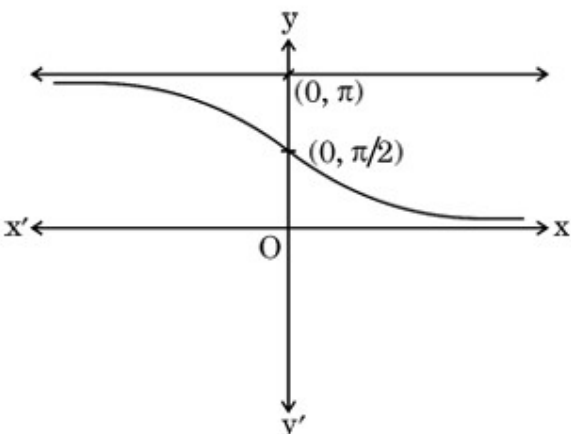


**MATHEMATICS CLASS XII (Subject Code–
041) (PAPER CODE: 65/4/2)
SOLUTIONS**

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Steps	Marks
	SECTION A		
	Q. Number 1 to 20 are multiple choice questions of 1 mark each.		
1.	The following graph represents :  (A) $y = \sec^{-1} x$ (B) $y = \cot^{-1} x$ (C) $y = \tan^{-1} x$ (D) $y = \operatorname{cosec}^{-1} x$		
Ans.	(B) $y = \cot^{-1} x$		1
2.	Let $A = [a_{ij}]$ be a 2×2 matrix whose elements are given by $a_{ij} = \frac{(2i - j)^2}{3}$. Then A' is : (A) $\begin{bmatrix} \frac{1}{3} & 3 \\ 0 & \frac{4}{3} \end{bmatrix}$ (B) $\begin{bmatrix} \frac{1}{3} & 0 \\ 3 & \frac{4}{3} \end{bmatrix}$ (C) $\begin{bmatrix} \frac{4}{3} & 3 \\ 0 & \frac{1}{3} \end{bmatrix}$ (D) $\begin{bmatrix} \frac{4}{3} & 0 \\ 3 & \frac{1}{3} \end{bmatrix}$		
Ans.	(A) $\begin{bmatrix} \frac{1}{3} & 3 \\ 0 & \frac{4}{3} \end{bmatrix}$		1

3.	If points (2, 3), (0, 4) and (p, 2) are collinear, then the value of p is : (A) $\frac{4}{7}$ (B) $\frac{-3}{7}$ (C) 4 (D) - 4		
Ans.	(C) 4		1
4.	Differential of e^{e^x} with respect to x is : (A) $\log x$ (B) e^{e^x} (C) $e^x \cdot e^{e^x}$ (D) $(e^x)^2$		
Ans.	(C) $e^x \cdot e^{e^x}$		1
5.	The principal value of $\sec^{-1}(\sqrt{2}) + 2 \operatorname{cosec}^{-1}(-\sqrt{2})$ is : (A) $-\frac{\pi}{2}$ (B) $-\frac{\pi}{4}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$		
Ans.	(B) $-\frac{\pi}{4}$		1
6.	If the distance travelled by a particle in t seconds is given by $S = 72t + 3t^2 - t^3$, then time taken by the particle to come to rest is : (A) 4 seconds (B) 6 seconds (C) 3 seconds (D) 0 seconds		
Ans.	(B) 6 seconds		1
7.	$\int \frac{dx}{\sqrt{25-16x^2}}$ is equal to : (A) $\frac{1}{5} \sin^{-1} 4x + c$ (B) $\frac{1}{25} \sin^{-1} 16x + c$ (C) $\frac{1}{4} \sin^{-1} \frac{4x}{5} + c$ (D) $\frac{1}{16} \sin^{-1} \frac{4x}{5} + c$		
Ans.	(C) $\frac{1}{4} \sin^{-1} \left(\frac{4x}{5} \right) + c$		1

