

### SECTION – A

1. A mosquito is moving with a velocity  $\vec{v} = 0.5t^2 \hat{i} + 3t\hat{j} + 9\hat{k}$  m/s and accelerating in uniform conditions. What will be the direction of mosquito after 2s?

- (1)  $\tan^{-1}\left(\frac{2}{3}\right)$  from x – axis                      (2)  $\tan^{-1}\left(\frac{5}{2}\right)$  from x – axis  
 (3)  $\tan^{-1}\left(\frac{2}{3}\right)$  from y – axis                      (4)  $\tan^{-1}\left(\frac{5}{2}\right)$  from y – axis

**Sol. Bonus**

Given

$$\vec{v} = 0.5t^2\hat{i} + 3t\hat{j} + 9\hat{k}$$

$$\vec{v}_{t=2} = 2\hat{i} + 6\hat{j} + 9\hat{k}$$

∴ angle made by direction of motion of mosquito will be –

$$\cos^{-1}\left(\frac{2}{11}\right) \text{ from x-axis} = \tan^{-1}\left(\frac{\sqrt{117}}{2}\right)$$

$$\cos^{-1}\left(\frac{6}{11}\right) \text{ from y-axis} = \tan^{-1}\left(\frac{\sqrt{85}}{9}\right)$$

$$\cos^{-1}\left(\frac{9}{11}\right) \text{ from z-axis} = \tan^{-1}\frac{\sqrt{40}}{9}$$

no. option is matching

2. **Statement I:** A cyclist is moving on an unbanked road with a speed of  $7 \text{ kmh}^{-1}$  and takes a sharp circular turn along a path of radius of 2m without reducing the speed. The static friction coefficient is 0.2. The cyclist will not slip and pass the curve. ( $g = 9.8 \text{ m/s}^2$ )

**Statement-II:** If the road is banked at an angle of  $45^\circ$ , cyclist can cross the curve of 2m radius with the speed of  $18.5 \text{ kmh}^{-1}$  without slipping.

In the light of the above statements, choose the correct answer from the options given below.

- (1) Both statement I and statement II are false  
 (2) Both statement I and statement II are true  
 (3) Statement I is correct and statement II is incorrect  
 (4) Statement I is incorrect and statement II is correct

**Sol. (2)**

On a horizontal ground,

$$v_{\max} = \sqrt{\mu Rg} = \sqrt{0.2 \times 2 \times 9.8} = 1.97 \text{ m/s}$$

$$1.97 \times \frac{18}{5}$$

$$= 7.12 \text{ km/hr} = 7.2 \text{ km/hr}$$

Statement-2

$$v_{\max} = \sqrt{gr\left(\frac{\tan\theta + \mu}{1 - \mu \tan\theta}\right)} = \sqrt{2 \times 9.8 \times \frac{12}{0.8}} = 19.5 \text{ km/hr}$$

$$v_{\min} = \sqrt{rg\left(\frac{\tan\theta - \mu}{1 + \mu \tan\theta}\right)} = \sqrt{2 \times 9.8 \times \frac{0.8}{1.2}} = 12.01 \text{ km/hr}$$

3. Calculate the time interval between 33% decay and 67% decay if half-life of a substance is:  
 (1) 40 minutes      (2) 20 minutes      (3) 60 minutes      (4) 13 minutes

Sol. (2)

$$T_{1/2} = 20 \text{ min} \Rightarrow \frac{\ln 2}{\lambda} = 20 \text{ min}$$

$$\Rightarrow \lambda = \frac{\ln 2}{20(\text{min})}$$

$$\therefore N_t = N_0 e^{-\lambda t}$$

$$\frac{N_t}{N_0} = e^{-\lambda t_1} \Rightarrow 0.67 = e^{-\lambda t_1}$$

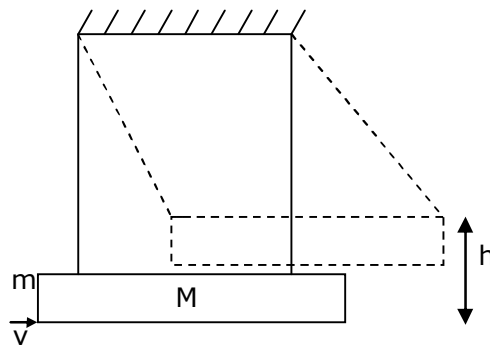
$$\Rightarrow \ln(0.67) = -\lambda t_1$$

