

MATHEMATICS (Subject Code –

041) (PAPER CODE: 65/1/1)

SOLUTIONS

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Steps	Marks
	SECTION A		
	Q. Number 1 to 20 are multiple choice questions of 1 mark each.		
1.	If $2 \cos^{-1}x = y$, then (A) $0 \leq y \leq \pi$ (B) $-\pi \leq y \leq \pi$ (C) $0 \leq y \leq 2\pi$ (D) $-\pi \leq y \leq 0$		
Sol.	(C) $0 \leq y \leq 2\pi$		1
2.	Which of the following cannot be the order of a row-matrix ? (A) 2×1 (B) 1×2 (C) 1×1 (D) $1 \times n$		
Sol.	(A) 2×1		1
3.	Which of the following properties is/are true for two matrices of suitable orders ? (i) $(A + B)' = A' + B'$ (ii) $(A - B)' = B' - A'$ (iii) $(AB)' = A'B'$ (iv) $(kAB)' = kB'A'$ (k is a scalar) (A) (i) only (B) (i), (ii) and (iii) (C) (i) and (ii) (D) (i) and (iv)		
Sol.	(D) (i) and (iv)		1

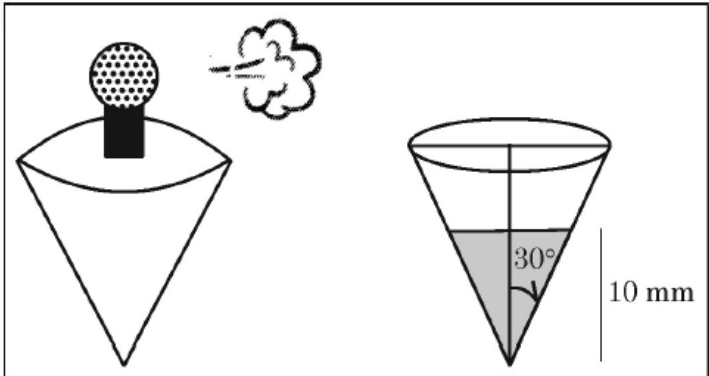
4.	If $\Delta_1 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 6 \end{vmatrix}$, then (A) $\Delta_1 = 2\Delta_2$ (B) $\Delta_2 = -2\Delta_1$ (C) $\Delta_1 = \Delta_2$ (D) $\Delta_2 = -\Delta_1$		
Sol.	(B) $\Delta_2 = -2\Delta_1$		1
5.	One of the values of x for which $\begin{vmatrix} \cos x & \sin x \\ -\cos x & \sin x \end{vmatrix} = 1$ is (A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$		
Sol.	(B) $\frac{\pi}{4}$		1
6.	If A and B are skew symmetric matrices of same order, then which of the following matrices is also skew symmetric ? (A) AB (B) AB + BA (C) $(A + B)^2$ (D) A – B		
Sol.	(D) A – B		1

7.	The least value of $f(x) = x^3 - 12x$, $x \in [0, 3]$ is (A) -16 (B) -9 (C) 0 (D) 16		
Sol.	(A) -16		1
8.	If $\int \frac{3ax}{b^2 + c^2x^2} dx = A \log b^2 + c^2x^2 + K$, then the value of A is (A) $3a$ (B) $\frac{3a}{2b^2}$ (C) $\frac{3a}{b^2c^2}$ (D) $\frac{3a}{2c^2}$		
Sol.	(D) $\frac{3a}{2c^2}$		1
9.	The value of $\int_{-1}^1 \frac{x^3}{x^2 + 2 x + 1} dx$ is (A) 0 (B) $\log 2$ (C) $2 \log 2$ (D) $\frac{1}{2} \log 2$		
Sol.	(A) 0		1

10.	The area bounded by the curve $y = x x$, x-axis and the ordinates $x = -1$ and $x = 1$ is given by (A) 0 (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{4}{3}$		
Sol.	(C) $\frac{2}{3}$		1
11.	The integrating factor of differential equation $R \frac{dx}{dy} + Px = Q$ where P, Q, R are functions of y is (A) $e^{\int \frac{P}{Q} dy}$ (B) $e^{\int P dy}$ (C) $e^{\int \frac{P}{R} dy}$ (D) $e^{\int \frac{P}{R} dx}$		
Sol.	(C) $e^{\int \frac{P}{R} dy}$		1
12.	The order and degree of the differential equation $\frac{d}{dx}(e^y) = 0$ respectively are (A) 0, 1 (B) 1, 1 (C) 2, 1 (D) 1, not defined		
Sol.	(B) 1, 1		1

