

PHYSICS**Max Marks: 100****(SINGLE CORRECT ANSWER TYPE)**

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 in all other cases.

1. The wavelength of the photon emitted by a hydrogen when an electron makes a transition from $n = 2$, to $n = 1$ state is:

1) 490.7 nm 2) 913.3 nm 3) 121.8 nm 4) 194.8 nm

Ans: 3

Sol: $E_n = \frac{-13.6}{n^2} eV, E_1 = -13.6 eV, E_2 = -3.4 eV, \Delta E = E_2 - E_1$

$$\Delta E = 10.2 eV, \lambda = \frac{1240}{\Delta E(eV)} nm, \lambda = \frac{1240}{10.2} nm, \lambda = 121.8 nm$$

2. The stopping potential for electrons from a photosensitive surface illuminated by light of wavelength 491 nm is 0.710V. When the incident wavelength is changed to a new value, the stopping potential is 1.43. The new wavelength is:

1) 309 nm 2) 329 nm 3) 382 nm 4) 400 nm

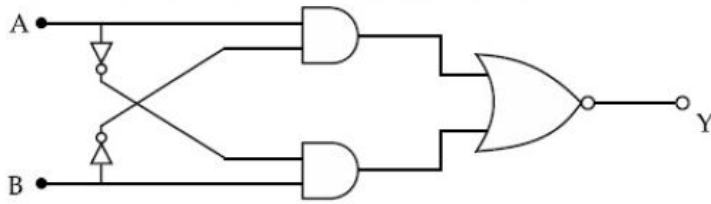
Ans: 3

Sol: energy of photon $E = \frac{1240}{\lambda(nm)} eV, E = \frac{1240}{491} eV, E = 2.53 eV, W_0 = E - V_0 e = 2.53 - 0.710 = 1.82 eV$

$$E = \frac{hc}{\lambda} - \omega_0, V_0 e = \frac{hc}{\lambda} - \omega_0, 0.710 + \omega_0 = \frac{hc}{\lambda}$$

$$0.710 + 1.82 = \frac{hc}{491}, 1.43 + 1.82 = \frac{hc}{\lambda}, \frac{2.53}{3.25} = \frac{\lambda}{491}, \lambda = 382 nm$$

3. The truth table for the following logic circuit is:



1)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

2)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

3)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

4)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

Ans: 2

Sol:

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

$$y = \overline{A} \cdot \overline{B} + \overline{A} \cdot B$$

$$y = \overline{A \cdot B} \cdot \overline{A \cdot B}$$

$$y = (\overline{A} + \overline{B}) \cdot (A + \overline{B})$$

$$y = \overline{A} \overline{A} + \overline{A} \overline{B} + \overline{B} A + \overline{B} \overline{B}$$

4. An electron with kinetic energy K_1 enters between parallel plates of a capacitor at an angle ' α ' with the plates. It leaves the plates at angle ' β ' with kinetic energy K_2 . Then the ratio of kinetic energies $K_1 : K_2$ will be:

1) $\frac{\sin^2 \beta}{\cos^2 \alpha}$

2) $\frac{\cos^2 \beta}{\cos^2 \alpha}$

3) $\frac{\cos \beta}{\cos \alpha}$

4) $\frac{\cos \beta}{\sin \alpha}$

Ans: 2

Sol:

$$u_1 \cos \alpha = u_2 \cos \beta$$

$$\frac{k_1}{k_2} = \frac{\frac{1}{2}mu_1^2}{\frac{1}{2}mu_2^2} = \left(\frac{u_1}{u_2}\right)^2 = \frac{\cos^2 \beta}{\cos^2 \alpha}$$

5. An LCR circuit contains resistance of 110Ω and a supply $220 V$ at 300 rad/s angular frequency. If only capacitance is removed from the circuit, current lags behind the voltage by 45° . If on the other hand, only inductor is removed the current leads by 45° with the applied voltage. The rms current flowing in the circuit will be:

- 1) 1.5A 2) 1A 3) 2A 4) 2.5A

Ans: 3

Sol: $R = 110V$

$$V_{\text{rms}} = 220V$$

$$\omega = 300 \text{ rad/s}, \tan \phi = \frac{X}{R}, \tan 45^\circ = \frac{X_L}{R}, \tan 45^\circ = \frac{X_C}{R}$$

$$X_L = X_C, \quad \therefore Z = R, \quad I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{220}{110} \quad I_{\text{rms}} = 2A$$

6. If e is the electronic charge, c is the speed of light in free space and h is plank's constant, the quantity $\frac{1}{4\pi\epsilon_0} \frac{|e|^2}{hc}$ has dimensions of:

- 1) $[M L T^{-1}]$ 2) $[M^0 L^0 T^0]$ 3) $[M L T^0]$ 4) $[L C^{-1}]$

Ans: 2

Sol: $\frac{1}{4\pi\epsilon_0} \frac{e^2}{hc} = \frac{Fr^2}{Er} = M^0 L^0 T^0$

7. For extrinsic semiconductors, when doping level is increased;

- 1) Fermi-level of p-type semiconductors will go downward and Fermi-level of n-type Semiconductor will go upward
- 2) Fermi-level of both p-type and n-type semiconductors will go upward for $T > T_F K$ and Downward $T < T_F K$, where T_F is Fermi temperature.
- 3) Fermi – level of p and n-type semiconductors will not be affected.
- 4) Fermi-level of p-type semiconductors will go upward for Fermi-level of n-type Semiconductors will go downward.

Ans: 1

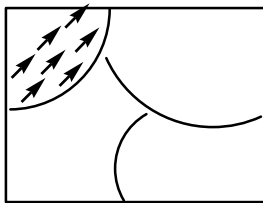
Sol: when doping level is increased Fermi level moves down in p- type and moves up in n - type

8. In a ferromagnetic material, below the Curie temperature, a domain is defined as:

- 1) a macroscopic region with randomly oriented magnetic dipoles
- 2) a macroscopic region with zero magnetization
- 3) a macroscopic region with consecutive magnetic dipoles oriented in opposite direction
- 4) a macroscopic region with saturation magnetization.

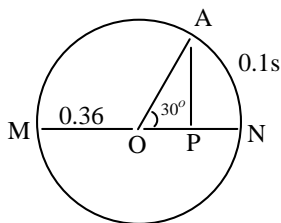
Ans: 4

Sol:



Below Curie temperature, the domain is a macroscopic region with saturation magnetization

9. The point A moves with a uniform speed along the circumference of a circle of radius 0.36 m and covers 30° in 0.1s. The perpendicular projection 'P' from 'A' on the diameter MN represents the simple harmonic motion of 'P'. The restoration force per unit mass when P touches M will be:



- 1) 0.49 N 2) 9.87 N 3) 50 N 4) 100 N

Ans: 2

Sol: when P touches M,

$$\begin{aligned} F_{\max} &= m\omega^2 A \\ &= 1 \times \left(\frac{\pi}{0.6} \right)^2 \times 0.36 \\ F_{\max} &= 9.87 N \end{aligned}$$

10. $Y = A\sin(\omega t + \phi_0)$ is the time –displacement equation of a SHM. At $t=0$ the displacement of the particle is $Y = \frac{A}{2}$ and it is moving along negative x -direction. Then the initial phase angle ϕ_0 will be:

- 1) $\frac{\pi}{3}$ 2) $\frac{2\pi}{3}$ 3) $\frac{\pi}{6}$ 4) $\frac{5\pi}{6}$

Ans: 4

Sol: $y = A\sin(\omega t + \phi_0)$, at $t = 0$: $y = -A/2$, $-\frac{A}{2} = A\sin\phi_0$, $\phi_0 = 5\frac{\pi}{6}$

11. A stone is dropped from the top of a building. When it crosses a point 5 m below the top, another stone starts to fall from a point 25m below the top. Both stones reach the bottom of building simultaneously. The height of the building is

- 1) 45 m 2) 50 m 3) 25 m 4) 35 m

Ans: 1

Sol: time of release of 2nd stone $t = \sqrt{\frac{2 \times 5}{10}} = 1 \therefore \frac{1}{2}gt^2 = \frac{1}{2}g(t-1)^2 + 25$, $5t^2 = 5(t^2 + 1 - 2t) + 25$

$$10t = 30, \quad t = 3\text{sec}, \quad h = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times 9 = 45\text{m}$$

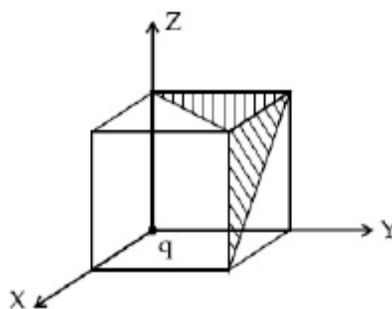
12. If a message signal of frequency ' f_m ' is amplitude modulated with a carrier signal of frequency f_c and radiated through an antenna, the wavelength of the corresponding signal in air is:

- 1) $\frac{c}{f_c}$ 2) $\frac{c}{f_m}$ 3) $\frac{c}{f_c + f_m}$ 4) $\frac{c}{f_c - f_m}$

Ans: 1

Sol: wave transmitted at carrier frequency $\lambda = \frac{C}{\text{frequencies of carrier wave}}, \lambda = \frac{C}{f_c}$

13. A charge 'q' is placed at one corner of a cube as show in figure. The flux of electrostatic field \vec{E} through the shaded area is:



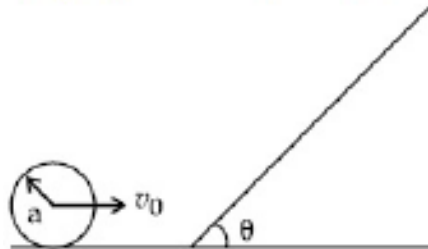
- 1) $\frac{q}{24\epsilon_0}$ 2) $\frac{q}{8\epsilon_0}$ 3) $\frac{q}{48\epsilon_0}$ 4) $\frac{q}{4\epsilon_0}$

Ans: 1

Sol: flux through 3 opposite faces = $\frac{Q}{8\epsilon_0}$

Flux through two half faces = flux through on full face $\phi = \frac{Q}{24\epsilon_0}$

14. A sphere of radius 'a' and mass 'm' rolls along a horizontal plane with constant speed v_0 . It encounters an inclined plane at angle θ and climbs upward. Assuming that it rolls without slipping, how far up, the sphere will travel ?



