MATHEMATICS – FULL SYLLABUS

MOCK TEST PAPER - 2

CBSE BOARD CLASS – XI (2025-26)

SUBJECT: MATHEMATICS **MAX. MARKS : 80**

CLASS : XI **DURATION : 3 HRS**

General Instructions:

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's** and 02 Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)**-type questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)**-type questions of 3 marks each.
5. **Section D** has 4 **Long Answer (LA)**-type questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

SECTION – A

Questions 1 to 20 carry 1 mark each.

1. The value of y will be, so that the line through (3, y) and (2, 7) is parallel to the line through (−1, 4) and (0, 6).
(a) 7 (b) 8 (c) 9 (d) 10

2. Two finite sets have *m* and *n* elements. The number of subsets of the first set is 112 more than that of second set. The values of *m* and *n* are respectively:
(a) 4, 7 (b) 7, 4 (c) 4, 4 (d) 7, 7

3. Let *A* and *B* be two sets such that $n(A) = 20$, $n(B) = 10$, $n(A \cup B) = 15$. Then, $n(A \cap B)$ is equal to:
(a) 30 (b) 40 (c) 15 (d) none of these

4. The mean and variance of a series containing 5 terms are 8 and 24 respectively. The variance of their combined series will be
(a) 20 (b) 24 (c) 25 (d) 42

5. The value of $(1 + i)^4 - (1 - i)^4$ is:
(a) 8 (b) 4 (c) –8 (d) –4

6. If ${}^{15}P_r = 2730$, then ${}^{5}P_r$.
(a) 3 (b) 30 (c) 15 (d) 20

7. If $\lim_{x \to 2} \frac{x^n - 2^n}{x - 2} = 80$ then n is:
(a) 1 (b) 3 (c) 5 (d) 7

8. If R is a relation on the set A = {1, 2, 3, 4, 6, 7, 8, 9, 11, 12} given by x R y ⇔ y = 2 x, then R is equal to:
(a) {(2, 1), (4, 2), (8, 2), (9, 3)}
(b) {(2, 1), (4, 2), (6, 3)}
(c) (5, 1), (2, 4), (3, 6)}
(d) none of these

9. The value of $\frac{1 - \tan^2 15^0}{1 + \tan^2 15^0}$
(a) 1 (b) √3 (c) √3/2 (d) 2

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10. If cos x = −1/2 and 0 < x < 2π, then the solutions are:
(a) x = $\pi/3, 4\pi/3$ (b) x = $2\pi/3, 4\pi/3$ (c) x = $2\pi/3, 7\pi/3$ (d) x = $2\pi/3, 5\pi/3$

11. If $\alpha + \beta = \frac{\pi}{4}$, then the value of (1 + tan α) (1 + tan β) is
(a) 1 (b) 2 (c) –2 (d) none of these

12. If the third term of G.P. is 4, then the product of its first 5 terms is:
(a) 4^3 (b) 4^4 (c) 4^5 (d) none of these

13. The 5th term from the end of the sequence 16, 8, 4, 2 ... $\frac{1}{16}$ is:
(a) 1 (b) 2 (c) 3 (d) 4

14. If A(2, 0) is the vertex and the y-axis is the directrix of a parabola, then its focus is
(a) F(2, 0) (b) F(−2, 0) (c) F(4, 0) (d) F(−4, 0)

15. 6 boys and 6 girls sit in a row at random. The probability that all the girls sit together is
(a) 1/432 (b) 12/431 (c) 1/132 (d) none of these

16. A single letter is selected at random from the word 'PROBABILITY'. The probability that it is a vowel is
(a) 1/3 (b) 4/11 (c) 2/11 (d) 3/11

17. Find the number of terms in the expansion of the following : $(1 + 2x + x^2)^{20}$
(a) 41 (b) 42 (c) 43 (d) 44

18. The ratio in which the line joining (2, 4, 5) and (3, 5, –4) is divided by the YZ-plane, is
(a) 2 : 3 (b) 3 : 2 (c) –2 : 3 (d) 4 : –3

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

19. Assertion (A): If (x – 1, y + 2) = (2, 4), then x = 3 and y = 2.
Reason (R): Two ordered pairs (x, y) and (p, q) equal, if their corresponding elements are equal.

