

MATHEMATICS (Subject Code–041)

(PAPER CODE: 65/1/3)

SOLUTIONS

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Steps	Marks
	SECTION A		
	Q. Number 1 to 20 are multiple choice questions of 1 mark each.		
1.	If $\vec{a} = 8$, $\vec{b} = 3$ and $\vec{a} \times \vec{b} = 12$, then the value of $\vec{a} \cdot \vec{b}$ (A) $6\sqrt{3}$ (B) $8\sqrt{3}$ (C) $12\sqrt{3}$ (D) $3\sqrt{12}$		
Sol.	(C) $12\sqrt{3}$		1
2.	The length of perpendicular drawn from the point (3, 4, 2) on the line $\frac{x}{0} = \frac{y}{0} = \frac{z}{1}$ is (A) 2 (B) 9 (C) 5 (D) $\sqrt{29}$		
Sol.	(C) 5		1

6.	If $A = [a_{ij}]_{3 \times 3}$ is a scalar matrix then which of the following must be true ? (A) A must be a symmetric matrix. (B) A must be a skew-symmetric matrix. (C) A must be an identity matrix. (D) A must be a null matrix.		
Sol.	(A) A must be a symmetric matrix.		1
7.	Which of the following properties is/are true for two matrices of suitable orders ? (i) $(A + B)' = A' + B'$ (ii) $(A - B)' = B' - A'$ (iii) $(AB)' = A'B'$ (iv) $(kAB)' = kB'A'$ (k is a scalar) (A) (i) only (B) (i), (ii) and (iii) (C) (i) and (ii) (D) (i) and (iv)		
Sol.	(D) (i) and (iv)		1
8.	If $\Delta_1 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 6 \end{vmatrix}$, then (A) $\Delta_1 = 2\Delta_2$ (B) $\Delta_2 = -2\Delta_1$ (C) $\Delta_1 = \Delta_2$ (D) $\Delta_2 = -\Delta_1$		
Sol.	(B) $\Delta_2 = -2\Delta_1$		1

9.	One of the values of x for which $\begin{vmatrix} \cos x & \sin x \\ -\cos x & \sin x \end{vmatrix} = 1$ is (A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$		
Sol.	(B) $\frac{\pi}{4}$		1
10.	If A and B are symmetric matrices of same order, then $(AB - BA)$ is a (A) Zero matrix (B) Identity matrix (C) Symmetric matrix (D) Skew symmetric matrix		
Sol.	(D) Skew symmetric matrix		1
11.	The least value of $f(x) = e^{-x}$ in $[0, 3]$ is (A) e^{-3} (B) -1 (C) 1 (D) $-e^3$		
Sol.	(A) e^{-3}		1

12.	If $\int \frac{3ax}{b^2 + c^2x^2} dx = A \log b^2 + c^2x^2 + K$, then the value of A is (A) $3a$ (B) $\frac{3a}{2b^2}$ (C) $\frac{3a}{b^2c^2}$ (D) $\frac{3a}{2c^2}$		
Sol.	(D) $\frac{3a}{2c^2}$		1
13.	The value of $\int_{-1}^1 \frac{x^3}{x^2 + 2 x + 1} dx$ is (A) 0 (B) $\log 2$ (C) $2 \log 2$ (D) $\frac{1}{2} \log 2$		
Sol.	(A) 0		1
14.	The area bounded by the curve $y = x x$, x-axis and the ordinates $x = -1$ and $x = 1$ is given by (A) 0 (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{4}{3}$		

Sol.	(C) $\frac{2}{3}$		1
15.	The integrating factor of differential equation $R \frac{dx}{dy} + Px = Q$ where P, Q, R are functions of y is (A) $e^{\int \frac{P}{Q} dy}$ (B) $e^{\int P dy}$ (C) $e^{\int \frac{P}{R} dy}$ (D) $e^{\int \frac{P}{R} dx}$		
Sol.	(C) $e^{\int \frac{P}{R} dy}$		1
16.	The order and degree of the differential equation : $\frac{d}{dx}(y')^3 + (y')^3 = 1$ respectively are where $y' = \frac{dy}{dx}$ (A) 1, 3 (B) 2, 1 (C) 3, 1 (D) 3, 2		
Sol.	(B) 2, 1		1
17.	The value of p for which vectors $\hat{i} + 2\hat{j} + 3\hat{k}$ and $2\hat{i} - p\hat{j} + \hat{k}$ are perpendicular to each other is (A) 0 (B) 1 (C) $\frac{5}{2}$ (D) $-\frac{5}{2}$		
Sol.	(C) $\frac{5}{2}$		1