1) $\frac{2}{5} \frac{v_0^2}{g \sin \theta}$

2) $\frac{v_0^2}{2g \sin \theta}$

3) $\frac{10v_0^2}{7g \sin \theta}$

4) $\frac{v_0^2}{5g \sin \theta}$

Ans: 3

Sol: $v^2 - u^2 = 2as, 0 - u_0^2 = \frac{2g \sin \theta \times s}{1 + \frac{2}{5}}, S = \frac{7v_0^2}{10g \sin \theta}$

15. Match list I with list II.

List I

List II

- | | |
|-----------------|---|
| (a) Rectifier | (i) Used either for stepping up or stepping down the a,c voltage |
| (b) Stabilizer | (ii) Used to convert a.c voltage into d.c voltage |
| (c) Transformer | (iii) Used to remove any ripple in the rectified output voltage |
| (d) Filter | (iv) Used to constant output voltage even when the input voltage or load current change |

Choose the correct answer from the options given below

- 1) (a) –(ii), (b)-(i), (c)-(iv), (d)-(iii) 2) (a) –(iii), (b)-(iv), (c)-(i), (d)-(ii)
3) (a)–(ii), (b)-(iv), (c)-(i), (d)-(iii) 4) (a)–(ii), (b)-(i), (c)-(iii), (d)-(iv)

Ans: 3

Sol: a. Rectifier – converts AC to DC

b. stabilizer – used for constant output voltage even when the input voltage (or) load current change

c. Transformer – used either for stepping up (or) stepping down the ac voltage

d. Filter - used to remove any ripple in the rectified output voltage

16. Given below are two statements:

Statement 1 : in a diatomic molecule, the rotational energy at a given temperature obeys Maxwell's distribution.

Statement 2: In a diatomic molecule, the rotational energy at a given temperature equals The translational kinetic energy for each molecule.

- 1) both statement 1 and statement 2 are true
- 2) both statement 1 and statement 2 are false
- 3) statement 1 is false but statement 2 is true
- 4) statement 1 is true but statement 2 is false

Ans: 4

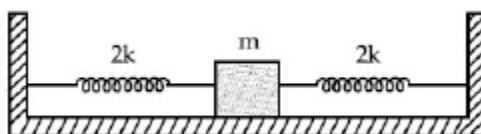
Sol: In a diatomic molecule even rotational energy at given temperature obeys Maxwell's distribution.

$$\text{In a diatomic molecule rotational energy} = \frac{2}{2}RT$$

$$\text{Translational energy} = \frac{3}{2}RT$$

$$E_R \neq E_T$$

17. Two identical springs constant '2K' are attached to a block of mass m and to fixed support (See figure). When the mass is displaced from equilibrium position on either side, it execute simple harmonic motion. The time period of oscillations of this system is:



- 1) $2\pi\sqrt{\frac{m}{2K}}$
- 2) $2\pi\sqrt{\frac{m}{K}}$
- 3) $\pi\sqrt{\frac{m}{K}}$
- 4) $\pi\sqrt{\frac{m}{2K}}$

Ans: 3

Sol: springs are in parallel

$$K_{eff} = 2k + 2k = 4k$$

$$T = 2\pi\sqrt{\frac{m}{K_{eff}}} = 2\pi\sqrt{\frac{m}{4K}} = \pi\sqrt{\frac{m}{K}}$$

18. An electron of mass m_e and a proton of mass $m_p = 1836 m_e$ are moving with the same speed. The ratio of their de Broglie wavelength $\frac{\lambda_{electron}}{\lambda_{proton}}$ will be:

- 1) 1836 2) 918 3) $\frac{1}{1836}$ 4) 1

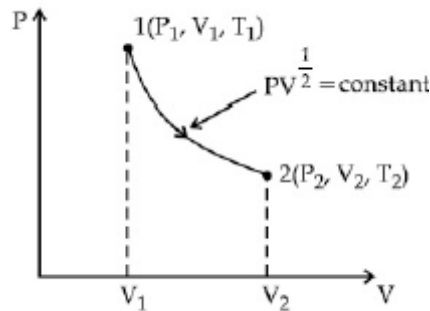
Ans: 1

$$\lambda = \frac{h}{p} = \frac{h}{mV} \propto \frac{1}{m}$$

Sol:

$$\frac{\lambda_{electron}}{\lambda_{proton}} = \frac{m_{proton}}{m_{electron}} = \frac{1836}{1}$$

19. Thermodynamics process is shown below on a P- V diagram for one mole of an ideal gas. If $V_2 = 2V_1$ then the ratio of temperature T_2/T_1 is:



- 1) 2 2) $\sqrt{2}$ 3) $\frac{1}{\sqrt{2}}$ 4) $\frac{1}{2}$

Ans: 2

Sol: $PV^{1/2} = const, PV = RT, P = \frac{RT}{V}, \frac{RT}{V} V^{1/2} = const$

$$T \propto V^{1/2}, \frac{T_2}{T_1} = \left(\frac{2V_1}{V_1}\right)^{1/2} = \sqrt{2}$$

20. Consider the diffraction pattern obtained from the sunlight incident on a pinhole of diameter $0.1 \mu\text{m}$. if the diameter of the pinhole is slightly increased, it will affect the diffraction pattern

- 1) Its size decreases, but intensity increases
 2) Its size decreases, but intensity decreases
 3) Its size increases, but intensity decreases
 4) Its size increases, but intensity increases

Ans: 1

Sol: $\beta = 1.22 \frac{f\lambda}{d}$

If d is increased

Its size (β) decreases

And intensity increases as it collects more light.

(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10. Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

21. The initial velocity v_i required to project a body vertically upward from the surface of the earth to each a height of $10R$, where R is the radius of the earth, may be described in terms of escape velocity v_e such that $v_i = \sqrt{\frac{x}{y}} \times v_e$. The value of x will be——

Ans: 10

Sol: If $V_i = KV_e$, $h = \frac{RK^2}{1-K^2}$, $10R = \frac{RK^2}{1-K^2}$, $10 = 11k^2$, $k = \sqrt{\frac{10}{11}}$, $V_i = KV_e$, $V_i = \sqrt{\frac{x}{y}} V_e$, $x = 10$

22. A reversible heat engine converts one – fourth of the heat input into work. When the temperature of the sink is reduced by $52K$, its efficiency is doubled. The temperature in Kelvin of the source will be ——

Ans: 208

Sol: $\eta = 1 - \frac{T_2}{T_1}$, $\frac{1}{4} = 1 - \frac{T_2}{T_1}$, $\frac{2}{4} = 1 - \frac{(T_2 - 52)}{T_1}$, $\frac{1}{2} = \frac{T_1 - T_2}{T_1 - T_2 + 52}$, $\frac{3}{4} = \frac{T_2}{T_1}$, $T_2 = \frac{3}{4}T_1$

Solving 1 & 2, $T_1 = 208K$

23. If $\vec{P} \times \vec{Q} = \vec{Q} \times \vec{P}$, the angle between \vec{P} and \vec{Q} is $\theta (0^\circ < \theta < 360^\circ)$. The value of ' θ ' will be ——

Ans: 180°

Sol: $\vec{P} \times \vec{Q} = \vec{Q} \times \vec{P}$, $\vec{P} \times \vec{Q} = -(\vec{P} \times \vec{Q})$, $PQ \sin \theta = -PQ \sin \theta$, $\therefore \theta = 180^\circ$

24. The wavelength of an X – ray beam is 10 \AA the mass of a frictions particle having the same energy as that of the X- ray photons is $\frac{x}{3} h \text{ kg}$. the value of x is ——

Ans: 10

Sol: energy of x – rays $= \frac{hc}{\lambda} = \frac{h \times 3 \times 10^8}{10 \times 10^{-10}} = 3h \times 10^{17} J$

$mc^2 = 3h \times 10^{17}$, $m \times 9 \times 10^{16} = 3h \times 10^{17}$, $m = \frac{10}{3} h = \frac{xh}{3}$, $x = 10$

25. The percentage increase in the speed of transverse waves produced in a stretched string if the tension is increased by 4%, will be ——%

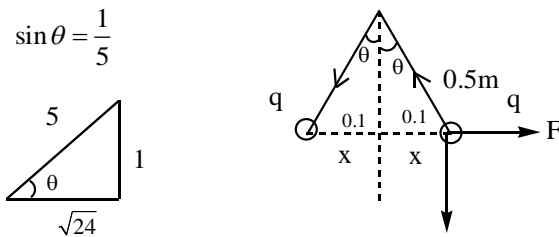
Ans: 2

Sol: $V = \sqrt{\frac{T}{\mu}}$, $\frac{\Delta V}{V} \times 100 = \frac{1}{2} \frac{\Delta T}{T} \times 100 = \frac{1}{2} \times 4\% = 2\%$