13.	The value of p for which vectors $\hat{i} + 2\hat{j} + 3\hat{k}$ and $2\hat{i} - p\hat{j} + \hat{k}$ are perpendicular to each other is (A) 0 (B) 1 (C) $\frac{5}{2}$ (D) $-\frac{5}{2}$		
Sol.	(C) $\frac{5}{2}$		1
14.	The value of m for which the points with position vectors $-\hat{i} - \hat{j} + 2\hat{k}$, $2\hat{i} + m\hat{j} + 5\hat{k}$ and $3\hat{i} + 11\hat{j} + 6\hat{k}$ are collinear, is (A) 8 (B) -8 (C) 2 (D) $\frac{5}{2}$		
Sol.	(A) 8		1
15.	If $\vec{a} = 8$, $\vec{b} = 3$ and $\vec{a} \times \vec{b} = 12$, then the value of $\vec{a} \cdot \vec{b}$ (A) $6\sqrt{3}$ (B) $8\sqrt{3}$ (C) $12\sqrt{3}$ (D) $3\sqrt{12}$		
Sol.	(C) $12\sqrt{3}$		1
16.	The length of perpendicular drawn from point (2, 5, 7) on line $\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$ is (A) 2 (B) 5 (C) $\sqrt{74}$ (D) $\sqrt{78}$		
Sol.	(C) $\sqrt{74}$		1

19.	Assertion (A) : In an experiment of throwing an unbiased die, the probability of getting a prime number given that number appearing on the die being odd is $\frac{2}{3}$. Reason (R) : For any two events A and B, $P(A B) = \frac{P(A \cap B)}{P(B)}$		
Sol.	(C) Assertion (A) is true and Reason (R) is false.		1
20.	Assertion (A) : Lines given by $x = py + q$, $z = ry + s$ and $x = p'y + q'$, $z = r'y + s'$ are perpendicular to each other when $pp' + rr' = 1$. Reason (R) : Two lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ are perpendicular to each other if $\vec{b}_1 \cdot \vec{b}_2 = 0$.		
Sol.	(D) Assertion (A) is false and Reason (R) is true.		1
SECTION B			
Q. Numbers 21 to 25 are very short answer questions of 2 marks each.			
21.(a)	Check whether function $f(x)$ defined as $f(x) = \begin{cases} \frac{ x-3 }{2(x-3)}, & x < 3 \\ \frac{x-6}{6}, & x \geq 3 \end{cases}$ is continuous at $x = 3$ or not ?		
Sol.	$f(x) = \begin{cases} \frac{-(x-3)}{2(x-3)}, & x < 3 \\ \frac{x-6}{6}, & x \geq 3 \end{cases} \Rightarrow f(x) = \begin{cases} \frac{-1}{2}, & x < 3 \\ \frac{x-6}{6}, & x \geq 3 \end{cases}$ $f(3) = \frac{-1}{2}$ $\text{LHL} = \lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} \frac{-1}{2} = \frac{-1}{2}$ $\text{RHL} = \lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} \frac{x-6}{6} = \frac{-1}{2}$	I II	1 $\frac{1}{2}$