18.	The value of m for which the points with position vectors $-\hat{i} - \hat{j} + 2\hat{k}$, $2\hat{i} + m\hat{j} + 5\hat{k}$ and $3\hat{i} + 11\hat{j} + 6\hat{k}$ are collinear, is (A) 8 (B) -8 (C) 2 (D) $\frac{5}{2}$		
Sol.	(A) 8		1
Assertion – Reason Based Questions Direction : Questions number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below. (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A). (C) Assertion (A) is true and Reason (R) is false. (D) Assertion (A) is false and Reason (R) is true.			
19.	Assertion (A) : In an experiment of throwing an unbiased die, the probability of getting a prime number given that number appearing on the die being odd is $\frac{2}{3}$. Reason (R) : For any two events A and B, $P(A B) = \frac{P(A \cap B)}{P(B)}$		
Sol.	(C) Assertion (A) is true and Reason (R) is false.		1
20.	Assertion (A) : Lines given by $x = py + q$, $z = ry + s$ and $x = p'y + q'$, $z = r'y + s'$ are perpendicular to each other when $pp' + rr' = 1$. Reason (R) : Two lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ are perpendicular to each other if $\vec{b}_1 \cdot \vec{b}_2 = 0$.		

Sol.	(D) Assertion (A) is false and Reason (R) is true.		1
SECTION B			
Q. Numbers 21 to 25 are very short answer questions of 2 marks each.			
21.(a)	Simplify : $\tan^{-1} \left(\frac{\cos 2x - \sin 2x}{\cos 2x + \sin 2x} \right), 0 < x < \frac{\pi}{4}$.		
Sol.	Given expression = $\tan^{-1} \left(\frac{1 - \tan 2x}{1 + \tan 2x} \right)$ $= \tan^{-1} \left[\tan \left(\frac{\pi}{4} - 2x \right) \right] = \frac{\pi}{4} - 2x$	I	1
		II	1
OR			
21.(b)	Evaluate : $\tan \left(\sin^{-1} 1 - \cos^{-1} \left(-\frac{1}{2} \right) \right)$		
Sol.	Given expression = $\tan \left(\frac{\pi}{2} - \frac{2\pi}{3} \right)$ $= \tan \left(-\frac{\pi}{6} \right) = -\frac{1}{\sqrt{3}}$	I	1
		II	1
22.(a)	Check whether function $f(x)$ defined as $f(x) = \begin{cases} \frac{ x-3 }{2(x-3)}, & x < 3 \\ \frac{x-6}{6}, & x \geq 3 \end{cases}$ is continuous at $x = 3$ or not ?		
Sol.	$f(x) = \begin{cases} \frac{-(x-3)}{2(x-3)}, & x < 3 \\ \frac{x-6}{6}, & x \geq 3 \end{cases} \Rightarrow f(x) = \begin{cases} -\frac{1}{2}, & x < 3 \\ \frac{x-6}{6}, & x \geq 3 \end{cases}$ $f(3) = \frac{-1}{2}$		

	$\text{LHL} = \lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} \frac{-1}{2} = \frac{-1}{2}$ $\text{RHL} = \lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} \frac{x-6}{6} = \frac{-1}{2}$ LHL = RHL = $f(3)$, hence the function $f(x)$ is continuous at $x = 3$.	I	1
		II	$\frac{1}{2}$
		III	$\frac{1}{2}$
	OR		
22.(b)	(b) If $\sqrt{3}(x^2 + y^2) = 4xy$, then find $\frac{dy}{dx}$ at $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.		
	Differentiating both sides w.r.to x, we get $\sqrt{3}\left(2x + 2y\frac{dy}{dx}\right) = 4\left(x\frac{dy}{dx} + y\right)$ $\Rightarrow \frac{dy}{dx} = \frac{2y - \sqrt{3}x}{\sqrt{3}y - 2x}$ $\Rightarrow \left(\frac{dy}{dx}\right)_{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)} = \sqrt{3}$	I	1
		II	$\frac{1}{2}$
		III	$\frac{1}{2}$
23.(a)	Simplify : $\sin^{-1} \sqrt{\frac{1 + \cos 2x}{2}}, 0 < x < \frac{\pi}{2}$.		
Sol.	Given expression = $\sin^{-1} \left(\sqrt{\frac{2\cos^2 x}{2}} \right)$ $= \sin^{-1}(\cos x)$ $= \sin^{-1} \left(\sin \left(\frac{\pi}{2} - x \right) \right) = \frac{\pi}{2} - x$	I	1
		II	1
	OR		
23.(b)	Evaluate : $\cos[\sin^{-1}(-1) - \tan^{-1}(-\sqrt{3})]$.		
Sol.	Given expression = $\cos \left(-\frac{\pi}{2} + \frac{\pi}{3} \right)$ $= \cos \left(-\frac{\pi}{6} \right) = \frac{\sqrt{3}}{2}$	I	1
		II	1