8.	If $\int_0^1 \frac{dx}{e^x + e^{-x}} = \tan^{-1} e + k$, then the value of k is : (A) e (B) $\frac{\pi}{4}$ (C) 0 (D) $-\frac{\pi}{4}$		
Ans.	(D) $-\frac{\pi}{4}$		1
9.	If $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is: (A) 0 (B) -2 (C) -1 (D) 2		
Ans.	(D) 2		1
10.	The area of the region bounded by the curve $y = x$ and x-axis, between $x = 0$ and $x = 2$ is : (A) 2 sq. units (B) $\frac{1}{2}$ sq. unit (C) 1 sq. unit (D) 4 sq. units		
Ans.	(A) 2 sq. units		1
11.	The greatest integer function, $f(x) = [x]$, $0 < x < 3$ is not differentiable at how many points ? (A) At only one point (B) At only two points (C) At no point (D) At three points		
Ans.	(B) At only two points		1
12.	The sum of the order and the degree of the differential equation $4 \left[\frac{d^2 y}{dx^2} \right]^2 + 3 \left[1 + \left(\frac{dy}{dx} \right)^2 - y \right] = 0$ is : (A) 2 (B) 3 (C) not defined (D) 4		
Ans.	(D) 4		1

13.	If $(3\hat{i} - 2\hat{j} + 5\hat{k}) \times (4\hat{i} + p\hat{j} + q\hat{k}) = \vec{0}$, then the values of p and q are : (A) $p = -\frac{2}{3}, q = \frac{5}{3}$ (B) $p = -\frac{8}{3}, q = \frac{20}{3}$ (C) $p = \frac{20}{3}, q = -\frac{8}{3}$ (D) $p = 0, q = 0$		
Ans.	(B) $p = -\frac{8}{3}, q = \frac{20}{3}$		1
14.	In the graph, the feasible region representing the Linear Programming Problem for maximising objective function $Z = px + qy$, $p, q > 0$ is shaded. If all points on segment AB give max (Z), then which of the following is true ? (A) $p = 2q$ (B) $p = 3q$ (C) $q = 3p$ (D) $q = 2p$		
Ans.	(C) $q = 3p$		1
15.	Three points A(0, 1, 1), B(2, 0, -1) and C(1, 0, 3) form ΔABC. The ar (ΔABC) is : (A) $\frac{\sqrt{53}}{2}$ sq. units (B) $\sqrt{53}$ sq. units (C) $\frac{\sqrt{11}}{2}$ sq. units (D) $\sqrt{11}$ sq. units		
Ans.	(A) $\frac{\sqrt{53}}{2}$ sq. units		1

16.	The region represented by the system of inequations $3x + y \geq 3$, $2x - y \geq -5$, $x, y \geq 0$ is : (A) unbounded in 1st quadrant (B) bounded in 1st quadrant (C) unbounded in 2nd quadrant (D) bounded in 2nd quadrant		
Ans.	(A) unbounded in 1 st Quadrant		1
17.	Which of the following is not a Linear Differential Equation? (A) $(1 + x^2)dy + 2xydx = \cot x dx$ (B) $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$ (C) $x(1 + y^2)dx - y(1 + x^2)dy = 0$ (D) $ydx - (x + 3y^2)dy = 0$		
Ans.	(C) $x(1 + y^2)dx - y(1 + x^2)dy = 0$		1
18.	The probability that it will rain tomorrow in cities A, B and C is 60%, 70% and 80% respectively. The probability that it will rain tomorrow in at least one of the cities is : (A) $\frac{3}{250}$ (B) $\frac{244}{250}$ (C) 1 (D) $\frac{9}{10}$		
Ans.	(B) $\frac{244}{250}$		1
19.	Assertion (A): If A and B are two square matrices such that AB and BA are defined, then it is not necessary that $AB = BA$. Reason (R): Product of two diagonal matrices of same order is commutative.		
Ans.	(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).		1