20. Assertion (A): The value of sin (–690°) cos (–300°) + cos (–750°) sin (–240°) = 1
Reason (R): The value of sin and cos is negative in the third and fourth quadrant respectively.

SECTION – B

Questions 21 to 26 carry 2 marks each.

21. If one end of the diameter of a circle $x^2 + y^2 – 4x – 6y + 11 = 0$ is (8, 4), show that coordinates of the other end are (– 4, 2).

OR

Find the coordinates of the focus, the equation of directrix, vertex and length of latus rectum for the parabola $y^2 = –12x$.

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22. Find the derivative of $(5x^3 + 3x - 1)(x - 1)$

23. If $X = \{5, 6, 7, 8\}$, $Y = \{7, 8, 9, 10\}$, $Z = \{3, 4, 5, 6\}$. Find: (a) $((X \cap Y) \cup Z)$ (b) $((X \cup Y) \cap Z)$

24. Find the value of tan 22° 30′.

25. Find r, if : ${}^{15}C_r : {}^{15}C_{r-1} = 11:5$

OR

How many words can be formed using all the letters of the word EQUATION so that (*i*) all the vowels are together, (*ii*) consonants occupy the odd places ?

SECTION – C

Questions 27 to 31 carry 3 marks each.

26. If $\sin x = \frac{3}{5}$, $\cos y = \frac{-12}{13}$ and x, y both lie in the second quadrant, find the values of $\cos (x - y)$

OR

Prove that $\frac{\sin A.\sin 2A + \sin 3A.\sin 6A}{\sin A.\cos 2A + \sin 3A.\cos 6A} = \tan 5A$

27. If $(x + iy)^3 = u + iv$, then show that, $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$.

28. Find the points of trisection of the segment joining the points A(1, 0, –6) and B(–5, 9, 6).

29. Find the domain and range of the real function $f(x) = \sqrt{9 - x^2}$

30. Simplify: $(x + \sqrt{x-1})^6 + (\sqrt{x} - \sqrt{x-1})^6$

OR

By using binomial theorem show that : $6^n - 5n - 1$ is divisible by 25, $n \in N$.

31. Solve for x: $|x + 1| + |x| > 3$

SECTION – D

Questions 32 to 35 carry 5 marks each.

32. In a survey it was found that 21 people liked product *A*, 26 liked product *B* and 29 liked product *C*. If 14 people liked products *A* and *B*, 12 people liked products *C* and *A*, 14 people liked products *B* and *C* and 8 liked all the three products. Find how many liked product *C* only.

33. The diameter of circles (in mm) drawn in a design are given below :

Diameter (in mm)	33 – 36	37 – 40	41 – 44	45 – 48	49 – 52
No. of circles	15	17	21	22	25

Calculate the standard deviation and mean diameter of the circles.

34. Three numbers are in AP their sum is 15. If 1, 3, 9 be added to them respectively they form a GP. Find the numbers.

OR

Find the value of n, so that $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ may be geometric mean between a and b.

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35. Find the derivative of $\frac{2}{x+1}-\frac{x^2}{3x-1}$

OR

Evaluate: $\lim\limits_{y\to 0}\frac{(x+y)\sec(x+y)-x\sec x}{y}$

SECTION – E(Case Study Based Questions)

Questions 36 to 38 carry 4 marks each.

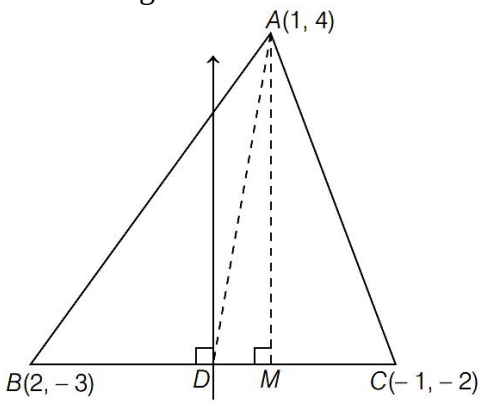
36. Case-Study 1:

Two students, Anil and Vijay, appeared in a highly competitive examination. Anil has been preparing part-time while managing a job, which has left him with limited preparation time. On the other hand, Vijay, though dedicated, has struggled with certain key concepts. Based on their preparation and past performance, the probability that Anil will qualify the examination is estimated to be 0.05, and the probability that Vijay will qualify is estimated at 0.10. Additionally, the probability that both students will qualify together, due to their independent preparation and individual strengths, is calculated as 0.02.

