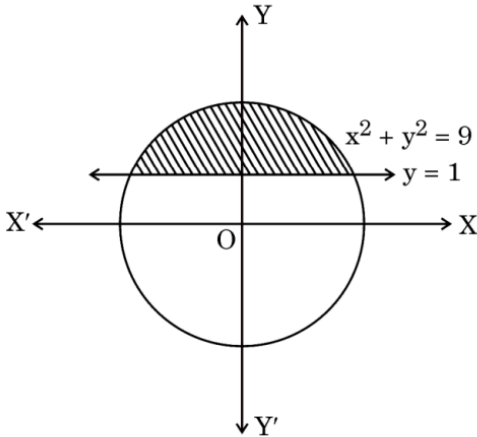


MATHEMATICS (Subject Code-041)
(PAPER CODE: 65/3/2)
SOLUTIONS

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	STEPS	MARKS
	SECTION A This section comprises 20 multiple choice questions (MCQs) of 1 mark each.		
1.	If $\tan^{-1} x = y$, then $\frac{dy}{dx}$ is equal to : (A) $(\sec^{-1} x)^2$ (B) $\sec^2 y$ (C) $\frac{1}{\sqrt{1+x^2}}$ (D) $\cos^2 y$		
Sol. 1	(D) $\cos^2 y$	I	1
2.	The rate of change of volume of a sphere with respect to its diameter, when its radius is 5 cm, is : (A) $400\pi \text{ cm}^3/\text{cm}$ (B) $100\pi \text{ cm}^3/\text{cm}$ (C) $50\pi \text{ cm}^3/\text{cm}$ (D) $25\pi \text{ cm}^3/\text{cm}$		
Sol. 2	(C) $50\pi \text{ cm}^3/\text{cm}$	I	1
3.	$\int \frac{dx}{\sqrt{e^{-2x}-1}}$ is equal to : (A) $\sin^{-1} e^{-x} + C$ (B) $\log e^{-x} + \sqrt{e^{-2x}-1} + C$ (C) $\sin^{-1} e^x + C$ (D) $\log e^{-x} - \sqrt{e^{-2x}-1} + C$		
Sol. 3	(C) $\sin^{-1} e^x + C$	I	1
4.	$\int_{-1}^1 (1 - x) dx$ is equal to : (A) $2 \int_0^1 (1+x) dx$ (B) $2 \int_{-1}^0 (1+x) dx$ (C) 0 (D) $2 \int_{-1}^0 (1-x) dx$		
Sol. 4	(B) $2 \int_{-1}^0 (1+x) dx$	I	1

5.	The area of the shaded region of the circle given below is equal to :  (A) $\int_1^3 \sqrt{9-y^2} \, dy$ (B) $2 \int_1^3 \sqrt{9-y^2} \, dy$ (C) $\int_0^3 \sqrt{9-x^2} \, dx$ (D) $2 \int_0^3 \sqrt{9-x^2} \, dx$		
Sol. 5	(B) $2 \int_1^3 \sqrt{9-y^2} \, dy$	I	1
6.	$\frac{dy}{dx} = F(x, y)$ will be a homogeneous differential equation for which of the following functions ? (i) $F(x, y) = 3x + 2y$ (ii) $F(x, y) = \sin \frac{y}{x} + \log y - \log x$ (iii) $F(x, y) = e^{y/x} + 1$ (iv) $F(x, y) = \sqrt{x^2 + y^2} - y$ (A) (i) and (ii) (B) (i), (ii) and (iii) (C) (ii), (iii) and (iv) (D) (ii) and (iii)		
Sol. 6	(D) (ii) and (iii)	I	1
7.	For any two vectors $\vec{a}$ and $\vec{b}$, which of the following statements is always true ? (A) $\vec{a} \cdot \vec{b} \leq \vec{a} \vec{b}$ (B) $\vec{a} + \vec{b} \geq \vec{a} + \vec{b}$ (C) $\vec{a} - \vec{b} = \vec{a} - \vec{b}$ (D) $\vec{a} \times \vec{b} \geq \vec{a} \vec{b}$		
Sol. 7	(A) $\vec{a} \cdot \vec{b} \leq \vec{a} \vec{b} $	I	1
8.	If $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 198$ and $\vec{a} = 10 \vec{b}$, then : (A) $\vec{a} = \sqrt{2}$ (B) $\vec{b} = \sqrt{2}$ (C) $\vec{b} = 10\sqrt{2}$ (D) $\vec{a} = \frac{10}{\sqrt{2}}$		
Sol. 8	(B) $ \vec{b} = \sqrt{2}$	I	1
9.	If l_1, m_1, n_1 and l_2, m_2, n_2 are direction cosines of lines L_1 and L_2 respectively and θ is the acute angle between them, then : (A) $\cos \theta = l_1 l_2 + m_1 m_2 + n_1 n_2$ (B) $\sin \theta = l_1 l_2 + m_1 m_2 + n_1 n_2$ (C) $\tan \theta = \frac{l_1}{l_2} + \frac{m_1}{m_2} + \frac{n_1}{n_2}$ (D) $\cos \theta = l_1 l_2 + m_1 m_2 + n_1 n_2$		

