

MATHEMATICS
(Subject Code-041) (PAPER CODE: 65/5/2)
SOLUTIONS

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Steps	Marks
	SECTION A Q. Number 1 to 20 are multiple choice questions of 1 mark each.		
1.	If $A = \begin{bmatrix} \sin \theta & \cos \theta \\ -\cos \theta & \sin \theta \end{bmatrix}$ and $A + A' = I$, then θ is equal to : (A) 0 (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) π		
Sol.	(B) $\frac{\pi}{6}$		1
2.	If A is a symmetric matrix, then for any matrix B of order same as A, BAB' is a/an : (A) Skew symmetric matrix (B) Identity matrix (C) Symmetric matrix (D) Null matrix		
Sol.	(C) Symmetric matrix		1
3.	Let $A = \begin{bmatrix} 0 & -3 & 4 \\ 1 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 0 & 1 \\ 2 & 4 & 0 \end{bmatrix}$. If $A + B + C = O$, then matrix C is : (A) $\begin{bmatrix} -3 & -3 & 5 \\ 3 & 4 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 3 & 3 & 5 \\ -3 & -4 & -2 \end{bmatrix}$ (C) $\begin{bmatrix} 3 & 3 & -5 \\ -3 & -4 & -2 \end{bmatrix}$ (D) $\begin{bmatrix} -3 & -3 & -5 \\ 3 & 4 & 2 \end{bmatrix}$		
Sol.	(C) $\begin{bmatrix} 3 & 3 & -5 \\ -3 & -4 & -2 \end{bmatrix}$		1

4.	For the inverse trigonometric functions, which of the following Principal Value Branch is not correctly defined ? (A) $\tan^{-1} : \mathbb{R} \rightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (B) $\sec^{-1} : \mathbb{R} - (-1, 1) \rightarrow [0, \pi] - \left\{\frac{\pi}{2}\right\}$ (C) $\cot^{-1} : \mathbb{R} \rightarrow (0, \pi)$ (D) $\operatorname{cosec}^{-1} : \mathbb{R} - (-1, 1) \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$		
Sol.	(D) $\operatorname{cosec}^{-1} : \mathbb{R} - (-1, 1) \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$		1
5.	If $f(x) = \begin{cases} \frac{x^2 - 4x - 5}{x + 1}, & x \neq -1 \\ k, & x = -1 \end{cases}$ is continuous at $x = -1$, then the value of k is : (A) Any real value (B) 6 (C) -1 (D) -6		
Sol.	(D) -6		1
6.	If A is a non-singular matrix, then which of the following is not true ? (A) $\operatorname{adj} A$ is singular (B) $(\operatorname{adj} A)^{-1} = (\operatorname{adj} A^{-1})$ (C) $ A \neq 0$ (D) A^{-1} exists		
Sol.	(A) $\operatorname{adj} A$ is singular		1
7.	Absolute minimum value of $f(x) = (x - 2)^2 + 5$ in the interval $[-3, 2]$ is : (A) -3 (B) 2 (C) 5 (D) 30		
Sol.	(C) 5		1
8.	If the area of ΔABC with vertices $A(3, 1)$, $B(-2, 1)$ and $C(0, k)$ is 5 sq. units, then values of k are : (A) 3, 1 (B) -1, 3 (C) -1, 2 (D) 0, 2		

Sol.	(B) $-1, 3$		1
9.	Derivative of $\cos^{-1} \left(\frac{\sin x + \cos x}{\sqrt{2}} \right)$, $-\frac{\pi}{4} < x < \frac{\pi}{4}$ with respect to x is : (A) -1 (B) 1 (C) $\frac{\pi}{4}$ (D) $-\frac{\pi}{4}$		
Sol.	(A) -1		1
10.	$\int \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}} dx$ is equal to : (A) $\log \sec^2 x + C$ (B) $\log \cos x + C$ (C) $\log 1 + \cos 2x + C$ (D) $\log \sec x + C$		
Sol.	(D) $\log \sec x + C$		1
11.	The value of $\int_0^1 \frac{1}{3x - 4} dx$ is : (A) $\frac{1}{3} \log 4$ (B) $-\frac{1}{3} \log 4$ (C) $\log (-4)$ (D) $\log 4$		
Sol.	(B) $-\frac{1}{3} \log 4$		1
12.	The general solution for the differential equation $\frac{dy}{dx} = e^{3x-y}$ is (A) $3e^y = e^{3x} + C$ (B) $\log (3x - y) = C$ (C) $e^{3x-y} = C$ (D) $-e^y + 3e^{3x} = C$		
Sol.	(A) $3e^y = e^{3x} + C$		1
13.	The order and degree of differential equation $\frac{d^2 y}{dx^2} = 1 - \left(\frac{d^3 y}{dx^3} \right)^2$ is (A) Order = 3, Degree = 3 (B) Order = 2, Degree = 2 (C) Order = 3, Degree = 2 (D) Order = 2, Degree = 1		

