Answers

Physics

1. (c)	**2.** (a)	**3.** (a)	**4.** (c)	**5.** (b)	**6.** (b)	**7.** (a)	**8.** (b)	**9.** (c)	**10.** (b)
11. (b)	**12.** (b)	**13.** (b)	**14.** (b)	**15.** (b)	**16.** (c)	**17.** (b)	**18.** (b)	**19.** (b)	**20.** (c)
21. (b)	**22.** (b)	**23.** (d)	**24.** (a)	**25.** (a)	**26.** (d)	**27.** (d)	**28.** (a)	**29.** (b)	**30.** (c)
31. (b)	**32.** (b)	**33.** (c)	**34.** (b)	**35.** (c)	**36.** (c)	**37.** (a)	**38.** (a)	**39.** (d)	**40.** (b)

Chemistry

41. (d)	**42.** (b)	**43.** (d)	**44.** (a)	**45.** (d)	**46.** (a)	**47.** (b)	**48.** (a)	**49.** (b)	**50.** (a)
51. (d)	**52.** (a)	**53.** (c)	**54.** (b)	**55.** (c)	**56.** (c)	**57.** (d)	**58.** (c)	**59.** (b)	**60.** (c)
61. (c)	**62.** (d)	**63.** (d)	**64.** (a)	**65.** (c)	**66.** (c)	**67.** (a)	**68.** (c)	**69.** (d)	**70.** (b)
71. (d)	**72.** (b)	**73.** (d)	**74.** (b)	**75.** (d)	**76.** (d)	**77.** (b)	**78.** (a)	**79.** (c)	**80.** (a)

English Proficiency

81. (b)	**82.** (b)	**83.** (c)	**84.** (a)	**85.** (b)	**86.** (c)	**87.** (b)	**88.** (a)	**89.** (a)	**90.** (c)
91. (b)	**92.** (d)	**93.** (c)	**94.** (b)	**95.** (c)					

Logical Reasoning

96. (d)	**97.** (c)	**98.** (c)	**99.** (d)	**100.** (b)	**101.** (d)	**102.** (d)	**103.** (c)	**104.** (b)	**105.** (b)

Mathematics

106. (a)	**107.** (c)	**108.** (c)	**109.** (b)	**110.** (a)	**111.** (c)	**112.** (a)	**113.** (d)	**114.** (c)	**115.** (a)
116. (c)	**117.** (b)	**118.** (b)	**119.** (c)	**120.** (a)	**121.** (d)	**122.** (a)	**123.** (c)	**124.** (a)	**125.** (d)
126. (c)	**127.** (d)	**128.** (b)	**129.** (b)	**130.** (a)	**131.** (b)	**132.** (b)	**133.** (d)	**134.** (b)	**135.** (d)
136. (c)	**137.** (d)	**138.** (a)	**139.** (b)	**140.** (b)	**141.** (a)	**142.** (b)	**143.** (b)	**144.** (b)	**145.** (a)
146. (a)	**147.** (b)	**148.** (d)	**149.** (a)	**150.** (a)					

Hints & Solutions

Physics

1. ***(c)*** As coil 2 with inductance L_2 has been wounded by a similar wire as coil 1 but the direction of windings is reversed in each layer, so flux through L_2 will be zero.

As, $L_2 \propto \phi \Rightarrow L_2 = 0$

Also, since the dimensions, number of turns and number of layers of windings are equal in coil 1 and coil 3.

So, $L_1 = L_3$

2. ***(a)*** Since, capacitors C_1 and C_2 are parallel, so their equivalent capacitance will be $(C_1 + C_2)$. Now, $(C_1 + C_2)$ and C_3 are in series, so the net equivalent capacitance of circuit will be

$$\frac{1}{C} = \frac{1}{C_3} + \frac{1}{C_1 + C_2} = \frac{C_1 + C_2 + C_3}{(C_1 + C_2)C_3}$$

or $$C = \frac{(C_1 + C_2)C_3}{C_1 + C_2 + C_3}$$

Since, V is the voltage of the battery, so charge on this system,

$$q = CV \Rightarrow q = \frac{(C_1 + C_2)C_3 V}{(C_1 + C_2 + C_3)}$$

If the capacitance C_3 breaks down, then total equivalent capacitance will become

$$C' = C_1 + C_2$$

$\therefore$ New charge stored,

$$q' = C'V$$

$$\Rightarrow \quad q' = (C_1 + C_2)V$$

Now, change in total charge,

$$\Delta q = q' - q \qquad [\because q' > q]$$

$$= (C_1 + C_2)V - \frac{(C_1 + C_2)C_3 V}{C_1 + C_2 + C_3}$$

or $$\Delta q = (C_1 + C_2)V\left[1 - \left(\frac{C_3}{C_1 + C_2 + C_3}\right)\right]$$

3. ***(a)*** Let E' be the new radiant energy.

According to Stefan's law,

$$E \propto T^4$$

$$\Rightarrow \quad \frac{E_2}{E_1} = \left(\frac{T_2}{T_1}\right)^4$$

$$\Rightarrow \quad \frac{E_2}{E} = \left\{\frac{(T/4)}{T}\right\}^4 = \left(\frac{1}{4}\right)^4$$

or $$E_2 = \frac{E}{256}$$

4. ***(c)*** Area of rectangle, $A = lb = 15.2 \times 2.9 = 44.08\,\text{cm}^2$

Minimum possible measurement of scale = 0.1 cm

As we know, the product should have significant figures that are present in the measurement with least number of significant figures which is 2 in this case. So, area measured by scale = 44 cm^2.

5. ***(b)*** In adiabatic process,

$$pV^\gamma = K$$

where, K is a constant.

Given that, $\gamma = \frac{4}{3} \Rightarrow pV^{4/3} = K$

Taking logarithm on both sides, we get

$$\log p + \frac{4}{3}\log V = \log K$$

$$\Rightarrow \frac{\Delta p}{p} + \frac{4}{3}\frac{\Delta V}{V} = 0 \Rightarrow \frac{\Delta V}{V} = -\frac{3}{4}\frac{\Delta p}{p}$$

Now, % change in volume,

$$\frac{\Delta V}{V} \times 100 = -\frac{3}{4}\left(\frac{\Delta p}{p} \times 100\right)$$

It is given that, pressure decreases by $\frac{3}{4}$%,

i.e. $$\frac{\Delta p}{p} \times 100 = -\frac{3}{4}$$

$$\therefore \quad \frac{\Delta V}{V} \times 100 = -\frac{3}{4} \times \left(-\frac{3}{4}\right) = \frac{9}{16}\%$$

Hence, the volume increases by $\frac{9}{16}$%.

6. ***(b)*** The given equation is

$$y = 4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t) \qquad \ldots(\text{i})$$

Now, the velocity of the string at a point x at time t is obtained by differentiating Eq. (i) w.r.t.

$$\therefore v = \frac{dy}{dt} = -(4 \times 96\pi) \times \sin\left(\frac{\pi x}{15}\right)\sin(96\pi t)$$

At $x = 7.5$ cm $= 7.5 \times 10^{-2}$ m and $t = 0.25$ s,

$$v = -384\pi\sin\left(\pi \times \frac{7.5 \times 10^{-2}}{15}\right)\sin\left(96\pi \times \frac{25}{100}\right)$$

$$= -384\pi\sin\left(\frac{\pi}{200}\right)\sin(24\pi)$$

$$= 0 \qquad (\because \sin 24\pi = 0)$$

Hence, the velocity of the particle is zero.

7. ***(a)*** Potential at point A,

$$V_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{a}$$

Potential at point B,

$$V_B = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{b}$$

Work done in taking a charge Q from A to B,

$$W = Q(V_B - V_A) = \frac{Qq}{4\pi\varepsilon_0}\left[\frac{1}{b} - \frac{1}{a}\right] = \frac{Qq}{4\pi\varepsilon_0}\left[\frac{a-b}{ab}\right]$$

8. ***(b)*** For hydrogen-like atom,

$$\frac{1}{\lambda'} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \quad \ldots\text{(i)}$$

Similarly, for hydrogen atom,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \quad \ldots\text{(ii)}$$

For the element to have hydrogen-like spectrum, the ratio of the wavelengths for transition of electron between any two levels n_1 and n_2 should be constant.

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{\lambda'}{\lambda} = \frac{R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}{RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)} \quad \text{or} \quad \frac{\lambda'}{\lambda} = \frac{1}{Z^2}$$

It is given that, wavelength of the hydrogen atom is four times the wavelength of the hydrogen-like atom, i.e.

$$\lambda = 4\lambda'$$

Hence, $Z = 2$

Now, the element with atomic number 2 is helium.

9. ***(c)*** Given, two small conducting spheres of equal radius R.

Charge on first sphere, $q_1 = +20\mu C$

Charge on second sphere, $q_2 = -40\mu C$

$$\therefore \quad F_1 = \frac{k(+20)(-40)}{R^2} = \frac{-k(800)}{R^2} \quad \ldots\text{(i)}$$

The spheres have equal radii, so their capacities will be same. Now, if they are brought in contact, then the net charge,

$$q_{\text{net}} = (+20-40)\mu C = -20\mu C$$

So, each sphere will have charge, $q = \frac{q_{\text{net}}}{2} = -10\mu C$

$$\text{Now,} \quad F_2 = \frac{k(q)(q)}{R^2} = \frac{k(-10)(-10)}{R^2} = \frac{k(100)}{R^2} \quad \ldots\text{(ii)}$$

$$\text{Ratio,} \quad \frac{F_1}{F_2} = -\frac{800}{100} = -8:1$$

Thus, ratio of F_1 to $F_2 = -8:1$

10. ***(b)*** Efficiency of Carnot engine is given by

$$\eta = 1 - \frac{T_2}{T_1}$$

Given, $T_1 = 600$ K, $T_2 = 300$K,

$T_1' = 1600$K and $T_2' = x$K

Carnot engine has same efficiency between 600 K to 300 K and 1600 K to xK, so

$$\left(1 - \frac{T_2}{T_1}\right) = \left(1 - \frac{T_2'}{T_1'}\right)$$

Substituting all the given values in above equation, we get

$$1 - \frac{300}{600} = 1 - \frac{x}{1600}$$

$$1 - \frac{1}{2} = 1 - \frac{x}{1600}$$

$$\frac{x}{1600} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\Rightarrow \quad x = \frac{1600}{2} = 800 \text{ K}$$

11. ***(b)*** Given, initial speed of ball, $v = 16 \text{ ms}^{-1}$

Maximum height, $h = 9$ m

Mass of ball, $m = 0.5$ kg

Energy supplied to ball while throwing up,

$$E_1 = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.5 \times 16 \times 16 = 64 \text{ J}$$

Energy stored when ball reaches the maximum height,

$$E_2 = mgh = 0.5 \times 9.8 \times 9 = 44.1 \text{ J}$$

Energy dissipated by air drag acting on the ball during the ascent,

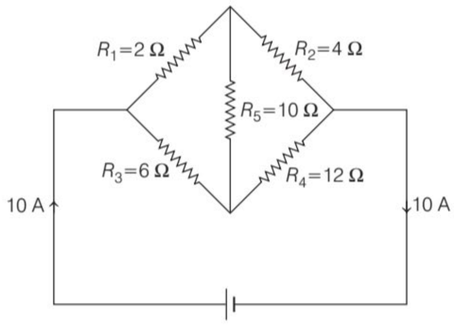
$$E = E_1 - E_2 = 64 - 44.1 = 19.9 \text{ J}$$

12. ***(b)*** From the following circuit diagram,

$\frac{R_1}{R_3} = \frac{R_2}{R_4}$, so the given Wheatstone bridge is balanced.

Hence, there is no current in the wire of 10 Ω resistance.

As,



Now, if the 10 Ω wire is replaced with 15 Ω wire, the bridge still remains balanced. Hence, current drawn from battery will be 10 A.

13. ***(b)*** Given, force, $F = 8t$ N

Mass, $m = 2\,\text{kg}$

From Newton's second law,

Force = Rate of change of momentum

$$F = \frac{dp}{dt}$$

i.e. $dp = Fdt$

$\therefore$ Momentum, $p = \int dp = \int_0^1 F \cdot dt = \int_0^1 8t \cdot dt$

$$= 8\left[\frac{t^2}{2}\right]_0^1 = 8\left(\frac{1}{2} - 0\right)$$

$\Rightarrow$ $p = 4\,\text{kg ms}^{-1}$

Change in kinetic energy,

$$\Delta K = \frac{p^2}{2m} = \frac{(4)^2}{2 \times 2} = 4\text{ J}$$

From work-energy theorem,

Work done = Change in kinetic energy

So, work done by the force during the first 1s,

$$W = \Delta K = 4\text{J}$$

14. ***(b)*** As, RC is the time constant, so it has the dimension of time and V has the dimensions of $[L]\left[\frac{di}{dt}\right]$.

$\therefore$ Dimensions of $\frac{L}{RCV}$, $\left[\frac{\text{L}}{\text{RCV}}\right] = \frac{[\text{L}]}{[\text{RC}]\,[\text{V}]}$

$$\frac{[\text{L}]}{[\text{T}][\text{L}]\left[\frac{\text{A}}{\text{T}}\right]} = \frac{[\text{L}][\text{T}]}{[\text{T}][\text{L}][\text{A}]} = \frac{1}{[\text{A}]}$$

So, $\left(\frac{\text{L}}{\text{RCV}}\right)$ has the dimensions $[\text{A}^{-1}]$, i.e. same as the dimensions of $\frac{1}{\text{current}}$.