	LHL = RHL = $f(3)$, hence the function $f(x)$ is continuous at $x = 3$.	III	$\frac{1}{2}$
	OR		
21.(b)	(b) If $\sqrt{3}(x^2 + y^2) = 4xy$, then find $\frac{dy}{dx}$ at $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.		
Sol.	Differentiating both sides w.r.to x , we get $\sqrt{3}\left(2x + 2y\frac{dy}{dx}\right) = 4\left(x\frac{dy}{dx} + y\right)$	I	1
	$\Rightarrow \frac{dy}{dx} = \frac{2y - \sqrt{3}x}{\sqrt{3}y - 2x}$	II	$\frac{1}{2}$
	$\Rightarrow \left(\frac{dy}{dx}\right)_{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)} = \sqrt{3}$	III	$\frac{1}{2}$
22.	A room freshner bottle in the shape of an inverted cone sprays the perfume at regular intervals such that volume of the perfume in the bottle decreases at the steady rate of $1 \text{ mm}^3/\text{min}$. Find the rate at which level of perfume is dropping at an instant when level of perfume in the bottle is 10 mm, if the semi-vertical angle of conical bottle is $\frac{\pi}{6}$. 		
Sol.	Let v , r , h , respectively, be the volume, radius and the height of perfume at any time t .		
	$\frac{dv}{dt} = -1 \text{ mm}^3/\text{min}$	I	$\frac{1}{2}$
	$\frac{r}{h} = \tan \frac{\pi}{6} \Rightarrow r = \frac{h}{\sqrt{3}}$	II	$\frac{1}{2}$
	$v = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \frac{h^3}{3}$	III	$\frac{1}{2}$
	$\Rightarrow \frac{dv}{dt} = \frac{\pi}{9} 3h^2 \frac{dh}{dt}$		

	$\Rightarrow \frac{dh}{dt} = -\frac{3}{\pi h^2}$ $\Rightarrow \left(\frac{dh}{dt}\right)_{h=10} = -\frac{3}{100\pi}$ The level of perfume is dropping at the rate of $\frac{3}{100\pi} \text{ mm/min}$	IV	$\frac{1}{2}$
23.	Find the vector of magnitude 14 in the direction of $\overrightarrow{QP}$, where P and Q are the points (1, 3, 2) and (-1, 0, 8) respectively.		
Sol.	$\overrightarrow{QP} = 2\hat{i} + 3\hat{j} - 6\hat{k}$ $ \overrightarrow{QP} = \sqrt{4 + 9 + 36} = 7$ The unit vector in the direction of $\overrightarrow{QP} = \frac{\overrightarrow{QP}}{ \overrightarrow{QP} } = \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}$ The required vector = $14\left(\frac{\overrightarrow{QP}}{ \overrightarrow{QP} }\right) = 4\hat{i} + 6\hat{j} - 12\hat{k}$	I II III	$\frac{1}{2}$ $\frac{1}{2}$ 1
24.	Vectors $\vec{a} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{k}$ represent the two adjacent sides of a parallelogram. Find the vectors representing its diagonals and hence find their lengths.		
Sol.	$\vec{a} + \vec{b} = 4\hat{i} - 2\hat{j} + 4\hat{k}, \vec{a} - \vec{b} = 2\hat{i} - 2\hat{j}$ The vector representing one of its diagonals is $4\hat{i} - 2\hat{j} + 4\hat{k}$ or $-(4\hat{i} - 2\hat{j} + 4\hat{k})$ The vector representing the other diagonal is $2\hat{i} - 2\hat{j}$ or $-(2\hat{i} - 2\hat{j})$ The lengths of the two diagonals are $\sqrt{16 + 4 + 16} = 6$ and $\sqrt{4 + 4} = 2\sqrt{2}$	I II III	$\frac{1}{2}$ $\frac{1}{2}$ 1
25. (a)	Simplify : $\tan^{-1} \left(\frac{\cos 2x - \sin 2x}{\cos 2x + \sin 2x} \right), 0 < x < \frac{\pi}{4}$.		
Sol.	Given expression = $\tan^{-1} \left(\frac{1 - \tan 2x}{1 + \tan 2x} \right)$ $= \tan^{-1} \left[\tan \left(\frac{\pi}{4} - 2x \right) \right] = \frac{\pi}{4} - 2x$	I II	1 1
	OR		

