

**MATHEMATICS (Subject Code–**

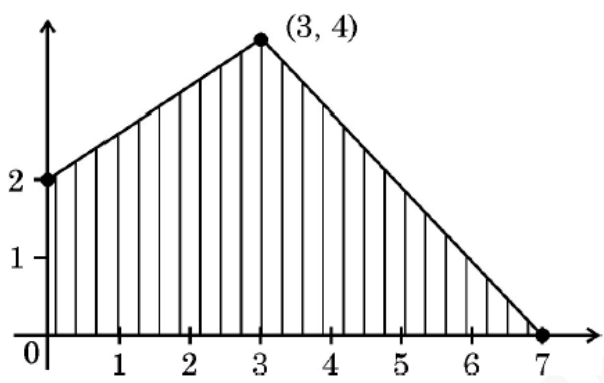
**041) (PAPER CODE: 65/1/2)**

**SOLUTIONS**

| Q. No.                                                          | EXPECTED OUTCOMES/VALUE POINTS                                                                                                                                                                                                                     | Steps | Marks |
|-----------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|-------|
| SECTION A                                                       |                                                                                                                                                                                                                                                    |       |       |
| Q. Number 1 to 20 are multiple choice questions of 1 mark each. |                                                                                                                                                                                                                                                    |       |       |
| 1.                                                              | <p>If <math>\int \frac{3ax}{b^2 + c^2x^2} dx = A \log  b^2 + c^2x^2  + K</math>, then the value of A is</p> <p>(A) <math>3a</math> (B) <math>\frac{3a}{2b^2}</math></p> <p>(C) <math>\frac{3a}{b^2c^2}</math> (D) <math>\frac{3a}{2c^2}</math></p> |       |       |
| Sol.                                                            | (D) $\frac{3a}{2c^2}$                                                                                                                                                                                                                              |       | 1     |
| 2.                                                              | <p>The value of <math>\int_{-1}^1 \frac{x^3}{x^2 + 2 x  + 1} dx</math> is</p> <p>(A) 0 (B) <math>\log 2</math></p> <p>(C) <math>2 \log 2</math> (D) <math>\frac{1}{2} \log 2</math></p>                                                            |       |       |
| Sol.                                                            | (A) 0                                                                                                                                                                                                                                              |       | 1     |

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| 3.   | <p>The area bounded by the curve <math>y = x x </math>, <math>x</math>-axis and the ordinates <math>x = -1</math> and <math>x = 1</math> is given by</p> <p>(A) 0 (B) <math>\frac{1}{3}</math></p> <p>(C) <math>\frac{2}{3}</math> (D) <math>\frac{4}{3}</math></p>                                                                |  |   |
| Sol. | (C) $\frac{2}{3}$                                                                                                                                                                                                                                                                                                                  |  | 1 |
| 4.   | <p>The integrating factor of differential equation <math>R \frac{dx}{dy} + Px = Q</math> where <math>P, Q, R</math> are functions of <math>y</math> is</p> <p>(A) <math>e^{\int \frac{P}{Q} dy}</math> (B) <math>e^{\int P dy}</math></p> <p>(C) <math>e^{\int \frac{P}{R} dy}</math> (D) <math>e^{\int \frac{P}{R} dx}</math></p> |  |   |
| Sol. | (C) $e^{\int \frac{P}{R} dy}$                                                                                                                                                                                                                                                                                                      |  | 1 |
| 5.   | <p>The order and degree of the differential equation : <math>\frac{d}{dx} (\sin y) = y^2</math> respectively are</p> <p>(A) 1, 1 (B) 2, 1</p> <p>(C) 2, 2 (D) 1, 2</p>                                                                                                                                                             |  |   |
| Sol. | (A) 1, 1                                                                                                                                                                                                                                                                                                                           |  | 1 |

|      |                                                                                                                                                                                                                                                                                                     |  |   |
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| 6.   | <p>The value of p for which vectors <math>\hat{i} + 2\hat{j} + 3\hat{k}</math> and <math>2\hat{i} - p\hat{j} + \hat{k}</math> are perpendicular to each other is</p> <p>(A) 0 (B) 1</p> <p>(C) <math>\frac{5}{2}</math> (D) <math>-\frac{5}{2}</math></p>                                           |  |   |
| Sol. | (C) $\frac{5}{2}$                                                                                                                                                                                                                                                                                   |  | 1 |
| 7.   | <p>The value of m for which the points with position vectors <math>-\hat{i} - \hat{j} + 2\hat{k}</math>, <math>2\hat{i} + m\hat{j} + 5\hat{k}</math> and <math>3\hat{i} + 11\hat{j} + 6\hat{k}</math> are collinear, is</p> <p>(A) 8 (B) -8</p> <p>(C) 2 (D) <math>\frac{5}{2}</math></p>           |  |   |
| Sol. | (A) 8                                                                                                                                                                                                                                                                                               |  | 1 |
| 8.   | <p>If <math> \vec{a}  = 8</math>, <math> \vec{b}  = 3</math> and <math> \vec{a} \times \vec{b}  = 12</math>, then the value of <math> \vec{a} \cdot \vec{b} </math></p> <p>(A) <math>6\sqrt{3}</math> (B) <math>8\sqrt{3}</math></p> <p>(C) <math>12\sqrt{3}</math> (D) <math>3\sqrt{12}</math></p> |  |   |
| Sol. | (C) $12\sqrt{3}$                                                                                                                                                                                                                                                                                    |  | 1 |
| 9.   | <p>The length of perpendicular drawn from the point (1, 2, 3) on line <math>\frac{x}{0} = \frac{y}{1} = \frac{z}{0}</math> is</p> <p>(A) 2 (B) 6</p> <p>(C) <math>\sqrt{10}</math> (D) <math>\sqrt{14}</math></p>                                                                                   |  |   |