15. ***(b)*** Given, magnetic field,

$$B = 10 \times 10^{-8} \sin(1 \times 10^7 z - 3.6 \times 10^{15} t)\text{ T} \quad \ldots\text{(i)}$$

Standard equation of magnetic field,

$$B = B_0 \sin(kz - \omega t) \quad \ldots\text{(ii)}$$

On comparing Eqs. (i) and (ii), we get

$$B_0 = 10 \times 10^{-8}\text{ T}$$

The average intensity of the beam,

$$I_{\text{av}} = \frac{B_0^2 c}{2\mu_0} = \frac{1}{2} \times \frac{(10 \times 10^{-8})^2 \times 3 \times 10^8}{4\pi \times 10^{-7}}$$

$$[\because c = 3 \times 10^8\text{ ms}^{-1}]$$

$$= 1.19\text{ W/m}^2$$

16. ***(c)*** Distance = area under speed-time graph as given in question.

$\therefore$ Distance travelled in 20 s = area of ΔOAB + area of ΔBCD

$$= \frac{1}{2} \times 6 \times 10 + \frac{1}{2} \times 6 \times 10 = 60\text{ m}$$

Displacement in 20 s = area of ΔOAB − area of ΔBCD

$$= \frac{1}{2} \times 6 \times 10 - \frac{1}{2} \times 6 \times 10 = 30 - 30 = 0$$

Hence, average velocity $= \frac{\text{displacement}}{\text{time}} = \frac{0}{t} = 0$

17. ***(b)*** The focal length of a convex lens of refractive index μ_g in air,

$$\frac{1}{f_{\text{air}}} = (\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots\text{(i)}$$

where, R_1, R_2 = radii of curvatures of its first and second surface.

When the lens is immersed in a liquid of refractive index μ_l, then refractive index of material of lens (glass) w.r.t. liquid is

$$_l\mu_g = \frac{\mu_g}{\mu_l} \quad \ldots\text{(ii)}$$

Now, focal length of lens in liquid,

$$\frac{1}{f'} = (_l\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots\text{(iii)}$$

Dividing Eq. (i) by Eq (iii), we get

$$\frac{f'}{f_{\text{air}}} = \frac{\mu_g - 1}{(_l\mu_g - 1)} = \frac{\mu_g - 1}{\left(\frac{\mu_g}{\mu_l} - 1\right)}$$

Given, $f' = -0.5$ m, $f_{\text{air}} = 0.2$ m, $\mu_g = 1.5$

Substituting all these values in above equation, we get

$$-\frac{0.5}{0.2} = \frac{1.5 - 1}{\left(\frac{1.5}{\mu_l} - 1\right)} = \frac{0.5}{\left(\frac{1.5}{\mu_l} - 1\right)}$$

$\Rightarrow$ $\frac{1.5}{\mu_l} - 1 = -0.2$

$\Rightarrow$ $\frac{1.5}{\mu_l} = 1 - 0.2 = 0.8$

$\Rightarrow$ $\mu_l = \frac{15}{8}$

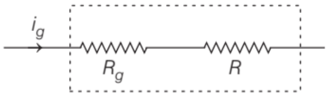
$\therefore$ Refractive index of liquid = 15/8

18. ***(b)*** Given, resistance of galvanometer, $R_g = 60\,\Omega$

Current, $I_g = 4.5\text{ mA} = 4.5 \times 10^{-3}\text{ A}$

Resistance used in converting a galvanometer into voltmeter, $R = 4.5\text{ k}\Omega = 4.5 \times 10^3\,\Omega$

To make the voltmeter, the galvanometer and the high resistance (R) have to be connected in series combination as shown below.

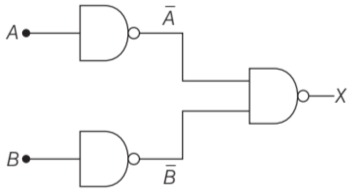


$\therefore$ Maximum current in galvanometer is

$$I_g = \frac{E}{R + R_g}$$

$$\Rightarrow \quad E = I_g(R + R_g)$$
$$= 4.5 \times 10^{-3} \times (4.5 \times 10^3 + 60)$$
$$= 20.5 \text{ V}$$

19. ***(b)*** The given combination of gates is shown below



The output of this combination can be determined by Boolean algebra.

$\therefore$ Output, $X = \overline{\bar{A} \cdot \bar{B}} = \bar{\bar{A}} + \bar{\bar{B}}$

[Using de-Morgan's theorem]

$$\Rightarrow \quad X = A + B$$

This is the output of OR gate. Hence, the given combination of gates yield OR gate.

20. ***(c)*** From gas equation,

$$pV = nRT \Rightarrow p = \frac{nRT}{V}$$

Pressure of one mole of O_2 gas,

$$(p)_{O_2} = \frac{1 \times RT}{V} \quad \ldots\text{(i)}$$

Pressure of one mole of He gas,

$$(p)_{\text{He}} = \frac{1 \times R(2T)}{V} \quad \ldots\text{(ii)}$$

Dividing Eq. (ii) by Eq. (i), we get

$$\therefore \quad \frac{(p)_{\text{He}}}{(p)_{O_2}} = 2 \Rightarrow (p)_{\text{He}} = 2(p)_{O_2} = 2p$$

21. ***(b)*** Acceleration, $a = \frac{dv}{dt} \Rightarrow dv = adt$

Integrating both sides, $\int dv = \int adt$

Given, $a = (3t^2 - 2t + 1) \text{ ms}^{-2}$

Substituting this value in above equation, we get

$$v = \int (3t^2 - 2t + 1)dt$$
$$= t^3 - t^2 + t + C \quad \ldots\text{(i)}$$

At $t = 1$ s, $v = 1$ m/s

Substituting these values in Eq. (i), we get

$$\because \quad 1 = 1 - 1 + 1 + C \quad \Rightarrow C = 0$$

Substituting the value of C in Eq. (i), we get

$$v = t^3 - t^2 + t$$

When $t = 4$ s, $v = (4)^3 - (4)^2 + 4 = 52$ m/s

22. ***(b)*** From ideal gas equation,

$$pV = nRT = \frac{mRT}{M}$$

where, $\quad n = \frac{\text{Molar mass } (m)}{\text{Molecular weight } (M)}$

Here, n = number of moles,

R = gas constant

and $\quad V$ = volume.

$$\Rightarrow \quad \frac{pV}{m} = \frac{RT}{M}$$

$$\Rightarrow \quad \frac{p}{(m/V)} = \frac{RT}{M}$$

$$\frac{p}{d} = \frac{RT}{M} \quad \left[\because \frac{m}{V} = d\right]$$

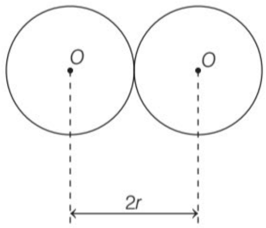
$$\Rightarrow \quad \frac{p}{d} \propto T$$

$\Rightarrow$ Slope of p *versus* d graph $\propto T$

Since, slope is less for the gas at temperature T_1.

Therefore, $\quad T_1 < T_2$

23. ***(d)*** The two spheres are touching each other as shown below.



Let m be the mass of each sphere.

Gravitational force between the spheres,

$$F = G\frac{m \cdot m}{(2r)^2} = G.\frac{m^2}{4r^2}$$

$$= G.\frac{\left[\frac{4}{3}\pi r^3 \cdot \rho\right]^2}{4r^2} \quad \text{[Mass = volume × density]}$$

$$= \frac{4G\pi^2 r^6 \rho^2}{9 \times 4r^2}$$

$$\Rightarrow \quad F = \left(\frac{4G\pi^2\rho^2}{9}\right) r^4 \Rightarrow F \propto r^4$$

24. ***(a)*** Power of first spherical lens, $P_1 = -4$ D

The power of this lens is negative, hence it is diverging lens.

Focal length of first spherical lens,

$$f_1 = \frac{1}{P_1} = \frac{1}{(-4\text{D})} = -0.25 \text{ m} = -25 \text{ cm}$$

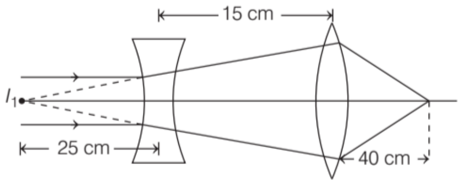
Power of second spherical lens, $P_2 = 5\text{D}$

The power of this lens is positive, hence it is converging lens.

Focal length of second spherical lens,

$$f_2 = \frac{1}{P_2} = \frac{1}{5\text{D}} = 0.2 \text{ m} = 20 \text{ cm}$$

The given situation is shown in the following figure.



Now, the image I_1 will be considered as object for converging lens.

For converging lens, distance of object,

$$u = -(25 + 15) = -40 \text{ cm}$$

and $\quad f_2 = 20$ cm

∴ Using lens formula,

$$\frac{1}{f_2} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \quad \frac{1}{20} = \frac{1}{v} - \frac{1}{-40}$$

$$\Rightarrow \quad v = 40 \text{ cm}$$

Since v is positive, so image formed will be real and will be formed at a distance of 40 cm from the lens of power 5 D (converging lens).

25. ***(a)*** From second equation of rotational motion,

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

Given, $\theta = 5$ rad, $t = 1$ s, $\omega_0 = 0$

Substituting all these values in above equation, we get

$$5 = 0 + \frac{1}{2}\alpha(1)^2$$

$$\Rightarrow \quad \alpha = 0 \text{ rad/s}^2$$

The angle rotated by wheel of car during first two seconds,

$$\theta_2 = 0 + \frac{1}{2} + \alpha t^2 = \frac{1}{2} \times 10 \times 2^2 = 20 \text{ rad}$$

Hence, angle rotated during the 2nd second

$$= \theta_2 - \theta$$
$$= 20 - 5 = 15 \text{ rad}$$

26. ***(d)*** Kinetic energy in photoelectric effect can be given as

$$\text{KE} = W - W_0 \qquad \begin{bmatrix} W \to \text{incident energy} \\ W_0 \to \text{work function} \end{bmatrix}$$

Given, $(\text{KE})_1 = 1.5 - 0.5 = 1$ eV

$(\text{KE})_2 = 2.5 - 0.5 = 1.5$ eV

$$\because \quad \frac{(\text{KE})_1}{(\text{KE})_2} = \frac{1}{1.5} = \frac{2}{3}$$

$$\Rightarrow \quad \frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2} = \frac{2}{3}$$

$$\Rightarrow \quad \frac{v_1}{v_2} = \sqrt{\frac{2}{3}}$$

$$\therefore \quad v_1 : v_2 = \sqrt{2} : \sqrt{3}$$

So, the ratio of maximum speeds of emitted electrons is $\sqrt{2}:\sqrt{3}$.

27. ***(d)*** Initial momentum, $p_i = mv$

$$p_i = m\sqrt{2gh} \qquad [\because v = \sqrt{2gh}]$$

New height of body, $h_1 = h + 3h = 4h$

Now, final momentum,

$$p_f = m\sqrt{2g.h_1} = m\sqrt{2g(4h)}$$
$$= m \cdot 2\sqrt{2gh} = 2p_i$$

$$\therefore \ \% \text{ change in momentum} = \frac{p_f - p_i}{p_i} \times 100$$
$$= \frac{2p_i - p_i}{p_i} \times 100 = 100\%$$

28. ***(a)*** In this case, the diode is forward biased. Potential drop in a germanium diode in forward bias is around 0.3 V.

Hence, current through the given circuit,

$$I = \frac{V_{\text{net}}}{R_{\text{eq}}} = \frac{5 - 0.3}{2 + 3} = \frac{4.7}{5}$$
$$= 0.94 \text{ A}$$

29. ***(b)*** Number of active nuclei after time t in a sample of radioactive substance is given by

$$N = N_0 e^{-\lambda t}$$

where, N_0 = initial number of nuclei at $t = 0$

and $\quad \lambda$ = decay constant.

