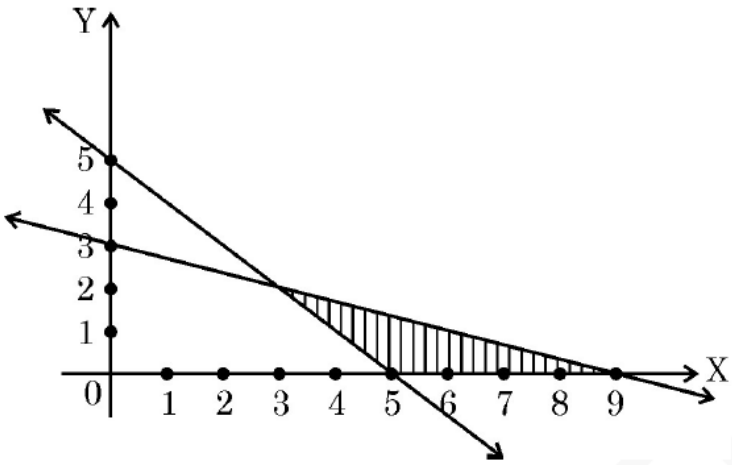


Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Steps	Marks
SECTION A			
Q. Number 1 to 20 are multiple choice questions of 1 mark each.			
1.	A relation R on set $A = \{1, 2, 3\}$ defined as $R = \{(1, 1), (2, 2), (1, 2)\}$ is (A) Reflexive only (B) Reflexive and Transitive (C) Symmetric and Transitive (D) Transitive only		
Sol.	(D) Transitive only		1
2.	If A and B are square matrices of same order, then which of the following statements is/are always true ? (i) $(A + B)(A - B) = A^2 - B^2$ (ii) $AB = BA$ (iii) $(A + B)^2 = A^2 + AB + BA + B^2$ (iv) $AB = 0 \Rightarrow A = 0$ or $B = 0$ (A) Only (i) and (iii) (B) Only (ii) and (iii) (C) Only (iii) (D) Only (iii) and (iv)		
Sol.	(C) Only (iii)		1
3.	If $A = \begin{bmatrix} 1 & a & b \\ -1 & 2 & c \\ 0 & 5 & 3 \end{bmatrix}$ is a symmetric matrix, then the value of $3a + b + c$ is (A) 2 (B) 6 (C) 4 (D) 0		
Sol.	(A) 2		1
4.	If $A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$ and $A + A' = I$, then the value of $x \in \left[0, \frac{\pi}{2}\right]$ is (A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$		
Sol.	(C) $\frac{\pi}{3}$		1

5.	For a square matrix A, $(3A)^{-1} =$ (A) $3A^{-1}$ (B) $9A^{-1}$ (C) $\frac{1}{3}A^{-1}$ (D) $\frac{1}{9}A^{-1}$	
Sol.	(C) $\frac{1}{3}A^{-1}$	1
6.	If $\begin{vmatrix} -1 & -2 & 5 \\ -2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = -86$, then the sum of all possible values of a is (A) 4 (B) 5 (C) -4 (D) 9	
Sol.	(C) -4	1
7.	If $e^{-x} + e^{-y} = 2$, then $\frac{dy}{dx}$ is (A) e^{x-y} (B) e^{y-x} (C) $-e^{x-y}$ (D) $-e^{y-x}$	
Sol.	(D) $-e^{y-x}$	1
8.	For $f(x) = x + \frac{1}{x}$ ($x \neq 0$) (A) local maximum value is 2 (B) local minimum value is -2 (C) local maximum value is -2 (D) local minimum value < local maximum value	
Sol.	(C) local maximum value is -2	1

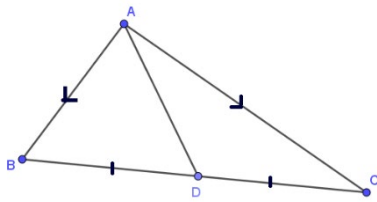
9.	If $\int_0^{2a} \frac{1}{1+4x^2} dx = \frac{\pi}{6}$, then the value of a is (A) $\frac{\sqrt{3}}{4}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\sqrt{3}$ (D) $2\sqrt{3}$	
Sol.	(A) $\frac{\sqrt{3}}{4}$	1
10.	Which of the following expressions will give the area of region bounded by the curve $y = x^2$ and line $y = 16$? (A) $\int_0^4 x^2 dx$ (B) $2 \int_0^4 x^2 dx$ (C) $\int_0^{16} \sqrt{y} dy$ (D) $2 \int_0^{16} \sqrt{y} dy$	
Sol.	(D) $2 \int_0^{16} \sqrt{y} dy$	1
11.	The general solution of the differential equation $\frac{dy}{dx} = \frac{\sqrt{y}}{\sqrt{x}}$ is (A) $\log \sqrt{y} = \log \sqrt{x} + C$ (B) $\sqrt{y} + \sqrt{x} = C$ (C) $\sqrt{y} - \sqrt{x} = C$ (D) $\log \sqrt{y} + \log \sqrt{x} = C$	
Sol.	(C) $\sqrt{y} - \sqrt{x} = C$	1
12.	The integrating factor of the differential equation $2x \frac{dy}{dx} - y = 3$ is (A) $\sqrt{x}$ (B) $\frac{1}{\sqrt{x}}$ (C) e^x (D) e^{-x}	
Sol.	(B) $\frac{1}{\sqrt{x}}$	1