Sol. 9	(D) $\cos\theta = l_1l_2 + m_1m_2 + n_1n_2 $	I	1
10.	Direction ratios of lines l_1 and l_2 respectively are $\langle 1, -2, 3 \rangle$ and $\langle -2, p, -6 \rangle$. The value of p for which $l_1 \parallel l_2$, is : (A) -4 (B) 4 (C) -10 (D) 10		
Sol. 10	(B) 4	I	1
11.	If E and F are two independent events such that $P(E) = \frac{3}{10}$, $P(E \cup F) = \frac{1}{2}$, then $P(E F) - P(F E)$ is equal to : (A) $\frac{2}{7}$ (B) $\frac{3}{35}$ (C) $\frac{1}{70}$ (D) $\frac{1}{7}$		
Sol. 11	(C) $\frac{1}{70}$	I	1
12.	The domain of $\sin^{-1}(1 - 2x)$ is : (A) $[-1, 1]$ (B) $[-1, 3]$ (C) $[-2, 2]$ (D) $[0, 1]$		
Sol. 12	(D) $[0, 1]$	I	1
13.	If $A^2 = 4A + 3I$ and $A^{-1} = xA + yI$, then the value of $(x + y)$ is : (A) -1 (B) 1 (C) $\frac{5}{3}$ (D) 7		
Sol. 13	(A) -1	I	1
14.	If A and B are skew-symmetric matrices of same order, then $AB' + BA'$ is a/an : (A) symmetric matrix (B) skew-symmetric matrix (C) null matrix (D) identity matrix		
Sol. 14	(A) symmetric matrix	I	1
15.	If a matrix B is such that $B \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$, then the order of matrix B is : (A) 1×3 (B) 3×1 (C) 3×3 (D) 1×1		
Sol. 15	(B) 3×1	I	1
16.	If a square matrix A is such that $A^2 = A$ and $(I - A)^3 = xA + I$, then value of x must be : (A) 7 (B) 5 (C) -7 (D) -1		
Sol. 16	(D) -1	I	1

17.	If $A(\text{adj } A) = \begin{bmatrix} 2026 & 0 & 0 \\ 0 & 2026 & 0 \\ 0 & 0 & 2026 \end{bmatrix}$, then the value of $\text{adj } A$ is equal to : (A) 2026 (B) $(2026)^{-1}$ (C) $(2026)^{-2}$ (D) $(2026)^2$		
Sol. 17	(D) $(2026)^2$	I	1
18.	The value of k for which the function $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ k(x+1), & x = 0 \end{cases}$ is a continuous function, is : (A) $\frac{1}{4}$ (B) 2 (C) $\frac{1}{2}$ (D) 0		
Sol. 18	(D) 0	I	1
<i>Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled as Assertion (A) and the other labelled as Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.</i> (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.			
19.	<i>Assertion (A) :</i> The vectors $\vec{a}$ and $(-2\vec{a})$, where $\vec{a} \neq \vec{0}$ are collinear vectors. <i>Reason (R) :</i> $\vec{a} \cdot (-2\vec{a}) = 0$.		
Sol. 19	(C) Assertion (A) is true, but Reason (R) is false.	I	1
20.	<i>Assertion (A) :</i> One of the particular solutions of the differential equation $\frac{dy}{dx} = e^{x+y}$ can be $e^x + e^{-y} = -2$. <i>Reason (R) :</i> $e^x + e^{-y} = C$ is the general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$.		
Sol. 20	(D) Assertion (A) is false, but Reason (R) is true.	I	1

