

SECTION - A

1. The area of the region  $\{(x, y) : x^2 \leq y \leq 8 - x^2, y \leq 7\}$  is.

- (1) 24 (2) 21 (3) 20 (4) 18

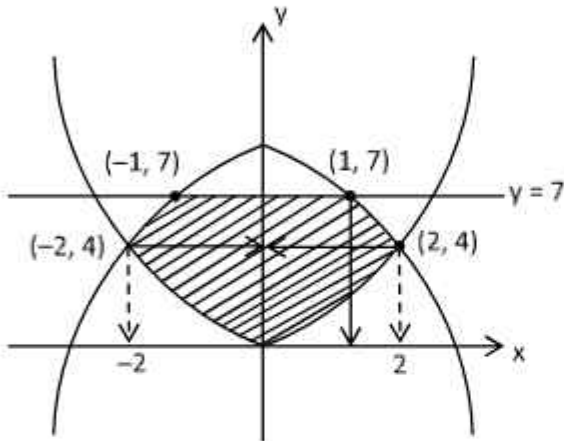
Sol. (3)

$$y \geq x^2 \quad y \leq 8 - x^2 \quad y \leq 7$$

$$x^2 = 8 - x^2$$

$$x^2 = 4$$

$$x = \pm 2$$



$$2 \left( 1 \cdot 7 + \int_1^2 (8 - 2x^2) dx \right) - 2 \int_0^1 (x^2) dx$$

$$= 2 \left[ 7 + \left( 8x - \frac{2x^3}{3} \right) \Big|_1^2 \right] - 2 \left( \frac{x^3}{3} \right) \Big|_0^1$$

$$= 2 \left[ 7 + \left( 16 - \frac{16}{3} \right) - \left( 8 - \frac{2}{3} \right) \right] - 2 \left( \frac{1}{3} \right)$$

$$= 2 \left[ 7 + \frac{32}{3} - \frac{22}{3} \right] = 2 \left[ 7 + \frac{10}{3} \right] = \frac{60}{3} = 20$$

2. Let  $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ ,  $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$  and  $Q = PAP^T$ . If  $P^T Q^{2007} P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ , then  $2a + b - 3c - 4d$  equal to

- (1) 2004 (2) 2007 (3) 2005 (4) 2006

Sol. (3)

$$Q = PAP^T$$

$$P^T \cdot Q^{2007} \cdot P = P^T \cdot Q \cdot Q \dots Q \cdot P$$

$$= P^T (PAP^T) (PAP^T) \dots (PAP^T) P$$

$$\Rightarrow (P^T P) A (P^T P) A \dots A (P^T P)$$

$$P^T P = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$\therefore P^T Q^{2007} P = A^{2007}$$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$\therefore A^{2007} = \begin{bmatrix} 1 & 2007 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$a = 1, b = 2007, c = 0, d = 1$$

$$2a + b - 3c - 4d = 2 + 2007 - 4 = 2005$$

3. Negation of  $(p \rightarrow q) \rightarrow (q \rightarrow p)$  is

(1)  $(\sim q) \wedge p$

(2)  $p \vee (\sim q)$

(3)  $(\sim p) \vee q$

(4)  $q \wedge (\sim p)$

Sol. (4)

$$(p \rightarrow q) \rightarrow (q \rightarrow p)$$

$$\sim [\sim p \rightarrow q \wedge q \rightarrow p]$$

$$\Rightarrow p \rightarrow q \wedge \sim q \rightarrow p$$

$$\Rightarrow \sim p \vee q \wedge q \wedge \sim p$$

$$\Rightarrow q \wedge \sim p$$

4. Let  $C(\alpha, \beta)$  be the circumcenter of the triangle formed by the lines

$$4x + 3y = 69,$$

$$4y - 3x = 17 \text{ and}$$

$$x + 7y = 61.$$

Then  $(\alpha - \beta)^2 + \alpha + \beta$  is equal to

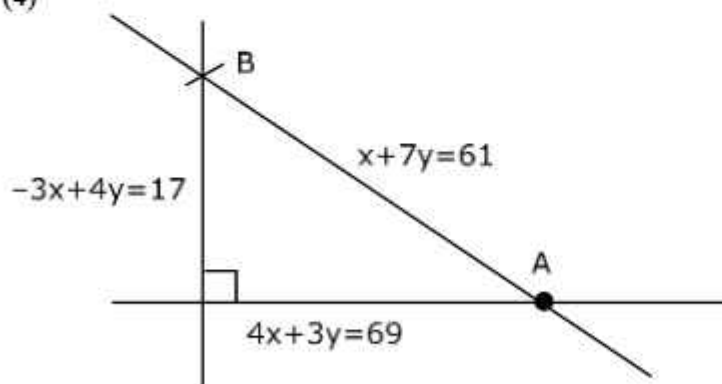
(1) 18

(2) 15

(3) 16

(4) 17

Sol. (4)



$$4x + 28y = 244$$

$$4x + 3y = 69$$

$$- - -$$

$$25y = 175$$

$$y = 7, x = 12$$

$$A(12, 7)$$

$$-3x + 4y = 17$$

$$3x + 21y = 183$$

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$$25y = 200$$
$$y = 8, x = 5$$

B(5, 8)

∴ Circumcenter

$$\alpha = \frac{17}{2}, \beta = \frac{15}{2}$$

$$\left(\frac{17}{2}, \frac{15}{2}\right)$$

$$(\alpha - \beta)^2 + \alpha + \beta$$

$$1 + 16 = 17$$

5. Let  $\alpha, \beta, \gamma$ , be the three roots of the equation  $x^3 + bx + c = 0$ . If  $\beta\gamma = 1 = -\alpha$ , then  $b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$  is equal to

(1)  $\frac{155}{8}$

(2) 21

(3) 19

(4)  $\frac{169}{8}$

Sol. (3)

$$x^3 + bx + c = 0 \begin{cases} \nearrow \alpha \\ \rightarrow \beta \\ \searrow \gamma \end{cases}$$

$$\beta\gamma = 1$$

$$\alpha = -1$$

Put  $\alpha = -1$

$$-1 - b + c = 0$$

$$c - b = 1$$

also

$$\alpha\beta\gamma = -c$$

$$-1 = -c \Rightarrow c = 1$$

$$\therefore b = 0$$

$$x^3 + 1 = 0$$

$$\alpha = -1, \beta = -w, \gamma = -w^2$$

$$\therefore b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$$

$$0 + 2 + 3 + 6 + 8 = 19$$

6. Let the number of elements in sets A and B be five and two respectively. Then the number of subsets of  $A \times B$  each having at least 3 and at most 6 elements is:

- (1) 752                      (2) 772                      (3) 782                      (4) 792

Sol. (4)

$$n(A \times B) = 10$$

$${}^{10}C_3 + {}^{10}C_4 + {}^{10}C_5 + {}^{10}C_6 = 792$$

7. If the coefficients of three consecutive terms in the expansion of  $(1 + x)^n$  are in the ratio 1 : 5 : 20, then the coefficient of the fourth term is

- (1) 5481                      (2) 3654                      (3) 2436                      (4) 1817

Sol. (2)

$$\frac{{}^nC_r}{{}^nC_{r-1}} = 5 \quad \frac{{}^nC_{r-1}}{{}^nC_r} = 4$$

$$\frac{n-r+1}{r} = 5 \quad n = 5r + 4 \dots (2)$$

$$n = 6r - 1 \dots (1)$$

$$\therefore n = 29, r = 5$$

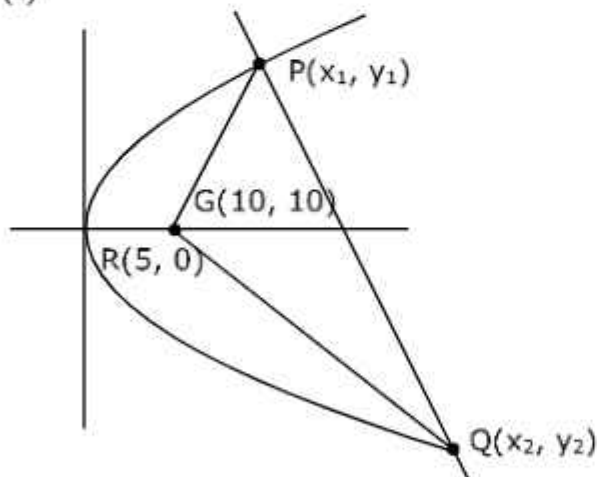
$$\text{Coeff of 4th term} = {}^{29}C_3$$

$$= 3654$$

8. Let R be the focus of the parabola  $y^2 = 20x$  and the line  $y = mx + c$  intersect the parabola at two points P and Q. Let the point G(10, 10) be the centroid of the triangle PQR. If  $c - m = 6$ , then  $(PQ)^2$  is

- (1) 325                      (2) 346                      (3) 296                      (4) 317

Sol. (1)



$$y^2 = 20x, y = mx + c$$

$$y^2 = 20\left(\frac{y-c}{m}\right)$$

$$y^2 - \frac{20y}{m} + \frac{20c}{m} = 0 \quad \frac{y_1 + y_2 + y_3}{3} = 10$$

$$\frac{20}{m} = 30$$

$$m = \frac{2}{3}$$

$$\text{and } c - m = 6$$

$$c = \frac{2}{3} + 6 \Rightarrow \frac{20}{3} = c$$

$$y^2 - 30y + \frac{20 \times 20}{\frac{2}{3}} = 0 \Rightarrow y^2 - 30y + 200 = 0$$

$$y = 10, y = 20$$

$$y = 20, x = 20 \quad P(5, 10); (20, 20)Q$$

$$\frac{20+5+x}{3} = 10 \Rightarrow x = 5 \quad PQ^2 = 15^2 + 10^2 = 225 + 100 = 325$$

9. Let  $S_K = \frac{1+2+\dots+K}{K}$  and  $\sum_{j=1}^n S_j^2 = \frac{n}{A} (Bn^2 + Cn + D)$ , where  $A, B, C, D \in \mathbb{N}$  and  $A$  has least value. Then

(1)  $A + B$  is divisible by  $D$

(2)  $A + B = 5(D - C)$

(3)  $A + C + D$  is not divisible by  $B$

(4)  $A + B + D$  is divisible by  $5$

Sol. (1)

$$S_k = \frac{k+1}{2}$$

$$S_k^2 = \frac{k^2 + 1 + 2k}{4}$$

$$\therefore \sum_{j=1}^n S_j^2 = \frac{1}{4} \left[ \frac{n(n+1)(2n+1)}{6} + n + n(n+1) \right]$$

$$= \frac{n}{4} \left[ \frac{(n+1)(2n+1)}{6} + 1 + n + 1 \right]$$

$$= \frac{n}{4} \left[ \frac{2n^2 + 3n + 1}{6} + n + 2 \right]$$

$$= \frac{n}{4} \left[ \frac{2n^2 + 9n + 13}{6} \right] = \frac{n}{24} [2n^2 + 9n + 13]$$

$$A = 24, B = 2, C = 9, D = 13$$

10. The shortest distance between the lines  $\frac{x-4}{4} = \frac{y+2}{5} = \frac{z+3}{3}$  and  $\frac{x-1}{3} = \frac{y-3}{4} = \frac{z-4}{2}$  is

(1)  $2\sqrt{6}$

(2)  $3\sqrt{6}$

(3)  $6\sqrt{3}$

(4)  $6\sqrt{2}$

Sol. (2)

$$S_d = \frac{|(\vec{a} - \vec{b}) \times (\vec{n}_1 \times \vec{n}_2)|}{|\vec{n}_1 \times \vec{n}_2|}$$

$$\vec{a} = (4, -2, -3)$$

$$\vec{b} = (1, 3, 4)$$

$$\vec{n}_1 = (4, 5, 3)$$

$$\vec{n}_2 = (3, 4, 2)$$

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 5 & 3 \\ 3 & 4 & 2 \end{vmatrix} = \hat{i}(-2) - \hat{j}(-1) + \hat{k}(1) = (-2, 1, 1)$$

$$S_d = \frac{(3, -5, -7) \cdot (-2, 1, 1)}{\sqrt{6}} = \frac{-6 - 5 - 7}{\sqrt{6}} = 3\sqrt{6}$$

11. The number of arrangements of the letters of the word "INDEPENDENCE" in which all the vowels always occur together is.