13.	If $\vec{a} = 5$ and $-2 \leq \lambda \leq 1$, then the sum of greatest and the smallest value of $\lambda \vec{a}$ is (A) -5 (B) 5 (C) 10 (D) 15	
Sol.	(C) 10	1
14.	Vector of magnitude 3 making equal angles with x and y axes and perpendicular to z axis is (A) $\hat{i} + 2\sqrt{2}\hat{j}$ (B) $3\hat{k}$ (C) $\frac{3\sqrt{2}}{2}\hat{i} + \frac{3\sqrt{2}}{2}\hat{j}$ (D) $\sqrt{3}\hat{i} + \sqrt{3}\hat{j} + \sqrt{3}\hat{k}$	
Sol.	(C) $\frac{3\sqrt{2}}{2}\hat{i} + \frac{3\sqrt{2}}{2}\hat{j}$	1
15.	Direction cosines of the line given by equations : $\frac{2x-1}{4} = \frac{1-y}{3} = \frac{-z}{6}$ are (A) $2, -3, -6$ (B) $\frac{2}{7}, \frac{-3}{7}, \frac{-6}{7}$ (C) $\frac{2}{7}, \frac{-3}{7}, \frac{6}{7}$ (D) $\frac{4}{\sqrt{61}}, \frac{-3}{\sqrt{61}}, \frac{-6}{\sqrt{61}}$	
Sol.	(B) $\frac{2}{7}, \frac{-3}{7}, \frac{-6}{7}$	1
16.	In a linear programming problem, the linear function which has to be maximized or minimized is called (A) a feasible function (B) an objective function (C) an optimal function (D) a constraint	
Sol.	(B) an objective function	1

17.	For the feasible region shown below, the non-trivial constraints of the linear programming problem are  (A) $x + y \leq 5, x + 3y \leq 9$ (B) $x + y \leq 5, x + 3y \geq 9$ (C) $x + y \geq 5, x + 3y \leq 9$ (D) $x + y \geq 5, 3x + y \leq 9$
Sol.	(C) $x + y \geq 5, x + 3y \leq 9$ 1
18.	For two events A and B such that $P(A) \neq 0$ and $P(B) \neq 1$, $P(A'/B') =$ (A) $1 - P(A/B)$ (B) $1 - P(A'/B)$ (C) $\frac{1 - P(A \cap B)}{P(B')}$ (D) $\frac{1 - P(A \cup B)}{P(B')}$
Sol.	(D) $\frac{1 - P(A \cup B)}{P(B')}$ 1
Assertion – Reason Based Questions Direction : Question numbers 19 and 20 are Assertion (A) and Reason (R) based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below. (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	

19.	For two vectors $\vec{a}$ and $\vec{b}$ Assertion (A) : $ \vec{a} \times \vec{b} ^2 + (\vec{a} \cdot \vec{b})^2 = \vec{a} ^2 \vec{b} ^2$ Reason (R) : $ \vec{a} \times \vec{b} = (\vec{a} \cdot \vec{b}) \tan \theta, \left(\theta \neq \frac{\pi}{2}\right)$		
Sol.	(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).		1
20.	Assertion (A) : A line can have direction cosines $< 1, 1, 1 >$ Reason (R) : $\cos \theta = 1$ is possible for $\theta = 0$.		
Sol.	(D) Assertion (A) is false, but Reason (R) is true.		1
SECTION B Q. Numbers 21 to 25 are very short answer questions of 2 marks each.			
21.	(a) Check whether $f : \mathbb{R} - \{3\} \rightarrow \mathbb{R}$ defined as $f(x) = \frac{x-2}{x-3}$ is onto or not. OR (b) Check whether $f : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z} \times \mathbb{Z}$ (where $\mathbb{Z}$ is the set of integers) defined as $f(x, y) = (2y, 3x)$ is injective or not.		
Sol. 21(a)	Let $y = \frac{x-2}{x-3}$ i.e. $x = \frac{3y-2}{y-1}$ Here $y \neq 1 \therefore R_f = \mathbb{R} - \{1\} \neq \text{Codomain}$ $\therefore f$ is not onto.	I II	1 1
OR			
Sol. 21(b)	Let $(x_1, y_1), (x_2, y_2) \in \mathbb{Z} \times \mathbb{Z}$ such that $f(x_1, y_1) = f(x_2, y_2)$ $\Rightarrow (2y_1, 3x_1) = (2y_2, 3x_2)$ $\Rightarrow y_1 = y_2, x_1 = x_2$ So, $(x_1, y_1) = (x_2, y_2)$ $\therefore f$ is injective.	I II	1 1
22.	If $x = a \sin^3 t, y = b \cos^3 t$, then find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$.		

