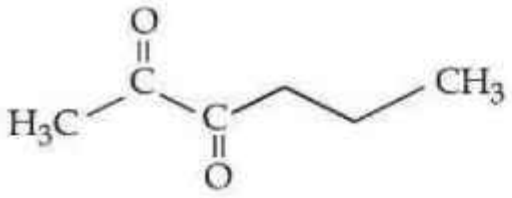
Options

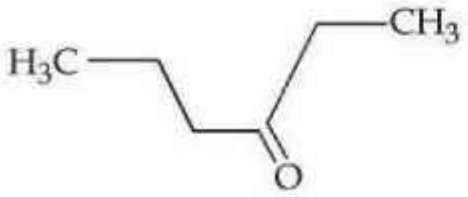
1. Both **Statement I** and **Statement II** are true
2. Both **Statement I** and **Statement II** are false
3. **Statement I** is false but **Statement II** is true
4. **Statement I** is true but **Statement II** is false

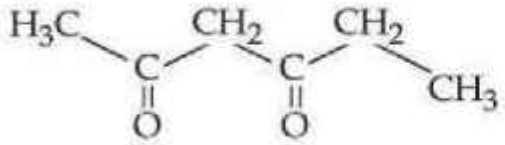
Ans. 68. S-1 is correct ∵ the conjugate base of ortho and para is more stable than meta. Secondly ortho has H-bonding b/w -OH & Polar oxygen of $-NO_2$ so it is much difficult to remove H. Thus order will be
m- < o- < p-nitrophenol
S-2 is wrong ∵ Prim. alcohol gives no turbidity with LUCAS Reagent
Correct option - (4)

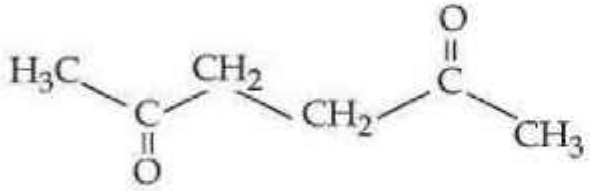
Q.69 Which of the following has highly acidic hydrogen ?

Options

1. 

2. 

3. 

4. 

Ans.69. correct option is (3), most acidic hydrogen

Q.70 Which of the following electronic configuration would be associated with the highest magnetic moment ?

Options

1. [Ar] $3d^6$
2. [Ar] $3d^3$
3. [Ar] $3d^8$
4. [Ar] $3d^7$

Ans.70 correct option is 1.
magnetic moment is directly proportional to number of unpaired electrons max. in $[Ar], 3d^6$

Q.71 Yellow compound of lead chromate gets dissolved on treatment with hot NaOH solution. The product of lead formed is a :

Options 1.

Tetraanionic complex with coordination number six

2. Neutral complex with coordination number four

3. Dianionic complex with coordination number six

4.

Dianionic complex with coordination number four

Ans-71 Correct option is 4.

$2\,NaOH + PbCrO_4 \longrightarrow Pb(OH)_2\downarrow + Na_2CrO_4$

or

$Na_2[CrO_4^{2-}]$

Dianionic with C.N = 4

Q.72 Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Melting point of Boron (2453 K) is unusually high in group 13 elements.

Reason (R) : Solid Boron has very strong crystalline lattice.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

Options

1. **(A)** is true but **(R)** is false

2. **(A)** is false but **(R)** is true

3.

Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**

4.

Both **(A)** and **(R)** are correct but **(R)** is **not** the correct explanation of **(A)**

Ans-72 option-3 is correct.

Because Boron is being smaller in size have high affinity to form strong covalent bond and strong crystalline lattice.

Q.73 Choose the polar molecule from the following :

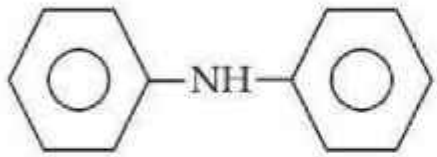
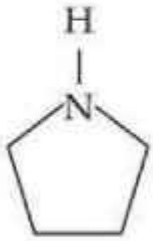
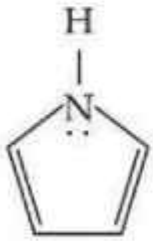
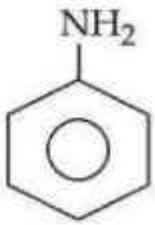
Options

1. CO_2
2. $CHCl_3$
3. $CH_2=CH_2$
4. CCl_4

Ans 73. Correct option is (2) ∵ $CHCl_3$ has unsymmetrical arrangement in which three C-Cl dipole moments are in the same direction but one C-H bond dipole in against it.
∴ Net $\mu_{CHCl_3} > 0$ → define the Polar nature of $CHCl_3$.

Q.74 Which of the following is strongest Bronsted base ?