|      |                                                                                                                                                                                                                                                                                                                                      |  |   |
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| Sol. | (C) $\sqrt{10}$                                                                                                                                                                                                                                                                                                                      |  | 1 |
| 10.  | <p>The feasible region of a linear programming problem with objective function <math>Z = 5x + 7y</math> is shown below :</p>  <p>The maximum value of <math>Z</math> – minimum value of <math>Z</math> is</p> <p>(A) 8 (B) 29<br/>(C) 35 (D) 43</p> |  |   |
| Sol. | (D) 43                                                                                                                                                                                                                                                                                                                               |  | 1 |
| 11.  | <p>The degree of an objective function of a linear programming problem is</p> <p>(A) 0 (B) 1<br/>(C) 2 (D) Any natural number</p>                                                                                                                                                                                                    |  |   |
| Sol. | (B) 1                                                                                                                                                                                                                                                                                                                                |  | 1 |
| 12.  | <p>If <math>\tan^{-1} x = 3y</math>, then</p> <p>(A) <math>-\frac{\pi}{2} &lt; y &lt; \frac{\pi}{2}</math> (B) <math>-\frac{3\pi}{2} &lt; y &lt; \frac{3\pi}{2}</math><br/>(C) <math>-\frac{\pi}{6} &lt; y &lt; \frac{\pi}{6}</math> (D) <math>-\frac{\pi}{6} \leq y \leq \frac{\pi}{6}</math></p>                                   |  |   |
| Sol. | (C) $-\frac{\pi}{6} < y < \frac{\pi}{6}$                                                                                                                                                                                                                                                                                             |  | 1 |

|      |                                                                                                                                                                                                                                                                                               |  |   |
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| 13.  | Which of the following cannot be an order of a column-matrix ?<br>(A) $1 \times 2$ (B) $2 \times 1$<br>(C) $1 \times 1$ (D) $m \times 1$ , where $m \in \mathbb{N}$                                                                                                                           |  |   |
| Sol. | (A) $1 \times 2$                                                                                                                                                                                                                                                                              |  | 1 |
| 14.  | Which of the following properties is/are true for two matrices of suitable orders ?<br>(i) $(A + B)' = A' + B'$ (ii) $(A - B)' = B' - A'$<br>(iii) $(AB)' = A'B'$ (iv) $(kAB)' = kB'A'$ (k is a scalar)<br>(A) (i) only (B) (i), (ii) and (iii)<br>(C) (i) and (ii) (D) (i) and (iv)          |  |   |
| Sol. | (D) (i) and (iv)                                                                                                                                                                                                                                                                              |  | 1 |
| 15.  | If $\Delta_1 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 6 \end{vmatrix}$ , then<br>(A) $\Delta_1 = 2\Delta_2$ (B) $\Delta_2 = -2\Delta_1$<br>(C) $\Delta_1 = \Delta_2$ (D) $\Delta_2 = -\Delta_1$ |  |   |
| Sol. | (B) $\Delta_2 = -2\Delta_1$                                                                                                                                                                                                                                                                   |  | 1 |

|      |                                                                                                                                                                                                                                                                     |  |   |
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| 16.  | <p>One of the values of <math>x</math> for which <math>\begin{vmatrix} \cos x &amp; \sin x \\ -\cos x &amp; \sin x \end{vmatrix} = 1</math> is</p> <p>(A) 0 (B) <math>\frac{\pi}{4}</math></p> <p>(C) <math>\frac{\pi}{3}</math> (D) <math>\frac{\pi}{2}</math></p> |  |   |
| Sol. | (B) $\frac{\pi}{4}$                                                                                                                                                                                                                                                 |  | 1 |
| 17.  | <p>If A and B are symmetric matrices of same order, then which of the following matrices is a skew-symmetric matrix ?</p> <p>(A) <math>A^2 + B^2</math> (B) <math>A^2 - B^2</math></p> <p>(C) <math>AB + BA</math> (D) <math>AB - BA</math></p>                     |  |   |
| Sol. | (D) $AB - BA$                                                                                                                                                                                                                                                       |  | 1 |
| 18.  | <p>The absolute maximum value of <math>f(x) = x^2 + 1</math> in <math>[-5, 2]</math> is</p> <p>(A) 26 (B) 1</p> <p>(C) 5 (D) 2</p>                                                                                                                                  |  |   |
| Sol. | (A) 26                                                                                                                                                                                                                                                              |  | 1 |

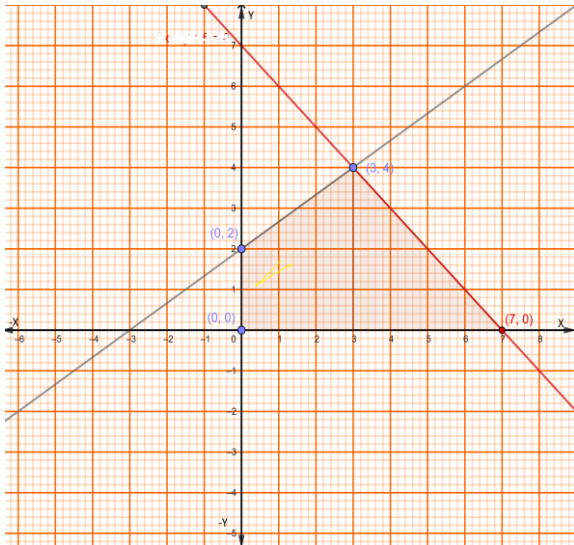
|                                                                      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |   |   |
|----------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|---|
|                                                                      | <p style="text-align: center;"><b>Assertion – Reason Based Questions</b></p> <p><b>Direction :</b> Questions number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R).<br/>Select the correct answer from the codes (A), (B), (C) and (D) as given below.</p> <p>(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).<br/>(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).<br/>(C) Assertion (A) is true and Reason (R) is false.<br/>(D) Assertion (A) is false and Reason (R) is true.</p> |   |   |
| 19.                                                                  | <p><b>Assertion (A) :</b> Lines given by <math>x = py + q</math>, <math>z = ry + s</math> and <math>x = p'y + q'</math>, <math>z = r'y + s'</math> are perpendicular to each other when <math>pp' + rr' = 1</math>.</p> <p><b>Reason (R) :</b> Two lines <math>\vec{r} = \vec{a}_1 + \lambda \vec{b}_1</math> and <math>\vec{r} = \vec{a}_2 + \mu \vec{b}_2</math> are perpendicular to each other if <math>\vec{b}_1 \cdot \vec{b}_2 = 0</math>.</p>                                                                                                                                                                                                                                                                                   |   |   |
| Sol.                                                                 | (D) Assertion (A) is false and Reason (R) is true.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |   | 1 |
| 20.                                                                  | <p><b>Assertion (A) :</b> In an experiment of throwing an unbiased die, the probability of getting a prime number given that number appearing on the die being odd is <math>\frac{2}{3}</math>.</p> <p><b>Reason (R) :</b> For any two events A and B, <math>P(A B) = \frac{P(A \cap B)}{P(B)}</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                               |   |   |
| Sol.                                                                 | (C) Assertion (A) is true and Reason (R) is false.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |   | 1 |
| SECTION B                                                            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |   |   |
| Q. Numbers 21 to 25 are very short answer questions of 2 marks each. |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |   |   |
| 21.                                                                  | A vector $\vec{a}$ of magnitude 14 has direction ratios $\langle 2, 3, -6 \rangle$ . Find the projection of the vector $\vec{a}$ on $\hat{i}$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |   |   |
| Sol.                                                                 | Getting $\vec{a} = 4\hat{i} + 6\hat{j} - 12\hat{k}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | I | 1 |