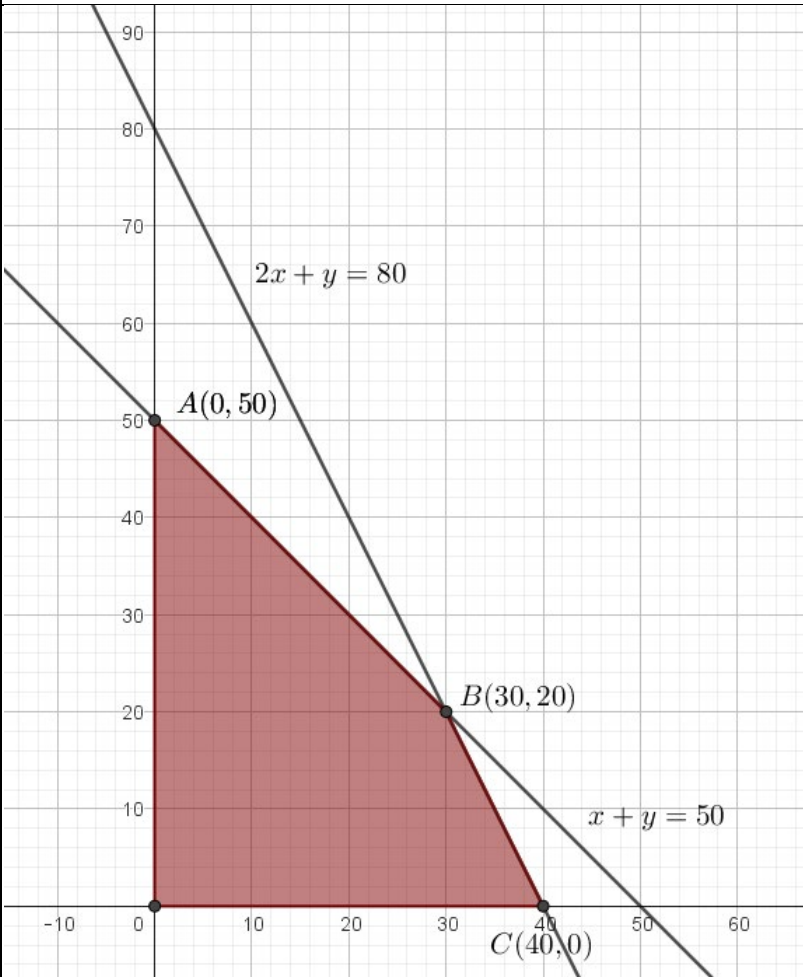
Sol.	$\frac{dx}{dt} = 3a \sin^2 t \cos t, \frac{dy}{dt} = -3b \cos^2 t \sin t$ $\frac{dy}{dx} = -\frac{b}{a} \cot t$ $\left. \frac{dy}{dx} \right _{t=\frac{\pi}{4}} = -\frac{b}{a}$	I	1
		II	$\frac{1}{2}$
		III	$\frac{1}{2}$
23.	(a) Find the absolute maximum value of $f(x) = \cos x + \sin^2 x, x \in [0, \pi]$ OR (b) If the volume of a solid hemisphere increases at a uniform rate, prove that its surface area varies inversely as its radius.		
Sol. 23(a)	$f(x) = \cos x + \sin^2 x, x \in [0, \pi]$ $f'(x) = -\sin x + 2 \sin x \cos x$ $f'(x) = 0 \Rightarrow \sin x (2 \cos x - 1) = 0$ $\Rightarrow \sin x = 0$ or $\cos x = \frac{1}{2}$ $\therefore x = \frac{\pi}{3}$ Now, $f(0) = 1, f\left(\frac{\pi}{3}\right) = \frac{5}{4}, f(\pi) = -1$ $\therefore$ Absolute maximum value of $f(x)$ is $\frac{5}{4}$.	I	1
		II	$\frac{1}{2}$
		III	$\frac{1}{2}$
OR			
Sol. 23(b)	$V = \frac{2}{3} \pi r^3 \Rightarrow \frac{dV}{dt} = 2\pi r^2 \frac{dr}{dt} = C$ (say) $\Rightarrow \frac{dr}{dt} = \frac{C}{2\pi r^2}$ Now, $S = 3\pi r^2$ $\Rightarrow \frac{dS}{dt} = 6\pi r \frac{dr}{dt} = 6\pi r \times \frac{C}{2\pi r^2} = \frac{3C}{r}$ $\therefore S$ varies inversely as its radius.	I	1
		II	1

