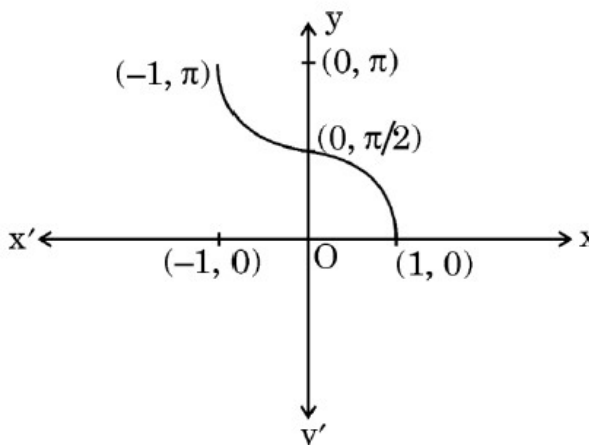


**MATHEMATICS CLASS XII (Subject Code–
041) (PAPER CODE: 65/4/3)
SOLUTIONS**

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Steps	Marks
	SECTION A		
	Q. Number 1 to 20 are multiple choice questions of 1 mark each.		
1.	The following graph represents :  (A) $y = \cos^{-1} x$ (B) $y = \sec^{-1} x$ (C) $y = \sin^{-1} x$ (D) $y = \tan^{-1} x$		
Ans.	(A) $y = \cos^{-1} x$		1
2.	Let $A = [a_{ij}]$ be a 2×2 matrix whose elements are given by $a_{ij} = \frac{(i + 2j)^2}{3}$. Then A' is given by : (A) $\begin{bmatrix} 12 & \frac{25}{3} \\ \frac{16}{3} & 3 \end{bmatrix}$ (B) $\begin{bmatrix} 12 & \frac{16}{3} \\ \frac{25}{3} & 3 \end{bmatrix}$ (C) $\begin{bmatrix} 3 & \frac{25}{3} \\ \frac{16}{3} & 12 \end{bmatrix}$ (D) $\begin{bmatrix} 3 & \frac{16}{3} \\ \frac{25}{3} & 12 \end{bmatrix}$		
Ans.	(D) $\begin{bmatrix} 3 & \frac{16}{3} \\ \frac{25}{3} & 12 \end{bmatrix}$		1

3.	The greatest integer function, $f(x) = [x]$, $0 < x < 3$ is not differentiable at how many points ? (A) At only one point (B) At only two points (C) At no point (D) At three points		
Ans.	(B) At only two points		1
4.	The area of the region bounded by the curve $y = x$ and x-axis, between $x = 0$ and $x = 2$ is : (A) 2 sq. units (B) $\frac{1}{2}$ sq. unit (C) 1 sq. unit (D) 4 sq. units		
Ans.	(A) 2 sq. units		1
5.	If $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is: (A) 0 (B) -2 (C) -1 (D) 2		
Ans.	(D) 2		1
6.	A cylindrical tank is being filled with sand at a rate of $314 \text{ m}^3/\text{h}$. If the radius of the tank is 10 m, then the height of sand in the tank increases at the rate of : (A) 1.1 m/h (B) 1 m/h (C) π m/h (D) $\frac{\pi}{2}$ m/h		
Ans.	(B) 1 m/h		1
7.	The principal value of $\sec^{-1}(\sqrt{2}) + 2 \operatorname{cosec}^{-1}(-\sqrt{2})$ is (A) $-\frac{\pi}{2}$ (B) $-\frac{\pi}{4}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$		
Ans.	(B) $-\frac{\pi}{4}$		1

