Answers

Physics

1. (c)	**2.** (a)	**3.** (b)	**4.** (d)	**5.** (c)	**6.** (a)	**7.** (b)	**8.** (c)	**9.** (a)	**10.** (c)
11. (d)	**12.** (b)	**13.** (b)	**14.** (a)	**15.** (c)	**16.** (c)	**17.** (a)	**18.** (b)	**19.** (d)	**20.** (c)
21. (a)	**22.** (d)	**23.** (a)	**24.** (d)	**25.** (*)	**26.** (b)	**27.** (c)	**28.** (d)	**29.** (d)	**30.** (c)
31. (a)	**32.** (d)	**33.** (b)	**34.** (a)	**35.** (c)	**36.** (a)	**37.** (c)	**38.** (a)	**39.** (c)	**40.** (d)

Chemistry

41. (a)	**42.** (a)	**43.** (c)	**44.** (b)	**45.** (b)	**46.** (a)	**47.** (d)	**48.** (c)	**49.** (c)	**50.** (c)
51. (b)	**52.** (a)	**53.** (a)	**54.** (a)	**55.** (d)	**56.** (c)	**57.** (c)	**58.** (a)	**59.** (c)	**60.** (a)
61. (b)	**62.** (d)	**63.** (c)	**64.** (a)	**65.** (b)	**66.** (c)	**67.** (c)	**68.** (c)	**69.** (b)	**70.** (a)
71. (c)	**72.** (a)	**73.** (a)	**74.** (b)	**75.** (b)	**76.** (c)	**77.** (b)	**78.** (c)	**79.** (a)	**80.** (a)

English Proficiency

81. (b)	**82.** (a)	**83.** (c)	**84.** (a)	**85.** (d)	**86.** (d)	**87.** (c)	**88.** (b)	**89.** (b)	**90.** (b)
91. (d)	**92.** (d)	**93.** (a)	**94.** (a)	**95.** (d)					

Logical Reasoning

96. (d)	**97.** (b)	**98.** (c)	**99.** (b)	**100.** (b)	**101.** (b)	**102.** (d)	**103.** (a)	**104.** (b)	**105.** (a)

Mathematics

106. (d)	**107.** (a)	**108.** (b)	**109.** (d)	**110.** (a)	**111.** (d)	**112.** (d)	**113.** (a)	**114.** (b)	**115.** (d)
116. (a)	**117.** (b)	**118.** (a)	**119.** (a)	**120.** (c)	**121.** (d)	**122.** (b)	**123.** (d)	**124.** (a)	**125.** (b)
126. (b)	**127.** (b)	**128.** (b)	**129.** (d)	**130.** (a)	**131.** (d)	**132.** (d)	**133.** (a)	**134.** (b)	**135.** (c)
136. (c)	**137.** (c)	**138.** (c)	**139.** (c)	**140.** (a)	**141.** (a)	**142.** (c)	**143.** (c)	**144.** (c)	**145.** (b)
146. (b)	**147.** (a)	**148.** (b)	**149.** (c)	**150.** (b)					

Note *(*) None of the option is correct.*

Hints & Solutions

Physics

1. ***(c)*** Given, half life $(T_{1/2}) = 23.1$ days

If N_0 be the initial amount of a radioactive sample, then active nuclei at $t = 15$ days

$$N_1 = N_0 e^{-15\lambda} \quad \ldots \text{(i)}$$

Active nuclei at $t = 16$ days

$$N_2 = N_0 e^{-16\lambda} \quad \ldots \text{(ii)}$$

Disintegration constant,

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{23.1} = 0.03$$

$\therefore$ % decay in 15th to 16th days $= \dfrac{N_1 - N_2}{N_1} \times 100$

$$= \frac{N_0 e^{-15\lambda} - N_0 e^{-16\lambda}}{N_0 e^{-15\lambda}} \times 100$$

[Using Eqs. (i) and (ii)]

$$= (1 - e^{-\lambda}) \times 100 = (1 - e^{-0.03}) \times 100$$

$$= \left(1 - \frac{1}{e^{0.03}}\right) \times 100 = \left(1 - \frac{1}{1.03}\right) \times 100 = 2.9\%$$

2. ***(a)*** Excess pressure inside first bubble of radius r_1,

$$p_1 = p_0 + \frac{4T}{r_1}$$

Excess pressure inside second bubble of radius r_2,

$$p_2 = p_0 + \frac{4T}{r_2}$$

Excess pressure inside double bubble,

$$p = p_2 - p_1$$

On putting values of p_1 and p_2, we get

$$p = 4T\left(\frac{r_1 - r_2}{r_1 r_2}\right) \quad \ldots \text{(i)}$$

If R be the radius of double bubble, then

$$p = \frac{4T}{R}$$

or $\quad 4T\left(\dfrac{r_1 - r_2}{r_1 r_2}\right) = \dfrac{4T}{R}$ [Using Eq. (i)]

$$\Rightarrow \quad R = \frac{r_1 r_2}{r_1 - r_2} = \frac{3 \times 10^{-3} \times 2 \times 10^{-3}}{(3-2) \times 10^{-3}} = 6 \times 10^{-3} \text{ m}$$

$$\Rightarrow \quad R = 6 \times 10^{-3} \text{ m}$$

3. ***(b)*** Magnetic field at the centre of circular loop,

$$B = \frac{\mu_0 I}{2r} \quad \ldots \text{(i)}$$

Period of revolution of electron is

$$T = \frac{2\pi}{\omega}$$

where, ω = angular velocity of electron.

Current produced due to movement of electron,

$$I = \frac{e}{T} = \frac{e}{\frac{2\pi}{\omega}} = \frac{e\omega}{2\pi}$$

On substituting the value of I in Eq. (i), we get

$$B = \frac{\mu_0}{2r} \cdot \frac{e\omega}{2\pi}$$

$$\Rightarrow \quad B = \frac{\mu_0 e\omega}{4\pi r}$$

4. ***(d)*** Given, elastic limit of steel wire, $Y = 2.2 \times 10^8$ m/s^2

Area of cross-section of steel wire,

$$A = 4 \text{ cm}^2 = 4 \times 10^{-4} \text{ m}^2$$

Mass of elevator, $m = 1000$ kg

The maximum tension in the steel wire that can support elevator is given by

$$T = \frac{1}{4} \times \text{stress} \times \text{area of cross-section}$$

$$= \frac{1}{4} \times 2.2 \times 10^8 \times 4 \times 10^{-4}$$

$$= 2.2 \times 10^4 \text{ N/m}^2$$

If f be the maximum upward acceleration of the elevator, then tension is given by

$$\Rightarrow \quad T = m(g + f)$$

$$2.2 \times 10^4 = 1000\,(10 + f)$$

$$\Rightarrow \quad f = 12 \text{ m/s}^2$$

5. ***(c)*** For an ideal monoatomic gas, the heat capacity ratio, $\gamma = \dfrac{5}{3}$

T_i, T_f = initial and final temperatures of gas.

In an adiabatic process,

$$TV^{\gamma - 1} = \text{constant}$$

i.e. $\quad T_f V_f^{\gamma-1} = T_i V_i^{\gamma-1}$

$$\Rightarrow \quad T_f = T_i\left(\frac{V_i}{V_f}\right)^{\gamma - 1} = 300\left(\frac{1}{2}\right)^{\frac{5}{3} - 1}$$

$$= 300\left(\frac{1}{2}\right)^{2/3}$$

6. ***(a)*** Given, frequency of sound produced by sources A and B, $f_A = f_B = 400$ Hz

Speed of sound in air, $v = 340$ m/s

Apparent frequency heard by the observer when source A is moving towards him,

$$f_A' = \frac{v}{v - v_S} \times f_A = \frac{340}{340 - v_S} \times 400 \quad \ldots\text{(i)}$$

Apparent frequency heard by the observer when source B is moving away from him,

$$f_B' = \frac{v}{v + v_S} \times f_B = \frac{340}{340 + v_S} \times 400 \quad \ldots\text{(ii)}$$

Given, beats $= f_A' - f_B' = 4$

On substituting the values of Eq. (i) and Eq. (ii), we get

$$\frac{340}{340 - v_S} \times 400 - \frac{340}{340 + v_S} \times 400 = 4$$

$$\Rightarrow \quad \frac{34000 \times 2v_S}{340^2 - v_S^2} = 1$$

[Neglecting the value of v_S^2 $\because v_S^2 << 340^2$]

$$\frac{34000 \times 2v_S}{340^2} = 1$$

$$\Rightarrow \quad v_S = 1.7 \text{ m/s}$$

7. ***(b)*** Given, density of gas at point A, $\rho_A = \rho_0$

General equation for an ideal gas is

$$pV = nRT \Rightarrow pV = \frac{m}{M}.RT$$

where m = mass and M = molecular mass.

$$\Rightarrow \quad p = \frac{m}{V}.\frac{R}{M}.T$$

$$\Rightarrow \quad p = \rho\left(\frac{R}{M}\right)T \quad \ldots\text{(i)} \left[\because \rho = \frac{m}{V}\right]$$

Applying Eq. (i) at point A and B, respectively, we get

$$p = \rho_A\left(\frac{R}{M}\right)T_0 \quad \ldots\text{(ii)}$$

$$2p = \rho_B\left(\frac{R}{M}\right)3T_0 \quad \ldots\text{(iii)}$$

From Eqs. (ii) and (iii), we get

$$\frac{1}{2} = \frac{\rho_A}{\rho_B}.\frac{1}{3}$$

$$\Rightarrow \quad \rho_B = \frac{2}{3}\rho_A = \frac{2}{3}\rho_0 \quad [\because \rho_A = \rho_0]$$

8. ***(c)*** Force constant K of rubber is given by

$$K = \frac{YA}{l} = \frac{5 \times 10^8 \times 10^{-6}}{0.1} = 5 \times 10^3 \text{ N/m}$$

Now, from law of conservation of energy

elastic potential energy of cord = kinetic energy of particle

i.e. $\frac{1}{2}K(\Delta l)^2 = \frac{1}{2}mv^2$

$$\Rightarrow \quad v = \sqrt{\frac{K}{m}}.\Delta l = \sqrt{\frac{5 \times 10^3}{5 \times 10^{-3}}}(0.125 - 0.1)$$

$$= 10^3 \times 0.025 = 25 \text{ m/s}$$

9. ***(a)*** By work-energy theorem,

$$W = P \times t = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v^2 = 2\frac{Pt}{m} \Rightarrow v = \left(\frac{2Pt}{m}\right)^{1/2}$$

As we know, $v = \frac{ds}{dt}$

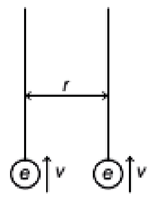
or $$\frac{ds}{dt} = \left(\frac{2Pt}{m}\right)^{1/2}$$

$$\int ds = \int\left(\frac{2Pt}{m}\right)^{1/2} dt$$

$$s = \left(\frac{2P}{m}\right)^{1/2}.\frac{2}{3}t^{3/2}$$

i.e. $s \propto t^{3/2} \Rightarrow s^2 \propto t^3$

10. ***(c)*** Given, speed of electrons, $v = 5 \times 10^5$ m/s



Electrostatic force between two electrons,

$$F_e = k.\frac{e^2}{r^2} \quad \ldots\text{(i)}$$

Magnetic field produced due to moving electron with velocity v,

$$B = \frac{\mu_0}{4\pi}.\frac{ev}{r^2} \quad \ldots\text{(ii)}$$

Magnetic force on electron due to magnetic field

$$F_m = Bev = \frac{\mu_0}{4\pi}.\frac{ev}{r^2}.ev \quad \text{(from Eq. (ii))}$$

$$F_m = \frac{\mu_0}{4\pi}.\frac{e^2v^2}{r^2} \quad \ldots\text{(iii)}$$

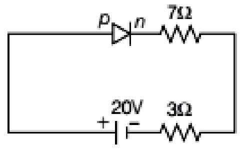
$\therefore$ From Eqs. (i) and (iii), we have

$$\frac{F_e}{F_m} = \frac{ke^2/r^2}{\frac{\mu_0 e^2 v^2}{4\pi.r^2}} = \frac{k.4\pi}{\mu_0 v^2}$$

$$\Rightarrow \quad \frac{F_e}{F_m} = \frac{9 \times 10^9 \times 4 \times 3.14}{4\pi \times 10^{-7}(5 \times 10^5)^2} = 3.6 \times 10^5$$

