c. $\ln\left|\frac{(x-y)^2+2}{2}\right| = 2(x+y)$

d. $\ln\left|\frac{(x+y)^2+2}{2}\right| = 2(x-y)$

145. If $y=\sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)$, then $\frac{dy}{dx}$ is

a. 1 b. -1 c. $\frac{x}{\sqrt{x^2-1}}$ d. $\frac{-x}{\sqrt{1-x^2}}$

146. The maximum value of the function $y = x(x-1)^2$, is

a. 0 b. 4/27 c. -4 d. None of these

147. The solution of $x^3\frac{dy}{dx}+4x^2\tan y = e^x \sec y$ satisfying $y(1)=0$, is

a. $\tan y = (x-2)e^x \log x$ b. $\sin y = e^x(x-1)x^{-4}$

c. $\tan y = (x-1)e^x x^{-3}$ d. $\sin y = e^x(x-1)x^{-3}$

148. The runs of two players for 10 innings each are as follows

A	58	59	60	54	65	66	52	75	69	52
B	94	26	92	65	96	78	14	34	98	13

The more consistent player is

a. player *A*
b. player *B*
c. both player *A* and *B*
d. None of the above

149. The linear programming problem minimise $z=3x+2y$ subject to constraints $x+y\geq 8$, $3x+5y\leq 15$, $x\geq 0$ and $y\geq 0$, has

a. one solution b. no feasible solution
c. two solutions d. infinitely many solutions

150. Find the area enclosed by the loop in the curve $4y^2 = 4x^2 - x^3$.

a. 128/5 b. 15/128 c. 130/17 d. 17/130

Answers

Physics

1. (a)	2. (a)	3. (c)	4. (d)	5. (b)	6. (c)	7. (a)	8. (a)	9. (c)	10. (b)
11. (a)	12. (b)	13. (b)	14. (c)	15. (a)	16. (d)	17. (c)	18. (b)	19. (a)	20. (c)
21. (d)	22. (a)	23. (b)	24. (d)	25. (b)	26. (a)	27. (b)	28. (c)	29. (b)	30. (b)
31. (b)	32. (d)	33. (c)	34. (b)	35. (a)	36. (c)	37. (a)	38. (a)	39. (b)	40. (d)

Chemistry

41. (a)	42. (a)	43. (a)	44. (a)	45. (c,d)	46. (a)	47. (a)	48. (a)	49. (a)	50. (d)
51. (d)	52. (b)	53. (a)	54. (a)	55. (c)	56. (a)	57. (b)	58. (a)	59. (b)	60. (c)
61. (b)	62. (a)	63. (c)	64. (c)	65. (a)	66. (a)	67. (a)	68. (a)	69. (a)	70. (c)
71. (c)	72. (a)	73. (c)	74. (d)	75. (c)	76. (a)	77. (c)	78. (b)	79. (d)	80. (a)

English Proficiency

81. (c)	82. (a)	83. (a)	84. (d)	85. (c)	86. (c)	87. (b)	88. (c)	89. (c)	90. (c)
91. (c)	92. (b)	93. (c)	94. (d)	95. (a)					

Logical Reasoning

96. (c)	97. (c)	98. (b)	99. (d)	100. (c)	101. (b)	102. (a)	103. (b)	104. (d)	105. (c)

Mathematics

106. (a)	107. (a)	108. (c)	109. (a)	110. (d)	111. (b)	112. (c)	113. (c)	114. (c)	115. (b)
116. (c)	117. (b)	118. (a)	119. (a)	120. (b)	121. (a)	122. (b)	123. (d)	124. (b)	125. (b)
126. (d)	127. (d)	128. (d)	129. (c)	130. (b)	131. (b)	132. (c)	133. (d)	134. (b)	135. (c)
136. (d)	137. (c)	138. (c)	139. (c)	140. (c)	141. (c)	142. (a)	143. (c)	144. (b)	145. (d)
146. (b)	147. (b)	148. (a)	149. (b)	150. (a)					

Hints & Solutions

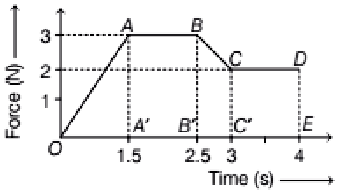
Physics

1. *(a)* Work done in the cyclic process

$= \text{Area of the loop } ABCD$

$= (2p - p) \times (2V - V) = pV$

2. *(a)* Impulse of a force = Area between the force-time graph



Hence, impulse of force = area of $\Delta OA'A$ + area of rectangle $A'B'BA$ + area of trapezium $BB'C'C$ + area of rectangle $CC'ED$

$= \frac{1}{2} \times 1.5 \times 3 + 1 \times 3 + \frac{1}{2}(3+2)(3-2.5) + 2 \times 1$

$= 2.25 + 3 + 1.25 + 2$

$= 8.50 \text{ Ns}$

3. *(c)* Bulk modulus $= \frac{\text{Hydraulic stress}}{\text{Volumetric strain}}$

$\Rightarrow \quad B = \frac{\text{Hydraulic stress}}{\frac{\Delta V}{V}}$

or $\quad \frac{\Delta V}{V} = \text{Hydraulic stress} \times \frac{1}{B}$

$\therefore$ For constant hydraulic stress, $\frac{\Delta V}{V} \propto \frac{1}{B}$

4. *(d)* A shunt is a low resistance which is connected in parallel with a galvanometer (or ammeter) to protect it from large current and to increase the range of ammeter.

5. *(b)* There is a phase difference of 180° between the signal voltage and output voltage in a common emitter amplifier. It is also known as phase reversal.

6. *(c)* $(n+1)$th divisions of vernier scale = n division of main scale

$\therefore \quad 1 \text{ VSD} = \frac{n}{n+1} \text{ MSD}$

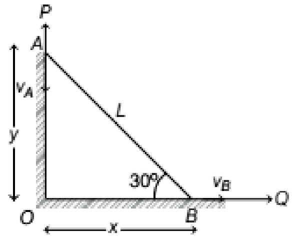
Least count = 1 MSD − 1 VSD

$= 1 \text{ MSD} - \frac{n}{n+1} \text{ MSD} = \frac{1}{n+1} \text{ MSD}$

$= \frac{1}{n+1} \times a \text{ units} = \frac{a}{n+1} \text{ units}$

[$\because$ given, 1MSD= a units]

7. *(a)* According to the question,



Let $OB = x$, $OA = y$ and $x = L\cos\theta$

So, by Pythagoras theorem,

$\Rightarrow \quad x^2 + y^2 = L^2$

Differentiating w.r.t. t, we get

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$2xv_B + 2yv_A = 0 \quad \left[\because \frac{dx}{dt} = v_B \text{ and } \frac{dy}{dt} = v_A\right]$$

$$\Rightarrow \quad v_A = -\frac{x}{y}v_B$$

$$\Rightarrow \quad |v_A| = \frac{x}{y}v_B = v_B\cot\theta \quad \left[\because \frac{x}{y} = \cot\theta\right]$$

$$= \sqrt{3} \times \cot 30°$$

$$= \sqrt{3} \times \frac{1}{\sqrt{3}} = 1 \text{ m/s}$$

8. *(a)* Let k be the force constant of spring.

Time period, $T = 2\pi\sqrt{\frac{M}{k}}$ ...(i)

When block of mass m is placed in the tray, the time period of oscillations becomes

$$T' = 2\pi\sqrt{\frac{M+m}{k}} \quad \text{...(ii)}$$

On dividing Eq. (ii) by Eq. (i), we get

$$\Rightarrow \quad \frac{T'}{T} = \frac{\sqrt{M+m}}{\sqrt{M}}$$

$$\Rightarrow \quad \frac{3}{1.5} = \sqrt{\frac{12+m}{12}} \quad \left[\begin{array}{r}\text{where, } M = 12 \text{ kg,} \\ T = 1.5 \text{ s} \\ \text{and } T' = 3 \text{ s,}\end{array}\right]$$

$$\Rightarrow \quad \sqrt{\frac{12+m}{12}} = 2$$

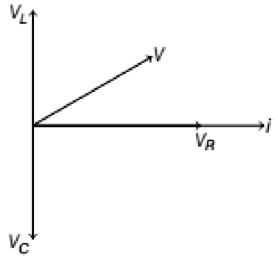
$$\Rightarrow \quad 12 + m = 48$$

$$\Rightarrow \quad m = 36 \text{ kg}$$

9. ***(c)*** As shown in phasor diagram, voltage leads the current.

As in purely inductive circuit, current lags behind the voltage by an angle of 90°, here angle is not 90° but it is lesser than that.

So, this type of case arise in *R-L-C* circuit when *R-L-C* load with inductive reactance X_L is more than the capacitive reactance X_C as shown below.



10. ***(b)*** Given,

$$Y = \sin\left[\pi\left(\frac{t}{5} - \frac{x}{9}\right) + \frac{\pi}{6}\right] \text{cm} \qquad \ldots\text{(i)}$$

The general equation of progressive wave is

$$Y = A\sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + \phi\right] \text{cm} \qquad \ldots\text{(ii)}$$

On comparing Eq. (i) with Eq. (ii), we get

$$A = 1 \text{ cm}, \; f = \frac{1}{T} = \frac{1}{10} = 0.1 \text{ Hz}$$

and $\quad \frac{2\pi}{\lambda} = \frac{\pi}{9}$

$\Rightarrow \quad \lambda = 18 \text{ cm}$

$v = f\lambda = 0.1 \times 18 = 1.8 \text{ m/s}$

11. ***(a)*** In fibre optic communication, signals are transmitted through an optical fibre. The property of light used in transmission through optical fibre cables is total internal reflection.

12. ***(b)*** Distance moved by man in 5 min with velocity of 45 m/s is

Distance = Speed × Time

$= 45 \text{ m/s} \times (5 \times 60) \text{ s}$

$= 13500 \text{ m}$

When he move back, it covers the same distance to come back to its original position.