8.	If points (2, 3), (0, 4) and (p, 2) are collinear, then the value of p is : (A) $\frac{4}{7}$ (B) $\frac{-3}{7}$ (C) 4 (D) - 4		
Ans.	(C) 4		1
9.	Differential of e^{e^x} with respect to x is : (A) $\log x$ (B) e^{e^x} (C) $e^x \cdot e^{e^x}$ (D) $(e^x)^2$		
Ans.	(C) $e^x \cdot e^{e^x}$		1
10.	$\int \frac{dx}{\sqrt{25-16x^2}}$ is equal to : (A) $\frac{1}{5} \sin^{-1} 4x + c$ (B) $\frac{1}{25} \sin^{-1} 16x + c$ (C) $\frac{1}{4} \sin^{-1} \frac{4x}{5} + c$ (D) $\frac{1}{16} \sin^{-1} \frac{4x}{5} + c$		
Ans.	(C) $\frac{1}{4} \sin^{-1} \frac{4x}{5} + c$		1
11.	If $\int_0^1 \frac{dx}{e^x + e^{-x}} = \tan^{-1} e + k$, then the value of k is : (A) e (B) $\frac{\pi}{4}$ (C) 0 (D) $-\frac{\pi}{4}$		
Ans.	(D) $-\frac{\pi}{4}$		1
12.	The sum of the order and the degree of the differential equation $y = x \frac{dy}{dx} + 3 \left[2 - \left(\frac{dy}{dx} \right)^2 \right]$ is : (A) not defined (B) 3 (C) 1 (D) 2		
Ans.	(B) 3		1

16.	Which of the following is not a Linear Differential Equation? $(A) (1 + x^2)dy + 2xydx = \cot x dx$ $(B) y + \frac{d}{dx}(xy) = x(\sin x + \log x)$ $(C) x(1 + y^2)dx - y(1 + x^2)dy = 0$ $(D) ydx - (x + 3y^2)dy = 0$		
Ans.	$(C) x(1 + y^2)dx - y(1 + x^2)dy = 0$		1
17.	The region represented by the system of inequations $3x + y \geq 3$, $2x - y \geq -5$, $x, y \geq 0$ is : (A) unbounded in 1 st quadrant (B) bounded in 1 st quadrant (C) unbounded in 2 nd quadrant (D) bounded in 2 nd quadrant		
Ans.	(A) unbounded in 1 st Quadrant		1
18.	The probability that a particular item is available in three shops A, B and C is $\frac{4}{5}, \frac{3}{4}$ and $\frac{2}{3}$ respectively. If a person visits all the three shops to buy the item, then what is the probability that it will be available in at least one of the shops ? $(A) \frac{59}{60}$ $(B) 1$ $(C) \frac{1}{60}$ $(D) \frac{9}{60}$		
Ans.	$(A) \frac{59}{60}$		1
19.	Assertion (A): If A and B are two square matrices such that AB and BA are defined, then it is not necessary that $AB = BA$. Reason (R): Product of two diagonal matrices of same order is commutative.		
Ans.	(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).		1

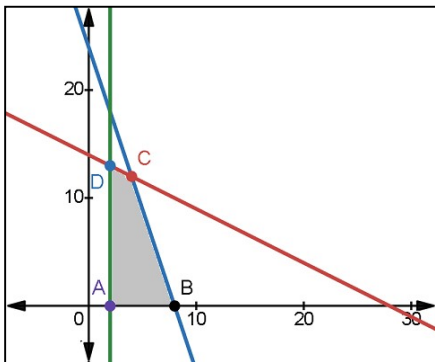
20.	Assertion (A): A function $f : \mathbb{N} \rightarrow \mathbb{N}$ given by $f(x) = x^3 + 2, \forall x \in \mathbb{N}$ is one-one but not onto. Reason (R): Since $\forall y \in \mathbb{N}$ (codomain), there does not exist $x = (y - 2)^{\frac{1}{3}}$ in $\mathbb{N}$ (Domain) such that $f(x) = x^3 + 2 = y$.		
Ans.	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).		1
SECTION B Q. Numbers 21 to 25 are very short answer questions of 2 marks each.			
21. (a)	Show that the function $f(x) = \begin{cases} \frac{\cos x}{-x + \frac{\pi}{2}}, & x \neq \frac{\pi}{2} \\ 1, & x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$.		
Sol.	For showing: $\lim_{x \rightarrow \frac{\pi}{2}} f(x) = 1$ For showing: $f\left(\frac{\pi}{2}\right) = 1$ $\therefore f(x)$ is continuous at $x = \frac{\pi}{2}$.	I II III	1 $\frac{1}{2}$ $\frac{1}{2}$
OR			
21. (b)	Find whether the function $f(x) = \begin{cases} x - 1, & x < 2 \\ 2x - 3, & x \geq 2 \end{cases}$ at $x = 2$ is differentiable or not.		
Sol.	Getting $LHD = 1$ Getting $RHD = 2$ $\therefore LHD \neq RHD$, so $f(x)$ is not differentiable at $x = 2$.	I II III	1 $\frac{1}{2}$ $\frac{1}{2}$