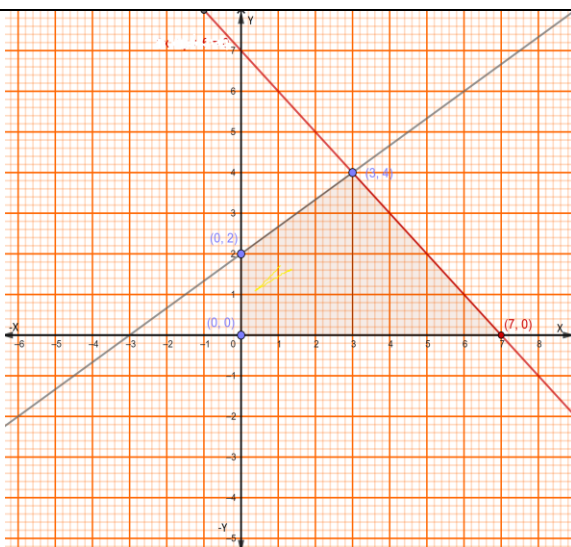
24.	Using vectors, find the area of ΔABC with vertices A(1, 2, 3), B(2, -1, 4) and C(4, 5, -1).		
Sol.	$\overrightarrow{AB} = \hat{i} - 3\hat{j} + \hat{k}$ $\overrightarrow{AC} = 3\hat{i} + 3\hat{j} - 4\hat{k}$ $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 3 & 3 & -4 \end{vmatrix}$ $= 9\hat{i} + 7\hat{j} + 12\hat{k}$ The required area $= \frac{1}{2} \overrightarrow{AB} \times \overrightarrow{AC} = \frac{1}{2} \sqrt{274}$	I II III IV	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
25.	Vectors $\vec{a} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{k}$ represent the two adjacent sides of a parallelogram. Find the vectors representing its diagonals and hence find their lengths.		
Sol.	$\vec{a} + \vec{b} = 4\hat{i} - 2\hat{j} + 4\hat{k}, \vec{a} - \vec{b} = 2\hat{i} - 2\hat{j}$ The vector representing one of its diagonals is $4\hat{i} - 2\hat{j} + 4\hat{k}$ or $-(4\hat{i} - 2\hat{j} + 4\hat{k})$ The vector representing the other diagonal is $2\hat{i} - 2\hat{j}$ or $-(2\hat{i} - 2\hat{j})$ The lengths of the two diagonals are $\sqrt{16 + 4 + 16} = 6$ and $\sqrt{4 + 4} = 2\sqrt{2}$	I II III	$\frac{1}{2}$ $\frac{1}{2}$ 1
SECTION C			
Q. Numbers 26 to 31 are short answer questions of 3 marks each.			
26.	Evaluate : $\int_0^1 \log(1 + x^2) dx$		
Sol.	$\int_0^1 \log(1 + x^2) dx = [\log(1 + x^2) \times x]_0^1 - \int_0^1 \frac{2x}{1 + x^2} \times x dx$ $= \log 2 - 2 \int_0^1 \frac{1 + x^2 - 1}{1 + x^2} dx$	I	1