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21.	(a) Find the value of $\sin [\cot^{-1} \sqrt{2} (\cos (\tan^{-1} 1))]$. OR (b) A relation R on $A = \{1, 2, 3\}$ is defined as $R = \{(1, 1) (3, 3), (1, 2)\}$. Is R a symmetric relation ? Justify. Write the smallest relation set R_1 such that $R \cup R_1$ becomes an equivalence relation on the set $\{1, 2, 3\}$.		
Sol. 21 (a)	$\sin[\cot^{-1}\sqrt{2}(\cos (\tan^{-1}1))] = \sin \left[\cot^{-1}\sqrt{2} \left(\cos \frac{\pi}{4} \right) \right] = \sin(\cot^{-1}1)$ $= \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$ OR (b) As $(1,2) \in R$, but $(2,1) \notin R \Rightarrow R$ is not symmetric relation. $R_1 = \{(2,2), (2,1)\}$	I II I II	1 1 1 1
22.	If for two unit vectors $\vec{a}$ and $\vec{b}$, $ \vec{a} + 2\vec{b} = 2\vec{a} - \vec{b} $, then find the angle between $\vec{a}$ and $\vec{b}$.		
Sol. 22	Given that $\vec{a} = \vec{b} = 1$ $ \vec{a} + 2\vec{b} = 2\vec{a} - \vec{b} \Rightarrow \vec{a} + 2\vec{b} ^2 = 2\vec{a} - \vec{b} ^2$ $\Rightarrow \vec{a} ^2 + 4 \vec{b} ^2 + 4\vec{a} \cdot \vec{b} = 4 \vec{a} ^2 + \vec{b} ^2 - 4\vec{a} \cdot \vec{b} \Rightarrow \vec{a} \cdot \vec{b} = 0$ $\Rightarrow \text{Angle between } \vec{a} \text{ and } \vec{b} = \frac{\pi}{2}$	I II III IV	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
23.	(a) If the lines $\frac{x-3}{1} = \frac{1-y}{1} = \frac{z+2}{p}$ and $\frac{2-x}{3} = \frac{y+1}{5} = \frac{z+56}{2p}$ are perpendicular to each other, then find the value(s) of p. OR (b) Find the vector equation of a line passing through the origin and perpendicular to both the lines $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and $\vec{r} = \mu(\hat{i} - \hat{j} + \hat{k})$.		
Sol. 23(a)	Direction Ratios of line $\frac{x-3}{1} = \frac{1-y}{1} = \frac{z+2}{p}$ are $\langle 1, -1, p \rangle$ Direction Ratios of line $\frac{2-x}{3} = \frac{y+1}{5} = \frac{z+56}{2p}$ are $\langle -3, 5, 2p \rangle$ $\therefore$ Given both lines are perpendicular to each other. $\therefore 1 \times (-3) + (-1) \times 5 + p \times 2p = 0$	I II III	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

	$\Rightarrow p = \pm 2$ OR (b) The vector parallel to required line is given by $\vec{b} = (3\hat{i} + 4\hat{j} + 2\hat{k}) \times (\hat{i} - \hat{j} + \hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 2 \\ 1 & -1 & 1 \end{vmatrix} = 6\hat{i} - \hat{j} - 7\hat{k}$ Equation of required line is given by $\vec{r} = \delta(6\hat{i} - \hat{j} - 7\hat{k})$	IV I II	$\frac{1}{2}$ $1\frac{1}{2}$ $\frac{1}{2}$
24.	If $x = e^{t+\frac{1}{t}}$ and $y = e^{t-\frac{1}{t}}$, then find $\frac{dy}{dx}$ at $t = -2$.		
Sol. 24	$x = e^{t+\frac{1}{t}} \Rightarrow \frac{dx}{dt} = \left(1 - \frac{1}{t^2}\right) e^{t+\frac{1}{t}}$ $y = e^{t-\frac{1}{t}} \Rightarrow \frac{dy}{dt} = \left(1 + \frac{1}{t^2}\right) e^{t-\frac{1}{t}}$ $\frac{dy}{dx} = \frac{t^2 + 1}{t^2 - 1} \cdot \frac{e^{t-\frac{1}{t}}}{e^{t+\frac{1}{t}}}$ $\left[\frac{dy}{dx}\right]_{t=-2} = \frac{5e}{3}$	I II III IV	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
25.	Find the sub-interval of $(0, \infty)$ in which $f(x) = x^2 e^{-x}$ is increasing.		
Sol. 25	$f(x) = x^2 e^{-x} \Rightarrow f'(x) = x(2 - x) e^{-x}$ Put $f'(x) = 0 \Rightarrow x = 0, 2$ $\begin{array}{c} + \qquad \qquad - \\ \hline 0 \qquad \qquad 2 \end{array}$ Hence, $f(x)$ is increasing in $(0, 2)$. Note: Interval $(0, 2]$ can be considered.	I II III	1 $\frac{1}{2}$ $\frac{1}{2}$
SECTION C			
<i>This section comprises 6 Short Answer (SA) type questions of 3 marks each.</i>			