22. (a)	Let two rods placed on the ground be represented by vectors $4\hat{i} - \hat{j} + 3\hat{k}$ and $-2\hat{i} + \hat{j} - 2\hat{k}$. Find a vector representing a flag-post of height 5 m that has to be erected perpendicular to both the rods.		
Sol.	Let $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$ $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 3 \\ -2 & 1 & -2 \end{vmatrix} = -\hat{i} + 2\hat{j} + 2\hat{k}$ Now, Required vector $= 5 \left(\frac{\vec{a} \times \vec{b}}{ \vec{a} \times \vec{b} } \right) = \begin{cases} -\frac{5}{3}\hat{i} + \frac{10}{3}\hat{j} + \frac{10}{3}\hat{k} \\ or \\ \frac{5}{3}\hat{i} - \frac{10}{3}\hat{j} - \frac{10}{3}\hat{k} \end{cases}$ Alternative approach: Let $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$ Let required vector be $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$ so, $25 = x^2 + y^2 + z^2, 4x - y + 3z = 0, -2x + y - 2z = 0$ on solving we get, $z = -2x, y = z \Rightarrow \frac{100}{9} = z^2 \Rightarrow z = \pm \frac{10}{3}$ $\therefore$ Required vector, $\vec{p} = \begin{cases} -\frac{5}{3}\hat{i} + \frac{10}{3}\hat{j} + \frac{10}{3}\hat{k} \\ or \\ \frac{5}{3}\hat{i} - \frac{10}{3}\hat{j} - \frac{10}{3}\hat{k} \end{cases}$	I II	1 1 (For any one result)
OR			
22. (b)	A unit vector $\vec{a}$ is such that it makes an angle $\frac{\pi}{4}$ with x-axis, $\frac{\pi}{3}$ with y-axis and an acute angle θ with z-axis. Find θ and the components of $\vec{a}$.		

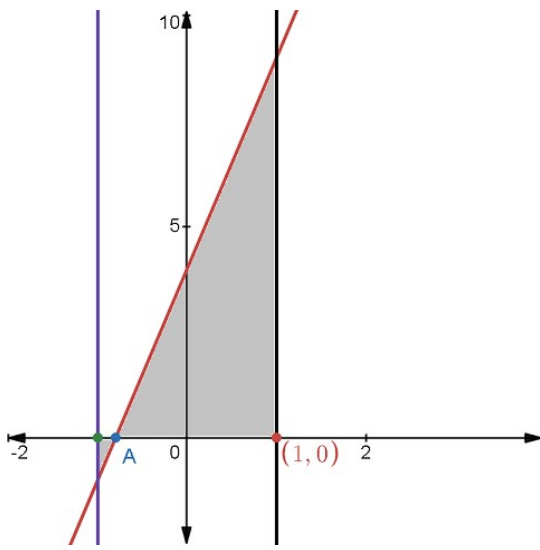
	$\cos^2\left(\frac{\pi}{4}\right) + \cos^2\left(\frac{\pi}{3}\right) + \cos^2\theta = 1 \Rightarrow \cos\theta = \frac{1}{2} \text{ (As, } \theta \text{ is acute)}$ $\Rightarrow \theta = \frac{\pi}{3} \text{ \& Components of } \vec{a} = \frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}$	I	1
		II	1
23.	Find the interval(s) in which the function $f(x) = \frac{x}{3} + \frac{3}{x}, x \neq 0$ is increasing.		
Sol.	$f'(x) = \frac{1}{3} - \frac{3}{x^2} = \frac{x^2 - 9}{3x^2}$ For increasing function, $f'(x) = 0 \Rightarrow x^2 - 9 = 0$ $\Rightarrow x = -3, 3$ $\therefore f(x)$ is increasing on $x \in (-\infty, -3) \cup (3, \infty)$.	I	1
		II	$\frac{1}{2}$
		III	$\frac{1}{2}$
24.	If in a parallelogram $PQRS, \overrightarrow{PQ} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\overrightarrow{QR} = \hat{i} - 2\hat{j} + \hat{k}$, then find the unit vectors parallel to the diagonals $\overrightarrow{PR}$ and $\overrightarrow{SQ}$.		
Sol.	$\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR} = 3\hat{i} + \hat{j} + 2\hat{k},$ So, unit vector parallel to $\overrightarrow{PR} = \frac{3}{\sqrt{14}}\hat{i} + \frac{1}{\sqrt{14}}\hat{j} + \frac{2}{\sqrt{14}}\hat{k}$ Now, $\overrightarrow{SQ} = \overrightarrow{PQ} + \overrightarrow{SP} = \overrightarrow{PQ} - \overrightarrow{PS} = \overrightarrow{PQ} - \overrightarrow{QR} = \hat{i} + 5\hat{j}$ Thus, unit vector parallel to $\overrightarrow{SQ} = \frac{1}{\sqrt{26}}\hat{i} + \frac{5}{\sqrt{26}}\hat{j}$	I	1
		II	1
25.	Evaluate: $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\left(\frac{-\pi}{2}\right)\right) + \tan^{-1}\left(\tan\frac{2\pi}{3}\right)$		2
Sol.	$\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\left(\frac{-\pi}{2}\right)\right) + \tan^{-1}\left(\tan\frac{2\pi}{3}\right)$ $= -\frac{\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4} - \frac{\pi}{3}$ $= \frac{-5\pi}{12}$	I	$1\frac{1}{2}$ (For any 3 correct result)
		II	$\frac{1}{2}$