|        |                                                                                                                                                                                                                                                                                                                                                                                                                                     |                        |                                             |
|--------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------|---------------------------------------------|
|        | Projection of $\vec{d}$ on $\hat{i} = \vec{d} \cdot \hat{i} = 4$                                                                                                                                                                                                                                                                                                                                                                    | II                     | 1                                           |
| 22.    | Vectors $\vec{a} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{k}$ represent the two adjacent sides of a parallelogram. Find the vectors representing its diagonals and hence find their lengths.                                                                                                                                                                                                                |                        |                                             |
| Sol.   | $\vec{a} + \vec{b} = 4\hat{i} - 2\hat{j} + 4\hat{k}, \vec{a} - \vec{b} = 2\hat{i} - 2\hat{j}$<br>The vector representing one of its diagonals is $4\hat{i} - 2\hat{j} + 4\hat{k}$ or $-(4\hat{i} - 2\hat{j} + 4\hat{k})$<br><br>The vector representing the other diagonal is $2\hat{i} - 2\hat{j}$ or $-(2\hat{i} - 2\hat{j})$<br><br>The lengths of the two diagonals are $\sqrt{16 + 4 + 16} = 6$ and $\sqrt{4 + 4} = 2\sqrt{2}$ | I<br><br>II<br><br>III | $\frac{1}{2}$<br><br>$\frac{1}{2}$<br><br>1 |
| 23.(a) | Simplify : $\tan^{-1} \left( \frac{\cos 2x - \sin 2x}{\cos 2x + \sin 2x} \right), 0 < x < \frac{\pi}{4}$ .                                                                                                                                                                                                                                                                                                                          |                        |                                             |
| Sol.   | Given expression = $\tan^{-1} \left( \frac{1 - \tan 2x}{1 + \tan 2x} \right)$<br>$= \tan^{-1} \left[ \tan \left( \frac{\pi}{4} - 2x \right) \right] = \frac{\pi}{4} - 2x$                                                                                                                                                                                                                                                           | I<br><br>II            | 1<br><br>1                                  |
|        | OR                                                                                                                                                                                                                                                                                                                                                                                                                                  |                        |                                             |
| 23.(b) | Evaluate : $\tan \left( \sin^{-1} 1 - \cos^{-1} \left( -\frac{1}{2} \right) \right)$                                                                                                                                                                                                                                                                                                                                                |                        |                                             |
|        | Given expression = $\tan \left( \frac{\pi}{2} - \frac{2\pi}{3} \right)$<br>$= \tan \left( -\frac{\pi}{6} \right) = -\frac{1}{\sqrt{3}}$                                                                                                                                                                                                                                                                                             | I<br><br>II            | 1<br><br>1                                  |

|         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                |                                     |
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| 24.(a)  | <p>Check whether function <math>f(x)</math> defined as</p> $f(x) = \begin{cases} \frac{ x-3 }{2(x-3)}, & x < 3 \\ \frac{x-6}{6}, & x \geq 3 \end{cases}$ <p>is continuous at <math>x = 3</math> or not ?</p>                                                                                                                                                                                                                                                                                                                      |                |                                     |
| Sol.    | $f(x) = \begin{cases} \frac{-(x-3)}{2(x-3)}, & x < 3 \\ \frac{x-6}{6}, & x \geq 3 \end{cases} \Rightarrow f(x) = \begin{cases} \frac{-1}{2}, & x < 3 \\ \frac{x-6}{6}, & x \geq 3 \end{cases}$ $f(3) = \frac{-1}{2}$ $\text{LHL} = \lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} \frac{-1}{2} = \frac{-1}{2}$ $\text{RHL} = \lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} \frac{x-6}{6} = \frac{-1}{2}$ $\text{LHL} = \text{RHL} = f(3), \text{ hence the function } f(x) \text{ is continuous at } x = 3.$ | I<br>II<br>III | 1<br>$\frac{1}{2}$<br>$\frac{1}{2}$ |
|         | OR                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                |                                     |
| 24. (b) | <p>(b) If <math>\sqrt{3}(x^2 + y^2) = 4xy</math>, then find <math>\frac{dy}{dx}</math> at <math>\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)</math>.</p>                                                                                                                                                                                                                                                                                                                                                                          |                |                                     |
| Sol.    | <p>Differentiating both sides w.r.to <math>x</math>, we get <math>\sqrt{3}\left(2x + 2y \frac{dy}{dx}\right) = 4\left(x \frac{dy}{dx} + y\right)</math></p> $\Rightarrow \frac{dy}{dx} = \frac{2y - \sqrt{3}x}{\sqrt{3}y - 2x}$ $\Rightarrow \left(\frac{dy}{dx}\right)_{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)} = \sqrt{3}$                                                                                                                                                                                                | I<br>II<br>III | 1<br>$\frac{1}{2}$<br>$\frac{1}{2}$ |
| 25. (a) | <p>Simplify : <math>\cot^{-1} \sqrt{\frac{1 + \cos 2x}{1 - \cos 2x}}, x \in \left(0, \frac{\pi}{2}\right)</math>.</p>                                                                                                                                                                                                                                                                                                                                                                                                             |                |                                     |
| Sol.    | <p>Given expression = <math>\cot^{-1} \left( \sqrt{\frac{2\cos^2 x}{2\sin^2 x}} \right)</math></p> $= \cot^{-1}(\cot x) = x$                                                                                                                                                                                                                                                                                                                                                                                                      | I<br>II        | 1<br>1                              |