26.	(a) Find : $\int \frac{\cos x}{(2 + \sin x)(4 + \sin x)} dx$ OR (b) Find : $\int \frac{x+3}{x^2 + 4x + 5} dx$		
Sol. 26(a)	Put $\sin x = t \Rightarrow \cos x dx = dt$ $\int \frac{\cos x}{(2 + \sin x)(4 + \sin x)} dx = \int \frac{1}{(2 + t)(4 + t)} dt$ $= \frac{1}{2} \int \left(\frac{1}{2 + t} - \frac{1}{4 + t} \right) dt = \frac{1}{2} \log \left \frac{2 + t}{4 + t} \right + c$ $= \frac{1}{2} \log \left \frac{2 + \sin x}{4 + \sin x} \right + c$ OR (b) $\int \frac{x+3}{x^2 + 4x + 5} dx = \frac{1}{2} \int \frac{(2x+4) + 2}{x^2 + 4x + 5} dx$ $= \frac{1}{2} \int \frac{2x+4}{x^2 + 4x + 5} dx + \int \frac{1}{x^2 + 4x + 5} dx$ $= \frac{1}{2} \log x^2 + 4x + 5 + \int \frac{1}{(x+2)^2 + 1} dx$ $= \frac{1}{2} \log x^2 + 4x + 5 + \tan^{-1}(x+2) + c$	I II III I II III	1 $1\frac{1}{2}$ $\frac{1}{2}$ 1 1 1
27.	(a) Find the general solution of the differential equation $(x^2 + y^2) dy = xy dx.$ OR (b) Find the particular solution of the differential equation $\frac{dy}{dx} - 3y \cot x = \sin 2x, \text{ given that } y = 2 \text{ when } x = \frac{\pi}{2}.$		
Sol. 27(a)	$\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$ Put $y = vx$ and $\frac{dy}{dx} = v + x \frac{dv}{dx}$ Hence, $v + x \frac{dv}{dx} = \frac{v}{1 + v^2} \Rightarrow x \frac{dv}{dx} = -\frac{v^3}{1 + v^2}$	I II	$\frac{1}{2}$ $\frac{1}{2}$

(b)	$\Rightarrow \int \frac{v^2 + 1}{v^3} dv = - \int \frac{dx}{x}$ $\Rightarrow \log v - \frac{1}{2v^2} = -\log x + c$ $\Rightarrow \log\left \frac{y}{x}\right - \frac{x^2}{2y^2} = -\log x + c \quad \text{or} \quad \log y = \frac{x^2}{2y^2} + c$ OR $\frac{dy}{dx} - 3y \cot x = \sin 2x$ $\text{I.F.} = e^{-3 \int \cot x dx} = \frac{1}{\sin^3 x} \text{ or } \operatorname{cosec}^3 x$ General solution is given by $\frac{y}{\sin^3 x} = \int \frac{\sin 2x}{\sin^3 x} dx$ $\Rightarrow y \operatorname{cosec}^3 x = -2 \operatorname{cosec} x + c$ Put $x = \frac{\pi}{2}$ and $y = 2$; we get $c = 4$ Hence, particular solution is given by $y \operatorname{cosec}^3 x = -2 \operatorname{cosec} x + 4 \text{ or } y = -2 \sin^2 x + 4 \sin^3 x$	III	1
		IV	1
		I	1
		II	$\frac{1}{2}$
		III	1
28.	If $(\sin x)^y = y^{\cos x}$, then find $\frac{dy}{dx}$.		
Sol. 28	$(\sin x)^y = y^{\cos x} \Rightarrow y \log \sin x = \cos x \log y$ On diff. wrt x both sides, we get $\frac{y \cos x}{\sin x} + \log \sin x \frac{dy}{dx} = \frac{\cos x}{y} \frac{dy}{dx} - \log y \cdot \sin x$ $\Rightarrow \frac{dy}{dx} = \frac{y(y \cos x + \log y \cdot \sin^2 x)}{\sin x (\cos x - y \log \sin x)} \text{ or } \frac{(y \cot x + \sin x \cdot \log y)y}{\cos x - y \log \sin x}$	I	1
		II	$1 \frac{1}{2}$
		III	$\frac{1}{2}$

29.	A survey was conducted on the patients who have undergone knee replacement surgeries.  It was found that, Robotic Knee replacement surgeries have 90% success rate. On a particular day, robotic surgery was performed on three patients, A, B and C, one after the other. Assuming that the success and failure of each surgery is independent of each other, find the probability that : (i) exactly one surgery is successful, (ii) at most two surgeries are successful.		
Sol. 29	$P(\text{success}) = \frac{9}{10}$ and $P(\text{not success}) = \frac{1}{10}$ (i) Probability that exactly one surgery is successful $= \frac{9}{10} \times \frac{1}{10} \times \frac{1}{10} \times 3 = \frac{27}{1000}$ (ii) Probability that all three surgeries are successful $= \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10} = \frac{729}{1000}$ Probability that at most two surgeries are successful $= 1 - \frac{729}{1000} = \frac{271}{1000}$	I II III IV	$\frac{1}{2}$ 1 1 $\frac{1}{2}$
30.	(a) Find : $\int \frac{dx}{x^{1/2} + x^{1/3}}$ OR (b) Find : $\int \tan^{-1} \left(\frac{1-x}{1+x} \right) dx$		
Sol. 30(a)	Put $x = t^6 \Rightarrow dx = 6t^5 dt$ $\int \frac{dx}{x^{1/2} + x^{1/3}} = 6 \int \frac{t^5}{t^3 + t^2} dt = 6 \int \frac{t^3}{t+1} dt$ $= 6 \int \left(t^2 - t + 1 - \frac{1}{t+1} \right) dt$	I II	1 1