SECTION C			
Q. Numbers 26 to 31 are short answer questions of 3 marks each.			
26. (a)	Find a point on the line $\frac{x-2}{3} = \frac{1-y}{2} = \frac{z-3}{2}$ at a distance of $\sqrt{2}$ units from the point (1, 2, 3).		
	Let $\frac{x-2}{3} = \frac{y-1}{-2} = \frac{z-3}{2} = k$ A general point P on the line be $(3k+2, -2k+1, 2k+3)$ It is given that $PQ = \sqrt{2}$ where $Q(1,2,3)$, so $PQ = \sqrt{(3k+1)^2 + (-2k-1)^2 + (2k)^2} = \sqrt{2}$ $\Rightarrow 17k^2 + 10k = 0$ $\therefore k = 0 \text{ or } \frac{-10}{17}$ Thus, required points are $(2,1,3) \& \left(\frac{4}{17}, \frac{37}{17}, \frac{31}{17}\right)$	I II III IV	1 1 $\frac{1}{2}$ $\frac{1}{2}$
OR			
26. (b)	Find the shortest distance between the lines: $\vec{r} = (4+\lambda)\hat{i} + (2\lambda-1)\hat{j} - 3\lambda\hat{k}$ $\vec{r} = (1+2\mu)\hat{i} + (2-\mu)\hat{k} + (4\mu-1)\hat{j}$		
	Here, $\vec{a}_1 = 4\hat{i} - \hat{j}, \vec{b}_1 = \hat{i} + 2\hat{j} - 3\hat{k}, \vec{a}_2 = \hat{i} - \hat{j} + 2\hat{k}, \vec{b}_2 = 2\hat{i} + 4\hat{j} - 5\hat{k}$ Now, $\vec{a}_2 - \vec{a}_1 = -3\hat{i} + 2\hat{k}, \quad \vec{b}_1 \times \vec{b}_2 = 2\hat{i} - \hat{j} \text{ and } \vec{b}_1 \times \vec{b}_2 = \sqrt{5}$ $\text{Required distance} = \frac{ (-3\hat{i} + 2\hat{k}) \cdot (2\hat{i} - \hat{j}) }{\sqrt{5}} = \frac{6}{\sqrt{5}} \text{ or } \frac{6\sqrt{5}}{5}$	I II III	1 $1\frac{1}{2}$ $\frac{1}{2}$