- (1) 16800                      (2) 14800                      (3) 18000                      (4) 33600

Sol. (1)  
 IEEEE,  
 NNN, DD, P, C  
 $\frac{8!}{3!2!} \times \frac{6!}{4!} = 16800$

12. If the points with position vectors  $\alpha\hat{i} + 10\hat{j} + 13\hat{k}$ ,  $6\hat{i} + 11\hat{j} + 11\hat{k}$ ,  $\frac{9}{2}\hat{i} + \beta\hat{j} - 8\hat{k}$  are collinear, then  $(19\alpha - 6\beta)^2$  is equal to

- (1) 49                      (2) 36                      (3) 25                      (4) 16

Sol. (2)  
 $(\alpha, 10, 13); (6, 11, 11), \left(\frac{9}{2}, \beta, -8\right)$

$$\frac{\alpha - 6}{\frac{3}{2}} = \frac{-1}{11 - \beta} = \frac{2}{19}$$

$$\alpha - 6 = \frac{3}{19} \qquad -19 = 22 - 2\beta$$

$$\alpha = 6 + \frac{3}{19} = \frac{117}{19} \qquad 2\beta = 41$$

$$\therefore (19\alpha - 6\beta)^2 = (117 - 123)^2 = 36$$

13. In a bolt factory, machines A, B and C manufacture respectively 20%, 30% and 50% of the total bolts. Of their output 3, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from the product. If the bolt drawn is found the defective, then the probability that it is manufactured by the machine C is.

- (1)  $\frac{5}{14}$                       (2)  $\frac{3}{7}$                       (3)  $\frac{9}{28}$                       (4)  $\frac{2}{7}$

Sol. (1)  
 $P(A) = \frac{2}{10}$     $P(B) = \frac{3}{10}$     $P(C) = \frac{5}{10}$

$$P(\text{Defective}/A) = \frac{3}{100}, \quad P(\text{Defective}/B) = \frac{4}{100}, \quad P(\text{Defective}/C) = \frac{2}{100}$$

$$P(E) = \frac{\frac{5}{10} \times \frac{2}{100}}{\frac{2}{10} \times \frac{3}{100} + \frac{3}{10} \times \frac{4}{100} + \frac{5}{10} \times \frac{2}{100}} = \frac{10}{6 + 12 + 10}$$

$$= \frac{10}{28}$$

$$= \frac{5}{14}$$

14. If for  $z = \alpha + i\beta$ ,  $|z + 2| = z + 4(1 + i)$ , then  $\alpha + \beta$  and  $\alpha\beta$  are the roots of the equation

- (1)  $x^2 + 3x - 4 = 0$                       (2)  $x^2 + 7x + 12 = 0$   
 (3)  $x^2 + x - 12 = 0$                       (4)  $x^2 + 2x - 3 = 0$

Sol. (2)

$$|z + 2| = |\alpha + i\beta + 2|$$

$$= \alpha + i\beta + 4 + 4i$$

$$\sqrt{(\alpha + 2)^2 + \beta^2} = (\alpha + 4) + i(\beta + 4) \quad \beta + 4 = 0$$

$$(\alpha + 2)^2 + 16 = (\alpha + 4)^2 \quad \beta = -4$$

$$\alpha^2 + 4 + 4\alpha + 16 = \alpha^2 + 16 + 8\alpha$$

$$4 = 4\alpha$$

$$\alpha = 1$$

$$\alpha = 1, \beta = -4$$

$$\alpha + \beta = -3, \alpha\beta = -4$$

$$\text{Sum of roots} = -7$$

$$\text{Product of roots} = 12$$

$$x^2 + 7x + 12 = 0$$

15.  $\lim_{x \rightarrow 0} \left( \left( \frac{1 - \cos^2(3x)}{\cos^3(4x)} \right) \left( \frac{\sin^3(4x)}{(\log_e(2x+1))^5} \right) \right)$  is equal to \_\_\_\_\_

(1) 24

(2) 9

(3) 18

(4) 15

Sol. (3)

$$\lim_{x \rightarrow 0} \left[ \frac{1 - \cos^2 3x}{9x^2} \right] \frac{9x^2}{\cos^3 4x} \cdot \frac{\left( \frac{\sin 4x}{4x} \right)^3 \times 64x^3}{\left[ \frac{\ln(1+2x)}{2x} \right]^5 \times 32x^5}$$

$$\lim_{x \rightarrow 0} 2 \left( \frac{1}{2} \times \frac{9}{1} \times \frac{1 \times 64}{1 \times 32} \right) = 18$$

16. The number of ways, in which 5 girls and 7 boys can be seated at a round table so that no two girls sit together, is

(1)  $7(720)^2$

(2) 720

(3)  $7(360)^2$

(4)  $126(5!)^2$

Sol. (4)

$$6 \times {}^7C_5 \times 5!$$

$$\Rightarrow 720 \times 21 \times 120$$

$$\Rightarrow 2 \times 360 \times 7 \times 3 \times 120$$

$$\Rightarrow 126 \times (5!)^2$$

17. Let  $f(x) = \frac{\sin x + \cos x - \sqrt{2}}{\sin x - \cos x}$ ,  $x \in [0, \pi] - \left\{ \frac{\pi}{4} \right\}$ . Then  $f\left(\frac{7\pi}{12}\right) f''\left(\frac{7\pi}{12}\right)$  is equal to

(1)  $-\frac{2}{3}$

(2)  $\frac{2}{9}$

(3)  $-\frac{1}{3\sqrt{3}}$

(4)  $\frac{2}{3\sqrt{3}}$

Sol. (2)

$$f(x) = -\tan\left(\frac{x}{2} - \frac{\pi}{8}\right)$$

$$f'(x) = -\frac{1}{2} \sec^2\left(\frac{x}{2} - \frac{\pi}{8}\right)$$

$$f''(x) = -\sec^2\left(\frac{x}{2} - \frac{\pi}{8}\right) \cdot \tan\left(\frac{x}{2} - \frac{\pi}{8}\right) \cdot \frac{1}{2}$$

$$f\left(\frac{7\pi}{12}\right) = -\tan\left(\frac{\pi}{6}\right) = -\frac{1}{\sqrt{3}}$$

$$f''\left(\frac{7\pi}{12}\right) = -\frac{1}{2} \sec^2 \frac{\pi}{6} \cdot \tan \frac{\pi}{6} = \frac{-1}{2} \cdot \frac{4}{3} \times \frac{1}{\sqrt{3}} = \frac{-2}{3\sqrt{3}}$$

$$f\left(\frac{7\pi}{12}\right) \cdot f''\left(\frac{7\pi}{12}\right) = \frac{2}{9}$$

18. If the equation of the plane containing the line  $x + 2y + 3z - 4 = 0$   $2x + y - z + 5$  and perpendicular to the plane  $\vec{r} = (\hat{i} - \hat{j}) + \lambda(\hat{i} + \hat{j} + \hat{k}) + \mu(\hat{i} - 2\hat{j} + 3\hat{k})$  is  $ax + by + cz = 4$ , then  $(a-b+c)$  is equal to

(1) 22

(2) 24

(3) 20

(4) 18

Sol. (1)

$$\text{D.R.'s of line } \vec{n}_1 = -5\hat{i} + 7\hat{j} - 3\hat{k}$$

D.R.'s of normal of second plane

$$\vec{n}_2 = 5\hat{i} - 2\hat{j} - 3\hat{k}$$

$$\vec{n}_1 \times \vec{n}_2 = -27\hat{i} - 30\hat{j} - 25\hat{k}$$

A point on the required plane is  $\left(0, -\frac{11}{5}, \frac{14}{5}\right)$

The equation of required plane is

$$27x + 30y + 25z = 4$$

$$\therefore a - b + c = 22$$

19. Let  $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ . If  $|\text{adj}(\text{adj}(\text{adj} 2A))| = (16)^n$ , then  $n$  is equal to

(1) 8

(2) 9

(3) 12

(4) 10

Sol. (4)

$$|A| = 2[3] - 1[2] = 4$$

$$\therefore |\text{adj}(\text{adj}(\text{adj} 2A))|$$

$$= |2A|^{(n-1)^3} \Rightarrow |2A|^8 = 16^n$$

$$\Rightarrow (2^3 |A|)^8 = 16^n$$

$$\Rightarrow (2^3 \times 2^2)^8 = 16^n$$





**SECTION - B**

21. Let  $A = \{0, 3, 4, 6, 7, 8, 9, 10\}$  and  $R$  be the relation defined on  $A$  such that  $R = \{(x, y) \in A \times A : x - y \text{ is odd positive integer or } x - y = 2\}$ . The minimum number of elements that must be added to the relation  $R$ , so that it is a symmetric relation, is equal to \_\_\_\_\_.

Sol. (19)

$$A = \{0, 3, 4, 6, 7, 8, 9, 10\} \quad 3, 7, 9 \rightarrow \text{odd}$$

$$R = \{x - y = \text{odd} + \text{ve or } x - y = 2\} \quad 0, 4, 6, 8, 10 \rightarrow \text{even}$$

$${}^3C_1 \cdot {}^5C_1 = 15 + (6, 4), (8, 6), (10, 8), (9, 7)$$

Min<sup>m</sup> ordered pairs to be added must be

$$\therefore 15 + 4 = 19$$

22. Let  $(t)$  denote the greatest integer  $\leq t$ . If the constant term in the expansion of  $\left(3x^2 - \frac{1}{2x^5}\right)^7$  is  $\alpha$ , then  $[\alpha]$  is equal to \_\_\_\_\_.