	$= \log 2 - 2 \left[\int_0^1 dx - \int_0^1 \frac{1}{1+x^2} dx \right]$ $= \log 2 - 2[x - \tan^{-1}x]_0^1$ $= \log 2 - 2 + \frac{\pi}{2}$	II	1
		III	1
27. (a)	Out of two bags, bag I contains 3 red and 4 white balls and bag II contains 8 red and 6 white balls. A die is thrown. If it shows a number less than 3 then a ball is drawn at random from bag I, otherwise a ball is drawn at random from bag II. Find the probability that the ball drawn from one of the bags is a red ball.		
Sol.	Let E_1: The number appearing on the die < 3 E_2: The number appearing on the die ≥ 3 A: Red ball is drawn $P(E_1) = \frac{2}{6}, P(A/E_1) = \frac{3}{7}, P(E_2) = \frac{4}{6}, P(A/E_2) = \frac{8}{14}$ The required probability $= P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2)$ $= \frac{2}{6} \times \frac{3}{7} + \frac{4}{6} \times \frac{8}{14}$ $= \frac{11}{21}$	I II III IV	1/2 1 1 1/2
	OR		
27. (b)	The probability of simultaneous occurrence of atleast one of the two events X and Y is a. If the probability that exactly one of the events X, Y occurs is b, prove that $P(X') + P(Y') = 2 - 2a + b$.		
Sol.	$P(X \cup Y) = a$ $P(X \cup Y) - P(X \cap Y) = b$ Hence, $P(X \cap Y) = a - b$ $P(X') + P(Y') = [1 - P(X)] + [1 - P(Y)]$	I II III	1/2 1 1

	$= 2 - [P(X) + P(Y)]$ $= 2 - [P(X \cup Y) + P(X \cap Y)]$ $= 2 - [a + a - b]$ $= 2 - 2a + b$	IV	1/2
28.(a)	Find $\int \sqrt{\frac{x+2}{x-2}} dx$		
Sol.	$I = \int \sqrt{\frac{x+2}{x-2}} dx = \int \sqrt{\frac{x+2}{x-2} \times \frac{x+2}{x+2}} dx = \int \frac{x+2}{\sqrt{x^2-4}} dx$ $= \frac{1}{2} \int \frac{2x}{\sqrt{x^2-4}} dx + 2 \int \frac{1}{\sqrt{x^2-4}} dx$ $I_1 = \frac{1}{2} \int \frac{2x}{\sqrt{x^2-4}} dx = \frac{1}{2} \int \frac{dt}{\sqrt{t}} \quad [(x^2-4) = t \Rightarrow 2x dx = dt]$ $= \sqrt{t} = \sqrt{x^2-4}$ $I_2 = 2 \int \frac{1}{\sqrt{x^2-4}} dx = 2 \log x + \sqrt{x^2-4} $ $I = I_1 + I_2 = \sqrt{x^2-4} + 2 \log x + \sqrt{x^2-4} + C$	I II III IV	1 1 1/2 1/2
	OR		
28.(b)	Find : $\int \frac{x^2}{(x^2+9)(x^2+16)} dx$		
	Put $x^2 = t$ to get $\frac{x^2}{(x^2+9)(x^2+16)} = \frac{t}{(t+9)(t+16)} = \frac{A}{t+9} + \frac{B}{t+16}$ Getting $A = \frac{-9}{7}, B = \frac{16}{7}$ Given integral $= \frac{-9}{7} \int \frac{1}{x^2+9} dx + \frac{16}{7} \int \frac{1}{x^2+16} dx$ $= \frac{-3}{7} \tan^{-1}\left(\frac{x}{3}\right) + \frac{4}{7} \tan^{-1}\left(\frac{x}{4}\right) + C$	I II III IV	1/2 1 1/2 1

29.	If $I_1 = \int_{-\pi/4}^{\pi/4} \frac{dx}{1 + \cos 2x}$ and $I_2 = \int_{-1/2}^{1/2} x dx$, then show that $I_1 - 4I_2 = 0$.		
Sol.	$I_1 = 2 \int_0^{\pi/4} \frac{dx}{1 + \cos 2x}, \quad \left(\frac{1}{1 + \cos 2x} \text{ is an even function}\right)$ $= 2 \int_0^{\pi/4} \frac{dx}{2 \cos^2 x} = \int_0^{\pi/4} \sec^2 x dx$ $= [\tan x]_0^{\pi/4} = 1$ Note: Students attempting I_1 without the application of any property should be marked proportionately. $I_2 = 2 \int_0^{1/2} x dx, \quad (x \text{ is an even function})$ $= 2 \int_0^{1/2} x dx$ $= [x^2]_0^{1/2} = \frac{1}{4}$ $I_1 - 4I_2 = 0$	I II III IV	1½ 1/2 ½ ½
30.(a)	Find the general solution of the differential equation $(y^2 - x^2)dx = 2xy dy$		
Sol.	Given differential equation is $\frac{dy}{dx} = \frac{1}{2} \left[\frac{y}{x} - \frac{x}{y} \right]$ Put $y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$ The given differential equation becomes $v + x \frac{dv}{dx} = \frac{1}{2} \left[v - \frac{1}{v} \right]$ $\Rightarrow \frac{2v dv}{1 + v^2} = -\frac{dx}{x}$ $\Rightarrow \int \frac{2v dv}{1 + v^2} = -\int \frac{dx}{x}$ $\Rightarrow \log(1 + v^2) = -\log x + \log C$	I II III IV	½ ½ ½ 1