Now, time taken = 1.5 min

$= 1.5 \times 60 \text{ s} = 90 \text{ s}$

Distance travelled = 13500 m

$\therefore \;$ Speed $= \frac{\text{Distance}}{\text{Time}} = \frac{13500}{90} = 150 \text{ m/s}$

13. ***(b)*** Potential at point *P* due to the solid sphere,

$$V_1 = -\frac{GM}{2R^3}\left[3R^2 - \left(\frac{R}{2}\right)^2\right]$$

$$= -\frac{11\,GM}{8R}$$

Potential at point *P* due to the cavity part,

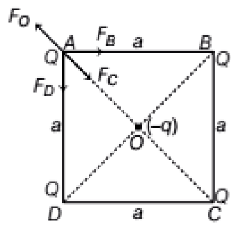
$$V_2 = -\frac{3}{2}\frac{G\left(\frac{M}{8}\right)}{\frac{R}{2}} = -\frac{3GM}{8R}$$

Potential due to the remaining part at point *P*,

$$V = V_1 - V_2 = \frac{-11GM}{8R} + \frac{3GM}{8R}$$

$$\Rightarrow \quad V = -\frac{GM}{R}$$

14. ***(c)*** Consider the equilibrium of charge *Q* at corner *A*



$$F_B = F_D = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{a^2} = \frac{kQ^2}{a^2} \qquad \left[\because \frac{1}{4\pi\varepsilon_0} = k\right]$$

$$F_C = \frac{kQ^2}{(\sqrt{2}a)^2} \qquad [\because AC = \sqrt{2}a]$$

$$= \frac{kQ^2}{2a^2}$$

$$F_O = \frac{k\,Qq}{\left(\frac{a}{\sqrt{2}}\right)^2} = \frac{k\,2Qq}{a^2} \qquad \left[\because OC = \frac{a}{\sqrt{2}}\right]$$

For equilibrium of charge *Q* at corner *A*, the net force on this charge along *AC* must be zero.

$\therefore \quad F_O = F_C + F_B\cos 45° + F_D \cos 45°$

$= F_C + 2F_B\cos 45° \qquad [\because F_B = F_D]$

$= F_C + 2F_B \times \frac{1}{\sqrt{2}} \qquad \left[\because \cos 45° = \frac{1}{\sqrt{2}}\right]$

$$\Rightarrow \quad \frac{k\,2Qq}{a^2} = \frac{kQ^2}{2a^2} + \frac{2 \times kQ^2}{a^2} \times \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad 2Qq = \frac{Q^2}{2} + \frac{Q^2}{\sqrt{2}} \times 2$$

$$\Rightarrow \quad 2q = \frac{Q}{2} + Q\sqrt{2}$$

$$\Rightarrow \quad q = \frac{Q}{4}(1 + 2\sqrt{2})$$

15. ***(a)*** Work done by the particle in *x*-direction is given by

$$dW = F\,dx = (a + bx)\,dx$$

Total work done in displacement from

$x = 0$ to $x = d$ will be

$$W = \int dW = \int_0^d (a + bx)\, dx = \left[ax + \frac{bx^2}{2}\right]_0^d$$

$$\Rightarrow \quad W = \left(a + \frac{bd}{2}\right)d$$

16. ***(d)*** Kinetic energy of a proton is given by

$$E = \frac{1}{2}m_p v_p^2$$

$$\Rightarrow \quad E = \frac{p^2}{2m_p} \qquad [\because p = mv]$$

$$\Rightarrow \quad p = \sqrt{2m_p E}$$

$\therefore$ Wavelength of proton is, $\lambda_1 = h/p$

$$\Rightarrow \quad \lambda_1 = \frac{h}{\sqrt{2m_p E}} \qquad \ldots(i)$$

Now, energy of photon is given by

$$E = h\nu_2 \Rightarrow E = \frac{hc}{\lambda_2}$$

$$\Rightarrow \quad \lambda_2 = \frac{hc}{E} \qquad \ldots(ii)$$

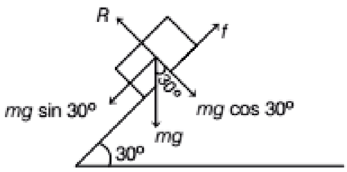
where, λ_2 is wavelength of photon.

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{\lambda_2}{\lambda_1} = \frac{hc}{E} \times \frac{\sqrt{2m_p E}}{h}$$

$$\Rightarrow \quad \frac{\lambda_2}{\lambda_1} = \frac{c}{\sqrt{E}}\sqrt{2m_p} \Rightarrow \frac{\lambda_2}{\lambda_1} \propto E^{-1/2}$$

17. ***(c)*** The FBD of block of mass 10 kg is



For the equilibrium of block, the frictional force must balanced the sine component of mg.

$$\therefore \quad f = mg \sin 30°$$

$$= 10 \times 9.8 \times \frac{1}{2} = 49 \text{ N}$$

18. ***(b)*** Conductivity (σ) of a metallic conductor is given by

$$\sigma = \frac{1}{\rho} = \frac{ne^2\tau}{m}$$

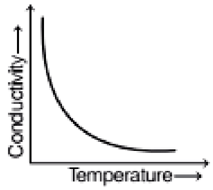
where, ρ = resistivity of conductor,

τ = relaxation time,

n = number of free electrons

and m = mass.

For metals, as the temperature increases, the relaxation time τ decreases because collisions become more frequent. Hence, the conductivity of a metallic conductor decreases with the increases of temperature.



19. ***(a)*** Given, radius of hydrogen atom, $r = 2.5 \times 10^{-13}$ m

Volume of one atom, $V = \frac{4}{3}\pi r^3$

Number of atoms in 1 mole $= 6.023 \times 10^{23}$

Volume of 1 mole of H-atoms $= N \times \frac{4}{3}\pi r^3$

$= 6.023 \times 10^{23} \times \frac{4}{3} \times 3.14 \times (2.5 \times 10^{-13})^3$

$= 3.94 \times 10^{-14}$ m^3

20. ***(c)*** Given, $m = 1$ kg, $\mathbf{r} = 3t\,\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$

Velocity of body, $\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{d}{dt}(3t\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$

$$\Rightarrow \quad \mathbf{v} = 3\hat{\mathbf{i}}$$

Angular momentum, $\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$

$$= 1 \times [(3t\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) \times 3\hat{\mathbf{i}}]$$

$$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3t & 4 & 0 \\ 3 & 0 & 0 \end{vmatrix}$$

$$= [3t \times 0 - 4 \times 3]\hat{\mathbf{k}}$$

$$\Rightarrow \quad \mathbf{L} = -12\hat{\mathbf{k}} \text{ J-s}$$

21. ***(d)*** The value of g at equator is expressed as

$$g_e = g - R\omega^2 \cos\phi = g - R\omega^2 \qquad [\because \cos\phi = 1]$$

Change in value of g is $g - g_e$

$$= R\omega^2 = R\left(\frac{2\pi}{T}\right)^2$$

$$= 6.4 \times 10^6 \times \left[\frac{2\pi}{24 \times 60 \times 60}\right]^2$$

$$= 3.37 \times 10^{-2} \text{ m/s}^2 \simeq 3.4 \text{ cm/s}^2$$

22. ***(a)*** Apparent frequency of car A = Apparent frequency of car B

$$\Rightarrow \frac{v}{v - v_s} \times \nu = \frac{v}{v - v_s'} \times \nu'$$

$$\Rightarrow \quad \frac{\nu}{v - 15} = \frac{504}{v - 30}$$

$\Rightarrow \quad v = \frac{340 - 15}{340 - 30} \times 504 \left[\because \text{velocity of sound in air is } 340 \text{ m/s.}\right]$

$\Rightarrow \quad v = 529.2 \text{ Hz}$

23. ***(b)*** Using KCL, $I_{DC} = 3 - 1 = \text{ZA}$

Using Kirchhoff's loop law, in loop $BDCR_1\ B$, we get

$$2 \times 2 + 3R_1 = 4 \quad \text{or} \quad R_1 = 0$$

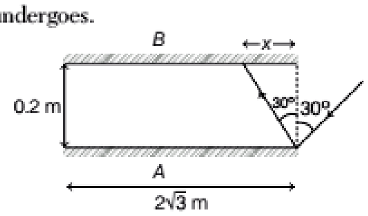
$\therefore$ Potential drop at B = Potential drop at C

$\Rightarrow \quad V_B = V_C = 2 + V_A$

$= 2 + 0 \qquad [\because V_A = 0]$

$\Rightarrow \quad V_B = 2\text{V}$

24. ***(d)*** Let N be the maximum number of reflection that ray undergoes.



Suppose it covers x distance in one reflection, then

$$Nx = 2\sqrt{3}$$

$\Rightarrow \quad N = \frac{2\sqrt{3}}{x} = \frac{2\sqrt{3}}{0.2 \tan 30°} \quad [\because x = 0.2 \tan 30°]$

$$N = \frac{6}{0.2} = 30$$

Number of reflections ray undergoes before it emerges excluding the first one are $N - 1$

$$= 30 - 1 = 29$$

25. ***(b)*** According to equation of motion, in vector from,

$$\mathbf{v} = \mathbf{u} - gt\ \hat{\mathbf{j}}$$

$\Rightarrow \quad \mathbf{v} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - 10t\ \hat{\mathbf{j}}$

Integrating w.r.t. t both sides, we get

$$\mathbf{r} = \int \mathbf{v}\ dt = \int (\hat{\mathbf{i}} + \hat{\mathbf{j}} - 10t\hat{\mathbf{j}})\ dt$$

or $\quad x\hat{\mathbf{i}} + y\hat{\mathbf{j}} = t\hat{\mathbf{i}} + t\hat{\mathbf{j}} - 5t^2\hat{\mathbf{j}} = t\hat{\mathbf{i}} + (t - 5t^2)\hat{\mathbf{j}}$

$\therefore$ On comparing, we get

$$x = t$$

and $\quad y = t - 5t^2$

$\Rightarrow \quad y = x - 5x^2$

This is the required equation of trajectory of particle.

26. ***(a)*** Volume of bigger drop = Volume of two smaller drops

$\Rightarrow \quad \frac{4}{3}\pi R'^3 = 2 \times \frac{4}{3}\pi R^3$

where, R' is the radius of bigger drop.

$\Rightarrow \quad R'^3 = 2R^3$

$\Rightarrow \quad R' = 2^{1/3} R$

Initial surface energy of two small drops,

$$U_1 = 8\pi R^2\sigma \qquad \ldots\text{(i)}$$

Final surface energy of big drop, $U_2 = 4\pi R'^2\sigma$

$$= 4\pi \times 2^{2/3} R^2\sigma \qquad \ldots\text{(ii)}$$

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{U_2}{U_1} = \frac{4\pi \times 2^{2/3} R^2\sigma}{8\pi R^2\sigma} = \frac{2^{2/3}}{2} = \frac{2^{-1/3}}{1}$$

Ratio of surface energy of bigger drop to smaller one is $2^{-1/3} : 1$.

27. ***(b)*** Force on charge particle in magnetic field is balanced by centripetal force, so $\frac{mv^2}{r} = qvB\sin 90°$

[$\theta = 90°$, therefore magnetic field is perpendicular]

$$\frac{mv^2}{r} = qvB \qquad [\sin 90° = 1]$$

$\Rightarrow \quad r = \frac{mv}{qB}$

For both particles, q and B are same.

Therefore, $\quad r \propto mv$

or $\quad \frac{r_A}{r_B} = \frac{m_A v_A}{m_B v_B}$

As, $\quad r_A > r_B \qquad$ [given]

$\Rightarrow \quad m_A v_A > m_B v_B$

28. ***(c)*** Minimum wavelength for X- ray is given by

$$\lambda_{\min} = \frac{12375}{V} \text{Å}$$

As the accelerating voltage V is decreased, $\lambda_{\min}$ increases.