Sol.	(C) Order = 3, Degree = 2		1
14.	An ant is observed crawling on a sheet of paper along a straight line given by equation $y = 2x - 4$. Area of the surface covered by the ant bounded by y-axis, x-axis and $x = 1$ is : (A) 1 sq. unit (B) 3 sq. units (C) 2 sq. units (D) 4 sq. units		
Sol.	(B) 3 sq. units		1
15.	If position vector $\vec{p}$ of a point (24, n) is such that $\vec{p} = 25$, then the value of n is : (A) ± 49 (B) ± 5 (C) ± 1 (D) ± 7		
Sol.	(D) ± 7		1
16.	The corner points of the feasible region determined by the system of linear constraints are (0, 0), (0, 40), (20, 40) (60, 20) and (60, 0). If the objective function of an LPP is $Z = 4x + 3y$, then the maximum value is : (A) 200 (B) 300 (C) 240 (D) 120		
Sol.	(B) 300		1
17.	If $3P(A) = P(B) = \frac{3}{5}$ and $P(A B) = \frac{2}{3}$, then $P(A \cup B)$ is : (A) $\frac{3}{5}$ (B) $\frac{1}{5}$ (C) $\frac{2}{15}$ (D) $\frac{2}{5}$		
Sol.	(D) $\frac{2}{5}$		1
18.	If vectors $\vec{a} = 3\hat{i} + 2\hat{j} + \lambda\hat{k}$ and $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$, represent the two strips of the Red Cross sign placed outside a doctor's clinic, then the value of λ is : (A) 1 (B) $\frac{5}{2}$ (C) $\frac{2}{5}$ (D) 0		

Sol.	(C) $\frac{2}{5}$		1
	Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below. (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.		
19.	Assertion (A) : A relation R on the set {1, 2, 3} defined as $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$ is an equivalence relation. Reason (R) : A relation that is reflexive, symmetric and transitive is an equivalence relation.		
Sol.	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).		1
20.	Assertion (A) : Consider a Linear Programming Problem with minimise $Z = x + 2y$ subject to constraints $2x + y \geq 3$, $x + 2y \geq 6$, $x, y \geq 0$ which gives minimum Z at infinitely many points. The corner points of feasible region are (0, 3) and (6, 0). Reason (R) : If two corner points produce the same minimum value of the objective function, then every point on the line segment joining the points will give the same minimum value.		
Sol.	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).		1
SECTION B			
Q. Numbers 21 to 25 are very short answer questions of 2 marks each.			
21.	Evaluate $\sin \left[\cos^{-1} \cos \left(\frac{7\pi}{6} \right) \right]$.		
Sol.	Given expression = $\sin \cos^{-1} \left(\frac{-\sqrt{3}}{2} \right)$ or $\sin \cos^{-1} \cos \left(2\pi - \frac{5\pi}{6} \right)$ $= \sin \frac{5\pi}{6}$ $= \frac{1}{2}$	I II III	1 $\frac{1}{2}$ $\frac{1}{2}$

22.	(a) Three honey bees were found flying along the vectors $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = 4\hat{j} - 2\hat{k}$ and $\vec{c} = 3\hat{i} + 2\hat{k}$ respectively. Find the value of λ such that the path for $\vec{a} + \lambda \vec{b}$ is perpendicular to $\vec{c}$. OR (b) If A, B and C be three non-collinear points such that $\vec{AB} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{AC} = 2\hat{i} - 3\hat{j}$, then find the area of ΔABC.		
Sol.	As $\vec{a} + \lambda \vec{b}$ is perpendicular to $\vec{c}$ $\therefore (\vec{a} + \lambda \vec{b}) \cdot \vec{c} = 0$ i.e., $\vec{a} \cdot \vec{c} + \lambda \vec{b} \cdot \vec{c} = 0$ which gives $8 + \lambda(-4) = 0$ $\lambda = 2$ OR Area of $\Delta ABC = \frac{1}{2} \vec{AB} \times \vec{AC}$ $= \frac{1}{2} -3\hat{i} - 2\hat{j} - 7\hat{k}$ $= \frac{\sqrt{62}}{2}$	I II III I II	$\frac{1}{2}$ 1 $\frac{1}{2}$ 1 1
23.	Determine the interval(s) in which $f(x) = 5x^{3/2} - 3x^{5/2}$, $x > 0$ is increasing.		
Sol.	$f'(x) = \frac{15x^{1/2}}{2} - \frac{15x^{3/2}}{2}$ $f'(x) = 0$ gives $x = 1$ as $x > 0$ Sign of $f'(x)$ $f(x)$ is increasing in $(0, 1)$ or $(0, 1]$	I II III	1 $\frac{1}{2}$ $\frac{1}{2}$