26. Two small spheres each of mass 10 mg are suspended from a point by threads 0.5m long. They are equally charged and repel each other to a distance of 0.20m. the charge on each of the sphere is $\frac{a}{21} \times 10^{-8} C$. the value of 'a' will be —— [given $g = 10ms^{-2}$]

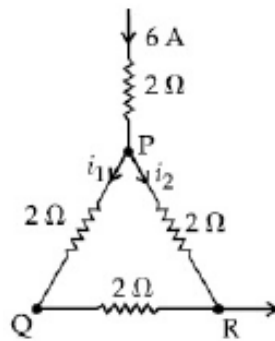
Ans: 20

Sol:



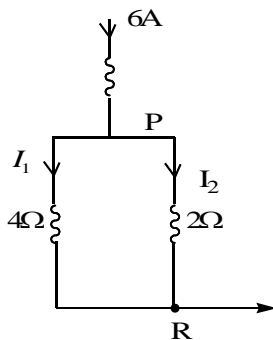
$F = mg \tan \theta$, $9 \times 10^{-3} \times \frac{q^2}{0.04} = 10 \times 10^{-3} \times 10^{-3} \times \frac{1}{\sqrt{24}} q = \frac{20}{21} \times 10^{-8} C = 20$

27. A current of 6A enters one corner P of an equilateral triangle PQR having 3 wires of resistance 2Ω each and leaves by the corner R. The current i_1 in ampere is ——



Ans: 2

Sol:



From Current dividing rule $I_1 = \frac{6 \times 2}{6} = 2A$

28. The peak electric field produced by the radiation coming from the 8W bulb at a distance of 10m is $\frac{x}{10} \sqrt{\frac{\mu_0 C}{\pi}} \frac{V}{m}$. the efficiency of the bulb is 10% and it is a point source. The value of x is

Ans: 0.632

Sol: $I = \frac{P}{4\pi r^2}$, $uc = \frac{P}{4\pi r^2}$, $\frac{1}{2} \epsilon_0 E_o^2 C = \frac{P}{4\pi r^2}$

$$E_o = \sqrt{\frac{2P}{4\pi r^2 \cdot \epsilon_0 \times c}} \quad , \quad E_o = \sqrt{\frac{2 \times 0.8}{4\pi \times 100 \times \epsilon_0}} \quad E_o = \sqrt{\frac{1.6 \mu_0 C}{4\pi \times 100}} = \frac{0.632}{10} \sqrt{\frac{\mu_0 C}{\pi}} v/m$$

29. Two particles having masses 4g and 16g respectively are moving with equal kinetic energies. The ratio of the magnitude of their linear momentum is n : 2. The value of n will be ——

Ans: 1

Sol: $k = \frac{p^2}{2m}$, $P \propto \sqrt{m}$, $\frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{4}{16}} = \frac{1}{2}$, $n = 1$

30. Two identical conducting spheres with negligible volume have 2.1 nC and -0.1 nC charges respectively. They are brought into contact and then separated by a distance of 0.5 m. The electrostatic force acting between the sphere is —— $\times 10^{-9}$ N

[Given: $4\pi \epsilon_0 = \frac{1}{9 \times 10^9}$ SI unit]

Ans: 36

Sol: $q_1 + q_2 = 2nc$, $q_1 = q_2 = 1nc$,

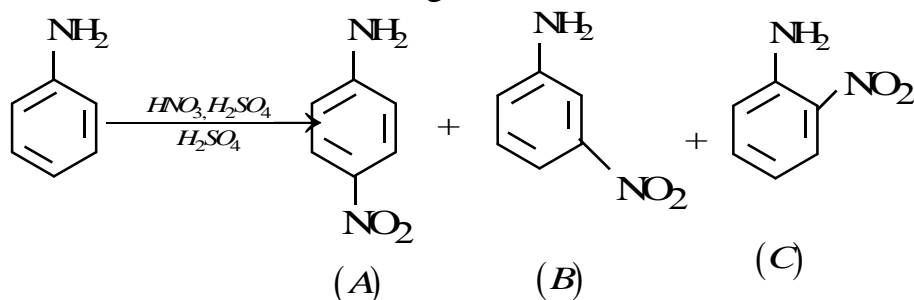
$$F = \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{r^2}, \quad F = 9 \times 10^9 \times \frac{10^{-18}}{0.25} = 36 \times 10^{-9} N = 36$$

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

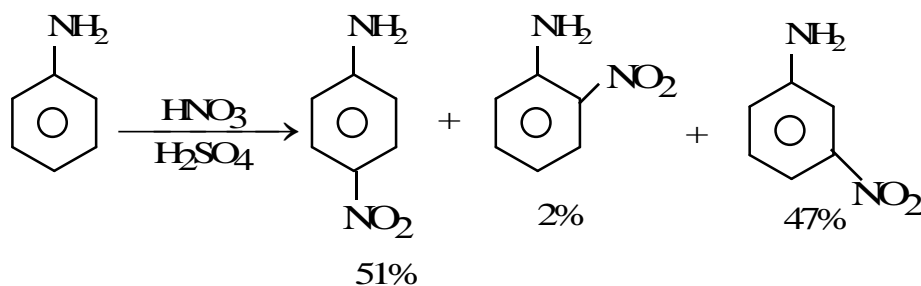
Marking scheme: +4 for correct answer, 0 if not attempted and -1 in all other cases.

31. Correct statement about the given chemical reaction is

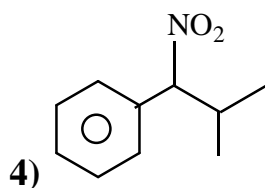
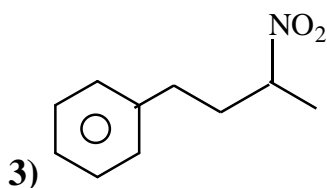
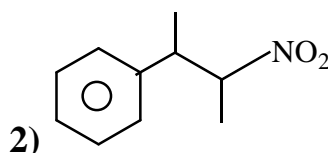
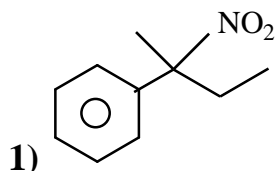
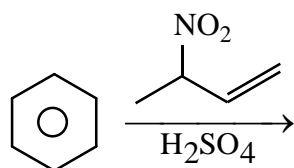


- 1) $-\ddot{N}H_2$ group is ortho and para directive, so product (B) is not possible
- 2) The reaction will form sulphonated product instead of nitration
- 3) Reaction is possible and compound (B) will be the major product
- 4) Reaction is possible and compound (A) will be major product

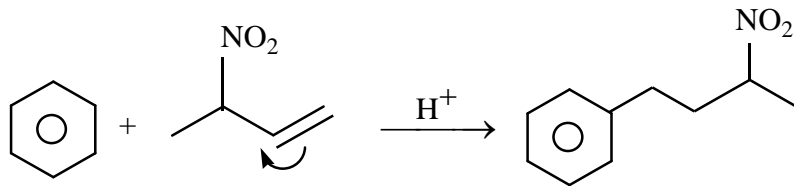
Ans 4
sol:



32. The major product of the following reaction is



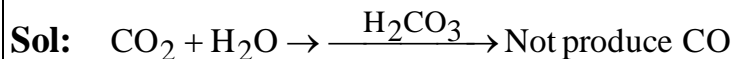
Ans: 3



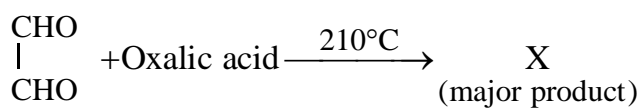
33. Water does not produce CO on reacting with:

- 1) C_3H_8 2) CO_2 3) C 4) CH_4

Ans: 2



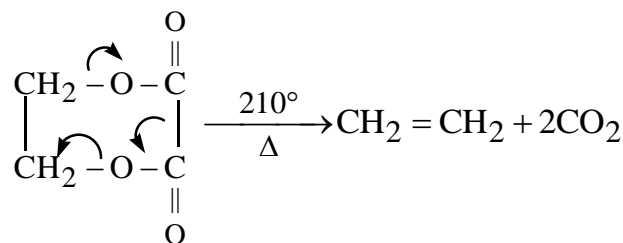
34. What is X in the given reaction?