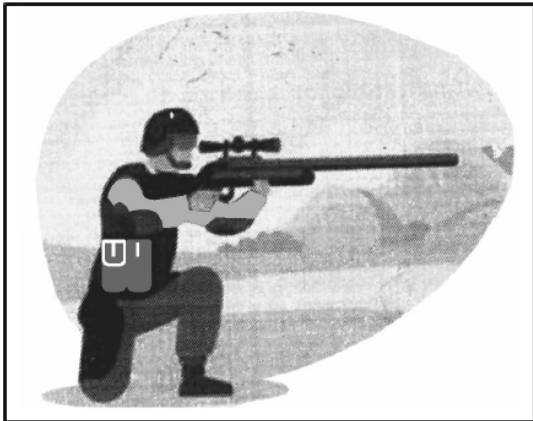
24.	If $\vec{AB} = \hat{j} + \hat{k}$ and $\vec{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$ represent the two vectors along the sides AB and AC of $\triangle ABC$, prove that the median $\vec{AD} = \frac{\vec{AB} + \vec{AC}}{2}$, where D is midpoint of BC. Hence, find the length of median AD.		
Sol.	Considering A as the origin, $\vec{AB}$ and $\vec{AC}$ are p.v. of B and C respectively. Hence p.v. of the mid point D is $\frac{\vec{AB} + \vec{AC}}{2}$  So, $\vec{AD} = \frac{(\hat{j} + \hat{k}) + (3\hat{i} - \hat{j} + 4\hat{k})}{2}$ $\vec{AD} = \frac{3\hat{i} + 5\hat{k}}{2}$ $\vec{AD} = \frac{1}{2} \sqrt{3^2 + 5^2} = \frac{\sqrt{34}}{2}$	I	1
		II	$\frac{1}{2}$
		III	$\frac{1}{2}$
25.	Find the co-ordinates of the point on the line $\vec{r} = -\hat{j} + 3\hat{k} + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ such that the sum of co-ordinates is 3.		
Sol.	$\vec{r} = -\hat{j} + 3\hat{k} + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ i.e. $\vec{r} = 2\lambda\hat{i} + (-2\lambda - 1)\hat{j} + (\lambda + 3)\hat{k}$ According to question, $2\lambda + (-2\lambda - 1) + (\lambda + 3) = 3 \Rightarrow \lambda = 1$ $\therefore \vec{r} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ Co-ordinates of the required point is $(2, -3, 4)$.	I	1
		II	1
SECTION C Q. Numbers 26 to 31 are short answer questions of 3 marks each.			
26.	Find : $\int \frac{x+2}{\sqrt{9x-x^2}} dx$		
Sol.	$I = \int \frac{x+2}{\sqrt{9x-x^2}} dx$ Let $x+2 = A(9-2x) + B$ $\Rightarrow A = -\frac{1}{2}, B = \frac{13}{2}$	I	1

	$I = -\frac{1}{2} \int \frac{9-2x}{\sqrt{9x-x^2}} dx + \frac{13}{2} \int \frac{1}{\sqrt{9x-x^2}} dx$ $= -\frac{1}{2} \left(2\sqrt{9x-x^2} \right) + \frac{13}{2} \int \frac{1}{\sqrt{\left(\frac{9}{2}\right)^2 - \left(x-\frac{9}{2}\right)^2}} dx$ $= -\sqrt{9x-x^2} + \frac{13}{2} \sin^{-1} \left(\frac{2x-9}{9} \right) + C$	II	1
		III	1
27.	(a) Evaluate : $\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{dx}{1+\sqrt{\cot x}}$ OR (b) Evaluate : $\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin x + \cos x) dx$		
Sol. 27(a)	$I = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{dx}{1+\sqrt{\cot x}} = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \quad (i)$ Applying $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$, we get $I = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \quad (ii)$ Adding (i) and (ii) $2I = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} 1 \cdot dx$ $= [x]_{\pi/12}^{5\pi/12} = \frac{\pi}{3}$ $\Rightarrow I = \frac{\pi}{6}$	I	$\frac{1}{2}$
		II	1
		III	1
		IV	$\frac{1}{2}$
OR			

Sol. 27(b)	$I = \int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin x + \cos x) dx$ $= \int_{-\frac{\pi}{6}}^0 (-\sin x + \cos x) dx + \int_0^{\frac{\pi}{2}} (\sin x + \cos x) dx$ $= [\cos x + \sin x]_{-\pi/6}^0 + [-\cos x + \sin x]_0^{\pi/2}$ $= \left(1 - \frac{\sqrt{3}}{2} + \frac{1}{2}\right) + (1 + 1)$ $= \frac{1}{2}(7 - \sqrt{3})$	I II III	1 1 1
28.	If $\frac{d}{dx}(F(x)) = \frac{1}{e^x + 1}$, then find $F(x)$ given that $F(0) = \log \frac{1}{2}$.		
Sol.	$\frac{d}{dx}(F(x)) = \frac{1}{e^x + 1}$ $\Rightarrow F(x) = \int \frac{1}{e^x + 1} dx$ $= \int \frac{e^{-x}}{1 + e^{-x}} dx$ $= -\log(1 + e^{-x}) + C$ Now, $F(0) = -\log 2 + C \Rightarrow \log \frac{1}{2} + \log 2 = C \Rightarrow C = 0$ $\therefore F(x) = -\log(1 + e^{-x})$ Note: The above expression is equivalent to $F(x) = x - \log(e^x + 1)$	I II III IV	1 1 $\frac{1}{2}$ $\frac{1}{2}$
29. (a)	(a) Solve the following differential equation : $x \frac{dy}{dx} = y - x \sin^2 \left(\frac{y}{x} \right), \text{ given that } y(1) = \frac{\pi}{6}$ OR (b) Find the general solution of the differential equation : $y \log y \frac{dx}{dy} + x = \frac{2}{y}$.		
Sol. 29(a)	Here, $\frac{dy}{dx} = \frac{y}{x} - \sin^2 \left(\frac{y}{x} \right)$ Put $\frac{y}{x} = v \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$	I II	$\frac{1}{2}$ $\frac{1}{2}$