24.	Find the angle between the following pair of lines : $\frac{x-2}{3} = \frac{y+5}{2} = \frac{1-z}{-6} \text{ and } \frac{x-7}{1} = \frac{y}{2} = \frac{6-z}{-2}$		
Sol.	Drs of given lines are $\langle 3, 2, 6 \rangle$ and $\langle 1, 2, 2 \rangle$ Let θ be angle between given lines $\therefore \cos \theta = \frac{3 \times 1 + 2 \times 2 + 6 \times 2}{7 \times 3}$ Thus, $\theta = \cos^{-1} \left(\frac{19}{21} \right)$	I II III	$\frac{1}{2}$ 1 $\frac{1}{2}$
25.	(a) Differentiate x^x with respect to $x \log x$. OR (b) If $y = P \cos ux + Q \sin ux$, show that $\frac{d^2y}{dx^2} + u^2y = 0$.		
Sol.	(a) Let $u = x^x$ and $v = x \log x$ $\frac{du}{dx} = x^x(1 + \log x)$ $\frac{dv}{dx} = 1 + \log x$ $\therefore \frac{du}{dv} = x^x$ OR (b) $\frac{dy}{dx} = -P u \sin ux + Q u \cos ux$ $\frac{d^2y}{dx^2} = -P u^2 \cos ux - Q u^2 \sin ux$ $\frac{d^2y}{dx^2} = -u^2(P \cos ux + Q \sin ux)$ $\therefore \frac{d^2y}{dx^2} + u^2y = 0$	I II III IV I II III IV	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

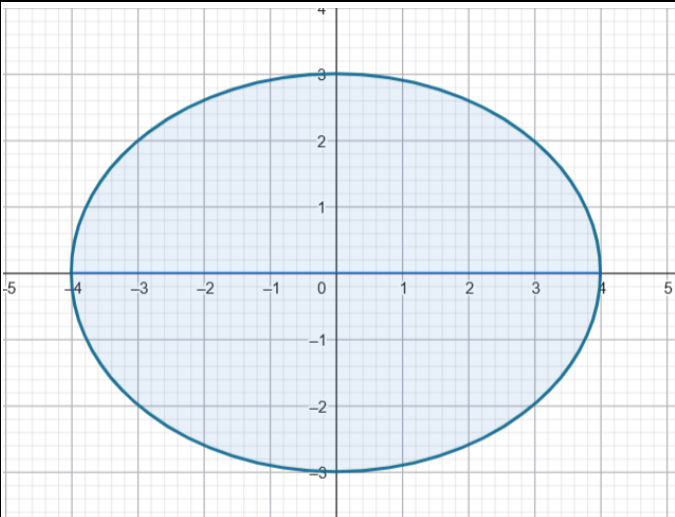
SECTION C			
Q. Numbers 26 to 31 are short answer questions of 3 marks each.			
26.	A thin metallic wire in the shape of a circular ring has its enclosed area increasing at a uniform rate when heated. Show that the rate of change of circumference varies inversely as the radius.		
Sol.	Let r be the radius of circular ring Area $A = \pi r^2$ $\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = K \quad (K > 0 \text{ is a constant}) \dots\dots\dots (i)$ Circumference $C = 2\pi r$ $\frac{dC}{dt} = 2\pi \frac{dr}{dt} = 2\pi \frac{K}{2\pi r} \quad \text{using (i)}$ $= \frac{K}{r}$ Thus, rate of change of circumference varies inversely with radius.	I II III IV	$\frac{1}{2}$ 1 1 $\frac{1}{2}$
27.	Solve the following Linear Programming Problem graphically : Maximize $Z = \frac{2x}{5} + \frac{3y}{10}$ subject to constraints $2x + y \leq 1000$ $x + y \leq 800$ $x, y \geq 0.$		
Sol.		I	Correct Lines and Shading $1\frac{1}{2}$