(a) Find the probability that at least one of them will qualify the exam.
(b) Find the probability that at least one of them will not qualify the exam.
(c) Find the probability that both Anil and Vijay will not qualify the exam.
(d) Find the probability that only one of them will qualify the exam.

37. Case-Study 2:

One day the mathematics teacher drew a triangle ΔABC while revising straight lines. He marked vertices A(1, 4), B (2, − 3) and C(− 1, − 2) as shown in the given below figure. AD is the median and AM is the altitude through A.



Based on the above information answer the following questions.
(i) Find the slope of BC. (1)
(ii) Find the equation of median through A. (1)
(iii) Find the equation of the altitude through A. (1)
(iv) Find the equation of right bisector of side BC. (1)

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38. Case-Study 3:

Raj works at a book store. While arranging some books on the book shelf, he observed that there are 5 History books, 3 Mathematics books and 4 Science books which are to be arranged on the shelf.

(i) In how many ways can he select either a History book or a Maths book? (1)

(ii) If he selects 2 History books, 1 Maths book and 1 Science book to arrange them, then find the number of ways in which selection can be made. (1)

(iii) Find the number of ways, if the books of same subject are put together. (1)

(iv) Find the number of arrangements, if he selects 3 History books, 2 Maths Books, 2 Science books. (1)

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SECTION – A

Questions 1 to 20 carry 1 mark each.

1. The value of y will be, so that the line through (3, y) and (2, 7) is parallel to the line through (−1, 4) and (0, 6).
(a) 7 (b) 8 (c) 9 (d) 10
Ans: Let A(3, y), B(2, 7), C(−1, 4) and D(0, 6) be the given points.
Then, m_1 = Slope of the line AB = $\frac{7-y}{2-3} = y-7$
and m_2 = Slope of the line CD = $\frac{6-4}{0-(-1)} = 2$
Since, AB and CD are parallel.
$\therefore m_1 = m_2 \Rightarrow y - 7 = 2 \Rightarrow y = 9$

2. Two finite sets have *m* and *n* elements. The number of subsets of the first set is 112 more than that of second set. The values of *m* and *n* are respectively:
(a) 4, 7 (b) 7, 4 (c) 4, 4 (d) 7, 7
Ans: (b) 7, 4
Given that the total number of subsets of the first set is 112 more than the total number of subsets of the second set.
$\Rightarrow 2^m - 2^n = 112 \Rightarrow 2^n (2^{m-n} - 1) = 2^4 \times 7$
$\therefore 2^n = 2^4$ and $(2^{m-n} - 1) = 7$
We can say that the value of $n = 4$
$\Rightarrow 2^{m-n} - 1 = 7 \Rightarrow 2^{m-n} = 8$
$\Rightarrow 2^{m-4} = 2^3$ [$\because n = 4$]
$\Rightarrow m - 4 = 3 \Rightarrow m = 7$
Therefore the value of $m = 7$ and $n = 4$.