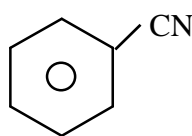
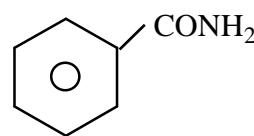
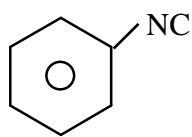
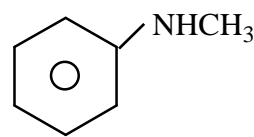
- 1) $\begin{array}{c} \text{CH}_2 \\ || \\ \text{CH}_2 \end{array}$ 2) $\begin{array}{c} \text{CHO} \\ | \\ \text{CHO} \end{array}$ 3) $\begin{array}{c} \text{CH}_2\text{OH} \\ | \\ \text{CHO} \end{array}$ 4) $\begin{array}{c} \text{CH-OH} \\ | \\ \text{CH}_2 \end{array}$

Ans: 1

Sol:



35. Carbylamine test is used to detect the presence of primary amino group in an organic compound. Which of the following compound is formed when this test is performed with aniline

- 1)  2) 
- 3)  4) 

Ans: 3

40. The method used for the purification of Indium is:

- 1) vapour phase refining
- 2) Van Arkel method
- 3) liquation
- 4) Zone refining

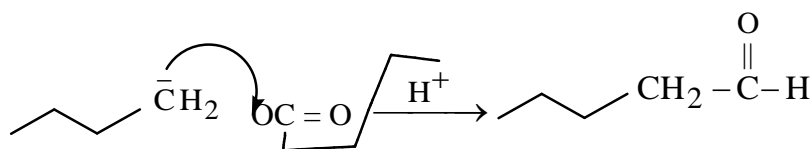
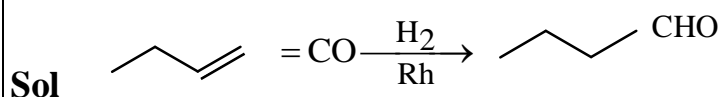
Ans: 4

Sol: Indium is purified by zone refining

41. The major product of the following reaction is $\text{CH}_3\text{CH}_2\text{CH}=\text{CH}_2 \xrightarrow[\text{Rh catalyst}]{\text{H}_2/\text{CO}}$

- 1) $\text{CH}_3\text{CH}_2\text{CH}_2\text{CHO}$
- 2) $\text{CH}_3\text{CH}_2\text{CH}_2\text{CH}_2\text{CHO}$
- 3) $\text{CH}_3\text{CH}_2\underset{\text{CHO}}{\text{C}}=\text{CH}_2$
- 4) $\text{CH}_3\text{CH}_2\text{CH}=\text{CH}-\text{CHO}$

Ans: 2



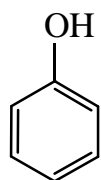
42. Which one of the following statement is FALSE for hydrophilic sols?

- 1) The sols cannot be easily coagulated
- 2) Their viscosity is of the order of that of H_2O
- 3) These sols are reversible in nature
- 4) They do not require electrolytes for stability

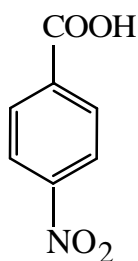
Ans: 2

Sol Their viscosity is of the order of that of H_2O is false statement

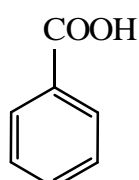
43. The correct order of acid character of the following compounds is



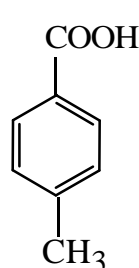
I



II

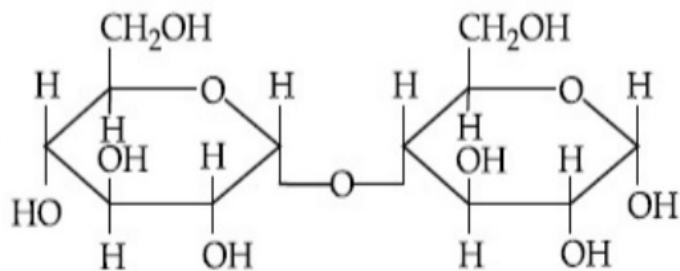


III



IV

- 1) IV > III > II > I
- 2) III > II > I > IV
- 3) II > III > IV > I
- 4) I > II > III > IV



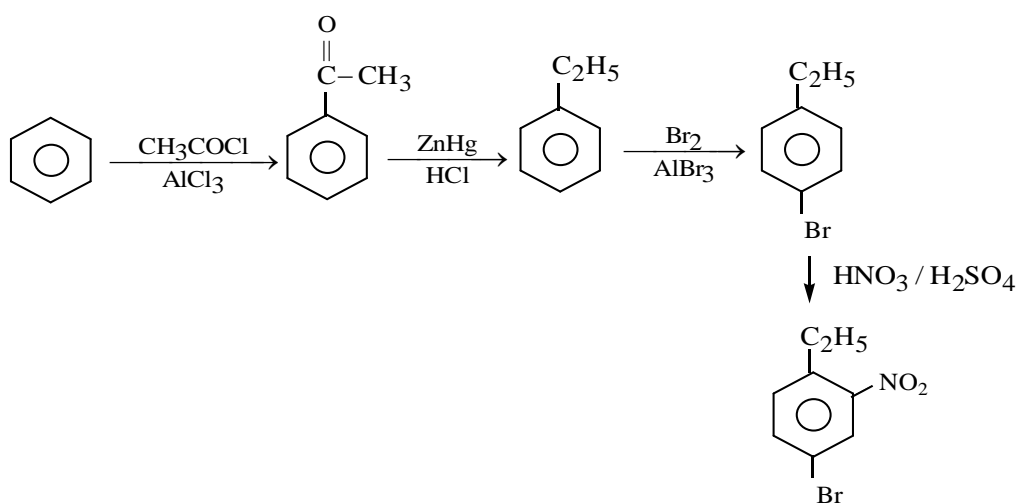
Sol:

47. The correct sequence of reagents used in the preparation of 4-bromo-2-nitroethyl benzene from benzene is?

- 1) $\text{CH}_3\text{COCl} / \text{AlCl}_3, \text{Zn} - \text{Hg} / \text{HCl}, \text{Br}_2 / \text{AlBr}_3, \text{HNO}_3 / \text{H}_2\text{SO}_4$
- 2) $\text{HNO}_3 / \text{H}_2\text{SO}_4, \text{Br}_2 / \text{AlCl}_3, \text{CH}_3\text{COCl} / \text{AlCl}_3, \text{Zn} - \text{Hg} / \text{HCl}$
- 3) $\text{CH}_3\text{COCl} / \text{AlCl}_3, \text{Br}_2 / \text{AlBr}_3, \text{HNO}_3 / \text{H}_2\text{SO}_4, \text{Zn} / \text{HCl}$
- 4) $\text{Br}_2 / \text{AlBr}_3, \text{CH}_3\text{COCl} / \text{AlCl}_3, \text{HNO}_3 / \text{H}_2\text{SO}_4, \text{Zn} / \text{HCl}$

Ans: 1

Sol:



48. Given below are two statements

Statement 1: The pH of rain water is normally – 5,6

Statement 2: If the pH of rain water drops below 5, 6, it is called acid rain.

In the light of the above statements, Choose the correct answer from the options given below

- 1) Both statement 1 and statement II are true
- 2) Statement I is true but statement II is false
- 3) Statement I is false but statement II is true
- 4) Both Statement I and statement II are false

Ans: 1

Sol: pH of rain water = 5.6

pH of rain water drops below 5.6 is called acid rain

49. The solubility of $\text{Ca}(\text{OH})_2$ in water is: [Given: The solubility product of $\text{Ca}(\text{OH})_2$ in water = 5.5×10^{-6}]

- 1) 1.11×10^{-2} 2) 1.77×10^{-2} 3) 1.77×10^{-6} 4) 1.11×10^{-6}

Ans: 1

Sol: $K_{sp} = 4s^3 = 5.5 \times 10^{-6}$

$$s^3 = \frac{5.5}{4} \times 10^{-6}$$

$$s = 1.11 \times 10^{-2}$$

50. Given below are two statements:

Statement 1: The identification of Ni^{2+} is carried out by dimethyl glyoxime in the present of NH_4OH

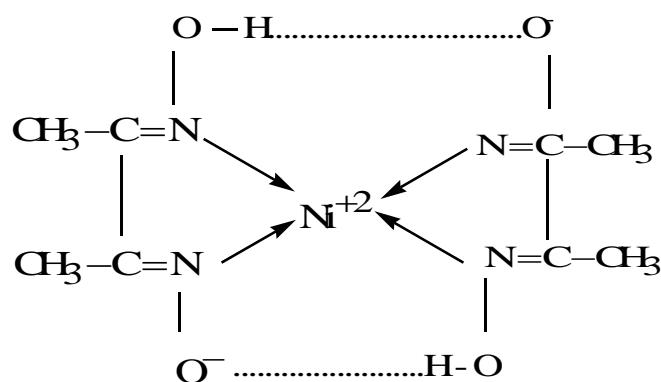
Statement 2: The dimethyl glyoxime is a bidentate neutral ligand.

In the light of the above statements, choose the correct answer from the options given below.

- 1) Both Statement I and statement II are false
2) Both statement 1 and statement II are true
3) Statement I is false but statement II is true
4) Statement I is true but statement II is false

Ans: 4

Sol: $\text{Ni}^{2+} + 2\text{DMG} \rightarrow [\text{Ni}(\text{DMG})_2]$



DMG is Charged bidentate ligand

(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10. Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

51. Electromagnetic radiation of wavelength 663 nm is just sufficient to ionize the atom of metal A. The ionization energy of metal A in kJ mol^{-1} is.....(Rounded off to the nearest integer) [$h = 6.63 \times 10^{-34} \text{ Js}$, $c = 3.00 \times 10^8 \text{ ms}^{-1}$, $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$]

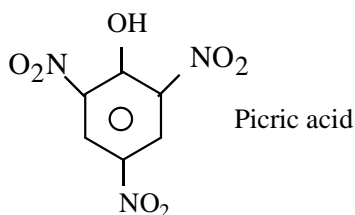
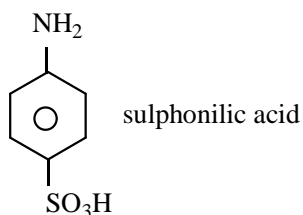
Ans: 181

Sol:
$$\text{I.E} = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{663 \times 10^{-9}} \times 6.02 \times 10^{23}$$
$$= 180.6 \text{ KJ/mole}$$
$$= 181 \text{ KJ}$$

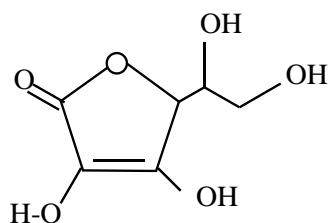
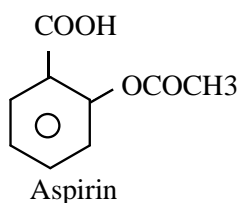
52. The number of compound/s given below which contains $-\text{COOH}$ group is.....[integer answer]

