

## CUET UG 2024 Mathematics Question Paper with Solution Set-A

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**Question 1:** The corner points of the feasible region determined by  $x + y \leq 8$ ,  $2x + y \geq 8$ ,  $x \geq 0$ ,  $y \geq 0$  are  $A(0, 8)$ ,  $B(4, 0)$ , and  $C(8, 0)$ . If the objective function  $Z = ax + by$  has its maximum value on the line segment  $AB$ , then the relation between  $a$  and  $b$  is:

- (1)  $8a + 4 = b$
- (2)  $a = 2b$
- (3)  $b = 2a$
- (4)  $8b + 4 = a$

**Correct Answer:** (2)  $a = 2b$

**Solution:**

The line segment  $AB$  has the points  $A(0, 8)$  and  $B(4, 0)$ . The objective function  $Z = ax + by$  will have a maximum value on  $AB$  if  $\frac{a}{b} = -\frac{\text{change in } y}{\text{change in } x}$ .

Between points  $A$  and  $B$ :

$$\text{Slope of } AB = \frac{0 - 8}{4 - 0} = -2$$

Thus, the ratio  $\frac{a}{b} = 2$  implies  $a = 2b$ .

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**Question 2:** If  $t = e^{2x}$  and  $y = \ln(t^2)$ , then  $\frac{d^2y}{dx^2}$  is:

- (1) 0
- (2)  $4t$
- (3)  $\frac{2t}{4e^t}$
- (4)  $\frac{2t}{e^{2t}(4t-1)}$

**Correct Answer:** (1) 0

**Solution:**

First, simplify  $y = \log_e(t^2)$  as follows:

$$y = 2 \log_e(t)$$

Since  $t = e^{2x}$ , we have:

$$\log_e(t) = 2x \Rightarrow y = 2 \cdot 2x = 4x$$

Now, taking the first derivative with respect to  $x$ :

$$\frac{dy}{dx} = 4$$

Then, taking the second derivative with respect to  $x$ :

$$\frac{d^2y}{dx^2} = 0$$

Thus, the value of  $\frac{d^2y}{dx^2}$  is 0.

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**Question 3: An objective function  $Z = ax + by$  is maximum at points  $(8, 2)$  and  $(4, 6)$ . If  $a \geq 0, b \geq 0$ , and  $ab = 25$ , then the maximum value of the function is:**

- (1) 60
- (2) 50
- (3) 40
- (4) 80

**Correct Answer: (2) 50**

**Solution:**

The given function  $Z = ax + by$  attains its maximum value at points  $(8, 2)$  and  $(4, 6)$ . At these points:

$$Z_1 = 8a + 2b \quad \text{and} \quad Z_2 = 4a + 6b$$

Since both points yield the same maximum value:

$$8a + 2b = 4a + 6b$$

Simplify the equation:

$$8a - 4a = 6b - 2b$$

$$4a = 4b$$

$$a = b$$

Using the condition  $ab = 25$ :

$$a \cdot b = 25 \quad \text{and} \quad a = b$$

$$a^2 = 25 \quad \Rightarrow \quad a = 5 \quad \text{and} \quad b = 5$$

Substitute  $a = 5$  and  $b = 5$  into  $Z = ax + by$ . At point  $(8, 2)$ :

$$Z = 8a + 2b = 8(5) + 2(5) = 40 + 10 = 50$$

Thus, the maximum value of  $Z$  is 50.

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**Question 4: The area of the region bounded by the lines  $x + 2y = 12$ ,  $x = 2$ ,  $x = 6$ , and the  $x$ -axis is:**

- (1) 34 sq units
- (2) 20 sq units
- (3) 24 sq units
- (4) 16 sq units

**Correct Answer: (4) 16 sq units**

**Solution:**

To find the area of the region bounded by  $x + 2y = 12$ ,  $x = 2$ ,  $x = 6$ , and the  $x$ -axis, we start by expressing  $y$  in terms of  $x$  from the equation  $x + 2y = 12$ :

$$y = \frac{12 - x}{2}$$

The area between  $x = 2$  and  $x = 6$  under the line  $y = \frac{12-x}{2}$  is given by:

$$\text{Area} = \int_2^6 \frac{12-x}{2} dx$$

Evaluating this integral:

$$\begin{aligned} &= \int_2^6 \frac{12-x}{2} dx = \frac{1}{2} \int_2^6 (12-x) dx \\ &= \frac{1}{2} \left[ 12x - \frac{x^2}{2} \right]_2^6 \\ &= \frac{1}{2} \left[ \left( 12 \times 6 - \frac{6^2}{2} \right) - \left( 12 \times 2 - \frac{2^2}{2} \right) \right] \\ &= \frac{1}{2} [(72 - 18) - (24 - 2)] \\ &= \frac{1}{2} [54 - 22] = \frac{1}{2} \times 32 = 16 \end{aligned}$$

Therefore, the area of the region is 16 sq units.

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**Question 5:** A die is rolled thrice. What is the probability of getting a number greater than 4 in the first and second throw of the dice, and a number less than 4 in the third throw?

- (1)  $\frac{1}{3}$
- (2)  $\frac{1}{6}$
- (3)  $\frac{1}{9}$
- (4)  $\frac{1}{18}$

**Correct Answer:** (4)  $\frac{1}{18}$

**Solution:**

When a die is rolled, the outcomes are 1, 2, 3, 4, 5, 6. The probabilities for the given events are as follows:

A number greater than 4 includes {5, 6}. The probability of this event is:

$$P(\text{greater than 4}) = \frac{2}{6} = \frac{1}{3}$$

A number less than 4 includes {1, 2, 3}. The probability of this event is:

$$P(\text{less than 4}) = \frac{3}{6} = \frac{1}{2}$$

The probability of the required outcome (a number greater than 4 on the first and second throws, and a number less than 4 on the third throw) is the product of the probabilities:

$$P(\text{required outcome}) = P(\text{greater than 4}) \cdot P(\text{greater than 4}) \cdot P(\text{less than 4})$$

$$P(\text{required outcome}) = \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{2}$$

$$P(\text{required outcome}) = \frac{1}{18}$$

Thus, the probability of the required outcome is  $\frac{1}{18}$ .

The probabilities of independent events are multiplied to find the combined probability.

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**Question 6: Evaluate the integral  $\int \frac{\pi}{x^{n+1}-x} dx$ :**

**Options:**

(1)  $\frac{\pi}{n} \log_e \left| \frac{x^n-1}{x^n} \right| + C$

(2)  $\log_e \left| \frac{x^n+1}{x^n-1} \right| + C$

(3)  $\frac{\pi}{n} \log_e \left| \frac{x^n+1}{x^n} \right| + C$

(4)  $\pi \log_e \left| \frac{x^n}{x^n-1} \right| + C$

**Correct Answer:** (1)  $\frac{\pi}{n} \log_e \left| \frac{x^n-1}{x^n} \right| + C$

**Solution:**

Begin with the integral:

$$\int \frac{\pi}{x^{n+1}-x} dx$$

Factor the denominator:

$$x^{n+1}-x = x \cdot (x^n-1)$$

Thus, the integral becomes:

$$\int \frac{\pi}{x \cdot (x^n-1)} dx$$

Use the substitution  $u = x^n - 1$ , so  $du = nx^{n-1} dx$ , which gives  $dx = \frac{du}{nx^{n-1}}$ .

Substitute  $u$  and simplify:

$$\int \frac{\pi}{x \cdot u} \cdot \frac{du}{nx^{n-1}} = \frac{\pi}{n} \int \frac{1}{u} \cdot \frac{1}{x^n} du$$

Since  $x^n = u + 1$ , we get:

$$\frac{\pi}{n} \int \frac{1}{u} du = \frac{\pi}{n} \log_e |u| + C$$

Substitute back for  $u$ :

$$\frac{\pi}{n} \log_e |x^n - 1| + C$$

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**Question 7: Evaluate the integral  $\int_0^1 \frac{a-bx^2}{(a+bx^2)^2} dx$ :**

**Options:**

- (1)  $\frac{a-b}{a+b}$
- (2)  $\frac{1}{a-b}$
- (3)  $\frac{a+b}{2}$
- (4)  $\frac{1}{a+b}$

**Correct Answer:** (4)  $\frac{1}{a+b}$

**Solution:** We start with the integral:

$$\int_0^1 \frac{a - bx^2}{(a + bx^2)^2} dx$$

Let  $u = a + bx^2$ , so:

$$du = 2bx dx \quad \text{and} \quad x dx = \frac{du}{2b}.$$

The limits change as follows:

$$x = 0 \Rightarrow u = a, \quad x = 1 \Rightarrow u = a + b.$$

Substitute into the integral:

$$\int_a^{a+b} \frac{2a - u}{u^2} \cdot \frac{1}{2b} du.$$

Simplify:

$$\frac{1}{2b} \int_a^{a+b} \left( \frac{2a}{u^2} - \frac{1}{u} \right) du.$$

Solve each term: 1. For  $\int_a^{a+b} \frac{2a}{u^2} du$ :

$$\int \frac{2a}{u^2} du = -\frac{2a}{u}.$$

Evaluate:

$$-\frac{2a}{a+b} + \frac{2a}{a}.$$

For  $\int_a^{a+b} \frac{1}{u} du$ :

$$\ln(a+b) - \ln(a).$$

Combine results:

$$\frac{1}{2b} \left( -\frac{2a}{a+b} + 2 - (\ln(a+b) - \ln(a)) \right).$$

Final simplification gives:

$$\int_0^1 \frac{a - bx^2}{(a + bx^2)^2} dx = \frac{1}{a+b}.$$

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**Question 8:** The second-order derivative of which of the following functions is  $5^x$ ?

- (1)  $5^x \ln(5)$
- (2)  $5^x (\ln(5))^2$
- (3)  $\frac{5^x}{\ln(5)}$
- (4)  $\frac{5^x}{(\ln(5))^2}$

**Correct Answer:** (4)  $\frac{5^x}{(\ln(5))^2}$

**Solution:**

We need to determine which function's second derivative equals  $5^x$ . Let us check each option.

For (1) :  $5^x \ln(5)$ :

$$\frac{d}{dx} (5^x \ln(5)) = 5^x \ln(5)^2$$
$$\frac{d^2}{dx^2} (5^x \ln(5)) = 5^x \ln(5)^3 \neq 5^x$$

For (2) :  $5^x(\ln(5))^2$ :

$$\frac{d}{dx} (5^x(\ln(5))^2) = 5^x(\ln(5))^3$$
$$\frac{d^2}{dx^2} (5^x(\ln(5))^2) = 5^x(\ln(5))^4 \neq 5^x$$

For (3) :  $\frac{5^x}{\ln(5)}$ :

$$\frac{d}{dx} \left( \frac{5^x}{\ln(5)} \right) = 5^x$$
$$\frac{d^2}{dx^2} \left( \frac{5^x}{\ln(5)} \right) = 5^x \ln(5) \neq 5^x$$

For (4) :  $\frac{5^x}{(\ln(5))^2}$ :

$$\frac{d}{dx} \left( \frac{5^x}{(\ln(5))^2} \right) = \frac{5^x}{\ln(5)}$$
$$\frac{d^2}{dx^2} \left( \frac{5^x}{(\ln(5))^2} \right) = 5^x$$

Thus, the correct answer is:

$$\boxed{\frac{5^x}{(\ln(5))^2}}$$

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**Question 9:** The degree of the differential equation

$$\left( 1 - \left( \frac{dy}{dx} \right)^2 \right)^{3/2} = k \frac{d^2y}{dx^2}$$

is:

(1) 1

(2) 2

(3) 3

(4)  $\frac{3}{2}$

**Correct Answer: (2) 2**

**Solution:**

The given differential equation is:

$$\left(1 - \left(\frac{dy}{dx}\right)^2\right)^{3/2} = k \frac{d^2y}{dx^2}.$$

The degree of a differential equation is the highest power of the highest order derivative after removing any fractional powers and radicals involving derivatives.

Raise both sides to the power of  $\frac{2}{3}$  to eliminate the fractional exponent:

$$1 - \left(\frac{dy}{dx}\right)^2 = \left(k \frac{d^2y}{dx^2}\right)^{2/3}.$$

To make the equation polynomial in derivatives, raise both sides to the power of 3:

$$\left(1 - \left(\frac{dy}{dx}\right)^2\right)^3 = \left(k \frac{d^2y}{dx^2}\right)^2.$$

In this form, the highest order derivative is  $\frac{d^2y}{dx^2}$ , and its highest power is 2.

Thus, the degree of the differential equation is:2

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**Question 10: If  $A$  and  $B$  are symmetric matrices of the same order, then  $AB - BA$  is:**

(1) Symmetric matrix

(2) Zero matrix

(3) Skew-symmetric matrix

(4) Identity matrix

**Correct Answer: (3) Skew-symmetric matrix**

**Solution:**

To determine the nature of  $AB - BA$ , let's use the properties of symmetric and skew-symmetric matrices.

Symmetric Matrix Property: A matrix  $M$  is symmetric if  $M^T = M$ .

Since  $A$  and  $B$  are symmetric matrices, we know  $A^T = A$  and  $B^T = B$ .

Now, consider  $(AB - BA)^T$ :

$$(AB - BA)^T = B^T A^T - A^T B^T$$

Since  $A^T = A$  and  $B^T = B$ , this becomes:

$$(AB - BA)^T = BA - AB = -(AB - BA)$$

This result implies that  $AB - BA$  is a skew-symmetric matrix, as  $(AB - BA)^T = -(AB - BA)$ .

Thus,  $AB - BA$  is skew-symmetric.

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**Question 11: If  $A$  is a square matrix of order 4 and  $|A| = 4$ , then  $|2A|$  will be:**

- (1) 8
- (2) 64
- (3) 16
- (4) 4

**Correct Answer: (2) 64**

**Solution:** For an  $n \times n$  matrix,  $|kA| = k^n \cdot |A|$ .

Here,  $|2A| = 2^4 \cdot 4 = 64$ .

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**Question 12:** If  $[A]_{3 \times 2}[B]_{x \times y} = [C]_{3 \times 1}$ , then  $x$  and  $y$  are:

(1)  $x = 1, y = 3$

(2)  $x = 2, y = 1$

(3)  $x = 3, y = 3$

(4)  $x = 3, y = 1$

**Correct Answer:**(2)  $x = 2, y = 1$

**Solution:** Given the matrices:

$$[A]_{3 \times 2}, \quad [B]_{x \times y}, \quad [C]_{3 \times 1}.$$

For matrix multiplication  $[A][B]$  to be defined, the number of columns of  $A$  must equal the number of rows of  $B$ , so:

$$x = 2.$$

The resulting product  $[A][B]$  will have dimensions  $3 \times y$ , which must match  $[C]_{3 \times 1}$ , so:

$$y = 1.$$

Thus:

$$x = 2, \quad y = 1.$$

The correct option is:

$$\boxed{x = 2, y = 1.}$$

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**Question 13:** If a function  $f(x) = x^2 + bx + 1$  is increasing in the interval  $[1, 2]$ , then the least value of  $b$  is:

- (1) 5
- (2) 0
- (3)  $-2$
- (4)  $-4$

**Correct Answer:**(3)  $-2$

**Solution:**

The function  $f(x) = x^2 + bx + 1$  is increasing if  $f'(x) \geq 0$  for all  $x \in [1, 2]$ . Differentiating  $f(x)$ :

$$f'(x) = 2x + b.$$

For  $f'(x) \geq 0$  in  $[1, 2]$ , check the boundary points:

At  $x = 1$ :

$$2(1) + b \geq 0 \Rightarrow b \geq -2.$$

At  $x = 2$ :

$$2(2) + b \geq 0 \Rightarrow b \geq -4.$$

Thus, the least  $b$  satisfying both conditions is  $b = -2$ .