$$\Rightarrow \ln\left(\frac{100}{67}\right) = \lambda t_1 \Rightarrow t_1 = \frac{\ln\left(\frac{100}{67}\right) \times 20(\text{min})}{(\ln 2)}$$

$$\text{Similarly, } t_2 = \frac{\ln\left(\frac{100}{34}\right) \times 20(\text{min})}{(\ln 2)}$$

$$t_2 - t_1 = 19.57 \text{ min} \approx 20 \text{ min.}$$

4. A large block of wood of mass  $M = 5.99 \text{ kg}$  is hanging from two long massless cords. A bullet of mass  $m = 10 \text{ g}$  is fired into the block and gets embedded in it. The (block + bullet) then swing upwards, their centre of mass rising a vertical distance  $h = 9.8 \text{ cm}$  before the (block + bullet) pendulum comes momentarily to rest at the end of its arc. The speed of the bullet just before collision is: (take  $g = 9.8 \text{ ms}^{-2}$ )



- (1) 831.4 m/s      (2) 841.4 m/s      (3) 811.4 m/s      (4) 821.4 m/s

Sol. (1)

$$P_i = 0.01 \times u + 0 = P_f = 6 \times v$$

$$v = \frac{0.01u}{6}$$

using energy conservation

$$\frac{1}{2} \times 6 \times \left(\frac{u}{600}\right)^2 = 6 \times 9.8 \times 9.8 \times 10^{-2}$$

$$u = 6 \times 98 \times \sqrt{2} = 588 \sqrt{2} \text{ m/s}$$

5. What will be the nature of flow of water from a circular tap, when its flow rate increased from 0.18 L/min to 0.48 L/min? The radius of the tap and viscosity of water are 0.5 cm and  $10^{-3}$  Pa s, respectively.

(Density of water:  $10^3$  kg/m<sup>3</sup>)

- (1) Remains steady flow (2) Unsteady to steady flow  
 (3) Steady flow to unsteady flow (4) Remains turbulent flow

Sol. (3)

The nature of flow is determined by Reynolds no.

$$R_e = \frac{\rho v D}{\eta}$$

if  $R_e < 1000 \rightarrow$  flow is steady

$1000 < R_e < 2000 \rightarrow$  flow becomes unsteady

$R_e > 2000 \rightarrow$  flow is turbulent

$$(R_e)_{\text{initial}} = 10^3 \times \frac{0.18 \times 10^{-3}}{\pi \times (0.5 \times 10^{-2})^2 \times 60} \times \frac{1 \times 10^{-2}}{10^{-3}} = 382.16$$

$$(R_e)_{\text{final}} = 10^3 \times \frac{0.48 \times 10^{-3}}{\pi \times (0.5 \times 10^{-2})^2 \times 60} \times \frac{1 \times 10^{-2}}{10^{-3}} = 1019.09$$

6. The refractive index of a converging lens is 1.4. What will be the focal length of this lens if it is placed in a medium of same refractive index? Assume the radii of curvature of the faces of lens are  $R_1$  and  $R_2$  respectively.

- (1) Zero (2)  $\frac{R_1 R_2}{R_1 - R_2}$  (3) Infinite (4) 1

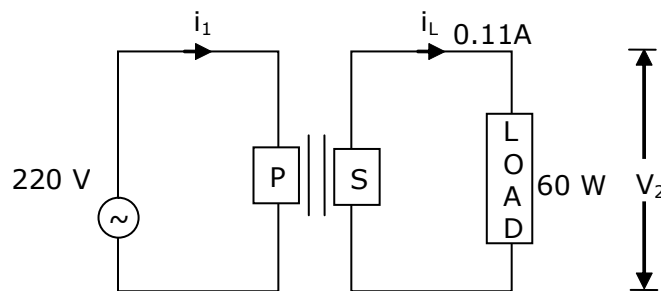
Sol. (3)

$$\frac{1}{f} = \left[ \frac{n_2}{n_1} - 1 \right] \left[ \frac{1}{R_1} - \frac{1}{R_2} \right]$$

$$\frac{1}{f} = 0$$

f = infinite.