3. Let *A* and *B* be two sets such that $n(A) = 20$, $n(B) = 10$, $n(A \cup B) = 15$. Then, $n(A \cap B)$ is equal to:
(a) 30 (b) 40 (c) 15 (d) none of these
Ans: (c) 15
Given, *A* and *B* are two sets such that $n(A) = 20$, $n(B) = 10$, $n(A \cup B) = 15$
We know that, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$\Rightarrow 15 = 20 + 10 - n(A \cap B)$
$\Rightarrow 15 = 30 - n(A \cap B)$
$\Rightarrow n(A \cap B) = 15$

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4. The mean and variance of a series containing 5 terms are 8 and 24 respectively. The variance of their combined series will be

(a) 20 (b) 24 (c) 25 (d) 42

Ans. (b) 24

5. The value of $(1 + i)^4 - (1 - i)^4$ is:

(a) 8 (b) 4 (c) –8 (d) –4

Ans. (c) –8

$(1 + i)^4 - (1 - i)^4$

$= ((1 + i)^2)^2 - ((1 - i)^2)^2$

$= (i^2 + 1 + 2i)^2 - (1 + i^2 - 2i)^2$

$= (-1 + 1 + 2i)^2 - (1 - 1 - 2i)^2$

$= (2i)^2 - (- 2i)^2$

$= 4i^2 + 4i^2$

$= 8i^2 = -8$

6. If ${}^{15}P_r = 2730$, then ${}^{5}P_r$.

(a) 3 (b) 30 (c) 15 (d) 20

Ans: (d) 20

$${}^{15}P_r = 2730 \Rightarrow {}^{15}P_r = 15 \times 182$$

$$\Rightarrow {}^{15}P_r = 15 \times 14 \times 13 = {}^{15}P_3 \Rightarrow r = 3$$

$$\Rightarrow {}^{5}P_3 = 5 \times 4 = 20$$

7. If $\lim_{x \to 2} \frac{x^n - 2^n}{x - 2} = 80$ then n is:

(a) 1 (b) 3 (c) 5 (d) 7

Ans. (c) 5

$n.2^{n-1} = 80$

$\Rightarrow n.2^{n-1} = 5 \times 16$

$\Rightarrow n.2^{n-1} = 5 \times 2^{5-1} \Rightarrow n = 5$

8. If R is a relation on the set A = {1, 2, 3, 4, 6, 7, 8, 9, 11, 12} given by x R y ⇔ y = 2 x, then R is equal to:

(a) {(2, 1), (4, 2), (8, 2), (9, 3)}

(b) {(2, 1), (4, 2), (6, 3)}

(c) (5, 1), (2, 4), (3, 6)}

(d) none of these

Ans. (b) {(2, 1), (4, 2), (6, 3)}

Given, R is a relation on the set

A = {1, 2, 3, 4, 6, 7, 8, 9, 11, 12} given by

x R y ⇔ y = 2x

Here, y = 2 x

if, x = 1, y = 2

x = 2, y = 4

x = 3, y = 6

So, R = {(2, 1), (4, 2), (6, 3)}

9. The value of $\frac{1 - \tan^2 15^0}{1 + \tan^2 15^0}$

(a) 1 (b) √3 (c) √3/2 (d) 2

Ans: (c) √3/2

Let θ = 15° ⇒ 2θ = 30°

Now, since we know that,

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$$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta} \Rightarrow \cos 30° = \frac{1-\tan^2 15°}{1+\tan^2 15°} \Rightarrow \frac{\sqrt{3}}{2} = \frac{1-\tan^2 15°}{1+\tan^2 15°}$$

10. If cos x = −1/2 and 0 < x < 2π, then the solutions are:
(a) x = π/3, 4π/3 (b) x = 2π/3, 4π/3 (c) x = 2π/3, 7π/3 (d) x = 2π/3, 5π/3
Ans: (b) x = 2π/3, 4π/3

11. If $\alpha + \beta = \frac{\pi}{4}$, then the value of (1 + tan α) (1 + tan β) is

(a) 1 (b) 2 (c) –2 (d) none of these
Ans: (b) 2

$$\alpha + \beta = \frac{\pi}{4} \quad \text{(given)}$$

$$\Rightarrow \quad \tan(\alpha+\beta) = \tan\frac{\pi}{4} \Rightarrow \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta} = 1$$

$$\Rightarrow \tan\alpha + \tan\beta + \tan\alpha\tan\beta = 1 \qquad ...(i)$$

Now (1 + tan α) (1 + tan β)