	$\Rightarrow \log(1 + v^2) x = \log C$ $\Rightarrow (1 + v^2)x = C$ $\Rightarrow x^2 + y^2 = Cx$	V	$\frac{1}{2}$
	OR		
30.(b)	Find the particular solution of the differential equation $(1 + e^{2x})dy + (1 + y^2)e^x dx = 0$, given that $y(1) = 0$.		
Sol.	Given differential equation is $\frac{dy}{1+y^2} = \frac{e^x}{1+e^{2x}} dx$ $\Rightarrow \int \frac{dy}{1+y^2} = - \int \frac{e^x}{1+e^{2x}} dx$ $\Rightarrow \tan^{-1}y = - \int \frac{1}{1+t^2} dt, \quad [e^x = t \Rightarrow e^x dx = dt]$ $\Rightarrow \tan^{-1}y + \tan^{-1}(e^x) = C$ When $x = 1, y = 0 \Rightarrow C = \tan^{-1}(e)$ Hence, the required particular solution is $\tan^{-1}y + \tan^{-1}(e^x) = \tan^{-1}(e)$	I II III	1 1 1
31.	Solve the following linear programming problem graphically : Minimize $Z = 13x - 15y$ Subject to constraints $x + y \leq 7,$ $2x - 3y + 6 \geq 0,$ $x \geq 0, y \geq 0$		
Sol.			



Corner point	Value of $Z = 13x - 15y$
(0, 0)	0
(7, 0)	91
(3, 4)	-21
(0, 2)	-30

The minimum value of Z is -30 at $x = 0, y = 2$

I	1½ (Correct graph and shading)
II	1
III	½

SECTION D

Q. Numbers 32 to 35 are long answer questions of 5 marks each.

32. Show that line AB passing through points A(0, 4, 1), B(2, 3, -1) and the line CD passing through points C(4, 5, 0), D(2, 6, 2) are parallel. Also, find distance between them.

Sol. Drs of AB are $\langle 2, -1, -2 \rangle$
 Drs of CD are $\langle -2, 1, 2 \rangle$
 These are proportional. Hence, the lines are parallel.

Lines in vector form are

$$\vec{r} = 4\hat{j} + \hat{k} + \lambda(2\hat{i} - \hat{j} - 2\hat{k})$$

$$\vec{r} = 4\hat{i} + 5\hat{j} + \mu(2\hat{i} - \hat{j} - 2\hat{k})$$

$$\vec{a}_2 - \vec{a}_1 = 4\hat{i} + \hat{j} - \hat{k}, \vec{b} = 2\hat{i} - \hat{j} - 2\hat{k}$$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 1 & -1 \\ 2 & -1 & -2 \end{vmatrix} = -3\hat{i} + 6\hat{j} - 6\hat{k}$$