The intensity of emitted radiation is determined by the number of electrons bombarding the target.

$\therefore$ Accelerating voltage does not change the intensity of X-rays emitted.

29. ***(b)*** As we know, $C_V = \frac{n}{2}R \qquad \ldots\text{(i)}$

$$C_p = \left(\frac{n}{2} + 1\right)R \qquad \ldots\text{(ii)}$$

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{C_p}{C_V} = \gamma = \frac{\left(\frac{n}{2} + 1\right)}{\frac{n}{2}} = 1 + \frac{2}{n}$$

$\Rightarrow \quad \frac{C_p}{C_V} = \gamma = 1 + \frac{2}{n}$

30. ***(b)*** Given, $v = 6$ m/s, $r = 18$ m, $g = 9.8$ m/s^2

Now, the velocity of cyclist is given by

$$v^2 = rg\tan\theta \quad \text{or} \quad \tan\theta = \frac{v^2}{rg} = \frac{6 \times 6}{18 \times 9.8}$$

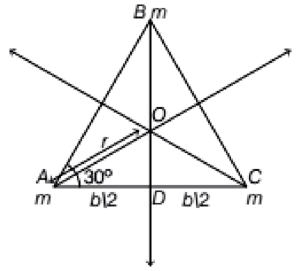
$\Rightarrow \quad \tan\theta = 0.2041$

The minimum value of coefficient of friction,

$$\mu = \tan\theta$$

$\Rightarrow \quad \mu = 0.2041$

31. ***(b)*** Let us consider an equilateral ΔABC of side b.



From figure,

In ΔAOD, $\cos 30° = \dfrac{AD}{AO}$

$\Rightarrow \quad \dfrac{\sqrt{3}}{2} = \dfrac{\frac{b}{2}}{r} \quad \left[\because \cos 30° = \dfrac{\sqrt{3}}{2}\right]$

$\Rightarrow \quad r = \dfrac{b}{2} \times \dfrac{2}{\sqrt{3}} = \dfrac{b}{\sqrt{3}}$

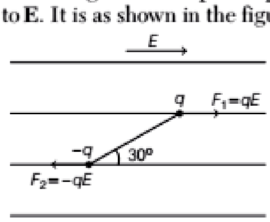
Moment of inertia about line passing through centroid,

$I = 3 \times mr^2$ [due to 3 masses present in system]

$\Rightarrow \quad = 3 \times m \times \left(\dfrac{b}{\sqrt{3}}\right)^2$

$\Rightarrow \quad I = mb^2$

32. ***(d)*** When a dipole is placed in a non-uniform electric field, opposite charges of the dipole experiences a force due to **E**. It is as shown in the figure below



These forces will act in opposite direction. Now, since the electric field is not uniform, so the force experienced by the charges will be unequal.

Hence, their will be a net force acting on the dipole, as they do not cancel each other.

Also, forces on the charges are not linear as shown above. So, they will experience a not non-zero torque. Hence, the dipole will experience both translational force and torque in a non-uniform electric field.

33. ***(c)*** Induced emf, $\varepsilon = -M\dfrac{di}{dt}$

where, i is given as, $i = i_0 \sin\omega t$

$\Rightarrow \quad \varepsilon = -M\dfrac{d}{dt}[i_0 \sin\omega t]$

$$= M\, i_0\, \omega \cos\omega t$$

For maximum value of emf in second coil, $\cos\omega t$ is maximum, i.e. 1.

$E_{max} = M\, i_0\, \omega \quad [\because \cos\omega t = 1]$

$\Rightarrow \quad = 0.001 \times 10 \times 10\pi$

$\Rightarrow \quad E_{max} = 0.1\,\pi V$

Hence, maximum emf value induced in second coil is 0.1 πV.

34. ***(b)*** Diode D_1 does not conduct because it is connected in reversed biased position.

Hence, only D_2 conducts as it is forward biased.

$\therefore$ Current in circuit is given by

$$i = \frac{V}{R}$$

$$= \frac{5}{30+20} = \frac{5}{50}\text{ A}$$

$\Rightarrow \quad i = 0.1$ A

35. ***(a)*** Kinetic energy at Q = 90% of potential energy at P

$\Rightarrow \quad \dfrac{1}{2}mv^2 = \dfrac{90}{100} \times mgh$

$\Rightarrow \quad v = \sqrt{1.8gh} \quad \Rightarrow \quad v = \sqrt{1.8 \times 10 \times 2}$

$\Rightarrow \quad v = 6$ m/s

36. ***(c)*** Let displacement, $x = A\sin\omega t$

Velocity, $v = \dfrac{dx}{dt} = A\omega\cos\omega t$

Kinetic energy of particle in SHM,

$K = \dfrac{1}{2}mv^2 \quad \Rightarrow \quad K = \dfrac{1}{2}m\omega^2A^2\cos^2\omega t$

$= \dfrac{1}{2}m\omega^2A^2\left(\dfrac{1+\cos 2\omega t}{2}\right) \quad \left[\because \cos^2\theta = \dfrac{1+\cos 2\theta}{2}\right]$

$= \dfrac{1}{4}m\,\omega^2A^2(1+\cos 2\omega t)$

$\therefore$ New angular frequency, $\omega_K = 2\omega$

$\therefore$ Frequency of oscillation of kinetic energy

$= 2f \quad \left[\because f = \dfrac{\omega}{2\pi}\right]$

37. ***(a)*** According to law of radioactivity, the rate of disintegration is given as

$$R = \lambda N = \lambda N_0\, e^{-\lambda t}$$

Given, $R = 5000$

$\Rightarrow \quad 5000 = \lambda N_0 e^{-\lambda t} \quad \ldots(i)$

After 5 min, $R = 1250$

$\therefore \quad 1250 = \lambda N_0 e^{-\lambda(t+5)} \quad \ldots(ii)$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{5000}{1250} = \frac{e^{-\lambda t}}{e^{-\lambda(t+5)}} \Rightarrow 4 = e^{5\lambda}$$

Taking log both sides, we get

$$\log_e 4 = \log_e(e^{5\lambda})$$

$\Rightarrow$ $\log_e 4 = 5\lambda$

$\Rightarrow$ $\lambda = \frac{1}{5}\log_e 2^2$

$\Rightarrow$ $\lambda = 0.4 \log_e 2$

38. ***(a)*** In a uniform electric field, potential difference across two parallel plates of capacitor, $V = Ed$

where, E is electric field and d is separation between plates.

$\Rightarrow$ $V = 2000 \times (1 \times 10^{-3}) = 2\text{V}$

39. ***(b)*** When light travel in meta-material, physical characteristics remain unchanged.

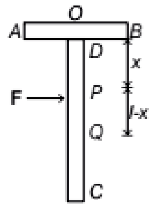
$\therefore$ $v = \frac{c}{|n|}$

where, $|n|$ is the relative refractive index of meta-material.

As, $|n| = \frac{c}{v}$

$\Rightarrow$ $|n| = \frac{\lambda_{air}}{\lambda_{med}} \Rightarrow \lambda_{med} = \frac{\lambda_{air}}{|n|}$

40. ***(d)*** Let O be the centre of mass of part AB and Q that of CD. Let M be the mass per unit length of the parts AB and CD.



As, no rotation is set up about point P, so

moment of part AB about P = moment of part CD about P

or $(ml)\,x = (2\,ml)(l - x)$ $\left[\because m = \frac{M}{l}\right]$

or $x = 2l - 2x$

or $x = \frac{2l}{3}$

Distance of P from the end

$$C = 2l - x = 2l - 2l/3 = 4l/3$$

Chemistry

41. ***(a)*** *o*-nitrophenol forms intramolecular H-bonding whereas molecules of *p*-nitrophenol get associated through intermolecular H-bond.
$\therefore$ *o*-nitrophenol is the most volatile compound.

42. ***(a)*** The magnetic moment is given by

$$\mu = \sqrt{n(n+2)}\ \text{BM}.$$

(where, n = number of unpaired electron).

Electronic configuration of

$$Ti^{2+}(22): 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^2$$

$\because$ Number of unpaired electrons = 2

$\therefore$ $\mu = \sqrt{2(2+2)} = \sqrt{8}$

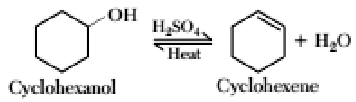
43. ***(a)*** Only Xe can form compounds with fluorine among noble gases because, Xe is large in size and have high atomic mass. Due to having larger atomic radius the force of attraction between the outer electrons and protons in the nucleous is weaker. Hence, they are easily available to form compound.

44. ***(a)*** Helium is used as a cryogenic agent due to its very low boiling point.

45. ***(c,d)*** The rate of reaction for the given reaction is

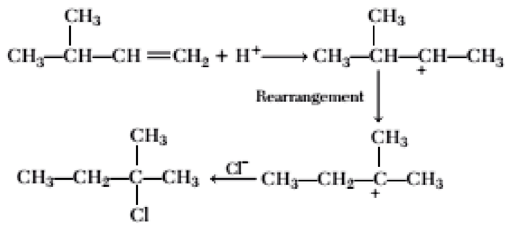
$$r = -\frac{1}{2}\frac{d[A]}{dt} = -\frac{1}{3}\frac{d[B]}{dt} = \frac{1}{3}\frac{d[C]}{dt} = \frac{1}{4}\frac{d[D]}{dt}$$

46. ***(a)*** Cyclohexanol on reaction with H_2SO_4 and heating gives cyclohexene as follows



47. ***(a)*** Addition of HCl to 3-methylbutene gives 2-chloro-2-methylbutane as major product.

The mechanism reaction is as follows



48. ***(a)*** Energy for atom, $E \propto \frac{Z^2}{n^2}$

For hydrogen, Z = 1, n = 1

For lithium Z = 3, n = ?

$\therefore$ For same energy,

$$E_H = E_{Li} \Rightarrow \frac{Z_H^2}{n_H^2} = \frac{Z_{Li}^2}{n_{Li}^2}$$

$$\therefore \quad n_{Li}^2 = \frac{Z_{Li}^2 \times n_H^2}{Z_H^2} \Rightarrow n_{Li}^2 = \frac{3^2 \times 1}{1}$$

$$n_{Li} = 3$$

While angular momentum $(nh/2\pi)$ is independent of atomic number.

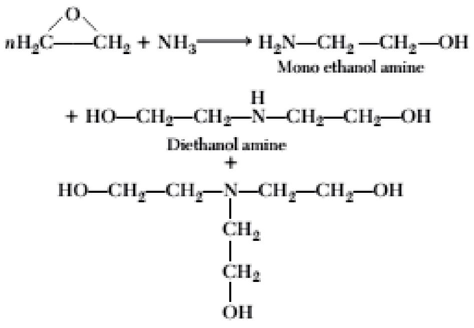
$\therefore$ For $3s$-orbital of lithium atom, energy and angular momentum is equal to that of $1s$ orbital of hydrogen atom.

