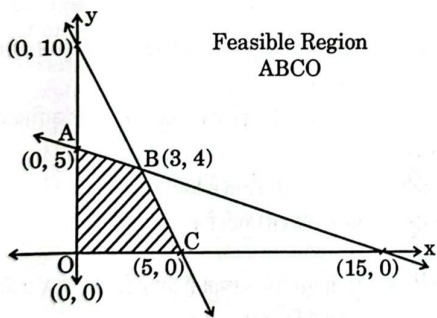


SOLUTIONS
MATHEMATICS CLASS XII (Subject Code-041)
(PAPER CODE: 65/4/1)

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Steps	Marks
	SECTION A		
	Q. Number 1 to 20 are multiple choice questions of 1 mark each.		
1.	The following graph represents : (A) $y = \cos^{-1} x$ (B) $y = \sec^{-1} x$ (C) $y = \tan^{-1} x$ (D) $y = \operatorname{cosec}^{-1} x$		
Ans.	(D) $y = \operatorname{cosec}^{-1} x$		1
2.	If $A = [a_{ij}]$ is a 2×2 matrix whose elements are given by $a_{ij} = \frac{ i - 3j }{2}$, then A' is : (A) $\begin{bmatrix} 1 & \frac{5}{2} \\ \frac{1}{2} & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & \frac{1}{2} \\ \frac{5}{2} & 2 \end{bmatrix}$ (C) $\begin{bmatrix} 2 & \frac{5}{2} \\ \frac{1}{2} & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & \frac{1}{2} \\ \frac{5}{2} & 1 \end{bmatrix}$		
Ans.	(B) $\begin{bmatrix} 1 & \frac{1}{2} \\ \frac{5}{2} & 2 \end{bmatrix}$		1

3.	The principal value of $\sec^{-1}(\sqrt{2}) + 2 \operatorname{cosec}^{-1}(-\sqrt{2})$ is : (A) $-\frac{\pi}{2}$ (B) $-\frac{\pi}{4}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$		
Ans.	(B) $-\frac{\pi}{4}$		1
4.	If points (2, 3), (0, 4) and (p, 2) are collinear, then the value of p is : (A) $\frac{4}{7}$ (B) $\frac{-3}{7}$ (C) 4 (D) - 4		
Ans.	(C) 4		1
5.	Differential of e^{e^x} with respect to x is : (A) $\log x$ (B) e^{e^x} (C) $e^x \cdot e^{e^x}$ (D) $(e^x)^2$		
Ans.	(C) $e^x \cdot e^{e^x}$		1
6.	The surface area of a sphere when its volume changes at the same rate as its radius is : (A) 4π sq. units (B) 1 sq. unit (C) 4 sq. units (D) π sq. units		
Ans.	(B) 1 sq. units		1
7.	If $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is: (A) 0 (B) - 2 (C) -1 (D) 2		
Ans.	(D) 2		1

8.	The greatest integer function, $f(x) = [x]$, $0 < x < 3$ is not differentiable at how many points ? (A) At only one point (B) At only two points (C) At no point (D) At three points		
Ans.	(B) At only two points		1
9.	$\int \frac{dx}{\sqrt{25-16x^2}}$ is equal to : (A) $\frac{1}{5} \sin^{-1} 4x + c$ (B) $\frac{1}{25} \sin^{-1} 16x + c$ (C) $\frac{1}{4} \sin^{-1} \frac{4x}{5} + c$ (D) $\frac{1}{16} \sin^{-1} \frac{4x}{5} + c$		
Ans.	(C) $\frac{1}{4} \sin^{-1} \left(\frac{4x}{5} \right) + c$		1
10.	If $\int_0^1 \frac{dx}{e^x + e^{-x}} = \tan^{-1} e + k$, then the value of k is : (A) e (B) $\frac{\pi}{4}$ (C) 0 (D) $-\frac{\pi}{4}$		
Ans.	(D) $-\frac{\pi}{4}$		1
11.	The area of the region bounded by the curve $y = x$ and x-axis, between $x = 0$ and $x = 2$ is : (A) 2 sq. units (B) $\frac{1}{2}$ sq. unit (C) 1 sq. unit (D) 4 sq. units		
Ans.	(A) 2 sq. units		1
12.	Product of the order and degree of differential equation $1 + \left(\frac{dy}{dx} \right)^3 = \lambda \left(\frac{d^3y}{dx^3} \right)^2$ is : (A) 5 (B) 6 (C) 2 (D) 3		
Ans.	(B) 6		1