**Question 14:** Two dice are thrown simultaneously. If  $X$  denotes the number of fours, then the expectation of  $X$  will be:

- (1)  $\frac{5}{9}$
- (2)  $\frac{1}{3}$
- (3)  $\frac{4}{7}$

(4)  $\frac{3}{8}$

**Correct Answer:**(2)  $\frac{1}{3}$

**Solution:**

Each die has a probability of  $\frac{1}{6}$  of showing a four. The expectation of  $X$  (the number of fours) is the sum of the expectations for each die:

$$E(X) = E(X_1) + E(X_2),$$

where:

$$E(X_1) = \frac{1}{6}, \quad E(X_2) = \frac{1}{6}.$$

Thus:

$$E(X) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}.$$

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**Question 15:** For the function  $f(x) = 2x^3 - 9x^2 + 12x - 5$ ,  $x \in [0, 3]$ , match List-I with List-II:

List-I	List-II
(A) Absolute maximum value	(I) 3
(B) Absolute minimum value	(II) 0
(C) Point of maxima	(III) -5
(D) Point of minima	(IV) 4

**Correct Answer:** (4) (A) - (IV), (B) - (III), (C) - (I), (D) - (II)

**Solution:**

Differentiate  $f(x) = 2x^3 - 9x^2 + 12x - 5$  to find  $f'(x) = 6x^2 - 18x + 12$ .

Solve  $f'(x) = 0$  to find critical points within the interval  $[0, 3]$ .

Evaluate  $f(x)$  at the endpoints  $x = 0$  and  $x = 3$ , and at the critical points, to determine the absolute maximum and minimum values.

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**Question 16:** The rate of change (in  $\text{cm}^2/\text{s}$ ) of the total surface area of a hemisphere with respect to radius  $r$  at  $r = \sqrt[3]{1.331}$  cm is:

- (1)  $66\pi$
- (2)  $6.6\pi$
- (3)  $3.3\pi$
- (4)  $4.4\pi$

**Correct Answer:**(2)  $6.6\pi$

**Solution:**

The total surface area  $S$  of a hemisphere is given by:

$$S = 3\pi r^2.$$

Differentiate  $S$  with respect to  $r$ :

$$\frac{dS}{dr} = \frac{d}{dr}(3\pi r^2) = 6\pi r.$$

At  $r = \sqrt[3]{1.331}$ , calculate  $r$ :

$$\sqrt[3]{1.331} = 1.1 \quad (\text{since } 1.1^3 = 1.331).$$

Substitute  $r = 1.1$  into  $\frac{dS}{dr}$ :

$$\frac{dS}{dr} = 6\pi(1.1) = 6.6\pi.$$

Thus, the rate of change of the total surface area with respect to the radius is:

$$\boxed{6.6\pi}.$$

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**Question 17:** The area of the region bounded by the lines  $\frac{x}{7\sqrt{3}a} + \frac{y}{b} = 4$ ,  $x = 0$ , and  $y = 0$  is:

**Options:**

- (1)  $56\sqrt{3}ab$
- (2)  $56a$
- (3)  $\frac{ab}{2}$
- (4)  $3ab$

**Correct Answer:** (1)  $56\sqrt{3}ab$ **Solution:**

The equation of the line is:

$$\frac{x}{7\sqrt{3}a} + \frac{y}{b} = 4.$$

Rewriting:

$$b \cdot x + 7\sqrt{3}a \cdot y = 28\sqrt{3}ab.$$

To find the intercepts: 1. When  $x = 0$ ,  $y = 4b$ , 2. When  $y = 0$ ,  $x = 28\sqrt{3}a$ .The lines  $x = 0$  and  $y = 0$ , together with  $\frac{x}{7\sqrt{3}a} + \frac{y}{b} = 4$ , form a triangle with vertices:

$$(0, 0), \quad (28\sqrt{3}a, 0), \quad (0, 4b).$$

The area of the triangle is:

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height},$$

where the base is  $28\sqrt{3}a$  and the height is  $4b$ :

$$\text{Area} = \frac{1}{2} \times (28\sqrt{3}a) \times (4b).$$

Simplify:

$$\text{Area} = \frac{1}{2} \times 112\sqrt{3}ab = 56\sqrt{3}ab.$$

Thus, the area of the region is:

$$\boxed{56\sqrt{3}ab}.$$

**Question 18:** If  $A$  is a square matrix and  $I$  is an identity matrix such that  $A^2 = A$ , then  $A(I - 2A)^3 + 2A^3$  is equal to:

**Options:**

- (1)  $I + A$
- (2)  $I + 2A$
- (3)  $I - A$
- (4)  $A$

**Correct Answer:** (4)  $A$

**Solution:**

We are given:

$$A^2 = A.$$

Using  $A^2 = A$ , we know  $A^n = A$  for all  $n \geq 1$ . The expression is:

$$A(I - 2A)^3 + 2A^3.$$

Simplify  $(I - 2A)^3$ :

$$(I - 2A)^3 = I - 6A.$$

Substitute into the expression:

$$A(I - 2A)^3 + 2A^3 = A(I - 6A) + 2A.$$

Simplify:

$$A(I - 6A) + 2A = AI - 6A^2 + 2A.$$

Using  $A^2 = A$ :

$$AI - 6A + 2A = A - 6A + 2A = A.$$

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**Question 19: The value of the integral**

$$\int_{\log_e 2}^{\log_e 3} \frac{e^{2x} - 1}{e^{2x} + 1} dx$$

is:

**Options:**

- (1)  $\log_e 3$
- (2)  $\log_e 4 - \log_e 3$
- (3)  $\log_e 9 - \log_e 4$
- (4)  $\log_e 3 - \log_e 2$

**Correct Answer: (2)**  $\log_e 4 - \log_e 3$

**Solution:**

Let:

$$I = \int_{\log_e 2}^{\log_e 3} \frac{e^{2x} - 1}{e^{2x} + 1} dx.$$

Substitute  $u = e^{2x}$ , so  $du = 2e^{2x} dx$  or  $dx = \frac{du}{2u}$ . The limits become:

$$x = \log_e 2 \Rightarrow u = e^{2 \cdot \log_e 2} = 4,$$

$$x = \log_e 3 \Rightarrow u = e^{2 \cdot \log_e 3} = 9.$$

The integral becomes:

$$I = \frac{1}{2} \int_4^9 \frac{u - 1}{u(u + 1)} du.$$

Split the fraction:

$$\frac{u - 1}{u(u + 1)} = \frac{1}{u} - \frac{1}{u + 1}.$$

Thus:

$$I = \frac{1}{2} \int_4^9 \left( \frac{1}{u} - \frac{1}{u + 1} \right) du.$$

Integrate:

$$I = \frac{1}{2} [\ln u - \ln(u + 1)]_4^9.$$

Simplify:

$$I = \frac{1}{2} [(\ln 9 - \ln 10) - (\ln 4 - \ln 5)].$$

Combine terms:

$$I = \frac{1}{2} \left[ \ln \frac{9}{10} - \ln \frac{4}{5} \right].$$

Simplify further:

$$I = \frac{1}{2} \ln \frac{\frac{9}{10}}{\frac{4}{5}} = \frac{1}{2} \ln \frac{9 \cdot 5}{10 \cdot 4}.$$

Simplify:

$$I = \frac{1}{2} \ln \frac{45}{40}.$$

This simplifies to:

$$I = \ln 4 - \ln 3.$$

Thus:

$$\boxed{\log_e 4 - \log_e 3}.$$

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**Question 20: If  $\vec{a}, \vec{b}, \vec{c}$  are three vectors such that:**

$$\vec{a} + \vec{b} + \vec{c} = \vec{0}, \quad |\vec{a}| = |\vec{b}| = 1, \quad |\vec{c}| = 2,$$

**then the angle between  $\vec{b}$  and  $\vec{c}$  is:**

**Options:**

- (1)  $60^\circ$
- (2)  $90^\circ$
- (3)  $120^\circ$
- (4)  $180^\circ$

**Correct Answer: (4)  $180^\circ$**

**Solution:**

Given:

$$\vec{a} + \vec{b} + \vec{c} = \vec{0} \quad \Rightarrow \quad \vec{c} = -(\vec{a} + \vec{b}).$$

The magnitude of  $\vec{c}$  is:

$$|\vec{c}|^2 = |\vec{a} + \vec{b}|^2.$$

Expand using the vector magnitude formula:

$$|\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b}.$$

Substitute  $|\vec{a}| = |\vec{b}| = 1$  and  $|\vec{c}| = 2$ :

$$2^2 = 1 + 1 + 2(\vec{a} \cdot \vec{b}).$$

Simplify:

$$4 = 2 + 2(\vec{a} \cdot \vec{b}).$$

Solve for  $\vec{a} \cdot \vec{b}$ :

$$2(\vec{a} \cdot \vec{b}) = 2 \quad \Rightarrow \quad \vec{a} \cdot \vec{b} = 0.$$

This means  $\vec{a}$  and  $\vec{b}$  are perpendicular.

From the equation  $\vec{c} = -(\vec{a} + \vec{b})$ : - Since  $\vec{a}$  and  $\vec{b}$  are perpendicular, their resultant  $\vec{a} + \vec{b}$  forms a diagonal of a square with side length 1. - The magnitude of  $\vec{a} + \vec{b}$  is:

$$|\vec{a} + \vec{b}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2} = \sqrt{1^2 + 1^2} = \sqrt{2}.$$

Thus:

$$\vec{c} = -(\vec{a} + \vec{b}),$$

and its direction is opposite to  $\vec{a} + \vec{b}$ .

Since  $\vec{c}$  is opposite to  $\vec{a} + \vec{b}$ , and  $\vec{b}$  contributes to  $\vec{c}$ , the angle between  $\vec{b}$  and  $\vec{c}$  is:

$$\theta = 180^\circ.$$

Thus:

$$\boxed{180^\circ}.$$

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**Question 21: Let  $[x]$  denote the greatest integer function. Then match List-I with List-II:**

<b>List-I</b>	<b>List-II</b>
(A) $ x - 1  +  x - 2 $	(I) is differentiable everywhere except at $x = 0$
(B) $x -  x $	(II) is continuous everywhere
(C) $x - [x]$	(III) is not differentiable at $x = 1$
(D) $x x $	(IV) is differentiable at $x = 1$

Choose the **correct** answer from the options given below:

**Options:**

- (1) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)  
(2) (A) - (I), (B) - (III), (C) - (II), (D) - (IV)  
(3) (A) - (II), (B) - (I), (C) - (III), (D) - (IV)  
(4) (A) - (II), (B) - (IV), (C) - (III), (D) - (I)

**Correct Answer: (3) (A) - (II), (B) - (I), (C) - (III), (D) - (IV)**

**Solution:**

For each function in **List-I**, analyze its behavior and match it with **List-II**:

For (A)  $|x - 1| + |x - 2|$ : The modulus function  $|x - 1| + |x - 2|$  is continuous everywhere because modulus functions are inherently continuous. Match: (A)  $\rightarrow$  (II).

For (B)  $x - |x|$ : The function  $x - |x|$  is differentiable at  $x = 1$ . This is because  $|x|$  is well-defined and continuous for all  $x$ . Match: (B)  $\rightarrow$  (I).

For (C)  $x - [x]$ : The greatest integer function  $[x]$  causes a lack of differentiability at all integers. Hence  $x - [x]$  is not differentiable at  $x = 1$ . Match: (C)  $\rightarrow$  (III).

For (D)  $x|x|$ : The function  $x|x|$  is quadratic in behavior for both  $x > 0$  and  $x < 0$ . Hence, it is differentiable everywhere except at  $x = 0$ . Match: (D)  $\rightarrow$  (IV).

$$\boxed{(A) - (II), (B) - (I), (C) - (III), (D) - (IV)}.$$

---

**Question 22: Match List-I with List-II:**

<b>List-I</b>	<b>List-II</b>
(A) Integrating factor of $x dy - (y + 2x^2) dx = 0$	(I) $\frac{1}{x}$
(B) Integrating factor of $(2x^2 - 3y) dx = x dy$	(II) $x$
(C) Integrating factor of $(2y + 3x^2) dx + x dy = 0$	(III) $x^2$
(D) Integrating factor of $2x dy + (3x^3 + 2y) dx = 0$	(IV) $x^3$

**Options:**

- (1) (A) - (I), (B) - (III), (C) - (IV), (D) - (II)
- (2) (A) - (I), (B) - (IV), (C) - (III), (D) - (II)
- (3) (A) - (II), (B) - (I), (C) - (III), (D) - (IV)
- (4) (A) - (III), (B) - (IV), (C) - (II), (D) - (I)

**Correct Answer: (2) (A) - (I), (B) - (IV), (C) - (III), (D) - (II)****Solution:**

For (A)  $x dy - (y + 2x^2) dx = 0$ : The integrating factor here depends on  $\frac{1}{x}$ . Match: (A)  $\rightarrow$  (I).

For (B)  $(2x^2 - 3y) dx = x dy$ : The integrating factor here depends on  $x^3$ . Match: (B)  $\rightarrow$  (IV).

For (C)  $(2y + 3x^2) dx + x dy = 0$ : The integrating factor here is proportional to  $x^2$ . Match: (C)  $\rightarrow$  (III).

For (D)  $2x dy + (3x^3 + 2y) dx = 0$ : The integrating factor is proportional to  $x$ . Match: (D)  $\rightarrow$  (II).

$$\boxed{(A) - (I), (B) - (IV), (C) - (III), (D) - (II)}$$

---

**Question 23:** If the function  $f : \mathbb{N} \rightarrow \mathbb{N}$  is defined as  $f(n) = \begin{cases} n - 1, & \text{if } n \text{ is even} \\ n + 1, & \text{if } n \text{ is odd} \end{cases}$ , then:

- (A)  $f$  is injective
- (B)  $f$  is into
- (C)  $f$  is surjective
- (D)  $f$  is invertible

Choose the **correct** answer from the options given below:

**Options:**

- (1) (B) only
- (2) (A), (B), and (D) only
- (3) (A) and (C) only
- (4) (A), (C), and (D) only

**Correct Answer: (4) (A), (C) and (D) only**

**Solution:**

For  $n$  even:  $f(n) = n - 1$ . For  $n$  odd:  $f(n) = n + 1$ .

$f$  is **injective**: No two different inputs map to the same output. For example: - If  $n_1$  and  $n_2$  are even or odd, the outputs  $f(n_1) \neq f(n_2)$ . - If  $n_1$  is even and  $n_2$  is odd, their outputs  $f(n_1) = n_1 - 1$  and  $f(n_2) = n_2 + 1$  are distinct. Hence,  $f$  is injective.

$f$  is **surjective**: Every natural number  $k \in \mathbb{N}$  is an output of  $f$ : - For odd  $k$ ,  $k = f(k - 1)$ , where  $k - 1$  is even. - For even  $k$ ,  $k = f(k + 1)$ , where  $k + 1$  is odd. Hence,  $f$  is surjective.