Here, at $t = 0$, number of nuclei in sample X and Y are equal,

i.e., $(N_0)_X = (N_0)_Y = N_0$

Also, $\lambda_X = 4\lambda$ and $\lambda_Y = \lambda$

So, after time t, number of active nuclei in X and Y are

$N_X = N_0 e^{-4\lambda t}$ and $N_Y = N_0 e^{-\lambda t}$

$$\therefore \quad \frac{N_X}{N_Y} = e^{-3\lambda t}$$

$$\Rightarrow \quad \frac{1}{e^3} = \frac{1}{e^{3\lambda t}} \qquad \left[\text{Given, } \frac{N_X}{N_Y} = \frac{1}{e^3}\right]$$

$$\Rightarrow \quad 3\lambda t = 3 \Rightarrow t = \frac{1}{2\lambda}$$

30. ***(c)*** Given, current in the wire, $I = 10\text{A}$

Radius of the wire, $R = 20\pi \text{ cm} = 20\pi \times 10^{-2}$ m

Magnetic field produced by semi-circular current carrying thin wire at centre O,

$$B = \frac{\mu_0 I}{4R}$$

Substituting all the given values in above equation, we get

$$B = \frac{4\pi \times 10^{-7} \times 10}{4 \times 20\pi \times 10^{-2}}$$

$$= 5 \times 10^{-6} \text{ T}$$

Now, magnitude of force per unit length,

$$F = IB = 10 \times 5 \times 10^{-6}$$

$$= 50 \times 10^{-6} \text{ N/m}$$

$$= 50\,\mu\text{N/m}$$

31. ***(b)*** Given, area, $A = 5 \text{ cm}^2 = 5 \times 10^{-4} \text{ m}^2$

Number of turns per unit length, $n = 4000$ turns/m

Magnetic field, $B = \pi T$

Current, $i = 5$ A

Now, magnetic intensity,

$$H = ni = 4000 \times 5 = 2 \times 10^4 \text{ Am}^{-1}$$

As, $B = \mu_0 (H + I)$ [I = intensity of magnetisation]

Substituting all the values in above equation, we get

$$\pi = 4\pi \times 10^{-7} \{(2 \times 10^4) + I\}$$

$$\frac{1}{4} \times 10^7 = (2 \times 10^4) + I$$

$$250 \times 10^4 = 2 \times 10^4 + I$$

$$I = 250 \times 10^4 - 2 \times 10^4$$

$$\Rightarrow \quad I = 248 \times 10^4 \text{ A/m}$$

Now, pole strength, $m = I \times A = 248 \times 10^4 \times 5 \times 10^{-4}$

$$= 1240 \text{ A-m}$$

32. ***(b)*** Given, mass of plane, $M = 10000$ kg

Coefficient of friction, $\mu = 0.2$

Distance, $s = 50$ m

Initial speed, $u = 0$

Final speed, $v = 72 \text{ kmh}^{-1} = 20 \text{ ms}^{-1}$

Let a be the acceleration of the plane.

From third equation of motion,

$$v^2 - u^2 = 2as$$

$$(20)^2 - (0)^2 = 2 \times a \times 50$$

$$\therefore \quad a = \frac{20 \times 20}{100} = 4 \text{ ms}^{-2}$$

Force required to provide necessary acceleration to the plane,

$$F_1 = ma = 10000 \times 4 = 4 \times 10^4 \text{ N}$$

Force required to overcome friction by engine,

$$F_2 = \mu R = \mu mg = 0.2 \times 10000 \times 9.8$$

$$= 1.96 \times 10^4 \text{ N}$$

Therefore, the minimum force required by the engine,

$$F_1 + F_2 = 4 \times 10^4 + 1.96 \times 10^4$$

$$= 5.96 \times 10^4 \text{ N}$$

33. ***(c)*** Given, reverse voltage, $E = 120$ V

Change in current, $\Delta i = 0.50 - 0.20 = 0.30\text{A}$

Time, $\Delta t = 0.030 \text{ ms} = 0.030 \times 10^{-3}$ s

In an inductor (choke),

Emf induced, $E_{\text{induced}} = L\dfrac{di}{dt}$

$\therefore$ Self inductance of choke, $L = \dfrac{E \times \Delta t}{\Delta i}$

$$= \frac{120 \times 0.030 \times 10^{-3}}{0.30}$$

$$\Rightarrow \quad = 12 \times 10^{-3} \text{ H}$$

34. ***(b)*** Given, linear dimensions increase by a factor of 8.

So, volume will become $(8)^3$ times and area of cross-section will become $(8)^2$ times.

Now, stress $= \dfrac{\text{weight}}{\text{area}}$

Weight = (volume × density × g) will also become $(8)^3$ times.

$$\therefore \text{ New stress} = \frac{(8)^3 \times w_0}{(8)^2 \times A_0} = 8\left(\frac{w_0}{A_0}\right)$$

Hence, the stress increases by a factor of 8.

35. ***(c)*** Let, γ be the adiabatic exponent for the mixture.

For monoatomic gas, $n_1 = 2$ and $\gamma_1 = 5/3$

For diatomic gas, $n_2 = 3$ and $\gamma_2 = 7/5$

For the mixture, $\dfrac{n_1 + n_2}{\gamma - 1} = \dfrac{n_1}{\gamma_1 - 1} + \dfrac{n_2}{\gamma_2 - 1}$

Substituting all the given values in above equation, we get

$$\frac{2+3}{\gamma - 1} = \frac{2}{\frac{5}{3} - 1} + \frac{3}{\frac{7}{5} - 1}$$

$$\Rightarrow \quad \frac{5}{\gamma - 1} = \frac{21}{2} \Rightarrow \gamma = \frac{31}{21}$$

36. ***(c)*** Let a be the acceleration of the lift.
Given, the ratio of time periods of pendulum, while the lift is moving upwards and downwards,

$$T_1 : T_2 = 1 : 3$$

When lift is moving upwards, then total time period,

$$T_1 = 2\pi\sqrt{\frac{l}{g+a}} \quad \ldots\text{(i)}$$

When lift is moving downwards, then total time period,

$$T_2 = 2\pi\sqrt{\frac{l}{g-a}} \quad \ldots\text{(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{T_1}{T_2} = \sqrt{\frac{g-a}{g+a}}$$

$$\Rightarrow \quad \frac{1}{3} = \sqrt{\frac{g-a}{g+a}}$$

$$\text{or} \quad \frac{g-a}{g+a} = \left(\frac{1}{3}\right)^2$$

$$\text{or} \quad \frac{g-a}{g+a} = \frac{1}{9}$$

$$\Rightarrow \quad 9g - 9a = g + a$$

$$\Rightarrow \quad 9g - g = 9a + a \Rightarrow 10a = 8g$$

$$\Rightarrow \quad a = \frac{8g}{10} = \frac{8 \times 10}{10} = 8\,\text{m/s}^2$$

37. ***(a)*** As, orbital velocity of a satellite, $v = \sqrt{\frac{GM}{r}}$

Hence, $v \propto \frac{1}{\sqrt{r}}$

% increase in speed $= \frac{1}{2}$(% decrease in radius)

$$= \frac{1}{2}(2\%) = 1\%$$

Thus, the speed of satellite will increase by 1%.

38. ***(a)*** Given, $\omega = 2000$ rad s^{-1}

$$L = 5\text{ mH} = 5 \times 10^{-3}\text{ H}$$

$$C = 50\mu\text{F} = 50 \times 10^{-6}\text{ F}$$

$$V = 30\text{ V}$$

Inductive reactance,

$$X_L = \omega L = 2000 \times 5 \times 10^{-3} = 10\,\Omega$$

and capactive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{50 \times 10^{-6} \times 2000} = \frac{100}{10} = 10\,\Omega$$

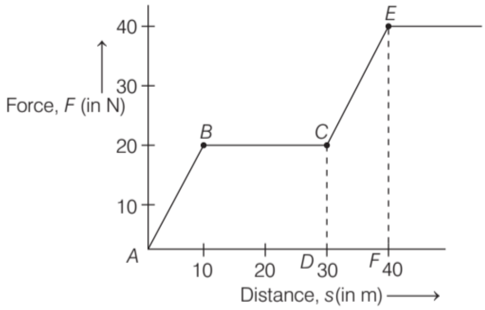
$$\Rightarrow \quad X_L = X_C$$

The given circuit is resonant circuit.
Hence, impedance of circuit, $Z = R$

$$= 4 + 6 + 0.5 = 10.5\,\Omega$$

Amplitude of current, $I_0 = \frac{V}{Z} = \frac{30}{10.5} = 2.85$ A

39. ***(d)*** Work done, W = area under F-s graph



$\therefore$ Total work done is covering an initial distance of 40 m,

W = area of trapezium $ABCD$ + area of trapezium $CEFD$

$$= \frac{1}{2} \times 20(20 + 30) + \frac{1}{2} \times 10 \times (20 + 40) = 800\text{ J}$$

[$\because$ Area of trapezium $= \frac{1}{2} \times$ height $\times$ sum of parallel sides]

$$\Rightarrow \quad W = 800\text{ J}$$

40. ***(b)*** Given, potential gradient $= 0.4$ mV cm^{-1}

$$= 0.4 \times 10^{-3}\text{ V cm}^{-1}$$

Length of wire, $l = 10$ m $= 1000$ cm

Electromagnetic force of cell, $E = 4$ V

Potential gradient along wire

$$= \frac{\text{Potential difference along wire}}{\text{Length of wire}}$$

$$\Rightarrow \quad 0.4 \times 10^{-3} = \frac{i \times 40}{1000}$$

$$\therefore \quad \text{Current in wire, } i = \frac{4}{400}\text{ A} \quad \ldots\text{(i)}$$

Let R be the required resistance unplugged in the box.

$$\therefore \quad i = \frac{E}{R + R'} = \frac{4}{40 + R} \quad \ldots\text{(ii)}$$

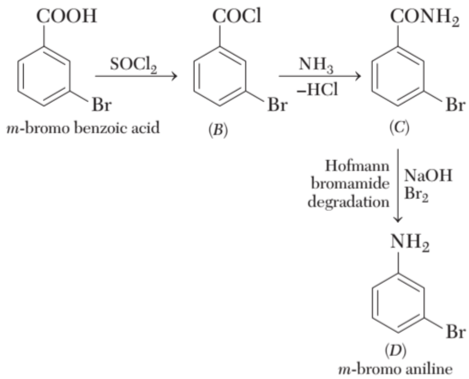
Equating Eqs. (i) and (ii), we get

$$\frac{4}{40 + R} = \frac{4}{400}$$

$$\Rightarrow \quad R = 400 - 40 = 360\,\Omega$$

Chemistry

41. ***(d)*** The given reactions take place as follows



42. ***(b)*** From molarity equation, $\underset{\text{Acid}}{M_1V_1} = \underset{\text{Base}}{M_2V_2}$

$$M_1 \times 50.0 = 0.10 \times 39.30$$

$$\therefore \quad M_1 = \frac{0.10 \times 39.30}{50}$$

$$= 0.0786 \text{ M}$$

$$\approx 0.079 \text{ M}$$

When half equivalence point is reached,

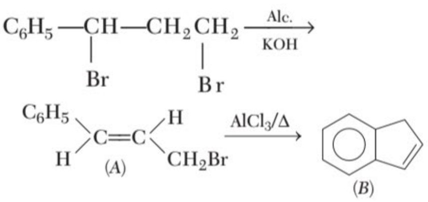
$$[HA] = [A^-]$$

Using Handerson-Hasselbalch equation

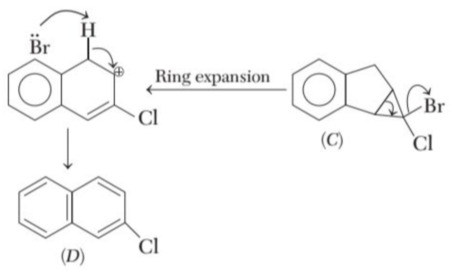
$$pH = pK_a + \log\frac{[A^-]}{[HA]}$$

$$\therefore \quad pH = pK_a = 4.85$$

43. ***(d)*** The complete equation is given as follows:



:CuBr compound gives :CClBr. This reaction is addition of carbene to double bond.



44. ***(a)*** Statement (a) is incorrect because α-D-fructose and β-D-fructose are anomers, diastereomers and geometrical isomers but not enantiomers.

Rest of the given statements are correct.

45. ***(d)*** Option (d) does not contain a pair of mixed oxides. Fe_2O_3 is not a mixed oxide.

While, Mn_3O_4 $(2MnO + MnO_2)$; Co_3O_4 (CoO + Co_2O_3); Fe_3O_4 (Fe_2O_3 + FeO) and Pb_3O_4 (2PbO + PbO_2), all are mixed salts.

46. ***(a)*** According to Faraday's first law of electrolysis:

The reaction at cathode,

$$\underset{63.5\text{ g}}{Cu^{2+}} + \underset{2 \times 96500\text{ C}}{2e^-} \longrightarrow Cu$$

The quantity of charge passed

$$= I \times t = (10 \text{ amp}) \times (60 \times 60\text{s}) = 36000 \text{ C.}$$

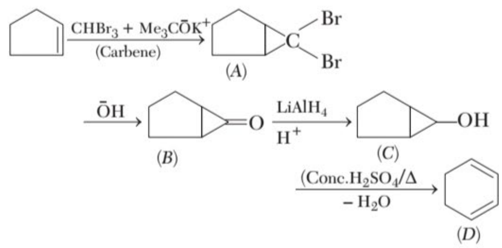
2×96500 C of charge deposit copper = 63.5 g

36000 C of charge deposit copper

$$= \frac{(63.5 \text{ g})}{(2 \times 96500 \text{ C})} \times (36000\text{C}) = 11.84 \text{ g}$$

Thus, 11.84 g of copper will dissolve from the anode and the same amount from the solution will get deposited on the cathode. The concentration of the solution will remain unchanged.

47. ***(b)*** The complete reaction is given as follows:



48. ***(a)*** Statement (a) is correct. Rest of the all statements are incorrect. These are explained as follows:

(a) $Cl_2(g) \longrightarrow 2Cl(g)$

As randomness is increasing. Therefore, ΔS is positive for this reaction.

∴ Statement is correct.

(b) In closed container, $\Delta V = 0$, hence work done is zero. There is no heat exchange as system is adiabatic. Hence, $\Delta E = q + W = 0$.

∴ Statement is incorrect.

(c) ΔG will be zero only when equilibrium is reached.

∴ Statement is incorrect.

(d) $\Delta G° = -RT \ln K_{eq}$, not a function of pressure. Thus, this statement is also incorrect.

49. ***(b)*** Statements I and III are correct and statement II is incorrect.