13.	Which of the following is not a Linear Differential Equation? (A) $(1+x^2)dy + 2xydx = \cot x dx$ (B) $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$ (C) $x(1+y^2)dx - y(1+x^2)dy = 0$ (D) $ydx - (x+3y^2)dy = 0$		
Ans.	(C) $x(1+y^2)dx - y(1+x^2)dy = 0$		1
14.	The region represented by the system of inequations $3x + y \geq 3$, $2x - y \geq -5$, $x, y \geq 0$ is : (A) unbounded in 1 st quadrant (B) bounded in 1 st quadrant (C) unbounded in 2 nd quadrant (D) bounded in 2 nd quadrant		
Ans.	(A) unbounded in 1 st Quadrant		1
15.	In the graph, the feasible region representing the Linear Programming Problem for maximising objective function $Z = px + qy$, $p, q > 0$ is shaded. If all points on segment AB give max (Z), then which of the following is true?  (A) $p = 2q$ (B) $p = 3q$ (C) $q = 3p$ (D) $q = 2p$		
Ans.	(C) $q = 3p$		1
16.	If $(3\hat{i} - 2\hat{j} + 5\hat{k}) \times (4\hat{i} + p\hat{j} + q\hat{k}) = \vec{0}$, then the values of p and q are : (A) $p = -\frac{2}{3}, q = \frac{5}{3}$ (B) $p = -\frac{8}{3}, q = \frac{20}{3}$ (C) $p = \frac{20}{3}, q = -\frac{8}{3}$ (D) $p = 0, q = 0$		
Ans.	(B) $p = -\frac{8}{3}, q = \frac{20}{3}$		1

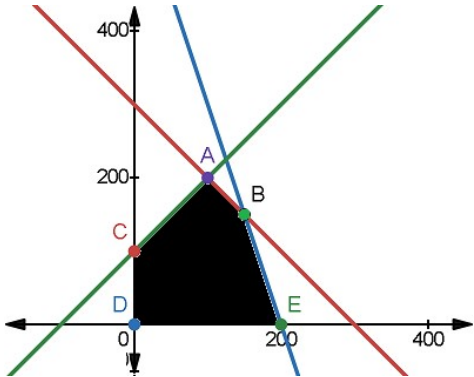
17.	Three points A(0, 1, 1), B(2, 0, -1) and C(1, 0, 3) form ΔABC . The ar (ΔABC) is : (A) $\frac{\sqrt{53}}{2}$ sq. units (B) $\sqrt{53}$ sq. units (C) $\frac{\sqrt{11}}{2}$ sq. units (D) $\sqrt{11}$ sq. units		
Ans.	(A) $\frac{\sqrt{53}}{2}$ sq. units		1
18.	A box contains 4 red, 5 blue and 1 green marble. A child randomly takes out a marble from the box, notes down the colour and puts it back in the box. If the activity is repeated 3 times, what is the probability that at least one marble is red ? (A) $\frac{27}{125}$ (B) $\frac{8}{125}$ (C) $\frac{2}{125}$ (D) $\frac{98}{125}$		
Ans.	(D) $\frac{98}{125}$		1
19.	Assertion (A): If A and B are two square matrices such that AB and BA are defined, then it is not necessary that $AB = BA$. Reason (R): Product of two diagonal matrices of same order is commutative.		
Ans.	(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).		1
20.	Assertion (A): A function $f : \mathbb{N} \rightarrow \mathbb{N}$ given by $f(x) = x^3 + 2, \forall x \in \mathbb{N}$ is one-one but not onto. Reason (R): Since $\forall y \in \mathbb{N}$ (codomain), there does not exist $x = (y - 2)^{\frac{1}{3}}$ in $\mathbb{N}$ (Domain) such that $f(x) = x^3 + 2 = y$.		
Ans.	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).		1
SECTION B			
Q. Numbers 21 to 25 are very short answer questions of 2 marks each.			