49. ***(a)*** Sodium iodide reacts with ammonia to produce tetraamminesodium iodide. The reaction is conducted in liquid ammonia. The reaction is as follows

$$\underset{\text{Sodium iodide}}{NaI} + \underset{\text{Ammonia}}{4NH_3} \longrightarrow \underset{\text{Tetraammine-sodium iodide}}{[Na(NH_3)_4]\,I}$$

50. ***(d)*** Sodium or ammonium thiosulphate is used as a fixing agent in photography. They converts the silver halide into soluble, complex silver salts that dissolve in the fixer. During this process the film loses its original silver halide milkiness overlaying the image and becomes clear.

51. ***(d)*** Epoxy ethane reacts with ammonia forming a mixture of mono, di, tri-ethanol amines, in presence of small amount of water as follows.



52. ***(b)*** Dialysis is a process of removing excess electrolyte from colloidal solution by means of diffusion through a suitable membrane.

53. ***(a)*** Maltase converts maltose into glucose by hydrolysis as follows

$$\text{Maltose} \xrightarrow[\text{Maltase}]{\text{Hydrolysis}} \text{glucose + glucose}$$

54. ***(a)*** Starch is a polymer of glucose in which glucopyranose units are bonded by α-linkage. It is made up mixture of amylose and amylopectin.

55. ***(c)*** Methyl alcohol can be distinguished from ethyl alcohol by iodoform test as follows by using sodium hydroxide and iodine.

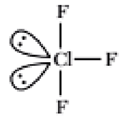
$$C_2H_5OH \xrightarrow[\text{Iodoform test}]{NaOH/I_2} CHI_3 \text{ (yellow ppt.)}$$

$$CH_3OH \xrightarrow{NaOH/I_2} \text{No ppt.}$$

56. ***(a)*** Sodium stearate $(C_{17}H_{35}COO^-Na^+)$ is an anionic soap. Anionic part of sodium stearate is involved in cleansing action.

57. ***(b)*** BaF_2 is not an anti-fluorite structure. It is of AB_2 type i.e. fluorite (CaF_2) structure. While Rb_2S, K_2O and Li_2O are anti-fluorite structures.

58. ***(a)*** The shape of ClF_3 is as follows



T-shape structure

It has two lone pairs at equatorial position.

59. ***(b)*** Molecular electronic configuration for given species are

(a) $O_2 = \sigma\, 1s^2\, \sigma^*1s^2\, \sigma 2s^2\, \sigma^*2s^2\, \sigma 2p_x^2\, \pi\, 2p_y^2\, \pi 2p_z^2$
$\pi^* 2p_y^1\, \pi^* 2p_z^1$

It has two unpaired electrons.

(b) O_2^{2-} : It has no unpaired electron.

(c) N_2^{2+} : It has one unpaired electron.

(d) B_2 : It has two unpaired electrons.

$\therefore$ O_2^{2-} is diamagnetic in nature.

60. ***(c)*** Bond order for the various compounds are as follows

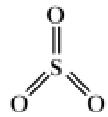
For SO_2,

Number of bonds = 4

Number of (SO) groups = 2

$\therefore$ Bond order $= \frac{4}{2} = 2$

For SO_3,

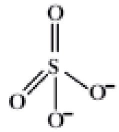


Number of bonds = 6

Number of (SO) groups = 3

$\therefore$ Bond order $= \frac{6}{3} = 2$

For SO_4^{2-},

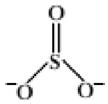


Number of bonds = 6

Number of (SO) groups = 4

$\therefore$ Bond order $= \frac{6}{4} = 1.5$

For SO_3^{2-},



Number of bonds = 4

Number of (SO) groups = 3

$\therefore$ Bond order $= \frac{4}{3} = 1.33$

Hence, correct order is $SO_2 = SO_3 > SO_4^{2-} > SO_3^{2-}$.

61. ***(b)*** The energy required for ionisation H-atom is given by, $E = \frac{-kZe^2}{2r}$

where, Z = atomic number

e = charge on proton/electron

r = radius of that electron

$\therefore \quad E \propto \frac{1}{r} \quad$ or $\quad E \propto r^{-1}$

$\therefore \quad n = -1$

62. ***(a)*** BHA is antioxidant (i.e. prevent oxidation). It help in food preservation by retarding the action of oxygen on food. As it is more reactive towards oxygen than the food material.

63. ***(c)*** In MnO_4^-,

$$x + (-2)4 = -1 \quad \Rightarrow \quad x = +7$$

$_{25}Mn^{+7}$ = [Ar], no unpaired electrons.

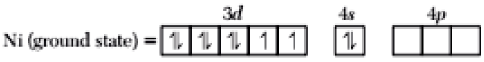
Thus, it will not show *d-d* transition. It is dark purple coloured due to charge transfer from ligand to metal.

64. ***(c)*** Rutherford model could not explain, the

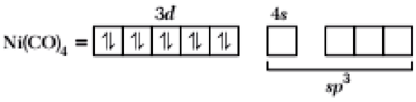
(a) electronic structure of an atom.

(b) stability of an atom.

65. ***(a)*** The oxidation state of Ni in $[Ni(CO)_4]$ is 0. The electronic configuration of Ni is $3d^8\ 4s^2$.



As CO is a strong ligand, pairing of electrons occur.



As, it has sp^3 hybridisation, so the geometry is tetrahedral. Since, it has no unpaired electrons. So, the complex is diamagnetic.

66. ***(a)*** The SI unit of Boltzmann's constant is JK^{-1}.

67. ***(a)*** Acetone does not undergo substitution reaction because it does not contain suitable leaving group.

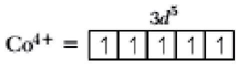
68. ***(a)*** Oxidation number of Co in $[CoF_6]^{2-}$

$$x + (-1 \times 6) = -2$$

$\therefore \quad x = 4$

$\therefore$ Electronic configuration of Co^{4+},

$$Co^{4+} = [Ar]\,3d^5$$



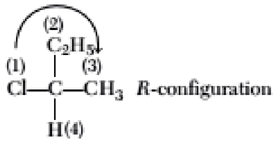
$\therefore$ Number of unpaired electrons = 5

$\therefore$ Magnetic moment $= \sqrt{n(n+2)} = \sqrt{5 \times 7} = \sqrt{35}$ BM

69. ***(a)*** Heavy water is used as a moderator in nuclear reactors. It has higher boiling point compare to ordinary water, thus it is more associated as compared to ordinary water. Dielectric constant of $H_2O > D_2O$. Therefore H_2O is more effective solvent.

70. ***(c)*** According to sequence rule, the priority order is

$\underset{(1)}{Cl} > \underset{(2)}{C_2H_5} > \underset{(3)}{CH_3} > \underset{(4)}{H}$, so in *R* configuration.



71. ***(c)*** Molecular mass

$$= \frac{\text{Weight of organic substance taken}}{\text{Air displaced at STP}} \times 22400$$

$$= \frac{0.2}{56} \times 22400 = 80$$

72. ***(a)*** $C_2H_5OH(l) + 3O_2(g) \longrightarrow 2CO_2(g) + 3H_2O(l)$

$$\Delta U = -1364.47 \text{ kJ/mol},$$

$\Rightarrow \quad \Delta n_g = -1,\ T = 25°C = 298$ K

$\therefore \quad \Delta H = \Delta U + \Delta n_g RT$

$$\Delta H = -1364.47 + \frac{-1 \times 8.314 \times 298}{1000}$$

[Here, value of R in unit of J must be converted into kJ]

$$= -1364.47 - 2.4776 = -1366.95 \text{ kJ/mol}$$

73. ***(c)*** (a) $\Delta G = \Delta H - T\,\Delta S$

For a system, total entropy change is ΔS_{total},

$$\Delta H_{total} = 0$$

$\therefore \quad \Delta G_{system} = -T\Delta S_{total}$

$\therefore \quad \frac{\Delta G_{system}}{\Delta S_{total}} = -T$

Thus, (a) is true,

(b) For isothermal reversible process, $\Delta E = 0$.

By first law of thermodynamics,

$$\Delta E = q \times W$$

$\therefore \quad W_{reversible} = -q = -\int_{V_i}^{V_f} p\,dV$

$$W_{reversible} = -nRT\ \text{In}\ \frac{V_f}{V_i}$$

Thus, (b) is also true.

(c) $\Delta G° = \Delta H° - T\Delta S°$

$\Delta G° = -nRT \ln K = -RT \ln K$ (for $n = 1$)

$\therefore \quad -RT \ln K = \Delta H° - T\Delta S°$

$\therefore \quad \ln K = -\left(\dfrac{\Delta H° - T\Delta S°}{RT}\right)$

Thus, (c) is false.

(d) $\Delta G° = -RT \ln K$

$\therefore \quad \ln K = -\dfrac{\Delta G°}{RT}$

$\therefore \quad K = e^{-\Delta G°/RT}$

Thus, (d) is also true.

74. ***(d)*** $Pb(OH)_2 \rightleftharpoons Pb^{2+} + 2OH^-$

$$K_{sp} = [Pb^{2+}][OH^-]^2 = S \times (2S)^2$$

$$K_{sp} = 4S^3$$

$$= 4 \times (6.7 \times 10^{-6})^3 = 1.20 \times 10^{-15}$$

In a solution with pH = 8

$$[H^+] = 10^{-8} \text{ and } [OH^-] = 10^{-6}$$

$$1.20 \times 10^{-15} = [Pb^{2+}][10^{-6}]^2$$

$$[Pb^{2+}] = \frac{1.2 \times 10^{-15}}{[10^{-6}]^2} = 1.2 \times 10^{-3} \text{ M}$$

75. ***(c)*** Molarity

$$= \frac{10 \times \text{density} \times \text{percentage weight of solute}}{\text{molecular weight of the solute}}$$

$$\Rightarrow \text{Density} = \frac{3.60 \times 98}{10 \times 29} = 1.216 = 1.22 \text{ g mol}^{-1}$$

76. ***(a)*** According to Raoult's law, relative lowering of vapour pressure is equal to mole fraction of solute,

$$\therefore \quad \frac{p - p_s}{p} = \frac{n}{n+N} = \frac{w/m}{\frac{w}{m} + \frac{W}{M}} \Rightarrow \frac{w}{m} <<< \frac{W}{M}$$

$$\therefore \quad \frac{p° - p_s}{p°} = \frac{w/m}{\frac{W}{M}} = \frac{w}{m} \times \frac{M}{W}$$

$$0.0125 = \frac{w \times 18}{m \times W} \Rightarrow \frac{w}{mW} = \frac{0.0125}{18}$$

$$m = \frac{\text{Weight of solute } (w) \times 1000}{\text{Molar mass of solute } (m) \times \text{Weight of } H_2O (W)}$$

Now, molality, $m = \dfrac{0.0125}{18} \times 1000$

$$= 0.69 = 0.7$$

77. ***(c)*** Suppose the equal mass of methane and oxygen, $w = 1$ g

$$\text{Mole fraction of oxygen} = \frac{\frac{w}{32}}{\frac{w}{32} + \frac{w}{16}} = \frac{\frac{1}{32}}{\frac{3}{32}} = \frac{1}{3}$$

Let the total pressure = p

Pressure exerted by oxygen (partial pressure)

$$= \chi_{O_2} \times p_{total} = p \times \frac{1}{3}$$

$\therefore$ The fraction of total pressure exerted by oxygen is $\frac{1}{3}$.