(I) $\dfrac{T_c}{p_c} = \dfrac{8a}{27Rb} \cdot \dfrac{27b^2}{a} = \dfrac{8b}{R}$

Thus, $\frac{T_c}{p_c} \propto b$

Thus, larger the T_c / p_c value, larger would be b i.e. included volume.

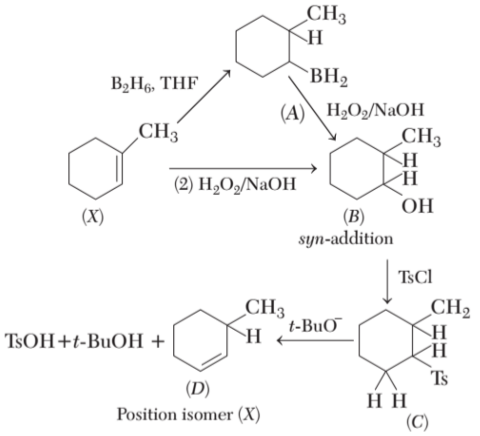
(II) $T_C = \frac{8a}{27Rb}$ and $T_B = \frac{a}{Rb}$

Thus, $T_B > T_C$

(III) When, T_C is attained.

$\therefore \quad (\partial p / \partial V)_{T_C} = 0$

50. ***(a)*** The reaction can be completed as follows:



51. ***(d)*** The reaction can be completed as follows

$$Na_2B_4O_7 + \text{conc.}H_2SO_4 + 5H_2O \longrightarrow Na_2SO_4 + \underset{(A)}{4H_3BO_3}$$

$$H_3BO_3 + 3C_2H_5OH \longrightarrow \underset{(B)}{B(OC_2H_5)_3}$$

Volatile

(Burn with green edged flame)

52. ***(a)*** Perhydrol means 30% solution of H_2O_2.

H_2O_2 decomposes as :

$$2H_2O_2 \longrightarrow 2H_2O + O_2$$

Volume strength of 30% H_2O_2 solution is 100 that means 1 mL of this solution on decomposition gives 100 mL oxygen.

$$SO_2 + \frac{1}{2}O_2 \longrightarrow SO_3$$

SO_2	$\frac{1}{2}O_2$	SO_3
1 L	1/2 L	1 L
2 L	1 L	2 L

Since, 100 mL of oxygen is obtained by 1 mL of H_2O_2.

$\therefore$ 1000 mL of oxygen will be obtained by

$$= \frac{1}{100} \times 1000 \text{ mL of } H_2O_2$$

$$= 10 \text{ mL of } H_2O_2$$

53. ***(c)*** At top of a mountain,

$T_1 = 0 + 273\text{K} = 273\text{K}, p_1 = \frac{710}{760}$ atm and density $= \rho_1$

At bottom of mountain,

$T_2 = 30 + 273 = 303$ K, $p_2 = 1$ atm and density $= \rho_2$

From ideal gas equation,

$\because \quad pV = \frac{w}{M}RT \qquad \left[p = \frac{w}{MV}RT; \rho = \frac{w}{V}\right]$

or $\quad p = \rho\frac{RT}{M}$

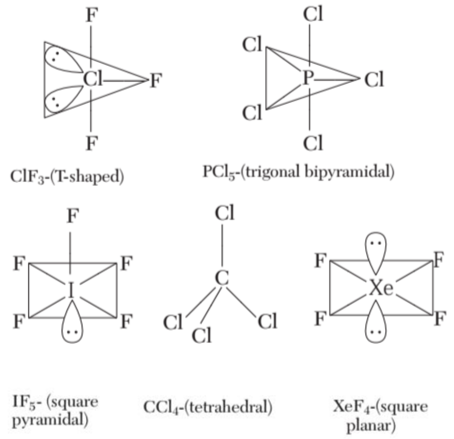
$\therefore \quad \frac{\rho_1}{\rho_2} = \frac{p_1}{T_1} \times \frac{T_2}{p_2}$

or $\quad \frac{\rho_1}{\rho_2} = \frac{710/760 \times 303}{273 \times 1} = \frac{1.04}{1}$

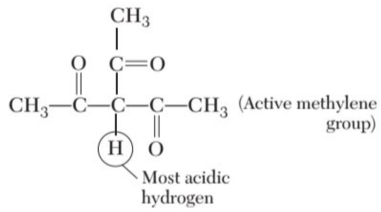
54. ***(b)*** The correct option is (b).

A $\rightarrow$ 5, B $\rightarrow$ 3, C $\rightarrow$ 4, D $\rightarrow$ 2, E $\rightarrow$ 1.

The shapes of molecules are given below.



55. ***(c)*** In option (c), the hydrogen of —CH group is surrounded by three $—\overset{O}{\overset{\|}{C}}—CH_3$ groups as:



Here, O of CO attracts, the electron density towards itself.

Or

The anion formed after removal of acidic proton in $\left(CH_3—\overset{O}{\overset{\|}{C}}—\right)_3$ is stablised by $-R$-effect of three C$=$O groups.

Thus, (c) has most acidic hydrogen among given compounds.

56. ***(c)*** The process by which the aquatic life gets deprived oxygen and results in subsequent loss of biodiversity is known as eutrophication. It is because of reduction in concentration of the dissolved oxygen in water due to phosphate pollution in water.

57. ***(d)*** As we know,

$$\nu = \frac{1}{\lambda} = \frac{(\text{IE})_H Z^2}{hc} x - \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

Given, $n_1 = 2, n_2 = 5$

$$(\text{IE})_H = 13.6 \text{ eV} = 13.6 \times 1.6 \times 10^{-19} \text{ J}$$

$$\nu = \frac{1}{\lambda} = \frac{13.6 \times 1.6 \times 10^{-19} \text{J}}{6.6 \times 10^{-34} \text{Js} \times 3 \times 10^8 \text{ ms}^{-1}}\left[\frac{1}{2^2} - \frac{1}{5^2}\right]$$

$$= 2.2988 \times 10^6 \text{ m}^{-1}$$

$$\therefore \quad \lambda = \frac{1}{\nu} \Rightarrow \lambda = 4.35 \times 10^{-7} \text{m} = 435 \times 10^{-9} \text{ m}$$

$$= 435 \text{ nm}$$

58. ***(c)*** Heat change at constant pressure means enthalpy change ($\Delta_r H = q_p$).

Heat of formation of CO is written as :

$$C(s) + \frac{1}{2}O_2(g) \longrightarrow CO(g);\ \Delta_f H^\ominus = ?$$

Now we have,

I. $C(g) + O_2(g) \longrightarrow CO_2(g)$;

$$\Delta_c H_1^\ominus = -393.5 \text{ kJ/mol}$$

II. $CO(g) + \frac{1}{2}O_2(g) \longrightarrow CO_2(g)$;

$$\Delta_c H_2^\ominus = -283 \text{ kJ/mol}$$

Substracting appropriately (i.e., I–II), we get,

$$C(s) + \frac{1}{2}O_2(g) \longrightarrow CO(g);\ \Delta_f H^\ominus$$

From Hess's law, $\Delta_f H^\ominus = \Delta_c H_1^\ominus - \Delta_c H_2^\ominus$

$\Rightarrow \Delta_f H^\ominus = -393.5 - (-283) = 110.5$ kJ/mol

Now, calculation of the heat of formation at constant volume means that we have to calculate change in internal energy (i.e. $\Delta_r U$).

Using, $\Delta_r H = \Delta_r U + p\Delta V$ [For a chemical reaction]

$= \Delta_r U + \Delta n_g RT$ [As $pV = nRT$]

[Δn_g = gaseous moles of products – gaseous moles of reactant]

$\Rightarrow \quad \Delta_r U = \Delta_r H - \Delta n_g RT$

Now putting the values : $\Delta n_g = 1 - \frac{1}{2} = \frac{1}{2}, T = 298$ K,

$R = 8.314$ J/K mol and $\Delta_f H^\ominus = -110.5$ kJ/mol

$$\Rightarrow \Delta_f U^\ominus = -110.5 - \frac{1}{2} \times 8.314 \times 2.98 \times 10^{-3}$$

$$= -111.7 \text{ kJ/mol}$$

59. ***(b)*** If $n = n$, then $l = 0, 1, 2 \ldots n$.

n	l	**Sub-orbit**	(n + l)
1	0	1*s*	1
	1	1*p*	2
	0	2 *s*	2
2	1	2*p*	3
	2	2*d*	4
	0	3*s*	3
3	1	3*p*	4
	2	3*d*	5
	3	3*f*	6

Thus, energy order is,
$1s < 1p < 2s < 2p < 3s < 2d < 3p$

Sc (21) will have electronic configuration as :

$$1s^2\, 1p^6\, 2s^2 2p^6\, 3s^2\, 2d^3$$

60. ***(c)***

	A	+	$2B$	$\rightleftharpoons$	$2C$	+	D	
Initial	1.1		2.2		0		0	
At equili.	$1.1-x$		$2.2-2x$		$2x$		x	$[2x = 0.2]$
	$1.1-0.1$		$2.2-0.2$		0.2		0.1	$[x = 0.1]$
	1		2		0.2		0.1	

$$K_C = \frac{[C^2][D]}{[A][B^2]} = \frac{(0.2)^2\,(0.1)}{(1)(2)^2} = 0.001$$

61. ***(c)*** Chemical reaction given in option 'c' is correct. Rest of the given chemical reactions are incorrect.

These can be corrected as:

(a) $CaCN_2 + 3H_2O \longrightarrow CaCO_3 + 2NH_3$

(b) $2NH_3 + CaSO_4 + CO_2 + H_2O \longrightarrow (NH_4)_2SO_4 + CaCO_3$

(d) $CaC_2 + H_2O \longrightarrow 2C_2H_2 + Ca(OH)_2$

62. ***(d)*** Disproportionation means a reaction in which a substance is oxidised as well as reduced simultaneously,

$$Na_2S_2O_3 + H_2SO_4 \longrightarrow Na_2SO_4 + SO_2 + S_8 + H_2O$$

Oxidation number of S in $Na_2S_2O_3 = +2$

Oxidation number of S in $H_2SO_4 = +6$

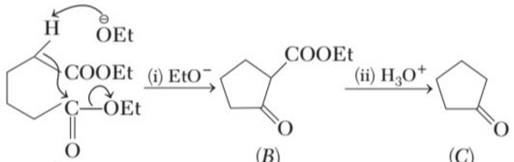
Oxidation number of S in $SO_2 = +4$

Oxidation number of S in $Na_2SO_4 = +6$

Oxidation number of S in $S_8 = 0$

So, sulphur can oxidise from + 2 to + 4 as well as reduce to 0 (S_8).

63. ***(d)*** Right answer is *d* because after heating (Δ), dcarboxylation occurs.



(*A*) (*B*) (*C*)

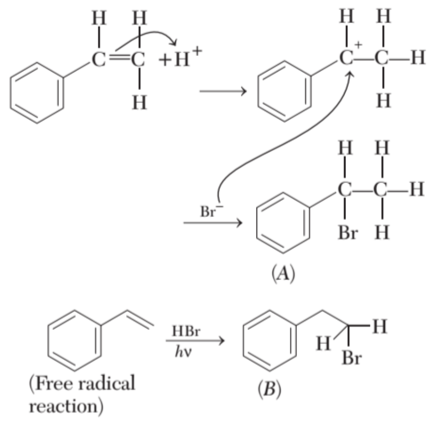
64. ***(a)*** $-\Delta S = \frac{\Delta G}{T}$, also $\frac{\Delta G}{T}$ is slope in Ellingham diagram. ΔS is different for different reactions as they have different slopes.

Rest of the statements (b), (c) and (d) are correct about Ellingham diagram.

65. ***(c)*** Phenolphthalein indicator changes into pink colour due to change in pH and not due to adsorption.

66. ***(c)*** The reaction take place as follows

$$HBr \longrightarrow H^+ + Br^-$$



67. ***(a)*** Z_{eff} for fcc = 4, Z_{eff} for bcc = 2

Atomic volume of α-form $= \frac{(3.68 \times 10^{-8})^3}{4} \times N_A$

Atomic volume of β-form $= \frac{(2.92 \times 10^{-8})^3}{2} \times N_A$

As atomic weight is same, element is same, so the density ratio is

$$\rho_\alpha : \rho_\beta = V_\beta : V_\alpha = \frac{(2.92)^3}{2} : \frac{(3.68)^2}{4}$$

$$= \frac{24.9}{2} : \frac{4}{49.8} = 1:1$$

68. ***(c)*** $\therefore \; K_b = \frac{RT_b^2 M_1}{\Delta H_{vap}}$

$$= \frac{8.314 \times (373)^2 \times 0.018}{40585 \text{ J/mol}}$$

$= 0.513$ K kg/mol

Now, molecular weight of $Al_2(SO_4)_3 = 342$

$$\underset{1-\alpha}{Al_2(SO_4)_3} \rightleftharpoons \underset{2\alpha}{2Al^{3+}} + \underset{3\alpha}{3SO_4^{2-}}$$

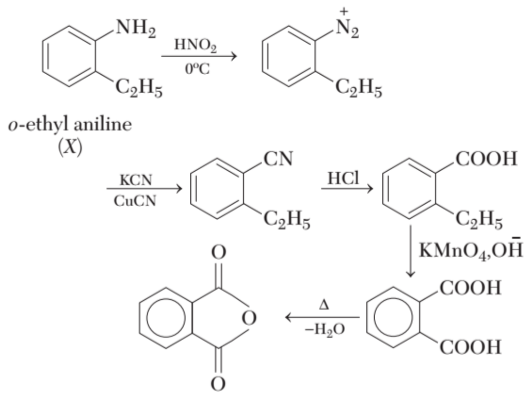
$i = 1 - \alpha + 2\alpha + 3\alpha = 1 + 4\alpha \Rightarrow i = 5 \qquad (\because \alpha = 1)$

$\Delta T_b = i \times K_b . m$

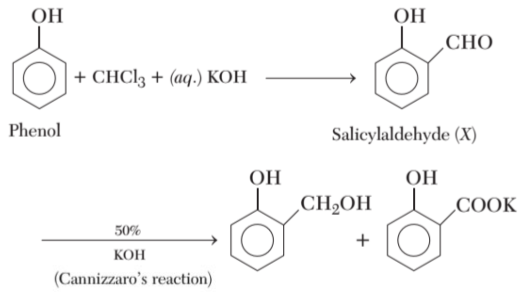
$$= 5 \times 0.513 \times \frac{5.6}{342 \times 1} = 0.042$$

Boiling point of solution = 100 + 0.042 = 100.042°C

69. ***(d)*** The reaction take place as follows



70. ***(b)*** Phenol on reaction with chloroform and KOH gives salicylaldehyde, which with 50% KOH solution undergoes Cannizzaro's reaction.