I	½
II	½
III	1
IV	1
V	1

	Distance between these parallel lines = $\frac{ (\vec{a_2} - \vec{a_1}) \times \vec{b} }{ \vec{b} } = \frac{\sqrt{81}}{\sqrt{9}} = 3$	VI	1
33. (a)	A relation R is defined on Z, the set of integers, as $R = \{(x, y) : x - y \text{ is divisible by a prime number 'p', } x, y \in \mathbb{Z}\}$ check whether R is an equivalence relation or not.		
Sol.	Let $x \in \mathbb{Z}$. Then $ x - x = 0$, which is divisible by a prime number p. Hence, $(x, x) \in R$. Thus, R is reflexive.	I	1½
	Let $x, y \in \mathbb{Z}$ such that $(x, y) \in R$. Then $ x - y $ is divisible by the prime number p. $\Rightarrow y - x $ is divisible by the prime number p as $ x - y = y - x $	II	1½
	Hence, $(y, x) \in R$. Thus, R is symmetric.		
	Let $x, y, z \in \mathbb{Z}$ such that $(x, y), (y, z) \in R$. Then $ x - y $ is divisible by the prime number p and $ y - z $ is divisible by the prime number p. $\Rightarrow x - y$ is divisible by the prime number p and $y - z$ is divisible by the prime number p. $\Rightarrow x - y + y - z$ is divisible by the prime number p $\Rightarrow x - z$ is divisible by the prime number p $\Rightarrow x - z $ is divisible by the prime number p	III	1½
	Hence, $(x, z) \in R$. Thus, R is transitive.	IV	½
	Since, R is reflexive, symmetric and transitive, therefore, R is an equivalence relation.		
	OR		
33. (b)	A function $f : \mathbb{R} - \left\{\frac{3}{5}\right\} \longrightarrow \mathbb{R} - \left\{\frac{3}{5}\right\}$ is defined as $f(x) = \frac{3x+2}{5x-3}$. Show that f is one-one and onto.		
Sol.	Let $x_1, x_2 \in \mathbb{R} - \left\{\frac{3}{5}\right\}$ such that $f(x_1) = f(x_2)$ $\Rightarrow \frac{3x_1+2}{5x_1-3} = \frac{3x_2+2}{5x_2-3}$		

	$\Rightarrow 15x_1x_2 - 9x_1 + 10x_2 - 6 = 15x_1x_2 + 10x_1 - 9x_2 - 6$ $\Rightarrow x_1 = x_2$ Hence, f is one-one. Let $y \in R - \left\{\frac{3}{5}\right\}$ (Codomain). Then $f(x) = y$ or, $\frac{3x+2}{5x-3} = y \Rightarrow x = \frac{3y+2}{5y-3} \in \text{Domain}$. Hence, for every $y \in R - \left\{\frac{3}{5}\right\}$ (codomain), there exists $x = \frac{3y+2}{5y-3} \in R - \left\{\frac{3}{5}\right\}$ (domain) such that $f\left(\frac{3y+2}{5y-3}\right) = y$ Thus, f is onto.	I	2½
34.(a)	If $A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & -1 & -2 \\ 1 & -1 & 0 \end{bmatrix}$, find A^{-1} and use it to solve the following system of equations : $-2y + z = 7, 2x - y - z = 8, x - 2y = 10$		
Sol.	$ A = -2(2) + 1(3) = -1 \neq 0$ $adjA = \begin{bmatrix} -2 & -1 & -3 \\ -2 & -1 & -2 \\ 3 & 2 & 4 \end{bmatrix}$ $A^{-1} = \frac{1}{ A } adjA = \begin{bmatrix} 2 & 1 & 3 \\ 2 & 1 & 2 \\ -3 & -2 & -4 \end{bmatrix}$ Given system of equations is equivalent to $A^T X = B$, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 7 \\ 8 \\ 10 \end{bmatrix}$ $X = (A^T)^{-1} B = (A^{-1})^T B$ $\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & 2 & -3 \\ 1 & 1 & -2 \\ 3 & 2 & -4 \end{bmatrix} \begin{bmatrix} 7 \\ 8 \\ 10 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$ $\therefore x = 0, y = -5, z = -3$	I II III IV V	1 1½ ½ ½ 1½
	OR		

34.(b)	If $\begin{bmatrix} 3 & -1 & \sin 3x \\ -7 & 4 & \cos 2x \\ -11 & 7 & 2 \end{bmatrix}$ is a singular matrix, then find all values of x where $x \in \left[0, \frac{\pi}{2}\right]$.		
	Given matrix is singular, hence, its determinant = 0, i. e., $\begin{vmatrix} 3 & -1 & \sin 3x \\ -7 & 4 & \cos 2x \\ -11 & 7 & 2 \end{vmatrix} = 0$ $\Rightarrow 2\cos 2x + \sin 3x = 2$ $\Rightarrow 4\sin^3 x + 4\sin^2 x - 3\sin x = 0$ $\Rightarrow \sin x(4\sin^2 x + 4\sin x - 3) = 0$ $\Rightarrow \sin x(2\sin x - 1)(2\sin x + 3) = 0$ Solving to get $x = 0, \frac{\pi}{6}$	I II III IV V	1 1 1 1 1
35.	If $x = a(\sin t - t \cos t)$ and $y = b(\cos t + t \sin t)$, then find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.		
Sol.	$\frac{dx}{dt} = a(\cos t + t \sin t - \cos t) = at \sin t,$ $\frac{dy}{dt} = b(-\sin t + t \cos t + \sin t) = bt \cos t$ $\frac{dy}{dx} = \frac{bt \cos t}{at \sin t}$ $= \frac{b}{a} \cot(t)$ $\frac{d^2y}{dx^2} = -\frac{b}{a} \operatorname{cosec}^2 t \times \frac{dt}{dx}$ $= -\frac{b}{a} \operatorname{cosec}^2 t \times \frac{1}{at \sin t} \text{ or } -\frac{b \operatorname{cosec}^3 t}{a^2 t}$	I II III IV V	1 1 1 1 1
SECTION E			
This section (Q. 36 to 38) has 3 case study-based questions of 4 marks each.			