	Differential equation reduces to $v + x \frac{dv}{dx} = v - \sin^2 v$ i.e. $x \frac{dv}{dx} = -\sin^2 v$ $\Rightarrow -\int \operatorname{cosec}^2 v \, dv = \int \frac{dx}{x}$ $\Rightarrow \cot v = \log x + C$ $\Rightarrow \cot\left(\frac{y}{x}\right) = \log x + C$ Now $y(1) = \frac{\pi}{6}$ gives $C = \sqrt{3}$ Required solution is : $\cot\left(\frac{y}{x}\right) = \log x + \sqrt{3}$	III	$\frac{1}{2}$
		IV	1
		V	$\frac{1}{2}$
OR			
Sol. 29(b)	Given differential equation can be written as $\frac{dx}{dy} + \frac{1}{y \log y} \cdot x = \frac{2}{y^2 \log y}$ I.F. = $e^{\int \frac{1}{y \log y} dy} = e^{\log(\log y)} = \log y$ Solution is given by $x \times \log y = \int \frac{2}{y^2 \log y} \times \log y \, dy = \int \frac{2}{y^2} dy$ $\Rightarrow x \log y = -\frac{2}{y} + C \quad \text{or} \quad xy \log y = -2 + Cy$	I	1
		II	1
		III	$\frac{1}{2}$
		IV	$\frac{1}{2}$

30.	Solve the following linear programming problem graphically : Maximize $Z = 10500x + 9000y$ Subject to constraints $x + y \leq 50$ $2x + y \leq 80$ $x, y \geq 0$												
Sol.	 <table data-bbox="173 1471 815 1771"><tr><th>Corner Point</th><th>Value of $Z = 10500x + 9000y$</th></tr><tr><td>$O(0,0)$</td><td>0</td></tr><tr><td>$A(0,50)$</td><td>4,50,000</td></tr><tr><td>$B(30,20)$</td><td>4,95,000</td></tr><tr><td>$C(40,0)$</td><td>4,20,000</td></tr></table> $Z_{\max} = 4,95,000$ when $x = 30$, $y = 20$	Corner Point	Value of $Z = 10500x + 9000y$	$O(0,0)$	0	$A(0,50)$	4,50,000	$B(30,20)$	4,95,000	$C(40,0)$	4,20,000	I	For correct graph and shading $1\frac{1}{2}$
Corner Point	Value of $Z = 10500x + 9000y$												
$O(0,0)$	0												
$A(0,50)$	4,50,000												
$B(30,20)$	4,95,000												
$C(40,0)$	4,20,000												
		II	For correct table 1										
		III	$\frac{1}{2}$										

31.	(a) The probability of hitting the target by a trained sniper is three times the probability of not hitting the target on a stormy day due to high wind speed.  The sniper fired two shots on the target on a stormy day when wind speed was very high. Find the probability that (i) target is hit (ii) atleast one shot misses the target. OR (b) Mother, Father and Son line up at random for a family picture. Let events E : Son on one end and F : Father in the middle. Find $P(E/F)$.		
Sol. 31(a)	(i) Let probability of <i>not</i> hitting = p $\therefore$ Probability of hitting = $3p$ $p + 3p = 1 \Rightarrow p = \frac{1}{4}$ So, Probability of hitting, $P(H) = \frac{3}{4}$, Probability of <i>not</i> hitting, $P(\overline{H}) = \frac{1}{4}$ Now, $P(\text{Target is hit}) = 1 - P(\overline{H} \overline{H}) = 1 - \frac{1}{4} \times \frac{1}{4} = \frac{15}{16}$ (ii) $P(\text{at least one shot misses the target})$ $= 1 - P(HH) = 1 - \frac{3}{4} \times \frac{3}{4} = \frac{7}{16}$	I II III	1 1 1
OR			
Sol. 31(b)	There are 3 people - Mother (M), Father (F), Son (S). Sample Space = $\{MFS, MSF, FMS, FSM, SMF, SFM\}$ Total possible arrangements = 6 Given E: Son on one end, F: Father in the middle $P(E \cap F) = P(\text{Son on one end and Father in the middle}) = \frac{2}{6}$, $P(F) = \frac{2}{6}$ $\therefore P(E F) = \frac{P(E \cap F)}{P(F)} = 1$	I II III	1 1 1

	Alternative Method : For $P(E F)$, as F has already occurred which means that the father is in the middle. So, son has to be on one end. $\therefore P(E F) = 1$ (sure event)	I	1
		II	2