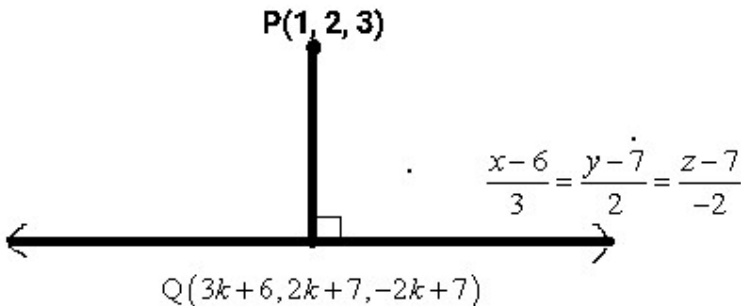
27.	If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then compute $A^2 - 5A + 4I$.		
Sol.	$A^2 = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$ $A^2 - 5A + 4I = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ $A^2 - 5A + 4I = \begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$	I II III	$1\frac{1}{2}$ ($\frac{1}{2}$ for any three correct elements) 1 $\frac{1}{2}$
28.	In a school, the probability of holding a debate competition is $\frac{1}{3}$ and that of a quiz competition is $\frac{2}{3}$. In the two participating teams, A has 4 girls and 6 boys and B has 7 girls and 3 boys. If a debate competition is held, the students are selected from team A and for the quiz competition they are selected from team B. If only two students are to be chosen from the teams, then find the probability that one will be a girl and the other a boy.		
Sol :	Let D be the event a debate competition is held, Q be the event a quiz competition is held and E be the event that one girl & one boy are chosen. $P(E D) = 2 \times \frac{4}{10} \times \frac{6}{9} = \frac{8}{15}, P(E Q) = 2 \times \frac{7}{10} \times \frac{3}{9} = \frac{7}{15}$ $\text{Now, } P(E) = \frac{1}{3} \times \frac{8}{15} + \frac{2}{3} \times \frac{7}{15} = \frac{22}{45}$	I II	1+1 1

29.	Solve the following Linear programming problem graphically: Maximize $Z = 20x + 10y$ subject to the constraints $x + 2y \leq 28$ $3x + y \leq 24$ $x \geq 2$ $x \geq 0, y \geq 0$												
Sol.	 <table><thead><tr><th>Corner point</th><th>Value of $Z=20x+10y$</th></tr></thead><tbody><tr><td>A (2, 0)</td><td>40</td></tr><tr><td>B (8,0)</td><td>160</td></tr><tr><td>C (4, 12)</td><td>200</td></tr><tr><td>D (2, 13)</td><td>170</td></tr></tbody></table> Thus, maximum value of Z is 200 at $x = 4$ and $y = 12$	Corner point	Value of $Z=20x+10y$	A (2, 0)	40	B (8,0)	160	C (4, 12)	200	D (2, 13)	170	I II III	1 ½ For correct graph & shading 1 ½
Corner point	Value of $Z=20x+10y$												
A (2, 0)	40												
B (8,0)	160												
C (4, 12)	200												
D (2, 13)	170												
30. (a)	Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. A function $f : A \rightarrow B$ is defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Find whether f is one-one and onto.												
Sol.	Let $f(a) = f(b)$ for some $a, b \in \mathbb{R} - \{3\}$ $\text{so, } \left(\frac{a-2}{a-3}\right) = \left(\frac{b-2}{b-3}\right) \Rightarrow ab - 2b - 3a + 6 = ab - 3b - 2a + 6$ $\Rightarrow a = b, \text{ Thus } f \text{ is one-one function.}$ Now, let us assume for some $y \in \mathbb{R} - \{1\}$, $y = f(x) = \frac{x-2}{x-3}$ For getting, $x = \frac{3y-2}{y-1}$ Range = $\mathbb{R} - \{1\}$ = co-domain, Thus f is onto function.	I II III IV	1 ½ 1 ½										
30. (b)	Let n be a fixed positive integer. A relation R is defined in set $\mathbb{Z}$ such that $R = \{(x, y) : (x - y) \text{ is divisible by } n, x, y \in \mathbb{Z}\}$. Determine if R is an equivalence relation.												