36.

Roundabouts are often made on busy roads to ease the traffic and avoid red lights.



One such round-about is made such that equation representing its boundary is given by $C_1 : x^2 + y^2 = 64$.

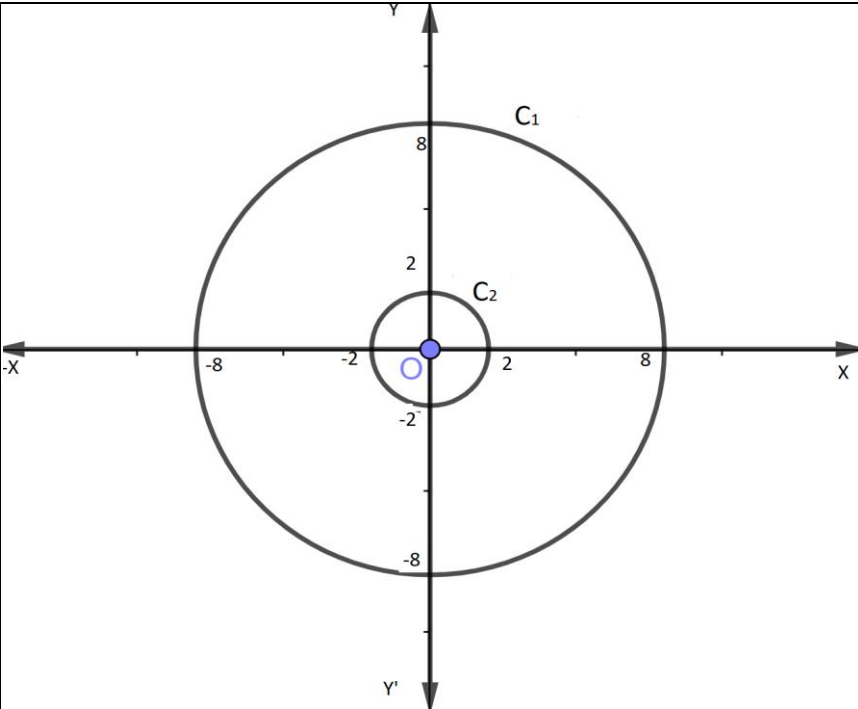
There is a circular pond with a fountain in the middle of the roundabout whose equation is given by $C_2 : x^2 + y^2 = 4$.

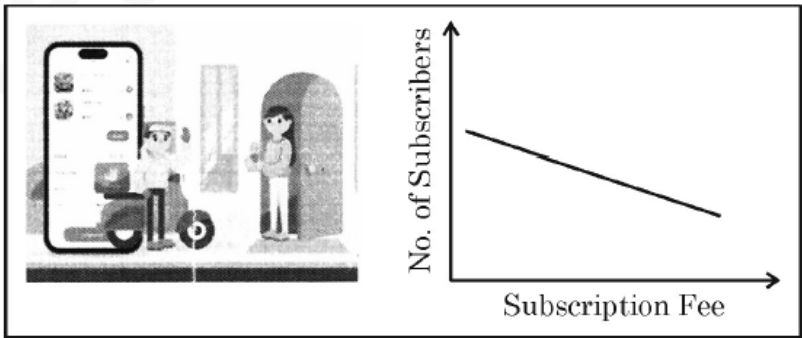
Based on the given information, answer the following questions :

- (i) Represent the given equations C_1 and C_2 with the help of a diagram.
- (ii) Express y as a function of x , ($y = f(x)$), for both C_1 and C_2 .
- (iii) (a) Using integration find the area of region covered by the roundabout.