= 1 + tan α + tan β + tan α tan β
= 1 + 1 {using (*i*)}
= 2

12. If the third term of G.P. is 4, then the product of its first 5 terms is:
(a) 4^3 (b) 4^4 (c) 4^5 (d) none of these
Ans. *(c)* 4^5
Here, $a_3 = 4 \Rightarrow ar^2 = 4$
Product of first 5 terms = $a.\ ar.\ ar^2.\ ar^3.\ ar^4 = a^5 . r^{10} = (ar^2)^5 = (4)^5$

13. The 5th term from the end of the sequence 16, 8, 4, 2 ... $\frac{1}{16}$ is:

(a) 1 (b) 2 (c) 3 (d) 4
Ans. (a) 1

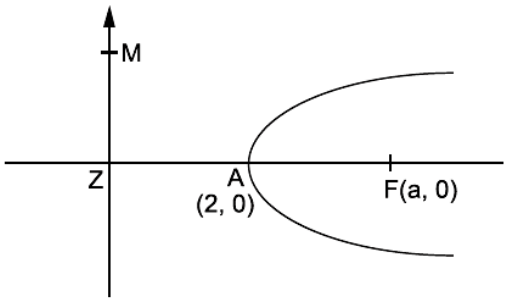
Given sequence is 16, 8, 4, 2, ... $\frac{1}{16}$

Here, $a = 16$ and $r = \frac{1}{2}$

We know that $a_n = ar^{n-1}$
So, n = 5th term ⇒ $a_5 = ar^4$

On putting $r = \frac{1}{2}$ we get $a_5 = 16\left(\frac{1}{2}\right)^4 = 16\times\frac{1}{16} = 1$

14. If A(2, 0) is the vertex and the y-axis is the directrix of a parabola, then its focus is
(a) F(2, 0) (b) F(−2, 0) (c) F(4, 0) (d) F(−4, 0)
Ans: (c) F(4, 0)
Let ZM be the directrix, where coordinates of Z are (0, 0).
Let the focus be F(a, 0).



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Then, vertex A is the midpoint of ZF.

$\therefore \frac{a+0}{2} = 2 \Rightarrow a = 4$

$\therefore$ focus is F(4, 0).

15. 6 boys and 6 girls sit in a row at random. The probability that all the girls sit together is

(a) 1/432 (b) 12/431 (c) 1/132 (d) none of these

Ans. (c) 1/132

16. A single letter is selected at random from the word 'PROBABILITY'. The probability that it is a vowel is

(a) 1/3 (b) 4/11 (c) 2/11 (d) 3/11

Ans. (b) 4/11

17. Find the number of terms in the expansion of the following : $(1 + 2x + x^2)^{20}$

(a) 41 (b) 42 (c) 43 (d) 44

Ans. (a) 41

$(1 + 2x + x^2)^{20} = ((1 + x)^2)^{20} = (1 + x)^{40}$

$\Rightarrow$ 41 terms in the expansion.

18. The ratio in which the line joining (2, 4, 5) and (3, 5, –4) is divided by the YZ-plane, is

(a) 2 : 3 (b) 3 : 2 (c) –2 : 3 (d) 4 : –3

Ans: (c) Let the point R divides the line joining the points P(2, 4, 5) and Q(3, 5, − 4) in the ratio m : n. Then, the coordinates of R are

$$\left(\frac{3m + 2n}{m + n}, \frac{5m + 4n}{m + n}, \frac{-4m + 5n}{m + n}\right)$$

For YZ-plane, x-coordinates will be zero.

$$\therefore \quad \frac{3m + 2n}{m + n} = 0 \Rightarrow \frac{m}{n} = \frac{-2}{3}$$

Hence, m : n = − 2 : 3

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

19. Assertion (A): If (x – 1, y + 2) = (2, 4), then x = 3 and y = 2.

Reason (R): Two ordered pairs (x, y) and (p, q) equal, if their corresponding elements are equal.

Ans: (a) Both A and R are true and R is the correct explanation of A.

20. Assertion (A): The value of sin (–690°) cos (–300°) + cos (–750°) sin (–240°) = 1

Reason (R): The value of sin and cos is negative in the third and fourth quadrant respectively.