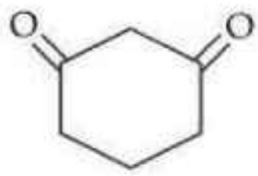
Options

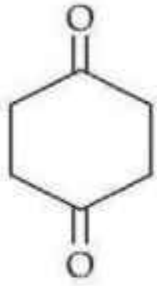
1. 
2. 
3. 
4. 

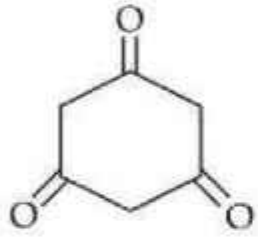
Ans 74. Bronsted base is H^+ acceptor and it is directly proportional to $I^{\pm}$ effect but inversely proportional to Resonance effect
Correct option - (2)

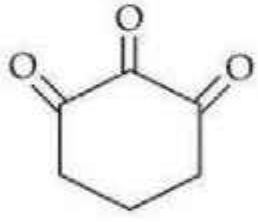
Q.75 Highest enol content will be shown by :

Options

1. 

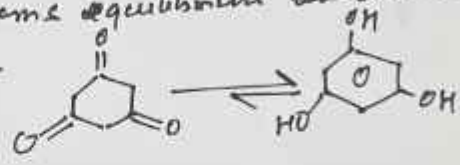
2. 

3. 

4. 

Ans 75. Correct option – (5)

∵ it forms equilibrium with reversibility



Q.76 Given below are two statements :

Statement (I) : Aqueous solution of ammonium carbonate is basic.

Statement (II) : Acidic/basic nature of salt solution of a salt of weak acid and weak base depends on K_a and K_b value of acid and the base forming it.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

Options

1. Both **Statement I** and **Statement II** are correct

2. **Statement I** is incorrect but **Statement II** is correct

3. **Statement I** is correct but **Statement II** is incorrect

4. Both **Statement I** and **Statement II** are incorrect

Ans 76. Correct option – (2)

Solution of $(NH_4)_2CO_3$ is neutral ∵ it is made by weak acid and weak base.

Strength of W.A/W.B ∝ K_a/K_b

Q.77 Given below are two statements :

Statement (I) : The 4*f* and 5*f* - series of elements are placed separately in the Periodic table to preserve the principle of classification.

Statement (II) : s-block elements can be found in pure form in nature.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

Options

1. Both **Statement I** and **Statement II** are false
2. Both **Statement I** and **Statement II** are true
3. **Statement I** is false but **Statement II** is true
4. **Statement I** is true but **Statement II** is false

Ans.77. Correct option -(4)
∵ s-block elements are most reactive metal are not found in the pure form in nature

Q.78 Consider the following complex ions

$P = [FeF_6]^{3-}$

$Q = [V(H_2O)_6]^{2+}$

$R = [Fe(H_2O)_6]^{2+}$

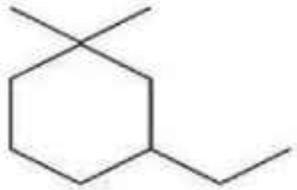
The correct order of the complex ions, according to their spin only magnetic moment values (in B.M.) is :

Options

1. $Q < P < R$
2. $Q < R < P$
3. $R < P < Q$
4. $R < Q < P$

Ans.78 Correct option -(2)
∵ magnetic moment ∝ no. of unpairing of e⁻(s)
order of unpairing in inc. order :-
$[V(H_2O)_6]^{3+} < [Fe(H_2O)_6]^{2+} < [FeF_6]^{3-}$

Q.79 IUPAC name of following compound (P) is :

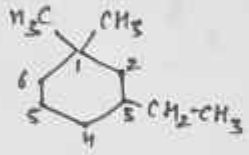


(P)

Options

1. 1-Ethyl-5,5-dimethylcyclohexane
2. 1,1-Dimethyl-3-ethylcyclohexane
3. 3-Ethyl-1,1-dimethylcyclohexane
4. 1-Ethyl-3,3-dimethylcyclohexane

Ans.79. correct option is (3)



3-Ethyl, 1,1-Dimethyl Cyclohexane

Q.80 Two nucleotides are joined together by a linkage known as :

Options

1. Peptide linkage
2. Disulphide linkage
3. Phosphodiester linkage
4. Glycosidic linkage

Ans.80. Correct option - (4) Glycosidic linkage

Q.81 From the given list, the number of compounds with +4 oxidation state of Sulphur is _____.
SO_3, H_2SO_3, $SOCl_2$, SF_4, $BaSO_4$, $H_2S_2O_7$

Given --

Ans.81. $H_2\overset{+4}{S}O_3$, $\overset{+4}{S}OCl_2$, $\overset{+4}{S}F_4$
Total no. of S-compounds with +4 O.S. in S.

Q.82 The mass of silver (Molar mass of Ag : 108 $gmol^{-1}$) displaced by a quantity of electricity which displaces 5600 mL of O_2 at S.T.P. will be _____ g.