SECTION D

Q. Numbers 32 to 35 are long answer questions of 5 marks each.

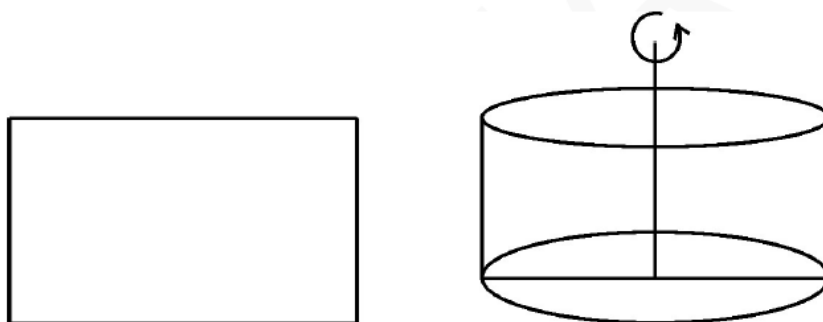
32.	(a) If $P = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $Q = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, find (QP) and hence solve the following system of equations using matrices : $x - y = 3, 2x + 3y + 4z = 17, y + 2z = 7$ OR (b) Obtain the value of $\Delta = \begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}$ in terms of x, y and z. Further, if $\Delta = 0$ and x, y, z are non-zero real numbers, prove that $x^{-1} + y^{-1} + z^{-1} = -1$.
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Sol. 32(a)	$QP = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} \text{ i.e. } QP = 6I$ $\Rightarrow P^{-1} = \frac{1}{6}Q$ Given system of equations is given by $PX = R$ where, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, R = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$ $PX = R \Rightarrow X = P^{-1}R$ $\Rightarrow X = \frac{1}{6}Q.R$ $= \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$ $\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$ $\Rightarrow x=2, y=-1, z=4$	I II III IV V	2 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $1\frac{1}{2}$
OR			
Sol. 32(b)	$\Delta = \begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}$ $= (1+x)[(1+y)(1+z)-1] - 1[(1+z)-1] + 1[1-(1+y)]$ $= (1+x)(1+z+y+yz-1) - z - y$ $= xy + yz + zx + xyz$ $\Delta = 0 \Rightarrow xy + yz + zx + xyz = 0$ $\Rightarrow yz + zx + xy = -xyz$ Dividing both sides by xyz, $x^{-1} + y^{-1} + z^{-1} = -1$ Note: Proportionate marks may be allotted if a student uses properties of determinants to solve this question.	I II III IV	2 1 1 1

33.	(a) Find the sub intervals in which $f(x) = \cot^{-1} (\sin x + \cos x)$, $x \in (0, \pi)$ is increasing and decreasing.
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OR

(b) A rectangle of perimeter 36 cm is revolved around one of its sides to sweep out a cylinder of maximum volume.



Find the dimensions of the rectangle.

Sol.
33(a)

$$f(x) = \cot^{-1}(\sin x + \cos x), x \in (0, \pi)$$

$$f'(x) = \frac{-(\cos x - \sin x)}{1 + (\sin x + \cos x)^2} = \frac{\sin x - \cos x}{1 + (\sin x + \cos x)^2}$$

For critical points of $f(x)$, put $f'(x) = 0$

$$\Rightarrow \sin x = \cos x \Rightarrow x = \frac{\pi}{4}$$


$$f'(x) > 0 \text{ when } x \in \left(\frac{\pi}{4}, \pi\right) \Rightarrow f(x) \text{ is increasing in } \left(\frac{\pi}{4}, \pi\right).$$

Note: Alternatively $f(x)$ is also increasing in the interval $\left[\frac{\pi}{4}, \pi\right)$.

$$f'(x) < 0 \text{ when } x \in \left(0, \frac{\pi}{4}\right) \Rightarrow f(x) \text{ is decreasing in } \left(0, \frac{\pi}{4}\right).$$

Note: Alternatively $f(x)$ is also decreasing in the interval $\left(0, \frac{\pi}{4}\right]$.