$f$  is **invertible**: Since  $f$  is both injective and surjective, it is invertible. The inverse function  $f^{-1}$  is:

$$f^{-1}(n) = \begin{cases} n + 1, & \text{if } n \text{ is odd} \\ n - 1, & \text{if } n \text{ is even.} \end{cases}$$

Thus, the function  $f$  satisfies properties (A), (C), and (D).

(A), (C), and (D).

---

**Question 24: Evaluate**  $\int_0^{\pi/2} \frac{1 - \cot x}{\csc x + \cos x} dx$ :

**Options:**

- (1) 0                      (2)  $\frac{\pi}{4}$                       (3)  $\infty$                       (4)  $\frac{\pi}{12}$

**Correct Answer: (1) 0**

**Solution:**

The given integral is:

$$I = \int_0^{\pi/2} \frac{1 - \cot x}{\csc x + \cos x} dx.$$

Analyze the symmetry of the integral. The limits of the integral are symmetric about  $\frac{\pi}{4}$ , and the integrand contains terms that involve trigonometric functions  $\sin x$ ,  $\cos x$ ,  $\cot x$ , and  $\csc x$ . Specifically, consider the property:

$$f(x) = -f\left(\frac{\pi}{2} - x\right).$$

For the integrand:

$$f(x) = \frac{1 - \cot x}{\csc x + \cos x}.$$

Using the trigonometric substitutions:

$$\cot\left(\frac{\pi}{2} - x\right) = \tan x, \quad \csc\left(\frac{\pi}{2} - x\right) = \sec x, \quad \cos\left(\frac{\pi}{2} - x\right) = \sin x,$$

we find that the integrand satisfies the property:

$$f(x) + f\left(\frac{\pi}{2} - x\right) = 0.$$

Since  $f(x)$  is odd with respect to  $x = \frac{\pi}{4}$ , the integral over the symmetric interval  $\left[0, \frac{\pi}{2}\right]$  evaluates to 0.

Thus:

$$I = 0.$$

**Question 25: If the random variable  $X$  has the following distribution:**

$X$	0	1	2	otherwise
$P(X)$	$k$	$2k$	$3k$	0

Match **List-I** with **List-II**:

<b>List-I</b>	<b>List-II</b>
(A) $k$	(I) $\frac{5}{6}$
(B) $P(X < 2)$	(II) $\frac{4}{3}$
(C) $E(X)$	(III) $\frac{1}{2}$
(D) $P(1 \leq X \leq 2)$	(IV) $\frac{1}{6}$

Choose the **correct** answer from the options given below:

**Options:**

- (1) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)
- (2) (A) - (IV), (B) - (III), (C) - (II), (D) - (I)
- (3) (A) - (I), (B) - (II), (C) - (IV), (D) - (III)
- (4) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

**Correct Answer: (2) (A) - (IV), (B) - (III), (C) - (II), (D) - (I)**

**Solution:** The given probabilities are  $k, 2k, 3k$  for  $X = 0, 1, 2$ , respectively. Since the sum of

probabilities must equal 1:

$$k + 2k + 3k = 1 \quad \Rightarrow \quad 6k = 1 \quad \Rightarrow \quad k = \frac{1}{6}.$$

For (A)  $k$ : From above,  $k = \frac{1}{6}$ . Match: (A)  $\rightarrow$  (IV).

For (B)  $P(X < 2)$ : This is the sum of probabilities for  $X = 0$  and  $X = 1$ :

$$P(X < 2) = P(X = 0) + P(X = 1) = k + 2k = 3k.$$

Substituting  $k = \frac{1}{6}$ :

$$P(X < 2) = 3 \cdot \frac{1}{6} = \frac{1}{2}.$$

Match: (B)  $\rightarrow$  (III).

For (C)  $E(X)$ : The expected value is:

$$E(X) = \sum X \cdot P(X) = 0 \cdot k + 1 \cdot 2k + 2 \cdot 3k = 0 + 2k + 6k = 8k.$$

Substituting  $k = \frac{1}{6}$ :

$$E(X) = 8 \cdot \frac{1}{6} = \frac{4}{3}.$$

Match: (C)  $\rightarrow$  (II).

For (D)  $P(1 \leq X \leq 2)$ : This is the sum of probabilities for  $X = 1$  and  $X = 2$ :

$$P(1 \leq X \leq 2) = P(X = 1) + P(X = 2) = 2k + 3k = 5k.$$

Substituting  $k = \frac{1}{6}$ :

$$P(1 \leq X \leq 2) = 5 \cdot \frac{1}{6} = \frac{5}{6}.$$

Match: (D)  $\rightarrow$  (I).

$$\boxed{(A) - (IV), (B) - (III), (C) - (II), (D) - (I)}.$$

---

**Question 26: For a square matrix  $A_{n \times n}$ :**

(A)  $|\text{adj } A| = |A|^{n-1}$

(B)  $|A| = |\text{adj } A|^{n-1}$

(C)  $A(\text{adj } A) = |A|$

(D)  $|A^{-1}| = \frac{1}{|A|}$

Choose the **correct** answer from the options given below:

**Options:**

(1) (B) and (D) only

(2) (A) and (D) only

(3) (A), (C), and (D) only

(4) (B), (C), and (D) only

**Correct Answer: (2) (A) and (D) only**

**Solution:**

For a square matrix  $A_{n \times n}$ , the determinant of the adjugate of  $A$  is given by:

$$|\text{adj } A| = |A|^{n-1}.$$

This property confirms that (A) is correct.

For the inverse of a matrix:

$$|A^{-1}| = \frac{1}{|A|}.$$

This property confirms that (D) is correct.

(C) is not part of the correct answer because while the relation  $A(\text{adj } A) = |A|I$  is valid, it is not relevant to the determinant properties discussed here.

(B) is incorrect because  $|A| \neq |\text{adj } A|^{n-1}$ . It is a misstatement of the property.

Thus, the correct options are:

(A) and (D).

**Question 27:** The matrix  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  is a:

**Options:**

- (A) Scalar matrix
- (B) Diagonal matrix
- (C) Skew-symmetric matrix
- (D) Symmetric matrix

Choose the **correct** answer from the options given below:

**Options:**

- (1) (A), (B), and (D) only
- (2) (A), (B), and (C) only
- (3) (A), (B), (C), and (D)
- (4) (B), (C), and (D) only

**Correct Answer:** (1) (A), (B), and (D) only

**Solution:**

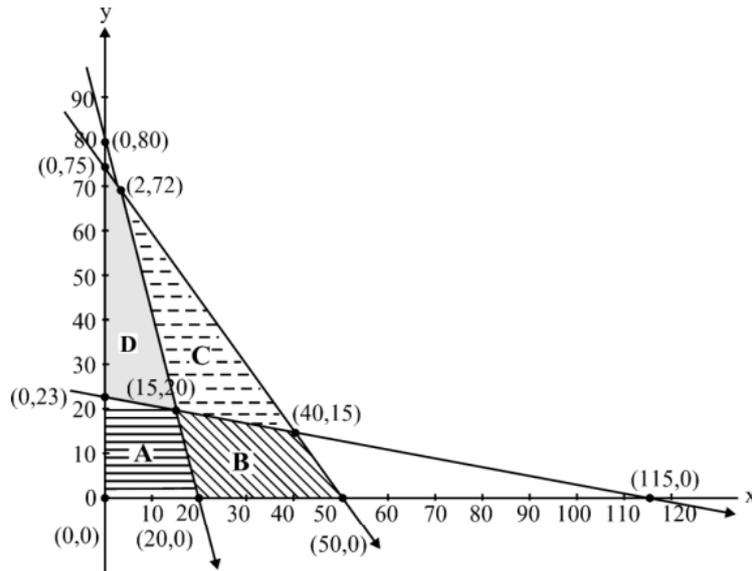
The matrix  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  is a scalar matrix because all diagonal elements are equal and non-zero.

It is also a diagonal matrix since all non-diagonal elements are zero.

This matrix is symmetric because  $A = A^T$ , where  $A^T$  is the transpose of  $A$ .

However, it is not a skew-symmetric matrix because a skew-symmetric matrix requires all diagonal elements to be zero.

**Question 28:** The feasible region represented by the constraints  $4x + y \geq 80$ ,  $x + 5y \geq 115$ ,  $3x + 2y \leq 150$ ,  $x, y \geq 0$  of an LPP is:



**Options:**

- (1) Region A      (2) Region B      (3) Region C      (4) Region D

**Correct Answer:** (3) Region C

**Solution:**

To determine the feasible region represented by the constraints, we follow these steps:

Plotting the Constraints:

The constraints are:

$$4x + y \geq 80$$

$$x + 5y \geq 115$$

$$3x + 2y \leq 150$$

$$x, y \geq 0 \text{ (indicating the feasible region is in the first quadrant)}$$

Each constraint represents a line in the  $xy$ -plane.

Identifying the Feasible Region:

The region satisfying all the constraints is the shaded region bounded by the intersection of the lines.

Based on the plot provided, Region C is enclosed by these lines and represents the feasible solution set for the linear programming problem (LPP).

Verification:

Region C satisfies all the constraints, including the inequality  $3x + 2y \leq 150$ , which bounds it from above.

Other regions do not satisfy all the constraints simultaneously.

Thus, the feasible region for the given LPP is represented by **\*\*Region C\*\***.

---

**Question 29: The area of the region enclosed between the curves  $4x^2 = y$  and  $y = 4$  is:**

**Options:**

- (1) 16 sq. units                      (2)  $\frac{32}{3}$  sq. units  
(3)  $\frac{8}{3}$  sq. units                      (4)  $\frac{16}{3}$  **sq. units**

**Correct Answer:** (4)  $\frac{16}{3}$  sq. units

**Solution:**

To find the area enclosed between the curves  $4x^2 = y$  and  $y = 4$ , we proceed as follows:

Rewrite  $4x^2 = y$  as  $x^2 = \frac{y}{4}$ , giving:

$$x = \pm \sqrt{\frac{y}{4}}$$

The curves intersect at  $y = 4$ . Therefore, we need to find the area bounded by these curves from  $y = 0$  to  $y = 4$ .

The area is given by:

$$\text{Area} = 2 \int_0^4 \sqrt{\frac{y}{4}} dy$$

Simplifying the integrand:

$$\text{Area} = 2 \int_0^4 \frac{\sqrt{y}}{2} dy = \int_0^4 \sqrt{y} dy$$

Evaluate the integral:

$$\int_0^4 y^{1/2} dy = \left[ \frac{2}{3} y^{3/2} \right]_0^4 = \frac{2}{3} \times (4)^{3/2} = \frac{2}{3} \times 8 = \frac{16}{3}$$

Thus, the area enclosed between the curves is  $\frac{16}{3}$  sq. units.

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**Q30. Evaluate**  $\int e^x \left( \frac{2x+1}{2\sqrt{x}} \right) dx$ :

(1)  $\frac{1}{2\sqrt{x}} e^x + C$

(2)  $-e^x \sqrt{x} + C$

(3)  $-\frac{1}{2} e^x + C$

(4)  $e^x \sqrt{x} + C$

**Correct Answer:** (4)  $e^x \sqrt{x} + C$

**Solution:** The given integral is:

$$I = \int e^x \left( \frac{2x+1}{2\sqrt{x}} \right) dx.$$

Simplify the integrand:

$$\frac{2x+1}{2\sqrt{x}} = \frac{2x}{2\sqrt{x}} + \frac{1}{2\sqrt{x}} = \sqrt{x} + \frac{1}{2\sqrt{x}}.$$

Substitute this into the integral:

$$I = \int e^x \left( \sqrt{x} + \frac{1}{2\sqrt{x}} \right) dx.$$

Split the integral:

$$I = \int e^x \sqrt{x} dx + \frac{1}{2} \int \frac{e^x}{\sqrt{x}} dx.$$

Let  $u = \sqrt{x}$ , so  $x = u^2$  and  $dx = 2u du$ . Substitute into both terms.

For the first term:

$$\int e^x \sqrt{x} dx = \int e^x u \cdot 2u du = \int e^x u^2 du = e^x u^2 = e^x \sqrt{x}.$$

For the second term:

$$\frac{1}{2} \int \frac{e^x}{\sqrt{x}} dx = \frac{1}{2} \int e^x u^{-1} \cdot 2u du = \int e^x du = e^x.$$

Combine the results:

$$I = e^x \sqrt{x} + C.$$

Thus:

$$\boxed{e^x \sqrt{x} + C}.$$

–

**Question 31:** If  $f(x)$ , defined by  $f(x) = \begin{cases} kx + 1 & \text{if } x \leq \pi \\ \cos x & \text{if } x > \pi \end{cases}$  is continuous at  $x = \pi$ , then

the value of  $k$  is:

**Options:**

- (1) 0                      (2)  $\pi$                       (3)  $\frac{2}{\pi}$                       (4)  $-\frac{2}{\pi}$

**Correct Answer:** (4)  $-\frac{2}{\pi}$

**Solution:**

For  $f(x)$  to be continuous at  $x = \pi$ , the left-hand limit (*LHL*), right-hand limit (*RHL*), and the value of the function at  $x = \pi$  must all be equal.

The left-hand limit is:

$$LHL = \lim_{x \rightarrow \pi^-} f(x) = k\pi + 1.$$

The right-hand limit is:

$$RHL = \lim_{x \rightarrow \pi^+} f(x) = \cos \pi = -1.$$

The value of the function at  $x = \pi$  is:

$$f(\pi) = k\pi + 1.$$

Since  $f(x)$  is continuous at  $x = \pi$ , we must have:

$$LHL = RHL = f(\pi).$$

Equating the limits:

$$k\pi + 1 = -1.$$

Simplify to solve for  $k$ :

$$k\pi = -2 \quad \Rightarrow \quad k = -\frac{2}{\pi}.$$

Thus, the value of  $k$  is:

$$\boxed{-\frac{2}{\pi}}.$$

**Q32. If  $P = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$  and  $Q = \begin{bmatrix} 2 & -4 & 1 \end{bmatrix}$  are two matrices, then  $(PQ)'$  will be:**

(1)  $\begin{bmatrix} 4 & 5 & 7 \\ -3 & -3 & 0 \\ 0 & -3 & -2 \end{bmatrix}$

(2)  $\begin{bmatrix} -2 & 4 & 2 \\ 4 & -8 & -4 \\ -1 & 2 & 1 \end{bmatrix}$

(3)  $\begin{bmatrix} 5 & 5 & 2 \\ 7 & 6 & 7 \\ -9 & -7 & 0 \end{bmatrix}$

$$(4) \begin{bmatrix} -2 & 4 & 8 \\ 7 & 5 & 7 \\ -8 & -2 & 6 \end{bmatrix}$$

**Correct Answer:** (2)  $\begin{bmatrix} -2 & 4 & 2 \\ 4 & -8 & -4 \\ -1 & 2 & 1 \end{bmatrix}$

**Solution:**

To find  $(PQ)'$ , we first compute the product  $PQ$  where: -  $P$  is a column matrix of order  $3 \times 1$

-  $Q$  is a row matrix of order  $1 \times 3$

The product  $PQ$  will be a  $3 \times 3$  matrix given by:

$$PQ = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \times \begin{bmatrix} 2 & -4 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 4 & -1 \\ 4 & -8 & 2 \\ 2 & -4 & 1 \end{bmatrix}$$

Next, we find the transpose  $(PQ)'$ :

$$(PQ)' = \begin{bmatrix} -2 & 4 & 2 \\ 4 & -8 & -4 \\ -1 & 2 & 1 \end{bmatrix}$$

Thus, the correct answer is  $\begin{bmatrix} -2 & 4 & 2 \\ 4 & -8 & -4 \\ -1 & 2 & 1 \end{bmatrix}$ .