21.	Evaluate: $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\left(\frac{-\pi}{2}\right)\right) + \tan^{-1}\left(\tan\frac{2\pi}{3}\right)$		2
Sol.	$\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\left(\frac{-\pi}{2}\right)\right) + \tan^{-1}\left(\tan\frac{2\pi}{3}\right)$ $= -\frac{\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4} - \frac{\pi}{3}$ $= \frac{-5\pi}{12}$	I II	1½ (For any 3 correct result) ½
22. (a)	Show that the function $f(x) = \begin{cases} \frac{\cos x}{-x + \frac{\pi}{2}}, & x \neq \frac{\pi}{2} \\ 1, & x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$.		
Sol.	For showing: $\lim_{x \rightarrow \frac{\pi}{2}} f(x) = 1$ For showing: $f\left(\frac{\pi}{2}\right) = 1$ $\therefore f(x)$ is continuous at $x = \frac{\pi}{2}$.	I II III	1 ½ ½
OR			
22. (b)	Find whether the function $f(x) = \begin{cases} x - 1, & x < 2 \\ 2x - 3, & x \geq 2 \end{cases}$ at $x = 2$ is differentiable or not.		
Sol.	Getting $LHD = 1$ Getting $RHD = 2$ $\therefore LHD \neq RHD$, so $f(x)$ is not differentiable at $x = 2$.	I II III	1 ½ ½

23.	Find the values of x for which $f(x) = x^x, x > 0$ is increasing.		
Sol.	$f'(x) = x^x(1 + \log x)$ Put, $f'(x) = 0 \Rightarrow x^x(1 + \log x) = 0$ As $x^x > 0$ for $x > 0, (1 + \log x) = 0 \Rightarrow x = \frac{1}{e}$ $\therefore f(x)$ is increasing when $x > \frac{1}{e}$ or $x \in \left(\frac{1}{e}, \infty\right)$ or $x \in \left[\frac{1}{e}, \infty\right)$	I	1
		II	$\frac{1}{2}$
		III	$\frac{1}{2}$
24.	If the position vectors of three points A, B and C are $3\hat{i} + \hat{j}$, $5\hat{i} + 6\hat{j} - 3\hat{k}$ and $4\hat{j}$ respectively, then show that they form an isosceles triangle.		
Sol.	$\overrightarrow{AB} = 2\hat{i} + 5\hat{j} - 3\hat{k}, \overrightarrow{BC} = -5\hat{i} - 2\hat{j} + 3\hat{k}, \overrightarrow{CA} = 3\hat{i} - 3\hat{j}$ Now, $ \overrightarrow{AB} = \overrightarrow{BC} = \sqrt{38}$ units, $ \overrightarrow{CA} = \sqrt{18}$ units As, $ \overrightarrow{AB} = \overrightarrow{BC} $ & sum of two sides is greater than the third so, given points form an isosceles triangle.	I	1
		II	$\frac{1}{2}$
		III	$\frac{1}{2}$
25. (a)	Let two rods placed on the ground be represented by vectors $4\hat{i} - \hat{j} + 3\hat{k}$ and $-2\hat{i} + \hat{j} - 2\hat{k}$. Find a vector representing a flag-post of height 5 m that has to be erected perpendicular to both the rods.		
Sol.	Let $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$ $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 3 \\ -2 & 1 & -2 \end{vmatrix} = -\hat{i} + 2\hat{j} + 2\hat{k}$ Now, Required vector = $5 \left(\frac{\vec{a} \times \vec{b}}{ \vec{a} \times \vec{b} } \right) = \begin{cases} \frac{-5}{3}\hat{i} + \frac{10}{3}\hat{j} + \frac{10}{3}\hat{k} \\ \text{or} \\ \frac{5}{3}\hat{i} - \frac{10}{3}\hat{j} - \frac{10}{3}\hat{k} \end{cases}$	I	1
		II	1 (For any one result)