1. Sulphanilic acid 2. Picric acid 3. Aspirin 4. Ascorbic acid

Ans: 1



Sol:



Ascarbic acid

53. Among the following number of metal/s which can be used as electrodes in the photoelectric cell is.....(integer answer)

1. Li 2. Na 3. Rb 4. Cs

Ans: 1

Sol: Cs Used as electrode in photoelectric cell

54. If a compound AB dissociates to the extent of 75% in an aqueous solution, the molality of the solution which shows a 2.5K rise in the boiling point of the solution is.....molal. ($K_b = 0.52 \text{ K kg mol}^{-1}$)

Ans: 3

Sol: $\Delta T_f = i \times k_f \times m$

$$\alpha = \frac{i-1}{n-1}, 0.75 = \frac{i-1}{2-1}, i = 1.75, 2.5 = 1.75 \times 0.52 \times m, \quad m = 2.74$$

55. Five moles of an ideal gas at 293K is expanded isothermally from an initial pressure of 2.1MPa to 1.3 MPa against at constant external pressure 4.3 MPa. The heat transferred in this process is..... kJ mol⁻¹ (Rounded – off to the nearest integer) [Use R = 8.314J mol⁻¹K⁻¹)

Ans: 15

Sol: $v_f = \frac{5 \times 8.314 \times 293}{1.3 \times 10^6} = 9.369 \times 10^{-3}$, $v_i = \frac{P_2 V_2}{V_1} = \frac{1.3 \times 9.369 \times 10^{-3}}{2.1}$

$$w = P_{\text{ext}}(\Delta V) = 4.3 \times 5 \times \frac{8.314}{10^6} \times 293 \left(\frac{1}{1.3} - \frac{1}{2.1} \right) = 15.3$$

56. Copper reduces NO₃⁻ into NO and NO₂ depending upon the concentration of HNO₃ in solution [Assuming fixed [Cu²⁺] and P_{NO} = P_{NO₂}), the HNO₃ concentration at which the thermodynamic tendency for reduction of NO₃⁻ into NO and NO₂ by copper is same is 10^x M. The value of 2x is..... (Rounded off to the nearest integer)

[Given E^o_{Cu²⁺/Cu = 0.34V, E^o_{NO₃⁻/NO = 0.96V, E^o_{NO₃⁻/NO₂ = 0.79V and 298K}}}

$$\frac{RT}{F} (2.303) = 0.059]$$

Ans: 1

Sol: $3\text{Cu} + 8\text{H}^+ + 2\text{NO}_3^- \rightarrow 3\text{Cu}^{2+} + 2\text{NO} + 4\text{H}_2\text{O}$

$\text{Cu} + 4\text{H}^+ + 2\text{NO}_3^- \rightarrow \text{Cu}^{2+} + 2\text{NO}_2 + 2\text{H}_2\text{O}$

Let con of HNO₃ = [H⁺] = x [NO₃⁻] = x

$$E_{\text{NO}_3^-/\text{NO}} - E_{\text{Cu}^{2+}/\text{Cu}} = E_{\text{NO}_3^-/\text{NO}_2} - E_{\text{Cu}^{2+}/\text{Cu}}, \quad E_{\text{NO}_3^-/\text{NO}} = E_{\text{NO}_3^-/\text{NO}_2}$$

$$0.96 - \frac{0.0591}{3} \log \frac{10^{-3}}{10^5} = 0.79 - \frac{0.059}{3} \log \frac{10^{-3}}{x^3}$$

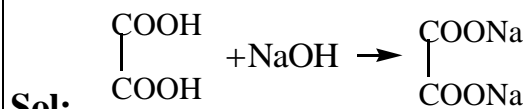
$$X=0.66 \quad 2x=1.32 \text{ nearest integer is } 1$$

57. Consider titration of NaOH solution versus 1.25 M oxalic acid solution. At the end point following burette readings were obtained.

1. 4.5ml 2. 4.5 ml 3. 4.4 ml 5. 4.4ml. 5. 4.4ml

If the volume of oxalic acid taken was 10.0ml then the molarity of the NaOH solution isM. (Rounded-off to the nearest integer)

Ans: 6



Milli eq oxalic acid = milli eq NaOH

$$v.f \times M_1 \times v_1 = v.f \times M_2 \times v_2, \quad 2 \times 1.25 \times 10 = 1 \times M_2 \times 4.4. \quad M_2 = \frac{2.5 \times 10}{4.4} = 5.8M$$

58. The rate constant of a reaction increases by five times on increase in temperature from 27°C to 52°C. The value of activation energy in KJ mol⁻¹ is..... (Rounded off to the nearest integer) [R = 8.314 Jk⁻¹ mol⁻¹]

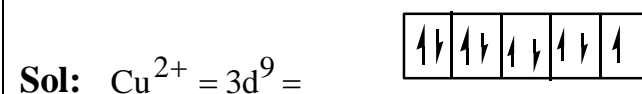
Ans: 52

Sol:
$$\log \frac{5}{1} = \frac{E_a}{2.303 \times 8.314} \left[\frac{25}{300 \times 325} \right]$$

$$E_a = \frac{0.7 \times 2.303 \times 8.314 \times 300 \times 325}{25 \times 100}. \quad E_a = 52.27KJ$$

59. The spin only magnetic moment of a divalent ion in aqueous solution (atomic number 29) is.....BM

Ans: 2



$$M.M = \sqrt{n(n+2)} = \sqrt{3} = 1.73$$

60. The unit cell of copper corresponds to a face centered cube of edge length 3.596Å with one copper atom at each lattice point. The calculated density of copper in kg/m³ is.....[Molar mass of Cu =63.54g: Avogadro number = 6.022×10²³]

Ans: 9079.28

Sol:
$$d = \frac{Z \times M}{a^3 \times N_A} = \frac{4 \times 63.54 \times 10^{-3}}{(3.596)^3 \times 6.02 \times 10^{23} \times 10^{-30}} = \frac{4 \times 63.54 \times 10^{+4}}{(3.596)^3 \times 6.02} = 9079.28$$

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 in all other cases.

61. A hyperbola passes through the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and its transverse and conjugate axes coincide with major and minor axes of the ellipse, respectively. If the product of their eccentricities is one, then the equation of the hyperbola is:
- 1) $\frac{x^2}{9} - \frac{y^2}{16} = 1$ 2) $\frac{x^2}{9} - \frac{y^2}{4} = 1$ 3) $x^2 - y^2 = 9$ 4) $\frac{x^2}{9} - \frac{y^2}{25} = 1$

Ans: 1

Sol: Equation of ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, $e = \frac{3}{5}$, $s(3,0)$, $s'(-3,0)$

Equation of hyperbola $\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1$

Satisfies $(\pm 3, 0) \Rightarrow A^2 = 9$

$eE = 1 \Rightarrow E = \frac{1}{e} = \frac{5}{3}$, Where "e" is eccentricity of ellipse and "E" is eccentricity of

hyperbola $B^2 = A^2(E^2 - 1) \Rightarrow B^2 = 16$, Hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$

62. The minimum value of $f(x) = a^{a^x} + a^{1-a^x}$, where $a, x \in \mathbb{R}$ and $a > 0$, is equal to
- 1) $2a$ 2) $a + \frac{1}{a}$ 3) $2\sqrt{a}$ 4) $a + 1$

Ans: 3

Sol: $f(x) = a^{a^x} + \frac{a}{a^{a^x}}$ is ($a > 0$)

$$AM \geq GM \Rightarrow \frac{a^{a^x} + \frac{a}{a^{a^x}}}{2} \geq \sqrt{a^{a^x} \cdot \frac{a}{a^{a^x}}} = 2\sqrt{a}$$

63. The following system of linear equations

$$2x + 3y + 2z = 9$$

$$3x + 2y + 2z = 9$$

$$x - y + 4z = 8$$

- 1) does not have any solution
 2) Has infinitely many solutions
 3) has a unique solution
 4) Has a solution (α, β, γ) satisfying $\alpha + \beta^2 + \gamma^2 = 12$

Ans: 3

$$\text{Sol: } \left. \begin{array}{l} 2x + 3y + 2z = 9 \\ 3x + 2y + 2z = 9 \\ x - y + 4z = 8 \end{array} \right\}$$

$$D = \begin{vmatrix} 2 & 3 & 2 \\ 3 & 2 & 2 \\ 1 & -1 & 4 \end{vmatrix} = 2(10) - 3(10) + 2(10) = 10 \neq 0$$

Unique solution

64. If $0 < x, y < \pi$ and $\cos x + \cos y - \cos(x + y) = \frac{3}{2}$, then $\sin x + \cos y$ is equal to

- 1) $\frac{\sqrt{3}}{2}$ 2) $\frac{1-\sqrt{3}}{2}$ 3) $\frac{1+\sqrt{3}}{2}$ 4) $\frac{1}{2}$

Ans: 3

Sol: $0 < x, y < \pi, \cos x + \cos y - \cos(x + y) = \frac{3}{2}$ $\sin x + \cos y = ?$

$$\cos x + \cos y - \cos(x + y) = \frac{3}{2}$$

$$\Rightarrow 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2} - 2 \cos^2 \frac{x+y}{2} + 1 = \frac{3}{2}$$

$$\Rightarrow 2 \cos \frac{x+y}{2} \left[\cos \frac{x-y}{2} - \cos \frac{x+y}{2} \right] = \frac{1}{2}$$

$$\Rightarrow 2 \cos \frac{x+y}{2} 2 \sin \frac{x}{2} \sin \frac{y}{2} = \frac{1}{2}$$

$$\Rightarrow \cos \frac{x+y}{2} \sin \frac{x}{2} \sin \frac{y}{2} = \frac{1}{8}$$

$$\Rightarrow \frac{x}{2} = 30^\circ, \frac{y}{2} = 30^\circ, \frac{x+y}{2} = 60^\circ \qquad \sin x + \cos y = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3}+1}{2}$$