11. ***(d)*** From the figure, It is clear that diodes D_1 and D_3 are reverse biased, therefore they will not conduct and resistance 11Ω and 5Ω will be not in use but diode D_2 is in forward biasing position. Therefore, current flows through diode D_2 is same as current through battery, according to the circuit shown after redrawing,

$\therefore$ Current, $I = \dfrac{20}{7+3} = 2\,A$



12. ***(b)*** The moment of inertia of disc of radius R about its axis is

$$I = \frac{M'R^2}{2}$$

where, M' = mass of complete disc $= 2M$

$$\therefore \quad I = \frac{2MR^2}{2}$$

$$I = MR^2 \quad \ldots \text{(i)}$$

If I_1 be the moment of inertia of semicircular disc about its axis, then disc may be assumed as combination of two semi-circular parts,

Thus, $I = I_1 + I_1 \Rightarrow I_1 = \dfrac{I}{2} = \dfrac{MR^2}{2}$ [From Eq. (i)]

13. ***(b)*** Given, total energy given to electron in third orbit,

$$E = 12\,\text{eV}$$

Energy of third orbit of H-atom, $E' = \dfrac{-13.6}{n^2}$

$$= \frac{-13.6}{3^2} = -1.51\,\text{eV}$$

Ionisation energy of electron of third orbit, $E'' = -E'$

$$= -(-1.51\,\text{eV}) \Rightarrow E'' = 1.51\,\text{eV}$$

Hence, final energy of electron when it comes out of H-atom from third orbit

$$E_F = E - E''$$
$$= 12 - 1.51 = 10.49\,\text{eV}$$

14. ***(a)*** Given, $D = 2\,\text{m}$, $d = 0.25\,\text{mm} = 2.5 \times 10^{-4}\,\text{m}$

Velocity of screen, $\dfrac{dD}{dt} = 5\,\text{m/s}$

Speed of first maxima $= \dfrac{d\beta}{dt} = ?$

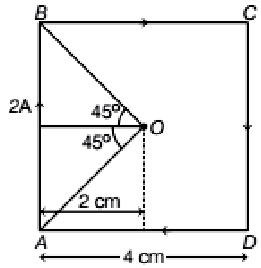
In Young's double slit experiment, fringe width is given by

$$\beta = \frac{D\lambda}{d}$$

On differentiating with respect to t, we get,

$$\frac{d\beta}{dt} = \frac{\lambda}{d} \cdot \frac{dD}{dt}$$
$$= \frac{800 \times 10^{-9}}{2.5 \times 10^{-4}} \cdot 5$$
$$= 16 \times 10^{-3}\,\text{m/s} = 16\,\text{mm/s}$$

15. ***(c)*** Given, current through the square loop is 2A.



$I = 2A$

The given situation is shown in the figure. Magnetic field due to wire AB at the centre O is

$$B_{AB} = \frac{\mu_0}{4\pi} \cdot \frac{I}{r} [\sin 45° + \sin 45°]$$
$$= 10^{-7} \times \frac{2}{2 \times 10^{-2}} \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right]$$
$$= \sqrt{2} \times 10^{-5}\,\text{T}$$

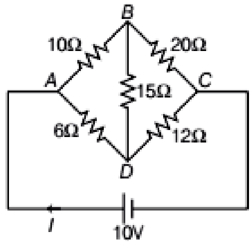
[Downward to the plane of square loop]

Since, magnetic field due to side AB, BC, CD and DA is same in magnitude and direction. Hence, net magnetic field at O

$$B = 4 \times B_{AB} = 4\sqrt{2} \times 10^{-5}\,\text{T}$$

16. ***(c)*** Wheatstone bridge in balanced condition maintains $\dfrac{P}{Q} = \dfrac{R}{S}$ i.e. no current flows through the galvanometer (i.e. $i_g = 0$)

The given circuit can be redrawn as



Since, $\dfrac{R_{AB}}{R_{BC}} = \dfrac{R_{AD}}{R_{DC}} = \dfrac{1}{2}$, hence it is a balanced Wheatstone bridge, therefore current flowing in arm $BD = 0$

$$\therefore \quad \frac{1}{R_{eq}} = \frac{1}{10+20} + \frac{1}{6+12} = \frac{1}{30} + \frac{1}{18}$$

$$\Rightarrow \quad R_{eq} = \frac{30 \times 18}{30+18} = 11.25\ \Omega$$

$$\therefore \quad I = \frac{V}{R_{eq}} = \frac{10}{11.25} = 0.89\text{ A} \simeq 0.9\text{ A}$$

17. ***(a)*** The flux linked with coil (made of wire of radius r) of area A and magnetic field B, is given by

$$\phi = BA\cos\theta = B\left(\frac{\pi r^2}{2}\right)\cos\omega t$$

$$\phi = \frac{\pi B r^2}{2}\cos\omega t \qquad \left[\because A = \frac{\pi r^2}{2} \text{ and } \theta = \omega t\right]$$

By Faraday's law of electromagnetic induction, induced emf, $e = -\frac{d\phi}{dt}$

Hence, after differentiating, we get,

$$e = -\frac{d}{dt}\left(\frac{\pi B r^2}{2}\cos\omega t\right) = \frac{\pi B r^2 \omega}{2}\sin\omega t$$

$$\therefore \quad \text{Power, } P = \frac{e^2}{R} = \frac{\pi^2 B^2 r^4 \omega^2 \sin^2\omega t}{4R}$$

(where, R = resistance of the circuit)

$$\therefore \text{ Mean power, } \overline{P} = \overline{\frac{\pi^2 B^2 r^4 \omega^2}{4R}} \cdot \overline{\sin^2\omega t}$$

$$= \frac{\pi^2 B^2 r^4 \omega^2}{4R} \cdot \frac{1}{2} \qquad \left[\because \overline{\sin^2\omega t} = \frac{1}{2}\right]$$

$$= \frac{(\pi B r^2 \omega)^2}{8R}$$

18. ***(b)*** For the given convex lens, focal length, $f = 25$ cm

Magnification of lens in term of u and f is given by

$$m = \frac{f}{f+u} = \frac{25}{25+u} \qquad \dots \text{(i)}$$

When the image is real, then $m = -2$ and $u = -x_1$

$$\therefore -2 = \frac{25}{25 - x_1} \Rightarrow x_1 = \frac{75}{2} \qquad \text{[from Eq. (i)]}$$

Again, when the image is virtual, then $m = 2$ and $u = -x_2$

$$\therefore \quad 2 = \frac{25}{25 - x_2} \Rightarrow x_2 = \frac{25}{2} \qquad \text{[from Eq. (i)]}$$

Hence, the ratio of x_2 and x_1 is

$$\therefore \quad \frac{x_2}{x_1} = \frac{25/2}{75/2} = \frac{1}{3}$$

19. ***(d)*** For perfectly elastic collision,

total momentum before collision

= total momentum after collision

i.e. $$m \cdot u + \frac{3m}{4} \times 0 = mv_x + \frac{3m}{4} \cdot v_y$$

$$\Rightarrow \quad 4u = 4v_x + 3v_y \qquad \dots \text{(i)}$$

Coefficient of restitution,

$$e = \frac{\text{Velocity of seperation } (v_y - v_x) \text{ after collision}}{\text{Velocity of approach } (u_x - u_y) \text{ before collision}}$$

$$\Rightarrow \quad 1 = \frac{v_y - v_x}{u - 0} \qquad \left[\because u_x = u \text{ and } u_y = 0\right]$$

$$u = v_y - v_x \qquad \dots \text{(ii)}$$

Solving Eqs. (i) and (ii), we have

$$v_x = \frac{u}{7} \text{ and } v_y = \frac{8u}{7}$$

$$\therefore \quad \frac{\lambda_y}{\lambda_x} = \frac{h/mv_y}{h/\frac{3m}{4}\cdot v_x} = \frac{3}{4}\cdot\frac{v_x}{v_y} = \frac{3}{4}\cdot\frac{u/7}{8u/7} = \frac{3}{32}$$

20. ***(c)*** We know that, velocity of light c is terms of ε_0 and μ_0 is given by

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \Rightarrow c^2 = \frac{1}{\mu_0\varepsilon_0} c^2 = \frac{1}{\mu_0^2}\cdot\frac{\mu_0}{\varepsilon_0}$$

$$\Rightarrow c = \frac{1}{\mu_0}\sqrt{\frac{\mu_0}{\varepsilon_0}} \Rightarrow \sqrt{\frac{\mu_0}{\varepsilon_0}} = c\mu_0 \qquad \dots \text{(i)}$$

Dimension of magnetic field, $B = \frac{\tau}{NIA} = \frac{[ML^2T^{-2}]}{[A][L^2]}$

$= [MA^{-1}T^{-2}]$

Again, $B = \mu_0 nI$

$$\therefore \quad \mu_0 = \frac{B}{nI} = \frac{[MA^{-1}\ T^{-2}]}{[L^{-1}][A]} = [MLA^{-2}T^{-2}]$$

$$\mu_0 = [MLA^{-2}T^{-2}] \qquad \dots \text{(ii)}$$

and $\quad c = [LT^{-1}] \qquad \dots$ (iii)

From Eqs. (i), (ii) and (iii), we get

$$\sqrt{\frac{\mu_0}{\varepsilon_0}} = [LT^{-1}][MLA^{-2}T^{-2}] = [ML^2T^{-3}A^{-2}]$$

21. ***(a)*** Given, $F = at$

when, $t = 0$, then linear momentum $= p$

when, $t = T$, then linear momentum $= 2p$

According to Newton's law of motion,

Applied force, $F = \frac{dp}{dt}$

or, $\quad dp = Fdt$

$\quad dp = at\,dt$

or, $$\int_p^{2p} dp = a\int_0^T t\,dt$$

$$\Rightarrow \quad (2p - p) = a\cdot\frac{T^2}{2}$$

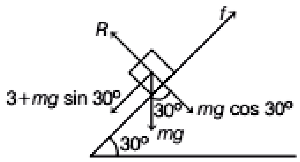
$$\Rightarrow \quad \frac{2p}{a} = T^2 \Rightarrow T = \sqrt{\frac{2p}{a}}$$

22. ***(d)*** Block does not move upto a maximum applied force of 3 N down the inclined plane.

Equation of motion is

$$3 + mg\sin 30° = f$$

$$3 + mg\sin 30° = \mu\, mg\cos 30° \quad \ldots \text{(i)}$$



Similarly, block also does not move upto a maximum applied force of 12 N up the plane.

Now, equation of motion is

$$mg\sin 30° + f = 12$$

or, $\quad mg\sin 30° + \mu\, mg\cos 30° = 12 \quad \ldots \text{(ii)}$

Solving Eqs. (i) and (ii), we have

$$mg\sin 30° = \frac{9}{2} \text{ and } \mu\, mg\cos 30° = \frac{15}{2}$$

Hence, dividing

$$\therefore \quad \frac{1}{\mu}\tan 30° = \frac{9/2}{15/2} = \frac{3}{2} \Rightarrow \mu = \frac{5}{3\sqrt{3}}$$

23. ***(a)*** If middle portion of lens is painted black, then less number of refracted rays will intersect, therefore intensity of the corresponding image of object will be less. Though, the rays coming from object refracted with small portion of lens, so complete image of object will be formed but of lesser intensity.

24. ***(d)*** Given, speed of sound, $v = 340$ m/s

Length of open organ pipe, $l = 2$ m

Frequency, $f = 1200$ Hz

For open organ pipe,

Fundamental frequency, $f_0 = \frac{v}{2l} = \frac{340}{2 \times 2} = 85$

$\therefore$ Number of tones present in the open organ pipe

$$= \frac{f}{f_0} = \frac{1200}{85} = 14.11 \simeq 14$$

25. ***(*)*** Given, charge on proton,

$$q = e = 1.6 \times 10^{-19} \text{ C}$$

Radius, $\quad r = 2 \text{ mm} = 2 \times 10^{-3}$ cm

Frequency, $\quad f = 10$ Hz

Current associated by proton is given by

$$I = \frac{q}{T} \quad [T \rightarrow \text{time period}]$$

$$= qf \quad \left[\because T = \frac{1}{f}\right]$$

$$= 1.6 \times 10^{-19} \times 10 = 1.6 \times 10^{-18} \text{ A}$$

$\therefore$ Magnetic dipole moment, $M = IA$

$$= I \cdot \pi r^2 = 1.6 \times 10^{-18} \times 3.14 \times (2 \times 10^{-3})^2$$

$$= 20.09 \times 10^{-24}$$

$$\simeq 2 \times 10^{-23} \text{ A-m}^2$$

26. ***(b)*** When capacitor is fully charged, then it acts like a open switch, therefore 8 kΩ resistor has no use. So, rest of the resistances 4 kΩ, 7kΩ and 9 kΩ will be in series.