25. (b)	Evaluate : $\tan\left(\sin^{-1} 1 - \cos^{-1}\left(-\frac{1}{2}\right)\right)$		
Sol.	Given expression = $\tan\left(\frac{\pi}{2} - \frac{2\pi}{3}\right)$ $= \tan\left(-\frac{\pi}{6}\right) = -\frac{1}{\sqrt{3}}$	I II	1 1
SECTION C Q. Numbers 26 to 31 are short answer questions of 3 marks each.			
26.	Evaluate : $\int_0^1 x \tan^{-1} x \, dx.$		
Sol.	$\int_0^1 x \tan^{-1} x \, dx = \left[\tan^{-1} x \times \frac{x^2}{2} \right]_0^1 - \int_0^1 \frac{1}{1+x^2} \times \frac{x^2}{2} \, dx$ $= \frac{\pi}{8} - \int_0^1 \frac{1+x^2-1}{1+x^2} \times \frac{1}{2} \, dx$ $= \frac{\pi}{8} - \frac{1}{2} \left[\int_0^1 dx - \int_0^1 \frac{1}{1+x^2} \, dx \right]$ $= \frac{\pi}{8} - \frac{1}{2} [x - \tan^{-1} x]_0^1$ $= \frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{8} = \frac{\pi}{4} - \frac{1}{2}$	I II III	1 1 1
27. (a)	Find $\int \sqrt{\frac{x+2}{x-2}} \, dx$		
Sol.	$I = \int \sqrt{\frac{x+2}{x-2}} \, dx = \int \sqrt{\frac{x+2}{x-2} \times \frac{x+2}{x+2}} \, dx = \int \frac{x+2}{\sqrt{x^2-4}} \, dx$ $= \frac{1}{2} \int \frac{2x}{\sqrt{x^2-4}} \, dx + 2 \int \frac{1}{\sqrt{x^2-4}} \, dx$	I II	1 1

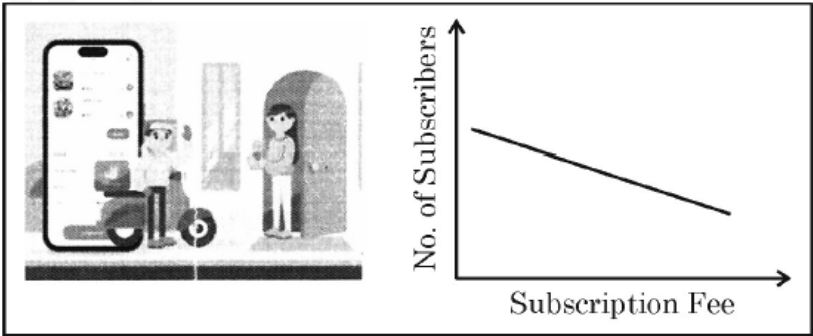
	$I_1 = \frac{1}{2} \int \frac{2x}{\sqrt{x^2-4}} dx = \frac{1}{2} \int \frac{dt}{\sqrt{t}} \quad [(x^2-4) = t \Rightarrow 2x dx = dt]$ $= \sqrt{t} = \sqrt{x^2-4}$	III	1/2
	$I_2 = 2 \int \frac{1}{\sqrt{x^2-4}} dx = 2 \log x + \sqrt{x^2-4} $ $I = I_1 + I_2 = \sqrt{x^2-4} + 2 \log x + \sqrt{x^2-4} + C$	IV	1/2
	OR		
27. (b)	Find : $\int \frac{x^2}{(x^2+9)(x^2+16)} dx$		
Sol.	Put $x^2 = t$ to get $\frac{x^2}{(x^2+9)(x^2+16)} = \frac{t}{(t+9)(t+16)} = \frac{A}{t+9} + \frac{B}{t+16}$ Getting $A = \frac{-9}{7}, B = \frac{16}{7}$ Given integral $= \frac{-9}{7} \int \frac{1}{x^2+9} dx + \frac{16}{7} \int \frac{1}{x^2+16} dx$ $= \frac{-3}{7} \tan^{-1}\left(\frac{x}{3}\right) + \frac{4}{7} \tan^{-1}\left(\frac{x}{4}\right) + C$	I II III IV	$\frac{1}{2}$ 1 $\frac{1}{2}$ 1
28.	If $I_1 = \int_{-\pi/4}^{\pi/4} \frac{dx}{1+\cos 2x}$ and $I_2 = \int_{-1/2}^{1/2} x dx$, then show that $I_1 - 4I_2 = 0$.		
Sol.	$I_1 = 2 \int_0^{\pi/4} \frac{dx}{1+\cos 2x}, \quad \left(\frac{1}{1+\cos 2x} \text{ is an even function}\right)$ $= 2 \int_0^{\pi/4} \frac{dx}{2\cos^2 x} = \int_0^{\pi/4} \sec^2 x dx$ $= [\tan x]_0^{\pi/4} = 1$ Note: Students attempting I_1 without the application of any property should be marked proportionately. $I_2 = 2 \int_0^{1/2} x dx, \quad (x \text{ is an even function})$ $= 2 \int_0^{1/2} x dx$	I II	$1\frac{1}{2}$ 1/2