Ans: (c) Assertion (A) is true but Reason (R) is false

SECTION – B

Questions 21 to 26 carry 2 marks each.

21. If one end of the diameter of a circle $x^2 + y^2 – 4x – 6y + 11 = 0$ is (8, 4), show that coordinates of the other end are (– 4, 2).

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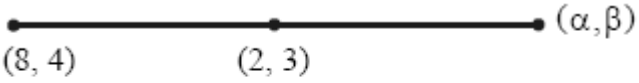
Ans: Centre is (2, 3).



$\frac{8+\alpha}{2}=2;\frac{4+\beta}{2}=3$

$\Rightarrow \alpha = -4, \beta = 2$, i.e. $(-4, 2)$

OR

Find the coordinates of the focus, the equation of directrix, vertex and length of latus rectum for the parabola $y^2 = -12x$.

Ans: Given parabola is $y^2 = -12x$

$\Rightarrow 4a = -12 \Rightarrow a = -3$

$\therefore$ Focus is $(-3, 0)$;

Directrix is $x + a = 0 \Rightarrow x - 3 = 0$

$\therefore$ Vertex is $(0, 0)$;

Length of latus rectum $= 4a = 12$

22. Find the derivative of $(5x^3 + 3x - 1)(x - 1)$

Ans.

Here $f(x) = (5x^3 + 3x - 1)(x - 1)$

$$\therefore f'(x) = \frac{d}{dx}[(5x^3 + 3x - 1)(x - 1)]$$

$$= (5x^3 + 3x - 1)\frac{d}{dx}(x - 1) + (x - 1)\frac{d}{dx}(5x^3 + 3x - 1)$$

$$= (5x^3 + 3x - 1) \times 1 + (x - 1)(15x^2 + 3)$$

$$= 5x^3 + 3x - 1 + 15x^3 + 3x - 15x^2 - 3 = 20x^3 - 15x^2 + 6x - 4$$

23. If $X = \{5, 6, 7, 8\}$, $Y = \{7, 8, 9, 10\}$, $Z = \{3, 4, 5, 6\}$. Find: (a) $((X \cap Y) \cup Z)$ (b) $((X \cup Y) \cap Z)$

Ans: Given, $X = \{5, 6, 7, 8\}$, $Y = \{7, 8, 9, 10\}$, $Z = \{3, 4, 5, 6\}$

(a) $X \cap Y = \{5, 6, 7, 8\} \cap \{7, 8, 9, 10\} = \{7, 8\}$

$\Rightarrow (X \cap Y) \cup Z = \{7, 8\} \cup \{3, 4, 5, 6\} = \{3, 4, 5, 6, 7, 8\}$

(b) $X \cup Y = \{5, 6, 7, 8\} \cup \{7, 8, 9, 10\} = \{5, 6, 7, 8, 9, 10\}$

$\Rightarrow (X \cup Y) \cap Z = \{5, 6, 7, 8, 9, 10\} \cap \{3, 4, 5, 6\} = \{5, 6\}$

24. Find the value of tan 22° 30′.

Ans:

$$\tan\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

Let $\theta = 45°$

$$\therefore \tan\frac{45°}{2} = \tan 22° 30'$$

$$= \sqrt{\frac{1-\cos 45°}{1+\cos 45°}} = \sqrt{\frac{1-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}} = \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$$

$$= \sqrt{\frac{(\sqrt{2}-1)^2}{2-1}} = \sqrt{2} - 1$$

25. Find r, if : ${}^{15}C_r : {}^{15}C_{r-1} = 11:5$

Ans:

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$$\frac{{}^{15}C_r}{{}^{15}C_{r-1}} = \frac{11}{5}$$

$$\frac{15!}{r!(15-r)!} \times \frac{(r-1)!(16-r)!}{15!} = \frac{11}{5}$$

$$\frac{16-r}{r} = \frac{11}{5}$$

$\Rightarrow 80 - 5r = 11r \Rightarrow r = 5$

OR

How many words can be formed using all the letters of the word EQUATION so that (*i*) all the vowels are together, (*ii*) consonants occupy the odd places ?