Hence, current drawn from cell,

$$I = \frac{10}{(4+7+9) \times 10^3} = \frac{10}{20 \times 10^3} = 0.5 \text{ mA}$$

27. ***(c)*** The charge on the capacitor at time t in a discharging circuit is given by

$$q = q_0 e^{-t/\tau}$$

where, $\tau = RC$ = time constant

$$\therefore \quad q = q_0\, e^{-t/RC}$$

when, $\quad q = \frac{q_0}{2}$

then $\quad \frac{q_0}{2} = q_0\, e^{-t/RC}$

$$\frac{1}{2} = e^{-t/RC}$$

Taking natural log on both side, we get

$$\log_e\left(\frac{1}{2}\right) = \log_e e^{-t/RC}$$

$$-\log_e 2 = -\frac{t}{RC}\log_e e \Rightarrow t = RC\log_e 2$$

$[\because \log_e e = 1]$

28. ***(d)*** If v be the velocity in the inter planetary space and U_i, U_f initial and final potential energy

Then, by the law of conservation of energy,

$$U_i + K_i = U_f + K_f$$

$$\frac{-GM_e m}{R_e} + \frac{1}{2}m(3v_e)^2 = 0 + \frac{1}{2}mv^2$$

$$\frac{-GM_e}{R_e} + \frac{9}{2}v_e^2 = \frac{1}{2}v^2$$

$$\Rightarrow \quad \frac{-2GM_e}{R_e} + 9v_e^2 = v^2$$

$$\Rightarrow \quad \frac{-2gR_e^2}{R_e} + 9v_e^2 = v^2 \quad [\because GM_e = gR_e^2]$$

$$-2gR_e + 9 \times 2 \times gR_e = v^2, \quad [\because v_e^2 = 2gR_e]$$

$$16\, gR_e = v^2$$

$$\Rightarrow \quad v = \sqrt{8}\,.\sqrt{2g\,R_e}$$

$$= 2\sqrt{2}\, v_e$$

29. *(d)* Given, density of bob, $\rho = \frac{4}{3} \times 10^3$ kg/m^3

Density of water, $\sigma = 10^3$ kg/m^3

If g' be gravitational acceleration in water, then

$$g' = g\left(1 - \frac{\sigma}{\rho}\right) = g\left(1 - \frac{10^3}{\frac{4}{3} \times 10^3}\right) = \frac{g}{4}$$

As, $T_{air} = 2\pi\sqrt{\frac{l}{g}}$

Similarly, $T_{water} = 2\pi\sqrt{\frac{l}{g'}}$ $\left[\because T_{water} = 2s \text{ and } g' = \frac{g}{4}\right]$

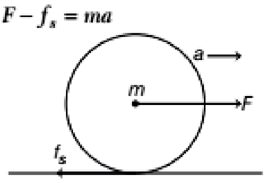
$\Rightarrow$ $2 = 2\pi\sqrt{\frac{l}{g/4}} = 2\pi\sqrt{\frac{l}{g}}.2$

$2 = 2\,T_{air} \Rightarrow T_{air} = 1$ s

30. *(c)* By Newton's law of motion,

$F - f_s = ma_{CM}$

$\Rightarrow$ $F - f_s = ma$ ... (i)



and torque, $\tau = I\alpha$

where, α = tangential component of acceleration

and I = moment of inertia.

$\Rightarrow$ $fR = I\alpha$

As the cylinder is rolling without slipping, the acceleration of point of contact will be zero.

$$a_{CM} - R\alpha = 0 \Rightarrow \alpha = \frac{a_{CM}}{R} = \frac{a}{R}$$

$$fR = \frac{MR^2}{2} \cdot \frac{a}{R} \qquad \left[\therefore I = \frac{MR^2}{2}\right]$$

$\Rightarrow$ $f = \frac{ma}{2}$

Substituting in Eq. (i), we get

$F = f + ma$

$\Rightarrow$ $F = \frac{ma}{2} + ma = \frac{3ma}{2}$

31. *(a)* Velocity of projectile at time t is given by

$\mathbf{v} = \mathbf{u} - \mathbf{g}\,t\,\hat{\mathbf{j}}$

$\mathbf{v} = \hat{\mathbf{i}} + \sqrt{3}\,\hat{\mathbf{j}} - 10t\,\hat{\mathbf{j}}$

$\therefore$ $\mathbf{r} = \int v\,dt$

$= \int (\hat{\mathbf{i}} + \sqrt{3}\,\hat{\mathbf{j}} - 10t\,\hat{\mathbf{j}})\,dt$

$= t\,\hat{\mathbf{i}} + \sqrt{3}\,t\,\hat{\mathbf{j}} - 10\frac{t^2}{2}\hat{\mathbf{j}}$

$= t\,\hat{\mathbf{i}} + \sqrt{3}\,t\,\hat{\mathbf{j}} - 5\,t^2\hat{\mathbf{j}}$

$\mathbf{r} = t\,\hat{\mathbf{i}} + (\sqrt{3}t - 5t^2)\,\hat{\mathbf{j}}$

or $x\hat{\mathbf{i}} + y\hat{\mathbf{j}} = t\,\hat{\mathbf{i}} + (\sqrt{3}t - 5t^2)\,\hat{\mathbf{j}}$

On comparing, we get

$x = t$ ... (i)

and $y = \sqrt{3}\,t - 5t^2$ ... (ii)

From Eqs (i) and (ii), we get

$y = \sqrt{3}\,x - 5x^2$

32. *(d)* $\mathbf{F} = 20\hat{\mathbf{i}} + 10\hat{\mathbf{j}}$

$\therefore$ $F_x = 20$ N and $F_y = 10$ N

$\therefore$ Acceleration is expressed as

$a_x = \frac{F_x}{m} = \frac{20}{1}$

$a_x = 20$ m/s^2

and $a_y = \frac{F_y}{m} = \frac{10}{1} = 10$ m/s^2

$\therefore$ Displacement of body in x-direction in 2s.

$x = u_x t + \frac{1}{2}a_x t^2$

$= 0 + \frac{1}{2} \times 20 \times 2^2 = 40$ m

$\therefore$ Displacement of body in y-direction in 2s.

$y = u_y t + \frac{1}{2}a_y t^2$

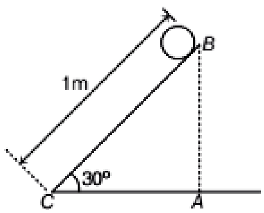
$= 0 + \frac{1}{2} \times 10 \times 2^2 = 20$m

$\therefore$ Position of the body at $t = 2$ s

$\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$

$\mathbf{r} = 40\hat{\mathbf{i}} + 20\hat{\mathbf{j}}$

33. *(b)* Potential energy of cylinder at top position



$U = mg(AB)$

$= 2 \times 10\,[1.\sin 30°]$ $\left[\because \sin 30° = \frac{AB}{BC} \quad \therefore AB = 1 \sin 30°\right]$

$= 10$J

If v is the linear speed of cylinder when it reaches at bottom, then at bottom, total potential energy of cylinder is converted into kinetic energy,

i.e. $K = U$

As the cylinder undergoes both rotational and translational motion,

$\therefore \quad \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = 10$

$\frac{1}{2}\cdot\frac{mr^2}{2}\cdot\frac{v^2}{r^2} + \frac{1}{2}mv^2 = 10 \quad \left[\because \omega = \frac{v}{r}\right]$

$\Rightarrow \quad \frac{3}{4}mv^2 = 10$

$$v = \sqrt{\frac{40}{3m}} = \sqrt{\frac{40}{3\times 2}} = \sqrt{\frac{20}{3}} \text{ m/s}$$

34. ***(a)*** According to the question,

$$F_1 = -k_1x \text{ and } F_2 = -k_2x$$

$\therefore$ Frequency of oscillation,

$$\nu_1 = \frac{1}{2\pi}\sqrt{\frac{k_1}{m}}$$

$$5 = \frac{1}{2\pi}\sqrt{\frac{k_1}{m}} \Rightarrow k_1 = 100\,\pi^2 m$$

Similarly, $12 = \frac{1}{2\pi}\sqrt{\frac{k_2}{m}}$

$\Rightarrow \quad k_2 = 576\,\pi^2 m$

Now, $\quad F = F_1 + F_2 = -(k_1 + k_2)\,x$

$\therefore$ Frequency of oscillation,

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}} = \frac{1}{2\pi}\sqrt{\frac{100\pi^2 m + 576\pi^2 m}{m}}$$

$\Rightarrow \quad \nu = 13$ Hz

35. ***(c)*** Number of moles in 2×10^{-4} kg hydrogen

$\therefore \quad n = \frac{\text{Mass of hydrogen in gram}}{\text{Molecules mass}}$

$= \frac{2\times 10^{-4}\times 10^3}{2} = 0.1$ mole

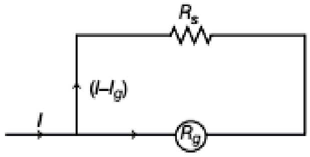
$\therefore \quad pV = nRT$

$\frac{pV}{T} = nR = 0.1\times R = \frac{R}{10}$

36. ***(a)*** Given, for an ammeter, $I_g = 1$ A, $R_g = 1.5\ \Omega$

Total current, $I = 4$ A

If R_s be the shunt resistance, then according to the question circuit diagram is given, ($R_s \parallel R_g$, hence have equal potential difference)



where, $I_gR_g = (I - I_g)\,R_s$

$$R_s = \frac{I_g}{I - I_g}\times R_g = \frac{1}{4-1}\times 1.5 = 0.5\ \Omega$$

37. ***(c)*** The electric field intensity at a point lying outside the sphere (non-conducting) is

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

i.e. $\quad E \propto \frac{1}{r^2} \quad \ldots$ (i)

The electric field intensity at the surface $(r = R)$,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2} \Rightarrow E \propto \frac{1}{R^2}$$

where, R being the radius of sphere.

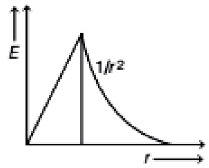
The electric field intensity inside the sphere is

$$E = \frac{qr}{4\pi\varepsilon_0 R^3}$$

$\Rightarrow \quad E \propto r$

At the centre of sphere, $r = 0, \therefore E = 0$

Hence, electric field will increase linearly with distance till R as $E \propto r$ after that it decreases as it is inversely proportional to the square of distance.



38. ***(a)*** Given, $R = (56 \pm 5.6)$ kΩ

$= 56$ k$\Omega \pm 10\%$ of 56 kΩ

$= 56\times 10^3 \pm 10\%\ \Omega$

As per the colour code for carbon resistors, the colour assigned to numbers

5 → Green
6 → Blue
3 → Orange

For 10% accuracy or tolerance, the colour is silver.

Hence, the bands of colours of carbon resistor in sequence are green, blue, orange and silver.

39. ***(c)*** When two waves with same frequency and constant phase difference interfere to each other, then after interference, intensity of resultant wave is distributed such that it is maximum at some points [constructive interference or bright fringes] and minimum at another points. [destructive interference or dark fringes].

Hence, energy is distributed and remains constant [in the form of bright and dark fringes].

40. ***(d)*** According to the given graph $(I - t)$ induced current (I) is obtained from the slope.

Hence, $I = \frac{1}{R}\frac{d\phi}{dt}$

$\Rightarrow d\phi = I\,dt.\,R =$ area of triangle $POS \times R$

$= \frac{8\times 0.2}{2}\times 50$

$= 40$ Wb

Chemistry

41. ***(a)*** Let atoms of Y in ccp structure = 100

Then, number of tetrahedral voids = 2×100

$\because$ Number of X atoms = $\frac{2}{3}$ rd of tetrahedral voids

$\therefore$ Number of X atoms = $\frac{2}{3} \times 200 = \frac{400}{3}$

Thus, $\frac{X}{Y} = \frac{400}{3 \times 100} = \frac{400}{300} = \frac{4}{3}$

Hence, formula of compound can be = X_4Y_3.