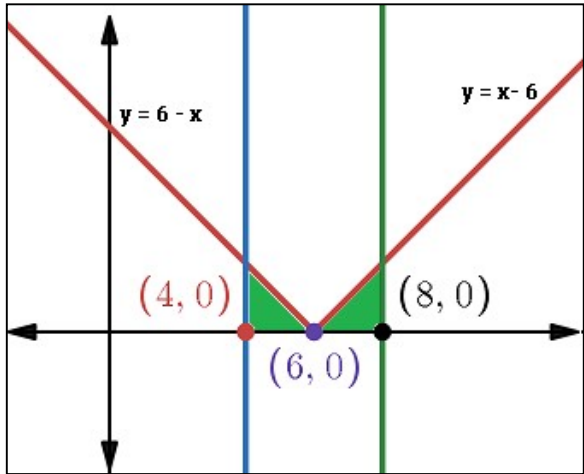
	Alternative approach: Let $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$ Let required vector be $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$ so, $25 = x^2 + y^2 + z^2, 4x - y + 3z = 0, -2x + y - 2z = 0$ on solving we get, $z = -2x, y = z \Rightarrow \frac{100}{9} = z^2 \Rightarrow z = \pm \frac{10}{3}$ $\therefore$ Required vector, $\vec{p} = \begin{cases} -\frac{5}{3}\hat{i} + \frac{10}{3}\hat{j} + \frac{10}{3}\hat{k} \\ or \\ \frac{5}{3}\hat{i} - \frac{10}{3}\hat{j} - \frac{10}{3}\hat{k} \end{cases}$	I	1
		II	1 (For any one result)
OR			
25. (b)	A unit vector $\vec{a}$ is such that it makes an angle $\frac{\pi}{4}$ with x-axis, $\frac{\pi}{3}$ with y-axis and an acute angle θ with z-axis. Find θ and the components of $\vec{a}$.		
Sol.	$\cos^2\left(\frac{\pi}{4}\right) + \cos^2\left(\frac{\pi}{3}\right) + \cos^2\theta = 1 \Rightarrow \cos\theta = \frac{1}{2}$ (As, θ is acute) $\Rightarrow \theta = \frac{\pi}{3}$ & Components of $\vec{a} = \frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}$	I	1
		II	1
SECTION C			
Q. Numbers 26 to 31 are short answer questions of 3 marks each.			
26. (a)	Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. A function $f: A \rightarrow B$ is defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Find whether f is one-one and onto.		
Sol :	Let $f(a) = f(b)$ for some $a, b \in \mathbb{R} - \{3\}$ so, $\left(\frac{a-2}{a-3}\right) = \left(\frac{b-2}{b-3}\right) \Rightarrow ab - 2b - 3a + 6 = ab - 3b - 2a + 6$ $\Rightarrow a = b$, Thus f is one-one function.	I	1
		II	$\frac{1}{2}$

Sol.	$A^2 = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 11 & 14 & 0 \\ 7 & 18 & 0 \\ 0 & 0 & 25 \end{bmatrix}$ $A^2 - 7A + 10I = \begin{bmatrix} 11 & 14 & 0 \\ 7 & 18 & 0 \\ 0 & 0 & 25 \end{bmatrix} - \begin{bmatrix} 21 & 14 & 0 \\ 7 & 28 & 0 \\ 0 & 0 & 35 \end{bmatrix} + \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$ $A^2 - 7A + 10I = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	I	1½ (½ for any three correct elements)
28. (a)	If $xy = e^{x-y}$, then find $\frac{dy}{dx}$.	II	1
Sol.	$xy = e^{x-y} \Rightarrow x \frac{dy}{dx} + y = e^{x-y} \left(1 - \frac{dy}{dx} \right)$ $\Rightarrow (x + e^{x-y}) \frac{dy}{dx} = e^{x-y} - y \Rightarrow \frac{dy}{dx} = \frac{e^{x-y} - y}{x + e^{x-y}} \text{ or } \frac{y(x-1)}{x(y+1)}$ Alternative method: $xy = e^{x-y} \Rightarrow \log x + \log y = (x - y)$ $\Rightarrow \frac{1}{x} + \frac{1}{y} \frac{dy}{dx} = 1 - \frac{dy}{dx}$ $\Rightarrow \frac{dy}{dx} = \frac{1 - \frac{1}{x}}{\frac{1}{y} + 1} = \frac{y(x-1)}{x(y+1)}$	I	1½
		II	1½
		I	1
		II	1
		III	1
OR			
28. (b)	Differentiate $\tan^{-1} \left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right)$ with respect to $\cos^{-1} x^2$.		