	$= [x^2]_0^{\frac{1}{2}} = \frac{1}{4}$ $I_1 - 4I_2 = 0$	III	$\frac{1}{2}$
		IV	$\frac{1}{2}$
29. (a)	Find the general solution of the following differential equation : $x^2 \frac{dy}{dx} = x^2 + xy + y^2$		
Sol.	Given differential equation is $\frac{dy}{dx} = 1 + \frac{y}{x} + \left(\frac{y}{x}\right)^2$ Put $y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$ The given differential equation becomes $x \frac{dv}{dx} = 1 + v^2$ $\Rightarrow \frac{dv}{1+v^2} = \frac{dx}{x}$ $\Rightarrow \int \frac{dv}{1+v^2} = \int \frac{dx}{x}$ $\Rightarrow \tan^{-1}v = \log x + C$ $\Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = \log x + C$	I	$\frac{1}{2}$
		II	$\frac{1}{2}$
		III	$\frac{1}{2}$
		IV	1
		V	$\frac{1}{2}$
	OR		
29. (b)	Find the particular solution of the differential equation $xy \frac{dy}{dx} = (x+2)(y+2), \text{ given that } y(1) = -1.$		
Sol.	Given differential equation is $\frac{y}{y+2} dy = \frac{(x+2)}{x} dx$ $\Rightarrow \int \left(1 - \frac{2}{y+2}\right) dy = \int \left(1 + \frac{2}{x}\right) dx$ $\Rightarrow y - 2\log y+2 = x + 2\log x + C \text{ or } y = x + 2\log x(y+2) + C.$ When $x = 1, y = -1 \Rightarrow C = -2$ The required particular solution is	I	1
		II	$1\frac{1}{2}$

31.(a)	Out of two bags, bag I contains 3 red and 4 white balls and bag II contains 8 red and 6 white balls. A die is thrown. If it shows a number less than 3 then a ball is drawn at random from bag I, otherwise a ball is drawn at random from bag II. Find the probability that the ball drawn from one of the bags is a red ball.		
Sol.	Let E_1: The number appearing on the die < 3 E_2: The number appearing on the die ≥ 3 A: Red ball is drawn $P(E_1) = \frac{2}{6}, P(A/E_1) = \frac{3}{7}, P(E_2) = \frac{4}{6}, P(A/E_2) = \frac{8}{14}$ The required probability $= P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2)$ $= \frac{2}{6} \times \frac{3}{7} + \frac{4}{6} \times \frac{8}{14}$ $= \frac{11}{21}$	I 1/2 II 1 III 1 IV 1/2	
	OR		
31.(b)	The probability of simultaneous occurrence of atleast one of the two events X and Y is a. If the probability that exactly one of the events X, Y occurs is b, prove that $P(X') + P(Y') = 2 - 2a + b$.		
Sol.	$P(X \cup Y) = a$ $P(X \cup Y) - P(X \cap Y) = b$ Hence, $P(X \cap Y) = a - b$ $P(X') + P(Y') = [1 - P(X)] + [1 - P(Y)]$ $= 2 - [P(X) + P(Y)]$ $= 2 - [P(X \cup Y) + P(X \cap Y)]$ $= 2 - [a + (a - b)]$	I 1/2 II 1 III 1	