	$= 6 \left(\frac{t^3}{3} - \frac{t^2}{2} + t - \log t + 1 \right) + c$ $= 2x^{\frac{1}{2}} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log \left x^{\frac{1}{6}} + 1 \right + c$ OR (b) $\int \tan^{-1} \left(\frac{1-x}{1+x} \right) dx$ Getting $\int \left(\frac{\pi}{4} - \tan^{-1} x \right) dx$ $= \frac{\pi x}{4} - \left[x \tan^{-1} x - \int \frac{x}{1+x^2} dx \right]$ $= \frac{\pi x}{4} - x \tan^{-1} x + \frac{1}{2} \log(1+x^2) + c$	III I II III	1 1 $1\frac{1}{2}$ $\frac{1}{2}$
31.	Evaluate : $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{2^x + 1} dx$		
Sol. 31	Let, $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{2^x + 1} dx \dots \dots \dots (1)$ Using property $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$, we get $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{2^x \cos^2 x}{2^x + 1} dx \dots \dots \dots (2)$ On adding equations (1) and (2), we get $2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 x dx = 2 \int_0^{\frac{\pi}{2}} \cos^2 x dx \Rightarrow I = \int_0^{\frac{\pi}{2}} \cos^2 x dx \dots \dots \dots (3)$ Getting $I = \int_0^{\frac{\pi}{2}} \sin^2 x dx \dots \dots \dots (4)$ On adding equations (3) and (4), we get	I II III	1 1 $\frac{1}{2}$

$2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx \Rightarrow I = \frac{\pi}{4}$	IV	$\frac{1}{2}$
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IV

SECTION D

32.



A total of 60 vouchers were given on the day. The number of ₹ 1,000 vouchers added to 3 times the number of ₹ 500 vouchers, gives 100. Express the given information as a system of linear equations in two variables. Hence, find the number of vouchers of each type by matrix method.

(b) Given that $P = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $Q = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$ and $R = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$, find a matrix S such that $PQ - RS$ is a null matrix.

and number of ₹ 500 vouchers = y

In matrix form, $\begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 60 \\ 100 \end{bmatrix}$

$$\Rightarrow X = A^{-1}B \quad \dots \dots \dots (1)$$
$$|A| = 2 \quad \therefore |A| \neq 0 \Rightarrow A \text{ is an invertible matrix.}$$
$$\text{adj } A = \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} \quad \text{and} \quad A^{-1} = \frac{1}{2} \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}$$

By eq. (1),
$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 60 \\ 100 \end{bmatrix} = \begin{bmatrix} 40 \\ 20 \end{bmatrix}$$