65. If $I_n = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^n x dx$, then:

- 1) $I_2 + I_4, I_3 + I_5, I_4 + I_6$ are in AP 2) $I_2 + I_4, (I_3 + I_5)^2, I_4 + I_6$ are in GP
3) $\frac{1}{I_2 + I_4}, \frac{1}{I_3 + I_5}, \frac{1}{I_4 + I_6}$ are in GP 4) $\frac{1}{I_2 + I_4}, \frac{1}{I_3 + I_5}, \frac{1}{I_4 + I_6}$ are in AP

Ans: 4

Sol: $I_n = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^n x dx$

$$\text{Consider } I_n + I_{n+2} = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\cot^n x + \cot^{n+2} x) dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^n x \cdot \operatorname{cosec}^2 x dx$$

$$= -\frac{\cot^{n+1} x}{n+1} \Bigg|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \frac{1}{n+1}$$

$$\frac{1}{I_2 + I_4} = 3; \frac{1}{I_3 + I_5} = 4; \frac{1}{I_4 + I_6} = 5 \text{ are in A.P.}$$

66. The contrapositive of the statement 'If you will work, you will earn money' is

- 1) To earn money, you need to work
- 2) If you will earn money, you will work
- 3) You will earn money if you will not work
- 4) If you will not earn money, you will not work

Ans: 4

Sol: Contrapositive of "If p then q is $\sim q \rightarrow \sim p$

67. A function f(x) is given by $f(x) = \frac{5^x}{5^x + 5}$, then the sum of the series

$$f\left(\frac{1}{20}\right) + f\left(\frac{2}{20}\right) + f\left(\frac{3}{20}\right) + \dots + f\left(\frac{39}{20}\right) \text{ is equal to}$$

- 1) $\frac{29}{2}$
- 2) $\frac{49}{2}$
- 3) $\frac{19}{2}$
- 4) $\frac{39}{2}$

Ans: 4

Sol: $f(x) = \frac{5^x}{5^x + 5}$; $f\left(\frac{1}{20}\right) + f\left(\frac{2}{20}\right) + f\left(\frac{3}{20}\right) + \dots + f\left(\frac{39}{20}\right) = ?$

$$f(x) + f(2-x) = \frac{5^x}{5^x + 5} + \frac{5^{2-x}}{5^{2-x} + 5} = \frac{5^x}{5^x + 5} + \frac{\frac{25}{5x}}{25 + 5.5x} = \frac{5^x}{5^x + 5} + \frac{5}{5^x + 5} = 1$$

$$f\left(\frac{1}{20}\right) + f\left(\frac{2}{20}\right) + f\left(\frac{3}{20}\right) + \dots + f\left(\frac{39}{20}\right) = 19 \times 1 + \frac{1}{2} = \frac{39}{2}$$

68. The integral $\int \frac{e^{3 \log_e 2x} + 5e^{2 \log_e 2x}}{e^{4 \log_e x} + 5e^{3 \log_e x} - 7e^{2 \log_e x}} dx, x > 0$ is equal to (where c is a constant of integration)

- 1) $4 \log_e |x^2 + 5x - 7| + c$
- 2) $\log_e |x^2 + 5x - 7| + c$
- 3) $\frac{1}{4} \log_e |x^2 + 5x - 7| + c$
- 4) $\log_e \sqrt{x^2 + 5x - 7} + c$

Ans: 1

Sol:
$$\int \frac{e^{3 \log_e 2x} + 5e^{2 \log_e 2x}}{e^{4 \log_e x} + 5e^{3 \log_e x} - 7e^{2 \log_e x}} dx; x > 0$$

$$= \int \frac{8x^3 + 5.4x^2}{x^4 + 5x^3 - 7x^2} dx = \int \frac{8x + 20}{x^2 + 5x - 7} dx = 4 \log |x^2 + 5x - 7| + C$$

69. A plane passes through the points A (1, 2, 3) B (2, 3, 1) and C(2, 4, 2). If O is the origin and P is (2, -1, 1), then the projection of \overline{OP} on this plane is of length

- 1) $\sqrt{\frac{2}{3}}$ 2) $\sqrt{\frac{2}{7}}$ 3) $\sqrt{\frac{2}{11}}$ 4) $\sqrt{\frac{2}{5}}$

Ans: 3

Sol: A(1,2,3), B(2,3,1), C(2,4,2). Eq of plane is
$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & 1 & -2 \\ 1 & 2 & -1 \end{vmatrix} = 0$$

$$(x-1)(3) - (y-2)(1) + (z-3)(1) = 0 \Rightarrow 3x - y + z - 4 = 0$$

Projection of O (0,0,0) on the plane is (h,k,l)

$$\frac{h-0}{3} = \frac{k-0}{-1} = \frac{l-0}{1} = \frac{-(-4)}{9+1+1} = \frac{4}{11} \Rightarrow O'(h,k,l) = \left(\frac{12}{11}, \frac{-4}{11}, \frac{4}{11}\right)$$

Projection of P (2,-1,1) on the plane is (h,k,l)

$$\frac{h-2}{3} = \frac{k+1}{-1} = \frac{l-1}{1} = \frac{-(6+1+1-4)}{9+1+1} = \frac{-4}{11}$$

$$h-2 = \frac{-12}{11}, k+1 = \frac{4}{11}, l-1 = \frac{-4}{11}$$

$$h = 2 - \frac{12}{11}, k = \frac{4}{11} - 1, l = 1 - \frac{4}{11}$$

$$h = \frac{10}{11}, k = \frac{-7}{11}, l = \frac{7}{11} = P' = \left(\frac{10}{11}, \frac{-7}{11}, \frac{7}{11}\right)$$

$$O'P' = \sqrt{\frac{4}{121} + \frac{9}{121} + \frac{9}{121}} = \sqrt{\frac{22}{121}} = \sqrt{\frac{2}{11}}$$

70. If for the matrix $A = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}$, $AA^T = I_2$, then the value of $\alpha^4 + \beta^4$ is:

- 1) 2 2) 1 3) 4 4) 3

Ans: 2

Sol: $A = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix} AA^T = I; \alpha^4 + \beta^4 = \dots\dots$

$$AA^T = \begin{pmatrix} 1 & -\alpha \\ \alpha & \beta \end{pmatrix} \begin{pmatrix} 1 & \alpha \\ -\alpha & \beta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1+\alpha^2 & \alpha-\alpha\beta \\ \alpha-\alpha\beta & \alpha^2+\beta^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \alpha = \alpha\beta \quad 1+\alpha^2 = 1 \Rightarrow \alpha = 0 \quad \Rightarrow \alpha(1-\beta) = 0 \Rightarrow \beta = 1$$

$$\alpha^4 + \beta^4 = 1$$

71. If $\alpha, \beta \in \mathbb{R}$ are such that $1 - 2i$ (here $i^2 = -1$) is a root of $z^2 + \alpha z + \beta = 0$, then $(\alpha - \beta)$ is equal to
- 1) 3 2) 7 3) -3 4) -7

Ans: 4

Sol $z^2 + \alpha z + \beta = 0$ has one root $1 - 2i$ other root $1 + 2i$
 $\alpha = -2; \beta = 5$
 $\alpha - \beta = -2 - 5 = -7$

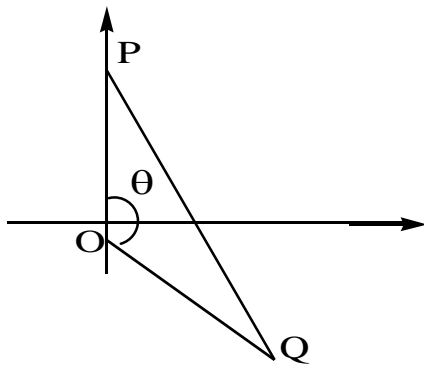
72. If the curve $x^2 + 2y^2 = 2$ intersects the line $x + y = 1$ at two points P and Q, then the angle subtended by the line segment PQ at the origin is

- 1) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{4}\right)$ 2) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{3}\right)$ 3) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$ 4) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{3}\right)$

Ans: 3

Sol $x^2 + 2y^2 = 2 \Rightarrow \frac{x^2}{2} + \frac{y^2}{1} = 1$

Line $x + y = 1$; $y = 1 - x$



Solving the equations $x^2 + 2(1 - x)^2 = 2$

$$x^2 + 2x^2 + 2 - 4x = 2$$

$$3x^2 - 4x = 0; x = 0, 3x - 4 = 0 \quad x = \frac{4}{3}$$

$$P(0, 1), Q\left(\frac{4}{3}, -\frac{1}{3}\right), \quad \theta = \frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$$