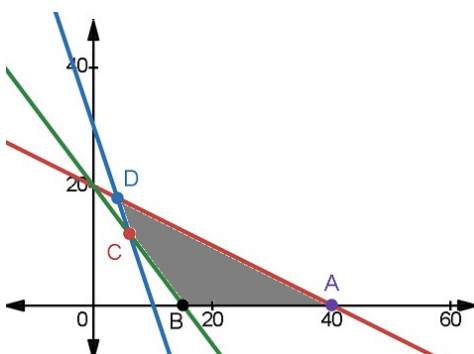
20.	Assertion (A): A function $f : \mathbb{N} \rightarrow \mathbb{N}$ given by $f(x) = x^3 + 2, \forall x \in \mathbb{N}$ is one-one but not onto. Reason (R): Since $\forall y \in \mathbb{N}$ (codomain), there does not exist $x = (y - 2)^{\frac{1}{3}}$ in $\mathbb{N}$ (Domain) such that $f(x) = x^3 + 2 = y$.		
Ans.	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).		1
SECTION B			
Q. Numbers 21 to 25 are very short answer questions of 2 marks each.			
21. (a)	Let two rods placed on the ground be represented by vectors $4\hat{i} - \hat{j} + 3\hat{k}$ and $-2\hat{i} + \hat{j} - 2\hat{k}$. Find a vector representing a flag-post of height 5 m that has to be erected perpendicular to both the rods.		
Sol.	Let $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$ $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 3 \\ -2 & 1 & -2 \end{vmatrix} = -\hat{i} + 2\hat{j} + 2\hat{k}$ Now, Required vector $= 5 \left(\frac{\vec{a} \times \vec{b}}{ \vec{a} \times \vec{b} } \right) = \begin{cases} \frac{-5}{3}\hat{i} + \frac{10}{3}\hat{j} + \frac{10}{3}\hat{k} \\ \text{or} \\ \frac{5}{3}\hat{i} - \frac{10}{3}\hat{j} - \frac{10}{3}\hat{k} \end{cases}$ Alternative approach: Let $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$ Let required vector be $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$ so, $25 = x^2 + y^2 + z^2, 4x - y + 3z = 0, -2x + y - 2z = 0$ on solving we get, $z = -2x, y = z \Rightarrow \frac{100}{9} = z^2 \Rightarrow z = \pm \frac{10}{3}$ $\therefore$ Required vector, $\vec{p} = \left\{ \frac{-5}{3}\hat{i} + \frac{10}{3}\hat{j} + \frac{10}{3}\hat{k} \text{ or } \frac{5}{3}\hat{i} - \frac{10}{3}\hat{j} - \frac{10}{3}\hat{k} \right\}$	I II I II	1 1 (For any one result) 1 1 (For any one result)

OR			
21. (b)	A unit vector $\vec{a}$ is such that it makes an angle $\frac{\pi}{4}$ with x-axis, $\frac{\pi}{3}$ with y-axis and an acute angle θ with z-axis. Find θ and the components of $\vec{a}$.		
	$\cos^2\left(\frac{\pi}{4}\right) + \cos^2\left(\frac{\pi}{3}\right) + \cos^2\theta = 1 \Rightarrow \cos\theta = \frac{1}{2}$ (As, θ is acute)	I	1
	$\Rightarrow \theta = \frac{\pi}{3}$ & Components of $\vec{a} = \frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}$	II	1
22.	Evaluate: $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\left(\frac{-\pi}{2}\right)\right) + \tan^{-1}\left(\tan\frac{2\pi}{3}\right)$		2
Sol.	$\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\left(\frac{-\pi}{2}\right)\right) + \tan^{-1}\left(\tan\frac{2\pi}{3}\right)$ $= -\frac{\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4} - \frac{\pi}{3}$ $= \frac{-5\pi}{12}$	I II	$1\frac{1}{2}$ (For any 3 correct result) $\frac{1}{2}$
23.	Find the interval(s) in which the function $f(x) = \frac{x}{\log x}$, where $x \in (0, 1) \cup (1, \infty)$, is increasing.		
Sol.	$f'(x) = \frac{\log x - 1}{(\log x)^2}$ Put $f'(x) = 0 \Rightarrow \log x = 1$ As, $\log x = 1 = \log e \Rightarrow x = e$ $\therefore f(x)$ is increasing on $x \in (e, \infty)$ or $x \in [e, \infty)$.	I II III	1 $\frac{1}{2}$ $\frac{1}{2}$