On comparing, $x = 40$ and $y = 20$

Hence, number of ₹ 1000 vouchers = 40

I

II

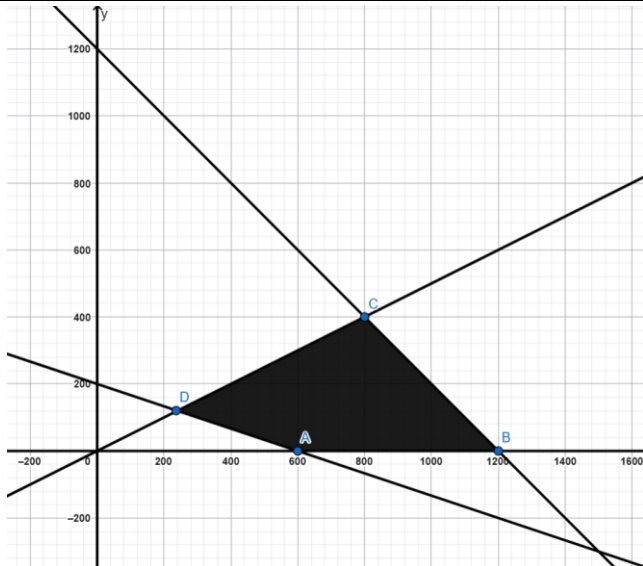
III

IV

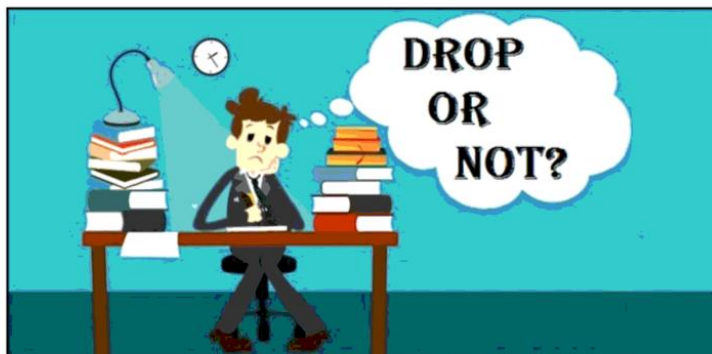
V

(b)	and number of ₹ 500 vouchers = 20 OR Let $S = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $PQ = \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix}$ $RS = \begin{bmatrix} 2a + 5c & 2b + 5d \\ 3a + 8c & 3b + 8d \end{bmatrix}$ Given that $PQ - RS = \text{Null matrix} \Rightarrow PQ = RS$ On comparing corresponding elements, $2a + 5c = 3, \quad 2b + 5d = 0, \quad 3a + 8c = 43, \quad 3b + 8d = 22$ By solving, $a = -191, \quad b = -110, \quad c = 77, \quad d = 44$ Hence, $S = \begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}$ Alternative Method: Given that $PQ - RS = \text{null matrix} \Rightarrow RS = PQ$ $\begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix} S = \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix}$ $\Rightarrow S = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}^{-1} \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix}$ $\Rightarrow S = \begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix}$ $\Rightarrow S = \begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}$	I II III IV V VI I II III IV V	$\frac{1}{2}$ 1 1 $\frac{1}{2}$ $1\frac{1}{2}$ $\frac{1}{2}$ 1 1 1 $1\frac{1}{2}$ $\frac{1}{2}$
33.	(a) Represent the equations of lines l_1 and l_2 in vector form and check whether they are intersecting or not. $l_1: \frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ $l_2: \frac{x+1}{-1} = \frac{2-y}{-2} = \frac{z-5}{5}$ OR (b) Opposite sides of a square are along the lines : $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ $\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$ Find the area of the square if direction ratios of other pair of opposite sides of the square are given by $\langle -3, 6, p \rangle$. Also, find the value of p.		
Sol. 33(a)	$l_1: \vec{r} = (-3\hat{i} + \hat{j} + 5\hat{k}) + \lambda(-3\hat{i} + \hat{j} + 5\hat{k})$ $l_2: \vec{r} = (-\hat{i} + 2\hat{j} + 5\hat{k}) + \mu(-\hat{i} + 2\hat{j} + 5\hat{k})$ $\vec{a}_1 = -3\hat{i} + \hat{j} + 5\hat{k}; \quad \vec{b}_1 = -3\hat{i} + \hat{j} + 5\hat{k}$ $\vec{a}_2 = -\hat{i} + 2\hat{j} + 5\hat{k}; \quad \vec{b}_2 = -\hat{i} + 2\hat{j} + 5\hat{k}$ $\vec{a}_2 - \vec{a}_1 = 2\hat{i} + \hat{j}$	I II III	1 1 1

	$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix} = -5\hat{i} + 10\hat{j} - 5\hat{k}$ $\therefore (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 0$ $\therefore \text{Both lines are intersecting.}$ OR $\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}; \quad \vec{a}_2 = 3\hat{i} + 3\hat{j} - 5\hat{k}; \quad \vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ $\vec{a}_2 - \vec{a}_1 = 2\hat{i} + \hat{j} - \hat{k}$ $(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix} = 9\hat{i} - 14\hat{j} + 4\hat{k}$ $ (\vec{a}_2 - \vec{a}_1) \times \vec{b} = \sqrt{293}$ $\text{Distance between given parallel lines} = \frac{ (\vec{a}_2 - \vec{a}_1) \times \vec{b} }{ \vec{b} } = \frac{\sqrt{293}}{7}$ $\therefore \text{Length of side of square} = \text{distance between parallel lines}$ $= \frac{\sqrt{293}}{7} \text{ or } \frac{\sqrt{293}}{7} \text{ units}$ $\text{Area of square} = \frac{293}{49} \text{ or } \frac{293}{49} \text{ square units}$ $\therefore \text{Adjacent sides of square are perpendicular to each other.}$ $\therefore -3 \times 2 + 6 \times 3 + 6p = 0 \Rightarrow p = -2$	IV V VI	1 $\frac{1}{2}$ $\frac{1}{2}$
(b)	$\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}; \quad \vec{a}_2 = 3\hat{i} + 3\hat{j} - 5\hat{k}; \quad \vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ $\vec{a}_2 - \vec{a}_1 = 2\hat{i} + \hat{j} - \hat{k}$ $(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix} = 9\hat{i} - 14\hat{j} + 4\hat{k}$ $ (\vec{a}_2 - \vec{a}_1) \times \vec{b} = \sqrt{293}$ $\text{Distance between given parallel lines} = \frac{ (\vec{a}_2 - \vec{a}_1) \times \vec{b} }{ \vec{b} } = \frac{\sqrt{293}}{7}$ $\therefore \text{Length of side of square} = \text{distance between parallel lines}$ $= \frac{\sqrt{293}}{7} \text{ or } \frac{\sqrt{293}}{7} \text{ units}$ $\text{Area of square} = \frac{293}{49} \text{ or } \frac{293}{49} \text{ square units}$ $\therefore \text{Adjacent sides of square are perpendicular to each other.}$ $\therefore -3 \times 2 + 6 \times 3 + 6p = 0 \Rightarrow p = -2$	I II III IV V VI	1 1 $\frac{1}{2}$ $\frac{1}{2}$ 1 1
34.	Show that $f: R_+ \rightarrow [-5, \infty)$ given by $f(x) = 4x^2 + 4x - 5$ is both one-one and onto where $R_+ = [0, \infty)$. Also, find $p \in R_+$ such that $f(p) = 3$.		
Sol. 34	One – One: For $x_1, x_2 \in R_+$, Let $f(x_1) = f(x_2) \Rightarrow 4x_1^2 + 4x_1 - 5 = 4x_2^2 + 4x_2 - 5$ $\Rightarrow (x_1 - x_2)(x_1 + x_2 + 1) = 0 \Rightarrow x_1 = x_2$ as $x_1 + x_2 + 1 \neq 0$ as $x_1, x_2 \in R_+$ Hence f is one – one. Onto: $y = 4x^2 + 4x - 5 \Rightarrow x = \frac{\sqrt{y+6} - 1}{2}$ As $x \geq 0 \Rightarrow \sqrt{y+6} \geq 1 \Rightarrow y \geq -5 \Rightarrow \text{Range} = [-5, \infty)$ As $\text{Range} = \text{codomain} = [-5, \infty) \Rightarrow f$ is onto. Given that $f(p) = 3 \Rightarrow 4p^2 + 4p - 5 = 3 \Rightarrow p = 1$ as $p \neq -2$	I II III IV V VI	1 1 1 $\frac{1}{2}$ $\frac{1}{2}$ 1