7. For the given circuit, comment on the type of transformer used.



- (1) Step down transformer (2) Auxilliary transformer  
 (3) Step-up transformer (4) Auto transformer

Sol. (3)

$$P_s = V_s \times I_s$$

$$60 = V_s \times 0.11$$

$$V_s = \frac{60}{0.11} = \frac{6000}{11}, V_p = 220 \text{ volt}$$

$V_s > V_p$  so it is step up transformer

- 8.** Red light differs from blue light as they have:  
 (1) Same frequencies and same wavelengths  
 (2) Different frequencies and different wavelengths  
 (3) Different frequencies and same wavelengths  
 (4) Same frequencies and different wavelengths

**Sol. (2)**  
 Red light and blue light have different wavelength and different frequency.

- 9.** The de-Broglie wavelength associated with an electron and a proton were calculated by accelerating them through same potential of 100 V. What should nearly be the ratio of their wavelengths? ( $m_p = 1.00727u$   $m_e = 0.00055u$ )

- (1)  $(1860)^2 : 1$       (2) 43 : 1      (3) 1860 : 1      (4) 41.4 : 1

**Sol. (2)**  

$$\lambda_e = \frac{12.27}{\sqrt{V}} \text{ \AA}$$

$$\lambda_p = \frac{0.286}{\sqrt{V}} \text{ \AA}$$

$$\frac{\lambda_e}{\lambda_p} = \frac{12.27}{0.286} = 43$$

- 10.** A charge Q is moving  $\vec{dl}$  distance in the magnetic field  $\vec{B}$ . Find the value of work done by  $\vec{B}$ .

- (1) Infinite      (2) 1      (3) -1      (4) Zero

**Sol. (4)**  

$$\vec{F} = q\vec{V} \times \vec{B}$$

$$P = \vec{F} \cdot \vec{V} = 0 \quad \Rightarrow W = 0$$

- 11.** Amplitude of a mass spring system, which is executing simple harmonic motion decreases with time. If mass = 500g, Decay constant = 20 g/s then how much time is required for the amplitude of the system to drop to half of its initial value?

( $\ln 2 = 0.693$ )

- (1) 15.01 s      (2) 17.32 s      (3) 0.034 s      (4) 34.65 s

**Sol. (4)**  

$$A = A_0 e^{-\frac{bt}{2m}}$$

$$\frac{bt}{2m} = \ln 2 = 0.693$$

$$t = \frac{2m}{b} \times 0.693$$

$$t = 2 \times \frac{500}{20} \times 0.693$$

$$t = 50 \times 0.693 = 34.6 \text{ sec.}$$

- 12.** A resistor develops 500 J of thermal energy in 20s when a current of 1.5A is passed through it. If the current is increased from 1.5A to 3A, what will be the energy developed in 20s.

- (1) 500 J      (2) 1000 J      (3) 2000 J      (4) 1500 J

**Sol.** (3)

$$I^2RT = H$$

$$\frac{H_1}{H_2} = \left(\frac{I_1}{I_2}\right)^2$$

$$\Rightarrow H_2 = 500 \times \left(\frac{3}{3/2}\right)^2 = 2000 \text{ J}$$

**13.** Calculate the value of mean free path ( $\lambda$ ) for oxygen molecules at temperature  $27^\circ\text{C}$  and pressure  $1.01 \times 10^5 \text{ Pa}$ . Assume the molecular diameter  $0.3 \text{ nm}$  and the gas is ideal. ( $k = 1.38 \times 10^{-23} \text{ Jk}^{-1}$ )