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	$(b) I = \int_0^2 \frac{1}{\sqrt{x^2+2x+3}} dx$ $= \int_0^2 \frac{1}{\sqrt{(x+1)^2+2}} dx$ $= \left[\log(x+1+\sqrt{x^2+2x+3}) \right]_0^2$ $= \log(3+\sqrt{11}) - \log(1+\sqrt{3}) \text{ or } \log\left(\frac{3+\sqrt{11}}{1+\sqrt{3}}\right)$	I	1
		II	1
		III	1
30.	(a) A die is rolled. Consider events : $A = \{1, 2, 5\}$, $B = \{3, 5\}$, $C = \{2, 3, 4, 5\}$ and hence find : (i) $P(A C)$ and $P(C A)$ (ii) $P(A \cap B C)$ and $P(A \cup B C)$ OR (b) A box contains 6 cards numbered 1 to 6. A student is asked to pick up two cards, one by one after replacement and note down the numbers on the cards. Let A be the event of getting sum of the numbers on two cards as 10, and B, the event of a number other than 4 on the first card selected. Find $P(A \text{ and } B)$ and find whether the events A and B are independent events or not.		
Sol.	(a) (i) $P(A C) = \frac{n(A \cap C)}{n(C)} = \frac{2}{4} \text{ or } \frac{1}{2}$ and $P(C A) = \frac{n(C \cap A)}{n(A)} = \frac{2}{3}$ (ii) $P(A \cap B C) = \frac{n(A \cap B \cap C)}{n(C)} = \frac{1}{4}$ and $P(A \cup B C) = \frac{n[(A \cup B) \cap C]}{n(C)} = \frac{3}{4}$ OR (b) $A = \{(4, 6), (6, 4), (5, 5)\}$ or $n(A) = 3$ $n(B) = 30$ $n(A \cap B) = 2$ $P(A \cap B) = \frac{2}{36}$ $P(A) = \frac{3}{36} = \frac{1}{12}$ $P(B) = \frac{30}{36} = \frac{5}{6}$ $P(A) \cdot P(B) = \frac{5}{72} \neq P(A \cap B)$ Hence A and B are not independent	I	1
		II	$\frac{1}{2}$
		III	1
		IV	$\frac{1}{2}$
		I	$\frac{1}{2}$
		II	$\frac{1}{2}$
		III	$\frac{1}{2}$
		IV	$\frac{1}{2}$
		V	$\frac{1}{2}$
		VI	$\frac{1}{2}$