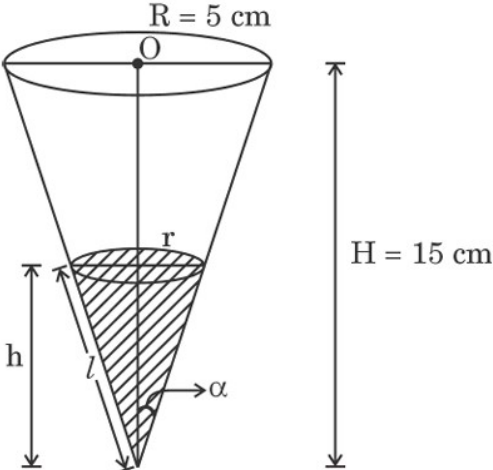
31. (b)	Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$ with respect to $\cos^{-1} x^2$.		
Sol.	Let $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$ & put $x^2 = \cos \theta$ $y = \tan^{-1}\left(\frac{\sqrt{1+\cos \theta}-\sqrt{1-\cos \theta}}{\sqrt{1+\cos \theta}+\sqrt{1-\cos \theta}}\right)$ $y = \tan^{-1}\left(\frac{\cos\left(\frac{\theta}{2}\right)-\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)+\sin\left(\frac{\theta}{2}\right)}\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\right)$ Thus, $y = \left(\frac{\pi}{4}-\frac{\theta}{2}\right) \Rightarrow \frac{dy}{d\theta} = -\frac{1}{2}$	I II III	$\frac{1}{2}$ $1\frac{1}{2}$ 1
SECTION D			
Q. Numbers 32 to 35 are long answer questions of 5 marks each.			
32. (a)	Find : $\int \frac{x}{(x-1)(x^2+4)} dx$		
	Let $\frac{x}{(x-1)(x^2+4)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+4}$ on solving we get, $A = \frac{1}{5}, B = \frac{-1}{5}, C = \frac{4}{5}$ so, $\int \frac{x}{(x-1)(x^2+4)} dx = \frac{1}{5} \int \frac{dx}{x-1} - \frac{1}{10} \int \frac{2x dx}{x^2+4} + \frac{4}{5} \int \frac{dx}{x^2+4}$ $\int \frac{x}{(x-1)(x^2+4)} dx = \frac{1}{5} \log x-1 - \frac{1}{10} \log(x^2+4) + \frac{2}{5} \tan^{-1}\left(\frac{x}{2}\right) + c$	I II III	1 $1\frac{1}{2}$ $2\frac{1}{2}$
OR			
32. (b)	Evaluate : $\int_0^1 \frac{x \tan^{-1} x}{(1+x^2)^{\frac{3}{2}}} dx$.		

	Let $x = \tan t \Rightarrow dx = \sec^2 t dt$ $I(\text{Let}) = \int_0^1 \frac{x \tan^{-1} x}{(1+x^2)^{\frac{3}{2}}} dx = \int_0^{\frac{\pi}{4}} \frac{t \tan t}{\sec^3 t} \sec^2 t dt = \int_0^{\frac{\pi}{4}} t \sin t dt$ $I = (-t \cos t + \sin t) \Big _0^{\frac{\pi}{4}}$ $I = \frac{-\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} \text{ or } \frac{-\pi+4}{4\sqrt{2}}$	I	1
		II	2
		III	1
		IV	1
33.	Using integration, find the area of the region bounded by $y = 5x + 4$, $y = 0$, $x = -1$ and $x = 1$.		
Sol.	 Required Area = $\left \int_{-1}^{-\frac{4}{5}} y dx \right + \int_{-\frac{4}{5}}^1 y dx$ Area = $-\int_{-1}^{-\frac{4}{5}} (5x + 4) dx + \int_{-\frac{4}{5}}^1 (5x + 4) dx$ Area = $-\left(\frac{(5x+4)^2}{10} \right)_{-1}^{-\frac{4}{5}} + \left(\frac{(5x+4)^2}{10} \right)_{-\frac{4}{5}}^1 = \frac{1}{10} + \frac{81}{10} = \frac{41}{5} \text{ or } \frac{41}{5} \text{ sq. units}$	I	1 (For correct graph)
		II	1
		III	1
		IV	2