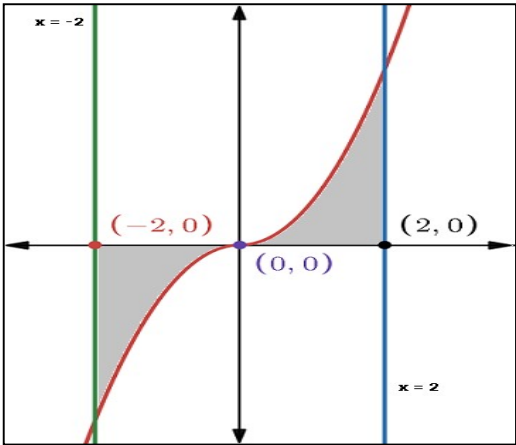
24.	If $(2\hat{i} + 3\hat{j} + \hat{k})$ and $(2\hat{i} + \hat{j} - \hat{k})$ represent the sides $\vec{AB}$ and $\vec{AC}$ respectively of ΔABC , find the vector representing the median through A.		
Sol.	$\vec{AB} + \vec{AC} = 4\hat{i} + 4\hat{j} + 0\hat{k}$ Now, $\vec{AD} = \frac{\vec{AB} + \vec{AC}}{2} = 2\hat{i} + 2\hat{j} + 0\hat{k}$	I	1
		II	1
25. (a)	Show that the function $f(x) = \begin{cases} \frac{\cos x}{-x + \frac{\pi}{2}}, & x \neq \frac{\pi}{2} \\ 1, & x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$.		
Sol.	For showing: $\lim_{x \rightarrow \frac{\pi}{2}} f(x) = 1$ For showing: $f\left(\frac{\pi}{2}\right) = 1$ $\therefore f(x)$ is continuous at $x = \frac{\pi}{2}$.	I	1
		II	$\frac{1}{2}$
		III	$\frac{1}{2}$
OR			
25. (b)	Find whether the function $f(x) = \begin{cases} x - 1, & x < 2 \\ 2x - 3, & x \geq 2 \end{cases}$ at $x = 2$ is differentiable or not.		
Sol.	Getting $LHD = 1$ Getting $RHD = 2$ $\therefore LHD \neq RHD$, so $f(x)$ is not differentiable at $x = 2$.	I	1
		II	$\frac{1}{2}$
		III	$\frac{1}{2}$
SECTION C			
Q. Numbers 26 to 31 are short answer questions of 3 marks each.			

26.	In a school, the probability of holding a debate competition is $\frac{1}{3}$ and that of a quiz competition is $\frac{2}{3}$. In the two participating teams, A has 4 girls and 6 boys and B has 7 girls and 3 boys. If a debate competition is held, the students are selected from team A and for the quiz competition they are selected from team B. If only two students are to be chosen from the teams, then find the probability that one will be a girl and the other a boy.		
Sol :	Let D be the event a debate competition is held, Q be the event a quiz competition is held and E be the event that one girl & one boy are chosen. $P(E D) = 2 \times \frac{4}{10} \times \frac{6}{9} = \frac{8}{15}, P(E Q) = 2 \times \frac{7}{10} \times \frac{3}{9} = \frac{7}{15}$ $\text{Now, } P(E) = \frac{1}{3} \times \frac{8}{15} + \frac{2}{3} \times \frac{7}{15} = \frac{22}{45}$	I II	1+1 1
27.	If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then compute $A^2 - 4A - 5I$.		
Sol.	$A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$ $A^2 - 4A - 5I = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 4 & 4 & 8 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ $A^2 - 4A - 5I = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	I II III	1½ (½ for any three correct elements) 1 ½