42. ***(a)*** Given, density (d_1) at 30°C (T_1) = 1.35 kg/m^3

To find density (d_2) at STP, (i.e at 760 torr and 273 K (T_2) temperature), we use formula

$$\frac{d_2}{d_1} = \frac{p_2 T_1}{p_1 T_2}$$

where, p_1 = 768 torr (Pressure at 30°C, i.e. at 303 K)

p_2 = 760 torr (Pressure at 273 K)

$$\therefore \quad d_2 = \frac{d_1 \times p_2 \times T_1}{p_1 \times T_2} = \frac{1.35 \times 760 \times 303}{768 \times 273} = 1.48 \text{ kg/m}^3$$

43. ***(c)*** $\Delta_o = (-0.4 \times nt_{2g}) + (0.6 \times ne_g)$

where, n = number of electrons in nt_{2g} and ne_g.

CFSE of given d-electron configurations are as follows

a. For high spin, d^6 ;

Δ_o (CFSE) = $-0.4 \times 4 + 0.6 \times 2 = -0.4$

b. For low spin, d^4;

Δ_o(CFSE) = $-0.4 \times 4 + 0.6 \times 0 = 1.6$

c. For low spin d^5

Δ_o (CFSE) = $-0.4 \times 5 + 0.6 \times 0 = -2.0$

d. For high spin, d^7

Δ_o (CFSE) = $-0.4 \times 5 + 0.6 \times 2 = -0.8$

Neglecting –ve sign (as it indicates that energy is released.

Hence, maximum Δ_o (CFSE) value is shown by low spin d^5 configuration.

44. ***(b)*** According to Rydberg's equation

$$\bar{\nu} = \frac{1}{\lambda} = RZ^2 \left[\frac{1}{n_L^2} - \frac{1}{n_H^2}\right]$$

where,

$\bar{\nu}$ = wave number

λ = wavelength

R = Rydberg constant (= 109677 cm^{-1})

Z = atomic number

n_L = lower energy state

n_H = higher energy state.

i.e. $\frac{1}{\lambda} \propto Z^2$

Thus, $\frac{\lambda(He^+)}{\lambda(H)} = \frac{Z^2(H)}{Z^2(He^+)}$

To find λ(He) = ?

Given, λ(H) = 91.2 nm

Z(H) = 1

Z(He) = 2

$\therefore \quad \lambda(He^+) = \frac{1 \times 91.2}{4} = 22.8$ nm

45. ***(b)*** $C_{18}H_8(s) + 12O_2(g) \longrightarrow 10CO_2(g) + 4H_2O(l)$

We know that, $\Delta H = \Delta E + \Delta n_g RT$

Δn_g = No. of moles of gaseous products – no. of moles of gaseous reactants.

$\Delta n_g = 10 - 12 = -2$

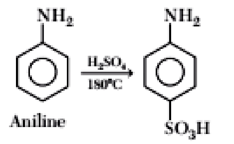
$\Delta H = -1228.2 \times 10^3 + (-2) \times 2 \times 298$

$= -1229393$ cal

$= -1229.39$ kcal

46. ***(a)*** Among the formation of *B*, *C*, *D* and *E*, the slowest step is the rate determining step. As formation of *B*, i.e. 0.002 mol/h per mole of *A* is the slowest step. Hence, step I is the rate determining step.

47. ***(d)*** In the given reaction as,

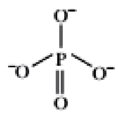


Aniline

Sulphonic acid

Sulphanilic acid exist as dipolar ion (Zwitter ion). It act as inner salt and —SO_3H group diminishes the basic character of —NH_2. Thus, statement (d) is true.

48. ***(c)*** The structure of PO_4^{3-} is as follows



As phosphorus central atom forms four (4) sigma bonds and has no lone pair of electrons. It uses four hybrid orbitals and thus, hybridisation of P-atom is sp^3.

49. ***(c)*** Electronic configuration of the given species are

a. Sm^{2+}(Z = 62) = [Xe] $4f^6$

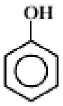
b. Eu^{2+} (Z = 63) = [Xe] $4f^7$

c. Yb^{2+}(Z = 70) = [Xe] $4f^{14}$

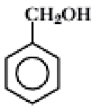
d. Ce^{2+}(Z = 58) = [Xe] $4f^1 5d^1$

On the basis of above electronic configuration, only Yb^{2+}(Z = 70) has fully-filled orbitals i.e. no unpaired electrons. Thus, it is dimagnetic in nature.

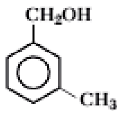
50. ***(c)*** Alcohol in which —OH group is directly bonded to benzene are aromatic alcohols.

I. 

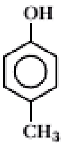
It is an aromatic alcohol, as —OH group is directly bonded with the benzene ring.

II. 

It is not an aromatic alcohol because —OH group is not bonded directly with the benzene group.

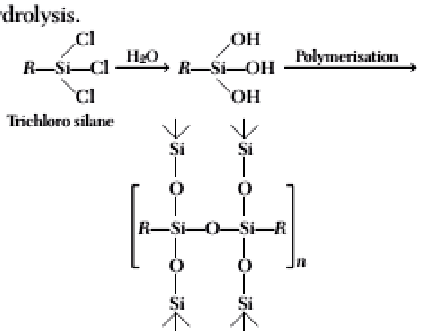
III. 

It is also not an aromatic alcohol because —OH group is not bonded directly with the benzene ring.

IV. 

It is an aromatic alcohol because —OH group is directly bonded with the benzene ring.

51. ***(b)*** $RSiCl_3$ gives cross-linked silicon polymer on hydrolysis.



Cross-linked silicon polymer

52. ***(a)*** Let k be the rate constant of given reaction

$$2O_3 \xrightarrow{k} 3O_2$$

From the slowest step, $r = k'[O][O_3]$ ... (i)

To eliminate [O], (From fast step)

$$k_{eq} = \frac{[O_2][O]}{[O_3]} \Rightarrow [O] = \frac{k_{eq}[O_3]}{[O_2]}$$

Now, substituting value of [O] in Eq (i)

$$r = \frac{k' k_{eq}.[O_3][O_3]}{[O_2]}$$

Let $k' k_{eq} = k$

Thus, $r = k[O_3]^2[O_2]^{-1}$

53. ***(a)*** Depression in freezing point is given as $\Delta T_f = K_f m$

$$\text{Molality of solution } (m) = \frac{\Delta T_f}{K_f}$$

$$= \frac{0.6}{1.86} = 0.322 \text{ m}$$

Elevation in boiling point of solution is given as

$$\Delta T_b = K_b \times m$$

$$= 0.512 \times 0.322 = 0.165$$

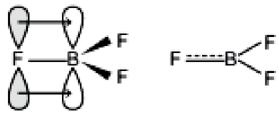
∴ Boiling point of solution

$$= 373 + 0.16$$

$$= 373.16 \text{ K}$$

54. ***(a)*** The strength of acidic character of boron trihalides depends upon $p\pi$ - $p\pi$ back bonding.

In boron trihalides, $p\pi$-$p\pi$ back bonding occurs due to empty orbital of boron and filled orbitals of halogen.



Phenomenon of back bonding in BF_3 molecule

The $p\pi$-$p\pi$ back bonding is shown maximum by BF_3, as the size of B and F are small and comparatively same. Due to this effect tendency of accepting lone-pair of electrons of boron decreases as size of halogen increases. The order of size of halogens are F < Cl < Br < I. Thus, acidic nature is in order

$$BF_3 < BCl_3 < BBr_3 < BI_3$$

55. ***(d)*** Mobility of alkali ions in aqueous solution ∝ size of ion

$$\propto \frac{1}{\text{size of hydrated ion}}.$$

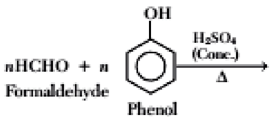
The mobility of alkali ions is inversely proportional to the size of ion in hydrated state, because smaller the ion, more it will get hydrated and becomes larger in size, thus its ionic mobility decreases.

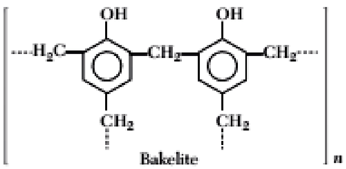
Hence, correct order of mobility is

$$Rb^+ > K^+ > Na^+ > Li^+$$

56. ***(c)*** Bakelite is a thermosetting plastic which is formed by the reaction of phenol with HCHO in the presence of H_2SO_4 (conc).

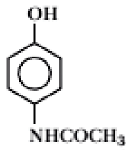
The reaction occurs as follows





A cross-linked condensation polymer (at *o*-and *p*-position) bakelite is thus produced.

57. ***(c)*** Paracetamol is the drug, that can be used as an antipyretic as well as an analgesic, i.e. to reduce the fever and gives relieve from pain. Its structure is as follows



Paracetamol

58. ***(a)*** In the given structure, ring (I) has one O-atom and is a six membered ring, thus is called pyranose ring, whereas ring (II) has one O-atom and is a five membered ring, thus is called furanose.

The disaccharide is linked by a glycosidic bond between C-1 of glucose (in the α-position, α-linkage) and C-2 of fructose (in the β-position, β-linkage).

59. ***(c)*** Due to size and geometries of bases, the only possible pairing in DNA are between G (guanine) and C (cytosine) through three H-bonds and between A (adenine) and T (thymine) through two H-bonds.

Thus, the complementary strand of DNA will be

| T A G C A T A C | (complementary strand)

60. ***(a)*** When Na_2SO_3 reacts with hot and dil. H_2SO_4 , it gives $SO_2(g)$ which decolourises bromine-water.

The reaction proceed as follows:

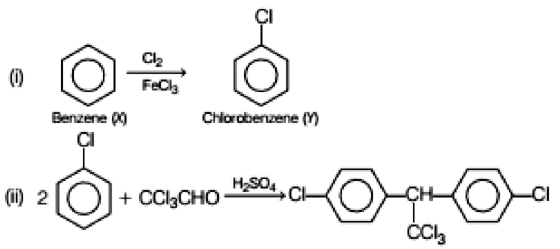
(i) $Na_2SO_3 + H_2SO_4 \longrightarrow Na_2SO_4 + SO_2 + H_2O$

(ii) $Br_2 + H_2O \longrightarrow 2HBr + [O]$

(iii) $SO_2 + [O] \longrightarrow SO_3$

(decolourisation of bromine water)

61. ***(b)*** The given reaction is for preparation of DDT (dichlorodiphenyl trichloroethane). Complete reaction is given as follows



62. ***(d)*** The mixture of $CO(g)$ and $H_2(g)$ is known as water gas as it is used in the synthesis of other compounds,

$$\underbrace{CO(g) + 2H_2(g)}_{\text{Water gas}} \xrightarrow[\text{Catalyst}]{\text{Cobalt}} \underset{\text{Methanol}}{CH_3OH(l)}$$

63. ***(c)*** Chromium hydroxide, i.e. $Cr(OH)_3$ when reacts with NaOH and Na_2O_2, gives yellow colour of sodium chromate. The whole reaction is as follows

$$3Na_2O_2 + 2Cr(OH)_3 \longrightarrow \underset{\text{(Yellow)}}{2Na_2CrO_4} + 2H_2O + 2NaOH$$

64. ***(a)*** Cl is the good leaving group, i.e. the weakest nucleophile among the given options.

∴ The rate of reaction is faster, when 'Z' is 'Cl'.

65. ***(b)*** Use Nernst equation, i.e.

$$E(\text{emf}) = E^\circ_{\text{cell}} - \frac{0.0591}{2}\log\frac{[\text{Product}]}{[\text{Reactant}]}$$

For reaction (i)

$$E_1 = E^\circ_{\text{cell}} - \frac{0.0591}{2}\log\frac{0.1}{1}$$

$$E_1 = E^\circ_{\text{cell}} + \frac{0.0591}{2}$$

For reaction (ii)

$$E_2 = E^\circ_{\text{cell}} - \frac{0.0591}{2}\log\frac{1}{1}$$

$$E_2 = E^\circ_{\text{cell}} - 0$$

$$E_2 = E^\circ_{\text{cell}}$$

For reaction (iii)

$$E_3 = E^\circ_{\text{cell}} - \frac{0.0591}{2}\log\frac{1}{0.1}$$

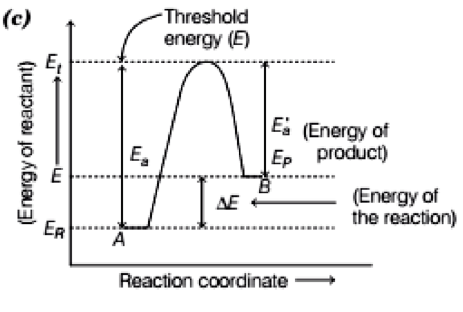
$$E_3 = E^\circ_{\text{cell}} - \frac{0.0591}{2}$$

Hence, $E_1 > E_2 > E_3$.