34. (a)	Solve the differential equation: $ye^y dx = (y^3 + 2xe^y)dy$, when $y(0) = 1$.		
Sol.	Given differential eq. can be written as $\frac{dx}{dy} - \frac{2}{y}x = y^2e^{-y}$	I	1
	Integrating factor $= e^{\int \frac{-2}{y} dy} = \frac{1}{y^2}$	II	1
	General solution: $x\left(\frac{1}{y^2}\right) = \int \frac{1}{y^2} \cdot y^2 e^{-y} dy$	III	1
	$x\left(\frac{1}{y^2}\right) = -e^{-y} + c$ or $x = y^2(-e^{-y} + c)$	IV	1
	Now, when $x = 0, y = 1: 0 = -e^{-1} + c \Rightarrow c = e^{-1}$	V	$\frac{1}{2}$
	Required particular solution is, $x = y^2(e^{-1} - e^{-y})$	VI	$\frac{1}{2}$
34. (b)	Find the general solution of the differential equation: $(x^3 - 3xy^2)dx = (y^3 - 3x^2y) dy$.		
Sol.	$\frac{dy}{dx} = \frac{x^3 - 3xy^2}{y^3 - 3x^2y}$	I	1
	Put $y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$, so	II	1
	$v + x \frac{dv}{dx} = \frac{1 - 3v^2}{v^3 - 3v} \Rightarrow x \frac{dv}{dx} = \frac{1 - 3v^2}{v^3 - 3v} - v = \frac{1 - v^4}{v^3 - 3v}$		
	$\Rightarrow \int \frac{3v - v^3}{v^4 - 1} dv = \int \frac{1}{x} dx$	III	1
	$\Rightarrow \frac{3}{4} \log \left \frac{v^2 - 1}{v^2 + 1} \right - \frac{1}{4} \log v^4 - 1 = \log x + c$	IV	$\frac{1}{2}$
	$\therefore \frac{3}{4} \log \left \frac{y^2 - x^2}{y^2 + x^2} \right - \frac{1}{4} \log \left \frac{y^4 - x^4}{x^4} \right = \log x + c$	V	$\frac{1}{2}$

35.	Find the length of the perpendicular drawn from the point P(1, 2, 3) to the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$. Also, find the equation of the perpendicular line joining P and the foot of the perpendicular.		
Sol :	Let $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2} = k$ So any general point on the given line is Q(3k + 6, 2k + 7, -2k + 7).  Direction ratio's of PQ are $\langle 3k + 5, 2k + 5, -2k + 4 \rangle$ Direction ratio's of line are $\langle 3, 2, -2 \rangle$ $\Rightarrow 3(3k + 5) + 2(2k + 5) - 2(-2k + 4) = 0$ $\Rightarrow k = -1$ So, Foot of the perpendicular Q(3, 5, 9) Required distance (PQ) = $\sqrt{4 + 9 + 36} = 7$ units Required equation of line PQ: $\left. \begin{aligned} \vec{r} &= (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}) \\ \text{or} \\ \frac{x-1}{2} &= \frac{y-2}{3} = \frac{z-3}{6} \end{aligned} \right\}$	I	1
		II	1
		III	1
		IV	1
		V	1
SECTION E			
This section (Q. 36 to 38) has 3 case study based questions of 4 marks each.			
36.	A carpenter needs to design a wooden box in the shape of a cuboid such that the sum of its length and breadth is 3 cm more than its height. Twice of its length, thrice of its breadth and its height add up to 10 cm. Its breadth added to 7 times its height is 1 cm less than 3 times its length.		