73. Let α and β be the roots of $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of

$$\frac{a_{10} - 2a_8}{3a_9} \text{ is}$$

- 1) 3 2) 1 3) 4 4) 2

Ans: 4

Sol: $x^2 - 6x - 2 = 0$; roots α, β

$$\left. \begin{array}{l} \alpha^2 - 6\alpha - 2 = 0 \\ \beta^2 - 6\beta - 2 = 0 \end{array} \right\} \Rightarrow \alpha^{10} = 6\alpha^9 + 2\alpha^8$$

$$\beta^{10} = 6\beta^9 + 2\beta^8 \quad a_8 = \alpha^8 - \beta^8$$

$$a_{10} = \alpha^{10} - \beta^{10} = 6(\alpha^9 - \beta^9) + 2(\alpha^8 - \beta^8)$$

$$a_{10} = 6a_9 + 2a_8 \Rightarrow \frac{a_{10} - 2a_8}{3a_9} = 2$$

74. Let A be a set of all 4 –digit natural numbers whose exactly one digit is 7. Then the probability that a randomly chosen element of A leaves remainder 2 when divided by 5 is

- 1) $\frac{97}{297}$ 2) $\frac{1}{5}$ 3) $\frac{2}{9}$ 4) $\frac{122}{297}$

Ans: 1

Sol

$$\boxed{7} \text{ --- --- } = 1 \times 9 \times 9 \times 9$$

$$\boxed{\boxed{7}} \text{ --- --- } = 8 \times 3C_1 \times 9 \times 9$$

$$n(s) = 9 \times 9 \times 9 + 24 \times 9 \times 9 = 81 \times (33) = 2673$$

$$7 \text{ --- --- } = 1 \times 9 \times 9 \times 1 = 81$$

$$\boxed{.} \text{ --- } 7 = 8 \times 9 \times 9 = 648$$

$$\boxed{\quad} \text{ --- } \text{ --- } = 8 \times 2C_1 \times 9 = 144$$

$$n(A) = 81 + 648 + 144 = 873$$

$$P(A) = \frac{873}{2673} = \frac{97}{297}$$

75. The shortest distance between the line $x - y = 1$ and the curve $x^2 = 2y$ is

- 1) $\frac{1}{2}$ 2) $\frac{1}{\sqrt{2}}$ 3) 0 4) $\frac{1}{2\sqrt{2}}$

Ans: 4

Sol Consider $x^2 = 2y$, Diff Wrt x, $2x = 2 \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = x = 1$

Point on the curve $P(1, \frac{1}{2})$

Eq of tangent to the curve at P is

$$y - \frac{1}{2} = 1(x - 1), \quad y = x - \frac{1}{2} \Rightarrow x - y = \frac{1}{2}$$

$$\text{S.D is } = \frac{1 - \frac{1}{2}}{\sqrt{1+1}} = \frac{1}{2\sqrt{2}}$$

76. Let A be a 3×3 matrix with $\det(A) = 4$ Let R_i denote the i th row of A. If a matrix B is obtained by performing the operation $R_2 \rightarrow 2R_2 + 5R_3$ on 2A, then $\det(B)$ is equal to

- 1) 64 2) 16 3) 128 4) 80

Ans: 1

Sol $(A)_{3 \times 3}; |A| = 4$

$$A = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$$

$$2A = \begin{pmatrix} 2a_1 & 2a_2 & 2a_3 \\ 2b_1 & 2b_2 & 2b_3 \\ 2c_1 & 2c_2 & 2c_3 \end{pmatrix}$$

$$R_2 \rightarrow 2R_2 + 5R_3$$

$$2A = \begin{pmatrix} 2a_1 & 2a_2 & 2a_3 \\ 4b_1 + 10c_1 & 4b_2 + 10c_2 & 4b_3 + 10c_3 \\ 2c_1 & 2c_2 & 2c_3 \end{pmatrix}$$

$$2A = \begin{pmatrix} 2a_1 & 2a_2 & 2a_3 \\ 4b_1 & 4b_2 & 4b_3 \\ 2c_1 & 2c_2 & 2c_3 \end{pmatrix} + \begin{pmatrix} 2a_1 & 2a_2 & 2a_3 \\ 10c_1 & 10c_2 & 10c_3 \\ 2c_1 & 2c_2 & 2c_3 \end{pmatrix}$$

$$|B| = |2A| = 16|A| \Rightarrow 16 \times 4 = 64$$

77. Let x denote the total number of one – one functions from a set A with 3 elements to a set B with 5 elements and y denotes the total number of one – one function from the set A to be set $A \times B$ then:

1) $y = 91x$ 2) $2y = 273x$ 3) $y = 273x$ 4) $2y = 91x$

Ans: 4

Sol x denotes total no of 1 – 1 function from A to B
A to B

$$n(A) = 3; n(B) = 5$$

$$x = {}^5P_3 = 5 \times 4 \times 3 = 60$$

Y denotes total no of 1 – 1 function from A to $A \times B$

$$n(A) = 3, n(A \times B) = 15$$

$$y = {}^{15}P_3 = 15 \times 14 \times 13 = 2730$$

$$\frac{Y}{X} = \frac{2730}{60} = \frac{91}{2}, 2Y = 91X$$

78. In a group of 400 people, 160 are smokers and non – vegetarian; 100 are smokers and vegetarian and the remaining 140 are non – smokers and vegetarian. Their chances of getting a particular chest disorder are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the chest disorder. The probability that the selected person is a smoker and non – vegetarian is:

1) $\frac{28}{45}$ 2) $\frac{7}{45}$ 3) $\frac{8}{45}$ 4) $\frac{14}{45}$

Ans: 1

Sol

$$\text{non veg +smokers} \rightarrow \frac{160}{400} \times \frac{35}{100}$$

$$\text{veg + smokers} \rightarrow \frac{100}{400} \times \frac{20}{100}$$

$$\text{Veg + non smokers} \rightarrow \frac{140}{400} \times \frac{10}{100}$$

$$\text{Req proba} = \frac{160 \times 35}{100 \times 20 + 140 \times 10 + 160 \times 35} = \frac{16 \times 35}{200 + 140 + 560}$$

$$\frac{560}{900} = \frac{28}{45}$$

79. $\operatorname{cosec} \left[2 \cot^{-1}(5) + \cos^{-1} \left(\frac{4}{5} \right) \right]$ is equal to

- 1) $\frac{56}{33}$ 2) $\frac{65}{56}$ 3) $\frac{75}{56}$ 4) $\frac{65}{33}$

Ans: 2

Sol $\operatorname{cosec} \left(2 \cot^{-1} 5 + \cos^{-1} \frac{4}{5} \right) = \operatorname{cosec}(2A + B)$

$$\sin(2A + B) = \sin 2A \cos B + \cos 2A \sin B$$

$$\sin 2A = 2 \sin A \cos A = \frac{2}{\sqrt{26}} \times \frac{5}{\sqrt{26}} = \frac{10}{26}$$

$$\cos 2A = 1 - 2 \sin^2 A = 1 - \frac{2}{26} = \frac{24}{26}$$

$$\sin(2A+B) = \frac{10}{26} \cdot \frac{4}{5} + \frac{24}{26} \cdot \frac{3}{5} = \frac{112}{130} = \frac{56}{65}$$

80. $\lim_{n \rightarrow \infty} \left[\frac{1}{n} + \frac{n}{(n+1)^2} + \frac{n}{(n+2)^2} + \dots + \frac{n}{(2n-1)^2} \right]$ is equal to

- 1) $\frac{1}{4}$ 2) $\frac{1}{2}$ 3) 1 4) $\frac{1}{3}$

Ans: 2

Sol $\operatorname{Lt}_{n \rightarrow \infty} \left(\frac{1}{n} + \frac{n}{(n+1)^2} + \frac{n}{(n+2)^2} + \dots + \frac{n}{(2n-1)^2} \right)$

$$\operatorname{Lt}_{n \rightarrow \infty} \frac{n}{n^2} \left\{ \sum_{r=1}^n \frac{1}{\left(1 + \frac{r}{n} \right)^2} \right\}$$

$$= \operatorname{Lt}_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n \frac{1}{\left(1 + \frac{r}{n} \right)^2} = \int_0^1 \frac{1}{(1+x)^2} dx = \left(\frac{-1}{1+x} \right)_0^1 = \frac{-1}{2} + 1 = \frac{1}{2}$$

(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10. Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

81. If the curves $x = y^4$ and $xy = k$ cut at right angles, then $(4k)^6$ is equal to

Ans: 4

$x = y^4, xy = k; \text{ Diff wrt } x$

$1 = 4y^3 \cdot \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{4y^3}; y = \frac{k}{x}$

$\frac{dy}{dx} = \frac{-k}{x^2}$ Two curves cut right angle

$\frac{1}{4y^3} \times \frac{-k}{x^2} = -1 \Rightarrow k = 4x^2y^3$

$\Rightarrow y^5 = 4x^2y^3$

$\Rightarrow y^2 = 4x^2 \Rightarrow y = \pm 2x$

$k = 4x^2 \times 8x^3 \Rightarrow 2x^2 = 4x^2 \times 8x^3 \Rightarrow x^3 = \frac{1}{16}$

$4^6 k^6 = 4^6 \times 4^6 \times 8^6 \times (x^3)^{10} = 4^6 \times 4^6 \times 8^6 = \frac{1}{16^{10}}$

$4^6 \times 4^6 \times 8^6 \times \frac{1}{4^{10} \times 4^{10}} = 4^4 \times 4^4 = \frac{4 \times 2}{4^4 \times 4^4} = \frac{64}{16} = 4$

$\int_{-2}^2 3x^2 - 3x - 6 dx$

82. The value of $\int_{-2}^2 3x^2 - 3x - 6 dx$ is.....

Ans: 19

Sol $\int_{-2}^2 |3x^2 - 3x - 6| dx = 3 \int_{-2}^2 |x^2 - x - 2| dx$

$= 3 \int_{-2}^2 |(x-2)(x+1)| dx = 3 \left\{ \int_{-2}^{-1} (x^2 - x - 2) dx + \int_{-1}^2 (2 + x - x^2) dx \right\} = 19$

83. If $\lim_{x \rightarrow 0} \frac{ax - (e^{4x} - 1)}{ax(e^{4x} - 1)}$ exists and is equal to b, then the value of $a - 2b$ is.....