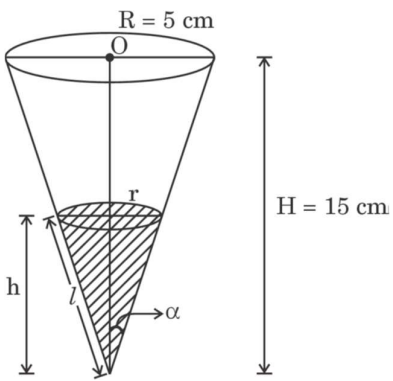
Sol.	Let $y = \tan^{-1} \left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right)$ & put $x^2 = \cos \theta$ $y = \tan^{-1} \left(\frac{\sqrt{1+\cos \theta} - \sqrt{1-\cos \theta}}{\sqrt{1+\cos \theta} + \sqrt{1-\cos \theta}} \right)$ $y = \tan^{-1} \left(\frac{\cos \left(\frac{\theta}{2} \right) - \sin \left(\frac{\theta}{2} \right)}{\cos \left(\frac{\theta}{2} \right) + \sin \left(\frac{\theta}{2} \right)} \right) = \tan^{-1} \left(\tan \left(\frac{\pi}{4} - \frac{\theta}{2} \right) \right)$ Thus, $y = \left(\frac{\pi}{4} - \frac{\theta}{2} \right) \Rightarrow \frac{dy}{d\theta} = \frac{-1}{2}$	I	$\frac{1}{2}$												
		II	$1\frac{1}{2}$												
		III	1												
29.	Solve the following Linear programming problem graphically: Maximize $Z = 200x + 120y$ subject to the constraints $x + y \leq 300$ $3x + y \leq 600$ $x - y \geq -100$ $x \geq 0, y \geq 0$														
Sol.	 <table><tr><th>Corner point</th><th>Value of $Z=200x+120y$</th></tr><tr><td>A (100,200)</td><td>44000</td></tr><tr><td>B (150,150)</td><td>48000</td></tr><tr><td>C (0, 100)</td><td>12000</td></tr><tr><td>D (0, 0)</td><td>0</td></tr><tr><td>E (200, 0)</td><td>40000</td></tr></table> Maximum value of Z is 48000 at $x = 150$ & $y = 150$.	Corner point	Value of $Z=200x+120y$	A (100,200)	44000	B (150,150)	48000	C (0, 100)	12000	D (0, 0)	0	E (200, 0)	40000	I	$1\frac{1}{2}$ For correct graph & shading
Corner point	Value of $Z=200x+120y$														
A (100,200)	44000														
B (150,150)	48000														
C (0, 100)	12000														
D (0, 0)	0														
E (200, 0)	40000														
		II	1												
		III	$\frac{1}{2}$												
30. (a)	Find a point on the line $\frac{x-2}{3} = \frac{1-y}{2} = \frac{z-3}{2}$ at a distance of $\sqrt{2}$ units from the point $(1, 2, 3)$.														

Sol.	Let $\frac{x-2}{3} = \frac{y-1}{-2} = \frac{z-3}{2} = k$ A general point P on the line be $(3k+2, -2k+1, 2k+3)$ It is given that $PQ = \sqrt{2}$ where $Q(1,2,3)$, so $PQ = \sqrt{(3k+1)^2 + (-2k-1)^2 + (2k)^2} = \sqrt{2}$ $\Rightarrow 17k^2 + 10k = 0$ $\therefore k = 0 \text{ or } \frac{-10}{17}$ Thus, required points are $(2,1,3) \& \left(\frac{4}{17}, \frac{37}{17}, \frac{31}{17}\right)$	I	1
	OR		
30. (b)	Find the shortest distance between the lines: $\vec{r} = (4 + \lambda)\hat{i} + (2\lambda - 1)\hat{j} - 3\lambda\hat{k}$ $\vec{r} = (1 + 2\mu)\hat{i} + (2 - \mu)\hat{k} + (4\mu - 1)\hat{j}$		
Sol	Here, $\vec{a}_1 = 4\hat{i} - \hat{j}, \vec{b}_1 = \hat{i} + 2\hat{j} - 3\hat{k}, \vec{a}_2 = \hat{i} - \hat{j} + 2\hat{k}, \vec{b}_2 = 2\hat{i} + 4\hat{j} - 5\hat{k}$ Now, $\vec{a}_2 - \vec{a}_1 = -3\hat{i} + 2\hat{k}, \quad \vec{b}_1 \times \vec{b}_2 = 2\hat{i} - \hat{j} \text{ and } \vec{b}_1 \times \vec{b}_2 = \sqrt{5}$ $\text{Required distance} = \left \frac{(-3\hat{i} + 2\hat{k}) \cdot (2\hat{i} - \hat{j})}{\sqrt{5}} \right = \frac{6}{\sqrt{5}} \text{ or } \frac{6\sqrt{5}}{5}$	I	1
31.	In a school, the probability of holding a debate competition is $\frac{1}{3}$ and that of a quiz competition is $\frac{2}{3}$. In the two participating teams, A has 4 girls and 6 boys and B has 7 girls and 3 boys. If a debate competition is held, the students are selected from team A and for the quiz competition they are selected from team B. If only two students are to be chosen from the teams,	II	$\frac{1}{2}$
		III	$\frac{1}{2}$