|                                                                 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                         |                                        |
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|                                                                 | OR                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |                                         |                                        |
| 25.(b)                                                          | Evaluate : $\sin (\tan^{-1} (-\sqrt{3}) - \sec^{-1} 2)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                         |                                        |
| Sol.                                                            | <p>Given expression = <math>\sin (-\frac{\pi}{3} - \frac{\pi}{3})</math></p> <p><math>= \sin (-\frac{2\pi}{3}) = -\frac{\sqrt{3}}{2}</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | I                                       | 1                                      |
|                                                                 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | II                                      | 1                                      |
| SECTION C                                                       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                         |                                        |
| Q. Numbers 26 to 31 are short answer questions of 3 marks each. |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                         |                                        |
| 26.                                                             | If $I_1 = \int_{-\pi/4}^{\pi/4} \frac{dx}{1 + \cos 2x}$ and $I_2 = \int_{-1/2}^{1/2}  x  dx$ , then show that $I_1 - 4I_2 = 0$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                         |                                        |
| Sol.                                                            | <p><math>I_1 = 2 \int_0^{\pi/4} \frac{dx}{1 + \cos 2x}</math>, (<math>\frac{1}{1 + \cos 2x}</math> is an even function)</p> <p><math>= 2 \int_0^{\pi/4} \frac{dx}{2 \cos^2 x} = \int_0^{\pi/4} \sec^2 x dx</math></p> <p><math>= [\tan x]_0^{\pi/4} = 1</math></p> <p>Note: Students attempting <math>I_1</math> without the application of any property should be marked proportionately.</p> <p><math>I_2 = 2 \int_0^{1/2}  x  dx</math>, (<math> x </math> is an even function)</p> <p><math>= 2 \int_0^{1/2} x dx</math></p> <p><math>= [x^2]_0^{1/2} = \frac{1}{4}</math></p> <p><math>I_1 - 4I_2 = 0</math></p> | <p>I</p> <p>II</p> <p>III</p> <p>IV</p> | <p>1½</p> <p>1/2</p> <p>½</p> <p>½</p> |

|         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                    |                                                  |
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| 27. (a) | Find the general solution of the differential equation :<br>$y^2 dx + (x^2 - xy + y^2) dy = 0$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                                    |                                                  |
| Sol.    | <p>Given differential equation is <math>\frac{dx}{dy} = -\left(\frac{x}{y}\right)^2 + \frac{x}{y} - 1</math></p> <p>Put <math>x = vy \Rightarrow \frac{dx}{dy} = v + y \frac{dv}{dy}</math></p> <p>The given differential equation becomes <math>y \frac{dv}{dy} = -1 - v^2</math></p> <p><math>\Rightarrow \frac{dv}{1+v^2} = -\frac{dy}{y}</math></p> <p><math>\Rightarrow \int \frac{dv}{1+v^2} = -\int \frac{dy}{y}</math></p> <p><math>\Rightarrow \tan^{-1} v = -\log y  + C</math></p> <p><math>\Rightarrow \tan^{-1} \frac{x}{y} = -\log y  + C</math></p> <p>Note: Students attempting the question with <math>\frac{dy}{dx}</math> should be given proportionate marks.</p> | I<br>II<br>III<br>IV<br>V                          | 1/2<br>1/2<br>1/2<br>1<br>1/2                    |
|         | OR                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |                                                    |                                                  |
| 27. (b) | Find the particular solution of the differential equation $\frac{dy}{dx} = y \tan x$ ,<br>given that $y = 2$ if $x = 0$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                    |                                                  |
| Sol.    | <p>Given differential equation is <math>\frac{dy}{y} = \tan x \, dx</math></p> <p><math>\Rightarrow \int \frac{dy}{y} = \int \tan x \, dx</math></p> <p><math>\Rightarrow \log y  = \log \sec x  + \log C,</math></p> <p><math>\Rightarrow \log \left  \frac{y}{\sec x} \right  = \log C</math></p> <p><math>\Rightarrow \frac{y}{\sec x} = C</math></p> <p><math>\Rightarrow y = C \sec x</math></p> <p>When <math>x = 0, y = 2 \Rightarrow C = 2</math></p> <p>Hence, the required particular solution is <math>y = 2 \sec x</math></p>                                                                                                                                             | I<br><br><br><br><br><br><br>II<br><br><br><br>III | 1<br><br><br><br><br><br><br>1½<br><br><br><br>½ |