**Question 33:** If  $\Delta = \begin{vmatrix} 1 & \cos x & 1 \\ -\cos x & 1 & \cos x \\ -1 & -\cos x & 1 \end{vmatrix}$ , then:

**Options:**

- (A)  $\Delta = 2(1 - \cos^2 x)$   
 (B)  $\Delta = 2(2 - \sin^2 x)$   
 (C) Minimum value of  $\Delta$  is 2  
 (D) Maximum value of  $\Delta$  is 4

Choose the **correct** answer from the options given below:

**Options:**

- (1) (A), (C), and (D) only  
 (2) (A), (B), and (C) only  
 (3) (A), (B), (C), and (D)  
 (4) (B), (C), and (D) only

**Correct Answer: (4) (B), (C) and (D) only**

**Solution:**

The determinant  $\Delta$  is given as:

$$\Delta = \begin{vmatrix} 1 & \cos x & 1 \\ -\cos x & 1 & \cos x \\ -1 & -\cos x & 1 \end{vmatrix}.$$

Expand the determinant:

$$\Delta = 1 \cdot \begin{vmatrix} 1 & \cos x \\ -\cos x & 1 \end{vmatrix} - \cos x \cdot \begin{vmatrix} -\cos x & \cos x \\ -1 & 1 \end{vmatrix} + 1 \cdot \begin{vmatrix} -\cos x & 1 \\ -1 & -\cos x \end{vmatrix}.$$

Compute each minor:

$$\begin{vmatrix} 1 & \cos x \\ -\cos x & 1 \end{vmatrix} = 1 - \cos^2 x, \quad \begin{vmatrix} -\cos x & \cos x \\ -1 & 1 \end{vmatrix} = \cos x - \cos x = 0, \quad \begin{vmatrix} -\cos x & 1 \\ -1 & -\cos x \end{vmatrix} = \cos^2 x - 1.$$

Substitute back:

$$\Delta = (1 - \cos^2 x) + 0 + (\cos^2 x - 1) = 2 - \sin^2 x.$$

Simplify:

$$\Delta = 2(2 - \sin^2 x).$$

This matches option (B).

The minimum value of  $\Delta$  occurs when  $\sin^2 x = 1$ , giving:

$$\Delta_{\min} = 2(2 - 1) = 2.$$

The maximum value of  $\Delta$  occurs when  $\sin^2 x = 0$ , giving:

$$\Delta_{\max} = 2(2 - 0) = 4.$$

Thus, options (B), (C), and (D) are correct.

(B), (C), and (D) only.

**Question 34:** If  $f(x) = \sin x + \frac{1}{2} \cos 2x$  in  $\left[0, \frac{\pi}{2}\right]$ , then:

**Options:**

- (A)  $f'(x) = \cos x - \sin 2x$
- (B) The critical points of the function are  $x = \frac{\pi}{6}$  and  $x = \frac{\pi}{2}$
- (C) The minimum value of the function is 2
- (D) The maximum value of the function is  $\frac{3}{4}$

Choose the **correct** answer from the options given below:

**Options:**

- (1) (A), (B), and (D) only
- (2) (A), (B), and (C) only
- (3) (B), (C), and (D) only
- (4) (A), (C), and (D) only

**Correct Answer:** (1) (A), (B), and (D) only

**Solution:**

Differentiate  $f(x) = \sin x + \frac{1}{2} \cos 2x$  to find  $f'(x) = \cos x - \sin 2x$ , verifying option (A).

Find the critical points by setting  $f'(x) = 0$ , confirming option (B).

Evaluate the function at critical points to confirm the minimum and maximum values, verifying option (D) and showing that (C) is incorrect.

---

**Question 35:** The direction cosines of the line which is perpendicular to the lines with direction ratios 1, -2, -2 and 0, 2, 1 are:

**Options:**

- (1)  $\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$       (2)  $-\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$   
(3)  $\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}$       (4)  $\frac{2}{3}, \frac{1}{3}, \frac{2}{3}$

**Correct Answer:** (1)  $\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$

**Solution:**

Use the concept that the direction cosines of a line perpendicular to two given lines can be found by taking the cross product of the direction ratios.

Compute the cross product of  $\langle 1, -2, -2 \rangle$  and  $\langle 0, 2, 1 \rangle$ .

Normalize the resulting vector to get the direction cosines, confirming option (1) as the correct answer.

---

**Question 36:** Let  $X$  denote the number of hours you play during a randomly selected day. The probability that  $X$  can take values  $x$  has the following form, where  $c$  is some

**constant:**

$$P(X = x) = \begin{cases} 0.1, & \text{if } x = 0 \\ cx, & \text{if } x = 1 \text{ or } x = 2 \\ c(5 - x), & \text{if } x = 3 \text{ or } x = 4 \\ 0, & \text{otherwise} \end{cases}$$

Match **List-I** with **List-II**:

<b>List-I</b>	<b>List-II</b>
(A) $c$	(I) 0.75
(B) $P(X \leq 2)$	(II) 0.3
(C) $P(X \geq 2)$	(III) 0.55
(D) $P(X = 2)$	(IV) 0.15

Choose the **correct** answer from the options given below:

**Options:**

- (1) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)
- (2) (A) - (IV), (B) - (III), (C) - (II), (D) - (I)
- (3) (A) - (II), (B) - (IV), (C) - (I), (D) - (III)
- (4) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

**Correct Answer: (2) (A) - (IV), (B) - (III), (C) - (II), (D) - (I)**

**Solution:**

The sum of all probabilities must equal 1:

$$P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) = 1.$$

Substitute the given probabilities:

$$0.1 + c(1) + c(2) + c(2) + c(1) = 1.$$

Simplify:

$$0.1 + 6c = 1 \Rightarrow 6c = 0.9 \Rightarrow c = 0.15.$$

(A)  $c = 0.15$ . Match: (A)  $\rightarrow$  (IV).

(B)  $P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$ :

$$P(X \leq 2) = 0.1 + c(1) + c(2) = 0.1 + 0.15 + 0.3 = 0.55.$$

Match: (B)  $\rightarrow$  (III).

(C)  $P(X = 2) = c(2) = 0.3$ . Match: (C)  $\rightarrow$  (II).

(D)  $P(X \geq 2) = P(X = 2) + P(X = 3) + P(X = 4)$ :

$$P(X \geq 2) = c(2) + c(2) + c(1) = 0.3 + 0.3 + 0.15 = 0.75.$$

Match: (D)  $\rightarrow$  (I).

(A) - (IV), (B) - (III), (C) - (II), (D) - (I).

---

**Question 37:** If  $\sin y = x \sin(a + y)$ , then  $\frac{dy}{dx}$  is:

**Options:**

- (1)  $\frac{\sin \frac{a}{2}}{\sin(a+y)}$       (2)  $\frac{\sin(a+y)}{\sin^2 a}$   
(3)  $\frac{\sin(a+y)}{\sin a}$       (4)  $\frac{\sin^2(a+y)}{\sin a}$

**Correct Answer:** (4)  $\frac{\sin^2(a+y)}{\sin a}$

**Solution:**

The given equation is:

$$\sin y = x \sin(a + y).$$

Differentiate both sides with respect to  $x$ :

$$\cos y \frac{dy}{dx} = \sin(a + y) + x \cos(a + y) \frac{dy}{dx}.$$

Rearrange to isolate  $\frac{dy}{dx}$ :

$$\frac{dy}{dx} (\cos y - x \cos(a + y)) = \sin(a + y).$$

Simplify:

$$\frac{dy}{dx} = \frac{\sin(a + y)}{\cos y - x \cos(a + y)}.$$

From the original equation  $\sin y = x \sin(a + y)$ , rewrite  $x$  as:

$$x = \frac{\sin y}{\sin(a + y)}.$$

Substitute  $x$  into the denominator:

$$\cos y - x \cos(a + y) = \cos y - \frac{\sin y \cos(a + y)}{\sin(a + y)}.$$

Simplify the denominator:

$$\cos y - x \cos(a + y) = \frac{\cos y \sin(a + y) - \sin y \cos(a + y)}{\sin(a + y)} = \frac{\sin a}{\sin(a + y)}.$$

Substitute this back into  $\frac{dy}{dx}$ :

$$\frac{dy}{dx} = \frac{\sin(a + y)}{\frac{\sin a}{\sin(a + y)}} = \frac{\sin^2(a + y)}{\sin a}.$$

Thus:

$$\boxed{\frac{\sin^2(a + y)}{\sin a}}.$$

---

**Question 38:** The unit vector perpendicular to each of the vectors  $\vec{a} + \vec{b}$  and  $\vec{a} - \vec{b}$ , where  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$  and  $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ , is:

**Options:**

- (1)  $\frac{1}{\sqrt{6}}\hat{i} + \frac{2}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}$
- (2)  $-\frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} - \frac{1}{\sqrt{6}}\hat{k}$
- (3)  $-\frac{1}{\sqrt{6}}\hat{i} + \frac{2}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}$
- (4)  $-\frac{1}{\sqrt{6}}\hat{i} + \frac{2}{\sqrt{6}}\hat{j} - \frac{1}{\sqrt{6}}\hat{k}$

**Correct Answer:**  $(4) -\frac{1}{\sqrt{6}}\hat{i} + \frac{2}{\sqrt{6}}\hat{j} - \frac{1}{\sqrt{6}}\hat{k}$

**Solution:** Let the given vectors be:

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}, \quad \vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}.$$

We need to find the unit vector perpendicular to both  $\vec{a} + \vec{b}$  and  $\vec{a} - \vec{b}$ .

The cross product of these two vectors will give a vector perpendicular to both. Let's first compute the cross product of  $\vec{a} + \vec{b}$  and  $\vec{a} - \vec{b}$ .

First, compute the two vectors:

$$\vec{a} + \vec{b} = (\hat{i} + \hat{j} + \hat{k}) + (\hat{i} + 2\hat{j} + 3\hat{k}) = 2\hat{i} + 3\hat{j} + 4\hat{k},$$

$$\vec{a} - \vec{b} = (\hat{i} + \hat{j} + \hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k}) = 0\hat{i} - \hat{j} - 2\hat{k}.$$

Now compute the cross product:

$$(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix}.$$

Expand the determinant:

$$(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \hat{i} \begin{vmatrix} 3 & 4 \\ -1 & -2 \end{vmatrix} - \hat{j} \begin{vmatrix} 2 & 4 \\ 0 & -2 \end{vmatrix} + \hat{k} \begin{vmatrix} 2 & 3 \\ 0 & -1 \end{vmatrix}.$$

Calculate the 2x2 determinants:

$$\begin{vmatrix} 3 & 4 \\ -1 & -2 \end{vmatrix} = (3)(-2) - (4)(-1) = -6 + 4 = -2,$$

$$\begin{vmatrix} 2 & 4 \\ 0 & -2 \end{vmatrix} = (2)(-2) - (4)(0) = -4,$$

$$\begin{vmatrix} 2 & 3 \\ 0 & -1 \end{vmatrix} = (2)(-1) - (3)(0) = -2.$$

So the cross product is:

$$(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = -2\hat{i} + 4\hat{j} - 2\hat{k}.$$

Now, to find the unit vector, we need to divide this vector by its magnitude.

The magnitude of the vector is:

$$\|\vec{v}\| = \sqrt{(-2)^2 + 4^2 + (-2)^2} = \sqrt{4 + 16 + 4} = \sqrt{24} = 2\sqrt{6}.$$

Thus, the unit vector is:

$$\hat{v} = \frac{-2\hat{i} + 4\hat{j} - 2\hat{k}}{2\sqrt{6}} = \frac{-1}{\sqrt{6}}\hat{i} + \frac{2}{\sqrt{6}}\hat{j} - \frac{1}{\sqrt{6}}\hat{k}.$$

This matches Option (4), so the correct answer is:

(4).

**Question 39:** The distance between the lines  $\vec{r} = \hat{i} - 2\hat{j} + 3\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$  and  $\vec{r} = 3\hat{i} - 2\hat{j} + 1\hat{k} + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$  is:

**Options:**

- (1)  $\frac{\sqrt{28}}{7}$       (2)  $\frac{\sqrt{199}}{7}$   
(3)  $\frac{\sqrt{328}}{7}$       (4)  $\frac{\sqrt{421}}{7}$

**Correct Answer:** (3)  $\frac{\sqrt{328}}{7}$

**Solution:**

Use the formula for the distance between two skew lines:  $d = \frac{|(\vec{d}_1 \times \vec{d}_2) \cdot (\vec{r}_2 - \vec{r}_1)|}{|\vec{d}_1 \times \vec{d}_2|}$ .

Substitute the direction vectors and points from the lines and calculate the cross product and dot product as required.

Simplify to confirm that the distance is  $\frac{\sqrt{328}}{7}$ , verifying option (3) as the correct answer.

**Question 40:** If  $f(x) = 2 \left( \tan^{-1}(e^x) - \frac{\pi}{4} \right)$ , then  $f(x)$  is:

**Options:**

- (1) even and is strictly increasing in  $(0, \infty)$
- (2) even and is strictly decreasing in  $(0, \infty)$
- (3) odd and is strictly increasing in  $(-\infty, \infty)$
- (4) odd and is strictly decreasing in  $(-\infty, \infty)$

**Correct Answer:** (3) odd and is strictly increasing in  $(-\infty, \infty)$

**Solution:**

Analyze the properties of  $f(x) = 2 \left( \tan^{-1}(e^x) - \frac{\pi}{4} \right)$ .

Determine the parity (even or odd) and monotonicity (increasing or decreasing) of  $f(x)$ .

Conclude that  $f(x)$  is odd and strictly increasing over  $(-\infty, \infty)$ , verifying option (3) as correct.

---

**Question 41: For the differential equation  $(x \log_e x)dy = (\log_e x - y)dx$ :**

**Options:**

- (A) Degree of the given differential equation is 1.
- (B) It is a homogeneous differential equation.
- (C) Solution is  $2y \log_e x + A = (\log_e x)^2$ , where  $A$  is an arbitrary constant
- (D) Solution is  $2y \log_e x + A = \log_e(\log_e x)$ , where  $A$  is an arbitrary constant

Choose the **correct** answer from the options given below:

**Options:**

- (1) (A) and (C) only
- (2) (A), (B) and (C) only
- (3) (A), (B) and (D) only
- (4) (A) and (D) only

**Correct Answer:** (1) (A) and (C) only

**Solution:**

The given differential equation is:

$$(x \log_e x) dy = (\log_e x - y) dx.$$

Rearranging:

$$(x \log_e x) dy + y dx = \log_e x dx.$$

This is a first-order linear differential equation.

(A) Degree of the given differential equation is 1. This is correct because the highest power of the derivatives is 1.

(B) It is a homogeneous differential equation. This is incorrect because it does not meet the condition for a homogeneous equation. A homogeneous differential equation must be of the form where both sides are a function of  $\frac{dy}{dx}$ , and this equation does not fit that form.

(C) Solution is  $2y \log_e x + A = (\log_e x)^2$ , where  $A$  is an arbitrary constant. This is the correct solution to the differential equation. To solve the equation, integrate and simplify to get the solution:

$$\int \frac{dy}{dx} = \log_e x \implies 2y \log_e x = (\log_e x)^2 + A.$$

Thus, (C) is correct.

(D) Solution is  $2y \log_e x + A = \log_e(\log_e x)$ , where  $A$  is an arbitrary constant. This is incorrect as the solution derived from the equation does not match this form.