78. ***(b)*** $NaCl + KNO_3 \longrightarrow NaNO_3 + KCl$

Sum of molar conductivity of reactant
= Sum of molar conductivity of products

$$\Lambda°(NaNO_3) = \Lambda°_m(NaCl) + \Lambda°_m(KNO_3) - \Lambda°_m(KCl)$$

$$= 128 + 111 - 152$$

$$= 87 \text{ S cm}^2 \text{ mol}^{-1}$$

79. ***(d)*** $Ca^{2+} + 2e^- \longrightarrow Ca$ (at cathode)

1 mole $CaCl_2 = 2F$

From $Q = it \Rightarrow n \cdot 2F = it$

$$\Rightarrow \quad t = \frac{n \cdot 2F}{i} = \frac{30}{40} \times \frac{2 \times 96500}{5} \quad \left(\because n = \frac{w}{m} = \frac{30}{40}\right)$$

$$= 28950 \text{ s} = \frac{28950}{60 \times 60}$$

$$= 8.04 = 8 \text{ h}$$

80. ***(a)*** As the value of reduction potential of metal ion increases, the tendency of metal oxide to get reduce into metal increases.

Since, reduction potential of only Ag is positive among the given, thus Ag_2O readily gets reduced to Ag metal. In other words, it can be said that Ag_2O is the least stable oxide among the given.

a. English Proficiency

81. ***(c)*** 'Immutable' means 'unchangeable'.

82. ***(a)*** 'Ignominious' means 'shameful'.

83. ***(a)*** 'Conjecture' means 'making a guess'.

84. ***(d)*** Institutional is the correct word to fill the blank. Institutional is the word which means relating to principles esp of law, so legally also every human has rights of freedom and equality.

85. ***(c)*** Volumes is the most appropriate option here. Other options do not match here.

86. ***(c)*** In the context of the sentence, the option 'skirted' is a appropriate word which means to avoid or evade.

87. ***(b)*** Immortal means living forever; never dying or decaying. So, among the given options, 'perishable' would be its correct opposite meaning word. Perishable means likely to decay easily.

88. ***(c)*** 'Opposed' is the correct option. Opposite meaning word to the given italicised word is 'patronised'.
Patronised means provide favour or support.
Opposed means disagreeing with someone/something.

89. ***(c)*** Barbarous' means extremely brutal, uncivilised. So, 'civilised' would be its correct opposite meaning word.

90. ***(c)*** 'Atheist' is the best alternative.

91. ***(c)*** 'Epicure' is the best alternative.

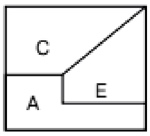
92. ***(b)*** 'Philanthropist' is the correct answer, other alternatives are not relevant.

93. ***(c)*** CBEDA **94.** (d) CDEAB **95.** ***(a)*** DEBAC

b. Logical Reasoning

96. ***(c)*** Figure shown in option (c) will make a complete square on joining with the problem figure.

97. ***(c)*** From figures (A), (C) and (E),



98. ***(b)*** The contents of the third figure in each row (and column) are determined by the contents of the first two figures. Lines are carried forward from the first two figures to the third one, except where two lines appear in the same position, in which they are cancelled out.

99. ***(d)*** From the given four positions of a single block,
Faces adjacent to face having red colour
= blue, pink, yellow, green
Clearly, black is opposite to red.

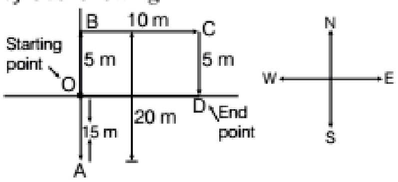
100. ***(c)*** In figure (A), the dot is placed in the region which is common to the circle and triangle. Now, we have to search similar common region in the four options. Only in figure (c), we find such a region which is common to the circle and triangle.

101. ***(b)*** The pattern is as following

$$G \xrightarrow{+3} J \xrightarrow{+3} M \xrightarrow{+3} P \xrightarrow{+3} S \xrightarrow{+3} V$$
$$4 \xrightarrow{\times 2+1} 9 \xrightarrow{\times 2+2} 20 \xrightarrow{\times 2+3} 43 \xrightarrow{\times 2+4} 90 \xrightarrow{\times 2+5} 185$$
$$T \xrightarrow{-2} R \xrightarrow{-2} P \xrightarrow{-2} N \xrightarrow{-2} L \xrightarrow{-2} J$$

So, V185J will replace the question mark.

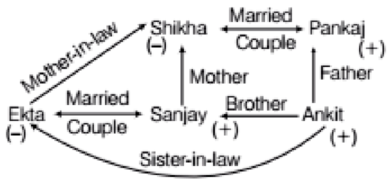
102. ***(a)*** According to the given information, the direction of Neeraj is as following



So, it is clearly shown that, Neeraj is 10 m for in East direction from his starting position.

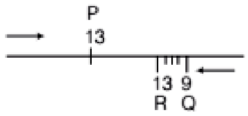
103. ***(b)*** Let the average age of 8 men = x yr
Total age of 8 man = Average × Total man
$= 8x$ yr
Now, new average age = $x + 2$ yr
Total age = $8(x+2)$ yr
Difference of ages = $8(x+2) - 8x$
$= 8x + 16 - 8x$
$= 16$ yr
$\therefore$ Age of new man = 20 + 16 = 36 yr
So, the new man is 36 yr older to the man by whom the new man is replaced.

104. ***(d)*** The relation is as following



It is clearly shown that, Shikha is the mother of Ankit.

105. ***(c)*** According to the question



From left end, position of R = Total children
− Position from right end +1
$= 40 - 13 + 1 = 28$th
Hence, number of children between P and R
$= (28 - 13) - 1 = 14$

Mathematics

106. ***(a)*** Let $z = x + iy$

$\text{Re}(z + 2) = |z - 2|$

$\text{Re}(x + iy + 2) = |x + iy - 2| \Rightarrow x + 2 = \sqrt{(x-2)^2 + y^2}$

Squaring on both sides,

$$(x+2)^2 = (x-2)^2 + y^2$$

$$x^2 + 4 + 4x = x^2 + 4 - 4x + y^2 \Rightarrow y^2 = 8x$$

The locus of z is a parabola.

107. ***(a)*** Let α, β be the roots of $x^2 - abx - a^2 = 0$ where, $\alpha + \beta = ab$ and $\alpha\beta = -a^2$, which shows product of roots < 0, i.e. one root must be negative and the other must be positive. Hence, equation has one positive root and one negative root.

108. ***(c)*** Given, $a + 2b + 3c = 12 \forall\ a, b, c \in R^+$

As, AM $\geq$ GM

$$\frac{a+b+b+c+c+c}{6} \geq \sqrt[6]{ab^2c^3}$$

$$\Rightarrow \quad \frac{12}{6} \geq \sqrt[6]{ab^2c^3} \quad \Rightarrow \quad ab^2c^3 \leq 2^6$$

Hence, the maximum value of ab^2c^3 is 2^6.

109. ***(a)*** a_n of the series $= n(2n+1)^2 \forall\ n \in N$

$$a_n = n[4n^2 + 4n + 1] \quad \Rightarrow \quad a_n = 4n^3 + 4n^2 + n$$

$$S_n = \Sigma a_n = 4\Sigma n^3 + 4\Sigma n^2 + \Sigma n$$

$$= 4\left(\frac{n(n+1)}{2}\right)^2 + \frac{4n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left[2n^2 + 2n + \frac{4(2n+1)}{3} + 1\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{6n^2 + 6n + (8n+4) + 3}{3}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{6n^2 + 14n + 7}{3}\right]$$

$$= \frac{n}{6}(n+1)(6n^2 + 14n + 7)$$

110. ***(d)***

$$\log_{225} 70 = \frac{\log_e 70}{\log_e 225} = \frac{\log_e 7 + \log_e 5 + \log_e 2}{\log_e 25 + \log_e 9}$$

$$= \frac{1 + \dfrac{\log_e 5}{\log_e 7} + \dfrac{\log_e 2}{\log_e 7}}{2\dfrac{\log_e 5}{\log_e 7} + 2\dfrac{\log_e 3}{\log_e 7}}$$

$$= \frac{1 + \log_7 5 + \log_3 2 \times \log_5 3 \times \log_7 5}{2[\log_7 5 + \log_5 3 \times \log_7 5]}$$

$$= \frac{1 + a + abc}{2[a + ab]} = \frac{1 + a + abc}{2a(1+b)}$$

111. ***(b)*** Two circles intersect maximum at two distinct points.

Now, two circles can be selected in ${}^{10}C_2$ ways.

The total number of points of intersection are ${}^{10}C_2 \times 2$

$$= \frac{10 \times 9}{1 \times 2} \times 2 = 90$$

112. ***(c)*** $\dfrac{C_1}{C_0} = n, \dfrac{C_2}{C_1} = \dfrac{(n-1)}{2}, \dfrac{C_3}{C_2} = \dfrac{n-2}{3}$ and so on.

$$\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + \ldots n\frac{C_n}{C_{n-1}} = \Sigma n = \frac{n(n+1)}{2}$$

On putting $n = 20$,

$$\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + \ldots + 20\frac{C_{20}}{C_{19}} = \frac{20 \times 21}{2} = 210$$

113. ***(c)*** Given, $\begin{vmatrix} 0 & x-p & x-q \\ x+p & 0 & x-r \\ x+q & x-r & 0 \end{vmatrix} = 0$

$$-(x-p)\begin{vmatrix} x+p & x-r \\ x+q & 0 \end{vmatrix} + (x-q)\begin{vmatrix} x+p & 0 \\ x+q & x-r \end{vmatrix} = 0$$

$$\Rightarrow (x-p)(x+q)(x-r) + (x-q)(x+p)(x-r) = 0$$

$$\Rightarrow (x-r)[(x-p)(x+q) + (x-q)(x+p)] = 0$$

$$\Rightarrow (x-r)[x^2 - px + qx - pq + x^2 - qx + px - pq] = 0$$

$$\Rightarrow (x-r)[2x^2 - 2pq] = 0 \Rightarrow x - r = 0 \text{ or } x^2 - pq = 0$$

either $x = r$ or pq

114. ***(c)*** $A \cdot \text{adj} A = |A| I$

$|A| = xyz - 8x - 3(z-8) + 2(2-2y)$

$|A| = xyz - (8x + 3z + 4y) + 28$

$\Rightarrow 60 - 20 + 28 = 68$

$\Rightarrow (\text{adj } A)^{-1}$ always exists whenever $(A)^{-1}$ exists.