71. ***(d)*** Some complex ions of nitrogen show similarities with halide ions. These are called pseudohalide ions e.g., OCN^-, NCO^-, CN^- etc.

Halogens, among themselves, form complex ions which are called polyhalide ions. e.g., I_3^-, BrI_2^- etc.

Similarly, interhalogens are the compound of halogens in which one halogen is cation and other halogen is anion. e.g., IF_7, ICl_5, BrF_5, IF_5 etc.

$\therefore$ The correct order of pseudohalide polyhalide and interhalogen is

$$OCN^-, BrI_2^-, IF_5$$

72. ***(b)*** The inorganic halide (*A*) is SO_2Cl_2.

$$\underset{(A)}{SO_2Cl_2} + 2H_2O \longrightarrow \underset{(B)}{H_2SO_4} + \underset{(C)}{2HCl}$$

$$\underset{(A)}{SO_2Cl_2} + 4NaOH \longrightarrow \underset{(D)}{Na_2SO_4} + \underset{(E)}{2NaCl} + 2H_2O$$

$$\underset{(D)}{Na_2SO_4} + BaCl_2 \longrightarrow \underset{\text{White ppt.}}{BaSO_4} + 2NaCl$$

$$\underset{(E)}{NaCl} + AgNO_3 \longrightarrow \underset{\text{White ppt.}}{AgCl} + NaNO_3$$

73. ***(d)*** In alkali metals, reactivity increases down the group as electropositivity increases but for halogens reactivity decreases down the group as molecular stability of halogens increases.

74. ***(b)*** IUPAC name of sodiumnitroprusside $Na_2[Fe(CN)_5NO]$ is sodium pentacyanonitrosyl ferrate (III) sodium pentacyanonitrosyl ferrate (III) because in it NO is neutral ligand and the oxidation number of Fe is III.

75. ***(d)***

$$\begin{array}{lcccc} & A(g) \longrightarrow & P(g) + & Q(g) + & R(g) \\ \text{At } t=0, & 0.4 \text{ atm} & 0 & 0 & 0 \\ \text{At time} = t, & 0.4-x & x & x & x \end{array}$$

$$P_t = 0.4 - x + x + x + x = 0.4 + 2x \text{ or } x = \frac{P_t - 0.4}{2}$$

$$P_A = P_0 - x = P_0 - \frac{P_t - 0.4}{2} = \frac{2 \times 0.4 - P_t + 0.4}{2}$$

$$= \frac{1.2 - P_t}{2}$$

For a first order reaction,

$$k = \frac{2.303}{t} \log \frac{P_0}{P_A}$$

$$\frac{0.693}{t_{1/2}} = \frac{2.303}{230} \log\left(\frac{0.4 \times 2}{1.2 - P_t}\right)$$

$$\frac{0.693}{t_{1/2}} = \frac{2.303}{230} \log\left(\frac{0.8}{1.2 - P_t}\right)$$

$$\log \frac{0.8}{1.2 - P_t} = \frac{0.693 \times 230}{69.3 \times 2.303} = 0.9987$$

$$\frac{0.8}{1.2 - P_t} = 10^{0.9987}$$

$$0.8 = (10)(1.2 - P_t)$$

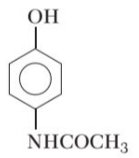
$$0.8 = 12 - 10P_t$$

$$10P_t = 12 - 0.8$$

$$10P_t = 11.2$$

$$P_t = \frac{11.2}{10} = 1.12 \text{ atm}$$

76. ***(d)*** The structure of an important antipyretic paracetamol is



So, *p*-amino phenol gives paracetamol on actylation.

77. ***(b)*** On passing H_2S gas in the aqueous solution of salt in the presence of dilute HCl, black ppt. of CuS is formed.

On boiling CuS(ppt.) with dil. HNO_3, $Cu(NO_3)_2$ (blue colour) is formed. Which gives deep blue solution of $[Cu(NH_3)_4]^{2+}$.

$$3CuS + 8HNO_3 \longrightarrow 3Cu(NO_3)_2 + 2NO + 3S + 4H_2O$$

$$Cu^{2+} + 4NH_4OH \longrightarrow \underset{\text{Deep blue complex}}{[Cu(NH_3)_4]^{2+}} + 4H_2O$$

78. ***(a)*** $5C_2O_4^{2-} + 2MnO_4^- + 16H^+ \longrightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$

Note In a reaction (redox/neutralisation) number of equivalents of different reactants are same.

Equivalents of $C_2O_4^{2-}$ = Equivalents of MnO_4^-

$$\text{Equivalent weight of } C_2O_4^{2-} = \frac{\text{Molecular weight}}{2}$$

$\therefore$ Molarity = 2 × Normality

$$\text{Equivalent weight of } MnO_4^- = \frac{\text{Molecular weight}}{5}$$

Molarity = 5 × Normality

$$\text{Number of equivalents of } MnO_4^- = \frac{28.85 \times 5\, M_1}{1000}$$

$$\text{Number of equivalents of } C_2O_4^{2-} = \frac{0.1467}{67}$$

$$\therefore \quad \frac{28.85 \times 5\, M_1}{1000} = \frac{0.1467}{67}$$

$$M_1 = 0.01518 \text{ M}$$

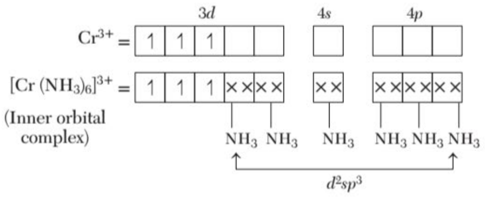
79. ***(c)*** $\underbrace{[Cr(NH_3)_6]}_{\text{Coordination sphere}} \underbrace{Cl_3}_{\text{Ionisable}} \rightleftharpoons [Cr(NH_3)_6]^{3+} + 3Cl^-$

$$[Cr(NH_3)_6]Cl_3 \xrightarrow{AgNO_3} \underset{\text{(White ppt.)}}{AgCl\downarrow}$$

Configuration of Cr (24) = $3d^5\ 4s^1$

Configuration of $Cr^{3+} = 3d^3 4s^0$

$\therefore$ It is paramagnetic nature due to 3 unpaired electrons.



(a) d^2sp^3 hybridisation, octahedral. Thus, option (a) is correct.

(b) Here are three unpaired electrons, hence it is paramagnetic. Thus, option (b) is correct.

(c) d^2sp^3 inner orbital complex. Thus option (c) is incorrect.

(d) Due to ionisable Cl^- ions, white ppt. with $AgNO_3$. Thus, option (d) is correct.

So, the incorrect statement is (c).

80. ***(a)*** $H_2SO_4(aq) \longrightarrow H^+(aq) + HSO_4^-(aq)$;

$H_2SO_4^-(aq) \longrightarrow H^+(aq) + SO_4^-(aq)$

Here, $K_{a_2} < K_{a_1}$ because the negatively charged HSO_4^- ion has much less tendency to donate a proton to H_2O as compared to neutral H_2SO_4.

a. English Proficiency

81. ***(b)*** Article 'the' should be used before 'famous' as the sentence refers to a particular thing i.e. a famous monument.

82. ***(b)*** As the sentence refers to a choice among more than two persons (servants), 'which' will be used in place of 'who'.

83. ***(c)*** The sentence is incomplete as it does not answer the question 'helped whom?', So we add 'him' at the end of the sentence.

84. ***(a)*** Use of preposition 'with' is suitable to fill the given blank.

85. ***(b)*** Use of preposition 'at' is suitable to fill the given blank.

86. ***(c)*** 'Benign' means not likely to cause death. 'Fatal' would be its opposite.

87. ***(b)*** 'Noted' is known by many people because of particular qualities. Its opposite would be 'Unknown'.

88. ***(a)*** 'Sagacious' means having or showing understanding and the ability to make good judgements. So, 'Foolish' would be its correct antonym.

89. ***(a)*** 'Getting peevish' means to get irritated.

90. ***(c)*** 'Roughed out' is a phrasal verb which means to draw a 'rough draft'. So, 'drew a quick plan' is its correct synonym.

91. ***(b)*** The need for an effective population policy is an urgent necessity in the country's planning strategy. Hence, option (b) is the correct answer.

92. ***(d)*** The development of human resource and the building up of an institutional framework would have to receive priority attention.

93. ***(c)*** The Centre and the States government must become partners in the planning process to determine national priorities together. Hence, the statement given in option (c) is not correct.

94. ***(b)*** Domestic economic situation and world trends would force the planning process to undergo a change.

95. ***(c)*** Important changes in the international scene is implied by the expression 'momentous trends.'

b. Logical Reasoning

96. ***(d)*** The pattern is as follows

$$2\times 6 = 12$$
$$6\times 12 = 72$$
$$12\times 72 = 864 \neq \boxed{865}$$
$$72\times 864 = 62208$$

Hence, number 865 is wrong and should be replaced by 864.

97. ***(c)*** According to the question,

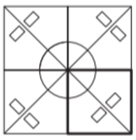
$$A / Q > T > P / B$$

But P is not the shortest

$$A / Q > T > P > B$$

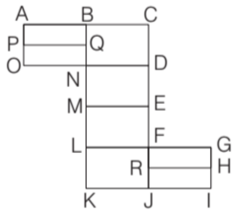
So, tallest is either A or Q.

98. ***(c)*** Option figure (c) will complete the given pattern as



99. ***(d)*** 'Spanner' is used by 'mechanic' as a tool to loose or tight nut and bolt. In the same way, 'saw' is used by 'carpenter' as a tool to cut wood. Hence, option (d) is correct.

100. ***(b)***



Total 18 rectangles are as follow

▭ ABQP,	▭ PQNO,	▭ BCDN,
▭ NDEM,	▭ MEFL,	▭ LFJK,
▭ FGHR,	▭ RHIJ,	▭ ABNO,
▭ BCEM,	▭ NDFL,	▭ MEJK,
▭ FGIJ,	▭ ACDO,	▭ BCFL,
▭ NDJK,	▭ LGIK and	▭ BCJK

101. ***(d)*** As, $B \xrightarrow{-1} A \xrightarrow{+3} D \xrightarrow{-1} C$

$$J \xrightarrow{-1} I \xrightarrow{+3} L \xrightarrow{-1} K$$
$$N \xrightarrow{-1} M \xrightarrow{+3} P \xrightarrow{-1} O$$

But, $\boxed{V \xrightarrow{-1} U \xrightarrow{+2} W \xrightarrow{+1} X}$

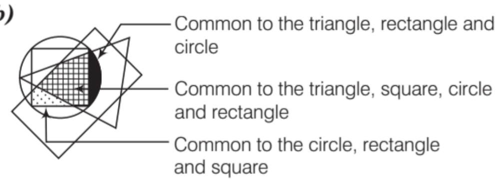
Except option (d), all others follow same pattern (– 1, + 3, – 1).

102. ***(d)*** Clearly, there are three types of shading of circles, one is unshaded another has its right half shaded and yet another has its upper half shaded. Also, two of the figures in each row have one triangle shaded.

103. ***(c)*** Option figure (c) can be formed from the cut out pieces given in question figure.

104. ***(b)*** After unfolding the folded and cut piece of paper it will look like as answer figure (b).

105. ***(b)***



Mathematics

106. ***(a)*** Given that, $A = \{x : x^2 = 1\} = \{-1, 1\}$ and $B = \{x : x^4 = 1\} = \{-1, 1, -i, i\}$

Now, $A - B = \phi$ and $B - A = \{-i, i\}$

$\therefore \quad A \Delta B = (A - B) \cup (B - A) = \{-i, i\}$

107. ***(c)*** We have, $2f(xy) = (f(x))^y + (f(y))^x$, $x, y \in R$ and $f(1) = a$.