35.	Solve the following Linear Programming Problem graphically : Maximise $Z = 12x + 18y$ subject to the constraints $x + y \leq 1200$ $x - 2y \geq 0$ $x + 3y \geq 600$ $x \geq 0, y \geq 0$											
Sol. 35	 <table border="1" data-bbox="464 1041 1018 1234"><thead><tr><th>Corner Points</th><th>Value of $Z=600x+400y$</th></tr></thead><tbody><tr><td>A (600,0)</td><td>7200</td></tr><tr><td>B (1200,0)</td><td>14400</td></tr><tr><td>C (800,400)</td><td>16800</td></tr><tr><td>D (240,120)</td><td>5040</td></tr></tbody></table> Hence, $Z_{max.} = 16800$ at point (800,400).	Corner Points	Value of $Z=600x+400y$	A (600,0)	7200	B (1200,0)	14400	C (800,400)	16800	D (240,120)	5040	I <
Corner Points	Value of $Z=600x+400y$											
A (600,0)	7200											
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C (800,400)	16800											
D (240,120)	5040											

A survey was conducted to find out the success rate of students who qualified the entrance examination by dropping a year after class XII.



As per the data collected, 40% students appearing in the examination were dropouts and the remaining students were regular students of class XII.

Of the dropouts, 5% qualify the examination while 10% of the regular students qualify the examination.

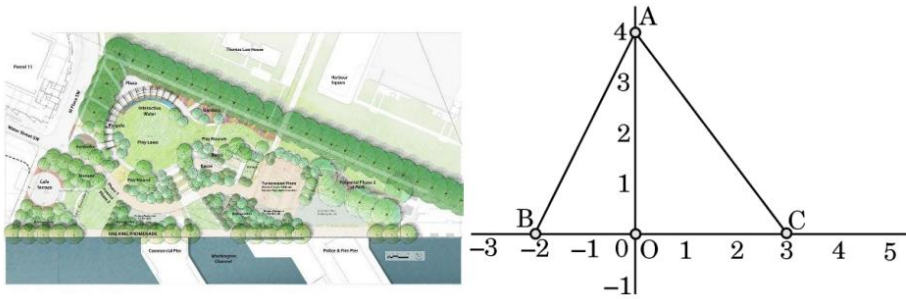
Based on the above information, answer the following questions.

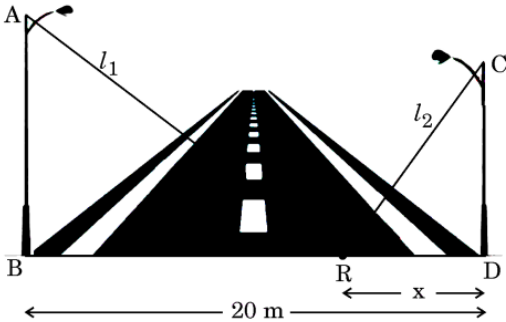
- (i) Find the probability that a student selected at random is a regular student.
- (ii) A student is selected at random from a group of dropout students. What is the probability that the student will not qualify the examination ?
- (iii) (a) A student selected at random qualified the examination. Find the probability that student is not a dropout.