I

2

II

1

III

1

IV

1

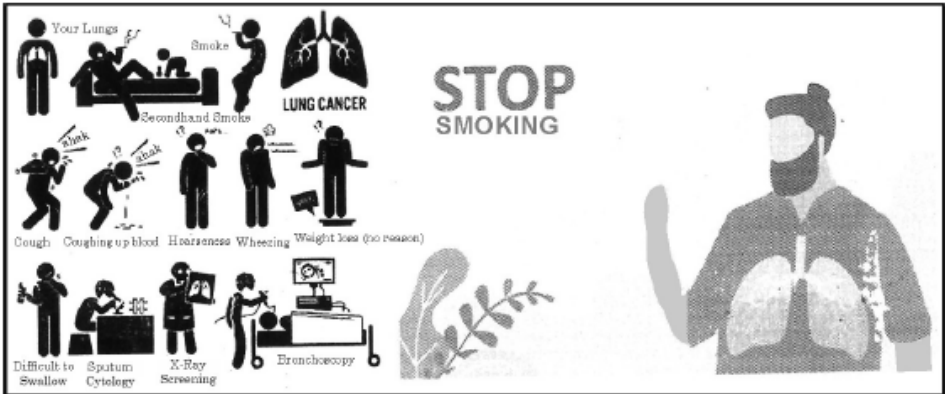
OR

Sol. 33(b)	Let the lengths of the sides of the rectangle be x and $(18 - x)$. Let it be revolved around the side of length $(18 - x)$, so that x becomes the radius of the cylinder. Volume of cylinder, $V = \pi x^2 (18 - x) = \pi (18x^2 - x^3)$ $\frac{dV}{dx} = \pi (36x - 3x^2)$ For critical points, put $\frac{dV}{dx} = 0$ $\Rightarrow x = 12 \text{ cm } (\because x \neq 0)$ Now $\left. \frac{d^2V}{dx^2} = \pi (36 - 6x) ; \frac{d^2V}{dx^2} \right _{x=12 \text{ cm}} = -36\pi < 0$ So, volume is maximum when $x = 12 \text{ cm}$. Hence the dimensions of rectangle are 12 cm and 6 cm.	I II III IV V VI	1 1 $\frac{1}{2}$ 1 1 $\frac{1}{2}$
34.	Find the domain of $g(x) = \cos^{-1} (x^2 - 1)$. Hence, find the value of x for which $g(x) = \frac{\pi}{3}$. Also, write the range of $\cos^{-1} x$ other than its principal branch.		
Sol.	Since domain of $\cos^{-1} x$ is $[-1, 1] \therefore -1 \leq x^2 - 1 \leq 1$ $\Rightarrow 0 \leq x^2 \leq 2 \Rightarrow -\sqrt{2} \leq x \leq \sqrt{2}$ $\therefore$ Domain of $g(x) = [-\sqrt{2}, \sqrt{2}]$ When $g(x) = \frac{\pi}{3}$, we have $\cos^{-1} (x^2 - 1) = \frac{\pi}{3}$ $\Rightarrow x^2 - 1 = \cos \frac{\pi}{3} = \frac{1}{2}$ $\Rightarrow x^2 = \frac{3}{2} \Rightarrow x = \sqrt{\frac{3}{2}}, -\sqrt{\frac{3}{2}}$ Range of $\cos^{-1} x$ other than its principal branch is $[-\pi, 0]$ or $[\pi, 2\pi]$. Note: Any valid branch like $[2\pi, 3\pi], [-2\pi, -\pi], \dots$ etc can be considered.	I II III IV V	1 1 1 1 1
35.	A line passing through the points A(1, 2, 3) and B(5, 8, 11) intersects the line $\vec{r} = 4\hat{i} + \hat{j} + \lambda(5\hat{i} + 2\hat{j} + \hat{k})$. Find the co-ordinates of the point of intersection. Hence, write the equation of a line passing through the point of intersection and perpendicular to both the lines.		

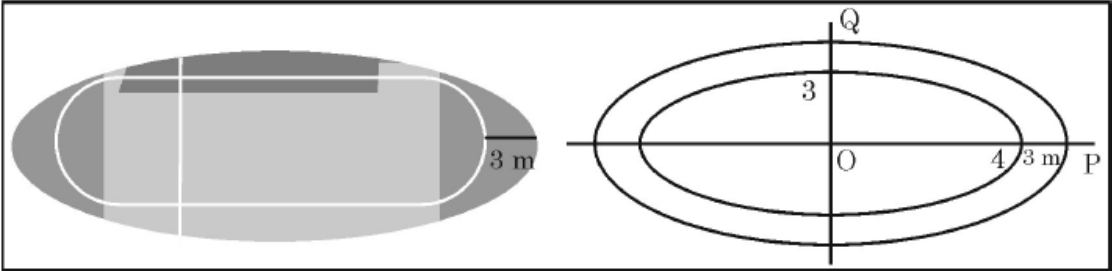
Sol.	The line passing through the points $A(1, 2, 3)$ and $B(5, 8, 11)$ is given by $l_1: \frac{x-1}{5-1} = \frac{y-2}{8-2} = \frac{z-3}{11-3} \text{ i.e. } \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \mu$ Also the given line in its cartesian form is, $l_2: \frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1} = \lambda$ Any point on l_1 is $P(2\mu + 1, 3\mu + 2, 4\mu + 3)$ Any point on l_2 is $Q(5\lambda + 4, 2\lambda + 1, \lambda)$ When the lines intersect, the points P and Q must coincide. $\therefore 2\mu + 1 = 5\lambda + 4, 3\mu + 2 = 2\lambda + 1 \text{ and } 4\mu + 3 = \lambda$ $\Rightarrow 2\mu - 5\lambda = 3 \dots (i), 3\mu - 2\lambda = -1 \dots (ii) \text{ and } 4\mu - \lambda = -3 \dots (iii)$ solving any two above equations, we get $\mu = \lambda = -1$ The point of intersection is $(-1, -1, -1)$. Let the required line(l) passing through $(-1, -1, -1)$ be $\frac{x+1}{a} = \frac{y+1}{b} = \frac{z+1}{c}$ Since $l \perp l_1$ and $l \perp l_2$, $2a + 3b + 4c = 0, 5a + 2b + c = 0$ $\Rightarrow \frac{a}{3-8} = \frac{b}{20-2} = \frac{c}{4-15} \text{ i.e. } \frac{a}{-5} = \frac{b}{18} = \frac{c}{-11}$ Hence, the required line is $\frac{x+1}{-5} = \frac{y+1}{18} = \frac{z+1}{-11}$ or $\frac{x+1}{5} = \frac{y+1}{-18} = \frac{z+1}{11}$ Note: The required line may also be written in the vector form. i.e. $\vec{r} = (-\hat{i} - \hat{j} - \hat{k}) + \lambda(5\hat{i} - 18\hat{j} + 11\hat{k})$ Note: Student can alternatively use point B to write the equation of line l_1.	I	1
		II	$\frac{1}{2}$
		III	1
		IV	$\frac{1}{2}$
		V	1
		VI	1

SECTION E

This section (Q. 36 to 38) has 3 case study-based questions of 4 marks each.