Putting $x = 1, y = 2$, we get

$$2f(2) = (f(1))^2 + f(2) \Rightarrow f(2) = a^2$$

Putting $x = 1, y = 3$, we get

$$2f(3) = (f(1))^3 + f(3) \Rightarrow f(3) = a^3, \text{ and so on.}$$

$\therefore \quad f(n) = a^n$

$$\sum_{i=1}^{n} f(i) = a + a^2 + .. + a^n = \frac{a(a^n - 1)}{a - 1}.$$

108. ***(c)*** The given inequality is equivalent to

$$1 < |x - 3| + |4 - x|$$

i.e., $$1 < |x - 3| + |x - 4| \quad ...(i)$$

For $x < 3$, we have

$$3 - x + 4 - x = 7 - 2x > 1$$

i.e., $x < 3$, which is true in the domain.

For $3 \le x < 4$, Eq. (i) gives

$$x - 3 + 4 - x = 1 \not> 1 \quad ...(ii)$$

For $4 \le x$, we have

$$2x - 7 > 1 \Rightarrow x > 4 \quad ...(iii)$$

From Eqs. (i) and (iii), we have the solution of the inequality as $x < 3$ or $x > 4$.

i.e., $x \in]-\infty, 3) \cup (4, \infty[$ which is equal to $R - [3, 4]$.

109. ***(b)*** $\because$ 13, $a_1, a_2, ..., a_{20}$, 67 are in AP.

$$\therefore \; a_1 + a_2 + a_3 + \ldots + a_{20} = 20\left(\frac{13 + 67}{2}\right) = 800$$

Also, AM > GM

$$\Rightarrow \quad \frac{a_1 + a_2 + \ldots + a_{20}}{20} \ge (a_1 \cdot a_2 \cdot a_3 \cdot \ldots \cdot a_{20})^{\frac{1}{20}}$$

$$\Rightarrow \quad 40 \ge (a_1 \cdot a_2 \cdot a_3 \ldots a_{20})^{\frac{1}{20}}$$

Hence, maximum value of $a_1 \cdot a_2 \cdot a_3 \ldots a_{20}$ is $(40)^{20}$.

110. ***(a)*** Since, p, q and r are positive and are in AP.

$$\therefore \quad q = \frac{p + r}{2} \quad ...(i)$$

$\because$ The roots of the equation, $px^2 + qx + r = 0$ are real.

$$\Rightarrow \quad q^2 \ge 4pr \Rightarrow \left[\frac{p + r}{2}\right]^2 \ge 4pr \quad \text{[from Eq. (i)]}$$

$$\Rightarrow \quad p^2 + r^2 - 14pr \ge 0$$

$$\Rightarrow \quad \left(\frac{r}{p}\right)^2 - 14\left(\frac{r}{p}\right) + 1 \ge 0 \; (\because p > 0)$$

$$\Rightarrow \quad \left(\frac{r}{p} - 7\right)^2 - 48 \ge 0$$

$$\Rightarrow \quad \left(\frac{r}{p} - 7\right)^2 - (4\sqrt{3})^2 \ge 0 \Rightarrow \left|\frac{r}{p} - 7\right| \ge 4\sqrt{3}$$

111. ***(c)*** $${}^{47}C_4 + \sum_{r=1}^{5} {}^{52-r}C_3 = {}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3$$

$$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + ({}^{47}C_3 + {}^{47}C_4) = {}^{52}C_4$$

112. ***(a)*** Possible even digits are 2,4,6,8,0.

Case **I** Number has digits 4,6,8,0.

(Here, sum of digits is 18, divisible by 3)

☐ ☐ ☐ ☐

Number of arrangements $= 3 \times 3!$

[Ist place can be filled using 4, 6, 8]

$$= 3 \times 6 = 18$$

Case **II** Number has digits 2, 4, 6, 0

(Here, sum of digits is 12, divisible by 3)

☐ ☐ ☐ ☐

1st place cannot be filled by 0.

Number of arrangements $= 3 \times 3! = 18$

$\therefore$ Number of numbers $= 18 + 18 = 36$

113. ***(d)*** We know that,

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\therefore \quad P = 1500 - n(A \cap B)$

$\Rightarrow \quad n(A \cap B) = 1500 - P$

Clearly, $1 \le n(A \cap B) \le 500$

$\Rightarrow \quad 1 \le 1500 - P \le 500$

$\Rightarrow \quad -1499 \le -P \le -1000$

$\Rightarrow \quad 1000 \le P \le 1499$

114. ***(c)*** Here, $\dfrac{2x-1}{x^3+4x^2+3x} \in R$ only when

$x^3 + 4x^2 + 3x \ne 0$

$\because \quad x(x^2+4x+3) = 0$

$\Rightarrow \quad x(x+1)(x+3) = 0$

$\Rightarrow \quad x = 0, -1, -3$

$$\therefore \quad \left\{x \in R : \frac{2x-1}{x^3+4x^2+3x} \in R\right\}$$

$$= R - \{0, -1, -3\}$$

115. ***(a)*** Let $f(x) = Ax^2 + Bx + C$

$\therefore \quad f(1) = A + B + C$

and $\quad f(-1) = A - B + C$

$\because \quad f(1) = f(-1)$

$\Rightarrow \quad A + B + C = A - B + C$

$\Rightarrow \quad 2B = 0 \Rightarrow B = 0$

$\therefore \quad f(x) = Ax^2 + C \Rightarrow f'(x) = 2Ax$

$\Rightarrow \quad f'(a) = 2Aa$

$f'(b) = 2Ab$ and $f'(c) = 2Ac$

Also, a, b, c are in AP

So, $2Aa$, $2Ab$ and $2Ac$ are in AP.

$\Rightarrow f'(a), f'(b)$ and $f'(c)$ are also in AP.

116. ***(c)*** $\displaystyle \lim_{n \to \infty} \frac{1}{n}\left\{\frac{1}{n+1} + \frac{2}{n+2} + \ldots + \frac{3n}{n+3n}\right\}$

$$= \lim_{n\to\infty} \sum_{r=1}^{3n} \frac{1}{n}\left(\frac{r}{n+r}\right) = \int_0^3 \frac{x}{1+x}\,dx$$

$$= \int_0^3 \left(1 - \frac{1}{1+x}\right) dx = [x - \log(1+x)]_0^3$$

$$= 3 - \log 4 = 3 - 2\log 2$$

117. ***(b)*** We have,

$$(x-1)(x-2)(x-3)\ldots(x-10)$$

Coefficient of x^8 = Sum of the terms taken two at a time i.e., $\displaystyle \sum_{1 \le i < j \le 10} x_i \cdot x_j = \frac{1}{2}\left[\left(\sum_{i=1}^{10} x_i\right)^2 - \left(\sum_{i=1}^{10} x_i^2\right)\right]$

$$= \frac{1}{2}[(1+2+3+..+10)^2 - (1^2+2^2+3^2+\ldots+10^2)]$$

$$= \frac{1}{2}\left[\left(\frac{10 \times 11}{2}\right)^2 - \left(\frac{10 \times 11 \times 21}{6}\right)\right]$$

$$= \frac{1}{2}((55)^2 - 385) = \frac{1}{2}(3025 - 385) = \frac{2640}{2} = 1320$$

118. ***(b)*** We have, $Z = \dfrac{7+i}{3+4i}$

$$\Rightarrow \quad Z = \left(\frac{7+i}{3+4i}\right)\left(\frac{3-4i}{3-4i}\right)$$

$$= \frac{21+4-28i+3i}{9+16} = 1 - i$$

$$\therefore \quad Z^{14} = (1-i)^{14} = [(1-i)^2]^7 = (1-1-2i)^7$$

$$= (-2)^7 (i)^7 = 2^7 i$$

119. ***(c)*** We have, $\dfrac{dy}{dx} + \dfrac{\tan y}{x} = \dfrac{\tan y \sin y}{x^2}$

Now, divide by $\tan y \sin y$ both sides, we get

$$\cot y \operatorname{cosec} y \frac{dy}{dx} + \frac{\operatorname{cosec} y}{x} = \frac{1}{x^2} \quad \ldots(i)$$

Put $\operatorname{cosec} y = z$

$$\therefore \quad -\operatorname{cosec} y \cot y \frac{dy}{dx} = \frac{dz}{dx}$$

$\therefore$ Eq. (i) reduces to

$$\frac{dz}{dx} - \frac{z}{x} = -\frac{1}{x^2}$$

$$\text{IF} = e^{\int -\frac{1}{x}dx} = e^{-\log x} = 1/x$$

Hence solution is

$$\frac{z}{x} = \int -\frac{1}{x^3}\,dx = \frac{1}{2x^2} - c$$

$$\Rightarrow \quad \frac{1}{x \sin y} = \frac{1 - 2cx^2}{2x^2}$$

$$\Rightarrow \quad 2x = \sin y(1 - 2cx^2)$$

120. ***(a)*** We have,

$$I = \int_0^{\pi/2} \frac{dx}{\tan x + \cot x + \operatorname{cosec} x + \sec x}$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{dx}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} + \frac{1}{\sin x} + \frac{1}{\cos x}}$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\sin x \cos x\,dx}{1 + \sin x + \cos x}$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{2\sin\frac{x}{2}\cos\frac{x}{2}\left(\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right)}{2\cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}\,dx$$

$$\Rightarrow I = \int_0^{\pi/2} \sin\frac{x}{2}\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right) dx$$

$$\Rightarrow I = \frac{1}{2}\int_0^{\pi/2} (\sin x + \cos x - 1)\,dx$$

$$\Rightarrow I = \frac{1}{2}[-\cos x + \sin x - x]_0^{\pi/2}$$

$$\Rightarrow I = \frac{1}{2}\left[\left(-\cos\frac{\pi}{2} + \sin\frac{\pi}{2} - \frac{\pi}{2}\right) - (-\cos 0 + \sin 0 - 0)\right]$$

$$\Rightarrow I = \frac{1}{2}\left[0 + 1 - \frac{\pi}{2} + 1\right] = 1 - \frac{\pi}{4}$$

121. ***(d)*** We have given,

$$|\mathbf{a}| = 1, |\mathbf{b}| = 4 \text{ and } \mathbf{a}\cdot\mathbf{b} = 2$$

Now, $\mathbf{c} = (2\mathbf{a}\times\mathbf{b}) - 3\mathbf{b}$

Multiplying by b both sides, we get

$$\mathbf{b}\cdot\mathbf{c} = (2\mathbf{a}\times\mathbf{b})\cdot\mathbf{b} - 3|\mathbf{b}|^2$$

$$\Rightarrow \quad \mathbf{b}\cdot\mathbf{c} = 0 - 3(4)^2 = -48$$

and $|\mathbf{c}|^2 = |(2\mathbf{a}\times\mathbf{b}) - 3\mathbf{b}|^2$

$$|\mathbf{c}|^2 = (2\mathbf{a}\times\mathbf{b})^2 + 9|\mathbf{b}|^2 = 4(\mathbf{a}\times\mathbf{b})^2 + 9(4)^2$$

$$\Rightarrow \quad |\mathbf{c}|^2 = 4(|\mathbf{a}|^2\cdot|\mathbf{b}|^2 - (\mathbf{a}\cdot\mathbf{b})^2) + 144$$

$$= 4(16-4) + 144 = 48 + 144 = 192$$

$$\therefore \quad \mathbf{b}\cdot\mathbf{c} = |\mathbf{b}||\mathbf{c}|\cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{\mathbf{b}\cdot\mathbf{c}}{|\mathbf{b}||\mathbf{c}|} = \frac{-48}{4\times\sqrt{192}} = -\frac{\sqrt{3}}{2}$$

$$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

122. ***(a)*** We have,

$$x^2 - (k-2)x + (k^2 + 3k + 5) = 0$$

x_1, x_2 are roots of equations

$$\therefore \quad x_1 + x_2 = k - 2, x_1x_2 = k^2 + 3k + 5$$

$$x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2$$

$$= (k-2)^2 - 2(k^2 + 3k + 5)$$

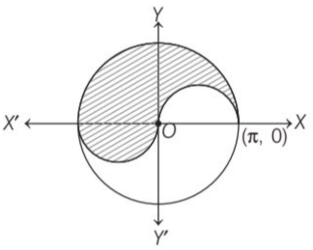
$$= k^2 - 4k + 4 - 2k^2 - 6k - 10$$

$$= -(k^2 + 10k + 6) = -[(k+5)^2 - 19]$$

$$= 19 - (k+5)^2 \le 19$$

$\therefore$ Maximum value of $x_1^2 + x_2^2 = 19$

123. ***(c)*** Graph of circle whose centre is origin and radius is π units and $y = \sin x$ is



Area of shaded region $= \frac{1}{2}$ Area of circle

$$= \frac{1}{2}(\pi r^2) = \frac{1}{2}\pi(\pi^2) \quad [\because r = \pi]$$

$$= \pi^3/2$$

124. ***(a)*** The given equation is

$$2(1+i)x^2 - 4(2-i)x - 5 - 3i = 0$$

$$\Rightarrow x = \frac{4(2-i) \pm \sqrt{16(2-i)^2 + 8(1+i)(5+3i)}}{4(1+i)}$$

$$= -\frac{i}{1+i} \text{ or } \frac{4-i}{1+i} = \frac{-1-i}{2} \text{ or } \frac{3-5i}{2}$$

Now, $\left|\frac{-1-i}{2}\right| = \sqrt{\frac{1}{4} + \frac{1}{4}} = \sqrt{\frac{1}{2}}$

and $\left|\frac{3-5i}{2}\right| = \sqrt{\frac{9}{4} + \frac{25}{4}} = \sqrt{\frac{17}{2}}$

Also, $\sqrt{\frac{17}{2}} > \sqrt{\frac{1}{2}}$

Hence, required root is $\frac{3-5i}{2}$.