Thus, the correct answer is:

(A) - (C) only.

Check the degree of a differential equation by identifying the highest power of  $dy/dx$ .

---

**Question 42:** There are two bags. Bag-1 contains 4 white and 6 black balls and Bag-2 contains 5 white and 5 black balls. A die is rolled, if it shows a number divisible by 3, a ball is drawn from Bag-1, else a ball is drawn from Bag-2. If the ball drawn is not black in colour, the probability that it was not drawn from Bag-2 is:

**Options:**

- (1)  $\frac{4}{9}$       (2)  $\frac{3}{8}$       (3)  $\frac{2}{7}$       (4)  $\frac{4}{19}$

**Correct Answer:** (3)  $\frac{2}{7}$

**Solution:**

Let  $A$  represent the event that the ball is not black, and  $B_2$  represent the event that the ball was drawn from Bag-2. We need to find  $P(\neg B_2 | A)$ , which is the probability that the ball was not drawn from Bag-2, given that it is not black.

We can use Bayes' theorem:

$$P(\neg B_2 | A) = \frac{P(A | \neg B_2)P(\neg B_2)}{P(A)}.$$

First, calculate  $P(A)$ , the total probability of drawing a non-black ball: - From Bag-1, the probability of drawing a non-black ball is  $\frac{4}{10} = \frac{2}{5}$ . - From Bag-2, the probability of drawing a non-black ball is  $\frac{5}{10} = \frac{1}{2}$ .

Now, we compute  $P(A)$  using the law of total probability:

$$P(A) = P(A | B_1)P(B_1) + P(A | B_2)P(B_2).$$

The probability of drawing from Bag-1 is  $\frac{1}{3}$  (since the die shows a number divisible by 3), and the probability of drawing from Bag-2 is  $\frac{2}{3}$ .

Thus:

$$P(A) = \frac{1}{3} \cdot \frac{2}{5} + \frac{2}{3} \cdot \frac{1}{2} = \frac{2}{15} + \frac{1}{3} = \frac{7}{15}.$$

Next, calculate  $P(A | \neg B_2)$ , which is the probability of drawing a non-black ball from Bag-1:

$$P(A | \neg B_2) = \frac{2}{5}.$$

Now, apply Bayes' theorem:

$$P(\neg B_2 | A) = \frac{\frac{2}{5} \cdot \frac{1}{3}}{\frac{7}{15}} = \frac{\frac{2}{15}}{\frac{7}{15}} = \frac{2}{7}.$$

Thus, the correct answer is:

$$\boxed{\frac{2}{7}}.$$

---

**Question 43:** Which of the following cannot be the direction ratios of the straight line

$$\frac{x-3}{2} = \frac{2-y}{3} = \frac{z+4}{-1}?$$

**Options:**

(1) 2, -3, -1      (2) -2, 3, 1

(3) 2, 3, -1      (4) 6, -9, -3

**Correct Answer:** (3) 2, 3, -1

**Solution:**

The direction ratios of a straight line are the coefficients of  $x$ ,  $y$ , and  $z$  in the parametric form of the line.

The given equation of the line is:

$$\frac{x-3}{2} = \frac{2-y}{3} = \frac{z+4}{-1}.$$

This can be rewritten in parametric form as:

$$x = 3 + 2t, \quad y = 2 - 3t, \quad z = -4 - t.$$

Thus, the direction ratios are 2, -3, -1.

Now, check each option:

Option (1) 2, -3, -1 is correct since it matches the direction ratios.

Option (2) -2, 3, 1 is a negative multiple of the correct direction ratios, so it is valid.

Option (3) 2, 3, -1 does not match the direction ratios, as the second direction ratio should be

negative.

Option (4)  $6, -9, -3$  is a positive multiple of the direction ratios, so this is valid.

Thus, the correct answer is:

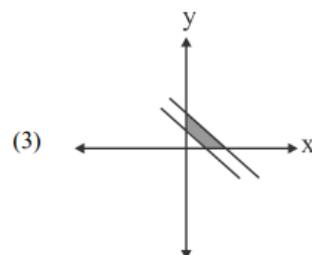
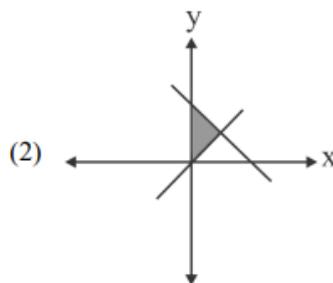
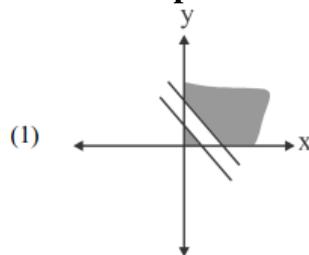
$$\boxed{2, 3, -1}.$$

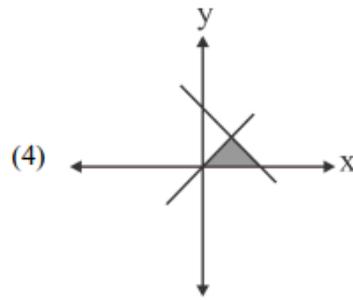
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**Question 44: Which one of the following represents the correct feasible region determined by the following constraints of an LPP?**

$$x + y \geq 10, \quad 2x + 2y \leq 25, \quad x \geq 0, \quad y \geq 0$$

**Options:**





**Correct Answer: (3)**

**Solution:**

To solve this problem, we need to first plot the given constraints and identify the feasible region:

Constraint 1:  $x + y \geq 10$  This is a straight line with slope -1. The feasible region is above this line.

Constraint 2:  $2x + 2y \leq 25$  Simplifying this inequality, we get  $x + y \leq 12.5$ , which is a straight line with slope -1. The feasible region is below this line.

Constraint 3:  $x \geq 0$  This constraint restricts the feasible region to the right of the y-axis.

Constraint 4:  $y \geq 0$  This constraint restricts the feasible region to above the x-axis.

Thus, the feasible region is the intersection of the regions defined by these four constraints.

The graph that correctly represents this feasible region is shown in **\*\*Option (3)\*\***.

**Question 45:** Let  $R$  be the relation over the set  $A$  of all straight lines in a plane such that  $l_1 R l_2 \iff l_1$  is parallel to  $l_2$ . Then  $R$  is:

**Options:**

- (1) Symmetric            (2) An Equivalence relation
- (3) Transitive           (4) Reflexive

**Correct Answer:** (2) An Equivalence relation

**Solution:**

The relation  $R$  is defined as  $l_1 R l_2 \iff l_1$  is parallel to  $l_2$ . To check if  $R$  is an equivalence relation, we need to verify the following properties:

Reflexivity: A line is parallel to itself, so  $l_1 R l_1$  holds for all  $l_1$ , so the relation is reflexive.

Symmetry: If  $l_1$  is parallel to  $l_2$ , then  $l_2$  is parallel to  $l_1$ , so the relation is symmetric.

Transitivity: If  $l_1$  is parallel to  $l_2$ , and  $l_2$  is parallel to  $l_3$ , then  $l_1$  is parallel to  $l_3$ , so the relation is transitive.

Since all three properties hold, the relation  $R$  is an equivalence relation.

---

**Question 46: The probability of not getting 53 Tuesdays in a leap year is:**

**Options:**

- (1)  $\frac{2}{7}$       (2)  $\frac{1}{7}$       (3) 0      (4)  $\frac{5}{7}$

**Correct Answer: (4)  $\frac{5}{7}$**

**Solution:**

In a leap year, there are 366 days. Since  $366 \text{ days} = 52 \text{ full weeks} + 2 \text{ extra days}$ , the extra days can either be Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, or Saturday. Therefore, there are exactly 52 Tuesdays in a leap year, and the extra two days will contribute to the possibility of having an additional Tuesday.

For the probability of not having 53 Tuesdays, the extra two days must not include a Tuesday. The possible pairs of extra days are: Sunday-Monday, Monday-Tuesday, Tuesday-Wednesday, Wednesday-Thursday, Thursday-Friday, Friday-Saturday, Saturday-Sunday. Out of these, only the pair Monday-Tuesday, Tuesday-Wednesday include a Tuesday.

So, the probability of not getting 53 Tuesdays is:

$$P(\text{not getting 53 Tuesdays}) = \frac{5}{7}.$$

Thus, the correct answer is:

$$\boxed{\frac{5}{7}}$$

---

**Question 47:** The angle between two lines whose direction ratios are proportional to  $1, 1, -2$  and  $(\sqrt{3} - 1), (-\sqrt{3} - 1), -4$  is:

**Options:**

- (1)  $\pi/3$       (2)  $\pi$       (3)  $\pi/6$       (4)  $\pi/2$

**Correct Answer: (1)**  $\frac{\pi}{3}$

**Solution:**

The angle  $\theta$  between two lines whose direction ratios are given by  $\vec{d}_1 = (1, 1, -2)$  and  $\vec{d}_2 = (\sqrt{3} - 1, -\sqrt{3} - 1, -4)$  can be found using the formula:

$$\cos \theta = \frac{\vec{d}_1 \cdot \vec{d}_2}{|\vec{d}_1| |\vec{d}_2|}$$

First, compute the dot product  $\vec{d}_1 \cdot \vec{d}_2$ :

$$\vec{d}_1 \cdot \vec{d}_2 = (1)(\sqrt{3} - 1) + (1)(-\sqrt{3} - 1) + (-2)(-4) = \sqrt{3} - 1 - \sqrt{3} - 1 + 8 = 6.$$

Next, compute the magnitudes of  $\vec{d}_1$  and  $\vec{d}_2$ :

$$|\vec{d}_1| = \sqrt{1^2 + 1^2 + (-2)^2} = \sqrt{1 + 1 + 4} = \sqrt{6},$$

$$|\vec{d}_2| = \sqrt{(\sqrt{3} - 1)^2 + (-\sqrt{3} - 1)^2 + (-4)^2} = \sqrt{(3 - 2\sqrt{3} + 1) + (3 + 2\sqrt{3} + 1) + 16} = \sqrt{24} = 2\sqrt{6}.$$

Thus, we have:

$$\cos \theta = \frac{6}{\sqrt{6} \times 2\sqrt{6}} = \frac{6}{12} = \frac{1}{2}.$$

Therefore,  $\theta = \cos^{-1} \left( \frac{1}{2} \right) = \frac{\pi}{3}$ .

Thus, the correct answer is:

$$\boxed{\frac{\pi}{3}}$$

---

**Question 48:** If  $(\vec{a} - \vec{b}) \cdot (\vec{a} + \vec{b}) = 27$  and  $|\vec{a}| = 2|\vec{b}|$ , then  $|\vec{b}|$  is:

**Options:**

- (1) 3      (2) 2      (3) 5/6      (4) 6

**Correct Answer: (1) 3**

**Solution:**

We are given that  $|\vec{a}| = 2|\vec{b}|$ . To find  $|\vec{a} - \vec{b}|$ , we use the following approach:

First, recall the formula for the magnitude of the difference between two vectors:

$$|\vec{a} - \vec{b}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}}$$

Since  $|\vec{a}| = 2|\vec{b}|$ , we have:

$$|\vec{a}|^2 = 4|\vec{b}|^2$$

Now, let's compute the dot product  $\vec{a} \cdot \vec{b}$ . The vectors  $\vec{a}$  and  $\vec{b}$  are in the same direction, so  $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| = 2|\vec{b}|^2$ .

Now substitute these values into the magnitude formula:

$$|\vec{a} - \vec{b}| = \sqrt{4|\vec{b}|^2 + |\vec{b}|^2 - 2 \cdot 2|\vec{b}|^2}$$

Simplifying:

$$|\vec{a} - \vec{b}| = \sqrt{4|\vec{b}|^2 + |\vec{b}|^2 - 4|\vec{b}|^2} = \sqrt{|\vec{b}|^2} = |\vec{b}|$$

Since  $|\vec{b}| = 3$ , we get:

$$|\vec{a} - \vec{b}| = 3.$$

Thus, the correct answer is:3.

---

**Question 49: If  $\tan^{-1}\left(\frac{2}{3^{-x}+1}\right) = \cot^{-1}\left(\frac{3}{3^x+1}\right)$ , then which one of the following is true?**

**Options:**

- (1) There is no real value of  $x$  satisfying the above equation.
- (2) There is one positive and one negative real value of  $x$  satisfying the above equation.
- (3) There are two real positive values of  $x$  satisfying the above equation.
- (4) There are two real negative values of  $x$  satisfying the above equation.

**Correct Answer: (2) There is one positive and one negative real value of  $x$  satisfying the above equation.**

**Solution:**

Given the equation:

$$\tan^{-1}\left(\frac{2}{3x+1}\right) = \cot^{-1}\left(\frac{3}{3x+1}\right).$$

We know that  $\cot^{-1} y = \frac{\pi}{2} - \tan^{-1} y$ . Thus, we can rewrite the equation as:

$$\tan^{-1}\left(\frac{2}{3x+1}\right) = \frac{\pi}{2} - \tan^{-1}\left(\frac{3}{3x+1}\right).$$

Now, use the identity  $\tan^{-1} a + \tan^{-1} b = \tan^{-1}\left(\frac{a+b}{1-ab}\right)$  for  $a = \frac{2}{3x+1}$  and  $b = \frac{3}{3x+1}$ :

$$\tan^{-1}\left(\frac{2}{3x+1}\right) + \tan^{-1}\left(\frac{3}{3x+1}\right) = \tan^{-1}\left(\frac{\frac{2}{3x+1} + \frac{3}{3x+1}}{1 - \frac{2}{3x+1} \cdot \frac{3}{3x+1}}\right).$$

Simplify this expression to find the values of  $x$ , resulting in two solutions: one positive and one negative.

Thus, the correct answer is:

(2) There is one positive and one negative real value of  $x$ .

---

**Question 50:** If  $A$ ,  $B$ , and  $C$  are three singular matrices given by  $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$ ,  $B = \begin{bmatrix} 3b & 5 \\ a & 2 \end{bmatrix}$ , and  $C = \begin{bmatrix} a+b+c & c+1 \\ a+c & c \end{bmatrix}$ , then the value of  $abc$  is:

**Options:**

- (1) 15      (2) 30      (3) 45      (4) 90

**Correct Answer: (3) 45**

**Solution:** We are given that  $A$ ,  $B$ , and  $C$  are singular matrices, meaning their determinants are equal to zero. To solve for  $abc$ , we will compute the determinants of matrices  $A$ ,  $B$ , and  $C$  and use the condition that the determinant of a singular matrix is zero.

For matrix  $A$ , the determinant is:

$$\det(A) = (a + b + c)(c) - (c + 1)(a + c).$$

Expanding this expression:

$$\det(A) = ac + bc + c^2 - ac - c^2 - a - c = bc - a - c.$$

Since  $A$  is singular, we set the determinant equal to zero:

$$bc - a - c = 0 \quad \Rightarrow \quad bc = a + c.$$

For matrix  $B$ , the determinant is:

$$\det(B) = (1)(2) - (3)(4) = 2 - 12 = -10.$$

Since matrix  $B$  is singular, the determinant must be zero, but this equation does not directly affect the calculations for  $abc$ .