66. ***(c)*** This test is used for the presence of sulphur in any compound. When sodium nitroprusside, react with sulphide ion, (e.g. Na_2S) purple colouration is produced, in which the following reaction takes place.

$$Na_2S + \underset{\text{(Sod. nitroprusside)}}{Na_2[Fe(CN)_5NO]} \longrightarrow \underset{\substack{\text{Purple colouration} \\ \text{(sod. thionitroprusside)}}}{Na_4[Fe(CN)_5NOS]}$$

67. ***(c)***



where, E_a = activation energy of forward reaction

E'_a = activation energy of backward reaction

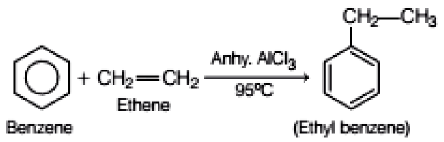
The above energy profile diagram shows that $E_a > E'_a$.

The potential energy of the product is greater than that of the reactant, so the reaction is endothermic.

$$E_a = E_a' + \Delta E \Rightarrow E_t = E_a \text{ or } E_t > E_a'$$

∴ The threshold energy is equal or greater than activation energy.

68. ***(c)*** Ethene and benzene on reaction with anhy.$AlCl_3$ produce ethyl benzene as follows



69. ***(b)*** Given, pH of solution $A = 3$

$pH = -\log[H^+]$

$[H^+] = 10^{-pH}$

$[H^+] = 10^{-3}M$

$[H^+]$ of $A = 10^{-3}M$ and, pH of solution $B = 2$

$[H^+] = 10^{-pH} = 10^{-2}M$

$[H^+]$ of $B = 10^{-2}$ M

Thus, total $[H^+]$ are $= 10^{-3} + 10^{-2} = 1.1 \times 10^{-3}$ M

$$\therefore \quad pH_{(final)} = -\log[H^+]$$

$$= -\log[1.1 \times 10^{-3}]$$

$$= 3 - \log 1.1$$

$$= 3 - 1.04 = 1.9$$

70. ***(a)*** Freundlich isotherm is related with the pressure of gas adsorbed as follows

$$\frac{x}{m} = k \cdot p^{1/n}$$

where, x/m = amount of gas (x) adsorbed over the surface of mass (m).

p = pressure of gas

k = Freundlich constant

Thus, value of 'n' is always greater than 1 and therefore value of $\frac{1}{n}$ lies between 0 and 1.

71. ***(c)*** Electronic configuration of Mn is $(Z = 25)$ $= [Ar]3d^5 4s^2$

∴ Electronic configuration of Mn^{4+} is $= [Ar]\,3d^3$

Thus, Mn^{4+} has 3 unpaired electrons and its spin only magnetic moment (μ) is $= \sqrt{n(n+2)}$ BM

where, n = number of unpaired electrons.

$$\mu = \sqrt{3(3+2)} \text{ BM}$$

$$\Rightarrow \quad \mu = \sqrt{15} = 3.87 \text{ BM}$$

$$= 4 \text{ BM}$$

72. ***(a)*** ∵ Gram equivalents of an acid

= gram equivalents of KOH i.e. $\frac{0.45}{E} = \frac{20 \times 0.5}{1000}$

where, E = equivalent mass $= \frac{0.45 \times 1000}{20 \times 0.5} = 45$

$$\text{Basicity} = \frac{\text{Molecular mass}}{\text{Equivalent mass}}$$

Thus, basicity $= \frac{90}{45} = 2$

73. ***(a)*** Given, $A + B \rightleftharpoons C + D$

	A	B	C	D
Initial conc.	4	4	0	0
At equilibrium	(4 − 2)	(4 − 2)	2	2
i.e.	2	2	2	2

$$K_{eq} = \frac{[C][D]}{[A][B]}$$

$$\therefore \quad K_{eq} = \frac{2 \times 2}{2 \times 2} = 1$$

74. ***(b)*** Given,

125 mL of 1 M $AgNO_3$ = 13.5 g of Ag

∵ 1000 mL of $AgNO_3$ = 108 g of Ag

∴ 125 mL of $AgNO_3 = \frac{108 \times 125}{1000} = 13.5$ g

Also, 108 g of Ag is deposited by = 96500 C

∴ 13.5 g of Ag is deposited by $= \frac{96500 \times 13.5}{108}$

$= 12062.5$ C

$$\because \quad Q = it$$

$\therefore\ t$ (time) $= Q / i = \frac{12062.50}{241.25}$, $t = 50$ s

75. ***(b)*** Orbitals of nearly equal energy are called degenerated orbitals. As, energy of,

a. $\sigma 2s > \sigma 1s$

b. $\pi\, 2p_x = \pi 2p_y$

c. $\pi^* 2p_x < \sigma^* 2p_z$

d. $\sigma 2p_x < \sigma^* 2p_z$

Thus, only $\pi\, 2p_x$ and $\pi 2p_y$ are degenerate orbitals.

76. ***(c)*** Total energy (E) of any photon is given by the relation : $E = \frac{hc}{\lambda}$

where, h = Planck's constant $= 6.6 \times 10^{-34}$ J-s

c = velocity of light $= 3 \times 10^{-10}$ m/s

λ = wavelength = 150 pm

$= 1.5 \times 10^{-10}$ m.

Thus, $E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{1.5 \times 10^{-10}}$;

$= 1.32 \times 10^{-15}$ J

and, energy of ejected electron

$$E' = \frac{1}{2}mv^2$$

where,

m = mass of electron = 9.1×10^{-31} kg

v = velocity of electron = 1.5×10^{7} m/s

$E' = \frac{1}{2}mv^2 = \frac{1}{2} \times 9.1 \times 10^{-31} \times (1.5 \times 10^{7})^2$

$= 1.024 \times 10^{-16}$

Thus, total energy of photon = binding energy of electron (B) + energy of ejected electron (E')

Thus, $1.32 \times 10^{-15} = B + E'$

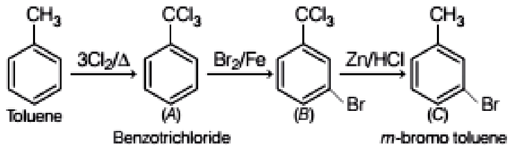
$\therefore \quad B = E - E'$

$= [1.32 \times 10^{-15}] - [1.024 \times 10^{-16}]$

$= 1.2176 \times 10^{-15}$ J

$= \frac{1.2176 \times 10^{-15}}{1.6 \times 10^{-19}}$ eV $= 7.6 \times 10^{3}$ eV

77. ***(b)*** The degree of unsaturation for compound C_7H_8 is three (3), as it reacts with Cl_2/Δ. Hence, C_7H_8 is toluene, i.e $C_6H_5CH_3$. The whole reaction occurs as follows:



78. ***(c)***

Element	Ratio (by weight)	Atomic weight	Mol ratio	Simplest mol ratio
C	9	12	$\frac{9}{12} = 0.75$	$\frac{0.75}{0.25} = 3$
H	1	1	$\frac{1}{1} = 1.00$	$\frac{1.00}{0.25} = 4$
N	3.5	14	$\frac{3.5}{14} = 0.25$	$\frac{0.25}{0.25} = 1$

$\therefore$ Empirical formula = C_3H_4N

The empirical formula mass = $3 \times 12 + 4 \times 1 + 1 \times 14$

= 54 g

Given, molecular mass = 108 g

Thus, $n = \frac{108}{54} = 2$

Hence, molecular formula = (Empirical formula) × 2

$(C_3H_4N) \times 2 = C_6H_8N_2$

79. ***(a)*** π-bond do not participate in hybridisation

a. $H—\overset{sp}{C}≡\overset{sp}{C}—\overset{sp}{C}≡\overset{sp}{C}—H$ b. $\overset{sp^3}{CH_3}—\overset{sp}{C}≡\overset{sp}{C}—\overset{sp^3}{CH_3}$

c. $\underset{sp^2}{CH_2}=\underset{sp}{C}=\overset{sp^2}{CH_2}$ d. $\underset{sp^2}{CH_2}=\underset{sp^2}{CH}—\underset{sp}{C}≡\underset{sp}{CH}$

$\therefore$ Option (a) is a compound in which all carbon atoms are *sp* hybridised.

80. ***(a)*** In bassemer converter, copper sulphide is partially oxidised to cuprous oxide, which further reacts with the remaining copper sulphide to give copper and sulphur dioxide,

i.e. $Cu_2S + 2Cu_2O \longrightarrow 6Cu + SO_2$

a. English Proficiency

81. (***b***) 'Many a' takes singular noun, as well as singular verb. So, replace 'have' by 'has'.

82. (***a***) The correct syntax for interrogative sentences is 'wh' word + do/does+sub+verb+?

So, we should replace 'it takes' by 'does it take'.

83. (***c***) Replace 'his' by 'ones' to make the possessive correct.

84. (***a***) Assured is followed by preposition 'of'.

85. (***d***) Preposition 'to' should be used with 'submit' here.

86. ***(d)*** ***Audacious*** means bold or fearless. So, its correct antonym will be 'timid', which means 'lacking in courage or confidence.'

87. ***(c)*** Cogent, which means reasonable and convincing. The correct antonym will be 'dissuasive', which means divert from the measure or purpose.

88. ***(b)*** Taint means a mark of disgrace or blemish. The correct antonym will be 'clear', which means free of ambiguity or doubt.

89. ***(b)*** Vivacious means full of life and energy or animated.

So, its correct synonym will be 'Lively'.

90. ***(b)*** Sporadic means rare and scattered in occurrence. Here 'scattered' has the same sense. It is the correct synonym.

91. (***d***) Both official and corporate India is allergic to 'mention of clean technology.'

92. (***d***) According to the passage, 'Failure in crops' is a life and death question to many Indians.

93. ***(a)*** The most similar in meaning to the word 'profligacy' is 'wastefulness'.

94. ***(a)*** India cannot tolerate any further 'crop failure'.

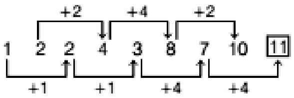
95. ***(d)*** The reason could be 'US wants to use it as a handle against the developing countries in the forthcoming meet'.

b. Logical Reasoning

96. ***(d)*** In the 'College', education is given to 'Students', in the same way, treatment is given to the 'Patient' in 'Hospital'.

97. ***(b)*** Except 35, all others are multiples of 9, but 35 is the multiple of 5 and 7.

98. ***(c)*** Pattern of the series is as shown below



This series contains two separate series. In first series, 1 is added in first two steps and 4 is added in third and fourth steps.
In second series, 2 and 4 is added alternately.

99. ***(b)*** From given information,

Ramesh > Mohan > Shyam ...(i)

Gautam > Ramesh > Rajat ...(ii)

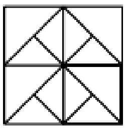
Shyam > Rajat ...(iii)

Combining Eqs. (i), (ii) and (iii), we get

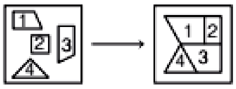
Gautam > Ramesh > Mohan > Shyam > Rajat

Hence, Rajat is the shortest among all.

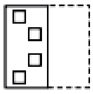
100. ***(b)*** If figure shown in option (b), is placed in the place of missing portion of the original figure, then it is completed as shown below



101. ***(b)*** Figure given in option (b) can be formed by joining the pieces given in question figure, as shown below.

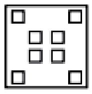


102. ***(d)*** In figure (X), the square sheet of paper is being folded along the vertical line of symmetry, so that right half of the sheet overlaps the left half.
In figure (Y), the sheet is folded further to a quarter.
In figure (Z), two squares are punched in the folded sheet. Clearly, the punched squares will be created in each quarter of the paper and after unfolding the first fold the figure will look like as,



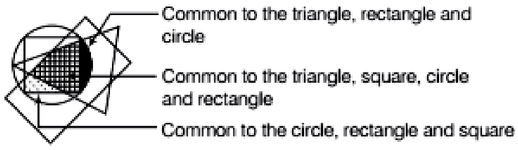
then after unfolding the last fold the transparent sheet will look like as given in option (d).

i.e.

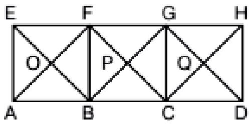


103. ***(a)*** In each row, the number of objects increases by 1 at each step from left to right. So, in third row, the missing segment contain 2 + 1 = 3 dots.