	then find the probability that one will be a girl and the other a boy.		
Sol.	Let D be the event a debate competition is held, Q be the event a quiz competition is held and E be the event that one girl & one boy are chosen. $P(E D) = 2 \times \frac{4}{10} \times \frac{6}{9} = \frac{8}{15}, P(E Q) = 2 \times \frac{7}{10} \times \frac{3}{9} = \frac{7}{15}$ $\text{Now, } P(E) = \frac{1}{3} \times \frac{8}{15} + \frac{2}{3} \times \frac{7}{15} = \frac{22}{45}$	I II	1+1 1
SECTION D			
Q. Numbers 32 to 35 are long answer questions of 5 marks each.			
32. (a)	Find : $\int \frac{x}{(x-1)(x^2+4)} dx$		
Sol.	Let $\frac{x}{(x-1)(x^2+4)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+4}$ on solving we get, $A = \frac{1}{5}, B = \frac{-1}{5}, C = \frac{4}{5}$ so, $\int \frac{x}{(x-1)(x^2+4)} dx = \frac{1}{5} \int \frac{dx}{x-1} - \frac{1}{10} \int \frac{2x dx}{x^2+4} + \frac{4}{5} \int \frac{dx}{x^2+4}$ $\int \frac{x}{(x-1)(x^2+4)} dx = \frac{1}{5} \log x-1 - \frac{1}{10} \log(x^2+4) + \frac{2}{5} \tan^{-1}\left(\frac{x}{2}\right) + c$	I II III	1 1½ 2½
OR			
32. (b)	Evaluate : $\int_0^1 \frac{x \tan^{-1} x}{(1+x^2)^{\frac{3}{2}}} dx.$		

Sol.	Let $x = \tan t \Rightarrow dx = \sec^2 t dt$ $I(\text{Let}) = \int_0^1 \frac{x \tan^{-1} x}{(1+x^2)^{\frac{3}{2}}} dx = \int_0^{\frac{\pi}{4}} \frac{t \tan t}{\sec^3 t} \sec^2 t dt = \int_0^{\frac{\pi}{4}} t \sin t dt$ $I = (-t \cos t + \sin t) \Big _0^{\frac{\pi}{4}}$ $I = \frac{-\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} \text{ or } \frac{-\pi + 4}{4\sqrt{2}}$	I	1
		II	2
		III	1
		IV	1
33.	Using integration, find the area of the region enclosed by the curve $y = x - 6 $, the x -axis, and between $x = 4$ and $x = 8$.		
Sol.	 Required Area = $\int_4^8 y dx = \int_4^8 x - 6 dx$ $\text{Area} = -\int_4^6 (x - 6) dx + \int_6^8 (x - 6) dx$ $\text{Area} = -\left(\frac{(x-6)^2}{2}\right)_4^6 + \left(\frac{(x-6)^2}{2}\right)_6^8 = 2 + 2 = 4 \text{ or } 4 \text{ sq. units}$	I	1 (For correct graph)
		II	1
		III	1
		IV	2
34. (a)	Solve the differential equation: $ye^y dx = (y^3 + 2xe^y) dy$, when $y(0) = 1$.		