$\therefore \quad A \cdot \text{adj} A = |A| I$

$$= 68\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 68 & 0 & 0 \\ 0 & 68 & 0 \\ 0 & 0 & 68 \end{bmatrix}$$

115. ***(b)*** Given, $f(x) = 4x - x^2$

$f(a+1) - f(a-1)$

$$= [4(a+1) - (a+1)^2] - [4(a-1) - (a-1)^2]$$

$$= [4a + 4 - a^2 - 1 - 2a] - [4a - 4 - a^2 - 1 + 2a]$$

$\Rightarrow \quad 8 - 4a = 0$

$\therefore \quad a = 2$

116. ***(c)*** Let $R = \{(a, b) : a + b$ is an even integer $a, b \in Z\}$

For $a \in Z$, $a + a = 2a$ is an even integer.

$\therefore (a, a) \in R\ \forall\ a \in Z$

$\therefore$ R is reflexive.

Let $(a, b) \in R \Rightarrow a + b$ is an even integer.

$\Rightarrow (b + a)$ is an even integer $\Rightarrow (b, a) \in R$

$\Rightarrow R$ is symmetric.

Let $(a, b), (b, c) \in R$

$\Rightarrow (a + b)$ and $(b + c)$ are even integers.

$\Rightarrow (a + b) + (b + c) = (a + c + 2b)$ is an even integer.

$\Rightarrow (a + c + 2b) - 2b = (a + c)$ is an even integer.

$\Rightarrow (a, c) \in R \Rightarrow R$ is transitive.

$\therefore$ R is an equivalence relation.

Let $R = \{(a, b) : (a - b)$ is an even integer, $a, b \in Z\}$.

For $a \in Z$, $a - a = 0$ is an even integer.

$\therefore (a, a) \in R \; \forall \; a \in Z$

$\therefore$ R is reflexive.

Let $(a, b) \in R \Rightarrow (a - b)$ is an even integer.

$\Rightarrow -(a - b)$ is an even integer.

$\Rightarrow (b - a)$ is an even integer $\Rightarrow (b, a) \in R$

$\Rightarrow R$ is symmetric.

Let $(a, b), (b, c) \in R$

$\Rightarrow (a - b), (b - c)$ are even integers.

$\Rightarrow (a - b) + (b - c)$ is an even integer.

$\Rightarrow (a - c)$ is an even integer.

$\Rightarrow (a, c) \in R$

$\Rightarrow R$ is transitive.

$\therefore$ R is an equivalence relation.

Let $R = \{(a,b) : a < b,\, a, b \in Z\}$

Let $a \in Z$. $a < a$ is false.

$\therefore$ R is not reflexive.

$\therefore$ R is not an equivalence relation.

Let $R = \{(a, b) : a = b,\, a, b \in Z\}$.

It is quite easy to check that R is an equivalence relation.

117. ***(b)*** Since, $\sim(p \vee q) \equiv (\sim p \wedge \sim q)$ and $\sim(p \wedge q) \equiv (\sim p \vee \sim q)$

So, options (a) and (d) are not true.

$\sim(p \to q) \equiv p \wedge \sim q$, so option (c) is not true.

Now, $p \to q \sim p \vee q$

$$\sim q \to \sim p \equiv [\sim(\sim q) \vee \sim p]$$

$$\equiv q \vee \sim p \equiv \sim p \vee q \quad \Rightarrow \quad p \to q \equiv (\sim q \to \sim p)$$

118. ***(a)*** $4^{-x+0.5} - 7.2^{-x} < 4$

Let $2^{-x} = t$

The equation becomes

$$2t^2 - 7t - 4 < 0 \quad \Rightarrow \quad (t - 4)(2t + 1) < 0$$

$$\Rightarrow \quad 2(t - 4)\left\{t - \left(-\frac{1}{2}\right)\right\} < 0 \Rightarrow -\frac{1}{2} < t < 4$$

Since, $t = 2^{-x} > 0 \; \forall \; x \in R$

$$\Rightarrow \qquad 0 < t < 4 \;\Rightarrow\; 0 < 2^{-x} < 2^2$$

As, 2^x is an increasing function,

$$-x < 2 \text{ or } x > -2$$

Solution is $(-2, \infty)$.

119. ***(a)*** $\cos^3 x \cdot \sin 2x = \dfrac{\cos 3x + 3\cos x}{4} \times \sin 2x$

$$= \frac{1}{8}(\sin 5x - \sin x) + \frac{3}{8}(\sin 3x + \sin x)$$

$$= \frac{1}{4}\sin x + \frac{3}{8}\sin 3x + \frac{1}{8}\sin 5x$$

Here, $n = 5$, $a_1 = \dfrac{1}{4}$, $a_2 = 0$, $a_3 = \dfrac{3}{8}$,

$a_4 = 0$, $a_5 = 1/8$

120. ***(b)*** $|\cot x| = \cot x + \dfrac{1}{\sin x}$

Let $\cot x > 0 \Rightarrow \cot x = \cot x + \dfrac{1}{\sin x} = 0$

$\Rightarrow \dfrac{1}{\sin x} = 0$ which is not possible.

Let $\cot x \le 0 \;\Rightarrow\; -\cot x = \cot x + \dfrac{1}{\sin x}$

$$\Rightarrow \quad -2\cot x = \frac{1}{\sin x}$$

$$\Rightarrow \qquad \cos x = -\frac{1}{2} \Rightarrow x = \frac{2\pi}{3}, \frac{8\pi}{3}$$

$\therefore$ The number of solutions is 2.

121. ***(a)*** Let $I = (\sin^{-1} x)^3 + (\cos^{-1} x)^3$

$$= (\sin^{-1} x + \cos^{-1} x)[(\sin^{-1} x)^2 + (\cos^{-1} x)^2 - (\sin^{-1} x)(\cos^{-1} x)]$$

$$= \frac{\pi}{2}\left[(\sin^{-1} x + \cos^{-1} x)^2 - 3\sin^{-1} x\left(\frac{\pi}{2} - \sin^{-1} x\right)\right]$$

$$= \frac{\pi}{2}\left(\frac{\pi^2}{4} - \frac{3\pi}{2}\sin^{-1} x + 3(\sin^{-1} x)^2\right)$$

$$= \frac{\pi}{2}\left[3\left((\sin^{-1} x)^2 - \frac{\pi}{2}(\sin^1 x) + \frac{\pi^2}{16} - \frac{\pi^2}{16}\right) + \frac{\pi^2}{4}\right]$$

$$= \frac{\pi}{2}\left(\frac{\pi^2}{16} + 3\left(\sin^{-1} x - \frac{\pi}{4}\right)^2\right)$$

$$\left(\sin^{-1} x - \frac{\pi}{4}\right)^2 \ge 0$$

For minimum value put $\left(\sin^{-1} x - \dfrac{\pi}{4}\right)^2 = 0$

Minimum values $= \dfrac{\pi}{2}\left[\dfrac{\pi^2}{16} + 0\right] = \dfrac{\pi^3}{32}$

122. ***(b)*** Let $P(x, y)$ be the original position of the point w.r.t. the original axes. Let us move the origin at new position to (h, k).

Hence, the position of the same point P in the new system is

$$x' = x - h \Rightarrow y' = y - k$$

Here, $(h, k) = (1, 2)$

$\therefore \quad x' = (x - 1), y' = (y - 2)$

According to the question,

$y^2 - 8x - 4y + 12 = (y - 2)^2 - 4a(x - 1)$

$\Rightarrow y^2 - 8x - 4y + 12 = y^2 - 4y + 4 - 4ax + 4a$

On comparing respective coefficients, we get

$$4a = 8 \Rightarrow a = 2$$

123. ***(d)*** Equations of the bisectors of the angles between the given straight lines are given by

$$\frac{3x + 4y + 7}{\sqrt{9 + 16}} = \pm \frac{12x + 5y - 8}{\sqrt{144 + 25}}$$

$\Rightarrow 13(3x + 4y + 7) = \pm 5(12x + 5y - 8)$

$\Rightarrow 39x + 52y + 91 = \pm(60x + 25y - 40)$

Taking positive signs,

$$39x + 52y + 91 = 60x + 25y - 40$$

$\Rightarrow \; -21x + 27y + 131 = 0 \Rightarrow 21x - 27y - 131 = 0$

Taking negative signs,

$$(39x + 52y + 91) = -(60x + 25y - 40)$$

$\Rightarrow \quad 99x + 77y + 51 = 0$

124. ***(b)*** Since, the circle touches X-axis,

$$(x - h)^2 + (y - k)^2 = k^2 \quad \dots \text{(i)}$$

Also, it passes through the points $(1, -2)$ and $(3, -4)$.

$$(1 - h)^2 + (-2 - k)^2 = k^2 \quad \dots \text{(ii)}$$

and $$(3 - h)^2 + (-4 - k)^2 = k^2 \quad \dots \text{(iii)}$$

Subtracting Eq. (iii) from Eq. (ii), we get

$$h = k + 5$$

On solving these equations, we get

$$k = -10, -2 \text{ and } h = -5, 3$$

By putting the values of $(h, k) = (-5, -10)$ or $(3, -2)$ in Eq. (i), we get

$$x^2 + y^2 + 10x + 20y + 25 = 0$$

or $$x^2 + y^2 - 6x + 4y + 9 = 0$$

125. ***(b)*** $x = 9$ meets the hyperbola at $(9, 6\sqrt{2})$ and $(9, -6\sqrt{2})$. Then, the equations of tangent at these points are $3x - 2\sqrt{2}y - 3 = 0$ and $3x + 2\sqrt{2y} - 3 = 0$.

The combined equation of these two tangent is $9x^2 - 8y^2 - 18x + 9 = 0$.

126. ***(d)*** Let the points be $A(10\hat{i} + 3\hat{j})$, $B(12\hat{i} - 15\hat{j})$ and $C(a\hat{i} + 11\hat{j})$.

$AB = |2\hat{i} - 18\hat{j}|$ and $AC = (a - 10)\,\hat{i} + 8\hat{j}$

Since, A, B and C are collinear, then

$$\frac{2}{a - 10} = -\frac{18}{8} \Rightarrow a = \frac{82}{9}$$

127. ***(d)*** We have, $|\mathbf{a}| = 3, |\mathbf{b}| = 4$ and $|\mathbf{c}| = 5$.