Ans: 5

Sol: $\text{Lt}_{x \rightarrow 0} \frac{ax - (e^{4x} - 1)}{ax(e^{4x} - 1)} = b$

$\text{Lt}_{x \rightarrow 0} \frac{a - 4e^{4x}}{ax4e^{4x} + ae^{4x} - a} = b$

$a - 4 = 0$

$$\Rightarrow a = 4, b = -\frac{1}{2}$$

$$a - 2b = 5$$

84. If the remainder when x is divided by 4 is 3, then the remainder when $(2020+x)^{2022}$ is divided by 8 is _____

Ans: 1

$$x = 4k + 3, k \in \mathbb{w}, \text{ let } x = 7, (2020 + 7)^{2022} = (2027)^{2022} = (2024 + 3)^{2022}$$

$$= {}^{2022}C_0 \cdot (2024)^{2022} + \dots + {}^{2022}C_{2021} (2024)^1 3^{2021} + {}^{2022}C_{2022} \cdot 3^{2022}, \text{ divided 8 remainder}$$

$$3^{2022} = (1+8)^{1011} \cdot \text{remainder} = 1$$

85. Let $\vec{a} = \hat{i} + \alpha\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - \alpha\hat{j} + \hat{k}$. If the area of the parallelogram whose adjacent sides are represented by the vectors \vec{a} and \vec{b} is $8\sqrt{3}$ square units, then $\vec{a} \cdot \vec{b}$ is equal to

Ans: 2

Sol : $\vec{a} = \vec{i} + \alpha\vec{j} + 3\vec{k}$ $\vec{b} = 3\vec{i} - \alpha\vec{j} + \vec{k}$

$$|\vec{a} \times \vec{b}| = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & \alpha & 3 \\ 3 & -\alpha & 1 \end{vmatrix}$$

$$= \vec{i}(\alpha + 3\alpha) - \vec{j}(-8) + \vec{k}(-\alpha - 3\alpha)$$

$$= |4\alpha\vec{i} + 8\vec{j} - 4\alpha\vec{k}| = \sqrt{16\alpha^3 + 64 + 16\alpha^2} = 32\alpha^2 + 64 = 64 \times 3$$

$$\Rightarrow 32\alpha^2 = 192 - 64 = 128$$

$$\alpha^2 = \frac{128}{32} = 4$$

$$\alpha = \pm 2$$

$$\vec{a} = \vec{i} + 2\vec{j} + 3\vec{k}$$

$$\vec{b} = 3\vec{i} - 2\vec{j} + \vec{k}$$

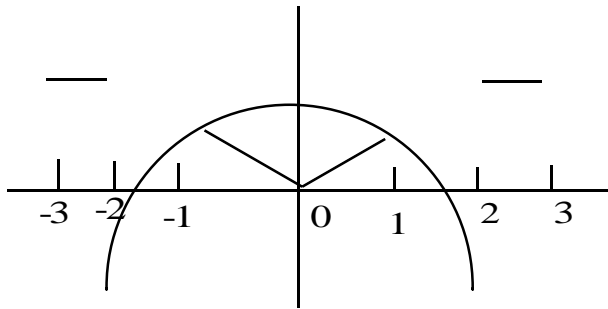
$$\vec{a} \cdot \vec{b} = 3 - 4 + 3 = 2$$

86. A function f is defined on $[-3, 3]$ as $f(x) = \begin{cases} \min\{|x|, 2 - x^2\}, & -2 \leq x \leq 2 \\ [|x|], & 2 < |x| \leq 3 \end{cases}$ where $[x]$ denotes

the greatest integer $\leq x$. The number of points, where f is not differentiable in $(-3, 3)$ is.....

Ans: 5

Sol: $f(x) = \min \begin{cases} |x|, 2 - x^2 & -2 \leq x \leq 2 \\ [|x|]; & 2 < |x| \leq 3 \end{cases}$



No of non diff points =5

87. A line is a common tangent to the circle $(x-3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$. If the two points of contact (a, b) and (c, d) are distinct and lie in the first quadrant then $2(a+c)$ is equal to

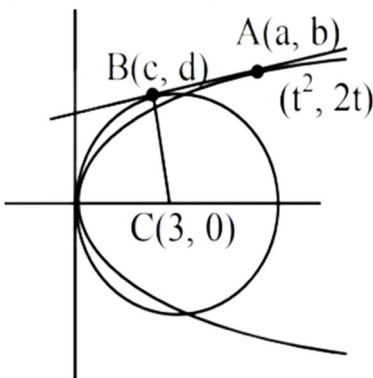
Ans: 9

Sol $(x-b)^2 + y^2 = 9$; $y^2 = 4x$

let $y = mx + \frac{1}{m}$ be a tangent to $y^2 = 4x$, $m^2x - my + 1 = 0$ is also tangent to circle

$C(3,0)$ $r=3$, $r=d$

$$3 = \frac{|3m^2 + 1|}{\sqrt{m^4 + m^2}} \Rightarrow 9(m^4 + m^2) = 9m^4 + 1 + 6m^2$$



$$\Rightarrow 9m^2 + 1 = 6m^2 + 1$$

$$m^2 = \frac{1}{3}$$

$$y = \frac{1}{\sqrt{3}}x + \sqrt{3} \quad (x-3)^2 + \left(\frac{x+3}{\sqrt{3}}\right)^2 = 9 \quad y = \frac{3}{2} \cdot \frac{1}{\sqrt{3}} + \sqrt{3}$$

$$\Rightarrow x - \sqrt{3}y + \sqrt{3} = 0$$

$$3(x-3)^2 + (x+3)^2 = 27 \quad y = \frac{\sqrt{3}}{2} + \sqrt{3} = 3 \frac{\sqrt{3}}{2}$$

$$\Rightarrow 3x^2 + 27 - 18x + x^2 + 6x + 9 = 27$$

$$\Rightarrow 4x^2 - 12x + 9 = 0$$

$$(2x - 3) = 0 \quad x = \frac{3}{2} \quad \text{and} \quad \left(\frac{x+3}{\sqrt{3}}\right)^2 = 4x$$

$$2(a+c) = 2\left(\frac{3}{2} + 3\right) = 9 \quad x = 3$$

88. If the curve $y = y(x)$ represented by the solution of the differential equation $(2xy^2 - y)dx + xdy = 0$, passes through the intersection of the lines $2x - 3y = 1$ and $3x + 2y = 8$ then $|y(1)|$ is equal to.....

Ans: 1

Sol: $(2xy^2 - y)dx + xdy = 0$

$$2xy^2 dx = ydx - xdy \quad 2x dx = \frac{ydx - xdy}{y^2}$$

$$x^2 = \frac{x}{y} + C, (2,1), 4 = 2 + C \Rightarrow C = 2, X^2 = \frac{X}{Y} + 2 \Rightarrow 1 = \frac{1}{Y} + 2 \Rightarrow Y = -1$$

$$\Rightarrow y = -1, |y| = 1$$

89. The total number of two digit numbers 'n', such that $3^n + 7^n$ is a multiple of 10, is.....

Ans: 45

Sol $3^n + 7^n$ is a multiple of 10

Since n is odd, $n = \{1, 3, 5, \dots, 99\}$

Number of values of n = 45

90. A line 'l' passing through origin is perpendicular to the lines

$$l_1: \vec{r} = (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\hat{k}$$

$$l_2: \vec{r} = (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}$$

If the coordinates of the point in the first octant on 'l' at a distance of $\sqrt{17}$ from the point of intersection of 'l' and 'l₁' are (a, b, c) then $18(a + b + c)$ is equal to.....

Ans: 44

Sol: $\vec{r} = (3\vec{i} - \vec{j} + 4\vec{k}) + t(\vec{i} + 2\vec{j} + 2\vec{k}) \quad \vec{r} = (3\vec{i} + 3\vec{j} + 2\vec{k}) + s(\vec{i} + 2\vec{j} + \vec{k})$

$$\vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}, \quad \vec{d} = 2\hat{i} + 2\hat{j} + \hat{k}$$

$$\vec{b} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{vmatrix} = \hat{i}(-2) - \hat{j}(-3) + \hat{k}(-2) = -2\hat{i} + 3\hat{j} - 2\hat{k}$$

$$\vec{r} = \lambda(-2\hat{i} + 3\hat{j} - 2\hat{k}), \quad 3 + t = -2\lambda, -1 + 2t = 3\lambda, 4 + 2t = -2\lambda$$

$$1 + t = 0 \Rightarrow t = -1, \lambda = -1 \quad \text{Point of l and } l_1 = (2, -3, 2)$$

$$17 = (2s+1)^2 + (6+2s)^2 + s^2, 17 = 4s^2 + 1 + 4s + 36 + 4s^2 + 24s + s^2, 17 = 9s^2 + 28s + 37$$

$$9s^2 + 28s + 20 = 0, 9s^2 + 18s + 10s + 20 = 0, 9s(s+2) + 10(s+2) = 0, s = -2, s = -\frac{10}{9}$$

$$(3-4, 3-4, 0) = (a, b, c) \quad \left(3 - \frac{20}{9}, 3 - \frac{20}{9}, 2 - \frac{10}{9}\right)$$

$$\text{Take } \left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right) = (a, b, c) \quad 18\left(\frac{7}{9} + \frac{7}{9} + \frac{8}{9}\right) = 44$$