	$\Rightarrow \frac{3x_1 + 2}{5x_1 - 3} = \frac{3x_2 + 2}{5x_2 - 3}$ $\Rightarrow 15x_1x_2 - 9x_1 + 10x_2 - 6 = 15x_1x_2 + 10x_1 - 9x_2 - 6$ $\Rightarrow x_1 = x_2$ Hence, f is one-one. Let $y \in R - \left\{\frac{3}{5}\right\}$ (Codomain). Then $f(x) = y$ or, $\frac{3x+2}{5x-3} = y \Rightarrow x = \frac{3y+2}{5y-3} \in \text{Domain}.$ Hence, for every $y \in R - \left\{\frac{3}{5}\right\}$ (codomain), there exists $x = \frac{3y+2}{5y-3} \in R - \left\{\frac{3}{5}\right\}$ (domain) such that $f\left(\frac{3y+2}{5y-3}\right) = y$ Thus, f is onto.	I	2½
		II	2½
33. (a)	If $A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & -1 & -2 \\ 1 & -1 & 0 \end{bmatrix}$, find A^{-1} and use it to solve the following system of equations : $-2y + z = 7, 2x - y - z = 8, x - 2y = 10$		
Sol.	$A = -2(2) + 1(3) = -1 \neq 0$ $adjA = \begin{bmatrix} -2 & -1 & -3 \\ -2 & -1 & -2 \\ 3 & 2 & 4 \end{bmatrix}$ $A^{-1} = \frac{1}{ A } adjA = \begin{bmatrix} 2 & 1 & 3 \\ 2 & 1 & 2 \\ -3 & -2 & -4 \end{bmatrix}$ Given system of equations is equivalent to $A^T X = B$, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 7 \\ 8 \\ 10 \end{bmatrix}$ $X = (A^T)^{-1} B = (A^{-1})^T B$ $\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & 2 & -3 \\ 1 & 1 & -2 \\ 3 & 2 & -4 \end{bmatrix} \begin{bmatrix} 7 \\ 8 \\ 10 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$ $\therefore x = 0, y = -5, z = -3$	I II III IV V	1 1½ ½ ½ 1½
	OR		

33. (b)	If $\begin{bmatrix} 3 & -1 & \sin 3x \\ -7 & 4 & \cos 2x \\ -11 & 7 & 2 \end{bmatrix}$ is a singular matrix, then find all values of x where $x \in \left[0, \frac{\pi}{2}\right]$.		
Sol.	Given matrix is singular, hence, its determinant = 0, $\text{i. e., } \begin{vmatrix} 3 & -1 & \sin 3x \\ -7 & 4 & \cos 2x \\ -11 & 7 & 2 \end{vmatrix} = 0$ $\Rightarrow 2\cos 2x + \sin 3x = 2$ $\Rightarrow 4\sin^3 x + 4\sin^2 x - 3\sin x = 0$ $\Rightarrow \sin x(4\sin^2 x + 4\sin x - 3) = 0$ $\Rightarrow \sin x(2\sin x - 1)(2\sin x + 3) = 0$ Solving to get $x = 0, \frac{\pi}{6}$	I II III IV V	1 1 1 1 1
34.	If $x = \cos t$, $y = \cos mt$, prove that $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + m^2y = 0$.		
Sol.	$\frac{dx}{dt} = -\sin t, \frac{dy}{dt} = -m \times \sin(mt)$ $\frac{dy}{dx} = \frac{m \times \sin(mt)}{\sin t} \text{ or } \sin t \frac{dy}{dx} = m \times \sin(mt)$ $\Rightarrow \sin t \frac{d^2y}{dx^2} + \frac{dy}{dx} \cos t \times \frac{dt}{dx} = m^2 \cos(mt) \times \frac{dt}{dx}$ $\Rightarrow (1 - \cos^2 t) \frac{d^2y}{dx^2} - x \frac{dy}{dx} = -m^2 \cos(mt)$ $\Rightarrow (1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + m^2y = 0$	I II III IV	1 1 2 1
35.	Check whether the lines given by $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ are parallel or not. If parallel, find the distance between them, otherwise find their point of intersection, if the lines are intersecting.		
Sol.	Drs of first line are $\langle 2, 3, 4 \rangle$		