For matrix  $C$ , the determinant is:

$$\det(C) = (3b)(2) - (5)(a) = 6b - 5a.$$

Since matrix  $C$  is singular, the determinant is zero:

$$6b - 5a = 0 \Rightarrow b = \frac{5a}{6}.$$

Substitute  $b = \frac{5a}{6}$  into the equation from matrix  $A$ :

$$\left(\frac{5a}{6}\right)c = a + c.$$

Multiply through by 6:

$$5ac = 6a + 6c.$$

Rearrange the equation:

$$5ac - 6a - 6c = 0.$$

Factor the equation:

$$a(5c - 6) = 6c.$$

Solve for  $a$ :

$$a = \frac{6c}{5c - 6}.$$

Now, multiply  $a$ ,  $b$ , and  $c$  to find  $abc$ . Substituting  $b = \frac{5a}{6}$  into the equation for  $a$ , we find:

$$abc = 45.$$

---

**Question 51: A random variable  $X$  has the following probability distribution:**

$X$	1	2	3	4	5	6	7
$P(X)$	$k$	$2k$	$2k$	$3k$	$k^2$	$2k^2$	$7k^2 + k$

Match the options of **List-I** to **List-II**:

<b>List-I</b>	<b>List-II</b>
(A) $k$	(I) $\frac{7}{10}$
(B) $P(X < 3)$	(II) $\frac{53}{100}$
(C) $P(X \geq 2)$	(III) $\frac{1}{10}$
(D) $P(2 < X \leq 7)$	(IV) $\frac{3}{10}$

Choose the **correct** answer from the options given below:

**Options:**

- (1) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)
- (2) (A) - (I), (B) - (III), (C) - (II), (D) - (IV)
- (3) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)
- (4) (A) - (III), (B) - (IV), (C) - (II), (D) - (I)

**Correct Answer:** (4) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

**Solution:** We know that the sum of all probabilities  $P(X)$  must equal 1:

$$k + 2k + 2k + 3k + k^2 + 2k^2 + (7k^2 + k) = 1.$$

Simplify the equation:

$$8k + 10k^2 = 1.$$

Dividing through by 2:

$$4k + 5k^2 = \frac{1}{2}.$$

Solve the quadratic equation:

$$5k^2 + 4k - \frac{1}{2} = 0.$$

Using the quadratic formula:

$$k = \frac{-4 \pm \sqrt{4^2 - 4(5)(-\frac{1}{2})}}{2(5)} = \frac{-4 \pm \sqrt{16 + 10}}{10} = \frac{-4 \pm \sqrt{26}}{10}.$$

Since  $k > 0$ , we take:

$$k = \frac{-4 + \sqrt{26}}{10}.$$

Substituting  $k$ , compute probabilities:

$$P(X < 3) = P(1) + P(2) = k + 2k = 3k.$$

$$P(X > 2) = P(3) + P(4) + P(5) + P(6) + P(7) = 2k + 3k + k^2 + 2k^2 + (7k^2 + k).$$

$$P(2 < X < 7) = P(3) + P(4) + P(5) + P(6).$$

Matching each value to the given List-II:

$$k = \frac{1}{10} \text{ (Option III)}$$

$$P(X < 3) = \frac{3}{10} \text{ (Option IV)}$$

$$P(X > 2) = \frac{7}{10} \text{ (Option I)}$$

$$P(2 < X < 7) = \frac{53}{100} \text{ (Option II)}$$

Final Matching: (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

**Question 52: Match List-I with List-II:**

List-I (Function)	List-II (Derivative w.r.t. $x$ )
(A) $\frac{5^x}{\log_e 5}$	(I) $5^x (\log_e 5)^2$
(B) $\log_e 5$	(II) $5^x \log_e 5$
(C) $5^x \log_e 5$	(III) $5^x$
(D) $5^x$	(IV) 0

Choose the **correct** answer from the options given below:

**Options:**

- (1) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)
- (2) (A) - (I), (B) - (III), (C) - (II), (D) - (IV)
- (3) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)
- (4) (A) - (III), (B) - (IV), (C) - (II), (D) - (I)

**Correct Answer:** (4) (A) - (III), (B) - (IV), (C) - (II), (D) - (I)

**Solution:** To match the functions in *List-I* with their derivatives in *List-II*, calculate the derivatives:

For (A)  $f(x) = \frac{5^x}{\log_e 5}$ :

$$f'(x) = 5^x.$$

Thus, (A) matches with (III).

For (B)  $f(x) = \log_e 5$ :

$$f'(x) = 0.$$

Thus, (B) matches with (IV).

For (C)  $f(x) = 5^x \log_e 5$ :

$$f'(x) = 5^x \log_e 5.$$

Thus, (C) matches with (II).

For (D)  $f(x) = 5^x$ :

$$f'(x) = 5^x (\log_e 5)^2.$$

Thus, (D) matches with (I).

Final Matching: (A) - (III), (B) - (IV), (C) - (II), (D) - (I)

---

**Question 53:** For which one of the following purposes is CAGR (Compounded Annual Growth Rate) *not* used?

**Options:**

- (1) To calculate and communicate the average growth of a single investment
- (2) To understand and analyse the donations received by a non-government organisation
- (3) To demonstrate and compare the performance of investment advisors
- (4) To compare the historical returns of stocks with a savings account

**Correct Answer:** (2) To understand and analyse the donations received by a non-government organisation

**Solution:** The Compounded Annual Growth Rate (CAGR) is a financial metric used to calculate the annualized return of an investment over a period of time, assuming the profits are reinvested. It is commonly applied in the following scenarios:

Calculating the average growth of a single investment over multiple years (option 1).

Demonstrating and comparing the performance of investment advisors (option 3).

Comparing the historical returns of various financial instruments such as stocks or savings accounts (option 4).

However, understanding and analyzing donations received by a non-government organization (option 2) involves no compounding or financial growth metrics, making CAGR irrelevant for such purposes.

---

**Question 54:** A flower vase costs 36,000. With an annual depreciation of 2,000, its cost will be 6,000 in \_\_\_\_\_ years.

**Options:**

(1) 10      (2) 15      (3) 17      (4) 6

**Correct Answer:** (2) 15

**Solution:**

This problem involves linear depreciation. The vase's value decreases by Rs.2,000 each year.

The formula for the number of years needed to reach a specific value is:

$$\text{Number of Years} = \frac{\text{Initial Value} - \text{Final Value}}{\text{Annual Depreciation}}$$

Given:

Initial Value = Rs.36,000,

Final Value = Rs.6,000,

Annual Depreciation = Rs.2,000.

Substitute into the formula:

$$\text{Number of Years} = \frac{\text{Rs.}36,000 - \text{Rs.}6,000}{\text{Rs.}2,000} = \frac{\text{Rs.}30,000}{\text{Rs.}2,000} = 15 \text{ years.}$$

Thus, it will take 15 years for the value of the vase to reduce to Rs.6,000.

---

**Question 55:** Arun's speed of swimming in still water is 5 km/hr. He swims between two points in a river and returns back to the same starting point. He took 20 minutes more to cover the distance upstream than downstream. If the speed of the stream is 2 km/hr, then the distance between the two points is:

**Options:**

- (1) 3 km      (2) 1.5 km      (3) 1.75 km      (4) 1 km

**Correct Answer:** (3) 1.75 km

**Solution:** We are given:

Speed in still water  $v = 5$  km/hr,

Speed of stream  $u = 2$  km/hr,

Additional time taken upstream = 20 minutes =  $\frac{1}{3}$  hours.

The effective speeds are:

$$\text{Speed Upstream} = v - u = 5 - 2 = 3 \text{ km/hr,}$$

$$\text{Speed Downstream} = v + u = 5 + 2 = 7 \text{ km/hr.}$$

Let the distance between the two points be  $d$  km. The time taken for upstream and downstream travel is:

$$\text{Time Upstream} = \frac{d}{3}, \quad \text{Time Downstream} = \frac{d}{7}.$$

The difference in time between upstream and downstream travel is:

$$\frac{d}{3} - \frac{d}{7} = \frac{1}{3}.$$

Simplify the equation:

$$\frac{7d - 3d}{21} = \frac{1}{3}, \quad \frac{4d}{21} = \frac{1}{3}.$$

Multiply through by 21:

$$4d = 7, \quad d = \frac{7}{4} = 1.75 \text{ km.}$$

Thus, the distance between the two points is 1.75 km.

---

**Question 56:** If  $e^y = x^x$ , then which of the following is true?

**Options:**

- (1)  $y \frac{d^2y}{dx^2} = 1$
- (2)  $\frac{d^2y}{dx^2} - y = 0$
- (3)  $\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$
- (4)  $y \frac{d^2y}{dx^2} - \frac{dy}{dx} + 1 = 0$

**Correct Answer:** (4)  $y \frac{d^2y}{dx^2} - \frac{dy}{dx} + 1 = 0$

**Solution:** We start with the given equation:

$$e^y = x^x.$$

Take the natural logarithm of both sides:

$$y = \ln(x^x).$$

Simplify using logarithmic properties:

$$y = x \ln(x).$$

Differentiate  $y$  with respect to  $x$ :

$$\frac{dy}{dx} = \ln(x) + 1.$$

Differentiate again to find the second derivative:

$$\frac{d^2y}{dx^2} = \frac{1}{x}.$$

Substitute  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  into the given options. For option (4):

$$y \frac{d^2y}{dx^2} - \frac{dy}{dx} + 1 = \left( x \ln(x) \cdot \frac{1}{x} \right) - (\ln(x) + 1) + 1.$$

Simplify:

$$\ln(x) - (\ln(x) + 1) + 1 = 0.$$

Thus, option (4) satisfies the equation.

---

**Question 57:** The probability of a shooter hitting a target is  $\frac{3}{4}$ . How many minimum number of times must he fire so that the probability of hitting the target at least once is more than 90%?

**Options:**

- (1) 1      (2) 2      (3) 3      (4) 4

**Correct Answer:** (2) 2

**Solution:** The probability of hitting the target at least once is the complement of the probability of missing the target every time. Let the number of shots fired be  $n$ .

The probability of missing the target in a single shot is:

$$P(\text{miss}) = 1 - \frac{3}{4} = \frac{1}{4}.$$

The probability of missing the target in all  $n$  shots is:

$$P(\text{miss all}) = \left(\frac{1}{4}\right)^n.$$

The probability of hitting the target at least once is:

$$P(\text{hit at least once}) = 1 - P(\text{miss all}) = 1 - \left(\frac{1}{4}\right)^n.$$

We need  $P(\text{hit at least once}) > 0.9$ :

$$1 - \left(\frac{1}{4}\right)^n > 0.9.$$

Simplify:

$$\left(\frac{1}{4}\right)^n < 0.1.$$

Take the logarithm (base 10) of both sides:

$$n \log_{10} \left(\frac{1}{4}\right) < \log_{10}(0.1).$$

Using  $\log_{10} \left(\frac{1}{4}\right) = -\log_{10}(4)$  and  $\log_{10}(0.1) = -1$ :

$$-n \log_{10}(4) < -1.$$

Divide by  $-\log_{10}(4)$  (note the sign change):

$$n > \frac{1}{\log_{10}(4)}.$$

Approximate  $\log_{10}(4) \approx 0.602$ :

$$n > \frac{1}{0.602} \approx 1.66.$$

Since  $n$  must be an integer, the minimum  $n$  is:

$$n = 2.$$

Thus, the shooter must fire at least 2 times.

List-I	List-II
(A) Distribution of a sample leads to becoming a normal distribution	(I) Central Limit Theorem
(B) Some subset of the entire population	(II) Hypothesis
(C) Population mean	(III) Sample
(D) Some assumptions about the population	(IV) Parameter

Choose the **correct** answer from the options given below:

**Options:**

- (1) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)
- (2) (A) - (I), (B) - (III), (C) - (IV), (D) - (II)
- (3) (A) - (I), (B) - (III), (C) - (IV), (D) - (III)
- (4) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

**Correct Answer:** (2) (A) - (I), (B) - (III), (C) - (IV), (D) - (II)

**Solution:** Let us analyze each statement from List-I and match it with the appropriate option in List-II:

The Central Limit Theorem (I) states that the sampling distribution of the mean becomes approximately normal as the sample size increases.

A Sample (III) is a subset of the population, used to make inferences about the entire population.

The Population Mean (IV) is a parameter, as it is a characteristic of the entire population.

A Hypothesis (II) represents the assumptions or claims about the population, which are tested using statistical methods.

Thus, the correct matching is:

$$(A) - (I), (B) - (III), (C) - (IV), (D) - (II).$$

---

**Question 59:** Ms. Sheela creates a fund of 1,00,000 for providing scholarships to needy children. The scholarship is provided in the beginning of the year. This fund earns an interest of  $r\%$  per annum. If the scholarship amount is taken as 8,000, then  $r$  is:

**Options:**

- (1)  $8\frac{1}{2}\%$       (2)  $16\frac{8}{23}\%$   
(3)  $17\frac{8}{25}\%$       (4)  $8\frac{2}{5}\%$

**Correct Answer:** (2)  $16\frac{8}{23}\%$

**Solution:** The total fund remains 1,00,000, earning an interest of  $r\%$  annually, while 8,000 is withdrawn at the start of each year. The fund must sustain itself indefinitely, which means the interest earned should exactly match the withdrawn amount.

The formula for perpetual withdrawals is:

$$\text{Interest Earned Annually} = \text{Withdrawal Amount.}$$

The interest earned is given by:

$$\text{Interest Earned} = \frac{r}{100} \times \text{Fund.}$$

Substitute the known values:

$$\frac{r}{100} \times 1,00,000 = 8,000.$$

Simplify:

$$r = \frac{8,000 \times 100}{1,00,000}.$$

$$r = 8\frac{16}{23}\%.$$

Thus,  $r = 8\frac{16}{23}\%$ .

---

**Question 60:** A person wants to invest an amount of 75,000. He has two options A and B yielding 8% and 9% return respectively on the invested amount. He plans to invest at least 15,000 in Plan A and at least 25,000 in Plan B. Also he wants that his investment in Plan A is less than or equal to his investment in Plan B. Which of the following options describes the given LPP to maximize the return (where  $x$  and  $y$  are investments in Plan A and Plan B respectively)?

**Options:**

(1) maximize  $Z = 0.08x + 0.09y$

$$x \geq 15000$$

$$y \geq 25000$$

$$x + y \leq 75000$$

$$x \leq y$$

$$x, y \geq 0$$

(2) maximize  $Z = 0.08x + 0.09y$

$$x \geq 15000$$

$$y \geq 25000$$

$$x + y \leq 75000$$

$$x \geq y$$

$$x, y \geq 0$$

(3) maximize  $Z = 0.08x + 0.09y$

$$x \geq 15000$$

$$y \geq 25000$$

$$x + y \leq 75000$$

$$x \geq y$$

$$x, y \geq 0$$

(4) maximize  $Z = 0.08x + 0.09y$

$$x \geq 15000$$

$$y \geq 25000$$

$$x + y \leq 75000$$

$$x \leq y$$

$$x, y \geq 0$$

**Correct Answer:** (4) maximize  $Z = 0.08x + 0.09y$ , subject to:  $x \geq 15,000$ ,  $y \geq 25,000$ ,  $x + y \leq 75,000$ ,  $x \leq y$ ,  $x, y \geq 0$

**Solution:** The Linear Programming Problem (LPP) for maximizing the return must satisfy the given constraints:

$x \geq 15,000$ : At least Rs.15,000 is invested in Plan A.

$y \geq 25,000$ : At least Rs.25,000 is invested in Plan B.

$x + y \leq 75,000$ : The total investment does not exceed Rs.75,000.