28. (a)	Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. A function $f : A \rightarrow B$ is defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Find whether f is one-one and onto.		
Sol.	Let $f(a) = f(b)$ for some $a, b \in \mathbb{R} - \{3\}$ so, $\left(\frac{a-2}{a-3}\right) = \left(\frac{b-2}{b-3}\right) \Rightarrow ab - 2b - 3a + 6 = ab - 3b - 2a + 6$ $\Rightarrow a = b$, Thus f is one-one function. Now, let us assume for some $y \in \mathbb{R} - \{1\}$, $y = f(x) = \frac{x-2}{x-3}$ For getting, $x = \frac{3y-2}{y-1}$ Range = $\mathbb{R} - \{1\}$ = co-domain, Thus f is onto function.	I II III IV	1 $\frac{1}{2}$ 1 $\frac{1}{2}$
OR			
28. (b)	Let n be a fixed positive integer. A relation R is defined in set $\mathbb{Z}$ such that $R = \{(x, y) : (x - y) \text{ is divisible by } n, x, y \in \mathbb{Z}\}$. Determine if R is an equivalence relation.		
Sol :	Let for some $a \in \mathbb{Z}$, $(a, a) \in R$ $\Rightarrow 0$ is divisible by n, which is true $\therefore (a, a) \in R$, so R is reflexive. Let $(a, b) \in R \Rightarrow a - b = np, p \in \mathbb{Z}$ $\Rightarrow b - a = n(-p) = nq, q \in \mathbb{Z}$ $\Rightarrow b - a$ is also divisible by n. $\therefore (b, a) \in R$, so R is symmetric. Let $(a, b) \in R$ & $(b, c) \in R$ for some $a, b, c \in \mathbb{Z}$ $\Rightarrow a - b = np$ & $b - c = nr$, where $p, r \in \mathbb{Z}$ $\Rightarrow a - b + b - c = n(p + r) \Rightarrow (a - c)$ is divisible by n. $\therefore (a, c) \in R$, so R is transitive. Thus, R is an equivalence relation.	I II III IV	$\frac{1}{2}$ 1 1 $\frac{1}{2}$

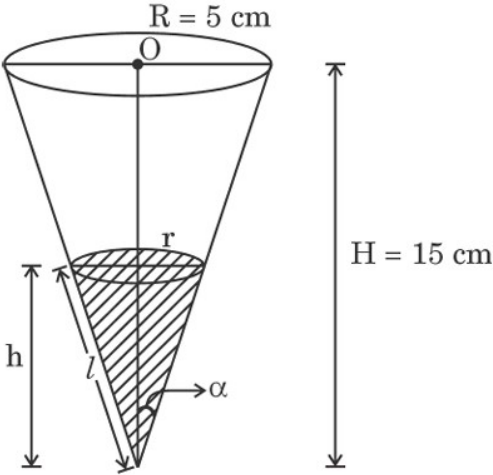
29.	Solve the following Linear programming problem graphically: Minimize $Z = 20x + 10y$ subject to the constraints $x + 2y \leq 40$ $3x + y \geq 30$ $4x + 3y \geq 60$ $x \geq 0, y \geq 0$												
Sol.	 <table><tr><th>Corner point</th><th>Value of $Z = 20x + 10y$</th></tr><tr><td>A (40, 0)</td><td>800</td></tr><tr><td>B (15, 0)</td><td>300</td></tr><tr><td>C (6, 12)</td><td>240</td></tr><tr><td>D (4, 18)</td><td>260</td></tr></table> Thus, minimum value of Z is 240 at $x = 6$ and $y = 12$.	Corner point	Value of $Z = 20x + 10y$	A (40, 0)	800	B (15, 0)	300	C (6, 12)	240	D (4, 18)	260	I II III	1 ½ For correct graph & shading 1 ½
Corner point	Value of $Z = 20x + 10y$												
A (40, 0)	800												
B (15, 0)	300												
C (6, 12)	240												
D (4, 18)	260												
30.													
(a)	If $xy = e^{x-y}$, then find $\frac{dy}{dx}$.												
Sol.	$xy = e^{x-y} \Rightarrow x \frac{dy}{dx} + y = e^{x-y} \left(1 - \frac{dy}{dx} \right)$ $\Rightarrow (x + e^{x-y}) \frac{dy}{dx} = e^{x-y} - y \Rightarrow \frac{dy}{dx} = \frac{e^{x-y} - y}{x + e^{x-y}} \text{ or } \frac{y(x-1)}{x(y+1)}$ Alternative method: $xy = e^{x-y} \Rightarrow \log x + \log y = (x - y)$ $\Rightarrow \frac{1}{x} + \frac{1}{y} \frac{dy}{dx} = 1 - \frac{dy}{dx}$ $\Rightarrow \frac{dy}{dx} = \frac{1 - \frac{1}{x}}{\frac{1}{y} + 1} = \frac{y(x-1)}{x(y+1)}$	I II I II III	1½ 1½ 1 1 1										
OR													