| 28.          | <p>Solve the following linear programming problem graphically :</p> <p>Minimize <math>Z = 13x - 15y</math></p> <p>Subject to constraints</p> $x + y \leq 7,$ $2x - 3y + 6 \geq 0,$ $x \geq 0, y \geq 0$                                                                                                                                                                                                                           |              |                          |        |   |        |    |        |     |        |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  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| Sol.         | <div></div> <table><tr><th>Corner point</th><th>Value of <math>Z = 13x - 15y</math></th></tr><tr><td>(0, 0)</td><td>0</td></tr><tr><td>(7, 0)</td><td>91</td></tr><tr><td>(3, 4)</td><td>-21</td></tr><tr><td>(0, 2)</td><td>-30</td></tr></table> <p>The minimum value of <math>Z</math> is <math>-30</math> at <math>x = 0, y = 2</math></p> | Corner point | Value of $Z = 13x - 15y$ | (0, 0) | 0 | (7, 0) | 91 | (3, 4) | -21 | (0, 2) | -30 | I<br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br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|
| Corner point | Value of $Z = 13x - 15y$                                                                                                                                                                                                                                                                                                                                                                                                          |              |                          |        |   |        |    |        |     |        |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  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| (0, 0)       | 0                                                                                                                                                                                                                                                                                                                                                                                                                                 |              |                          |        |   |        |    |        |     |        |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  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| (7, 0)       | 91                                                                                                                                                                                                                                                                                                                                                                                                                                |              |                          |        |   |        |    |        |     |        |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  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| (3, 4)       | -21                                                                                                                                                                                                                                                                                                                                                                                                                               |              |                          |        |   |        |    |        |     |        |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  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| (0, 2)       | -30                                                                                                                                                                                                                                                                                                                                                                                                                               |              |                          |        |   |        |    |        |     |        |     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  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|---------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------|--|
| 29. (a) | Out of two bags, bag I contains 3 red and 4 white balls and bag II contains 8 red and 6 white balls. A die is thrown. If it shows a number less than 3 then a ball is drawn at random from bag I, otherwise a ball is drawn at random from bag II. Find the probability that the ball drawn from one of the bags is a red ball.                                                                                                                                                                                            |                                                                                               |  |
| Sol.    | <p>Let <math>E_1</math>: The number appearing on the die <math>&lt; 3</math></p> <p><math>E_2</math>: The number appearing on the die <math>\geq 3</math></p> <p>A: Red ball is drawn</p> <p><math>P(E_1) = \frac{2}{6}, P(A/E_1) = \frac{3}{7}, P(E_2) = \frac{4}{6}, P(A/E_2) = \frac{8}{14}</math></p> <p>The required probability <math>= P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2)</math></p> <p><math>= \frac{2}{6} \times \frac{3}{7} + \frac{4}{6} \times \frac{8}{14}</math></p> <p><math>= \frac{11}{21}</math></p> | <p>I <math>\frac{1}{2}</math></p> <p>II 1</p> <p>III 1</p> <p>IV <math>\frac{1}{2}</math></p> |  |
|         | OR                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                               |  |
| 29. (b) | The probability of simultaneous occurrence of atleast one of the two events X and Y is a. If the probability that exactly one of the events X, Y occurs is b, prove that $P(X') + P(Y') = 2 - 2a + b$ .                                                                                                                                                                                                                                                                                                                    |                                                                                               |  |
| Sol.    | <p><math>P(X \cup Y) = a</math></p> <p><math>P(X \cup Y) - P(X \cap Y) = b</math></p> <p>Hence, <math>P(X \cap Y) = a - b</math></p> <p><math>P(X') + P(Y') = [1 - P(X)] + [1 - P(Y)]</math></p> <p><math>= 2 - [P(X) + P(Y)]</math></p> <p><math>= 2 - [P(X \cup Y) + P(X \cap Y)]</math></p> <p><math>= 2 - [a + (a - b)]</math></p> <p><math>= 2 - 2a + b</math></p>                                                                                                                                                    | <p>I <math>\frac{1}{2}</math></p> <p>II 1</p> <p>III 1</p> <p>IV <math>\frac{1}{2}</math></p> |  |



|                                                                |                                                                                                                                                                                                                                                                                                                                                                                         |                                           |                                                               |
|----------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------|---------------------------------------------------------------|
|                                                                | OR                                                                                                                                                                                                                                                                                                                                                                                      |                                           |                                                               |
| 31.(b)                                                         | Find : $\int \frac{x^2}{(x^2 + 9)(x^2 + 16)} dx$                                                                                                                                                                                                                                                                                                                                        |                                           |                                                               |
| Sol.                                                           | Put $x^2 = t$ to get $\frac{x^2}{(x^2+9)(x^2+16)} = \frac{t}{(t+9)(t+16)} = \frac{A}{t+9} + \frac{B}{t+16}$<br><br>Getting $A = \frac{-9}{7}, B = \frac{16}{7}$<br><br>Given integral $= \frac{-9}{7} \int \frac{1}{x^2+9} dx + \frac{16}{7} \int \frac{1}{x^2+16} dx$<br><br>$= \frac{-3}{7} \tan^{-1}\left(\frac{x}{3}\right) + \frac{4}{7} \tan^{-1}\left(\frac{x}{4}\right) + C$    | I<br><br><br>II<br>III<br>IV              | $\frac{1}{2}$<br><br><br>1<br>$\frac{1}{2}$<br>1              |
| SECTION D                                                      |                                                                                                                                                                                                                                                                                                                                                                                         |                                           |                                                               |
| Q. Numbers 32 to 35 are long answer questions of 5 marks each. |                                                                                                                                                                                                                                                                                                                                                                                         |                                           |                                                               |
| 32.                                                            | If $x = 3 \sin t - \sin 3t, y = 3 \cos t - \cos 3t$<br><br>find $\frac{dy}{dx}$ and prove that $\frac{d^2y}{dx^2} = \frac{-\operatorname{cosec}^3 2t \cdot \operatorname{cosec} t}{3}$                                                                                                                                                                                                  |                                           |                                                               |
| Sol.                                                           | $\frac{dx}{dt} = 3 \cos t - 3 \cos 3t,$<br><br>$\frac{dy}{dt} = -3 \sin t + 3 \sin 3t$<br><br>$\frac{dy}{dx} = \frac{3(\sin 3t - \sin t)}{3(\cos t - \cos 3t)}$<br><br>$= \frac{2 \cos 2t \sin t}{2 \sin 2t \sin t}$<br><br>$= \cot 2t$<br><br>$\frac{d^2y}{dx^2} = -2 \operatorname{cosec}^2 2t \times \frac{dt}{dx}$<br><br>$= -\frac{2 \operatorname{cosec}^2 2t}{6 \sin 2t \sin t}$ | I<br><br><br>II<br>III<br><br>IV<br><br>V | 1<br><br><br>1<br>$\frac{1}{2}$<br><br>$\frac{1}{2}$<br><br>1 |