(1) 102 nm

(2) 32 nm

(3) 58 nm

(4) 86 nm

**Sol.** (1)

Given :  $P = 1.01 \times 10^5 \text{ Pa}$

$d = 0.3 \text{ nm}$

$k = 1.38 \times 10^{-23} \text{ Jk}^{-1}$

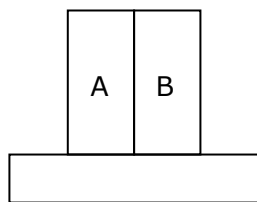
$T = 27^\circ\text{C}$

$$\Rightarrow \lambda_{\text{mean}} = \frac{1}{\sqrt{2}\pi d^2 \left[\frac{N}{V}\right]} \text{ and } PV = nKT$$

$$\therefore \lambda_{\text{mean}} = \frac{KT}{\sqrt{2}\pi d^2 p}$$

$$\Rightarrow \frac{1.38 \times 10^{-23} \times 300}{\sqrt{2} \times 3.14 \times 0.09 \times 10^{-18} \times 1.01 \times 10^5}$$
$$\Rightarrow 102 \text{ nm}$$

**14.** A bimetallic strip consists of metals A and B. It is mounted rigidly as shown. The metal A has higher coefficient of expansion compared to that of metal B. When the bimetallic strip is placed in a cold bath, it will:



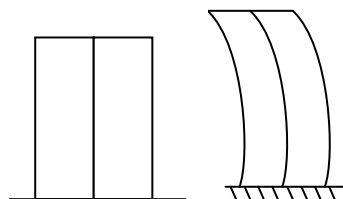
(1) Not bend but shrink

(3) Bend towards the right

(2) Neither bend nor shrink

(4) Bend towards the left

**Sol.** (4)

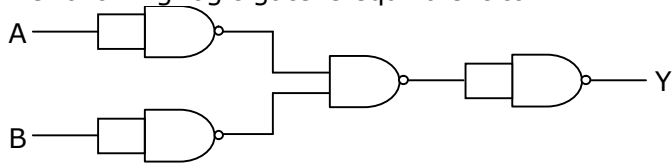


$$\alpha_A > \alpha_B$$

length of both strips will decrease

$$\Delta L_A > \Delta L_B$$

15. The following logic gate is equivalent to:



- (1) NOR Gate      (2) AND Gate      (3) OR Gate      (4) NAND Gate

Sol. (1)

$$Y = \overline{\overline{A \cdot B}} = \overline{A \cdot B}$$

So, given logic gates circuit is a NOR gate

16. Two identical antennas mounted on identical towers are separated from each other by a distance of 45 km. What should nearly be the minimum height of receiving antenna to receive the signals in line of sight ?

(Assume radius of earth is 6400 km)

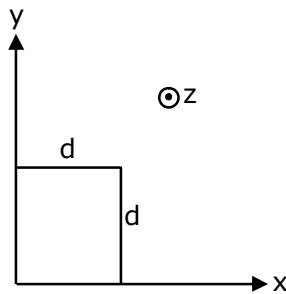
- (1) 19.77 m      (2) 79.1 m      (3) 158.2 m      (4) 39.55 m

Sol. (4)

$$D = 2\sqrt{2Rh}$$

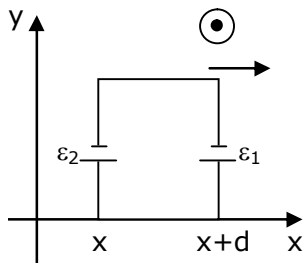
$$H = \frac{D^2}{8R} = \frac{45^2}{8 \times 6400} \text{ km} = 39.55 \text{ m}$$

17. The magnetic field in a region is given by  $\vec{B} = B_0 \left( \frac{x}{a} \right) \hat{k}$ . A square loop of side  $d$  is placed with its edges along the  $x$  and  $y$  axes. The loop is moved with a constant velocity  $\vec{v} = v_0 \hat{i}$ . The emf induced in the loop is :



- (1)  $\frac{B_0 v_0 d^2}{2a}$       (2)  $\frac{B_0 v_0 d^2}{a}$       (3)  $\frac{B_0 v_0 d}{2a}$       (4)  $\frac{B_0 v_0^2 d}{2a}$