Ans. (*i*) $4! \times 5! = 4 \times 3 \times 2 \times 5 \times 4 \times 3 \times 2 \times 1 = 2880$

(*ii*) ${}^4P_3 \times {}^5P_5 = 4! \times 5! = 4 \times 3 \times 2 \times 1 \times 5 \times 4 \times 3 \times 2 \times 1 = 2880$

SECTION – C

Questions 27 to 31 carry 3 marks each.

26. If $\sin x = \frac{3}{5}$, $\cos y = \frac{-12}{13}$ and *x*, *y* both lie in the second quadrant, find the values of $\cos(x - y)$

Ans: Given, $\sin x = \frac{3}{5}$, $\cos y = \frac{-12}{13}$ and *x*, *y* both lie in the second quadrant.

We know that $\cos^2 x = 1 - \sin^2 x = 1 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} \Rightarrow \cos x = \pm \frac{4}{5}$

Since, *x* lies in 2nd quadrant, $\cos x$ is (–ve).

$\therefore \cos x = \frac{-4}{5}$

Also, $\sin^2 y = 1 - \cos^2 y = 1 - \left(\frac{-12}{13}\right)^2 = \frac{25}{169} \Rightarrow \sin y = \pm \frac{5}{13}$

Since, *y* lies in 2nd quadrant, $\sin y$ is (+ve)

$\therefore \sin y = \frac{5}{13}$

$\cos(x - y) = \cos x \cdot \cos y + \sin x \cdot \sin y$

$$= \left(\frac{-4}{5}\right)\left(\frac{-12}{13}\right) + \frac{3}{5} \times \frac{5}{13} = \frac{48}{65} + \frac{15}{65} = \frac{63}{65}$$

OR

Prove that $\frac{\sin A.\sin 2A + \sin 3A.\sin 6A}{\sin A.\cos 2A + \sin 3A.\cos 6A} = \tan 5A$

Ans: LHS = $\frac{\sin A.\sin 2A + \sin 3A.\sin 6A}{\sin A.\cos 2A + \sin 3A.\cos 6A}$

$$= \frac{2\sin 2A \sin A + 2\sin 6A \sin 3A}{2\cos 2A \sin A + 2\cos 6A \sin 3A}$$

$$= \frac{\cos A - \cos 3A + \cos 3A - \cos 9A}{\sin 3A - \sin A + \sin 9A - \sin 3A}$$

$$= \frac{\cos A - \cos 9A}{\sin 9A - \sin A} = \frac{2\sin 5A \sin 4A}{2\cos 5A \sin 4A}$$

$= \tan 5A = \text{RHS}$

27. If $(x + iy)^3 = u + iv$, then show that, $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$.

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Ans: Given, $(x + iy)^3 = u + iv$

$\Rightarrow x^3 + 3x^2\, iy + 3x\,(iy)^2 + (iy)^3 = u + iv$

$\Rightarrow (x^3 - 3xy^2) + i(3x^2 y - y^3) = u + iv$

Equating real and imaginary parts on both sides, we get,

$x^3 - 3xy^2 = u$ and $3x^2 y - y^3 = v$

$\Rightarrow \frac{u}{x} = x^2 - 3y^2$ and $\frac{v}{y} = 3x^2 - y^2$

$\therefore \frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$.

28. Find the points of trisection of the segment joining the points A(1, 0, –6) and B(–5, 9, 6).

Ans: Let P and Q be the points of trisection of the segment [AB], then P divides [AB] in the ratio 1 : 2 and Q divides [AB] in the ratio 2 : 1.

$\therefore \quad P \equiv \left(\frac{1\times(-5)+2\times1}{1+2}, \frac{1\times9+2\times0}{1+2}, \frac{1\times6+2\times(-6)}{1+2}\right)$,

i.e., P is $(-1, 3, -2)$

and $Q \equiv \left(\frac{2\times(-5)+1\times1}{2+1}, \frac{2\times9+1\times0}{2+1}, \frac{2\times6+1\times(-6)}{2+1}\right)$,

i.e., Q is $(-3, 6, 2)$.