Sol. (1275)

$$\left(3x^2 - \frac{1}{2x^5}\right)^7$$

$$T_{r+1} = {}^7C_r (3x^2)^{7-r} \left(-\frac{1}{2x^5}\right)^r$$

$$14 - 2r - 5r = 14 - 7r = 0$$

$$\therefore r = 2$$

$$\therefore T_3 = {}^7C_2 \cdot 3^5 \left(-\frac{1}{2}\right)^2 = \frac{21 \times 243}{4} = 1275.75$$

$$\therefore [\alpha] = 1275$$

23. Let  $\lambda_1, \lambda_2$  be the values of  $\lambda$  for which the points  $\left(\frac{5}{2}, 1, \lambda\right)$  and  $(-2, 0, 1)$  are at equal distance from the plane  $2x + 3y - 6z + 7 = 0$ . If  $\lambda_1 > \lambda_2$ , then the distance of the point  $(\lambda_1 - \lambda_2, \lambda_2, \lambda_1)$  from the line  $\frac{x-5}{1} = \frac{y-1}{2} = \frac{z+7}{2}$  is \_\_\_\_\_.

Sol. 9

$$2x + 3y - 6z + 7 = 0 \quad \left(\frac{5}{2}, 1, \lambda\right), (-2, 0, 1)$$

$$d_1 = \left|\frac{5+3-6\lambda+7}{7}\right| = d_2 = \left|\frac{-4-6+7}{7}\right|$$

$$\Rightarrow |15-6\lambda| = |3|$$

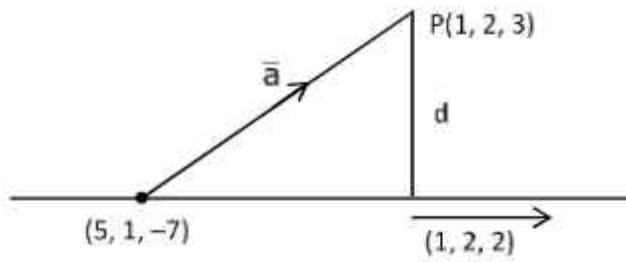
$$15 - 6\lambda = 3 \text{ or } 15 - 6\lambda = -3$$

$$6\lambda = 12 \quad 6\lambda = 18$$

$$\lambda = 2 \quad \lambda = 3$$

$$\lambda_1 = 3, \quad \lambda_2 = 2$$

$$\therefore P(1, 2, 3) \quad \frac{x-5}{1} = \frac{y-1}{2} = \frac{z+7}{2}$$



$$d = \left| \frac{(4, -1, -10) \times (1, 2, 2)}{3} \right| = \left| \frac{18\hat{i} - 18\hat{j} + 9\hat{k}}{3} \right| = 9$$

24. If the solution curve of the differential equation  $(y - 2 \log_e x)dx + (x \log_e x^2) dy = 0$ ,  $x > 1$  passes through the points  $\left(e, \frac{4}{3}\right)$  and  $(e^4, \alpha)$ , then  $\alpha$  is equal to \_\_\_\_\_.

Sol. (3)

$$(y - 2 \ln x)dx + (2x \ln x)dy = 0$$

$$dy(2x \ln x) = [(2 \ln x) - y] dx$$

$$\frac{dy}{dx} = \frac{1}{x} - \frac{y}{2x \ln x}$$

$$\frac{dy}{dx} + \frac{y}{2x \ln x} = \frac{1}{x}$$

$$\text{I.F.} = e^{\int \frac{1}{2x \ln x} dx}$$

$$= e^{\frac{1}{2} \int \frac{dx}{x}} = e^{\frac{1}{2} \ln(\ln x)}$$

$$\Rightarrow \text{I.F.} = (\ln x)^{1/2}$$

$$\therefore y\sqrt{\ln x} = \int \frac{\sqrt{\ln x}}{x} dx \quad (\text{Let, } \ln x = u^2)$$

$$= 2 \int u^2 du \quad \frac{1}{x} dx = 2u du$$

$$y\sqrt{\ln x} = \frac{2}{3} (\ln x)^{3/2} + c \leftarrow \left(e, \frac{4}{3}\right)$$

$$\frac{4}{3} = \frac{2}{3} + c \Rightarrow c = \frac{2}{3}$$

$$y\sqrt{\ln x} = \frac{2}{3} (\ln x)^{3/2} + \frac{2}{3} \leftarrow (e^4, \alpha)$$

$$\alpha \cdot 2 = \frac{2}{3} \times 8 + \frac{2}{3}$$

$$\alpha = 3$$

25. Let  $\vec{a} = 6\hat{i} + 9\hat{j} + 12\hat{k}$ ,  $\vec{b} = \alpha\hat{i} + 11\hat{j} - 2\hat{k}$  and  $\vec{c}$  be vectors such that  $\vec{a} \times \vec{c} = \vec{a} \times \vec{b}$ . If  $\vec{a} \cdot \vec{c} = -12$ ,  $\vec{c} \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 5$ , then  $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k})$  is equal to \_\_\_\_.

Sol. (11)

$$\vec{a} \times \vec{c} = \vec{a} \times \vec{b}$$

$$\Rightarrow \vec{a} \times (\vec{c} - \vec{b}) = 0$$

$$\vec{a} \parallel (\vec{c} - \vec{b})$$

$$\therefore \vec{a} = \lambda(\vec{c} - \vec{b})$$

$$(6, 9, 12) = \lambda[x - \alpha, y - 11, z + 2]$$

$$\frac{x - \alpha}{2} = \frac{y - 11}{3} = \frac{z + 2}{4}$$

$$4y - 44 = 3z + 6$$

$$4y - 3z = 50$$

$$6x + 9y + 12z = -12$$

$$2x + 3y + 4z = -4$$

$$2x - 4y + 2z = 10$$

$$+ \quad - \quad -$$

$$(\because x - 2y + z = 5)$$

$$7y + 2z = -14 \dots(2)$$

$$8y - 6z = 100$$

$$21y + 6z = -42$$

$$29y = 58$$

$$y = 2, z = -14$$

$$\therefore x - 4 - 14 = 5$$

$$x = 23$$

$$\vec{c} = (23, 2, -14)$$

$$\vec{c} \cdot (1, 1, 1) = 23 + 2 - 14 = 11$$

26. The largest natural number  $n$  such that  $3^n$  divides  $66!$  is \_\_\_\_\_.

Sol. (31)

$$\left[ \frac{66}{3} \right] + \left[ \frac{66}{9} \right] + \left[ \frac{66}{27} \right]$$

$$22 + 7 + 2 = 31$$

27. If  $a_n$  is the greatest term in the sequence  $a_n = \frac{n^3}{n^4 + 147}$ ,  $n = 1, 2, 3, \dots$ , then  $a$  is equal to \_\_\_\_\_.

Sol. (0.158)

$$f(x) = \frac{x^3}{x^4 + 147}$$

$$f'(x) = \frac{(x^4 + 147)3x^2 - x^3(4x^3)}{(x^4 + 147)^2}$$

$$= \frac{3x^6 + 147 \times 3x^2 - 4x^6}{+ve} = x^2(44 - x^4)$$

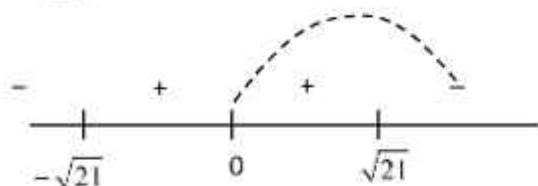
$$f'(x) = 0 \text{ at } x^6 = 147 \times 3x^2$$

$$x^2 = 0, x^4 = 147 \times 3$$

$$x = 0, x^2 = \pm \sqrt{147 \times 3}$$

$$x^2 = \pm 21$$

$$x = \pm \sqrt{21}$$



$f_{\max}$  at  $f(4)$  or  $f(5)$

$$f(4) = \frac{64}{403} \approx 0.158 \quad f(5) = \frac{125}{772} \approx 0.161$$

$$\therefore a = 5$$

28. Let the mean and variance of 8 numbers  $x, y, 10, 12, 6, 12, 4, 8$  be 9 and 9.25 respectively. If  $x > y$ , then  $3x - 2y$  is equal to \_\_\_\_\_.

Sol. (25)

$$\frac{x + y + 52}{8} = 9 \Rightarrow x + y = 20$$

For variance

$$x - 9, y - 9, 3, 3, 1, -5, -1, -3$$

$$\bar{x} = 0$$

$$\therefore \frac{(x-9)^2 + (y-9)^2 + 54}{8} - \bar{0}^2 = 9.25$$

$$(x-9)^2 + (11-x)^2 = 20$$

$$x = 7 \text{ or } 13 \therefore y = 13, 7$$

$$3x - 2y = 3 \times 13 - 2 \times 7 = 25$$

29. Consider a circle  $C_1 : x^2 + y^2 - 4x - 2y = \alpha - 5$ . Let its mirror image in the line  $y = 2x + 1$  be another circle  $C_2 : 5x^2 + 5y^2 - 10fx - 10gy + 36 = 0$ . Let  $r$  be the radius of  $C_2$ . Then  $\alpha + r$  is equal to \_\_\_\_\_.

Sol. (2)

$$x^2 + y^2 - 4x - 2y + 5 - \alpha = 0,$$

$$C_1(2, 1) \quad r_1 = \sqrt{\alpha}$$

$$2x - y + 1 = 0$$

Image of  $(2, 1)$

$$\frac{x-2}{2} = \frac{y-1}{-1} = \frac{-2(4-1+1)}{5}$$

$$\frac{x-2}{2} = \frac{y-1}{-1} = \frac{-8}{5}$$

$$x = 2 - \frac{16}{5} = \frac{-6}{5}, y = 1 + \frac{8}{5} = \frac{13}{5}$$

$$x^2 + y^2 - 2fx - 2gy + \frac{36}{5} = 0$$

$$C_2(f, g)$$

$$r_2 = \sqrt{f^2 + g^2 - \frac{36}{5}}$$

$$\alpha = f^2 + g^2 - \frac{36}{5}$$

$$\therefore f = -\frac{6}{5}, g = \frac{13}{5}$$

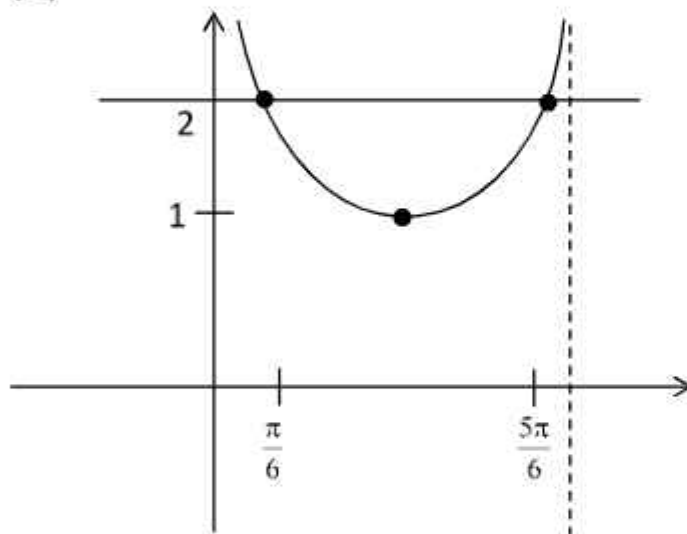
$$\alpha = \frac{36}{25} + \frac{169}{25} - \frac{36}{5}$$

$$= \frac{36 + 169 - 180}{25} \Rightarrow \alpha = 1 \Rightarrow r = 1$$

$$\therefore \alpha + r = 2$$

30. Let  $[t]$  denote the greatest integer  $\leq t$ . The  $\frac{2}{\pi} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (8[\operatorname{cosec} x] - 5[\cot x]) dx$  is equal to \_\_\_\_\_.