Sol.	Given differential eq. can be written as $\frac{dx}{dy} - \frac{2}{y}x = y^2 e^{-y}$	I	1
	Integrating factor = $e^{\int \frac{-2}{y} dy} = \frac{1}{y^2}$	II	1
	General solution: $x \left(\frac{1}{y^2} \right) = \int \frac{1}{y^2} \cdot y^2 e^{-y} dy$	III	1
	$x \left(\frac{1}{y^2} \right) = -e^{-y} + c$ or $x = y^2 (-e^{-y} + c)$	IV	1
	Now, when $x = 0, y = 1: 0 = -e^{-1} + c \Rightarrow c = e^{-1}$	V	$\frac{1}{2}$
	Required particular solution is, $x = y^2 (e^{-1} - e^{-y})$	VI	$\frac{1}{2}$
OR			
34. (b)	Find the general solution of the differential equation: $(x^3 - 3xy^2)dx = (y^3 - 3x^2y) dy.$		
Sol.	$\frac{dy}{dx} = \frac{x^3 - 3xy^2}{y^3 - 3x^2y}$	I	1
	Put $y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$, so	II	1
	$v + x \frac{dv}{dx} = \frac{1 - 3v^2}{v^3 - 3v} \Rightarrow x \frac{dv}{dx} = \frac{1 - 3v^2}{v^3 - 3v} - v = \frac{1 - v^4}{v^3 - 3v}$		
	$\Rightarrow \int \frac{3v - v^3}{v^4 - 1} dv = \int \frac{1}{x} dx$	III	1
	$\Rightarrow \frac{3}{4} \log \left \frac{v^2 - 1}{v^2 + 1} \right - \frac{1}{4} \log v^4 - 1 = \log x + c$	IV	$1\frac{1}{2}$
	$\therefore \frac{3}{4} \log \left \frac{y^2 - x^2}{y^2 + x^2} \right - \frac{1}{4} \log \left \frac{y^4 - x^4}{x^4} \right = \log x + c$	V	$\frac{1}{2}$

35.	Find the equation of a line (in vector and cartesian form) that passes through the point of intersection of lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ and is parallel to the vector $3\hat{i} + 2\hat{j} - 8\hat{k}$.		
Sol.	Let $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = a$ & $\frac{x-4}{5} = \frac{y-1}{2} = z = b$ So any general point on these are $P(2a+1, 3a+2, 4a+3)$ and $Q(5b+4, 2b+1, b)$ respectively. For intersection points: $2a+1 = 5b+4, 4a+3 = b$ on solving we get, $a = b = -1$ Thus, required point of intersection is $(-1, -1, -1)$. Required equation of line (cartesian form): $\frac{x+1}{3} = \frac{y+1}{2} = \frac{z+1}{-8}$ Required equation of line (Vector form): $\vec{r} = (-\hat{i} - \hat{j} - \hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 8\hat{k})$	I II III IV V	1 1 1 1 1
SECTION E			
This section (Q. 36 to 38) has 3 case study based questions of 4 marks each.			
36.	At a birthday party, children are being served orange juice in conical cups, as shown in the figure. 		

	Each cup is 15 cm deep and has a radius 5 cm. The juice is being poured into this cup at a rate of $0.1 \text{ cm}^3/\text{s}$. On the basis of the above information, answer the following questions: (i) Establish a relation between the height h of the juice in the cup and radius r of the surface of the juice in the cup, if the semi-vertical angle of the cone is α. (ii) At what rate is the juice level in the cup rising when the juice is 6 cm deep ? (iii) (a) When the juice is 6 cm deep, then find at what rate is the upper surface area of juice increasing ? OR (iii) (b) When the juice is 6 cm deep, then find the rate at which the wetted surface area of the cup is increasing.		
Sol.	(i) $\tan \alpha = \frac{r}{h} = \frac{5}{15} \Rightarrow r = \frac{h}{3}$ (ii) Volume, $V = \frac{1}{27} \pi h^3 \Rightarrow \frac{dV}{dt} = \frac{1}{9} \pi h^2 \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{40\pi} \text{ cm/sec}$ (iii) (a) Upper surface area of juice, $A = \pi r^2 = \frac{\pi h^2}{9}$ $\frac{dA}{dt} = \frac{2\pi h}{9} \frac{dh}{dt} = \frac{1}{30} \text{ cm}^2/\text{sec}$ OR (iii) (b) Wetted surface area, $S = \pi r l = \frac{\sqrt{10}\pi h^2}{9}$ $\frac{dS}{dt} = \frac{2\sqrt{10}\pi h}{9} \frac{dh}{dt} = \frac{\sqrt{10}}{30} \text{ cm}^2/\text{sec}$	I I I II I II	1 1 1 1 1 1
37.	A carpenter needs to design a wooden box in the shape of a cuboid such that the sum of its length and breadth is 3 cm more than its height. Twice of its length, thrice of its breadth and its height add up to 10 cm. Its breadth added to 7 times its height is 1 cm less than 3 times its length. On the basis of the above information, answer the following questions:		