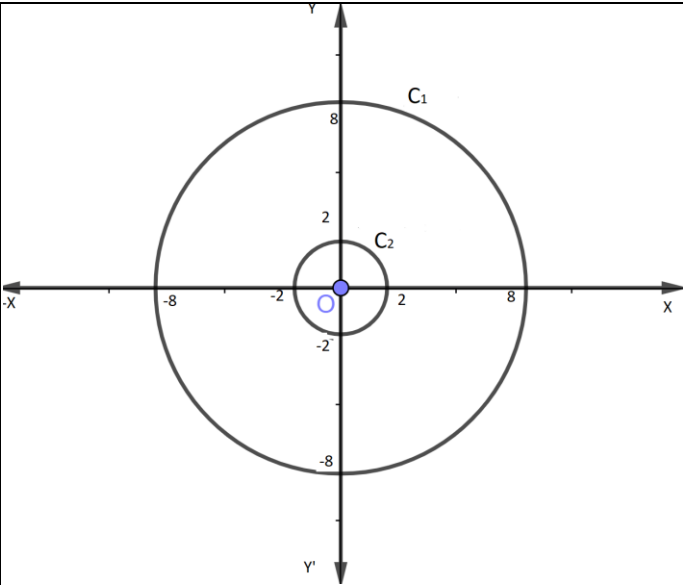
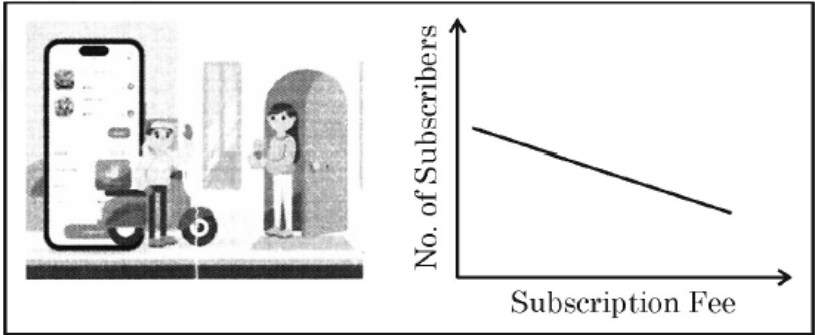
|        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |                                                            |                                                                                                     |
|--------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------|-----------------------------------------------------------------------------------------------------|
|        | $= -\frac{\operatorname{cosec}^3 2t \operatorname{cosec} t}{3}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | VI                                                         | 1                                                                                                   |
| 33.    | Prove that the line through points A(0, -1, -1) and B(4, 5, 1) intersects the line through points C(3, 9, 4) and D(-4, 4, 4). Hence, write the equation of line passing through the point of intersection of lines AB and CD as well as origin.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |                                                            |                                                                                                     |
| Sol.   | <p>Lines AB and CD respectively are</p> $\frac{x}{2} = \frac{y+1}{3} = \frac{z+1}{1}$ <p>and</p> $\frac{x-3}{-7} = \frac{y-9}{-5} = \frac{z-4}{0}$ <p>Any point on line AB is <math>(2\lambda, 3\lambda - 1, \lambda - 1)</math></p> <p>Any point on line CD is <math>(-7\mu + 3, -5\mu + 9, 4)</math></p> <p>For the lines to intersect, we must have some <math>\lambda</math> and <math>\mu</math>, for which these coordinates must coincide, i.e., we must have <math>\lambda - 1 = 4, 2\lambda = -7\mu + 3, 3\lambda - 1 = -5\mu + 9</math>, for some <math>\lambda</math> and <math>\mu</math></p> <p>The first two equations, when solved, give us <math>\lambda = 5</math> and <math>\mu = -1</math>. These values satisfy the third equation. Hence, the lines intersect.</p> <p>The point of intersection is (10, 14, 4)</p> <p>The required equation is</p> $\frac{x}{10} = \frac{y}{14} = \frac{z}{4} \text{ or } \frac{x}{5} = \frac{y}{7} = \frac{z}{2} \text{ or } \frac{x-10}{5} = \frac{y-14}{7} = \frac{z-4}{2}$ | <p>I</p> <p>II</p> <p>III</p> <p>IV</p> <p>V</p> <p>VI</p> | <p>1</p> <p>1</p> <p>1</p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p>1</p> |
| 34.(a) | <p>A relation R is defined on Z, the set of integers, as</p> $R = \{(x, y) :  x - y  \text{ is divisible by a prime number 'p', } x, y \in \mathbb{Z}\}$ <p>check whether R is an equivalence relation or not.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |                                                            |                                                                                                     |
| Sol.   | <p>Let <math>x \in \mathbb{Z}</math>. Then <math> x - x  = 0</math>, which is divisible by a prime number p.</p> <p>Hence, <math>(x, x) \in R</math>. Thus, R is reflexive.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | I                                                          | $1\frac{1}{2}$                                                                                      |