Sol. (14)



$$8 \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} [\operatorname{cosec} x] dx$$

$$8 \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} dx = \frac{16\pi/3}{16\pi/3}$$

$$I = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} [\cot x] dx$$

$$x \rightarrow \pi - x$$

$$I = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} [-\cot x] dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} ([\cot x] + [-\cot x]) dx$$

$$I = -\frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} dx \Rightarrow -\frac{1}{2} \left( \frac{4\pi}{6} \right)$$

$$= -\frac{\pi}{3}$$

$$\therefore \frac{2}{\pi} \left[ \frac{16\pi}{3} + \frac{5\pi}{3} \right] = \frac{2}{\pi} \left( \frac{21\pi}{3} \right)$$

$$= 14$$

**SECTION - A**

31. A cylindrical wire of mass  $(0.4 \pm 0.01)$ g has length  $(8 \pm 0.04)$  cm and radius  $(6 \pm 0.03)$  mm. The maximum error in its density will be:

(1) 4%                      (2) 1%                      (3) 3.5%                      (4) 5%

Sol. (1)

Cylindrical wire  $m = (0.4 \pm 0.01)$  g

$\ell = (8 \pm 0.04)$  cm

$r = (6 \pm 0.03)$  mm

Density  $\rho = \frac{m}{\pi r^2 \ell} \Rightarrow \rho r^2 \ell m^{-1} = \frac{1}{\pi} = \text{const.}$

Differentiating after taking log on both side

$$\frac{d\rho}{\rho} + \frac{2dr}{r} + \frac{d\ell}{\ell} - \frac{dm}{m} = 0$$

$$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} - \frac{\Delta\ell}{\ell} - \frac{2\Delta r}{r}$$

$$\left(\frac{\Delta\rho}{\rho}\right)_{\text{max}} = \left[\frac{0.01}{0.4} + \frac{0.04}{8} + 2\left(\frac{0.03}{6}\right)\right]$$

$$\left(\frac{\Delta\rho}{\rho}\right)_{\text{max}} = 0.04$$

Percentage error =  $0.04 \times 100 = 4\%$

32. The engine of a train moving with speed  $10 \text{ ms}^{-1}$  towards a platform sounds a whistle at frequency 400 Hz. The frequency heard by a passenger inside the train is : (neglect air speed. Speed of sound in air =  $330 \text{ ms}^{-1}$ )

(1) 400 Hz                      (2) 388 Hz                      (3) 200 Hz                      (4) 412 Hz

Sol. (1)

The passenger inside the train is at rest wrt train so frequency heard by passenger inside the train is same as the source frequency i.e., 400 Hz.

33. The weight of a body on the earth is 400 N. Then weight of the body when taken to a depth half of the radius of the earth will be:

(1) 300 N                      (2) Zero                      (3) 100 N                      (4) 200 N

Sol. (4)

Weight on the earth surface =  $mg$

$mg = 400 \text{ N}$  (given)

Weight at a depth  $d$   $w = m \left(\frac{GM(R-d)}{R^3}\right)$

$$W = mg \left(1 - \frac{d}{R}\right)$$

$$d = \frac{R}{2} \Rightarrow w = mg \left(1 - \frac{1}{2}\right) \Rightarrow w = \frac{mg}{2}$$

$w = 200 \text{ N}$

34. A TV transmitting antenna is 98 m high and the receiving antenna is at the ground level. If the radius of the earth is 6400 km, the surface area covered by the transmitting antenna is approximately:

(1)  $1240 \text{ km}^2$                       (2)  $1549 \text{ km}^2$                       (3)  $4868 \text{ km}^2$                       (4)  $3942 \text{ km}^2$

Sol. (4)

Max. distance covered  $d = \sqrt{2Rh_T}$

( $R$  = radius of earth,  $h_T$  = height of antenna)

Area  $A = \pi d^2$

$A = \pi (2Rh_T)$

$A = 2 \times 3.14 \times 6400 \times 98 \times 10^{-3}$

$A \approx 3942 \text{ km}^2$



35. Certain galvanometers have a fixed core made of non magnetic metallic material. The function of this metallic material is

- (1) To produce large deflecting torque on the coil
- (2) To bring the coil to rest quickly
- (3) To oscillate the coil in magnetic field for longer period of time
- (4) To make the magnetic field radial

Sol. (2)  
To bring the coil at rest quickly

36. Dimension of  $\frac{1}{\mu_0 \epsilon_0}$  should be equal to

- (1) T/L
- (2) T<sup>2</sup>/L<sup>2</sup>
- (3) L/T
- (4) L<sup>2</sup>/T<sup>2</sup>

Sol. (4)

Dimension of  $\frac{1}{\mu_0 \epsilon_0}$

$$C = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \Rightarrow \frac{1}{\mu_0 \epsilon_0} = c^2$$

$$\left[ \frac{1}{\mu_0 \epsilon_0} \right] = [c^2]$$

$$= \left[ \frac{L^2}{T^2} \right]$$

37. Two projectiles A and B are thrown with initial velocities of 40 m/s and 60 m/s at angles 30° and 60° with the horizontal respectively. The ratio of their ranges respectively is ( $g = 10 \text{ m/s}^2$ )

- (1) 2 :  $\sqrt{3}$
- (2)  $\sqrt{3} : 2$
- (3) 4 : 9
- (4) 1 : 1

Sol. (3)

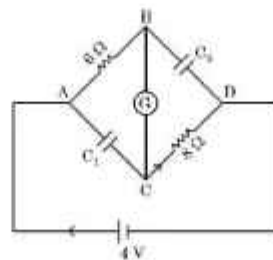
$$R = \frac{u^2 \sin 2\theta}{g}$$

$$\{u_1 = 40 \text{ m/s}, \theta_1 = 30^\circ, u_2 = 60 \text{ m/s}, \theta_2 = 60^\circ\}$$

$$\frac{R_1}{R_2} = \left( \frac{u_1}{u_2} \right)^2 \frac{\sin 2\theta_1}{\sin 2\theta_2}$$

$$\frac{R_1}{R_2} = \left( \frac{40}{60} \right)^2 \times \frac{\sin 60^\circ}{\sin 120^\circ} \Rightarrow \frac{R_1}{R_2} = \frac{4}{9}$$

38. In this figure the resistance of the coil of galvanometer G is 2 Ω. The emf of the cell is 4 V. The ratio of potential difference across C<sub>1</sub> and C<sub>2</sub> is:



(1)  $\frac{5}{4}$

(2) 1

(3)  $\frac{4}{5}$

(4)  $\frac{3}{4}$

Sol. (3)

At steady state current will not be in the capacitor branch.

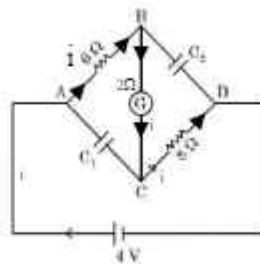
$$i = \frac{4}{6+2+8}$$

$$i = \frac{1}{4} \text{ A}$$

$$\Delta V_{C_1} = i(6+2)$$

$$\Delta V_{C_2} = i(2+8)$$

$$\frac{\Delta V_{C_1}}{\Delta V_{C_2}} = \frac{4}{5}$$



39. A charge particle moving in magnetic field  $B$ , has the components of velocity along  $B$  as well as perpendicular to  $B$ . The path of the charge particle will be

- (1) Helical path with the axis along magnetic field  $B$
- (2) Straight along the direction of magnetic field  $B$
- (3) Helical path with the axis perpendicular to the direction of magnetic field  $B$
- (4) Circular path

Sol. (1)

Path will be helical with axis along uniform  $\vec{B}$ .

40. Proton ( $P$ ) and electron ( $e$ ) will have same de-Broglie wavelength when the ratio of their momentum is (assume,  $m_p = 1849 m_e$ ):

- (1) 1 : 43
- (2) 43 : 1
- (3) 1 : 1849
- (4) 1 : 1

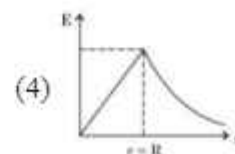
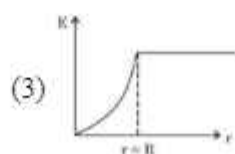
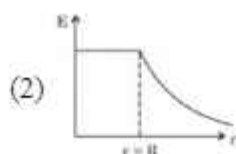
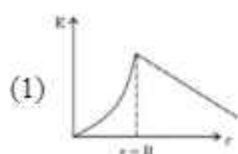
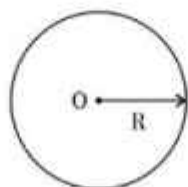
Sol. (4)

$$\text{Debroglie wavelength } \lambda = \frac{h}{p}$$

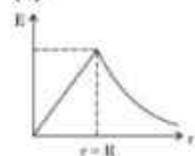
$$\lambda_p = \lambda_e$$

$$\frac{h}{p_p} = \frac{h}{p_e} \Rightarrow \frac{p_p}{p_e} = 1$$

41. Graphical variation of electric field due to a uniformly charged insulating solid sphere of radius  $R$ , with distance  $r$  from the centre  $O$  is represented by:



Sol. (4)



Electric field due to uniformly charged insulating solid sphere

$$E = \begin{cases} \frac{kQr}{R^3} & r \leq R \\ \frac{kQ}{r^2} & r \geq R \end{cases}$$

42. For a nucleus  ${}^A_ZX$  having mass number A and atomic number Z
- A. The surface energy per nucleon  $(b_s) = -a_1 A^{2/3}$ .
- B. The Coulomb contribution to the binding energy  $b_c = -a_2 \frac{Z(Z-1)}{A^{4/3}}$
- C. The volume energy  $b_v = a_3 A$
- D. Decrease in the binding energy is proportional to surface area.
- E. While estimating the surface energy, it is assumed that each nucleon interacts with 12 nucleons. ( $a_1, a_2$  and  $a_3$  are constants)