Given --

Ans.82 $\text{mole} = \frac{\text{given mass}}{\text{molar mass}} = \frac{\text{given vol.}}{22,400\ mL}$

$\frac{x}{32} = \frac{5600}{22400}$

given mass of oxygen in 8 gm.

Acc. to Faraday's law

$\frac{W_{Ag}}{W_O} = \frac{E_{Ag}}{E_{O^{2-}}} \Rightarrow W_{Ag} = \frac{100 \times 8}{8}$

$W_{Ag} = 100$ gm

Q.83 Sum of bond order of CO and NO^+ is _____.

Given 6

Ans.83 CO and NO^+ are heteroatomic molecule and ion

Heteroatomic molecular orbital electronic configuration,

$= (\sigma_{1s})^2 . (\sigma^*_{1s})^2 . (\sigma_{2s})^2 . (\sigma^*_{2s})^2 . \left\{\begin{matrix}(\pi_{2p_x})^2 \\ (\pi_{2p_y})^2\end{matrix}\right\} . (\sigma_{2p_z})^2 . \left\{\begin{matrix}(\pi^*_{2p_x}) \\ (\pi^*_{2p_y})\end{matrix}\right\} . (\sigma^*_{2p_z})$

$\left.\begin{matrix}\{CO\} \\ \{NO\}^+\end{matrix}\right\}$ (14) =

B.O of each $= \frac{1}{2}(N_b - N_a)$

$= \frac{1}{2}(10 - 4) = \frac{6}{2} = 3$

Sum of Bond order of CO & $NO^+ = 3 + 3 = 6$

Q.84 Consider the following data for the given reaction

$2\,HI_{(g)} \rightarrow H_{2(g)} + I_{2(g)}$

	1	2	3
HI (mol L^{-1})	0.005	0.01	0.02
Rate (mol $L^{-1}s^{-1}$)	7.5×10^{-4}	3.0×10^{-3}	1.2×10^{-2}

The order of the reaction is _____.

Given –

Ans. 04

$$\text{Rate} = K[HI]^x$$

$$\frac{7.5\times10^{-4}}{3.0\times10^{-3}} = \frac{K[0.005]^x}{K[0.01]^x} \quad \text{— (1)/(2)}$$

$$\frac{7.5}{30} = \left(\frac{5}{10}\right)^x$$

$$\frac{1}{4} = \left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^2 \Rightarrow x = 2$$

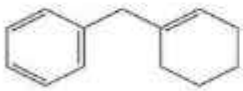
Q.85 3-Methylhex-2-ene on reaction with HBr in presence of peroxide forms an addition product (A). The number of possible stereoisomers for 'A' is _____.

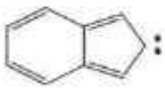
Given 4

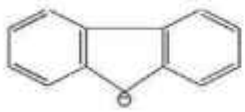
Ans. 04 Total number of stereoisomer is 4.

Q.86 Among the given organic compounds, the total number of aromatic compounds is _____.

(A) 

(B) 

(C) 

(D) 

Given –

Ans. 06. Total no. of aromatic compounds is 2.

Q.87 The number of electrons present in all the completely filled subshells having n = 4 and $s = +\frac{1}{2}$ is _____.

(Where n = principal quantum number and
s = spin quantum number)

Given 16

Ans. 87.

$n = 4$, $l = 0$ = ↑↓

$l = 1$ = ↑↓ ↑↓ ↑↓

$l = 2$ = ↑↓ ↑↓ ↑↓ ↑↓ ↑↓

$l = 3$ = ↑↓ ↑↓ ↑↓ ↑↓ ↑↓ ↑↓ ↑↓

Total = 16

Q.88 Among the following, total number of meta directing functional groups is _____.
(Integer based)

$-OCH_3$, $-NO_2$, $-CN$, $-CH_3$, $-NHCOCH_3$, $-COR$, $-OH$, $-COOH$, $-Cl$

Given 4

Ans. 88.

Total m- directing groups are 4.

$-NO_2$, $-CN$, $-COR$, $-COOH$

Q.89 Mass of methane required to produce 22 g of CO_2 after complete combustion is _____ g.
(Given Molar mass in g mol^{-1} C = 12.0
H = 1.0
O = 16.0)

Given 8

Ans. 89.

$CH_4 + 2O_2 \xrightarrow{\Delta} CO_2 + 2H_2O$

16 gm ⟵ 44 gm

Answer 8 gm ⟵ 22 gm

Q.90 If three moles of an ideal gas at 300 K expand isothermally from 30 dm^3 to 45 dm^3 against a constant opposing pressure of 80 kPa, then the amount of heat transferred is _____ J.

Given –

Ans. 90

By 1st Law of Thermodynamics.

$\Delta U = Q + W$

Process is isothermal, $\Delta U = 0$

$\therefore Q = -W$

$= -P_{ext.} \Delta V$

$Q = +80 \times 10^3 (45 - 300) \times 10^{-3}$

$Q = 12000$ Joule