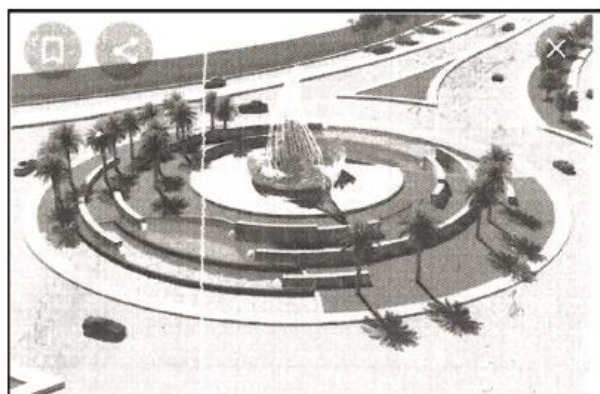
	Drs of second line are $\langle 5, 2, 1 \rangle$ Drs of the two lines are not proportional. Hence, the lines are not parallel. Any point on the first line is $(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$ Any point on the second line is $(5\mu + 4, 2\mu + 1, \mu)$ For the lines to intersect, we must have some λ and μ, for which these coordinates must coincide, i.e., we must have $2\lambda + 1 = 5\mu + 4, 3\lambda + 2 = 2\mu + 1, 4\lambda + 3 = \mu$, for some λ and μ The first two equations, when solved, give us $\lambda = -1$ and $\mu = -1$. These values satisfy the third equation. Hence, the lines intersect. The point of intersection is $(-1, -1, -1)$	I II III IV V	1 1 1 1 1
	SECTION E This section (Q. 36 to 38) has 3 case study-based questions of 4 marks each.		
36.	An online delivery company in a city has 5000 subscribers and collects annual subscription fees of ₹ 300 per subscriber for unlimited free deliveries.  The company wishes to increase the annual subscription fee. It is predicted that, for every increase of ₹ 1, ten subscribers will discontinue. Assume that the company increased the annual fee by ₹ x.		

	Based on the given information, answer the following questions : (i) How many subscribers will discontinue after an increase of ₹ x in annual fee ? (ii) If $R(x)$ denotes the total revenue collected after the increase of ₹ x in subscription fee, express $R(x)$ as a function of x. (iii) (a) Find the value of x for which $R(x)$ is maximum. OR (iii) (b) Find the sub-intervals of $(0, 5000)$ in which $R(x)$ is increasing and decreasing.		
Sol.(i)	$10x$	I	1
Sol.(ii)	$R(x) = (5000 - 10x)(300 + x)$	I	1
Sol.(iii)	$R'(x) = 2000 - 20x$	I	$\frac{1}{2}$
(a)	$R'(x) = 0 \Rightarrow x = 100$	II	$\frac{1}{2}$
	$R''(x) = -20 < 0$ at $x = 100$	III	$\frac{1}{2}$
	At $x = 100$, $R(x)$ is maximum	IV	$\frac{1}{2}$
	OR		
Sol.(iii)	$R'(x) = 2000 - 20x$	I	$\frac{1}{2}$
(b)	$R'(x) = 0 \Rightarrow x = 100$	II	$\frac{1}{2}$
	In $(0, 100)$, $R'(x) > 0$. Therefore, $R(x)$ is increasing in $(0, 100)$ or $(0, 100]$	III	$\frac{1}{2}$
	In $(100, 5000)$, $R'(x) < 0$. Therefore, $R(x)$ is decreasing in $(100, 5000)$ or $[100, 5000)$	IV	$\frac{1}{2}$

37.	In an online jackpot, there is one first prize of ₹ 3,00,000, two second prizes of ₹ 2,00,000 each and three third prizes of ₹ 50,000 each.  A total of 1,00,000 jackpot tickets each costing ₹ 100 were sold there by raising a fund of ₹ 1,00,00,000. Rohan bought one ticket. Based on given information, answer the following questions : (i) What are the possible amounts, the person can win ? (ii) (a) What is the probability that the person wins atleast ₹ 2,00,000 ? OR (ii) (b) What is the probability that the person does not win any amount ? (iii) In another jackpot, Rohan also bought a ticket having a prize money of ₹ 5,00,000. The chances of winning the jackpot are 1 in 1,00,000. Find the probability that on exactly one of tickets he wins the jackpot.		
Sol.(i)	₹ 300000, ₹ 200000, ₹ 50000	I	1
Sol.(ii) (a)	Required probability = $\frac{3}{100000}$	I	2
	OR		
Sol.(ii) (b)	Required probability = $1 - \frac{6}{100000} = \frac{99994}{100000}$ or $\frac{49997}{50000}$	I	2
Sol.(iii)	P (Exactly on one of the tickets Rohan wins the jackpot) = P [(He wins first and he loses second) or (He loses first and wins second)] $= \frac{6}{100000} \times \frac{99999}{100000} + \frac{99994}{100000} \times \frac{1}{100000} = \frac{699988}{10^{10}} \text{ or } \frac{174997}{25000000000}$	I	1

38.