125. ***(d)*** For real roots, discriminant,

$$D = b^2 - 4ac \ge 0$$

$$= \cos^2\beta - 4(\cos\beta - 1)\sin\beta \ge 0$$

$$= \cos^2\beta + 4(1 - \cos\beta)\sin\beta \ge 0$$

So, $\sin\beta$ should be > 0. $[\because \cos^2\beta \ge 0, 1 - \cos\beta \ge 0]$

$\Rightarrow \beta \in (0, \pi)$

126. ***(c)*** We have,

$$(1+\alpha)x + \beta y + z = 2$$

$$\alpha x + (1+\beta)y + z = 3$$

$$\alpha x + \beta y + 2z = 2$$

For unique solution $\begin{vmatrix} 1+\alpha & \beta & 1 \\ \alpha & 1+\beta & 1 \\ \alpha & \beta & 2 \end{vmatrix} \ne 0$

Apply $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$, we get

$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \alpha & \beta & 2 \end{vmatrix} \ne 0$$

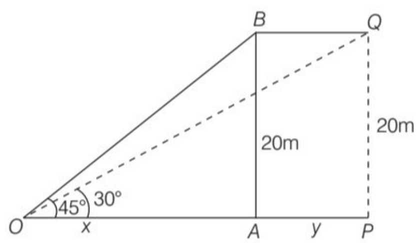
Expanding along R_1, we get

$$1(2+\beta) + 1(0+\alpha) \ne 0 = \alpha + \beta + 2 \ne 0$$

Only option (c) satisfied above equation.

127. ***(d)*** Let $AB = h =$ Height of vertical pole

$OA = x =$ Horizontal distance from O to pole



In ΔOAB

$$\tan 45° = \frac{AB}{OA} \Rightarrow 1 = \frac{20}{x}$$

$\Rightarrow \quad x = 20$ m

In ΔOPQ

$$\tan 30° = \frac{PQ}{OP} \Rightarrow \frac{1}{\sqrt{3}} = \frac{20}{x+y}$$

$\Rightarrow \quad x + y = 20\sqrt{3}$

$\Rightarrow \quad y = 20\sqrt{3} - x = 20\sqrt{3} - 20 = 20(\sqrt{3} - 1)$

$\therefore$ Speed of bird is $20(\sqrt{3} - 1)$ m/s.

128. ***(b)*** Since, $g = \sqrt{ab}$. Also, a, p, q and b are in AP.

So, common difference d is $\frac{b-a}{3}$

$$\therefore \quad p = a + d = a + \frac{b-a}{3} = \frac{2a+b}{3}$$

$$q = b - d = b - \frac{b-a}{3} = \frac{a+2b}{3}$$

Now, $(2p - q)(p - 2q)$

$$= \frac{(4a + 2b - a - 2b)}{3} \cdot \frac{(2a + b - 2a - 4b)}{3}$$

$$= -ab = -g^2$$

129. ***(b)*** Let $f(x) = x^{100}$

$g(x) = x^2 - 3x + 2$

$q(x)$ = quotient

$r(x)$ = remainder = $(ax + b)$

$\therefore \quad f(x) = g(x)\,q(x) + r(x)$

$x^{100} = (x^2 - 3x + 2)\,q(x) + ax + b$

Put $\quad x = 1 \Rightarrow 1 = a + b \quad$ ...(i)

Put $\quad x = 2 \Rightarrow 2^{100} = 2a + b \quad$ ...(ii)

From Eqs. (i) and (ii), we get

$a = 2^{100} - 1,\ b = 2 - 2^{100}$

Remainder $= (2^{100} - 1)x + (2 - 2^{100})$

$= (2^{99+1} - 1)x - 2(2^{99} - 1)$

$\therefore \quad k = 99$

130. ***(a)*** Let two observations be x and y.

$$\therefore \text{ Mean} = \frac{1 + 3 + 8 + x + y}{5} = \frac{12 + x + y}{5}$$

$$\Rightarrow \quad 5 = \frac{x + y + 12}{5} \Rightarrow x + y = 13 \quad \text{...(i)}$$

$$\text{Now, variance} = \frac{\Sigma x_i^2}{n} - (\bar{x})^2$$

$$\Rightarrow \quad 9.20 = \frac{1^2 + 3^2 + 8^2 + x^2 + y^2}{5} - 25$$

$$\Rightarrow \quad 46 = 74 + x^2 + y^2 - 125$$

$\Rightarrow \quad x^2 + y^2 = 97 \quad$ ...(ii)

From Eqs. (i) and (ii), we get

$(x + y)^2 - 2xy = 97$

$\Rightarrow \quad (13)^2 - 2xy = 97$

$\Rightarrow \quad 2xy = 169 - 97 = 72$

$\Rightarrow \quad xy = 36$

$\therefore \quad x : y = 4 : 9$

131. ***(b)*** Let the three digit number be abc.

Given, middle digit is AM of first and last

$$\therefore \quad b = \frac{a+c}{2}$$

$\Rightarrow \quad 2b = a + c$

$2b$ is an even.

$\therefore$ a and c should be both even or both odd.

$\therefore$ Total number of ways $= 5C_1 \times 5C_1 + 4C_1 \times 5C_1$

$= 5 \times 5 + 4 \times 5 = 25 + 20 = 45$

132. ***(b)*** We have,

$Z = re^{i\theta}$

$Z = r(\cos\theta + i\sin\theta)$

$e^{iz} = e^{ir(\cos\theta + i\sin\theta)}$

$\Rightarrow \quad e^{iz} = e^{ir\cos\theta + i^2 r\sin\theta}$

$\Rightarrow \quad e^{iz} = e^{ir\cos\theta - r\sin\theta}$

$\Rightarrow \quad e^{iz} = e^{-r\sin\theta} \cdot e^{ir\cos\theta}$

$\therefore \quad \arg(e^{iz}) = r\cos\theta$

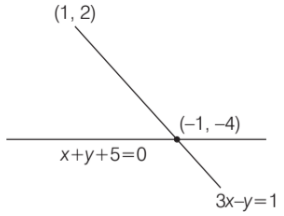
133. ***(d)*** With four 6-sided dics, there are possible combinations that can be rolled. Out of these, only 80 sum to 10 as the table shows,

$6 \times 6 \times 6 \times 6 = 6^4 = 1296$

Combinations	Count
$1-1-2-6$	$\frac{4!}{2!} = 12$
$1-1-3-5$	$\frac{4!}{2!} = 12$
$1-1-4-4$	$\frac{4!}{2!2!} = 6$
$1-2-2-5$	$\frac{4!}{2!} = 12$
$1-2-3-4$	$4! = 24$
$1-3-3-3$	$\frac{4!}{3!} = 4$
$2-2-2-4$	$\frac{4!}{3!} = 4$
$2-2-3-3$	$\frac{4!}{2!2!} = 6$
	Total Sum = 80

134. ***(b)*** A line passing through (1, 2) and parallel to the line $3x - y = 10$ is

$$3(x-1)-(y-2)=0$$



$$\Rightarrow \quad 3x-3-y+2=0 \quad \Rightarrow \quad 3x-y-1=0$$

The intersection point of line

$$x+y+5=0 \quad \ldots(i)$$

and $\quad 3x-y-1=0 \quad \ldots(ii)$

are $\quad (-1,-4)$

$\therefore$ Distance from the line $x+y+5$ to the point (1, 2) measured parallel to the line $3x - y = 10$ is

$$\Rightarrow \quad \sqrt{(-1-1)^2+(-4-2)^2}$$

$$\Rightarrow \quad \sqrt{4+36}=\sqrt{40}=2\sqrt{10}$$

135. ***(d)*** We have,

$$f(x)=a_0+a_1x^2+a_2x^4+a_3x^6+\ldots+a_nx^{2n}$$

$$f'(x)=2a_1x+4a_2x^3+6a_3x^5+\ldots+2na_nx^{2n-1}$$

$$f'(x)=2x(a_1+2a_2x^2+3a_3x^4+\ldots+na_nx^{2n-2})$$

For maxima or minima

Put $f'(x)=0 \Rightarrow x=0$

$$f''(x)=2(a_1+6a_2x^2+15a_3x^4+\ldots+n(2n-1)a_nx^{2n-2})$$

$$(f''(x))_{x=0}=2a_1>0$$

$\therefore$ $f(x)$ has only one minima at $x = 0$.

136. ***(c)*** We have,

$$f(x)=2x^3+x^4+\log x$$

Put $\quad x = 1$, we get

$\therefore \quad f(1)=3 \Rightarrow 1=f^{-1}(3)$

Now, given g is the inverse of f

$\therefore \quad g(x)=f^{-1}(x) \Rightarrow f(g(x))=x$

Differentiating with respect to x, we get

$$f'(g(x))\cdot g'(x)=1$$

$$g'(x)=\frac{1}{f'(g(x))}$$

$$g'(x)=\frac{1}{6(g(x))^2+4(g(x))^3+\dfrac{1}{g(x)}}$$

$$\left[\because f'(x)=6x^2+4x^3+\frac{1}{x}\right]$$

$$g'(3)=\frac{1}{6(g(3))^2+4(g(3))^3+\dfrac{1}{g(3)}}$$

$$g'(3)=\frac{1}{6+4+1} \qquad [\because g(3)=f^{-1}(3)=1]$$

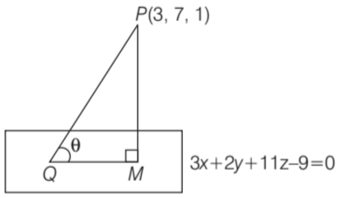
$$g'(3)=\frac{1}{11}$$

137. ***(d)*** We have $P(3, 7, 1)$ and $R(2, 5, 7)$

DR's of $PR = (-1, -2, 6)$

Angle between line PQ and plane is

$$\sin\theta=\frac{(-1)(3)+(-2)\times 2+6\times 11}{\sqrt{1+4+36}\,\sqrt{9+4+121}}$$



$$\sin\theta=\frac{59}{\sqrt{41}\,\sqrt{134}}$$

$$PM=\left|\frac{3(3)+2(7)+11(\times 1)-9}{\sqrt{9+4+121}}\right|$$

$$=\left|\frac{9+14+11-9}{\sqrt{134}}\right|=\frac{25}{\sqrt{134}}$$

$$\sin\theta=\frac{PM}{PQ}$$

$$\Rightarrow \quad PQ=\frac{PM}{\sin\theta}=\frac{25}{\sqrt{134}}\times\frac{\sqrt{41}\times\sqrt{134}}{59}=\frac{25\sqrt{41}}{59}$$

138. ***(a)*** We have,

$$I=\int\frac{8x^{43}+13x^{38}}{(x^{13}+x^5+1)^4}\,dx$$

$$=\int\frac{8x^{43}+13x^{38}}{x^{52}\left(1+\dfrac{1}{x^8}+\dfrac{1}{x^{13}}\right)^4}\,dx=\int\frac{8\left(\dfrac{1}{x^9}\right)+13\left(\dfrac{1}{x^{14}}\right)}{\left(1+\dfrac{1}{x^8}+\dfrac{1}{x^{13}}\right)^4}\,dx$$

Put, $\quad 1+\dfrac{1}{x^8}+\dfrac{1}{x^{13}}=t$

$$\Rightarrow \quad \left(\frac{-8}{x^9}-\frac{13}{x^{14}}\right)dx=dt$$

$$\therefore \quad I=-\int\frac{dt}{t^4}=\frac{1}{3t^3}+C$$

$$\Rightarrow I=\frac{1}{3\left(1+\dfrac{1}{x^8}+\dfrac{1}{x^{13}}\right)^3}+C=\frac{x^{39}}{3(x^{13}+x^5+1)^3}+C$$

139. ***(b)*** We have,

Vector r is coplanar with vector a and b

$\therefore \quad \mathbf{r} = \lambda(\mathbf{b} \times (\mathbf{a} \times \mathbf{b}))$

$\Rightarrow \quad \mathbf{r} = \lambda((\mathbf{b}\cdot\mathbf{b})\mathbf{a} - (\mathbf{b}\cdot\mathbf{a})\mathbf{b})$

$\Rightarrow \quad \mathbf{r} = \lambda(|\mathbf{b}|^2\mathbf{a} - (\mathbf{a}\cdot\mathbf{b})\mathbf{b})$

$\Rightarrow \quad \mathbf{r} = \lambda[5\mathbf{a} + \mathbf{b}] \quad [\because |\mathbf{b}| = \sqrt{2^2 + 1^2} = \sqrt{5}$

and $\quad \mathbf{a}\cdot\mathbf{b} = (-\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})\cdot(2\hat{\mathbf{i}} + \hat{\mathbf{k}}) = -1]$