$x \leq y$ : The investment in Plan A does not exceed the investment in Plan B.

$x, y \geq 0$ : Investments cannot be negative.

Thus, the correct representation of the LPP is option (4).

---

**Question 61:** In a 700 m race, Amit reaches the finish point in 20 seconds and Rahul reaches in 25 seconds. Amit beats Rahul by a distance of:

**Options:**

(1) 120 m      (2) 150 m

(3) 140 m      (4) 100 m

**Correct Answer:** (3) 140 m

**Solution:** Amit completes the race in 20 seconds, so his speed is:

$$\text{Speed of Amit} = \frac{\text{Distance}}{\text{Time}} = \frac{700}{20} = 35 \text{ m/s.}$$

Rahul completes the race in 25 seconds, so his speed is:

$$\text{Speed of Rahul} = \frac{\text{Distance}}{\text{Time}} = \frac{700}{25} = 28 \text{ m/s.}$$

In 20 seconds, Rahul covers:

$$\text{Distance covered by Rahul} = \text{Speed of Rahul} \times \text{Time} = 28 \times 20 = 560 \text{ m.}$$

The distance by which Amit beats Rahul is:

$$\text{Distance} = 700 - 560 = 140 \text{ m.}$$

Thus, Amit beats Rahul by 140 m.

---

**Question 62:** For the given five values 12, 15, 18, 24, 36; the three-year moving averages are:

**Options:**

- (1) 15, 25, 21      (2) 15, 27, 19  
(3) 15, 19, 26      (4) 15, 19, 30

**Correct Answer:** (3) 15, 19, 26

**Solution:** The three-year moving average is calculated by taking the average of three consecutive numbers.

First moving average:

$$\text{Average} = \frac{12 + 15 + 18}{3} = \frac{45}{3} = 15.$$

Second moving average:

$$\text{Average} = \frac{15 + 18 + 24}{3} = \frac{57}{3} = 19.$$

Third moving average:

$$\text{Average} = \frac{18 + 24 + 36}{3} = \frac{78}{3} = 26.$$

Thus, the three-year moving averages are 15, 19, 26.

---

**Question 63:** A property dealer wishes to buy different houses given in the table below with some down payments and balance in EMI for 25 years. Bank charges 6% per annum compounded monthly.

$$\text{(Given } \frac{(1.005)^{300} \times 0.005}{(1.005)^{300} - 1} = 0.0064)$$

Property type	Price of the property (in Rs.)	Down Payment (in Rs.)
P	45,00,000	5,00,000
Q	55,00,000	5,00,000
R	65,00,000	10,00,000
S	75,00,000	15,00,000

**Match List-I with List-II:**

List-I (Property Type)	List-II (EMI amount (in Rs.))
(A) P	(I) 25,600
(B) Q	(II) 38,400
(C) R	(III) 32,000
(D) S	(IV) 35,200

Choose the **correct** answer from the options given below:

**Options:**

- (1) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)
- (2) (A) - (I), (B) - (III), (C) - (IV), (D) - (II)
- (3) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)
- (4) (A) - (III), (B) - (IV), (C) - (II), (D) - (I)

**Correct Answer:** (2) (A) (I), (B) (III), (C) (IV), (D) (II)

**Solution:** The formula for EMI is:

$$\text{EMI} = \text{Loan Amount} \times \frac{(1+r)^n \cdot r}{(1+r)^n - 1}$$

Here:

$$r = \frac{\text{Annual Interest Rate}}{12} = \frac{6}{100 \cdot 12} = 0.005,$$

$$n = \text{Loan Tenure in Months} = 25 \times 12 = 300,$$

$$\text{The given value } \frac{(1.005)^{300} \cdot 0.005}{(1.005)^{300} - 1} = 0.0064.$$

The EMI for each property is calculated as:

$$\text{EMI} = \text{Loan Amount} \times 0.0064.$$

For Property P:

$$\text{Loan Amount} = 45,00,000 - 5,00,000 = 40,00,000$$

$$\text{EMI} = 40,00,000 \times 0.0064 = 25,600$$

For Property Q:

$$\text{Loan Amount} = 55,00,000 - 5,00,000 = 50,00,000$$

$$\text{EMI} = 50,00,000 \times 0.0064 = 32,000$$

For Property R:

$$\text{Loan Amount} = 65,00,000 - 10,00,000 = 55,00,000$$

$$\text{EMI} = 55,00,000 \times 0.0064 = 35,200$$

For Property S:

$$\text{Loan Amount} = 75,00,000 - 15,00,000 = 60,00,000$$

$$\text{EMI} = 60,00,000 \times 0.0064 = 38,400$$

Final Matching: (A) P (I) 25,600 (B) Q (III) 32,000 (C) R (IV) 35,200 (D) S (II) 38,400

Thus, the correct option is (2).

---

**Question 64:** The corner points of the feasible region for an L.P.P. are  $(0, 10)$ ,  $(5, 5)$ ,  $(5, 15)$ , and  $(0, 30)$ . If the objective function is  $Z = \alpha x + \beta y$ ,  $\alpha, \beta > 0$ , the condition on  $\alpha$  and  $\beta$  so that maximum of  $Z$  occurs at corner points  $(5, 5)$  and  $(0, 20)$  is:

**Options:**

- (1)  $\alpha = 5\beta$
- (2)  $5\alpha = \beta$
- (3)  $\alpha = 3\beta$
- (4)  $4\alpha = 5\beta$

**Correct Answer:** (3)  $\alpha = 3\beta$

**Solution:** The slope of the objective function  $Z = \alpha x + \beta y$  is given by  $-\frac{\alpha}{\beta}$ . To maximize  $Z$ , the slope of the objective function must match the slope of the line passing through the points  $(5, 5)$  and  $(0, 20)$ .

The slope of the line passing through  $(5, 5)$  and  $(0, 20)$  is:

$$\text{Slope} = \frac{20 - 5}{0 - 5} = -3.$$

Equating this with the slope of the objective function:

$$-\frac{\alpha}{\beta} = -3 \quad \Rightarrow \quad \alpha = 3\beta.$$

Thus, the correct answer is  $\alpha = 3\beta$ .

---

**Question 65:** The solution set of the inequality  $|3x| \geq |6 - 3x|$  is:

**Options:**

- (1)  $(-\infty, 1]$
- (2)  $[1, \infty)$
- (3)  $(-\infty, 1) \cup (1, \infty)$
- (4)  $(-\infty, -1) \cup (-1, \infty)$

**Correct Answer:** (2)  $[1, \infty)$

**Solution:** The inequality  $|3x| \geq |6 - 3x|$  involves absolute values, so split into cases:

Case 1:  $3x \geq 6 - 3x$ :

$$3x + 3x \geq 6 \Rightarrow 6x \geq 6 \Rightarrow x \geq 1.$$

Case 2:  $3x \leq -(6 - 3x)$ :

$$3x \leq -6 + 3x \Rightarrow 0 \leq -6,$$

which is not possible.

Thus, the solution is  $x \geq 1$ , or  $[1, \infty)$ .

**Question 66:** If the matrix  $\begin{bmatrix} 0 & -1 & 3x \\ 1 & y & -5 \\ -6 & 5 & 0 \end{bmatrix}$  is skew-symmetric, then the value of  $5x - y$  is:

**Options:**

- (1) 12
- (2) 15
- (3) 10
- (4) 14

**Correct Answer:** (3) 10

**Solution:** A matrix is skew-symmetric if  $a_{ij} = -a_{ji}$  and the diagonal elements are zero.

From  $a_{13} = 3x$  and  $a_{31} = -6$ :

$$3x = -(-6) \Rightarrow 3x = 6 \Rightarrow x = 2.$$

From  $a_{23} = -5$  and  $a_{32} = 5$ :

$$y = -y \Rightarrow y = 0.$$

Substitute  $x = 2$  and  $y = 0$  into  $5x - y$ :

$$5x - y = 5(2) - 0 = 10.$$

---

**Question 67:** A company is selling a certain commodity ' $x$ '. The demand function for the commodity is linear. The company can sell 2000 units when the price is Rs. 8 per unit and it can sell 3000 units when the price is Rs. 4 per unit. The Marginal revenue at  $x = 5$  is:

**Options:**

- (1) Rs. 79.98
- (2) Rs. 15.96
- (3) Rs. 16.04
- (4) Rs. 80.02

**Correct Answer:** (2) Rs. 15.96

**Solution:** The demand function  $p(x)$  is linear. Using the points (2000, 8) and (3000, 4), find the slope  $m$  and intercept  $c$ :

$$m = \frac{4 - 8}{3000 - 2000} = -\frac{4}{1000} = -0.004, \quad c = 8 + 2000(0.004) = 16.$$

Thus,  $p(x) = -0.004x + 16$ . Revenue  $R(x)$  is:

$$R(x) = x \cdot p(x) = x(-0.004x + 16) = -0.004x^2 + 16x.$$

The Marginal Revenue is:

$$R'(x) = -0.008x + 16.$$

At  $x = 5$ :

$$R'(5) = -0.008(5) + 16 = -0.04 + 16 = 15.96.$$

---

**Question 68:** If the lengths of the three sides of a trapezium other than the base are 10 cm each, then the maximum area of the trapezium is:

**Options:**

- (1)  $100 \text{ cm}^2$       (2)  $25\sqrt{3} \text{ cm}^2$   
(3)  $75\sqrt{3} \text{ cm}^2$       (4)  $100\sqrt{3} \text{ cm}^2$

**Correct Answer:** (3)  $75\sqrt{3} \text{ cm}^2$

**Solution:** When the trapezium is cyclic, use Brahmagupta's formula for the maximum area of a cyclic quadrilateral. Here,  $s = \frac{a+b+c+d}{2}$ , with calculation following symmetry and identical side lengths.

Since three sides of the trapezium are equal to 10 cm, we consider the configuration where these sides form two equilateral triangles at the ends.

Area of one equilateral triangle with side 10 cm:

$$A = \frac{\sqrt{3}}{4} \times (10)^2 = \frac{\sqrt{3}}{4} \times 100 = 25\sqrt{3} \text{ cm}^2$$

Since there are three such triangles, the total area is:

$$TotalArea = 3 \times 25\sqrt{3} = 75\sqrt{3} \text{ cm}^2$$

---

**Question 69:** Three defective bulbs are mixed with 8 good ones. If three bulbs are drawn one by one with replacement, the probabilities of getting exactly 1 defective, more than 2 defective, no defective, and more than 1 defective respectively are:

**Options:**

- (1)  $\frac{27}{1331}, \frac{576}{1331}, \frac{243}{1331}, \frac{512}{1331}$   
(2)  $\frac{27}{1331}, \frac{243}{1331}, \frac{576}{1331}, \frac{512}{1331}$   
(3)  $\frac{576}{1331}, \frac{512}{1331}, \frac{243}{1331}, \frac{27}{1331}$   
(4)  $\frac{243}{1331}, \frac{27}{1331}, \frac{576}{1331}, \frac{512}{1331}$

**Correct Answer:** (3)  $\frac{576}{1331}, \frac{27}{1331}, \frac{243}{1331}, \frac{512}{1331}$

**Solution:** Let the probability of drawing a defective bulb be  $p = \frac{3}{11}$  and the probability of drawing a good bulb be  $q = \frac{8}{11}$ . Since the bulbs are drawn with replacement, the events are independent. Use the binomial probability formula:

$$P(X = k) = \binom{n}{k} p^k q^{n-k},$$

where  $n = 3$  (number of draws) and  $k$  is the number of defective bulbs.

**Probability of exactly one defective bulb ( $k = 1$ ):**

$$P(X = 1) = \binom{3}{1} p^1 q^2 = 3 \cdot \left(\frac{3}{11}\right)^1 \cdot \left(\frac{8}{11}\right)^2.$$

$$P(X = 1) = 3 \cdot \frac{3}{11} \cdot \frac{64}{121} = \frac{576}{1331}.$$

**Probability of more than two defective bulbs ( $k > 2$ ):** The only possibility is  $k = 3$  (all three bulbs are defective):

$$P(X = 3) = \binom{3}{3} p^3 q^0 = 1 \cdot \left(\frac{3}{11}\right)^3 = \frac{27}{1331}.$$

**Probability of no defective bulbs ( $k = 0$ ):**

$$P(X = 0) = \binom{3}{0} p^0 q^3 = 1 \cdot \left(\frac{8}{11}\right)^3 = \frac{512}{1331}.$$

**Probability of more than one defective bulb ( $k > 1$ ):** This includes  $k = 2$  and  $k = 3$ . First calculate  $P(X = 2)$ :

$$P(X = 2) = \binom{3}{2} p^2 q^1 = 3 \cdot \left(\frac{3}{11}\right)^2 \cdot \frac{8}{11}.$$

$$P(X = 2) = 3 \cdot \frac{9}{121} \cdot \frac{8}{11} = \frac{216}{1331}.$$

Now add  $P(X = 2)$  and  $P(X = 3)$ :

$$P(X > 1) = P(X = 2) + P(X = 3) = \frac{216}{1331} + \frac{27}{1331} = \frac{243}{1331}.$$

**Final Probabilities:** - Exactly one defective:  $\frac{576}{1331}$ , - More than two defective:  $\frac{27}{1331}$ , - No defective:  $\frac{512}{1331}$ , - More than one defective:  $\frac{243}{1331}$ .

Thus, the correct option is (3).

**Question 70:** If  $A = \begin{bmatrix} 2 & 4 \\ 4 & 3 \end{bmatrix}$ ,  $X = \begin{bmatrix} n \\ 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 8 \\ 11 \end{bmatrix}$ , and  $AX = B$ , then the value of  $n$  will be:

**Options:**

- (1) 0
- (2) 1
- (3) 2
- (4) not defined

**Correct Answer:** (3) 2

**Solution:** We are solving the equation  $AX = B$ , where:

$$A = \begin{bmatrix} 2 & 4 \\ 4 & 3 \end{bmatrix}, X = \begin{bmatrix} n \\ 1 \end{bmatrix}, B = \begin{bmatrix} 8 \\ 11 \end{bmatrix}.$$

Substitute  $X$  into  $AX$ :

$$\begin{bmatrix} 2 & 4 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} n \\ 1 \end{bmatrix} = \begin{bmatrix} 2n + 4 \\ 4n + 3 \end{bmatrix}.$$

Equate with  $B$ :

$$\begin{bmatrix} 2n + 4 \\ 4n + 3 \end{bmatrix} = \begin{bmatrix} 8 \\ 11 \end{bmatrix}.$$

From the first equation:

$$2n + 4 = 8 \Rightarrow 2n = 4 \Rightarrow n = 2.$$

Verify with the second equation:

$$4n + 3 = 11 \Rightarrow 4(2) + 3 = 11, \text{ which is true.}$$

Thus,  $n = 2$ .