Roundabouts are often made on busy roads to ease the traffic and avoid red lights.



One such round-about is made such that equation representing its boundary is given by $C_1 : x^2 + y^2 = 64$.

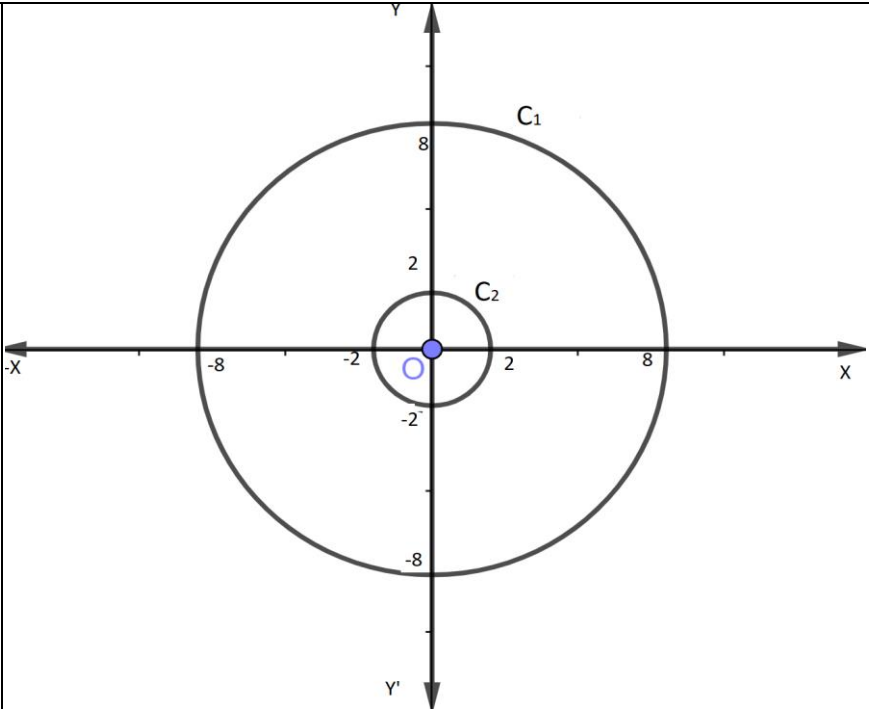
There is a circular pond with a fountain in the middle of the roundabout whose equation is given by $C_2 : x^2 + y^2 = 4$.

Based on the given information, answer the following questions :

- (i) Represent the given equations C_1 and C_2 with the help of a diagram.
- (ii) Express y as a function of x , ($y = f(x)$), for both C_1 and C_2 .
- (iii) (a) Using integration find the area of region covered by the roundabout.

OR

- (iii) (b) Using integration, find the area of region covered by circular pond.

Sol.(i)		I	1
Sol.(ii)	Circle C_1: $y = \sqrt{64 - x^2}$ or $y = -\sqrt{64 - x^2}$ Circle C_2: $y = \sqrt{4 - x^2}$ or $y = -\sqrt{4 - x^2}$	I II	$\frac{1}{2}$ $\frac{1}{2}$
Sol.(iii) (a)	Required area = $4 \times \int_0^8 \sqrt{64 - x^2} dx$ $= 2[x\sqrt{64 - x^2} + 64\sin^{-1}\frac{x}{8}]_0^8 = 64\pi$	I II	1 1
	OR		
Sol.(iii) (b)	Required area = $4 \times \int_0^2 \sqrt{4 - x^2} dx$ $= 2[x\sqrt{4 - x^2} + 4\sin^{-1}\frac{x}{2}]_0^2 = 4\pi$	I II	1 1