It is given that

$$\mathbf{a} \perp (\mathbf{b} + \mathbf{c}), \mathbf{b} \perp (\mathbf{c} + \mathbf{a}) \text{ and } \mathbf{c} \perp (\mathbf{a} + \mathbf{b})$$

$\Rightarrow \mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = 0, \mathbf{b}.(\mathbf{c} + \mathbf{a}) = 0$ and $\mathbf{c} \cdot (\mathbf{a} + \mathbf{b}) = 0$

$\Rightarrow \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c} = \mathbf{b} \cdot \mathbf{c} + \mathbf{b} \cdot \mathbf{a} = \mathbf{c} \cdot \mathbf{a} + \mathbf{c} \cdot \mathbf{b} = 0$

or $\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a} = 0$

(adding all the above equations)

Now, $|\mathbf{a} + \mathbf{b} + \mathbf{c}|^2 = |\mathbf{a}|^2 + |\mathbf{b}|^2 + |\mathbf{c}|^2 + 2$ $(\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a})$

$$= 3^2 + 4^2 + 5^2 = 50$$

$\therefore \quad |\mathbf{a} + \mathbf{b} + \mathbf{c}| = 5\sqrt{2}$

128. ***(d)*** We have, $|(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}| = |\mathbf{a}||\mathbf{b}||\mathbf{c}|$

$\Rightarrow ||\mathbf{a}||\mathbf{b}||\mathbf{c}|\sin\theta\cos\alpha| = |\mathbf{a}||\mathbf{b}||\mathbf{c}|$

$\Rightarrow |\sin\theta||\cos\alpha| = 1 \Rightarrow \theta = \frac{\pi}{2}$ and $\alpha = 0$

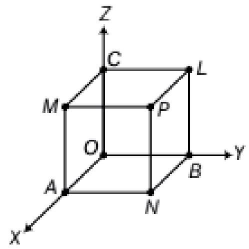
$\Rightarrow \mathbf{a} \perp \mathbf{b}$ and $\mathbf{c} \parallel \hat{\mathbf{n}} \Rightarrow \mathbf{a} \perp \mathbf{b}$ and $\mathbf{c} \perp$ both $\mathbf{a}$ and $\mathbf{b}$.

$\Rightarrow \mathbf{a}, \mathbf{b}, \mathbf{c}$ are mutually perpendicular.

$\Rightarrow \mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{c} = \mathbf{c} \cdot \mathbf{a} = 0$

129. ***(c)*** Let each edge of cube be a, then coordinates of the vertices of cube are

$O(0, 0, 0)$, $A(a, 0, 0)$, $B(0, a, 0)$, $C(0,0, a)$, $N(a, a, 0)$, $P(a, a, a)$, $L(0, a, a)$, $M(a, 0, a)$



Direction ratios of the diagonals OP, AL, BM and CN are (a, a, a), $(-a, a, a)$, $(a, -a, a)$ and $(a, a, -a)$.

Let θ be the acute angle between diagonals OP and AL.

$$\therefore \quad \cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{a \times (-a) + a \times a + a \times a}{\sqrt{a^2 + a^2 + a^2}\sqrt{(-a)^2 + a^2 + a^2}}$$

$$= \frac{-a^2 + a^2 + a^2}{\sqrt{3a^2}\,\sqrt{3a^2}} = \frac{a^2}{a\sqrt{3} \times a\sqrt{3}}$$

$$\Rightarrow \quad \cos\theta = \frac{1}{3} \quad \Rightarrow \quad \theta = \cos^{-1}\left(\frac{1}{3}\right)$$

130. ***(b)*** Given lines are

$L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}$ and $L_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$

Now, convert into vector form

$L_1 : (-\hat{i} - 2\hat{j} - \hat{k}) + \lambda(3\hat{i} + \hat{j} + 2\hat{k})$

$L_2 : (2\hat{i} - 2\hat{j} + 3\hat{k}) + \mu(\hat{i} + 2\hat{j} + 3\hat{k})$

Line L_1 comparing with $\mathbf{a}_1 + \lambda\mathbf{b}_1$,

and L_2 comparing with $\mathbf{a}_2 + \mu\mathbf{b}_2$, then we have

$\mathbf{b}_1 = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\mathbf{b}_2 = \hat{i} + 2\hat{j} + 3\hat{k}$

Perpendicular to both $\mathbf{b}_1$ and $\mathbf{b}_2 = \mathbf{b}_1 \times \mathbf{b}_2$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix}$$

$= \hat{i}(3-4) - \hat{j}(9-2) + \hat{k}(6-1) = -\hat{i} - 7\hat{j} + 5\hat{k}$

$\therefore$ Required unit vector $= \frac{-\hat{i} - 7\hat{j} + 5\hat{k}}{\sqrt{(-1)^2 + (-7)^2 + (5)^2}}$

$= \frac{-\hat{i} - 7\hat{j} + 5\hat{k}}{\sqrt{75}} = \frac{-\hat{i} - 7\hat{j} + 5\hat{k}}{5\sqrt{3}}$

131. ***(b)*** The given line is $\mathbf{r} = \mathbf{a} + t\mathbf{b}$, where $\mathbf{a} = 2\hat{i} - 2\hat{j} + 3\hat{k}$, $\mathbf{b} = \hat{i} - \hat{j} + 4\hat{k}$ and given plane is $\mathbf{r} \cdot \mathbf{n} = d$, where $\mathbf{n} = \hat{i} + 5\hat{j} + \hat{k}$, $d = 5$.

Since, $\mathbf{b} \cdot \mathbf{n} = 1 - 5 + 4 = 0$

$\therefore$ Given line is parallel to given plane.

$\therefore$ The distance between the line and the plane is equal to length of the perpendicular from the point $\mathbf{a} = 2\hat{i} - 2\hat{j} + 3\hat{k}$ on the line to the given plane.

$\therefore$ Required distance

$$= \left| \frac{(2\hat{i} - 2\hat{j} + 3\hat{k}) \cdot (\hat{i} + 5\hat{j} + \hat{k}) - 5}{\sqrt{1 + 25 + 1}} \right|$$

$$= \left| \frac{2 - 10 + 3 - 5}{\sqrt{27}} \right| = \frac{10}{3\sqrt{3}}$$

132. ***(c)*** Let E_1 = event of getting both red cards

E_2 = event of getting both kings

and $E_1 \cap E_2$ = event of getting 2 kings of red cards

$\therefore \; P(E_1) = \frac{{}^{26}C_2}{{}^{52}C_2} = \frac{325}{1326}$, $P(E_2) = \frac{{}^{4}C_2}{{}^{52}C_2} = \frac{6}{1326}$

and $P(E_1 \cap E_2) = \frac{{}^{2}C_2}{{}^{52}C_2} = \frac{1}{1326}$

$\therefore$ P(both red or both kings) $= P(E_1 \cup E_2)$

$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= \frac{325}{1326} + \frac{6}{1326} - \frac{1}{1326} = \frac{330}{1326} = \frac{55}{221}$

133. ***(d)*** Since, A and B are independent events.

$\therefore \quad P(A/B) = P(A) = \frac{1}{2}$

$\Rightarrow P\left(\frac{A}{A \cup B}\right) = \frac{P[A \cap (A \cup B)]}{P(A \cup B)}$

$= \frac{P(A)}{P(A \cup B)} = \frac{1/2}{\frac{1}{2} + \frac{1}{5} - \frac{1}{10}} = \frac{1/2}{\frac{1}{2} + \frac{1}{5} - \frac{1}{10}} = \frac{1/2}{\frac{6}{10}} = \frac{5}{6}$

Similarly, $P\left(\frac{A \cap B}{A' \cup B'}\right) = P\left(\frac{A \cap B}{(A \cap B)'}\right) = 0$

134. ***(b)*** Let E_1 = coin shows head, E_2 = coin shows tail,

A = drawn ball is blue

$P(E_1) = \frac{1}{2} = P(E_2)$

$P(A/E_1)$ = Probability of drawing a blue ball from bag I = 2/7

$P(A/E_2)$ = Probability of drawing a blue ball from bag II = 6/8

By Baye's theorem, we have

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\,P\left(\frac{A}{E_1}\right)}{P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2} \times \frac{2}{7}}{\frac{1}{2} \times \frac{2}{7} + \frac{1}{2} \times \frac{6}{8}} = \frac{\frac{2}{7}}{\frac{2}{7} + \frac{6}{8}} = \frac{2}{7} \times \frac{56}{58} = \frac{8}{29}$$

135. ***(c)*** Given, $E(X) = 3$ and $E(X^2) = 11$

Variable of $X = E(X^2) - [E(X)]^2 = 11 - 3^2 = 11 - 9 = 2$

136. ***(d)*** Given, $x_1 + x_2 + x_3 + \ldots + x_{10} = 12$

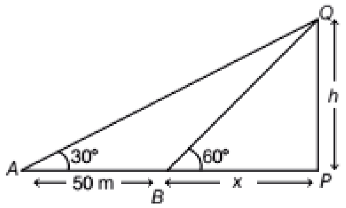
and $x_1^2 + x_2^2 + \ldots + x_{10}^2 = 18$

$\sigma^2 = \frac{1}{n} \sum x^2 - \left(\frac{1}{n} \sum x\right)^2$

$= \frac{1}{10} \times 18 - \left(\frac{1}{10} \times 12\right)^2 = \frac{9}{5} - \frac{36}{25} = \frac{9}{25}$

$\therefore$ Standard deviation = 3/5

137. ***(c)*** Let PQ be the chimney whose height is h metres.