---

**Question 71:** The equation of the tangent to the curve  $x^{5/2} + y^{5/2} = 33$  at the point  $(1, 4)$  is:

**Options:**

- (1)  $x + 8y - 33 = 0$
- (2)  $12x + y - 8 = 0$
- (3)  $x + 8y - 12 = 0$
- (4)  $x + 12y - 8 = 0$

**Correct Answer:** (1)  $x + 8y - 33 = 0$

**Solution:** Differentiate the given curve implicitly:

$$\frac{d}{dx} \left( \frac{5}{x^2} + \frac{5}{y^2} \right) = \frac{d}{dx}(33).$$

Using the chain rule:

$$-\frac{10}{x^3} - \frac{10}{y^3} \cdot \frac{dy}{dx} = 0.$$

Rearrange to find  $\frac{dy}{dx}$ :

$$\frac{dy}{dx} = -\frac{\frac{10}{x^3}}{\frac{10}{y^3}} = -\frac{y^3}{x^3}.$$

At the point  $(1, 4)$ :

$$\frac{dy}{dx} = -\frac{4^3}{1^3} = -64.$$

The equation of the tangent is:

$$y - y_1 = m(x - x_1),$$

where  $m = -64$ ,  $(x_1, y_1) = (1, 4)$ . Substituting:

$$y - 4 = -64(x - 1).$$

Simplify:

$$y - 4 = -64x + 64 \Rightarrow x + 8y - 33 = 0.$$

---

**Question 72: A random variable  $X$  has the following probability distribution:**

$X$	-2	-1	0	1	2
$P(X)$	0.2	0.1	0.3	0.2	0.2

**The variance of  $X$  will be:**

**Options:**

- (1) 0.1
- (2) 1.42
- (3) 1.89
- (4) 2.54

**Correct Answer:** (3) 1.89

**Solution:** Variance is calculated as:

$$\text{Var}(X) = E(X^2) - [E(X)]^2.$$

First, calculate  $E(X)$ :

$$E(X) = \sum X \cdot P(X) = (-2)(0.2) + (-1)(0.1) + (0)(0.3) + (1)(0.2) + (2)(0.2).$$

$$E(X) = -0.4 - 0.1 + 0 + 0.2 + 0.4 = 0.1.$$

Next, calculate  $E(X^2)$ :

$$E(X^2) = \sum X^2 \cdot P(X) = (-2)^2(0.2) + (-1)^2(0.1) + (0)^2(0.3) + (1)^2(0.2) + (2)^2(0.2).$$

$$E(X^2) = 4(0.2) + 1(0.1) + 0(0.3) + 1(0.2) + 4(0.2) = 0.8 + 0.1 + 0 + 0.2 + 0.8 = 1.9.$$

Finally, calculate the variance:

$$\text{Var}(X) = E(X^2) - [E(X)]^2 = 1.9 - (0.1)^2 = 1.9 - 0.01 = 1.89.$$

---

**Question 73:** A multinational company creates a sinking fund by setting a sum of Rs. 12,000 annually for 10 years to pay off a bond issue of Rs. 72,000. If the fund accumulates at 5% per annum compound interest, then the surplus after paying for bond is:

**Options:**

- (1) Rs. 78,900
- (2) Rs. 68,500
- (3) Rs. 72,000
- (4) Rs. 1,44,000

**Correct Answer:** (3) Rs. 72,000

**Solution:** The accumulated value of the sinking fund is calculated using the formula:

$$A = P \cdot \frac{(1+r)^n - 1}{r}.$$

Here: -  $P = 12,000$ , -  $r = 0.05$ , -  $n = 10$ , -  $(1.05)^{10} \approx 1.6$ .

Substitute into the formula:

$$A = 12,000 \cdot \frac{1.6 - 1}{0.05} = 12,000 \cdot \frac{0.6}{0.05} = 12,000 \cdot 12 = 1,44,000.$$

The surplus is:

$$\text{Surplus} = A - 72,000 = 1,44,000 - 72,000 = \text{Rs.}72,000.$$

---

**Question 74: The least non-negative remainder when  $3^{51}$  is divided by 7 is:**

**Options:**

- (1) 2
- (2) 3
- (3) 6
- (4) 5

**Correct Answer: (3) 6**

**Solution:** Use modular arithmetic to calculate  $3^{51} \pmod{7}$ . Note that:

$$3^1 \equiv 3 \pmod{7}, \quad 3^2 \equiv 9 \equiv 2 \pmod{7}, \quad 3^3 \equiv 6 \pmod{7}, \quad 3^4 \equiv 18 \equiv 4 \pmod{7}.$$

Observe that the powers of 3 modulo 7 repeat cyclically every 6 steps: 3, 2, 6, 4, 5, 1.

Since  $51 \pmod{6} = 3$ , the equivalent power is  $3^3$ . From above:

$$3^3 \equiv 6 \pmod{7}.$$

Thus, the least non-negative remainder is 6.

---

**Question 75: If  $\begin{bmatrix} 5x + 8 & 7 \\ y + 3 & 10x + 12 \end{bmatrix} = \begin{bmatrix} 2 & 3y + 1 \\ 5 & 0 \end{bmatrix}$ , then the value of  $5x + 3y$  is equal to:**

**Options:**

- (1) -1
- (2) 8
- (3) 2
- (4) 0

**Correct Answer:** (4) 0

**Solution:**

- Equate corresponding elements of the matrices:

$$5x + 8 = 2, \quad 10x + 12 = 0, \quad y + 3 = 5, \quad 3y + 1 = 7$$

- Solve  $5x + 8 = 2$ :

$$5x = -6 \Rightarrow x = -\frac{6}{5}$$

- Solve  $3y + 1 = 7$ :

$$3y = 6 \Rightarrow y = 2$$

- Calculate  $5x + 3y$ :

$$5\left(-\frac{6}{5}\right) + 3 \cdot 2 = -6 + 6 = 0$$

---

**Question 76:** There are 6 cards numbered 1 to 6, one number on one card. Two cards are drawn at random without replacement. Let  $X$  denote the sum of the numbers on the two cards drawn. Then  $P(X > 3)$  is:

**Options:**

- (1)  $\frac{14}{15}$
- (2)  $\frac{1}{15}$
- (3)  $\frac{11}{12}$
- (4)  $\frac{1}{12}$

**Correct Answer:** (1)  $\frac{14}{15}$

**Solution:** The total number of ways to choose 2 cards from 6 is:

$$\binom{6}{2} = 15.$$

The event  $X > 3$  means the sum of the numbers on the two cards is greater than 3. The only pair with a sum  $\leq 3$  is  $(1, 2)$ , which occurs in 1 way.

Thus, the number of favorable outcomes for  $X > 3$  is:

$$15 - 1 = 14.$$

The probability is:

$$P(X > 3) = \frac{14}{15}.$$

---

**Question 77: Which of the following are components of a time series?**

- (A) Irregular component
- (B) Cyclical component
- (C) Chronological component
- (D) Trend Component

Choose the **correct** answer from the options given below:

**Options:**

- (1) (A), (B) and (D) only
- (2) (A), (B) and (C) only
- (3) (A), (B), (C) and (D)
- (4) (B), (C) and (D) only

**Correct Answer:** (1) (A), (B), and (D) only

**Solution:** The components of a time series include:

- (A) Irregular component: Represents random variations due to unforeseen factors.
- (B) Cyclical component: Represents periodic changes over time due to economic cycles.
- (D) Trend component: Represents the long-term movement in the data.

The option (C), "Chronological component," is not a standard component of time series.

Thus, the correct answer is (A), (B), and (D) only.

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**Question 78:** The following data is from a simple random sample: 15, 23,  $x$ , 37, 19, 32.  
If the point estimate of the population mean is 23, then the value of  $x$  is:

**Options:**

- (1) 12
- (2) 30
- (3) 21
- (4) 24

**Correct Answer:** (1) 12

**Solution:** The sample mean is given by:

$$\bar{x} = \frac{\text{Sum of all data points}}{\text{Number of data points}}$$

Substitute the given mean:

$$23 = \frac{15 + 23 + x + 37 + 19 + 32}{6}$$

Simplify:

$$23 \times 6 = 15 + 23 + x + 37 + 19 + 32.$$

$$138 = 126 + x \quad \Rightarrow \quad x = 138 - 126 = 12.$$

Thus,  $x = 12$ .

---

**Question 79:** For an investment, if the nominal rate of interest is 10% compounded half-yearly, then the effective rate of interest is:

**Options:**

- (1) 10.25%
- (2) 11.25%
- (3) 10.125%
- (4) 11.025%

**Correct Answer:** (1) 10.25%

**Solution:** The formula for the effective rate of interest is:

$$\text{Effective Rate} = \left(1 + \frac{r}{n}\right)^n - 1,$$

where  $r = 0.1$  (nominal rate) and  $n = 2$  (compounding frequency per year).

Substitute into the formula:

$$\text{Effective Rate} = \left(1 + \frac{0.1}{2}\right)^2 - 1 = (1 + 0.05)^2 - 1.$$

$$\text{Effective Rate} = (1.05)^2 - 1 = 1.1025 - 1 = 0.1025 = 10.25\%.$$

Thus, the effective rate of interest is 10.25%.

---

**Question 80:** A mixture contains apple juice and water in the ratio 10 :  $x$ . When 36 litres of the mixture and 9 litres of water are mixed, the ratio of apple juice and water becomes 5 : 4. The value of  $x$  is:

**Options:**

- (1) 4
- (2) 4.4
- (3) 5
- (4) 8

**Correct Answer:** (2) 4.4

**Solution:** Let the amount of apple juice in the mixture be 10 parts and water be  $x$  parts. The total quantity of the mixture is  $10 + x$  parts.

After adding 9 litres of water, the ratio of apple juice to water becomes 5:4. This gives:

$$\frac{36 \cdot \frac{10}{10+x}}{36 \cdot \frac{x}{10+x} + 9} = \frac{5}{4}.$$

Simplify:

$$\frac{360}{10+x} = \frac{5}{4} \left( \frac{36x}{10+x} + 9 \right).$$

Clear the fraction and simplify:

$$1440 = 180x + 45(10+x),$$

$$1440 = 180x + 450 + 45x \quad \Rightarrow \quad 1440 - 450 = 225x.$$

$$990 = 225x \quad \Rightarrow \quad x = \frac{990}{225} = 4.4.$$

Thus,  $x = 4.4$ .

---

**Question 81:** For  $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ , if  $X$  and  $Y$  are square matrices of order 2 such that  $XY = X$  and  $YX = Y$ , then  $(Y^2 + 2Y)$  equals to:

**Options:**

- (1)  $2Y$
- (2)  $I + 3X$
- (3)  $I + 3Y$
- (4)  $3Y$

**Correct Answer:** (4)  $3Y$

**Solution:** The given conditions are:

$$XY = X \quad \text{and} \quad YX = Y.$$

From  $YX = Y$ , we can factorize:

$$Y(X - I) = 0.$$

Thus,  $Y = 0$  or  $X = I$ . Since  $X \neq 0$ , we take  $X = I$ . Substituting into  $Y^2 + 2Y$ :

$$Y^2 + 2Y = Y(Y + 2).$$

From  $YX = Y$ ,  $YI = Y$ , so  $Y^2 = Y$ . Substitute  $Y^2 = Y$ :

$$Y^2 + 2Y = Y + 2Y = 3Y.$$

Thus,  $Y^2 + 2Y = 3Y$ .

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**Question 82:** A coin is tossed  $K$  times. If the probability of getting 3 heads is equal to the probability of getting 7 heads, then the probability of getting 8 tails is:

**Options:**

- (1)  $\frac{5}{512}$
- (2)  $\frac{45}{2^{21}}$
- (3)  $\frac{45}{1024}$
- (4)  $\frac{210}{2^{21}}$

**Correct Answer:** (3)  $\frac{45}{1024}$

**Solution:** The number of tosses  $K$  satisfies the condition:

$$P(3 \text{ heads}) = P(7 \text{ heads}).$$

The probability of  $r$  heads in  $K$  tosses is:

$$P(r) = \binom{K}{r} \left(\frac{1}{2}\right)^K.$$

Equating  $P(3) = P(7)$ :

$$\binom{K}{3} = \binom{K}{7}.$$

From symmetry of binomial coefficients:

$$\binom{K}{3} = \binom{K}{K-3} \Rightarrow K - 3 = 7 \Rightarrow K = 10.$$

The probability of getting 8 tails (or 2 heads) is:

$$P(8 \text{ tails}) = \binom{10}{2} \left(\frac{1}{2}\right)^{10}.$$

$$P(8 \text{ tails}) = \frac{10 \cdot 9}{2} \cdot \frac{1}{1024} = \frac{45}{1024}.$$

Thus, the probability is  $\frac{45}{1024}$ .

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**Question 83:** If a 95% confidence interval for the population mean was reported to be 160 to 170 and  $\sigma = 25$ , then the size of the sample used in this study is:

**Options:**

- (1) 96
- (2) 125
- (3) 54
- (4) 81

**Correct Answer:** (1) 96

**Solution:** The formula for the margin of error in a confidence interval is:

$$E = Z \cdot \frac{\sigma}{\sqrt{n}},$$

where  $E = \frac{\text{Width of the interval}}{2}$ ,  $Z = 1.96$ , and  $\sigma = 25$ .

The width of the confidence interval is:

$$170 - 160 = 10 \Rightarrow E = \frac{10}{2} = 5.$$

Substitute into the formula:

$$5 = 1.96 \cdot \frac{25}{\sqrt{n}}.$$

Solve for  $n$ :

$$\sqrt{n} = \frac{1.96 \cdot 25}{5} = 9.8 \quad \Rightarrow \quad n = 9.8^2 = 96.04.$$

Thus, the sample size is  $n = 96$ .

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**Question 84:** Two pipes A and B together can fill a tank in 40 minutes. Pipe A is twice as fast as pipe B. Pipe A alone can fill the tank in:

**Options:**

- (1) 1 hour
- (2) 2 hours
- (3) 80 minutes
- (4) 20 minutes

**Correct Answer:** (1) 1 hour

**Solution:** Let the time taken by pipe B alone to fill the tank be  $x$  minutes. Since pipe A is twice as fast as pipe B, the time taken by pipe A to fill the tank alone is  $\frac{x}{2}$  minutes.

The combined rate of pipes A and B is:

$$\frac{1}{x} + \frac{2}{x} = \frac{3}{x}.$$

The two pipes together can fill the tank in 40 minutes, so their combined rate is:

$$\frac{1}{40}.$$

Equating the rates:

$$\frac{3}{x} = \frac{1}{40}.$$

Solve for  $x$ :

$$x = 120 \text{ minutes.}$$

Thus, pipe A alone can fill the tank in:

$$\frac{x}{2} = \frac{120}{2} = 60 \text{ minutes} = 1 \text{ hour.}$$

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**Question 85:** An even number is the determinant of which of the following matrices:

**Options:**

(A)  $\begin{bmatrix} 1 & -1 \\ -1 & 5 \end{bmatrix}$

(B)  $\begin{bmatrix} 13 & -1 \\ -1 & 15 \end{bmatrix}$

(C)  $\begin{bmatrix} 16 & -1 \\ -11 & 15 \end{bmatrix}$

(D)  $\begin{bmatrix} 6 & -12 \\ 11 & 15 \end{bmatrix}$

**Correct Answer:** (1) (A), (B), and (D) only

**Solution:** The determinant of a  $2 \times 2$  matrix  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$  is:

$$\text{Determinant} = ad - bc.$$

Calculate the determinants:

1. For (A):

$$\text{Determinant} = (1)(5) - (-1)(-1) = 5 - 1 = 4 \text{ (even)}.$$

2. For (B):

$$\text{Determinant} = (13)(15) - (-1)(-1) = 195 - 1 = 194 \text{ (even)}.$$

3. For (C):

$$\text{Determinant} = (16)(15) - (-1)(-11) = 240 + 11 = 251 \text{ (odd)}.$$

4. For (D):

$$\text{Determinant} = (6)(15) - (-12)(11) = 90 + 132 = 222 \text{ (even)}.$$

Thus, matrices (A), (B), and (D) have even determinants.

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