|        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |    |    |
|--------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|----|
|        | <p>Let <math>x, y \in Z</math> such that <math>(x, y) \in R</math>.</p> <p>Then <math> x - y </math> is divisible by the prime number <math>p</math>.</p> <p><math>\Rightarrow  y - x </math> is divisible by the prime number <math>p</math> as <math> x - y  =  y - x </math></p> <p>Hence, <math>(y, x) \in R</math>. Thus, <math>R</math> is symmetric.</p> <p>Let <math>x, y, z \in Z</math> such that <math>(x, y), (y, z) \in R</math>.</p> <p>Then <math> x - y </math> is divisible by the prime number <math>p</math> and <math> y - z </math> is divisible by the prime number <math>p</math>.</p> <p><math>\Rightarrow x - y</math> is divisible by the prime number <math>p</math> and <math>y - z</math> is divisible by the prime number <math>p</math>.</p> <p><math>\Rightarrow x - y + y - z</math> is divisible by the prime number <math>p</math></p> <p><math>\Rightarrow x - z</math> is divisible by the prime number <math>p</math></p> <p><math>\Rightarrow  x - z </math> is divisible by the prime number <math>p</math></p> <p>Hence, <math>(x, z) \in R</math>. Thus, <math>R</math> is transitive.</p> <p>Since, <math>R</math> is reflexive, symmetric and transitive, therefore, <math>R</math> is an equivalence relation.</p> | II | 1½ |
|        | OR                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |    |    |
| 34.(b) | <p>A function <math>f : R - \left\{ \frac{3}{5} \right\} \longrightarrow R - \left\{ \frac{3}{5} \right\}</math> is defined as <math>f(x) = \frac{3x+2}{5x-3}</math>. Show that <math>f</math> is one-one and onto.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |    |    |
| Sol.   | <p>Let <math>x_1, x_2 \in R - \left\{ \frac{3}{5} \right\}</math> such that <math>f(x_1) = f(x_2)</math></p> <p><math>\Rightarrow \frac{3x_1+2}{5x_1-3} = \frac{3x_2+2}{5x_2-3}</math></p> <p><math>\Rightarrow 15x_1x_2 - 9x_1 + 10x_2 - 6 = 15x_1x_2 + 10x_1 - 9x_2 - 6</math></p> <p><math>\Rightarrow x_1 = x_2</math></p> <p>Hence, <math>f</math> is one-one.</p> <p>Let <math>y \in R - \left\{ \frac{3}{5} \right\}</math> (Codomain).</p> <p>Then <math>f(x) = y</math></p> <p>or, <math>\frac{3x+2}{5x-3} = y \Rightarrow x = \frac{3y+2}{5y-3} \in \text{Domain}</math>.</p> <p>Hence, for every <math>y \in R - \left\{ \frac{3}{5} \right\}</math> (codomain),</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | I  | 2½ |

|        |                                                                                                                                                                                                                                                                                                    |     |    |
|--------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|----|
|        | there exists $x = \frac{3y+2}{5y-3} \in R - \left\{\frac{3}{5}\right\}$ (domain) such that $f\left(\frac{3y+2}{5y-3}\right) = y$<br>Thus, f is onto.                                                                                                                                               | II  | 2½ |
| 35.(a) | If $A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & -1 & -2 \\ 1 & -1 & 0 \end{bmatrix}$ , find $A^{-1}$ and use it to solve the following system of equations :<br>$-2y + z = 7, 2x - y - z = 8, x - 2y = 10$                                                                                               |     |    |
| Sol.   | $ A  = -2(2) + 1(3) = -1 \neq 0$                                                                                                                                                                                                                                                                   | I   | 1  |
|        | $adjA = \begin{bmatrix} -2 & -1 & -3 \\ -2 & -1 & -2 \\ 3 & 2 & 4 \end{bmatrix}$                                                                                                                                                                                                                   | II  | 1½ |
|        | $A^{-1} = \frac{1}{ A } adjA = \begin{bmatrix} 2 & 1 & 3 \\ 2 & 1 & 2 \\ -3 & -2 & -4 \end{bmatrix}$                                                                                                                                                                                               | III | ½  |
|        | Given system of equations is equivalent to $A^T X = B$ , where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 7 \\ 8 \\ 10 \end{bmatrix}$                                                                                                                                     | IV  | ½  |
|        | $X = (A^T)^{-1} B = (A^{-1})^T B$<br>$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & 2 & -3 \\ 1 & 1 & -2 \\ 3 & 2 & -4 \end{bmatrix} \begin{bmatrix} 7 \\ 8 \\ 10 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$<br>$\therefore x = 0, y = -5, z = -3$ | V   | 1½ |
|        | OR                                                                                                                                                                                                                                                                                                 |     |    |
| 35.(b) | If $\begin{bmatrix} 3 & -1 & \sin 3x \\ -7 & 4 & \cos 2x \\ -11 & 7 & 2 \end{bmatrix}$ is a singular matrix, then find all values of $x$ where $x \in \left[0, \frac{\pi}{2}\right]$ .                                                                                                             |     |    |
| Sol.   | Given matrix is singular, hence, its determinant = 0,<br>i.e., $\begin{vmatrix} 3 & -1 & \sin 3x \\ -7 & 4 & \cos 2x \\ -11 & 7 & 2 \end{vmatrix} = 0$                                                                                                                                             | I   | 1  |

|         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                      |                  |
|---------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------|------------------|
|         | $\Rightarrow 2\cos 2x + \sin 3x = 2$<br>$\Rightarrow 4\sin^3 x + 4\sin^2 x - 3\sin x = 0$<br>$\Rightarrow \sin x(4\sin^2 x + 4\sin x - 3) = 0$<br>$\Rightarrow \sin x(2\sin x - 1)(2\sin x + 3) = 0$<br>Solving to get $x = 0, \frac{\pi}{6}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | II<br>III<br>IV<br>V | 1<br>1<br>1<br>1 |
|         | SECTION E<br>This section (Q. 36 to 38) has 3 case study-based questions of 4 marks each.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                      |                  |
| 36.     | <p>In an online jackpot, there is one first prize of ₹ 3,00,000, two second prizes of ₹ 2,00,000 each and three third prizes of ₹ 50,000 each.</p>  <p>A total of 1,00,000 jackpot tickets each costing ₹ 100 were sold there by raising a fund of ₹ 1,00,00,000.</p> <p>Rohan bought one ticket.</p> <p>Based on given information, answer the following questions :</p> <p>(i) What are the possible amounts, the person can win ?</p> <p>(ii) (a) What is the probability that the person wins atleast ₹ 2,00,000 ?</p> <p style="text-align: center;"><b>OR</b></p> <p>(ii) (b) What is the probability that the person does not win any amount ?</p> <p>(iii) In another jackpot, Rohan also bought a ticket having a prize money of ₹ 5,00,000. The chances of winning the jackpot are 1 in 1,00,000. Find the probability that on exactly one of tickets he wins the jackpot.</p> |                      |                  |
| Sol.(i) | ₹ 300000, ₹ 200000, ₹ 50000                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | I                    | 1                |