31. (b)	Find the shortest distance between the lines: $\vec{r} = (4 + \lambda)\hat{i} + (2\lambda - 1)\hat{j} - 3\lambda\hat{k}$ $\vec{r} = (1 + 2\mu)\hat{i} + (2 - \mu)\hat{k} + (4\mu - 1)\hat{j}$		
Sol.	Here, $\vec{a}_1 = 4\hat{i} - \hat{j}$, $\vec{b}_1 = \hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{a}_2 = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b}_2 = 2\hat{i} + 4\hat{j} - 5\hat{k}$ Now, $\vec{a}_2 - \vec{a}_1 = -3\hat{i} + 2\hat{k}$, $\vec{b}_1 \times \vec{b}_2 = 2\hat{i} - \hat{j}$ and $ \vec{b}_1 \times \vec{b}_2 = \sqrt{5}$ Required distance = $\left \frac{(-3\hat{i} + 2\hat{k}) \cdot (2\hat{i} - \hat{j})}{\sqrt{5}} \right = \frac{6}{\sqrt{5}}$ or $\frac{6\sqrt{5}}{5}$	I II III	1 1½ ½
SECTION D			
Q. Numbers 32 to 35 are long answer questions of 5 marks each.			
32. (a)	Solve the differential equation: $ye^y dx = (y^3 + 2xe^y)dy$, when $y(0) = 1$.		
Sol.	Given differential eq. can be written as $\frac{dx}{dy} - \frac{2}{y}x = y^2e^{-y}$ Integrating factor = $e^{\int \frac{-2}{y} dy} = \frac{1}{y^2}$ General solution: $x \left(\frac{1}{y^2} \right) = \int \frac{1}{y^2} \cdot y^2 e^{-y} dy$ $x \left(\frac{1}{y^2} \right) = -e^{-y} + c$ or $x = y^2 (-e^{-y} + c)$ Now, when $x = 0$, $y = 1$: $0 = -e^{-1} + c \Rightarrow c = e^{-1}$ Required particular solution is, $x = y^2 (e^{-1} - e^{-y})$	I II III IV V VI	1 1 1 1 ½ ½
OR			

32. (b)	Find the general solution of the differential equation: $(x^3 - 3xy^2)dx = (y^3 - 3x^2y) dy$.		
Sol.	$\frac{dy}{dx} = \frac{x^3 - 3xy^2}{y^3 - 3x^2y}$ Put $y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$, so $v + x \frac{dv}{dx} = \frac{1 - 3v^2}{v^3 - 3v} \Rightarrow x \frac{dv}{dx} = \frac{1 - 3v^2}{v^3 - 3v} - v = \frac{1 - v^4}{v^3 - 3v}$ $\Rightarrow \int \frac{3v - v^3}{v^4 - 1} dv = \int \frac{1}{x} dx$ $\Rightarrow \frac{3}{4} \log \left \frac{v^2 - 1}{v^2 + 1} \right - \frac{1}{4} \log v^4 - 1 = \log x + c$ $\therefore \frac{3}{4} \log \left \frac{y^2 - x^2}{y^2 + x^2} \right - \frac{1}{4} \log \left \frac{y^4 - x^4}{x^4} \right = \log x + c$	I II III IV V	1 1 1 1½ ½
33.	Using integration, find the area of the region bounded by the curve $y = x x $, x -axis, $x = -2$ and $x = 2$.		
Sol.	 Required Area = $-\int_{-2}^0 x x dx + \int_0^2 x x dx$ Area = $\int_{-2}^0 -x^2 dx + \int_0^2 x^2 dx$	I II III	1 (For correct graph) 1 1