104. ***(b)*** One of the three dots occupies the region which is common to the circle, rectangle and triangle; another dot occupies the region which is common to the triangle, circle, rectangle and square and the third dot occupies the region which is common to the circle, rectangle and square. These three characteristics as shown by these three dots are found in figure (b). It possesses region which is common to the circle, rectangle and triangle, a region which is common to the triangle circle, rectangle, and square and a region which is common to the circle, rectangle and square



105. ***(a)*** Naming the figure,



Clearly, there are 28 triangles in the given figure namely, ΔEOF, ΔAOE, ΔAOB, ΔBOF, ΔABF, ΔBEF, ΔABE, ΔAEF, ΔBPF, ΔFPG, ΔCPG, ΔBPC, ΔBFG, ΔBCG, ΔCFG, ΔBCF, ΔGQC, ΔCDQ, ΔDQH, ΔGQH, ΔGDC, ΔGDH, ΔGHC, ΔCDH, ΔAFC, ΔBGD, ΔEBG and ΔFCH.

Mathematics

106. ***(d)*** Given, $a = 3b$

$$e = \sqrt{1 - \frac{b^2}{a^2}} \Rightarrow e = \sqrt{1 - \frac{b^2}{9b^2}}$$

$$\Rightarrow \quad e = \sqrt{1 - \frac{1}{9}} = \frac{\sqrt{8}}{3} = \frac{2\sqrt{2}}{3}$$

107. ***(a)*** We have, $\sum_{m=1}^{n} \tan^{-1}\left(\frac{2m}{m^4 + m^2 + 2}\right)$

$$= \sum_{m=1}^{n} \tan^{-1}\left(\frac{(m^2+m+1)-(m^2-m+1)}{1+(m^2+m+1)(m^2-m+1)}\right)$$

$$= \sum_{m=1}^{n} [\tan^{-1}(m^2+m+1) - \tan^{-1}(m^2-m+1)]$$

$$\left[\because \tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)\right]$$

$$= (\tan^{-1}3 - \tan^{-1}1) + (\tan^{-1}7 - \tan^{-1}3) + (\tan^{-1}(13) - \tan^{-1}7) + \ldots + (\tan^{-1}(n^2+n+1) - \tan^{-1}(n^2-n+1))$$

$$= \tan^{-1}(n^2+n+1) - \tan^{-1}1$$

$$= \tan^{-1}\left(\frac{n^2+n+1-1}{1+(n^2+n+1)}\right) = \tan^{-1}\left(\frac{n^2+n}{n^2+n+2}\right)$$

108. ***(b)*** Given, $(2x^2+3x+4)^{10} = \sum_{r=0}^{20} a_r x^r \quad \ldots(i)$

Replacing x by $\frac{2}{x}$, we get

$$\left(\frac{8}{x^2} + \frac{6}{x} + 4\right)^{10} = \sum_{r=0}^{20} a_r \left(\frac{2}{x}\right)^r$$

$$\Rightarrow 2^{10}(2x^2+3x+4)^{10} = \sum_{r=0}^{20} a_r 2^r x^{20-r}$$

$$\Rightarrow \quad 2^{10}\sum_{r=0}^{20} a_r x^r = \sum_{r=0}^{20} a_r 2^r x^{20-r} \quad \text{[from Eq. (i)]}$$

Comparing coefficient of x^8 both sides, we get

$$2^{10}a_8 = a_{12}2^{12} \Rightarrow \frac{a_8}{a_{12}} = \frac{2^{12}}{2^{10}} = 4$$

109. ***(d)*** In ΔADB,

$$\angle ABD = \angle ADB$$

[Angles opposite to equal sides are equal]

$\therefore \quad \angle ADC = \pi - \angle ABD$

Using 2nd formula in $m - n$ cot theorem

Here, $m = 1$ and $n = 1$

$$(1+1)\cot(\pi - B) = 1\cot B - 1\cot C$$

$$\Rightarrow \quad 3\cot B = \cot C$$

$$\Rightarrow \quad \frac{\tan B}{\tan C} = 3$$

110. ***(a)*** We have,

$$L = \lim_{n\to\infty} \sqrt{n}\int_0^1 \frac{dx}{(1+x^2)^n}$$

We know that, $\quad (1+x^2)^n > 1 + nx^2$

$$\Rightarrow \quad \frac{1}{(1+x^2)^n} < \frac{1}{1+nx^2}$$

$$\Rightarrow \quad \int_0^1 \frac{dx}{(1+x^2)^n} < \int_0^1 \frac{1}{1+nx^2}\,dx$$

$$\Rightarrow \quad \int_0^1 \frac{1}{(1+x^2)^n} < \frac{1}{\sqrt{n}}[\tan^{-1}\sqrt{n}\,x]_0^1$$

$$= \frac{1}{\sqrt{n}}\tan^{-1}\sqrt{n}$$

$$\therefore L = \lim_{n\to\infty} \sqrt{n}\int_0^1 \frac{1}{(1+x^2)^n}\,dx < \lim_{n\to\infty} \sqrt{n}\frac{1}{\sqrt{n}}\tan^{-1}\sqrt{n}$$

$$\Rightarrow \quad L < \frac{\pi}{2}$$

$$\therefore \quad \frac{1}{2} < L < 2$$

111. ***(d)*** We have,

$$\sqrt{x+3-4\sqrt{x-1}} + \sqrt{x+8-6\sqrt{x-1}} = 1$$

$$\sqrt{(\sqrt{x-1})^2 - 2\times 2\sqrt{x-1} + 4} + \sqrt{(\sqrt{x-1})^2 - 2\times 3\sqrt{x-1} + 9} = 1$$

$$\Rightarrow \quad \sqrt{(\sqrt{x-1}-2)^2} + \sqrt{(\sqrt{x-1}-3)^2} = 1$$

$$\Rightarrow \quad |\sqrt{x-1}-2| + |\sqrt{x-1}-3| = 1$$

$$\Rightarrow \quad |\sqrt{x-1}-2| + |\sqrt{x-1}-3| = (\sqrt{x-1}-2) - (\sqrt{x-1}-3)$$

We know that,

If $\quad |x-a| + |x-b| = (x-a) - (x-b)$

Then, $\quad (x-a)(x-b) < 0$

$\therefore \quad (\sqrt{x-1}-2)(\sqrt{x-1}-3) < 0$

$\Rightarrow \quad 2 < \sqrt{x-1} < 3 \Rightarrow 5 < x < 10$

$\therefore$ Equation have infinite many solutions.

112. ***(d)*** We have,

$$ax + y + z = a \quad \ldots(i)$$

$$x + by + z = b \quad \ldots(ii)$$

$$x + y + cz = c \quad \ldots(iii)$$

are inconsistent

$$\therefore \quad \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0 \text{ and } \begin{vmatrix} a & 1 & 1 \\ b & b & 1 \\ c & 1 & c \end{vmatrix} \neq 0$$

Now, $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$

$$a(bc-1) - 1(c-1) + 1(1-b) = 0$$
$$\Rightarrow \quad abc - a - c + 1 + 1 - b = 0$$
$$\Rightarrow \quad abc - a - b - c + 2 = 0$$

and $\begin{vmatrix} a & 1 & 1 \\ b & b & 1 \\ c & 1 & c \end{vmatrix} \neq 0$

It is possible only when $a \neq 1$, $b \neq 1$ and $c \neq 1$.

113. ***(a)*** P = Probability of getting an even number $= \frac{1}{2}$

$$q = 1 - \frac{1}{2} = \frac{1}{2}$$

For atleast 4 times an even number, probability is

$$^7C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^3 + {}^4C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^2 + {}^7C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^1 + {}^7C_7\left(\frac{1}{2}\right)^7$$

$= \frac{1}{2}$

114. ***(b)*** Equation of plane containing line $x - y - 1 = 0$ and $z - 1 = 0$ is $(x - y - 1) + \lambda(z - 1) = 0$

$$\Rightarrow \quad x - y + \lambda z - 1 - \lambda = 0 \quad \text{...(i)}$$

Since, this plane is parallel to line

$$\frac{x}{2} - \frac{z}{3} = 1 \text{ and } y = 3$$
$$\frac{x-1}{2} = \frac{y-3}{0} = \frac{z}{3} \quad \text{...(ii)}$$
$$\therefore \quad 2(1) + (0)(-1) + 3(\lambda) = 0$$
$$\Rightarrow \quad \lambda = -2/3$$

Putting the value of λ in Eq. (i), we get

$$x - y - \frac{2}{3}z - 1 + \frac{2}{3} = 0 \text{ and } 3x - 3y - 2z = 1$$

115. ***(d)*** We have, $(1 + x + x^2)^{20} = \sum_{r=0}^{40} a_r x^r$

x is replaced by $-\frac{1}{x}$, we get

$$\left(1 - \frac{1}{x} + \frac{1}{x^2}\right)^{20} = \sum_{r=0}^{40}(-1)^r a_r \left(\frac{1}{x}\right)^r$$
$$\Rightarrow \quad (1 - x + x^2)^{20} = \sum_{r=0}^{40}(-1)^r a_r x^{40-r}$$

$\sum_{r=0}^{39}(-1)^r a_r \cdot a_{r+1}$ is coefficient of x in product of

$$(1 + x + x^2)^{20}\left(1 - \frac{1}{x} + \frac{1}{x^2}\right)^{20} = 0$$

116. ***(a)*** Let

$$L = \lim_{x \to \frac{3\pi}{4}} \frac{4\sin^2 x \cos x - \cos x + \sin x}{\sin x + \cos x}$$
$$L = \lim_{x \to \frac{3\pi}{4}} \frac{2\sin x(\sin^2 x + \cos^2 x + 2\sin x \cos x - 1) - \cos x + \sin x}{\sin x + \cos x}$$
$$L = \lim_{x \to \frac{3\pi}{4}} \frac{2\sin x[(\sin x + \cos x)^2 - 1] - \cos x + \sin x}{\sin x + \cos x}$$
$$L = \lim_{x \to \frac{3\pi}{4}} \frac{2\sin x(\sin x + \cos x)^2 - 2\sin x - \cos x + \sin x}{\sin x + \cos x}$$
$$L = \lim_{x \to \frac{3\pi}{4}} \frac{(\sin x + \cos x)(2\sin x(\sin x + \cos(x - 1))}{\sin x + \cos x}$$
$$L = \lim_{x \to \frac{3\pi}{4}} 2\sin x(\sin x + \cos x) - 1 = -1$$

117. ***(b)*** Given,

$$\mathbf{a} \times \mathbf{b} = \mathbf{c}$$
$$\mathbf{b} \times \mathbf{c} = \mathbf{a}$$
$$\mathbf{c} \times \mathbf{a} = \mathbf{b}$$
$$(\mathbf{b} \times \mathbf{c}) \cdot \mathbf{a} = \mathbf{a} \cdot \mathbf{a}$$
$$(\mathbf{c} \times \mathbf{a}) \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{b}$$

Now, $\mathbf{a}^2 - \mathbf{b}^2 = (\mathbf{b} \times \mathbf{c}) \cdot \mathbf{a} - (\mathbf{c} \times \mathbf{a}) \cdot \mathbf{b}$

$$= [\mathbf{b}\,\mathbf{c}\,\mathbf{a}] - [\mathbf{c}\,\mathbf{a}\,\mathbf{b}] = 0$$
$$\therefore \quad |\mathbf{a}|^2 - |\mathbf{b}|^2 = 0$$
$$\Rightarrow \quad |\mathbf{a}| - |\mathbf{b}| = 0$$

We know that,

$$|\mathbf{a} \times \mathbf{b}\;\mathbf{b} \times \mathbf{c}\;\mathbf{c} \times \mathbf{a}| = [\mathbf{a}\;\mathbf{b}\,\mathbf{c}]^2$$
$$[\mathbf{a}\;\mathbf{b}\,\mathbf{c}] = [\mathbf{a}\;\mathbf{b}\,\mathbf{c}]^2$$
$$|\mathbf{a}\,\mathbf{b}\mathbf{c}|[-[\mathbf{a}\;\mathbf{b}\,\mathbf{c}] - 1] = 0$$
$$[\mathbf{a}\;\mathbf{b}\;\mathbf{c}] = 1$$
$$|\mathbf{a}||\mathbf{b}||\mathbf{c}| = 1$$
$$\therefore \quad |\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}| \neq 2$$

118. ***(a)*** Let $z = x + iy$

Then $\arg z = \tan^{-1}\left(\frac{y}{x}\right)$

Given, $\arg(z) = \frac{\pi}{6}$

$$\therefore \quad \tan^{-1}\frac{y}{x} = \frac{\pi}{6}$$
$$\Rightarrow \quad \frac{y}{x} = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$
$$\Rightarrow \quad x = \sqrt{3}\,y$$

Also given, $|z - 2\sqrt{3}i| = \lambda$

$$|x + iy - 2\sqrt{3}\, i| = \lambda$$

$$x^2 + (y - 2\sqrt{3})^2 = \lambda^2$$

This is the equation of circle whose centre is $(0, 2\sqrt{3})$ and $r = \lambda$.