	(i) Write the equations representing the various dimensions & express them as the matrix equation $AX = B$. (ii) Find if A^{-1} exists. Justify your answer. (iii) (a) Find A^{-1}. OR (iii) (b) Find $A^2 + 7I$.		
Sol.	(i) $\begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ -3 & 1 & 7 \end{pmatrix} \begin{pmatrix} l \\ b \\ h \end{pmatrix} = \begin{pmatrix} 3 \\ 10 \\ -1 \end{pmatrix}$ $A \quad X = B$ (ii) $A = -8 \neq 0$, so A^{-1} exists. (iii) (a) $A^{-1} = \frac{1}{ A }(\text{adj}A) = \frac{1}{-8} \begin{pmatrix} 20 & -8 & 4 \\ -17 & 4 & -3 \\ 11 & -4 & 1 \end{pmatrix}$ OR (iii) (b) $A^2 = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ -3 & 1 & 7 \end{pmatrix} \begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ -3 & 1 & 7 \end{pmatrix} = \begin{pmatrix} 6 & 3 & -7 \\ 5 & 12 & 8 \\ -22 & 7 & 53 \end{pmatrix}$ Thus, $A^2 + 7I = \begin{pmatrix} 13 & 3 & -7 \\ 5 & 19 & 8 \\ -22 & 7 & 60 \end{pmatrix}$ Alternative : If $A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{pmatrix}$, then $A = 8$ & $A^{-1} = \frac{1}{8} \begin{pmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{pmatrix}$ $A^2 = \begin{pmatrix} 0 & 5 & 7 \\ 11 & 10 & -6 \\ -20 & 7 & 45 \end{pmatrix}$, $A^2 + 7I = \begin{pmatrix} 7 & 5 & 7 \\ 11 & 17 & -6 \\ -20 & 7 & 52 \end{pmatrix}$ Note: In this question, students may take different A by rearranging of terms so check the correctness of the solution accordingly.	I I I I I II I II	1 1 2 (1 Mark for any 4 correct entries) 1½ ½ 1 + 2 1 1

38.	An NGO organises a charity event in which they decide to distribute woollen caps to protect children from winter. The caps to be distributed are in three separate boxes, Box I has 30 red caps, Box II has 20 red and 10 green caps, & Box III has 30 green caps. The probability that a Box i is selected and a cap picked out is $\frac{i}{6}$, where $i = 1, 2, 3$. Based on the above information, answer the following questions : A person selects a cap. (i) What is the probability that he selects a red cap ? (ii) If he selects a green cap, what is the probability that the cap has come from Box II ?		
Sol.	(i) Let B_1, B_2 & B_3 be the event that box I, box II and box III selected respectively. $P(B_1) = \frac{1}{6}, P(B_2) = \frac{2}{6}, P(B_3) = \frac{3}{6}$ $\text{Now, } P(\text{selecting a red cap}) = \frac{1}{6} \times 1 + \frac{2}{6} \times \frac{2}{3} + \frac{3}{6} \times 0 = \frac{7}{18}$ (ii) $P(\text{selecting a green cap}) = 1 - \frac{7}{18} = \frac{11}{18}$ $\text{Now, } P\left(\frac{B_2}{\text{Green cap}}\right) = \frac{\frac{2}{6} \times \frac{1}{3}}{\frac{11}{18}} = \frac{2}{11}$	I II I II	1 1 1 1