	$\text{Area} = -\left(\frac{x^3}{3}\right)_{-2}^0 + \left(\frac{x^3}{3}\right)_0^2 = \frac{8}{3} + \frac{8}{3} = \frac{16}{3} \text{ or } \frac{16}{3} \text{ sq. units}$ Alternative Approach: Area = $2 \times \int_0^2 x^2 dx$ & proceed accordingly	IV	2
34. (a)	Find : $\int \frac{x}{(x-1)(x^2+4)} dx$		
Sol.	Let $\frac{x}{(x-1)(x^2+4)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+4}$ on solving we get, $A = \frac{1}{5}, B = \frac{-1}{5}, C = \frac{4}{5}$ so, $\int \frac{x}{(x-1)(x^2+4)} dx = \frac{1}{5} \int \frac{dx}{x-1} - \frac{1}{10} \int \frac{2xdx}{x^2+4} + \frac{4}{5} \int \frac{dx}{x^2+4}$ $\int \frac{x}{(x-1)(x^2+4)} dx = \frac{1}{5} \log x-1 - \frac{1}{10} \log(x^2+4) + \frac{2}{5} \tan^{-1}\left(\frac{x}{2}\right) + c$	I II III	1 1½ 2½
OR			
34. (b)	Evaluate : $\int_0^1 \frac{x \tan^{-1} x}{(1+x^2)^{\frac{3}{2}}} dx.$		
Sol.	Let $x = \tan t \Rightarrow dx = \sec^2 t dt$ $I(\text{Let}) = \int_0^1 \frac{x \tan^{-1} x}{(1+x^2)^{\frac{3}{2}}} dx = \int_0^{\frac{\pi}{4}} \frac{t \tan t}{\sec^3 t} \sec^2 t dt = \int_0^{\frac{\pi}{4}} t \sin t dt$ $I = (-t \cos t + \sin t) \Big _0^{\frac{\pi}{4}}$ $I = \frac{-\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} \text{ or } \frac{-\pi+4}{4\sqrt{2}}$	I II III IV	1 2 1 1

35.	Find the vector and cartesian equations of the line passing through the point of intersection of the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ and parallel to the line $\frac{x-1}{-2} = \frac{7-y}{-3} = z$.		
Sol.	Lines are $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ So any general point on these lines are $P(3\lambda + 1, -\lambda + 1, -1)$ & $Q(2\mu + 4, 0, 3\mu - 1)$ respectively. For intersection points: $-\lambda + 1 = 0, 3\mu - 1 = -1$ & $3\lambda + 1 = 2\mu + 4$ on solving we get, $\lambda = 1, \mu = 0$ Thus, required point of intersection is $(4, 0, -1)$. Required equation of line (cartesian form): $\frac{x-4}{-2} = \frac{y}{3} = \frac{z+1}{1}$ Required equation of line (Vector form): $\vec{r} = (4\hat{i} - \hat{k}) + \omega(-2\hat{i} + 3\hat{j} + \hat{k})$	I II III IV V	1 1 1 1 1
SECTION E			
This section (Q. 36 to 38) has 3 case study based questions of 4 marks each.			
36.	An NGO organises a charity event in which they decide to distribute woollen caps to protect children from winter. The caps to be distributed are in three separate boxes, Box I has 30 red caps, Box II has 20 red and 10 green caps, & Box III has 30 green caps. The probability that a Box i is selected and a cap picked out is $\frac{i}{6}$, where $i = 1, 2, 3$. Based on the above information, answer the following questions : A person selects a cap. (i) What is the probability that he selects a red cap ? (ii) If he selects a green cap, what is the probability that the cap has come from Box II ?		

Sol.	(i) Let B_1, B_2 & B_3 be the event that box I, box II and box III selected respectively. $P(B_1) = \frac{1}{6}, P(B_2) = \frac{2}{6}, P(B_3) = \frac{3}{6}$ Now, $P(\text{selecting a red cap}) = \frac{1}{6} \times 1 + \frac{2}{6} \times \frac{2}{3} + \frac{3}{6} \times 0 = \frac{7}{18}$ (ii) $P(\text{selecting a green cap}) = 1 - \frac{7}{18} = \frac{11}{18}$ Now, $P\left(\frac{B_2}{\text{Green cap}}\right) = \frac{\frac{2}{6} \times \frac{1}{3}}{\frac{11}{18}} = \frac{2}{11}$	I II I II	1 1 1 1
37.	At a birthday party, children are being served orange juice in conical cups, as shown in the figure.  Each cup is 15 cm deep and has a radius 5 cm. The juice is being poured into this cup at a rate of $0.1 \text{ cm}^3/\text{s}$. On the basis of the above information, answer the following questions: (i) Establish a relation between the height h of the juice in the cup and radius r of the surface of the juice in the cup, if the semi-vertical angle of the cone is α. (ii) At what rate is the juice level in the cup rising when the juice is 6 cm deep ? (iii) (a) When the juice is 6 cm deep, then find at what rate is the upper surface area of juice increasing ?		