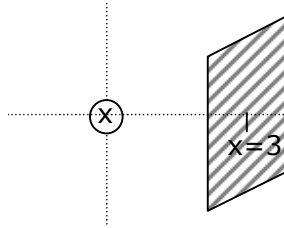
Sol. (2)



$$\epsilon_1 = \frac{B_0(x+d)}{a} v_0 d$$



**Sol.** (3)



Electric field due to wire is given by  $E = \frac{2k\lambda}{r}$

Electric field with surface charge density  $E' = \frac{\sigma}{\epsilon_0}$

$$\frac{2k\lambda}{r} = \frac{\sigma}{\epsilon_0}$$

$$2 \frac{1}{4\pi \epsilon_0} \frac{\lambda}{r} = \frac{\sigma}{\epsilon_0}$$

$$\frac{8 \times 10^{-9}}{2 \times 3.14 \times 3} = \sigma$$

$$\sigma = 0.424 \text{ n Cm}^{-2}$$

### SECTION - B

- 1.** A closed organ pipe of length  $L$  and an open organ pipe contain gases of densities  $\rho_1$  and  $\rho_2$  respectively. The compressibility of gases are equal in both the pipes. Both the pipes are vibrating in their first overtone with same frequency. The length of the open pipe is  $\frac{x}{3} L \sqrt{\frac{\rho_1}{\rho_2}}$ .

where  $x$  is \_\_\_\_\_ (Round off to the Nearest Integer)

**Sol.** **4**

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{1}{k\rho}} \quad (\text{compressibility } K = \frac{1}{\gamma})$$

$$\text{for closed pipe} = \frac{3}{4L} \sqrt{\frac{1}{k_1\rho_1}}$$

$$= \frac{3}{2L} \sqrt{\frac{1}{k_2\rho_2}}$$

$$= \frac{3}{4L_1} \sqrt{\frac{1}{k_1\rho_1}} = \frac{1}{L_2} \sqrt{\frac{1}{k_2\rho_2}} \quad (\text{for open pipe } \frac{2v}{2L})$$

$$\Rightarrow \frac{3}{4L_1} \sqrt{\frac{1}{\rho_1}} = \sqrt{\frac{1}{\rho_2}} \times \frac{1}{L_2}$$

$$\Rightarrow \frac{3}{4L} \sqrt{\frac{\rho_2}{\rho_1}} = \frac{1}{L_2}$$

$$\Rightarrow L_2 = \frac{4L}{3} \sqrt{\frac{\rho_1}{\rho_2}}$$

Now, comparing with given eq =  $\frac{x}{2} L \sqrt{\frac{\rho_1}{\rho_2}}$ , we get

$$x = 4$$



2. A deviation of  $2^\circ$  is produced in the yellow ray when prism of crown and flint glass are achromatically combined. Taking dispersive powers of crown and flint glass as 0.02 and 0.03 respectively and refractive index for yellow light for these glasses are 1.5 and 1.6 respectively. The refracting angles for crown glass prism will be \_\_\_\_\_  $^\circ$  (in degree).  
(Round off to the Nearest Integer)

**Sol. 12**

$$\delta_{\text{net}} = (\mu_1 - 1)A_1 - (\mu_2 - 1)A_2$$

$$2^\circ = (\mu_1 - 1)A_1 - (\mu_2 - 1)A_2 \quad \dots(1)$$

$$\text{and } \omega_1 (\mu_1 - 1)A_1 = \omega_2 (\mu_2 - 1)A_2 \quad \dots(2)$$

Substituting the values in equation (1) and (2), we get

$$2^\circ = 0.5A_1 - 0.6A_2 \quad \dots(3)$$

$$10A_1 = 18A_2 \quad \dots(4)$$

From equation (3) and (4)

$$2^\circ = \frac{A_1}{2} - \frac{A_1}{3}$$

$$A_1 = 12^\circ$$

3. If one wants to remove all the mass of the earth to infinity in order to break it up completely.

The amount of energy that needs to be supplied will be  $\frac{x}{5} \frac{GM^2}{R}$  where x is \_\_\_\_\_

(Round off to the Nearest Integer)

(M is the mass of earth, R is the radius of earth, G is the gravitational constant)