Choose the most appropriate answer from the options given below:

- (1) B, C only                      (2) A, B, C, D only                      (3) B, C, E only                      (4) C, D only

Sol. (4)

$$E_B = a_v A - a_s A^{2/3} - a_a \frac{(A-2Z)^2}{A^{1/3}} - a_c \frac{Z(Z-1)}{A^{1/3}} + \delta(A, Z)$$

Volume term
Surface term
Asymmetry term
Coulomb term
Pairing term

Most appropriate is option (4)

43. At any instant the velocity of a particle of mass 500 g is  $(2t\hat{i} + 3t^2\hat{j}) \text{ ms}^{-1}$ . If the force acting on the particle at  $t = 1 \text{ s}$  is  $(\hat{i} + x\hat{j}) \text{ N}$ . Then the value of x will be:

- (1) 2                                      (2) 6                                      (3) 3                                      (4) 4

Sol. (3)

$$\vec{v} = (2t\hat{i} + 3t^2\hat{j}) \text{ m/s, mass } m = 500 \text{ gm}$$

$$\text{Force, } \vec{F} = m\vec{a}$$

$$\vec{F} = \frac{1}{2} \left( \frac{d\vec{v}}{dt} \right) \Rightarrow \vec{F} = \frac{1}{2} (2\hat{i} + 6t\hat{j})$$

$$\vec{F} = (\hat{i} + 3t\hat{j})$$

$$\text{At } t = 1 \text{ s} \Rightarrow \vec{F} = (\hat{i} + 3\hat{j})$$

$$x = 3$$

44. Given below are two statements:

**Statement I :** If E be the total energy of a satellite moving around the earth, then its potential energy will be  $\frac{E}{2}$

**Statement II :** The kinetic energy of a satellite revolving in an orbit is equal to the half the magnitude of total energy E.

In the light of the above statements, choose the most appropriate answer from the options given below

- (1) Both Statement I and Statement II are incorrect
- (2) Statement I is incorrect but Statement II is correct
- (3) Statement I is correct but Statement II is incorrect
- (4) Both Statement I and Statement II are correct

Sol. (1)

$$\text{For satellite K.E.} = \frac{1}{2} mv^2 = \frac{1}{2} m \left( \sqrt{\frac{GM}{r}} \right)^2$$

$$\text{K.E.} = \frac{GMm}{2r}$$

$$\text{Potential energy } U = - \frac{GMm}{r}$$

$$\text{Total energy} = \text{K.E.} + U$$

$$E = - \frac{GMm}{2r}$$

$$U = 2E \quad \text{St I - incorrect}$$

$$\text{K.E.} = |E| \quad \text{St II - incorrect}$$

45. Two forces having magnitude  $A$  and  $\frac{A}{2}$  are perpendicular to each other. The magnitude of their resultant is:

- (1)  $\frac{5A}{2}$                       (2)  $\frac{\sqrt{5}A^2}{2}$                       (3)  $\frac{\sqrt{5}A}{4}$                       (4)  $\frac{\sqrt{5}A}{2}$

Sol. (4)

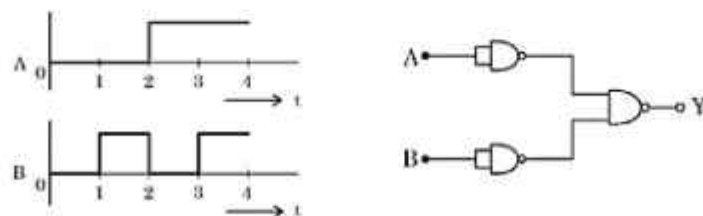
$$|\vec{F}_1| = A, \quad |\vec{F}_2| = \frac{A}{2} \quad \theta = \frac{\pi}{2}$$

$$|\vec{F}_{\text{net}}| = \sqrt{F_1^2 + F_2^2}$$

$$= \sqrt{A^2 + \left(\frac{A}{2}\right)^2}$$

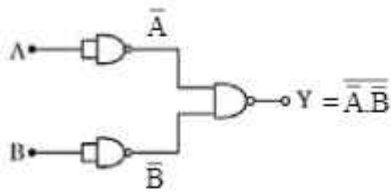
$$|\vec{F}_{\text{net}}| = \frac{\sqrt{5}A}{2}$$

46. For the logic circuit shown, the output waveform at Y is:



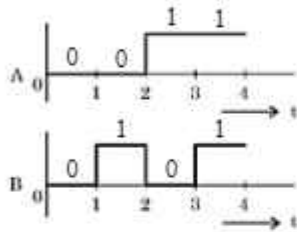
- (1)                      (2)                      (3)                      (4)

Sol. (2)

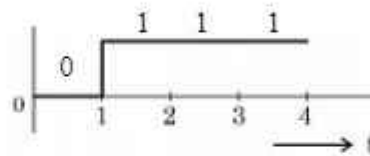


$$y = \overline{A \cdot B} \Rightarrow y = \overline{A} + \overline{B}$$

$$y = A + B$$



A	B	y = A + B
0	0	0
0	1	1
1	0	1
1	1	1



47. An aluminium rod with Young's modulus  $Y = 7.0 \times 10^{10} \text{ N/m}^2$  undergoes elastic strain of 0.04%. The energy per unit volume stored in the rod in SI unit is:

- (1) 5600                      (2) 2800                      (3) 11200                      (4) 8400

Sol. (1)

Aluminium rod Young's modulus

$$y = 7.0 \times 10^{10} \frac{\text{N}}{\text{m}^2}$$

strain 0.04%

$$\text{strain} = \frac{0.04}{100}$$

$$\text{Energy per unit volume} = \frac{1}{2} \text{ stress} \times \text{strain}$$

$$= \frac{1}{2} y \text{ strain} \times \text{strain}$$

$$= \frac{1}{2} y (\text{strain})^2$$

$$= \frac{1}{2} \times 7 \times 10^{10} \times \left(\frac{0.04}{100}\right)^2$$

$$\text{Energy per unit volume} = 5600 \frac{\text{J}}{\text{m}^3}$$

48. Given below are two statements:

**Statement I** : If heat is added to a system, its temperature must increase.

**Statement II** : If positive work is done by a system in a thermodynamic process, its volume must increase.

In the light of the above statements, choose the correct answer from the options given below

- (1) Both Statement I and Statement II are true      (2) Both Statement I and Statement II are false  
 (3) Statement I is true but Statement II is false      (4) Statement I is false but Statement II is true

Sol. (4)

St I False

Ex. in isothermal process temp. is constant but heat can be added.

ST II True

$$w = \int P dV$$

If volume increases the  $w = +ve$

49. An air bubble of volume  $1 \text{ cm}^3$  rises from the bottom of a lake  $40 \text{ m}$  deep to the surface at a temperature of  $12^\circ\text{C}$ . The atmospheric pressure is  $1 \times 10^5 \text{ Pa}$ , the density of water is  $1000 \text{ kg/m}^3$  and  $g = 10 \text{ m/s}^2$ . There is no difference of the temperature of water at the depth of  $40 \text{ m}$  and on the surface. The volume of air bubble when it reaches the surface will be:

- (1)  $3 \text{ cm}^3$                       (2)  $4 \text{ cm}^3$                       (3)  $2 \text{ cm}^3$                       (4)  $5 \text{ cm}^3$

Sol. (4)

Pressure at surface =  $P_{\text{atm}} = 1 \times 10^5 \text{ Pa}$

$V_{\text{surface}} = ?$

Pressure at  $h = 40 \text{ m}$  depth

$$P = P_{\text{atm}} + \rho gh$$

$$P = 10^5 + 10^3 \times 10 \times 40$$

$$P = 5 \times 10^5 \text{ Pa}$$

$$v = 1 \text{ cm}^3$$

Temp. is constant

$$P_1 V_1 = P_2 V_2$$

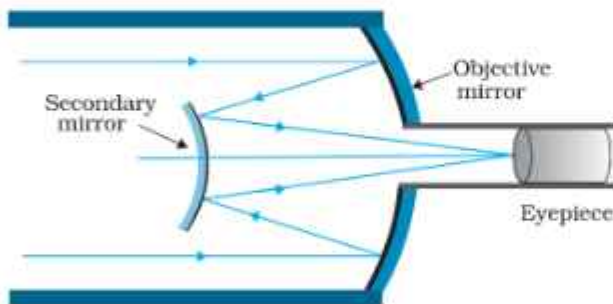
$$10^5 \times v = 5 \times 10^5 \times 1$$

$$v = 5 \text{ cm}^3$$

50. In a reflecting telescope, a secondary mirror is used to:

- (1) Make chromatic aberration zero  
 (2) Reduce the problem of mechanical support  
 (3) Move the eyepiece outside the telescopic tube  
 (4) Remove spherical aberration

Sol. (3)



To move the eye piece outside the telescopic tube

### SECTION – B

51. The momentum of a body is increased by  $50\%$ . The percentage increase in the kinetic energy of the body is \_\_\_\_\_ %.

Sol. (125)

$$K_i = \frac{P_i^2}{2m}$$

$$K_f = \frac{\left(P_i + \frac{P_i}{2}\right)^2}{2m} \Rightarrow K_f = \frac{9 P_i^2}{4 \cdot 2m}$$

$$\text{Percentage increase in K.E.} = \frac{K_f - K_i}{K_i} \times 100$$

$$= \frac{\frac{9}{4} - 1}{1} \times 100$$

$$= \frac{5}{4} \times 100 = 125\%$$

52. A nucleus with mass number 242 and binding energy per nucleon as 7.6 MeV breaks into two fragment each with mass number 121. If each fragment nucleus has binding energy per nucleon as 8.1 MeV, the total gain in binding energy is \_\_\_\_\_ MeV.

Sol. (121)

$$\begin{aligned} \text{Gain in binding energy} &= B.E_f - B.E_i \\ &= 2(121 \times 8.1) - 242 \times 7.6 \\ &= 121 \text{ MeV} \end{aligned}$$

53. An electric dipole of dipole moment is  $6.0 \times 10^{-6} \text{ C m}$  placed in a uniform electric field of  $1.5 \times 10^3 \text{ NC}^{-1}$  in such a way that dipole moment is along electric field. The work done in rotating dipole by  $180^\circ$  in this field will be \_\_\_\_\_ mJ.

Sol. (18)

$$\begin{aligned} W_{\text{ext}} &= U_f - U_i \quad \{U = -\vec{P} \cdot \vec{E}\} \\ &= -PE \cos\pi - (-PE \cos 0) \\ &= 2PE \\ &= 2 \times 6 \times 10^{-6} \times 1.5 \times 10^3 \\ &= 18 \text{ mJ} \end{aligned}$$

54. An organ pipe 40 cm long is open at both ends. The speed of sound in air is  $360 \text{ ms}^{-1}$ . The frequency of the second harmonic is \_\_\_\_\_ Hz.