Now, $x - \sqrt{3}y = 0$ touches the circle.

$$\therefore \quad \lambda = \left|\frac{0 - \sqrt{3}\,(2\sqrt{3})}{\sqrt{1+3}}\right|$$

$$\Rightarrow \quad \lambda = \left|\frac{6}{2}\right| = 3$$

$$\therefore \quad \lambda = 3$$

119. ***(a)*** We have,

$$l + m + n = 0 \quad \ldots\text{(i)}$$

and $$l^2 = m^2 + n^2 \quad \ldots\text{(ii)}$$

Putting the value of l in Eq. (ii), we get

$$(m+n)^2 = m^2 + n^2$$

$$\Rightarrow \quad m^2 + n^2 + 2mn = m^2 + n^2$$

$$\Rightarrow \quad mn = 0$$

$$\Rightarrow \quad m = 0, n = 0$$

when $m = 0, l = -n$

$\therefore$ Direction cosines are $\left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$.

When $n = 0, l = -m$

$\therefore$ Direction cosines are $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$.

Angle between lines are

$$\cos\theta = \left(\frac{-1}{\sqrt{2}} \times -\frac{1}{\sqrt{2}}\right) + \left(0 \times \frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}} \times 0\right)$$

$$\cos\theta = \frac{1}{2} \Rightarrow \theta = \pi/3$$

120. ***(c)*** Let the foot of perpendicular be (h, k).

Equation of tangent with slope m passing (h, k) is $y = mx \pm \sqrt{6m^2 + 2}$, where $m = -h/k$

$$\Rightarrow \quad \sqrt{\frac{6h^2}{k^2} + 2} = \frac{h^2 + k^2}{k}$$

$$6h^2 + 2k^2 = (h^2 + k^2)^2$$

So, required locus is $6x^2 + 2y^2 = (x^2 + y^2)^2$.

121. ***(d)*** Let

$$h(x) = f(x) - 2g(x)$$

Now, $$h(0) = f(0) - 2g(0)$$

$$\Rightarrow \quad h(0) = 2 - (0)$$

$$[\because f(0) = 2, g(0) = 0]$$

$$\Rightarrow \quad h(0) = 2$$

Also, $$h(1) = f(1) - 2g(1)$$

$$\Rightarrow \quad h(1) = 6 - 2(2)$$

$$[\because f(1) = 6, g(1) = 2]$$

$$\Rightarrow \quad h(1) = 2$$

$$\therefore \quad h(0) = h(1)$$

$$h'(x) = f'(x) - 2g'(x)$$

By Rolle's theorem $h'(c) = 0$

$\therefore f'(c) - 2g'(c) = 0, f'(c) = 2g'(c)$

122. ***(b)*** We have,

$$I = \int\left(1 + x - \frac{1}{x}\right)e^{x + \frac{1}{x}}\,dx$$

$$I = \int e^{x+\frac{1}{x}}dx + \int\left(x - \frac{1}{x}\right)e^{x+\frac{1}{x}}\,dx$$

$$I = \int e^{x+\frac{1}{x}}dx + \int x\left(1 - \frac{1}{x^2}\right)e^{x+\frac{1}{x}}dx$$

$$I = \int e^{x+\frac{1}{x}}\,dx + xe^{x+\frac{1}{x}} - \int e^{x+\frac{1}{x}}\,dx$$

$$I = xe^{x+\frac{1}{x}} + C$$

123. ***(d)*** We have,

$$(1 + ax + bx^2)(1 - 2x)^{18}$$

$$\Rightarrow (1 + ax + bx^2)(1 - {}^{18}C_1 2x + {}^{18}C_2(2x^2) - {}^{18}C_3(2x)^3 + {}^{18}C_4(2x)^4 \ldots)$$

Coefficient of x^3 is $-{}^{18}C_3(2)^3 + a \cdot {}^{18}C_2(2)^2 - b\,{}^{18}C_1(2)$

and coefficient of x^4 is

$${}^{18}C_4(2)^4 - {}^{18}C_3(2)^3 a + {}^{18}C_2(2)^2 b$$

Since, coefficient of x^3 and x^4 are zero.

$$\therefore -{}^{18}C_3(2)^3 + {}^{18}C_2(2)^2(a) - {}^{18}C_1(2)\,b = 0$$

$$\Rightarrow \quad \frac{4 \times 17 \times 16}{3 \times 2} - 17a + b = 0 \quad \ldots\text{(i)}$$

and $$80 - \frac{32}{3}a + b = 0 \quad \ldots\text{(ii)}$$

Solving Eqs. (i) and (ii), we get

$$a = 16 \text{ and } b = \frac{272}{3}$$

124. ***(a)*** Equation of plane parallel to the plane $x - 2y + 2z - 5 = 0$ is $x - 2y + 2z + \lambda = 0$.

Since, this plane is unit distance from origin.

$$\therefore \quad 1 = \left|\frac{\lambda}{\sqrt{1+4+4}}\right|$$

$$|\lambda| = 3 \Rightarrow \lambda = \pm 3$$

$\therefore$ Equation of plane is

$$x - 2y + 2z = 3$$

125. ***(b)*** Let A be the event that maximum is 6.

B be the event that minimum is 3.

$P(A) = \frac{{}^5C_2}{{}^8C_3}$ (number less than 6 is 5)

$P(B) = \frac{{}^5C_2}{{}^8C_3}$ (number greater than 3 is 5)

$P(A \cap B) = \frac{{}^2C_1}{{}^8C_3}$

$\therefore$ Required probability

$= P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{{}^2C_1}{{}^5C_2} = \frac{2}{10} = \frac{1}{5}$

126. ***(b)*** We have, $\frac{\sin(\lambda\alpha)}{\sin\alpha} - \frac{\cos(\lambda\alpha)}{\cos\alpha} = \lambda - 1$

$\Rightarrow \quad \sin(\lambda\alpha)\cos\alpha - \cos(\lambda\alpha)\sin\alpha = (\lambda - 1)\sin\alpha\cos\alpha$

$\Rightarrow \quad \sin(\lambda - 1)\alpha = (\lambda - 1)\sin\alpha\cos\alpha$

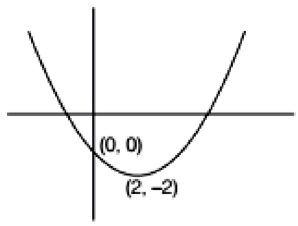
$\Rightarrow \quad \sin(\lambda - 1)\alpha = \frac{\lambda - 1}{2}\sin 2\alpha$

It is possible, when $\lambda - 1 = 0 \Rightarrow \lambda = 1$

and $\quad \frac{\lambda - 1}{2} = 1 \Rightarrow \lambda = 3$

$\therefore \lambda$ has only two solution.

127. ***(b)*** Given, $y = ax^2 + bx + c$ has two x-intercepts one is positive and one is negative and vertex is $(2, -2)$.The graph of $y = ax^2 + bx + c$ is



Clearly from graph

$c > 0, a > 0$

and $\quad -\frac{b}{a} > 0 \quad \Rightarrow \quad -b > 0$

$\Rightarrow \quad b < 0$

$f(1) < 0 \Rightarrow a + b + c < 0$

$ab < 0, ac < 0, bc > 0$

128. ***(b)*** Let A = (1, 2) equation of line is

$\frac{x - 1}{\cos\theta} = \frac{y - 2}{\sin\theta} = r = \frac{\sqrt{6}}{3}$

$\therefore \quad x = \frac{\sqrt{6}}{3}\cos\theta + 1, y = \frac{\sqrt{6}}{3}\sin\theta + 2$

This point lies on line $x + y = 4$

$\therefore \frac{\sqrt{6}}{3}\cos\theta + \frac{\sqrt{6}}{3}\sin\theta = 1$

$\Rightarrow \quad \sin\theta + \cos\theta = \frac{\sqrt{3}}{\sqrt{2}}$

$\Rightarrow \quad \sin\left(\frac{\pi}{4} + \theta\right) = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad \frac{\pi}{4} + \theta = 60° \text{ or } 120°$

$\Rightarrow \quad \theta = 15° \text{ or } 75°$

$\therefore \quad$ Larger angle = 75°.

129. ***(d)*** $[P + 1] = [P] + 1$, Let $[P] = n$, then n is integer

$\therefore$ $([P + 1], [P]) = (n + 1, n)$ lie inside the region of circles

$S_1 = x^2 + y^2 - 2x - 15 = 0, C_1 = (1, 0), r_1 = 4$

and $S_2 = x^2 + y^2 - 2x - 7 = 0, C_2 = (1, 0), r_2 = 2\sqrt{2}$

Both circles are concentric.

$\therefore (n + 1)^2 + n^2 - 2(n + 1) - 7 > 0$

and $(n + 1)^2 + n^2 - 2(n + 1) - 15 < 0 \Rightarrow 4 < n^2 < 8$

Which is not possible for any integer.

130. ***(a)*** We have,

$$\frac{dy}{dx} = e^{x - y}(e^x - e^y)$$

$\Rightarrow \quad e^y\frac{dy}{dx} + e^y e^x = e^{2x}$

Put $\quad e^y = v$

$\therefore \quad e^y\frac{dy}{dx} = \frac{dv}{dx}$

$\therefore \quad \frac{dv}{dx} + ve^x = e^{2x}$

This is linear equation

$$\text{I.F.} = e^{\int e^x dx} = e^{e^x}$$

$\therefore$ Required solution is

$$v \cdot e^{e^x} = \int e^{e^x} \cdot e^{2x}dx + C$$

$$v \cdot e^{e^x} = e^{e^x}(e^x - 1) + C$$

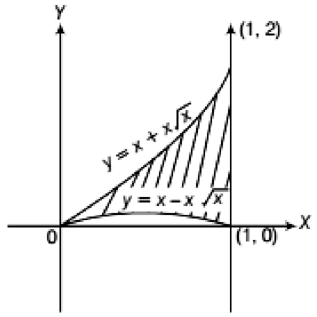
$\Rightarrow \quad e^y \cdot e^{e^x} = e^{e^x}(e^x - 1) + C$

$\Rightarrow \quad e^y = e^x - 1 + ce^{-e^x}$

131. ***(d)*** Given, curve is $(y - x)^2 = x^3$

$\Rightarrow \quad y - x = \pm x\sqrt{x}$

$\Rightarrow \quad y = x \pm x\sqrt{x}$



$\therefore$ Area bounded $= \left|\int_0^1 \{(x + x\sqrt{x}) - (x - x\sqrt{x})\} dx\right|$

$= \left|2\int_0^1 x^{3/2} dx\right| = \frac{4}{5}$ sq unit

132. ***(d)*** We have, $f(x) = \int_0^x (3\sin x + 4\cos x) dx$

$\Rightarrow \quad f'(x) = 3\sin x + 4\cos x = 5\sin\left(x + \tan^{-1}\frac{4}{3}\right)$

Since, $\frac{5\pi}{4} < x < \frac{4\pi}{3}$

$\Rightarrow \quad \pi < x + \tan^{-1} 4/3 < 2\pi$

$\because f'(x) < 0$, i.e. $f(x)$ is decreasing.