**Sol. 3**

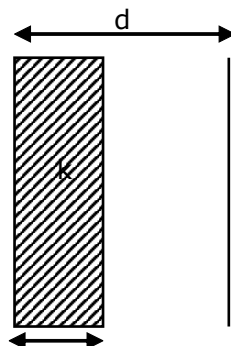
$$\text{Energy given} = U_f - U_i$$

$$E = U_f - U_i$$

$$E = 0 - \left( \frac{3}{5} \frac{GM_e^2}{R_e} \right) = \frac{3}{5} \frac{GM_e^2}{R_e}$$

$$\text{So, } x = 3$$

4. In a parallel plate capacitor set up, the plate area of capacitor is  $2 \text{ m}^2$  and the plates are separated by 1 m. If the space between the plates are filled with a dielectric material of thickness 0.5 m and area  $2 \text{ m}^2$  (see fig) the capacitance of the set-up will be \_\_\_\_\_  $\epsilon_0$ .  
(Dielectric constant of the material = 3.2) (Round off to the Nearest Integer)



**Sol. 3.04**

$$C_1 = \frac{K\epsilon_0 A}{d/2}; C_2 = \frac{\epsilon_0 A}{d/2}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{2K\epsilon_0 A} + \frac{d}{2\epsilon_0 A}$$

$$\frac{1}{C} = \frac{d}{2\epsilon_0 A} \left( \frac{K+1}{K} \right)$$

$$C = \frac{2\epsilon_0 AK}{d(K+1)} = \frac{2 \times 2 \times 3.2}{1 \times 4.2} \epsilon_0 = 3.04\epsilon_0$$

5. For an ideal heat engine, the temperature of the source is  $127^\circ\text{C}$ . In order to have 60% efficiency the temperature of the sink should be \_\_\_\_\_ $^\circ\text{C}$ . (Round off to the Nearest Integer)

**Sol. 113**

$$\eta = \left( 1 - \frac{T_2}{T_1} \right) = 0.6 = 1 - \frac{T_2}{400}$$

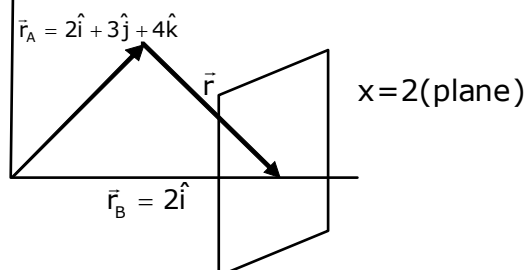
$$\frac{T_2}{400} = 0.4$$

$$T_2 = 160 \text{ K}$$

$$T_2 = 160 - 273$$

$$T_2 = -113^\circ\text{C}$$

6. A force  $\vec{F} = 4\hat{i} + 3\hat{j} + 4\hat{k}$  is applied on an intersection point of  $x = 2$  plane and  $x$ -axis. The magnitude of torque of this force about a point  $(2, 3, 4)$  is \_\_\_\_\_. (Round off to the Nearest Integer)

**Sol. 20**

given :  $\vec{F} = 4\hat{i} + 3\hat{j} + 4\hat{k}$

$$\vec{r}_A + \vec{r} = \vec{r}_B$$

$$\vec{r} = \vec{r}_B - \vec{r}_A$$

$$\vec{r} = (-3\hat{j} - 4\hat{k})$$

$$|\vec{\tau}| = |\vec{r} \times \vec{F}|$$

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -3 & -4 \\ 4 & 3 & 4 \end{vmatrix}$$

$$\vec{\tau} = -16\hat{j} - 12\hat{k}$$

$$|\vec{\tau}| = \sqrt{(-16)^2 + (-12)^2} = \sqrt{400} = 20$$

7. The energy dissipated by a resistor is 10 mJ in 1 s when an electric current of 2mA flows through it. The resistance is \_\_\_\_\_ $\Omega$ . (Round off to the Nearest Integer)