Sol. (900)

Open organ pipe  $\ell = 40 \text{ cm}$

Speed of sound  $v = 360 \text{ m/s}$

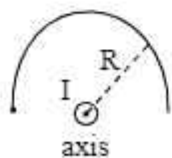
$$\text{Frequency of second harmonics } f_2 = \frac{2v}{2\ell}$$

$$f_2 = \frac{v}{\ell} \Rightarrow f_2 = \frac{360}{0.4}$$

$$f_2 = 900 \text{ Hz}$$

55. The moment of inertia of a semicircular ring about an axis, passing through the center and perpendicular to the plane of ring, is  $\frac{1}{x} MR^2$ , where R is the radius and M is the mass of the semicircular ring. The value of x will be \_\_\_\_\_.

Sol. (1)



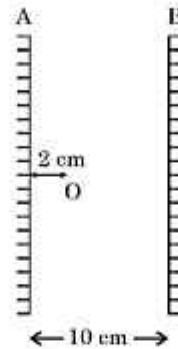
$$I = \int dm R^2 \Rightarrow R^2 \int dm = MR^2$$

$$I = MR^2$$

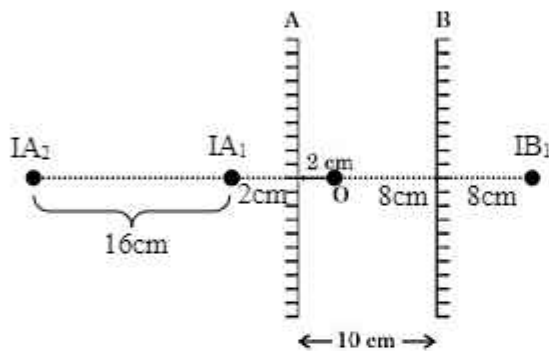
$$\text{Given } I = \frac{1}{x} MR^2$$

$$x = 1$$

56. Two vertical parallel mirrors A and B are separated by 10 cm. A point object O is placed at a distance of 2 cm from mirror A. The distance of the second nearest image behind mirror A from the mirror A is \_\_\_\_\_ cm.



Sol. (18)



$$d = 2 + 16$$

$$d = 18 \text{ cm}$$

57. The magnetic intensity at the center of a long current carrying solenoid is found to be  $1.6 \times 10^3 \text{ Am}^{-1}$ . If the number of turns is 8 per cm, then the current flowing through the solenoid is \_\_\_\_\_ A.

Sol. (2)

$$H = 1.6 \times 10^3 \text{ A/m}, n = 8 \text{ per cm} = 800 \text{ per m}$$

$$H = nI \Rightarrow I = \frac{H}{n}$$

$$I = \frac{1.6 \times 10^3}{8 \times 10^2} \Rightarrow I = 2 \text{ A}$$

58. A current of 2 A through a wire of cross-sectional area  $25.0 \text{ mm}^2$ . The number of free electrons in a cubic meter are  $2.0 \times 10^{28}$ . The drift velocity of the electrons is \_\_\_\_\_  $\times 10^{-6} \text{ ms}^{-1}$  (given, charge on electron =  $1.6 \times 10^{-19} \text{ C}$ ).

Sol. (25)

$$I = neAV_d$$

$$V_d = \frac{I}{neA} \Rightarrow V_d = \frac{2}{2 \times 10^{28} \times 1.6 \times 10^{-19} \times 25 \times 10^{-6}}$$

$$V_d = 25 \text{ m/s}$$



59. An oscillating LC circuit consists of a 75 mH inductor and a 1.2  $\mu\text{F}$  capacitor. If the maximum charge to the capacitor is 2.7  $\mu\text{C}$ . The maximum current in the circuit will be \_\_\_\_\_ mA.

Sol. (9)

LC oscillation  $L = 75 \text{ mH}$

$C = 1.2 \mu\text{F}$

$U_{\text{max}L} = U_{\text{max}C}$

$$\frac{1}{2} LI_{\text{max}}^2 = \frac{1}{2} \frac{q_{\text{max}}^2}{C}$$

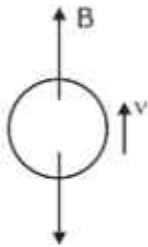
$$I_{\text{max}} = \frac{q_{\text{max}}}{\sqrt{LC}} \Rightarrow I_{\text{max}} = \frac{2.7 \times 10^{-6}}{\sqrt{75 \times 10^{-3} \times 1.2 \times 10^{-6}}}$$

$$I_{\text{max}} = 9 \times 10^{-3} \text{ A}$$

$$I_{\text{max}} = 9 \text{ mA}$$

60. An air bubble of diameter 6 mm rises steadily through a solution of density 1750  $\text{kg/m}^3$  at the rate of 0.35 cm/s. The co-efficient of viscosity of the solution (neglect density of air) is \_\_\_\_\_ Pas (given,  $g = 10 \text{ ms}^{-2}$ ).

Sol. (10)



$$F_v = 6\pi\eta r v$$

For uniform velocity net force = 0

$$B = 6\pi\eta r v$$

$$\rho \frac{4}{3} \pi r^3 g = 6\pi\eta r v$$

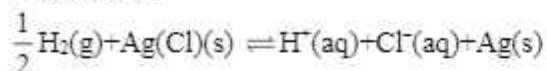
$$\eta = \frac{2\rho r^2 g}{9v}$$

$$\eta = \frac{2 \times 1750 \times (3 \times 10^{-3})^2 \times 10}{9 \times 0.35 \times 10^{-2}}$$

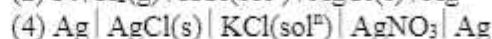
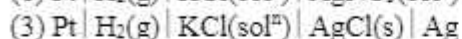
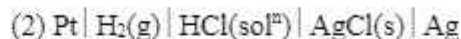
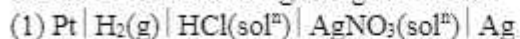
$$\eta = 10 \text{ Pa-s}$$

## SECTION - A

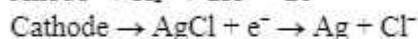
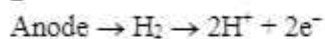
61. The reaction



occurs in which of the given galvanic cell.



Sol. 2



62. Sulphur (S) containing amino acids from the following are:

(a) isoleucine

(b) cysteine

(c) lysine

(d) methionine

(e) glutamic acid

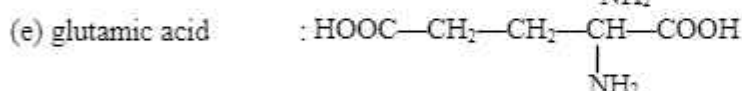
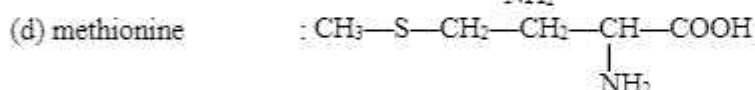
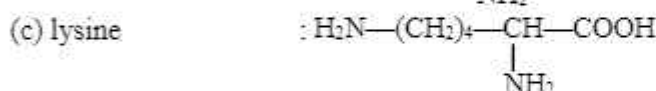
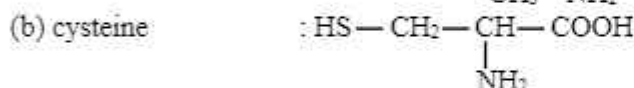
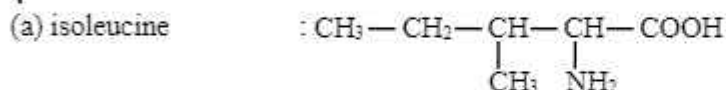
(1) b, c, e

(2) a, d

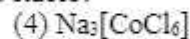
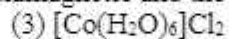
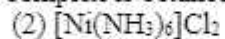
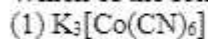
(3) a, b, c

(4) b, d

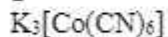
Sol. 4



63. Which of the following complex is octahedral, diamagnetic and the most stable?

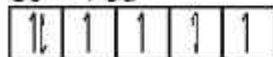
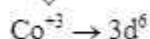


Sol. 1

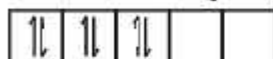


$$+3 + x - 6 = 0$$

$$\boxed{x = +3}$$



$\therefore \text{CN}^-$  is SFL so pairing occur so



$$u - e = 0$$



So diamagnetic

64. Which of the following metals can be extracted through alkali leaching technique?

(1) Cu

(2) Au

(3) Pb

(4) Sn

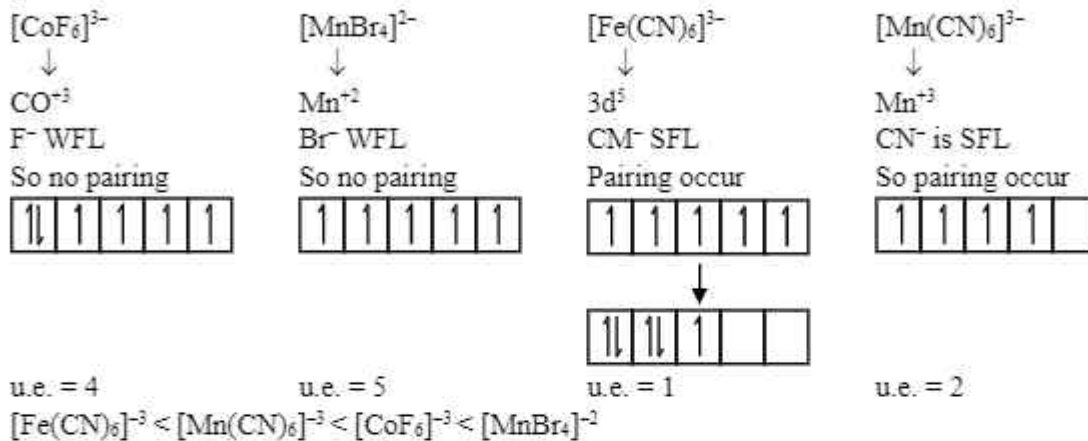
Sol. 4

Sn due to Amphoteric nature.