$\because$ Least value of $f(x) is \left(\frac{5\pi}{4}, \frac{4\pi}{3}\right) = f\left(\frac{4\pi}{3}\right)$

Least $f(x) = \int_0^{4\pi/3} (3\sin x + 4\cos x)\, dx$

$= [-3\cos x + 4\sin x]_0^{4\pi/3}$

$= -3\left(-\frac{1}{2} - 1\right) + 4\left(-\frac{\sqrt{3}}{2}\right)$

$= \frac{9 - 4\sqrt{3}}{2}$

133. ***(a)*** Let $z_1 = re^{i\theta}$

Then, $\quad \bar{z}_1 = re^{-i\theta}$

$\frac{z_1}{\bar{z}_1} = e^{2i\theta} = e^{2\pi i/n}$

$\Rightarrow \quad \theta = \frac{\pi}{n}$

Also, $\frac{I_m(z_1)}{R_e(z_1)} = \tan\theta = \tan\frac{\pi}{n} = \sqrt{2} - 1$

$\Rightarrow \quad \tan\frac{\pi}{8} = \sqrt{2} - 1$

$\therefore \quad n = 8$

134. ***(b)*** $(1 + x + x^3 + x^4)^{10} = [(1 + x)(1 + x^3)]^{10}$

$= (1 + x)^{10}(1 + x^3)^{10}$

$= (1 + {}^{10}C_1 + {}^{10}C_2\, x^2 + {}^{10}C_3\, x^3 + {}^{10}C_4\, x^4 \ldots)$

$\times (1 + {}^{10}C_1\, x^3 + {}^{10}C_2\, x^6 + \ldots)$

$\therefore$ Coefficient of $x^4 = ({}^{10}C_1)({}^{10}C_1) + {}^{10}C_4$

$= 100 + 210 = 310$

135. ***(c)*** Since only one letters can not be in wrong envelope so that at least two letters are in wrong envelope means all the letters are not in right envelopes.

$\therefore \; x = 6! - 1 = 720 - 1 = 719$

Y = number of ways, so that all the letters are in wrong envelopes

$= 6!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!}\right)$

$= 360 - 120 + 30 - 6 + 1 = 265$

$\therefore \quad x - y = 719 - 265 = 454$

136. ***(c)*** We have, $x = \log_5 3 + \log_7 5 + \log_9 7$

Since, AM $\geq$ GM

$\therefore \frac{\log_5 3 + \log_7 5 + \log_9 7}{3}$

$\geq (\log_5 3 \cdot \log_7 5 \cdot \log_9 7)^{1/3}$

$\Rightarrow \quad x \geq 3\left(\frac{\log 3}{\log 5} \times \frac{\log 5}{\log 7} \times \frac{\log 7}{\log 9}\right)^{1/3}$

$\Rightarrow \quad x > 3\left(\frac{1}{2}\right)^{1/3} \quad \Rightarrow x > \frac{3}{\sqrt[3]{2}}$

137. ***(c)*** $\quad x^2 - 2x + A = 0$

$\Rightarrow \quad x = 1 \pm \sqrt{1 - A}$

$\therefore \quad p = 1 - \sqrt{1 - A},\; q = 1 + \sqrt{1 - A}$

Similarly, $r = 9 - \sqrt{81 - B},\; s = 9 + \sqrt{81 - B}$

Since, $\quad p, q, r, s$ are in AP.

$\therefore \quad q - p = s - r$

$\Rightarrow \quad 2\sqrt{1 - A} = 2\sqrt{81 - B}$

$\Rightarrow \quad B = 80 + A$

and $\quad q - p = r - q$

$\Rightarrow \quad 3\sqrt{1 - A} = 8 - \sqrt{81 - B}$

$\Rightarrow \quad A = -3, B = 77$

138. ***(c)*** We have,

$(1 - a)x + y + z = 0$

$x + (1 - b)\,y + z = 0$

$x + y + (1 - c)\,z = 0$

has many solutions

$\therefore \quad \begin{vmatrix} 1 - a & 1 & 1 \\ 1 & 1 - b & 1 \\ 1 & 1 & 1 - c \end{vmatrix} = 0$

$\Rightarrow \quad ab + ac + bc = abc$

$\therefore \quad \frac{ab+bc+ac}{3} \geq (a^2 b^2 c^2)^{1/3} = (abc)^{1/3} \geq 3$

$\therefore \quad$ Minimum of $abc = (3)^3 = 27$

139. ***(c)*** Given,

$$AA' = A'A$$

and $B = A^{-1}A'$

So,

$$\begin{aligned} BB' &= (A^{-1}A')(A^{-1}A')' \\ &= (A^{-1}A')[(A')'(A^{-1})'] \\ &= (A^{-1}A')[A(A')^{-1}] \\ & \qquad [\because (A')' = A, (A^{-1})' = (A')^{-1}] \\ &= A^{-1}(A'A)(A')^{-1} \\ &= A^{-1}(AA'(A')^{-1} \\ &= (A^{-1}A)(A^{-1}A)' \\ &= I \cdot I = I \end{aligned}$$

140. ***(a)*** We have,

$$\sin^{-1}x + \tan^{-1}x = \frac{\pi}{2} \Rightarrow 0 < x < 1$$

$\therefore \quad \tan^{-1}x = \frac{\pi}{2} - \sin^{-1}x = \cos^{-1}x$

$\Rightarrow \quad \tan^{-1}x = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$

$\Rightarrow \quad x^2 = \sqrt{1-x^2}$

$\Rightarrow \quad x^4 + x^2 - 1 = 0 \quad \Rightarrow x^2 = \frac{-1 \pm \sqrt{5}}{2}$

$\Rightarrow \quad x^2 > 0$

$\therefore \quad x^2 = \frac{\sqrt{5}-1}{2}$

$\Rightarrow \quad = 2x^2 + 1 = \sqrt{5}$

141. ***(a)*** Given, $|\cos x| = 2[x]$

$$0 \leq |\cos x| \leq 1$$

$\therefore \quad 0 \leq 2[x] \leq 1$

$$0 \leq [x] \leq \frac{1}{2}$$

$$x = 0$$

$\therefore \quad |\cos x| = 0$

$$x : n\pi + \frac{\pi}{2}$$

For any value of $n \in N, [n] \neq 0$

Hence, the equation has no solution.

142. ***(c)*** $0 \leq \alpha, \beta \leq \frac{\pi}{4} \Rightarrow \alpha + \beta \leq \frac{\pi}{2}$

$\cos(\alpha+\beta) = 4/5 \Rightarrow \tan(\alpha+\beta) = 3/4$

$\sin(\alpha-\beta) = 5/13 \Rightarrow \alpha - \beta \in \theta_1$

and $\tan(\alpha-\beta) = 5/12$

Now, $\tan 2\alpha = \tan(\alpha+\beta+\alpha-\beta)$

$$= \frac{\tan(\alpha+\beta) + \tan(\alpha-\beta)}{1 - \tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$= \frac{3/4 + 5/12}{1 - 3/4 \times \frac{5}{12}} = \frac{56}{33}$$

143. ***(c)*** 18 draws are required for 2 aces means in the first 17 draws, there is one ace and 16 non-ace and 18th draw 2nd ace.

$\therefore$ Required probability $= \frac{{}^{48}C_{16} \times {}^{4}C_1}{{}^{52}C_{17}} \times \frac{3}{35} = \frac{561}{15925}$

144. ***(c)*** Since, $(\mathbf{a} + p\mathbf{b})$ is normal to $\mathbf{c}$.

$\therefore \quad (\mathbf{a} + p\mathbf{b}) \cdot \mathbf{c} = 0$

$$((1-p)\hat{\mathbf{i}} + (2+2p)\hat{\mathbf{j}} + (3+p)\hat{\mathbf{k}}) \cdot (3\hat{\mathbf{i}} + \hat{\mathbf{j}}) = 0$$

$$3 - 3p + 2 + 2p = 0 \Rightarrow p = 5$$

145. ***(b)*** We have,

$$f(x) = x[x]$$

$$f(x) = px \qquad [\because [x] = p]$$

$$f'(x) = p = [x]$$

$\therefore \quad f'(x) = [x]$

146. ***(b)*** We have,

$$f(x) = x^2 e^{-x^2}$$

$\Rightarrow \quad f'(x) = 2xe^{-x^2} - 2x^3 e^{-x^2}$

$\Rightarrow \quad f'(x) = 2xe^{-x^2}(1-x^2)$

$\Rightarrow \quad f'(x) = 0 \Rightarrow x = 0, -1, 1$

$f'(-h) < 0, f'(h) > 0$, So minimum at $x = 0$

$f'(1-h) > 0, f'(1+h) < 0$, So maximum at $x = 1$

$f'(-1-h) > 0, f'(-1+h) < 0$, So maximum at $x = -1$

$\therefore$ Minimum $f(x) = f(0) = 0$

Maximum $f(x) = e^{-1} = \frac{1}{e}$

Since, $\quad f(x) \geq 0 \; \forall \; x$

So, difference between maximum and minimum values $= \frac{1}{e} - 0 = \frac{1}{e}$

147. ***(a)*** Given,

$$f'(x) = f(x) + \int_0^1 f(x)\,dx, f(0) = 1 \qquad \ldots\text{(i)}$$

$\Rightarrow \quad f''(x) = f'(x) + 0 \Rightarrow \frac{f''(x)}{f'(x)} = 1$

$\Rightarrow \quad \int \frac{f''(x)}{f'(x)}dx = \int dx$

$\Rightarrow \quad \log f'(x) = x + C$

$\Rightarrow \quad f'(x) = Ae^x$

$\Rightarrow \quad f(x) = Ae^x + K$

$$f(0) = A + K = 1 \quad \ldots\text{(ii)}$$

$\therefore \quad Ae^x = Ae^x + K + \int_0^1 (Ae^x + k)\,dx$

$\Rightarrow \quad k + [Ae^x + Kx]_0^1 = 0$

$\Rightarrow \quad k + Ae - A + K = 0$

$\Rightarrow \quad A(e-1) + 2k = 0 \quad \ldots\text{(iii)}$

From Eqs. (ii) and (iii), we get

$$A = \frac{2}{3-e},\ k = \frac{1-e}{3-e}$$

$$\therefore \quad f(x) = \frac{2e^x}{3-e} + \frac{1-e}{3-e}$$

148. ***(b)*** We have,

$$y = e^{2x} + x^2$$

$$\Rightarrow \quad \frac{dy}{dx} = 2e^{2x} + 2x$$

$$\left(\frac{dy}{dx}\right)_{x=0} = 2$$

$\therefore$ Equation of normal at (0, 1) is

$$y - 1 = -\frac{1}{2}(x)$$

$$\Rightarrow \quad x + 2y - 2 = 0$$

$\therefore$ Distance from (0, 0) from normal $= \dfrac{2}{\sqrt{5}}$

149. ***(c)*** Given, line $\dfrac{x-1}{-1} = \dfrac{y-2}{3} = \dfrac{z-4}{1}$ is passes through $P(1, 2, 4)$.

To find the reflection of line we need one more point on the line clearly M (0, 5, 5) also lie on line.

Let $N(\alpha, \beta, \gamma)$ be the reflection M in the

$$x + y + z = 7$$

$$\therefore \quad \frac{\alpha}{2} + \frac{\beta+5}{2} + \frac{\gamma+5}{2} + 7$$

$$\Rightarrow \quad \alpha + \beta + \gamma = 4$$

Also, MN is perpendicular, i.e. parallel to the normal of the plane

$$\therefore \quad \frac{\alpha}{1} = \frac{\beta-5}{1} = \frac{\gamma-5}{1} = \lambda$$

$\Rightarrow \quad \alpha = \lambda, \beta = \lambda + 5, \gamma = \lambda - 5$

$\therefore \lambda + \lambda + 5 + \lambda + 5 = 4$

$\Rightarrow \quad \lambda = -2$

$\therefore \quad N = (-2, 3, 5)$

Equation PN is $\dfrac{x-1}{-3} = \dfrac{y-2}{1} = \dfrac{z-4}{-1}$

150. ***(b)*** We have,

$$x + y + z = 5$$

$$x^2 + y^2 + z^2 = 9$$

$\Rightarrow \quad x^2 + y^2 + (5 - x - y)^2 = 9$

$\Rightarrow \quad y^2 + x^2 - 5x - 5y + xy + 8 = 0$

$\Rightarrow \quad y^2 + y(x-5) + (x^2 - 5x + 8) = 0$

Since $y \in R$

$\therefore \quad (x-5)^2 - 4(x^2 - 5x + 8) \geq 0$

$\Rightarrow \quad x^2 - 10x + 25 - 4x^2 + 20x - 32 \geq 0$

$\Rightarrow \quad 3x^2 - 10x + 7 \leq 0$

$\Rightarrow \quad 3x^2 - 7x - 3x + 7 \leq 0$

$\Rightarrow \quad (3x-7)(x-1) \leq 0$

$\Rightarrow \quad x \in \left[1, \dfrac{7}{3}\right]$

Length of interval $= \left(\dfrac{7}{3} - 1\right) = \dfrac{4}{3}$