Sol. **2500**

$$H = i^2 R t$$

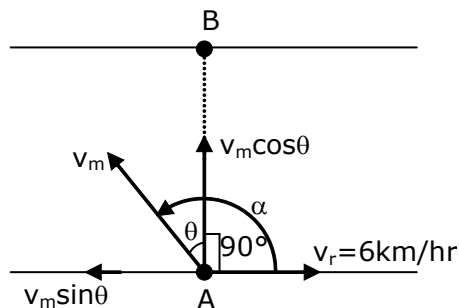
$$10 \times 10^{-3} = 4 \times 10^{-6} R$$

$$\therefore 10 \times \frac{10^3}{4} = R$$

$$\therefore R = 2500 \Omega$$

8. A swimmer can swim with velocity of 12 km/h in still water. Water flowing in a river has velocity 6 km/h. The direction with respect to the direction of flow of river water he should swim in order to reach the point on the other bank just opposite to his starting point is \_\_\_\_\_ $^\circ$ . (Round off to the Nearest Integer)  
(Find the angles in degrees)

Sol. **120**



$$v_m = 12 \text{ km/hr}$$

$$v_r = 6 \text{ km/hr}$$

$$v_m \sin \theta = v_r$$

$$\sin \theta = \frac{6}{12}$$

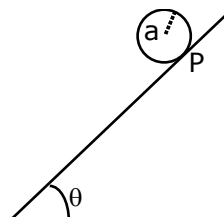
$$\theta = 30^\circ$$

$$\alpha = 90 + \theta$$

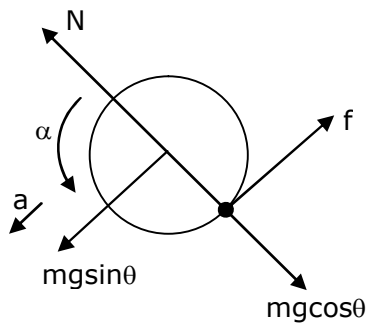
$$\alpha = 90 + 30$$

$$\alpha = 120^\circ$$

9. A solid disc of radius of 'a' and mass 'm' rolls down without slipping on an inclined plane making an angle  $\theta$  with the horizontal. The acceleration of the disc will be  $\frac{2}{b} g \sin \theta$  where b is \_\_\_\_\_  
(Round off to the Nearest Integer)  
(g = acceleration due to gravity  
 $\theta$  = angles as shown in figure)



**Sol. 3**



$$mg \sin \theta - f = ma_{\text{cm}} \dots\dots(1)$$

$$f \cdot a = \frac{ma^2}{2} \cdot \alpha \dots\dots(2)$$

$$a_{\text{cm}} = \alpha a \dots\dots(3)$$

on solving

$$a_{\text{cm}} = \frac{2}{3} g \sin \theta \Rightarrow b = 3$$

- 10.** A body of mass 2 kg moves under a force of  $(2\hat{i} + 3\hat{j} + 5\hat{k})$  N. It starts from rest and was at the origin initially. After 4s, its new coordinates are (8, b, 20). The value of b is \_\_\_\_\_.  
(Round off to the Nearest Integer)

**Sol. 12**

$$\vec{F} = (2\hat{i} + 3\hat{j} + 5\hat{k})\text{N}$$

time = 4 sec

As body start from rest therefore position vector initially

$$\vec{r}_i = (0\hat{i} + 0\hat{j} + 0\hat{k}) \text{ \& u (initial velocity) = 0}$$

$$\text{given, } r_f = (x\hat{i} + y\hat{j} + z\hat{k})$$

Now, from second equation of motion

$$\vec{s} = \vec{u}t + \frac{1}{2} \vec{a}t^2$$

$$r_f - r_i = \frac{1}{2} \times \left( \frac{2\hat{i} + 3\hat{j} + 5\hat{k}}{2} \right) \times (4)^2$$

$$(x\hat{i} + y\hat{j} + z\hat{k}) - (0\hat{i} + 0\hat{j} + 0\hat{k}) = 8\hat{i} + 12\hat{j} + 20\hat{k}$$

$$x\hat{i} + y\hat{j} + z\hat{k} = 8\hat{i} + 12\hat{j} + 20\hat{k}$$

$\therefore$  The value of b = 12