65. The correct order of spin only magnetic moments for the following complex ions is

- (1)  $[\text{CoF}_6]^{3-} < [\text{MnBr}_4]^{2-} < [\text{Fe}(\text{CN})_6]^{3-} < [\text{Mn}(\text{CN})_6]^{3-}$   
 (2)  $[\text{Fe}(\text{CN})_6]^{3-} < [\text{CoF}_6]^{3-} < [\text{MnBr}_4]^{2-} < [\text{Mn}(\text{CN})_6]^{3-}$   
 (3)  $[\text{MnBr}_4]^{2-} < [\text{CoF}_6]^{3-} < [\text{Fe}(\text{CN})_6]^{3-} < [\text{Mn}(\text{CN})_6]^{3-}$   
 (4)  $[\text{Fe}(\text{CN})_6]^{3-} < [\text{Mn}(\text{CN})_6]^{3-} < [\text{CoF}_6]^{3-} < [\text{MnBr}_4]^{2-}$

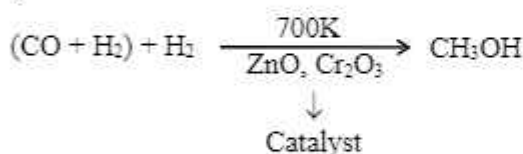
Sol. 4



66. The water gas on reacting with cobalt as a catalyst forms

- (1) Methanoic acid      (2) Methanal      (3) Ethanol      (4) Methanol

Sol. 4



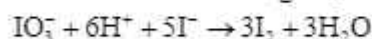
67.  $2\text{IO}_3^- + x\text{I}^- + 12\text{H}^+ \rightarrow 6\text{I}_2 + 6\text{H}_2\text{O}$

What is the value of x?

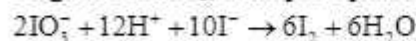
- (1) 12      (2) 10      (3) 2      (4) 6

Sol. 2

n factor of  $\text{IO}_3^-$  and  $\text{I}^-$  in the given redox reaction are 5 and 1 respectively. Therefore,  $\text{IO}_3^-$  will always react in the molar ratio 1 : 5 to get  $\text{I}_2$ .



To get 6 molar  $\text{I}_2$ , multiple equation by 2

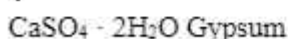


So, x = 10

68. What is the purpose of adding gypsum to cement?

- (1) To give a hard mass      (2) To speed up the process of setting  
 (3) To facilitate the hydration of cement      (4) To slow down the process of setting

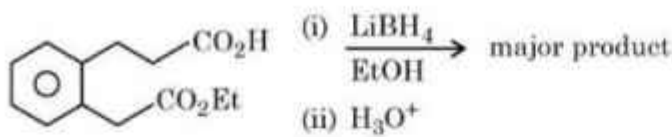
Sol. 4



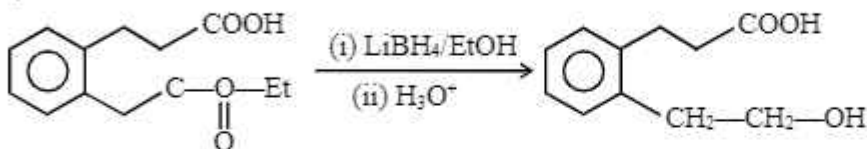
To slow down the process of setting.

Gypsum is added to control the 'setting of cement'. If not added, the cement will set immediately after mixing of water leaving no time the concrete placing.

69. The major product formed in the following reaction is:



Sol. 4



**Note:** Lithium borohydride is commonly used for selective reduction of esters and lactones to the corresponding alcohol.

70. Match list I with list II:

List I (species)	List II (Maximum allowed concentration in ppm in drinking water)
A. $F^-$	I. < 50 ppm
B. $SO_4^{2-}$	II. < 5 ppm
C. $NO_3^-$	III. < 2 ppm
D. Zn	IV. < 500 ppm

(1) A-III, B-II, C-I, D-IV  
 (3) A-IV, B-III, C-II, D-I

(2) A-II, B-I, C-III, D-IV  
 (4) A-I, B-II, C-III, D-IV

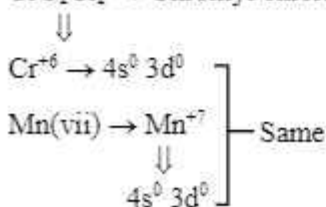
Sol. **Bouns**  
 Data based

	Maximum allowed (ppm)
$F^-$	< 2 ppm
$SO_4^{2-}$	< 5 ppm
$NO_3^-$	< 50 ppm
Zn	< 500 ppm

71. In chromyl chloride, the number of d-electrons present on chromium is same as in (Given at no. of Ti : 22, V : 23, Cr : 24, Mn : 25, Fe : 26)

(1) Fe (III)      (2) V (IV)      (3) Ti (III)      (4) Mn (VII)

Sol. 4  
 $CrO_2Cl_2 \rightarrow$  Chromyl chloride



72. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Butan-1-ol has higher boiling point than ethoxyethane.

Reason R : Extensive hydrogen bonding leads to stronger association of molecules.

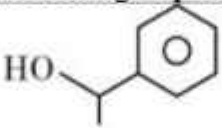

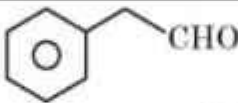
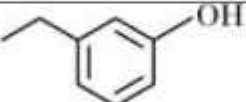
In the light of the above statements, choose the correct answer from the options given below:

- (1) Both A and R are true but R is not the correct explanation of A
- (2) Both A and R are true and R is the correct explanation of A
- (3) A is false but R is true
- (4) A is true but R is false

Sol. 2

At comparable molecular mass, alcohol has higher b.p. than ether due to H-bond, because H-bond leads to stronger associated of molecules.

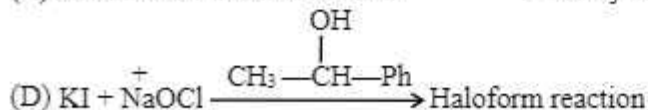
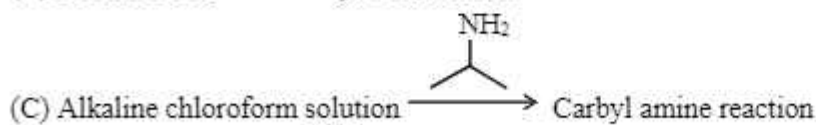
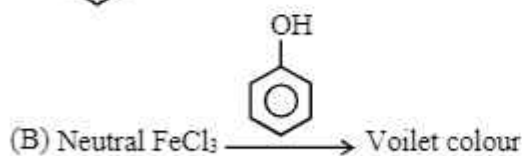
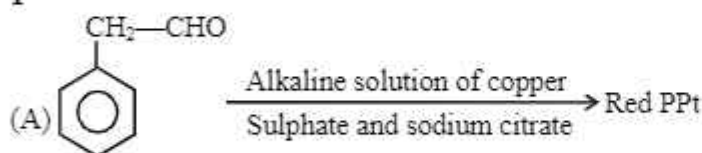
73. Match List I with List II:

List I (Reagents used)	List II (Compound with Functional group detected)
A. Alkaline solution of copper sulphate and sodium citrate	I. 
B. Neutral FeCl <sub>3</sub> solution	II. 
C. Alkaline chloroform solution	III. 
D. Potassium iodide and sodium hypochlorite	IV. 

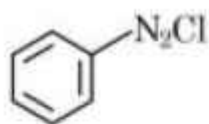
Choose the correct answer from the options given below:

- (1) A-III, B-IV, C-II, D-I
- (2) A-II, B-IV, C-III, D-I
- (3) A-IV, B-I, C-II, D-III
- (4) A-III, B-IV, C-I, D-II

Sol. 1



74. Match List I with List II:



is reacted with reagents in List I to form products in List II.

List I (Reagent)	List II (Product)
A.	I.
B. $\text{HBF}_4, \Delta$	II.
C. $\text{Cu}, \text{HCl}$	III.
D. $\text{CuCN/KCN}$	IV.

Choose the correct answer from the options given below:

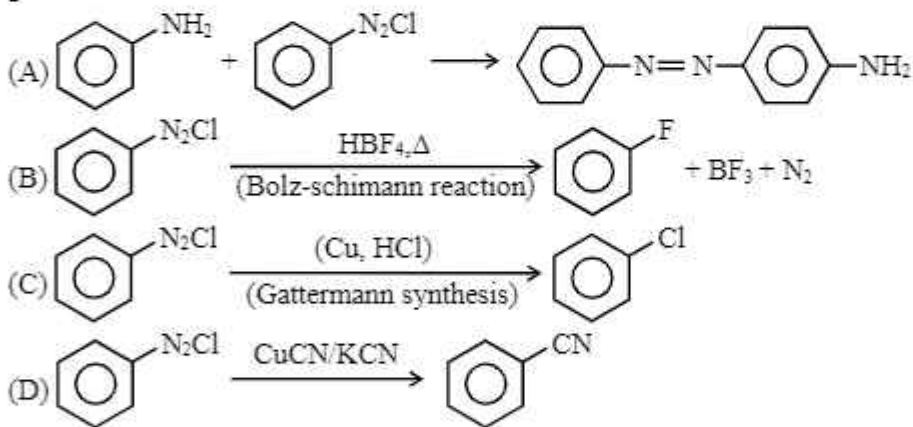
(1) A-I, B-III, C-IV, D-II

(2) A-III, B-I, C-II, D-IV

(3) A-III, B-I, C-IV, D-II

(4) A-IV, B-III, C-II, D-I

Sol. 3



75. Match List I with List II:

List I	List II
A. Saccharin	I. High potency sweetener
B. Aspartame	II. First artificial sweetening agent
C. Alitame	III. Stable at cooking temperature
D. Sucralose	IV. Unstable at cooking temperature

Choose the **correct** answer from the options given below:

(1) A-II, B-III, C-IV, D-I

(2) A-II, B-IV, C-I, D-III

(3) A-IV, B-III, C-I, D-II

(4) A-II, B-IV, C-III, D-I

Sol. 2

- (A) Saccharin → First artificial sweetening agent  
 (B) Aspartame → Unstable at cooking temperature used in soft drink and cold drink.  
 (C) Alitame → High potency sweetener (2000 more sweeter than cane sugar)  
 (D) Sucralose → Stable at cooking temperature. Also it does not provide calories.

76. The correct order of electronegativity for given elements is:

- (1)  $P > Br > C > At$     (2)  $C > P > At > Br$     (3)  $Br > P > At > C$     (4)  $Br > C > At > P$

Sol. **4**  
 $C (2.5)$   
 $P (2.1)$   
 $Br (2.8)$   
 $At (2.2)$

$$\Rightarrow Br > C > At > P$$

77. Given below are two statements :

**Statement I** : Lithium and Magnesium do not form superoxide

**Statement II** : The ionic radius of  $Li^+$  is larger than ionic radius of  $Mg^{2+}$

In the light of the above statements, choose the **most appropriate** answer from the options given below:

- (1) Statement I is correct but Statement II is incorrect  
 (2) Statement I is incorrect but Statement II is correct  
 (3) Both statement I and Statement II are correct  
 (4) Both statement I and Statement II are incorrect

Sol. **3 (Fact-based)**

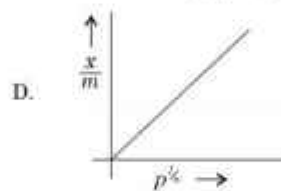
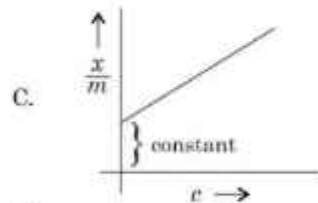
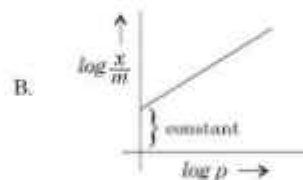
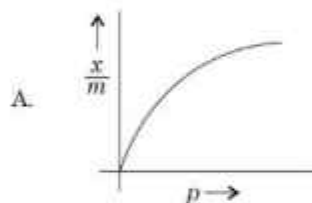
Due to small in size  $Li$  and  $Mg$  do not form superoxide.