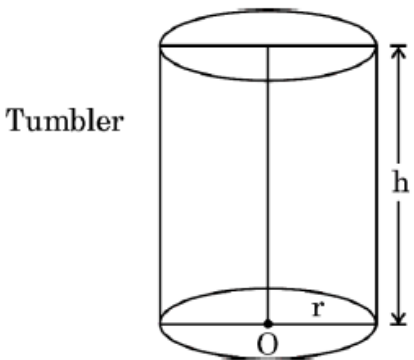
31.	(a) Let three toys A, B and C be placed in the same straight line. If the position vectors of A, B and C are $55\hat{i} - 2\hat{j}$, $5\hat{i} + 8\hat{j}$ and $a\hat{i} - 52\hat{j}$ respectively, find the value of 'a'. OR (b) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors, then prove that $ \vec{a} - \vec{b} ^2 + \vec{b} - \vec{c} ^2 + \vec{c} - \vec{a} ^2 \leq 9.$		
Sol.	(a) $\overrightarrow{AB} = -50\hat{i} + 10\hat{j} \quad \text{and} \quad \overrightarrow{BC} = (a - 5)\hat{i} - 60\hat{j}$ As $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are collinear vectors $\frac{-50}{a-5} = \frac{10}{-60}$ $a = 305$ OR (b) $\vec{a} = \vec{b} = \vec{c} = 1$ Now, consider $ \vec{a} - \vec{b} ^2 + \vec{b} - \vec{c} ^2 + \vec{c} - \vec{a} ^2$ $= 2 \vec{a} ^2 + 2 \vec{b} ^2 + 2 \vec{c} ^2 - 2\vec{a} \cdot \vec{b} - 2\vec{b} \cdot \vec{c} - 2\vec{c} \cdot \vec{a}$ $= 3 \vec{a} ^2 + 3 \vec{b} ^2 + 3 \vec{c} ^2 - (\vec{a} ^2 + \vec{b} ^2 + \vec{c} ^2 + 2\vec{a} \cdot \vec{b} + 2\vec{b} \cdot \vec{c} + 2\vec{c} \cdot \vec{a})$ $= 9 - \vec{a} + \vec{b} + \vec{c} ^2 \leq 9$ $ \vec{a} - \vec{b} ^2 + \vec{b} - \vec{c} ^2 + \vec{c} - \vec{a} ^2 \leq 9$	I II III I II III IV	1 1 1 $\frac{1}{2}$ $1\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
SECTION D			
Q. Numbers 32 to 35 are long answer questions of 5 marks each.			
32.	Find cost (per kg) of each fertilizer A, B and C that the farmer needs to buy, such that 1 kg each of fertilizer A and C added to 2 kg of B costs him ₹ 400. Also, cost of each kg of fertilizer B and C added together is equal to cost of 1 kg of fertilizer A. However, cost of 3 kg of fertilizer B added to ₹ 200 is the same as cost of 1 kg of fertilizer A and C together. Use matrix method to find the solution.		
Sol.	Let cost of one Kg of fertilizers A, B and C be ₹x, ₹y and ₹z respectively $\left. \begin{array}{l} x + 2y + z = 400 \\ -x + y + z = 0 \\ x - 3y + z = 200 \end{array} \right\}$ Let $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 400 \\ 0 \\ 200 \end{bmatrix}$ $A = 10 \neq 0 \Rightarrow A^{-1}$ exists	I II	1 1

	System becomes $AX = B$. So, $X = A^{-1}B$ $adjA = \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}$ $A^{-1} = \frac{1}{10} \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}$ $X = \frac{1}{10} \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix} \begin{bmatrix} 400 \\ 0 \\ 200 \end{bmatrix}$ $X = \begin{bmatrix} 180 \\ 40 \\ 140 \end{bmatrix}$ $\therefore$ cost of one Kg of fertilizers A, B and C is ₹180, ₹40 and ₹140 respectively	III	$1\frac{1}{2}$
33.	(a) Find the foot of the perpendicular from the point (0, 2, 3) on the line $\frac{-x-3}{-5} = \frac{1-y}{-2} = \frac{3z+12}{9}$ and hence find the length of the perpendicular. OR (b) Find the value of p if the shortest distance between the lines $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k}) \text{ and}$ $\vec{r} = (p\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$ is $\frac{3}{\sqrt{2}}$ units.		
Sol.	(a) Given line is $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3} = \lambda$ Any general point on line l is $(5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$ Drs of given line are $\langle 5, 2, 3 \rangle$ Drs of perpendicular line are $\langle 5\lambda - 3, 2\lambda - 1, 3\lambda - 7 \rangle$ As lines are perpendicular $\therefore 5(5\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 7) = 0$ gives, $\lambda = 1$ $\therefore$ coordinates of foot of perpendicular are $(2, 3, -1)$ length of perpendicular = $\sqrt{21}$ OR (b) Let $\vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{a}_2 = p\hat{i} - \hat{j} - \hat{k}$, and $\vec{b}_1 = \hat{i} - \hat{j} + \hat{k}, \vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k}$ Here $(\vec{a}_2 - \vec{a}_1) = (p-1)\hat{i} - 3\hat{j} - 2\hat{k}$ $\vec{b}_1 \times \vec{b}_2 = -3\hat{i} + 3\hat{k}$ $\text{S.D.} = \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{ \vec{b}_1 \times \vec{b}_2 }$	I II III IV V VI I II	1 $\frac{1}{2}$ $\frac{1}{2}$ 1 1 1 1 $1\frac{1}{2}$