On the basis of the above information, answer the following questions: (i) Write the equations representing the various dimensions & express them as the matrix equation $AX = B$. (ii) Find if A^{-1} exists. Justify your answer. (iii) (a) Find A^{-1}. OR (iii) (b) Find $A^2 + 7I$.		
(i) $\begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ -3 & 1 & 7 \end{pmatrix} \begin{pmatrix} l \\ b \\ h \end{pmatrix} = \begin{pmatrix} 3 \\ 10 \\ -1 \end{pmatrix}$ $A \quad X = B$ (ii) $A = -8 \neq 0$, so A^{-1} exists. (iii) (a) $A^{-1} = \frac{1}{ A }(\text{adj}A) = \frac{1}{-8} \begin{pmatrix} 20 & -8 & 4 \\ -17 & 4 & -3 \\ 11 & -4 & 1 \end{pmatrix}$ OR (iii) (b) $A^2 = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ -3 & 1 & 7 \end{pmatrix} \begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ -3 & 1 & 7 \end{pmatrix} = \begin{pmatrix} 6 & 3 & -7 \\ 5 & 12 & 8 \\ -22 & 7 & 53 \end{pmatrix}$ Thus, $A^2 + 7I = \begin{pmatrix} 13 & 3 & -7 \\ 5 & 19 & 8 \\ -22 & 7 & 60 \end{pmatrix}$ Alternative : If $A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{pmatrix}$, then $A = 8$ & $A^{-1} = \frac{1}{8} \begin{pmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{pmatrix}$ $A^2 = \begin{pmatrix} 0 & 5 & 7 \\ 11 & 10 & -6 \\ -20 & 7 & 45 \end{pmatrix}$, $A^2 + 7I = \begin{pmatrix} 7 & 5 & 7 \\ 11 & 17 & -6 \\ -20 & 7 & 52 \end{pmatrix}$ Note: In this question, students may take different A by rearranging of terms so check the correctness of the solution accordingly	I I I I I II I II	1 1 2 (1 Mark for any 4 correct entries) 1½ ½ 1+2 1 1

37.	An NGO organises a charity event in which they decide to distribute woollen caps to protect children from winter. The caps to be distributed are in three separate boxes, Box I has 30 red caps, Box II has 20 red and 10 green caps, & Box III has 30 green caps. The probability that a Box i is selected and a cap picked out is $\frac{i}{6}$, where $i = 1, 2, 3$. Based on the above information, answer the following questions : A person selects a cap. (i) What is the probability that he selects a red cap ? (ii) If he selects a green cap, what is the probability that the cap has come from Box II ?		
Sol.	(i) Let B_1, B_2 & B_3 be the event that box I, box II and box III selected respectively. $P(B_1) = \frac{1}{6}, P(B_2) = \frac{2}{6}, P(B_3) = \frac{3}{6}$ Now, $P(\text{selecting a red cap}) = \frac{1}{6} \times 1 + \frac{2}{6} \times \frac{2}{3} + \frac{3}{6} \times 0 = \frac{7}{18}$ (ii) $P(\text{selecting a green cap}) = 1 - \frac{7}{18} = \frac{11}{18}$ Now, $P\left(\frac{B_2}{\text{Green cap}}\right) = \frac{\frac{2}{6} \times \frac{1}{3}}{\frac{11}{18}} = \frac{2}{11}$	I II I II	1 1 1 1
38.	At a birthday party, children are being served orange juice in conical cups, as shown in the figure. 		

	Each cup is 15 cm deep and has a radius 5 cm. The juice is being poured into this cup at a rate of $0.1 \text{ cm}^3/\text{s}$. On the basis of the above information, answer the following questions: (i) Establish a relation between the height h of the juice in the cup and radius r of the surface of the juice in the cup, if the semi-vertical angle of the cone is α. (ii) At what rate is the juice level in the cup rising when the juice is 6 cm deep ? (iii) (a) When the juice is 6 cm deep, then find at what rate is the upper surface area of juice increasing ? OR (iii) (b) When the juice is 6 cm deep, then find the rate at which the wetted surface area of the cup is increasing.		
Sol.	(i) $\tan \alpha = \frac{r}{h} = \frac{5}{15} \Rightarrow r = \frac{h}{3}$ (ii) Volume, $V = \frac{1}{27} \pi h^3 \Rightarrow \frac{dV}{dt} = \frac{1}{9} \pi h^2 \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{40\pi} \text{ cm/sec}$ (iii) (a) Upper surface area of juice, $A = \pi r^2 = \frac{\pi h^2}{9}$ $\frac{dA}{dt} = \frac{2\pi h}{9} \frac{dh}{dt} = \frac{1}{30} \text{ cm}^2/\text{sec}$ OR (iii) (b) Wetted surface area, $S = \pi r l = \frac{\sqrt{10}\pi h^2}{9}$ $\frac{dS}{dt} = \frac{2\sqrt{10}\pi h}{9} \frac{dh}{dt} = \frac{\sqrt{10}}{30} \text{ cm}^2/\text{sec}$	I I I II I II	1 1 1 1 1 1