$Li^+ > Mg^{2+}$  - radius

$2e^-$      $10e^-$

↓  
 Due to diagonal relationship.

78. Which of the following represent the Freundlich adsorption isotherms?



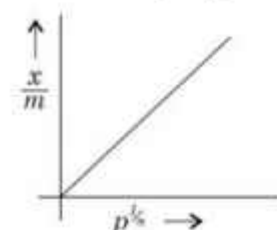
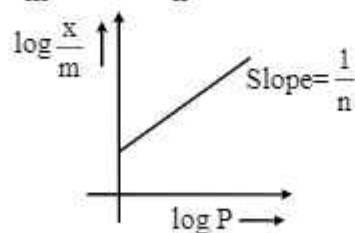
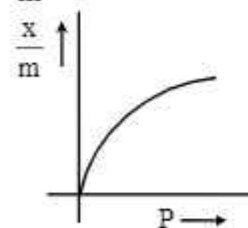
Choose the correct answer from the options given below:

- (1) A, C, D only    (2) A, B only    (3) A, B, D only    (4) B, C, D only

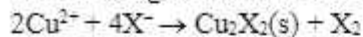
Sol. **3**

$$\frac{x}{m} = Kp^{1/n}$$

$$\log \frac{x}{m} = \log K + \frac{1}{n} \log P$$



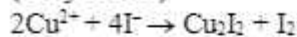
79. Which halogen is known to cause the reaction given below:



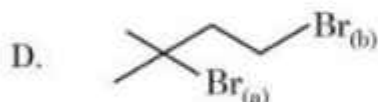
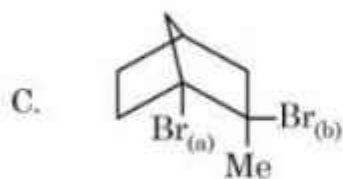
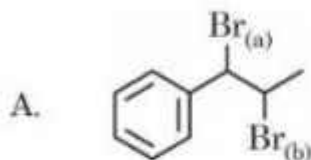
- (1) All halogens (2) Only chlorine (3) Only Bromine (4) Only Iodine

Sol. 4

(Only iodine)



80. Choose the halogen which is most reactive towards  $\text{S}_{\text{N}}1$  reaction in the given compounds (A, B, C, & D)




(1) A-Br(a); B-I(a); C-Br(b); D-Br(a)

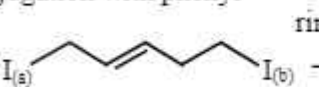
(2) A-Br(b); B-I(a); C-Br(a); D-Br(a)

(3) A-Br(b); B-I(b); C-Br(b); D-Br(b)

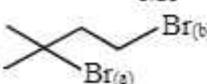
(4) A-Br(a); B-I(a); C-Br(a); D-Br(a)

Sol. 1

(A)  → Because formed intermediate carbocation formed by Br(a) get stabilised by conjugation with phenyl ring

(B)  → Because the intermediate carbocation formed by I(a) become more stable by conjugation

(C)  → Because, we can't remove Br(a) from bridge head carbon (Bredt's rule)

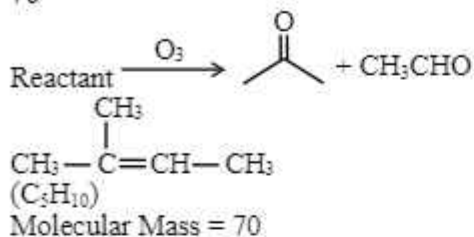
(D)  → Because, formed intermediate by Br(a),  $3^\circ$  carbocation is more stable (stability of carbocation  $3^\circ > 2^\circ > 1^\circ$ )

## SECTION - B



81. Molar mass of the hydrocarbon (X) which on ozonolysis consumes one mole of  $O_3$  per mole of (X) and gives one mole each of ethanol and propanone is \_\_\_\_\_  $g\ mol^{-1}$  (Molar mass of C :  $12\ g\ mol^{-1}$ , H :  $1\ gmol^{-1}$ )

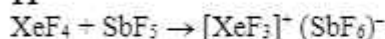
Sol. 70



82.  $XeF_4$  reacts with  $SbF_5$  to form  $[XeF_m]^{n+} [SbF_y]^{z-}$

$$m+n+y+z = \underline{\hspace{2cm}}$$

Sol. 11



$$m + n + x + y = 3 + 1 + 6 + 1 = 11$$

Xenon fluoride act as  $F^-$  donor and  $F^-$  acceptor.

83. The number of following statements which is/are incorrect is \_\_\_\_\_

- (1) Line emission spectra are used to study the electronic structure
- (2) The emission spectra of atoms in the gas phase show a continuous spread of wavelength from red to violet
- (3) An absorption spectrum is like the photographic negative of an emission spectrum
- (4) The element helium was discovered in the sun by spectroscopic method

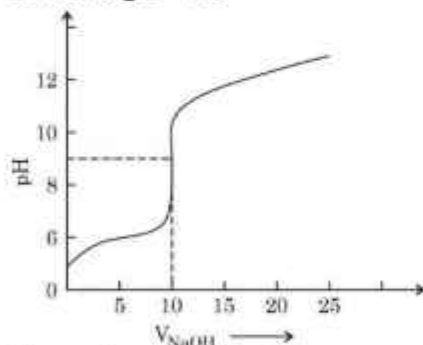
Sol. 1

Fact

84. The titration curve of weak acid vs. strong base with phenolphthalein as indicator) is shown below. The

$$K_{\text{phenolphthalein}} = 4 \times 10^{-10}$$

$$\text{Given : } \log 2 = 0.3$$



The number of following statements/s which is/are correct about phenolphthalein is \_\_\_\_\_

- (1) It can be used as an indicator for the titration of weak acid with weak base.
- (2) It begins to change colour at  $pH = 8.4$
- (3) It is a weak organic base
- (4) It is colourless in acidic medium

Sol. 2

$$(B) \text{ } pK_{in} = -\log(4 \times 10^{-10}) = 9.4$$

Indicator range

$$\Rightarrow pK_{in} \pm 1$$

i.e. 8.4 to 10.4

(D) In acidic medium, phenolphthalein is in unionized form and is colourless.

85. When a 60 W electric heater is immersed in a gas for 100s in a constant volume container with adiabatic walls, the temperature of the gas rises by 5°C. The heat capacity of the given gas is \_\_\_\_\_ J K<sup>-1</sup> (Nearest integer)

Sol. 1200

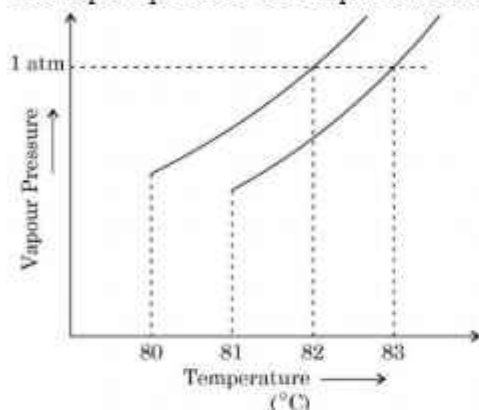
Adiabatic wall {no heat exchange between system and surrounding}

$$C_v \times \Delta T = P \times t / \text{sec}$$

$$C_v \times 5 = 60 \times 100$$

$$C_v = 1200$$

86. The vapour pressure vs. temperature curve for a solution solvent system is shown below:



The boiling point of the solvent is \_\_\_\_\_ °C

Sol. 82

Boiling point of solvent is 82°C

Boiling point of solvent is 83°C

87. 0.5 g of an organic compound (X) with 60% carbon will produce \_\_\_\_\_ × 10<sup>-1</sup> g of CO<sub>2</sub> on complete combustion.

Sol. 11

$$\text{Moles of carbon} = \frac{0.5 \times 0.6}{12}$$

$$\text{Moles of CO}_2 = \frac{0.5 \times 0.6}{12}$$

$$\text{Mass of CO}_2 = \frac{0.5 \times 0.6}{12} \times 44 = 11 \times 10^{-1} \text{ gram}$$

88. The number of following factors which affect the percent covalent character of the ionic bond is \_\_\_\_\_

(1) Polarising power of cation

(2) Extent of distortion of anion

(3) Polarisability of the anion

(4) Polarising power of anion

Sol. 3

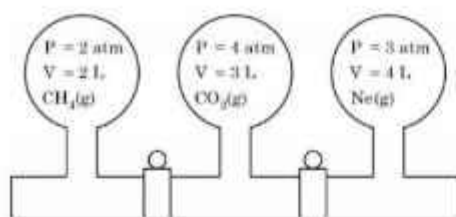
Percent covalent character of the ionic bond

(1) Polarising power of cation

(2) Extent of distortion of anion

(3) Polarisability of the anion

89.



Three bulbs are filled with  $\text{CH}_4$ ,  $\text{CO}_2$  and  $\text{Ne}$  as shown the picture. The bulbs are connected through pipes of zero volume. When the stopcocks are opened and the temperature is kept constant throughout, the pressure of the system is found to be \_\_\_\_\_ atm. (Nearest integer)

Sol. 3

$$P_f V_f = P_1 V_1 + P_2 V_2 + P_3 V_3$$

$$P_f \times 9 = 2 \times 2 + 4 \times 3 + 3 \times 4$$

$$P_f = \frac{28}{9} = 3.11 \approx 3$$

90. The number of given statements/s which is/are correct is \_\_\_\_\_

- (1) The stronger the temperature dependence of the rate constant, the higher is the activation energy.
- (2) If a reaction has zero activation energy, its rate is independent of temperature.
- (3) The stronger the temperature dependence of the rate constant, the smaller is the activation energy
- (4) If there is no correlation between the temperature and the rate constant then it means that the reaction has negative activation energy.

Sol. 2

Clearly, if  $E_a = 0$ ,  $K$  is temperature independent

if  $E_a > 0$ ,  $K$  increase with increase in temperature

if  $E_a < 0$ ,  $K$  decrease with increase in temperature