OR

- (iii) (b) Using integration, find the area of region covered by circular pond.

Sol.(i)		I	1
Sol.(ii)	Circle C_1: $y = \sqrt{64 - x^2}$ or $y = -\sqrt{64 - x^2}$ Circle C_2: $y = \sqrt{4 - x^2}$ or $y = -\sqrt{4 - x^2}$	I II	$\frac{1}{2}$ $\frac{1}{2}$
Sol.(iii) (a)	Required area $= 4 \times \int_0^8 \sqrt{64 - x^2} dx$ $= 2[x\sqrt{64 - x^2} + 64\sin^{-1}\frac{x}{8}]_0^8 = 64\pi$	I II	1 1
	OR		
Sol.(iii) (b)	Required area $= 4 \times \int_0^2 \sqrt{4 - x^2} dx$ $= 2[x\sqrt{4 - x^2} + 4\sin^{-1}\frac{x}{2}]_0^2 = 4\pi$	I II	1 1

37.	An online delivery company in a city has 5000 subscribers and collects annual subscription fees of ₹ 300 per subscriber for unlimited free deliveries.  The company wishes to increase the annual subscription fee. It is predicted that, for every increase of ₹ 1, ten subscribers will discontinue. Assume that the company increased the annual fee by ₹ x. Based on the given information, answer the following questions : (i) How many subscribers will discontinue after an increase of ₹ x in annual fee ? (ii) If $R(x)$ denotes the total revenue collected after the increase of ₹ x in subscription fee, express $R(x)$ as a function of x. (iii) (a) Find the value of x for which $R(x)$ is maximum. OR (iii) (b) Find the sub-intervals of $(0, 5000)$ in which $R(x)$ is increasing and decreasing.		
Sol.(i)	$10x$	I	1
Sol.(ii)	$R(x) = (5000 - 10x)(300 + x)$	I	1
Sol.(iii)	$R'(x) = 2000 - 20x$	I	$\frac{1}{2}$
(a)	$R'(x) = 0 \Rightarrow x = 100$	II	$\frac{1}{2}$
	$R''(x) = -20 < 0$ at $x = 100$	III	$\frac{1}{2}$
	At $x = 100, R(x)$ is maximum	IV	$\frac{1}{2}$
	OR		
Sol.(iii)	$R'(x) = 2000 - 20x$	I	$\frac{1}{2}$
(b)			

	$R'(x) = 0 \Rightarrow x = 100$ In $(0, 100)$, $R'(x) > 0$. Therefore, $R(x)$ is increasing in $(0, 100)$ or $(0, 100]$ In $(100, 5000)$, $R'(x) < 0$. Therefore, $R(x)$ is decreasing in $(100, 5000)$ or $[100, 5000)$	II III IV	1/2 1/2 1/2
38.	In an online jackpot, there is one first prize of ₹ 3,00,000, two second prizes of ₹ 2,00,000 each and three third prizes of ₹ 50,000 each.  A total of 1,00,000 jackpot tickets each costing ₹ 100 were sold there by raising a fund of ₹ 1,00,00,000. Rohan bought one ticket. Based on given information, answer the following questions : (i) What are the possible amounts, the person can win ? (ii) (a) What is the probability that the person wins atleast ₹ 2,00,000 ? OR (ii) (b) What is the probability that the person does not win any amount ? (iii) In another jackpot, Rohan also bought a ticket having a prize money of ₹ 5,00,000. The chances of winning the jackpot are 1 in 1,00,000. Find the probability that on exactly one of tickets he wins the jackpot.		
Sol.(i)	₹ 300000, ₹ 200000, ₹ 50000	I	1
Sol.(ii) (a)	Required probability = $\frac{3}{100000}$	I	2
	OR		
Sol.(ii) (b)	Required probability = $1 - \frac{6}{100000} = \frac{99994}{100000}$ or $\frac{49997}{50000}$	I	2

Sol.(iii)	P (Exactly on one of the tickets Rohan wins the jackpot) = P [(He wins 1st and he loses 2nd) or (He loses first and wins 2nd)] $= \frac{6}{100000} \times \frac{99999}{100000} + \frac{99994}{100000} \times \frac{1}{100000} = \frac{699988}{10^{10}} \text{ or } \frac{174997}{25000000000}$	I	1
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