	OR (iii) (b) When the juice is 6 cm deep, then find the rate at which the wetted surface area of the cup is increasing.		
Sol.	(i) $\tan \alpha = \frac{r}{h} = \frac{5}{15} \Rightarrow r = \frac{h}{3}$ (ii) Volume, $V = \frac{1}{27} \pi h^3 \Rightarrow \frac{dV}{dt} = \frac{1}{9} \pi h^2 \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{40\pi} \text{ cm/sec}$ (iii) (a) Upper surface area of juice, $A = \pi r^2 = \frac{\pi h^2}{9}$ $\frac{dA}{dt} = \frac{2\pi h}{9} \frac{dh}{dt} = \frac{1}{30} \text{ cm}^2/\text{sec}$ OR (iii) (b) Wetted surface area, $S = \pi r l = \frac{\sqrt{10} \pi h^2}{9}$ $\frac{dS}{dt} = \frac{2\sqrt{10} \pi h}{9} \frac{dh}{dt} = \frac{\sqrt{10}}{30} \text{ cm}^2/\text{sec}$	I I I II I II	1 1 1 1 1 1
38.	A carpenter needs to design a wooden box in the shape of a cuboid such that the sum of its length and breadth is 3 cm more than its height. Twice of its length, thrice of its breadth and its height add up to 10 cm. Its breadth added to 7 times its height is 1 cm less than 3 times its length. On the basis of the above information, answer the following questions: (i) Write the equations representing the various dimensions & express them as the matrix equation $AX = B$. (ii) Find if A^{-1} exists. Justify your answer. (iii) (a) Find A^{-1}. OR (iii) (b) Find $A^2 + 7I$.		
Sol.	(i) $\begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ -3 & 1 & 7 \end{pmatrix} \begin{pmatrix} l \\ b \\ h \end{pmatrix} = \begin{pmatrix} 3 \\ 10 \\ -1 \end{pmatrix}$ $A \quad X = B$	I	1

(ii) $ A = -8 \neq 0$, so A^{-1} exists.	I	1	
(iii) (a) $A^{-1} = \frac{1}{ A }(\text{adj}A) = \frac{1}{-8} \begin{pmatrix} 20 & -8 & 4 \\ -17 & 4 & -3 \\ 11 & -4 & 1 \end{pmatrix}$	I	2 (1 Mark for any 4 correct entries)	
OR			
(iii) (b) $A^2 = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ -3 & 1 & 7 \end{pmatrix} \begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ -3 & 1 & 7 \end{pmatrix} = \begin{pmatrix} 6 & 3 & -7 \\ 5 & 12 & 8 \\ -22 & 7 & 53 \end{pmatrix}$	I	1½	
Thus, $A^2 + 7I = \begin{pmatrix} 13 & 3 & -7 \\ 5 & 19 & 8 \\ -22 & 7 & 60 \end{pmatrix}$	II	½	
Alternative : If $A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{pmatrix}$, then $A = 8$ & $A^{-1} = \frac{1}{8} \begin{pmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{pmatrix}$ $A^2 = \begin{pmatrix} 0 & 5 & 7 \\ 11 & 10 & -6 \\ -20 & 7 & 45 \end{pmatrix}, A^2 + 7I = \begin{pmatrix} 7 & 5 & 7 \\ 11 & 17 & -6 \\ -20 & 7 & 52 \end{pmatrix}$ Note: In this question, students may take different A by rearranging of terms so check the correctness of the solution accordingly.			1+ 2
	I II	1 1	