OR

- (iii) (b) A student selected at random did not qualify the examination. Find the probability that the student was a regular student.

Sol. 36(i)	Probability that a student selected at random is a regular student = $\frac{3}{5}$	I	1
(ii)	Probability that the selected student from the group of dropout will not qualify the examination = $\frac{95}{100}$ or $\frac{19}{20}$	I	1
(iii) (a)	Let A: The student has qualified the examination. E_1: The student appearing in the examination is dropout. E_2: The student is regular. Using Bayes' Theorem, $P\left(\frac{E_1}{A}\right) = P\left(\frac{E_2}{A}\right) = \frac{\frac{60}{100} \times \frac{10}{100}}{\frac{40}{100} \times \frac{5}{100} + \frac{60}{100} \times \frac{10}{100}}$	I	$1\frac{1}{2}$

	$= \frac{3}{4}$	II	$\frac{1}{2}$
	OR		
(iii) (b)	$P\left(\frac{E_2}{A'}\right) = \frac{\frac{60}{100} \times \frac{90}{100}}{\frac{40}{100} \times \frac{95}{100} + \frac{60}{100} \times \frac{90}{100}}$ $= \frac{27}{46}$	I	$1\frac{1}{2}$
		II	$\frac{1}{2}$
	Case Study – 2		
37.	There is a triangular park in the society. The park is divided into two sections as shown in the figure.  In the region OAC, children are allowed to play games like cricket, football, while in the region AOB, activities which involve running are not allowed. The vertices of the triangular park ABC are A(0, 4), B(– 2, 0) and C(3, 0). Based on the above information, answer the following questions : (i) Write the equation of the boundary line AB of the park. (ii) Write the equation of the boundary line AC of the park. (iii) (a) Using integration, find the area of region OAC, in which children are allowed to play cricket, football. OR (iii) (b) Using integration, find the area of region AOB.		
Sol. 37(i)	Equation of the boundary line AB is $2x - y + 4 = 0$	I	1
(ii)	Equation of the boundary line AC is $4x + 3y = 12$	I	1
(iii) (a)	$\text{Area of region OAC} = \int_0^3 \frac{12 - 4x}{3} dx$ $= \frac{1}{3} [12x - 2x^2]_0^3 = 6$	I	1
		II	1
	OR		

(iii) (b)	$\text{Area of region AOB} = \int_{-2}^0 (2x + 4) dx$ $= [x^2 + 4x]_{-2}^0 = 4$	I II	1 1
	Case Study – 3		
38.	Two vertical light poles of height 22 m and 16 m stand on the opposite sides of a 20 m wide road as shown below in the figure.  Two ladders of length l_1 and l_2 are placed from a common point R on the road at a distance of x m from the smaller pole. Based on the above information, answer the following questions : (i) Express $p(x) = l_1 + l_2$ in terms of x. (ii) Find $p'(x)$. (iii) (a) Find the value of x for which $l_1^2 + l_2^2$ is minimum. OR (iii) (b) If the 22 m long pole is also replaced by a 16 m long pole, at what distance from either pole should the ladders be kept so that the sum of squares of lengths of ladders needed to reach the top of the pole is minimum ?		
Sol. 38 (i)	$p(x) = \sqrt{22^2 + (20 - x)^2} + \sqrt{16^2 + x^2}$ or $p(x) = \sqrt{x^2 - 40x + 884} + \sqrt{x^2 + 256}$	I	1
(ii)	$p'(x) = \frac{x - 20}{\sqrt{x^2 - 40x + 884}} + \frac{x}{\sqrt{x^2 + 256}}$	I	1
(iii) (a)	Let $q(x) = l_1^2 + l_2^2 = 2x^2 - 40x + 1140$ $q'(x) = 4x - 40$ Put $q'(x) = 0 \Rightarrow x = 10$ $q''(x = 10) = 4 > 0 \Rightarrow q(x)$ is minimum at $x = 10$.	I II III IV	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
	OR		
(iii) (b)	Let, ladder is kept at y m distance from one pole. Suppose, $r(y)$ = sum of squares of lengths of ladders $\Rightarrow r(y) = (16^2 + y^2) + [16^2 + (20 - y)^2]$ $\Rightarrow r(y) = 2y^2 - 40y + 912$ On diff. wrt y, we get $r'(y) = 4y - 40$	I II	$\frac{1}{2}$ $\frac{1}{2}$

	Put $r'(y) = 0 \Rightarrow y = 10$ $r''(y) = 4 > 0 \Rightarrow r(y)$ is minimum at $y = 10$. Hence, the required distance of the ladder from each pole = 10 m	III IV	$\frac{1}{2}$ $\frac{1}{2}$
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