	$\frac{-3p-3}{3\sqrt{2}} = \pm \frac{3}{\sqrt{2}}$ gives $p = -4, 2$	III	1½
		IV	1
34.	Sketch the curve described by $\{(x, y) : 9x^2 + 16y^2 = 144\}$ and find the area of the region enclosed by it, using integration.		
Sol.	 Required area $= 4 \int_0^4 \frac{3}{4} \sqrt{16 - x^2} dx$ $= 3 \left[\frac{x}{2} \sqrt{16 - x^2} + \frac{16}{2} \sin^{-1} \frac{x}{4} \right]_0^4$ $= 12\pi$	I	1½
		II	1½
		III	1
		IV	1
35.	(a) If $y\sqrt{x^2+1} = \log \sqrt{x^2+1} - x$, show that $(x^2 + 1) \frac{dy}{dx} + xy + 1 = 0.$ OR (b) Find the differential of $x^{\cot x} + \frac{2x^2 - 3}{2x^2 - x + 2}$ with respect to x.		
Sol.	(a) Differentiating both sides with respect to x we get, $\sqrt{x^2 + 1} \frac{dy}{dx} + y \cdot \frac{x}{\sqrt{x^2 + 1}} = \frac{1}{\sqrt{x^2 + 1}} \cdot \frac{x}{\sqrt{x^2 + 1}} - 1$ $(x^2 + 1) \frac{dy}{dx} + y \cdot x = \frac{x}{\sqrt{x^2 + 1}} - \sqrt{x^2 + 1}$	I	2 + 2
		II	1

	Note: In case student, considers $\sqrt{x^2 + 1} - x$ as part of log and writes following solution Differentiating both sides with respect to x we get, $\sqrt{x^2 + 1} \frac{dy}{dx} + y \cdot \frac{x}{\sqrt{x^2 + 1}} = \frac{1}{\sqrt{x^2 + 1} - x} \cdot \left(\frac{x}{\sqrt{x^2 + 1}} - 1 \right)$ $\sqrt{x^2 + 1} \frac{dy}{dx} + y \cdot \frac{x}{\sqrt{x^2 + 1}} = \frac{1}{\sqrt{x^2 + 1} - x} \cdot \left(\frac{x - \sqrt{x^2 + 1}}{\sqrt{x^2 + 1}} \right)$ $\sqrt{x^2 + 1} \frac{dy}{dx} + y \cdot \frac{x}{\sqrt{x^2 + 1}} = \frac{-1}{\sqrt{x^2 + 1}}$ gives $(x^2 + 1) \frac{dy}{dx} + y \cdot x = -1 \quad \text{or} \quad (x^2 + 1) \frac{dy}{dx} + xy + 1 = 0$ OR (b) Let $u = x^{\cot x}$ and $v = \frac{2x^2 - 3}{2x^2 - x + 2}$ $\log u = \cot x \cdot \log x$ $\frac{1}{u} \frac{du}{dx} = \frac{\cot x}{x} + \log x (-\operatorname{cosec}^2 x)$ $\frac{du}{dx} = x^{\cot x} \left(\frac{\cot x}{x} - \operatorname{cosec}^2 x \cdot \log x \right)$ $\frac{dv}{dx} = \frac{(2x^2 - x + 2)(4x) - (2x^2 - 3)(4x - 1)}{(2x^2 - x + 2)^2}$ $= \frac{-2x^2 + 20x - 3}{(2x^2 - x + 2)^2}$ Required derivative = $x^{\cot x} \left(\frac{\cot x}{x} - \operatorname{cosec}^2 x \cdot \log x \right) + \frac{-2x^2 + 20x - 3}{(2x^2 - x + 2)^2}$	I II III IV I II III IV V VI VII	2 1 1 1 $\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$ 1 1 $\frac{1}{2}$
	SECTION E This section (Q. 36 to 38) has 3 case study based questions of 4 marks each.		