36.	Smoking increases the risk of lung problems. 
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	A study revealed that 170 in 1000 males who smoke develop lung complications, while 120 out of 1000 females who smoke develop lung related problems. In a colony, 50 people were found to be smokers of which 30 are males. A person is selected at random from these 50 people and tested for lung related problems. Based on the given information, answer the following questions : (i) What is the probability that selected person is a female ? 1 (ii) If a male person is selected, what is the probability that he will not be suffering from lung problems ? 1 (iii) (a) A person selected at random is detected with lung complications. Find the probability that selected person is a female. 2 OR (iii) (b) A person selected at random is not having lung problems, find the probability that the person is a male. 2		
Sol.	Total number of smokers in the colony = 50, Number of male smokers = 30, Number of female smokers = 20 Let L represent the event that the person is suffering from the lung complications. (i) $P(F) = \frac{20}{50} = \frac{2}{5}$ (ii) $P(L' M) = 1 - P(L M) = 1 - \frac{170}{1000} = \frac{83}{100}$ (iii) (a) $P(L M) = \frac{170}{1000} = \frac{17}{100}$ and $P(L F) = \frac{120}{1000} = \frac{12}{100}$ Using Baye's Theorem, $P(F L) = \frac{P(F).P(L F)}{P(M).P(L M) + P(F).P(L F)}$ $= \frac{\frac{2}{5} \cdot \frac{12}{100}}{\frac{3}{5} \cdot \frac{17}{100} + \frac{2}{5} \cdot \frac{12}{100}}$ $= \frac{24}{75} \text{ or } \frac{8}{25}$	I I I II	1 1 1½ ½

	OR (iii)(b) Here $P(L' M) = \frac{83}{100}$, $P(L' F) = \frac{88}{100}$ Using Baye's Theorem, $P(M L') = \frac{P(M) \cdot P(L' M)}{P(M) \cdot P(L' M) + P(F) \cdot P(L' F)}$ $= \frac{\frac{3}{5} \cdot \frac{83}{100}}{\frac{3}{5} \cdot \frac{83}{100} + \frac{2}{5} \cdot \frac{88}{100}}$ $= \frac{249}{425}$	I	1½
		II	½
37.	A racing track is build around an elliptical ground whose equation is given by $9x^2 + 16y^2 = 144$. The width of the track is 3 m as shown below :  Based on given information, answer the following questions : (i) Express y as a function of x from the given equation of ellipse. 1 (ii) Integrate the function obtained in (i) with respect to x. 1 (iii) (a) Find the area of the region enclosed within the elliptical ground excluding the track using integration. 2 OR (iii) (b) Write the co-ordinates of the points P and Q where the outer edge of the track cuts x axis and y axis in first quadrant and find the area of the triangle formed by points P, O, Q using integration. 2		
Sol.	(i) $9x^2 + 16y^2 = 144 \Rightarrow y = \frac{3}{4} \sqrt{16 - x^2}$ (ii) $\int y dx = \int \frac{3}{4} \sqrt{16 - x^2} dx$ $= \frac{3}{4} \left[\frac{x}{2} \sqrt{16 - x^2} + 8 \sin^{-1} \left(\frac{x}{4} \right) \right] + C$	I	1
		I	1

	(iii)(a) Required Area $= 4 \times \int_0^4 \frac{3}{4} \sqrt{16 - x^2} dx$ $= 3 \left[\frac{x}{2} \sqrt{16 - x^2} + 8 \sin^{-1} \left(\frac{x}{4} \right) \right]_0^4$ $= 3 (8 \sin^{-1} 1 - 0) = 24 \times \frac{\pi}{2} = 12\pi$ OR (iii)(b) The track has a width of 3 m. Clearly $P(7,0)$ and $Q(0,6)$ The equation of line passing through $P(7,0)$ and $Q(0,6)$ is $\frac{x}{7} + \frac{y}{6} = 1$ Here, $y = \frac{6}{7}(7 - x)$ Required area $= \int_0^7 y dx = \int_0^7 \frac{6}{7}(7 - x) dx$ $= \frac{6}{7} \left[\frac{(7 - x)^2}{-2} \right]_0^7$ $= 21$	I	1
		II	1
		I	1
		II	$\frac{1}{2}$
		III	$\frac{1}{2}$
38.	Sports car racing is a form of motorsport which uses sports car prototypes. The competition is held on special tracks designed in various shapes.  The equation of one such track is given as follows : $f(x) = \begin{cases} x^4 - 4x^2 + 4, & 0 \leq x < 3 \\ x^2 + 40, & x \geq 3 \end{cases}$		

	Based on given information, answer the following questions :		
	(i) Find $f'(x)$ for $0 < x < 3$.		1
	(ii) Find $f'(4)$		1
	(iii) (a) Test for continuity of $f(x)$ at $x = 3$.		2
	OR		
	(iii) (b) Test for differentiability of $f(x)$ at $x = 3$.		2
Sol.	(i) For $0 < x < 3$, $f(x) = x^4 - 4x^2 + 4$ $\therefore f'(x) = 4x^3 - 8x$ (ii) For $x \geq 3$, $f(x) = x^2 + 40$ $\therefore f'(x) = 2x$ Hence, $f'(4) = 2 \times 4 = 8$ (iii)(a) Here $f(3) = 49$ LHL: $\lim_{x \rightarrow 3^-} (x^4 - 4x^2 + 4) = 49$ RHL: $\lim_{x \rightarrow 3^+} (x^2 + 40) = 49$ Since $\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x) = f(3)$, so $f(x)$ is continuous at $x = 3$. OR (iii)(b) Here $f(3) = 49$ getting LHD = 84 getting RHD = 6 Since LHD $\neq$ RHD at $x = 3$, so $f(x)$ is not differentiable at $x = 3$.	I I I II I II	1 1 1½ ½ 1½ ½