Given, $\quad \mathbf{r}\cdot\mathbf{a} = 7$

$\therefore \quad \lambda(5\mathbf{a} + \mathbf{b})\cdot\mathbf{a} = 7$

$\Rightarrow \quad \lambda[5|\mathbf{a}|^2 + \mathbf{a}\cdot\mathbf{b}] = 7$

$\Rightarrow \quad \lambda[15 - 1] = 7 \quad [\because |a|^2 = 1 + 1 + 1 = 3]$

$\Rightarrow \quad \lambda = 1/2$

$\therefore \quad \mathbf{r} = \frac{1}{2}(5\mathbf{a} + \mathbf{b})$

$$\mathbf{r} = \frac{1}{2}(-5\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 5\hat{\mathbf{k}} + 2\hat{\mathbf{i}} + \hat{\mathbf{k}})$$

$$\mathrm{r} = \frac{-3}{2}\hat{\mathbf{i}} + \frac{5}{2}\hat{\mathbf{j}} + \frac{6}{2}\hat{\mathbf{k}}$$

z component of $\mathbf{r} = \frac{6}{2} = 3$

140. ***(b)*** Since x, y and z are three consecutive integers

$\therefore \quad 2y = x + z$

$\Rightarrow \quad (4y^2) = (x + z)^2$

$= (x - z)^2 + 4xz$

$= (-2)^2 + 4xz \quad [\because x - z = -2]$

$\Rightarrow \quad 4y^2 = 4 + 4xz$

$\Rightarrow \quad y^2 = 1 + xz$

Now, $\frac{1}{2}\log_e x + \frac{1}{2}\log_e z + \frac{1}{2zx + 1} + \frac{1}{3(2xz + 1)^3} + \dots$

$$= \frac{1}{2}\left[\log_e(xz) + 2\left(\frac{1}{2zx + 1} + \frac{1}{3}\cdot\frac{1}{(2zx + 1)^3} + \dots\right)\right]$$

$$= \frac{1}{2}\left[\log_e(xz) + \log_e\left(\frac{1 + \frac{1}{2zx + 1}}{1 - \frac{1}{2zx + 1}}\right)\right]$$

$$= \frac{1}{2}\left[\log_e(xz) + \log_e\left(\frac{1 + xz}{xz}\right)\right]$$

$$= \frac{1}{2}[\log_e(1 + xz)] = \frac{1}{2}[\log_e(y)^2] = \log_e(y)$$

141. ***(a)*** We have, $\quad (xy^5 + 2y)dx = x\,dy$

$\Rightarrow \quad x\frac{dy}{dx} - 2y = xy^5$

$\Rightarrow \quad \frac{dy}{dx} - \frac{2y}{x} = y^5$

$\Rightarrow \quad y^{-5}\frac{dy}{dx} - \frac{2y^{-4}}{x} = 1 \quad \dots(\text{i})$

Put, $\quad y^{-4} = t$

$\Rightarrow \quad -4y^{-5}\frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \quad y^{-5}\frac{dy}{dx} = \frac{-1}{4}\frac{dt}{dx} \quad \dots(\text{ii})$

From Eqs. (i) and (ii), we get

$$-\frac{1}{4}\frac{dt}{dx} - \frac{2t}{x} = 1$$

$\Rightarrow \quad \frac{dt}{dx} + \frac{8t}{x} = -4$

Now, IF $= e^{\int \frac{8}{x}dx} = e^{8\log x} = x^8$

and the solution is $t\cdot x^8 = \int(-4)x^8 dx + C$

$\Rightarrow \quad \frac{x^8}{y^4} = -\frac{4\cdot x^9}{9} + C$

$\Rightarrow \quad 9x^8 + 4x^9\cdot y^4 = 9y^4 C$

142. ***(b)*** Given, $\frac{|x - 2| - 1}{|x - 2| - 2} \le 0$

Let $|x - 2| = k$

Then, given equation,

$$\frac{k - 1}{k - 2} \le 0 \Rightarrow \frac{(k - 1)(k - 2)}{(k - 2)^2} \le 0$$

$\Rightarrow \quad (k - 1)(k - 2) \le 0 \Rightarrow 1 \le k \le 2$

$\Rightarrow \quad 1 \le |x - 2| \le 2$

Case I When $1 \le |x - 2|$

$\Rightarrow \quad |x - 2| \ge 1 \Rightarrow x - 2 \ge 1$ or $x - 2 \le -1$

$\Rightarrow \quad x \ge 3$ and $x \le 1 \quad \dots(\text{i})$

Case II When $|x - 2| \le 2$

$\Rightarrow \quad -2 \le x - 2 \le 2 \Rightarrow -2 + 2 \le x \le 2 + 2$

$\Rightarrow \quad 0 \le x \le 4 \quad \dots(\text{ii})$

From Eqs. (i) and (ii), $x \in [0, 1] \cup [3, 4]$

143. ***(b)*** We have, $f(x) = \frac{x}{\sqrt{1 + x^2}}$

$$\Rightarrow f(f(x)) = \frac{f(x)}{\sqrt{1 + (f(x))^2}} = \frac{\frac{x}{\sqrt{1 + x^2}}}{\sqrt{1 + \frac{x^2}{1 + x^2}}} = \frac{x}{\sqrt{1 + 2x^2}}$$

Similarly, $f(f(f(x))) = \frac{x}{\sqrt{1 + 3x^2}}$

$$\vdots \qquad \vdots$$

$$\underbrace{fofo \dots of\ of(x)}_{n \text{ times}} = \frac{x}{\sqrt{1 + nx^2}} = \frac{x}{\sqrt{1 + \left(\sum_{r=1}^{n} 1\right)x^2}}$$

144. ***(b)*** We have,

$$\log_5\left(\frac{a+b}{3}\right) = \frac{\log_5 a + \log_5 b}{2}$$

$$\Rightarrow \quad 2\log_5\left(\frac{a+b}{3}\right) = \log_5 ab$$

$$\Rightarrow \quad \log_5\left(\frac{a+b}{3}\right)^2 = \log_5 ab$$

$$\Rightarrow \quad \frac{(a+b)^2}{9} = ab \Rightarrow (a+b)^2 = 9ab$$

$$\Rightarrow \quad a^2 + b^2 = 7ab$$

$$\Rightarrow \quad \frac{a}{b} + \frac{b}{a} = 7$$

$$\Rightarrow \quad \left(\frac{a}{b} + \frac{b}{a}\right)^2 = 49$$

$$\Rightarrow \quad \frac{a^2}{b^2} + \frac{b^2}{a^2} + 2 = 49$$

$$\Rightarrow \quad \frac{a^4 + b^4}{a^2b^2} = 49 - 2 = 47$$

145. ***(a)*** We have,

$$\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$$

$$\Rightarrow \frac{5}{4}\cos^2 2x + (\cos^2 x + \sin^2 x)^2 - 2\sin^2 x\cos^2 x + (\cos^2 x + \sin^2 x)^3 - 3\sin^2 x\cos^2 x(\sin^2 x + \cos^2 x) = 2$$

$$\Rightarrow \frac{5}{4}\cos^2 2x + 1 - 2\sin^2 x\cos^2 x + 1 - 3\sin^2 x\cos^2 x = 2$$

$$\Rightarrow \frac{5}{4}\cos^2 2x - \frac{1}{2}\sin^2 2x - \frac{3}{4}\sin^2 2x = 0$$

$$\Rightarrow \frac{5}{4}(\cos^2 2x - \sin^2 2x) = 0$$

$$\Rightarrow \cos 4x = 0$$

$$\Rightarrow 4x = 2n\pi \pm \frac{\pi}{2}, n \in Z$$

$$\Rightarrow x = (4n \pm 1)\frac{\pi}{8}, n \in Z$$

$$\Rightarrow x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$$

So, number of solution = 8

146. ***(a)*** Given equation of ellipse,

$$16x^2 + 11y^2 = 256 \quad \Rightarrow \frac{x^2}{16} + \frac{11y^2}{256} = 1$$

Equation of tangent of ellipse at $\left(4\cos\phi, \frac{16}{\sqrt{11}}\sin\phi\right)$ is

$$\frac{4x\cos\phi}{16} + \frac{11 \times 16y}{256\sqrt{11}}\sin\phi = 1$$

$$\Rightarrow \quad \frac{x\cos\phi}{4} + \frac{\sqrt{11}}{16}\sin\phi\, y = 1$$

is also a tangent of circle $x^2 + y^2 - 2x = 15$

$$\Rightarrow \quad (x-1)^2 + y^2 = 16$$

radius of circle = 4 and centre = (1, 0)

Now, $\frac{x\cos\phi}{4} + \frac{\sqrt{11}}{16}\sin\phi\, y = 1$ is tangent of circle

$$\therefore \; 4 = \left|\frac{\frac{\cos\phi}{4} - 1}{\sqrt{\frac{\cos^2\phi}{16} + \frac{11}{256}\sin^2\phi}}\right| \Rightarrow \phi = \pm\frac{\pi}{3}$$

$$\therefore \quad \phi = \frac{\pi}{3}$$

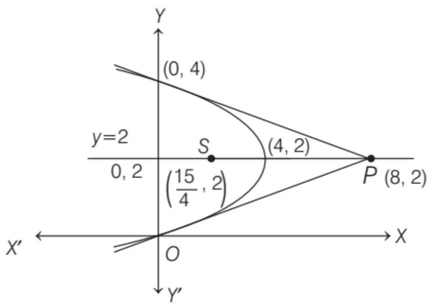
147. ***(b)*** Given equation of parabola is

$$x = 4y - y^2$$

$$\Rightarrow \quad y^2 - 4y + 4 = -x + 4$$

$$\Rightarrow \quad (y-2)^2 = -(x-4)$$

$$a = \frac{1}{4}$$



Focus $= \left(\frac{15}{4}, 2\right)$

Parabola cuts Y-axis at $(0, 0)$ and $(0, 4)$.

Now, equation of tangent at $(0, 0)$ is

$$x = 4y \qquad \text{...(i)}$$

and equation of tangent at $(0, 4)$ is

$$x + 4y = 16 \qquad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$x = 8, y = 2$$

∴ Intersection point of tangent is $P(8, 2)$.

Distance from point P to focus S

$$\text{Distance} = \sqrt{\left(8 - \frac{15}{4}\right)^2 + (2-2)^2}$$

$$= 8 - \frac{15}{4} = \frac{17}{4}$$

148. ***(d)*** We have, $S_\infty = \sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\frac{k}{2^{n+k}}$

$$S_\infty = \sum_{k=1}^{\infty}\frac{k}{2^k}\cdot\sum_{n=1}^{\infty}\frac{1}{2^n}$$

$$S_\infty = \sum_{k=1}^{\infty}\frac{k}{2^k}\left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+..\right)$$

$$S_\infty = \sum_{k=1}^{\infty}\frac{k}{2^k}\left[\frac{1}{2}\left(1+\frac{1}{2}+\frac{1}{2^2}\ldots\right)\right]$$

$$S_\infty = \sum_{k=1}^{\infty}\frac{k}{2^k}\left(\frac{1}{2}\times\frac{1}{1-\frac{1}{2}}\right) = \sum_{k=1}^{\infty}\frac{k}{2^k}$$

$$S_\infty = \frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\frac{4}{2^4}+\ldots \quad \text{...(i)}$$

$$\frac{1}{2}S_\infty = \frac{1}{2^2}+\frac{2}{2^3}+\frac{3}{2^4}+\ldots \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{1}{2}S_\infty = \frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\ldots$$

$$\frac{1}{2}S_\infty = \frac{1}{2}\left(\frac{1}{1-\frac{1}{2}}\right) = 1$$

$$S_\infty = 2$$

149. ***(a)*** We have,

Slope of tangent of the curve is $\frac{(x+1)^2+y-3}{x+1}$

$$\therefore \quad \frac{dy}{dx} = \frac{(x+1)^2+y-3}{x+1}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{(x+1)^2-3}{x+1}+\frac{y}{x+1}$$

$$\Rightarrow \quad \frac{dy}{dx}-\frac{y}{x+1} = \frac{(x+1)^2-3}{x+1}$$

This is a form of linear differential equations

$$\therefore \quad \text{IF} = e^{\int\frac{-1}{x+1}dx} = e^{\log\frac{1}{1+x}} = \frac{1}{1+x}$$

Solution of given differential equation is

$$\frac{y}{x+1} = \int\frac{(x+1)^2-3}{(x+1)^2}dx$$

$$\frac{y}{x+1} = \int\left(1-\frac{3}{(x+1)^2}\right)dx$$

$$\frac{y}{x+1} = x+\frac{3}{x+1}+c$$

$\because$ Curve passes through $(2, 0)$

$$\therefore \quad 0 = 2+\frac{3}{3}+c$$

$$\Rightarrow \quad c = -3$$

Hence, equation of curve is

$$\frac{y}{x+1} = x+\frac{3}{x+1}-3$$

$$y = x^2+x+3-3x-3$$

$$y = x^2-2x$$

150. ***(a)*** We have,

$$A = \begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}$$

We know that, $|A-\lambda I| = 0$

$$\left|\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}-\begin{bmatrix}\lambda & 0\\0 & \lambda\end{bmatrix}\right| = 0 = \begin{vmatrix}2-\lambda & 1\\1 & 2-\lambda\end{vmatrix} = 0$$

$$\Rightarrow \quad (2-\lambda)^2-1 = 0$$

$$\Rightarrow \quad 4-4\lambda+\lambda^2-1 = 0$$

$$\Rightarrow \quad \lambda^2-4\lambda+3 = 0$$

$$\therefore \quad A^2-4A+3I = 0$$

Multiply by A^{-1}, we get

$$A-4I+3A^{-1} = 0$$

$$\Rightarrow \quad A^{-1} = \frac{-1}{3}A+\frac{4}{3}I$$

$$\therefore \quad \alpha = -\frac{1}{3} \text{ and } \beta = \frac{4}{3}$$

$$\alpha+\beta = -\frac{1}{3}+\frac{4}{3} = \frac{3}{3} = 1$$