36.	There are three types of vaccines A_1, A_2, A_3, available in the market to protect the population of the country from spread of certain infection. According to a survey conducted, it was found that 25% of the population was given Vaccine A_1, 35% of the population was given Vaccine A_2 and 40% of the population was given Vaccine A_3. The survey also stated that the probabilities that Vaccines A_1, A_2 and A_3 would protect against the infection were 60%, 55% and 50% respectively. Based on the above information, answer the following questions : Find the probability that : (i) The person taking vaccine A_2 will get infected. 1 (ii) If a person is chosen randomly, he/she will be protected from the infection. 1 (iii) (a) The person was given Vaccine A_1, given that the randomly chosen person is infected. 2 OR (iii) (b) The person was given Vaccine A_3, given that the randomly chosen person is not infected. 2		
Sol.	(i) $P(\text{Person taking vaccine } A_2 \text{ will be infected}) = 45\% \text{ or } 0.45$ (ii) $P(\text{Person is protected from infection})$ $= \frac{25}{100} \cdot \frac{60}{100} + \frac{35}{100} \cdot \frac{55}{100} + \frac{40}{100} \cdot \frac{50}{100}$ $= \frac{5425}{10000} \text{ or } 0.5425$ (iii) (a) $P(A_1/\text{person is infected}) = \frac{\frac{25}{100} \cdot \frac{40}{100}}{\frac{25}{100} \cdot \frac{40}{100} + \frac{35}{100} \cdot \frac{45}{100} + \frac{40}{100} \cdot \frac{50}{100}}$ $= \frac{1000}{4575} \text{ or } \frac{40}{183}$ OR (iii) (b) $P(A_3/\text{person is not infected}) = \frac{\frac{40}{100} \cdot \frac{50}{100}}{\frac{25}{100} \cdot \frac{60}{100} + \frac{35}{100} \cdot \frac{55}{100} + \frac{40}{100} \cdot \frac{50}{100}}$ $= \frac{2000}{5425} \text{ or } \frac{80}{217}$	I I II I II I II	1 $\frac{1}{2}$ $\frac{1}{2}$ $1\frac{1}{2}$ $\frac{1}{2}$ $1\frac{1}{2}$ $\frac{1}{2}$

37.	Case Study - 2 A company produces cylindrical tumblers, open from the top. Since they want uniformity in the product, they fix the surface area of the tumblers produced.  Based on the above information, answer the following questions : If for a tumbler, V is its volume, h the height and r the radius of the circular base, then : (i) Differentiate its volume with respect to radius of the base, where the surface area is constant. 2 (ii) If the company wants to maximize the volume of each tumbler, then establish a relation between its height and the radius of the base. 2		
Sol.	(i) Surface area $S = 2\pi rh + \pi r^2$ $V = \pi r^2 h = \frac{\pi r^2 (S - \pi r^2)}{2\pi r} = \frac{(Sr - \pi r^3)}{2}$ $\frac{dV}{dr} = \frac{(S - 3\pi r^2)}{2}$ (ii) $\frac{dV}{dr} = 0$ gives $S = 3\pi r^2$ $\frac{d^2V}{dr^2} = -3\pi r < 0$ V is maximum when $S = 3\pi r^2$ Thus, $h = \frac{3\pi r^2 - \pi r^2}{2\pi r} = r$	I II III I II III	$\frac{1}{2}$ 1 $\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$

38.	Case Study – 3 A school wants the students of class XII to do a project on ‘Sustainability’ keeping the world environment in mind. They select the student participants on the basis of an essay writing competition. 7 students out of 80 are selected for the project and are categorized into two sets such that : Girl students belong to Set $A = \{G_1, G_2, G_3, G_4\}$, Boy students belong to Set $B = \{B_1, B_2, B_3\}$. Based on the above information, answer the following questions : (i) How many relations are possible from Set $A \rightarrow$ Set B ? 1 (ii) Let R be a relation from $A \rightarrow B$ such that $R = \{(G_1, B_1), (G_2, B_2), (G_3, B_2), (G_4, B_3), (G_1, B_2)\}$. Is R an injective function ? Justify your answer. 1 (iii) (a) Let the relation R from $A \rightarrow A$ be such that $R = \{(x, y), x, y \in A, x \text{ and } y \text{ are students from the same colony in the city}\}$ Verify if R is an equivalence relation. 2 OR (iii) (b) Verify if any function $f : B \rightarrow A$ is bijective. Give reason to support your answer. 2		
Sol.	(i) Number of relations from A to B is 2^{12} or 4096 (ii) No R is not injective It is not a function as G_1 has two images B_1 and B_2 (iii) (a) R is reflexive as $(x, x) \in R \forall x \in A$ because x and x are students from same colony Let $(x, y) \in R$ so x and y are students from same colony Hence y and x are students from same colony Thus, $(y, x) \in R$ $\therefore R$ is symmetric Let $(x, y) \in R$ and $(y, z) \in R$ so x and y are students from same colony, y and z are students from same colony	I I II I II III	1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

	Thus, x and z are students from same colony $\therefore (x, z) \in R$ Thus, R is transitive As R is reflexive, symmetric and transitive, hence R is an equivalence relation OR	IV	$\frac{1}{2}$
(iii) (b)	Any function f from B to A will not be bijective	I	1
	As f from B to A cannot be surjective because $n(B) < n(A)$ or As A and B are finite and $n(A) \neq n(B)$	II	1