In ΔBPQ, $\tan 60° = \frac{PQ}{BP} \Rightarrow \sqrt{3} = \frac{h}{x}$

$\Rightarrow \quad h = \sqrt{3}\, x \quad \ldots$(i) and in ΔAPQ, $\tan 30° = \frac{PQ}{AP}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{50 + x}$

$\Rightarrow \quad 50 + x = h\sqrt{3}$

$\Rightarrow \quad 50 + x = 3x \Rightarrow x = 25$ [using Eq. (i)]

$\therefore$ Height of the chimney $= 25\sqrt{3}$ m

138. ***(c)*** The required limit

$$= \lim_{x \to 0} \frac{\sqrt{(1 - \cos x^2)(1 + \cos x^2)}}{(1 - \cos x)\sqrt{1 + \cos x^2}}$$

$$= \lim_{x \to 0} \frac{\left(\frac{\sin x^2}{x^2}\right)}{2\left(\frac{\sin^2(x/2)}{x^2}\right)} \cdot \frac{1}{\sqrt{1 + \cos x^2}}$$

$$= \frac{1}{2 \times \frac{1}{4} \times \sqrt{2}} = \sqrt{2}$$

139. ***(c)*** $f'(1-0) = \lim_{h \to 0} \frac{f(1-h) - f(1)}{-h}$

$$= \lim_{h \to 0} \frac{a(1-h)^2 + 1 - (1+a)}{-h}$$

$$= \lim_{h \to 0} \frac{a(h^2 - 2h)}{-h} = 2a$$

$$f'(1+0) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{(1+h)^2 + a(1+h) + b - (1+a)}{h}$$

$$= \lim_{h \to 0} \frac{h^2 + 2h + ah + b}{h}$$

$= 2 + a$, if $b = 0$

Thus, $2a = 2 + a$, $b = 0$

$\Rightarrow \quad a = 2, b = 0$

140. ***(c)*** We have, $\frac{dx}{dt} = 2t + 3$ and $\frac{dy}{dt} = 4t - 2$

$$\therefore \quad \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4t - 2}{2t + 3}$$

Thus, slope of the tangent to the curve at the point $t = 2$ is

$$\left[\frac{dy}{dx}\right]_{t=2} = \frac{4(2) - 2}{2(2) + 3} = \frac{6}{7}$$

141. ***(c)*** Let $I = \int \frac{1}{1 - 2\sin x} dx$

On putting $\sin x = \frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$, we get

$$I = \int \frac{1}{1 - \frac{4\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}} dx$$

$$= \int \frac{1 + \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2} - 4\tan\frac{x}{2}} dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{1 + \tan^2\frac{x}{2} - 4\tan\frac{x}{2}} dx$$

$$= 2\int \frac{1}{\left(\tan\frac{x}{2} - 2\right)^2 - (\sqrt{3})^2} d\left(\tan\frac{x}{2}\right)$$

$$= 2\int \frac{dt}{(t-2)^2 - (\sqrt{3})^2}, \text{ where } t = \tan\frac{x}{2}$$

$$= 2 . \frac{1}{2\sqrt{3}} \log\left|\frac{t - 2 - \sqrt{3}}{t - 2 + \sqrt{3}}\right| + c$$

$$= \frac{1}{\sqrt{3}} \log\left|\frac{\tan\frac{x}{2} - 2 - \sqrt{3}}{\tan\frac{x}{2} - 2 + \sqrt{3}}\right| + c$$

142. ***(a)*** Let

$$I = \int_0^1 \frac{\log(1+x)}{1+x^2} dx = \int_0^{\pi/4} \frac{\log(1+\tan\theta)}{(1+\tan^2\theta)} \sec^2\theta \, d\theta$$

[Let $x = \tan\theta \Rightarrow dx = \sec^2\theta d\theta$]

$$= \int_0^{\pi/4} \log(1+\tan\theta) d\theta$$

$$= \int_0^{\pi/4} \log\left[1 + \tan\left(\frac{\pi}{4} - \theta\right)\right] d\theta$$

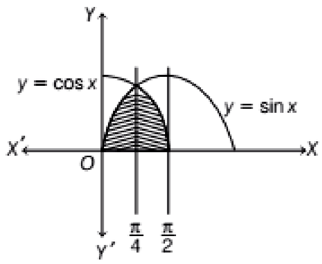
$$= \int_0^{\pi/4} \log\left(1 + \frac{1 - \tan\theta}{1 + \tan\theta}\right) d\theta$$

$$= \int_0^{\pi/4} \log\left(\frac{2}{1 + \tan\theta}\right) d\theta$$

$$= \log 2 \int_0^{\pi/4} 1 \, d\theta - I$$

$$\Rightarrow \quad 2I = \frac{\pi}{4}\log 2 \Rightarrow I = \frac{\pi}{8}\log 2$$

143. ***(c)*** Required area



$$= \int_0^{\pi/4} \sin x \, dx + \int_{\pi/4}^{\pi/2} \cos x \, dx$$

$$= -[\cos x]_0^{\pi/4} + [\sin x]_{\pi/4}^{\pi/2}$$

$$= 2\left(1 - \frac{1}{\sqrt{2}}\right) = (2 - \sqrt{2}) \text{ sq units}$$

144. ***(b)*** $\left(\frac{x+y-1}{x+y-2}\right)\frac{dy}{dx} = \left(\frac{x+y+1}{x+y+2}\right)$

Put $x + y = v$

$\therefore \quad 1 + \frac{dy}{dx} = \frac{dv}{dx}$ or $\left(\frac{v-1}{v-2}\right)\left(\frac{dv}{dx} - 1\right) = \left(\frac{v+1}{v+2}\right)$

$\Rightarrow \frac{dv}{dx} - 1 = \frac{(v+1)(v-2)}{(v-1)(v+2)} = \frac{v^2 - v - 2}{v^2 + v - 2}$

or $\quad \frac{dv}{dx} = \frac{2v^2 - 4}{(v^2 + v - 2)}$

$\Rightarrow \frac{(v^2 + v - 2)}{(v^2 - 2)} dv = 2dx$

$\Rightarrow \left(1 + \frac{v}{v^2 - 2}\right) dv = 2dx$

On integrating, we get

$$v + \frac{1}{2}\log|v^2 - 2| = 2x + c$$

or $\quad (y - x) + \frac{1}{2}\log\left|(x+y)^2 - 2\right| = c$

Given, $y = 1$ when $x = 1$

$\therefore \quad 0 + \frac{1}{2}\log 2 = c$ or $(y - x) + \frac{1}{2}\log\left|\frac{(x+y)^2 - 2}{2}\right| = 0$

or $\quad \log\left|\frac{(x+y)^2 - 2}{2}\right| = 2(x - y)$

145. ***(d)*** Given, $y = \sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)$

Let $x = \cos 2\theta \Rightarrow y = \sin\left(2\tan^{-1}\sqrt{\frac{1 - \cos 2\theta}{1 + \cos 2\theta}}\right)$

$\Rightarrow \quad y = \sin\left(2\tan^{-1}\sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}}\right)$

$\Rightarrow \quad y = \sin[2\tan^{-1}(\tan\theta)]$

$\Rightarrow \quad y = \sin 2\theta \Rightarrow y = \sqrt{1 - \cos^2 2\theta}$

$\Rightarrow \quad y = \sqrt{1 - x^2}$

$\therefore \quad \frac{dy}{dx} = \frac{1}{2\sqrt{1 - x^2}}(-2x) = -\frac{x}{\sqrt{1 - x^2}}$

146. ***(b)*** We have, $y = x(x-1)^2$

$\therefore \frac{dy}{dx} = (x-1)^2 \cdot 1 + 2x(x-1)$

$\quad = 3x^2 - 4x + 1 = (x-1)(3x-1)$

For maximum or minimum, $\frac{dy}{dx} = 0$

$\Rightarrow \quad (x-1)(3x-1) = 0$

$\Rightarrow \quad x = \frac{1}{3}, 1$

Now, $\frac{d^2y}{dx^2} = 6x - 4 \Rightarrow \left[\frac{d^2y}{dx^2}\right]_{x=\frac{1}{3}} = -2 < 0$

$\therefore$ y is maximum when $x = \frac{1}{3}$ and maximum value is

$$[y]_{x=\frac{1}{3}} = \frac{4}{27}$$

Also, $\quad \left[\frac{d^2y}{dx^2}\right]_{x=1} = 2 > 0$

$\therefore$ y is minimum when $x = 1$.

147. ***(b)*** We have, $\cos y\frac{dy}{dx} + \frac{4}{x}\sin y = \frac{e^x}{x^3}$

Let $\quad \sin y = t$

$\Rightarrow \quad \cos y\frac{dy}{dx} = \frac{dt}{dx}$

$\therefore \quad \frac{dt}{dx} + \frac{4}{x}t = \frac{e^x}{x^3}$

$$\text{IF} = e^{\int \frac{4}{x}dx} = e^{4\log x} = x^4$$

The solution is

$\therefore tx^4 = \int x^4 \cdot \frac{e^x}{x^3} dx = xe^x - e^x + c$

$\because \sin y \, x^4 = xe^x - e^x + c$

$\because x = 1, y = 0$

$\therefore \sin y = e^x(x-1)x^{-4}$ $\quad [\because c = 0]$

148. ***(a)*** Let us make the table from the given data

Player *A*			Player *B*		
x_i	$x_i - 61$	$(x_i - 61)^2$	y_i	$y_i - 61$	$(y_i - 61)^2$
58	− 3	9	94	33	1089
59	− 2	4	26	− 35	1225
60	− 1	1	92	31	961
54	− 7	49	65	4	16
65	4	16	96	35	1225
66	5	25	78	17	289
52	− 9	81	14	−47	2209
75	14	196	34	−27	729
69	8	64	98	37	1369
52	− 9	81	13	−48	2304
$\sum x_i = 610$		$\sum (x_i - 61)^2 = 526$	$\sum_{yi} = 610$		$\sum (y_i - 61)^2 = 11416$

For player *A*, Mean $= \frac{\sum x_i}{n} = \frac{610}{10} = 61$

$$SD = \sqrt{\frac{\sum (x_i - 61)^2}{N}}$$

$$= \sqrt{\frac{526}{10}} = 7.25$$

For player *B*, Mean $= \frac{\sum y_i}{n} = \frac{610}{10} = 61$

$$SD = \sqrt{\frac{\sum (y_i - 61)^2}{N}}$$

$$= \sqrt{\frac{11416}{10}}$$

$$= 33.79$$

Since, SD for player *A* is 7.25 < SD for player *B* is 33.79.

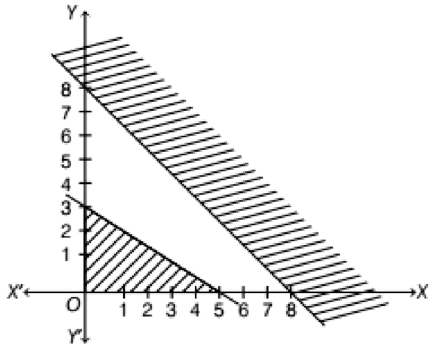
Hence, player A is more consistent player.

149. ***(b)*** Table for equation $x + y = 8$ is

x	0	8
$y = 8 - x$	8	0

Table for equation $3x + 5y = 15$ is

x	0	5
$y = \frac{15 - 3x}{5}$	3	0



It can be concluded from the graph, that there is no point which can satisfy all the constraints simultaneously. Therefore, the problem has no feasible solution.

150. ***(a)*** Substitute 0 for y in the equation $4y^2 = 4x^2 - x^3$,

$$0 = x^2(4 - x) \Rightarrow x = 0, x = 4$$

It means curve makes the loop symmetric about *X*-axis between 0 and 4.

Area $= 2\int_0^4 y\,dx = \frac{2}{2}\int_0^4 \sqrt{4x^2 - x^3}\,dx = \int_0^4 x\sqrt{4 - x}\,dx$

Let $4 - x = t \Rightarrow -dx = dt$

Area $= -\int_4^0 (4 - t)\sqrt{t}\,dt$

$$= \int_0^4 (4\sqrt{t} - t\sqrt{t})dt = \left[4 \times \frac{2}{3}t^{3/2} - \frac{2}{5}t^{5/2}\right]_0^4$$

$$= \frac{8}{3} \times 8 - \frac{2}{5} \times 32 = \frac{64}{3} - \frac{64}{5} = 64 \times \frac{2}{15} = \frac{128}{15}$$