|                 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |   |   |
|-----------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|---|
|                 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |   |   |
| Sol.(ii)<br>(a) | Required probability = $\frac{3}{100000}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | I | 2 |
|                 | OR                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |   |   |
| Sol.(ii)<br>(b) | Required probability = $1 - \frac{6}{100000} = \frac{99994}{100000}$ or $\frac{49997}{50000}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | I | 2 |
| Sol.(iii)       | <p>P (Exactly on one of the tickets Rohan wins the jackpot) = P [(He wins first and he loses second) or (He loses first and wins second)]</p> $= \frac{6}{100000} \times \frac{99999}{100000} + \frac{99994}{100000} \times \frac{1}{100000} = \frac{699988}{10^{10}} \text{ or } \frac{174997}{2500000000}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | I | 1 |
| 37.             | <p>Roundabouts are often made on busy roads to ease the traffic and avoid red lights.</p>  <p>One such round-about is made such that equation representing its boundary is given by <math>C_1 : x^2 + y^2 = 64</math>.</p> <p>There is a circular pond with a fountain in the middle of the roundabout whose equation is given by <math>C_2 : x^2 + y^2 = 4</math>.</p> <p>Based on the given information, answer the following questions :</p> <p>(i) Represent the given equations <math>C_1</math> and <math>C_2</math> with the help of a diagram.</p> <p>(ii) Express <math>y</math> as a function of <math>x</math>, (<math>y = f(x)</math>), for both <math>C_1</math> and <math>C_2</math>.</p> <p>(iii) (a) Using integration find the area of region covered by the roundabout.</p> <p style="text-align: center;"><b>OR</b></p> <p>(iii) (b) Using integration, find the area of region covered by circular pond.</p> |   |   |

|                  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |         |                                |
|------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------|--------------------------------|
| Sol.(i)          |                                                                                                                                                                                                                                                                                                                                                                                                                                                               | I       | 1                              |
| Sol.(ii)         | <p>Circle <math>C_1</math>: <math>y = \sqrt{64 - x^2}</math> or <math>y = -\sqrt{64 - x^2}</math></p> <p>Circle <math>C_2</math>: <math>y = \sqrt{4 - x^2}</math> or <math>y = -\sqrt{4 - x^2}</math></p>                                                                                                                                                                                                                                                                                                                                      | I<br>II | $\frac{1}{2}$<br>$\frac{1}{2}$ |
| Sol.(iii)<br>(a) | <p>Required area <math>= 4 \times \int_0^8 \sqrt{64 - x^2} dx</math></p> <p><math>= 2[x\sqrt{64 - x^2} + 64\sin^{-1}\frac{x}{8}]_0^8 = 64\pi</math></p>                                                                                                                                                                                                                                                                                                                                                                                        | I<br>II | 1<br>1                         |
|                  | OR                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |         |                                |
| Sol.(iii)<br>(b) | <p>Required area <math>= 4 \times \int_0^2 \sqrt{4 - x^2} dx</math></p> <p><math>= 2[x\sqrt{4 - x^2} + 4\sin^{-1}\frac{x}{2}]_0^2 = 4\pi</math></p>                                                                                                                                                                                                                                                                                                                                                                                            | I<br>II | 1<br>1                         |
| 38.              | <p>An online delivery company in a city has 5000 subscribers and collects annual subscription fees of ₹ 300 per subscriber for unlimited free deliveries.</p> <div data-bbox="323 1516 1141 1850" data-label="Figure">  </div> <p>The company wishes to increase the annual subscription fee. It is predicted that, for every increase of ₹ 1, ten subscribers will discontinue. Assume that the company increased the annual fee by ₹ <math>x</math>.</p> |         |                                |

|           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |     |               |
|-----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|---------------|
|           | <p>Based on the given information, answer the following questions :</p> <p>(i) How many subscribers will discontinue after an increase of ₹ <math>x</math> in annual fee ?</p> <p>(ii) If <math>R(x)</math> denotes the total revenue collected after the increase of ₹ <math>x</math> in subscription fee, express <math>R(x)</math> as a function of <math>x</math>.</p> <p>(iii) (a) Find the value of <math>x</math> for which <math>R(x)</math> is maximum.</p> <p style="text-align: center;"><b>OR</b></p> <p>(iii) (b) Find the sub-intervals of <math>(0, 5000)</math> in which <math>R(x)</math> is increasing and decreasing.</p> |     |               |
| Sol.(i)   | $10x$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | I   | 1             |
| Sol.(ii)  | $R(x) = (5000 - 10x)(300 + x)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | I   | 1             |
| Sol.(iii) | $R'(x) = 2000 - 20x$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | I   | $\frac{1}{2}$ |
| (a)       | $R'(x) = 0 \Rightarrow x = 100$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | II  | $\frac{1}{2}$ |
|           | $R''(x) = -20 < 0$ at $x = 100$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | III | $\frac{1}{2}$ |
|           | At $x = 100, R(x)$ is maximum                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | IV  | $\frac{1}{2}$ |
|           | OR                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |     |               |
| Sol.(iii) | $R'(x) = 2000 - 20x$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | I   | $\frac{1}{2}$ |
| (b)       | $R'(x) = 0 \Rightarrow x = 100$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | II  | $\frac{1}{2}$ |
|           | In $(0, 100), R'(x) > 0$ . Therefore, $R(x)$ is increasing in $(0, 100)$ or $(0, 100]$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | III | $\frac{1}{2}$ |
|           | In $(100, 5000), R'(x) < 0$ . Therefore, $R(x)$ is decreasing in $(100, 5000)$ or $[100, 5000)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | IV  | $\frac{1}{2}$ |