Hence, the required points of trisection are P(–1, 3, –2) and Q(–3, 6, 2).

29. Find the domain and range of the real function $f(x) = \sqrt{9 - x^2}$

Ans: Given function is $f(x) = \sqrt{9 - x^2}$

For domain of 'f', $9 - x^2 \geq 0$

$\Rightarrow 9 \geq x^2 \Rightarrow x^2 \leq 9 \Rightarrow -3 \leq x \leq 3$

$\therefore$ Domain is $\{x \in R \mid -3 \leq x \leq 3\}$, i.e. $[-3, 3]$

For range : $f(x) = \sqrt{9 - x^2} \Rightarrow y = \sqrt{9 - x^2}$

$\sqrt{9 - x^2}$ is always +ve

$\Rightarrow y$ is always +ve.

$\Rightarrow y^2 = 9 - x^2 \Rightarrow x^2 = 9 - y^2$

$\Rightarrow x = \sqrt{9 - y^2}$

For x to exist $9 - y^2 \geq 0 \Rightarrow y^2 \leq 9 \Rightarrow -3 \leq y \leq 3$

As $y \geq 0$

$\therefore$ Range = [0, 3]

30. Simplify: $(x + \sqrt{x-1})^6 + (\sqrt{x} - \sqrt{x-1})^6$

Ans: Use $(x + y)^n + (x - y)^n$

$= 2[{}^nC_0\, x^n + {}^nC_2\, x^{n-2} y^2 + {}^nC_4\, x^{n-4} y^4 + ...]$

$= 2[{}^6C_0\, x^6 + {}^6C_2\, x^4(x-1) + {}^6C_4\, x^2\,(x-1)^2\ {}^6C_6(x-1)^3]$

$= 2(x^6 + 15x^5 - 29x^3 + 12x^2 + 3x - 1)$

OR

By using binomial theorem show that : $6^n - 5n - 1$ is divisible by 25, $n \in N$.

Ans: $6^n - 5n - 1 = (1 + 5)^n - 5n - 1$

$= [1 + 5n + {}^nC_2 \,.\, 5^2 + {}^nC_3 \,.\, 5^3 + ...5^n] - 5n - 1$

$= 25[{}^nC_2 + 5.{}^nC_3 + ...5^{n-2}]$

which is divisible by 25.

31. Solve for x: $|x + 1| + |x| > 3$

Ans. LHS $= |x + 1| + |x|$

As both the terms contain modulus by equating the expression within modulus to zero,

We get x = –1, 0 as critical points.

These critical points divide the line in three parts as $(-\infty, -1)$, $[-1, 0)$, $[0, \infty)$.

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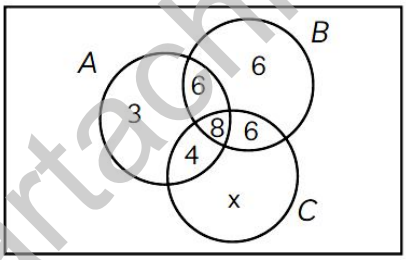
Case I: when $-\infty < x < -1$
$|x + 1| + |x| > 3$
$\Rightarrow -x - 1 - x > 3 \Rightarrow -2x > 4 \Rightarrow x < -2$
Case II: when $-1 \le x < 0$
$|x + 1| + |x| > 3$
$\Rightarrow x + 1 - x > 3 \Rightarrow 1 > 3$ (not possible)
Case III: when $0 \le x < \infty$
$|x + 1| + |x| > 3$
$\Rightarrow x + 1 + x > 3 \Rightarrow 2x > 2 \Rightarrow x > 1$
Combining the results of cases, we get $x \in (-\infty, -2) \cup (1, \infty)$

SECTION – D

Questions 32 to 35 carry 5 marks each.

32. In a survey it was found that 21 people liked product *A*, 26 liked product *B* and 29 liked product *C*. If 14 people liked products *A* and *B*, 12 people liked products *C* and *A*, 14 people liked products *B* and *C* and 8 liked all the three products. Find how many liked product *C* only.
Ans: Let *A*, *B* and *C* be the sets of people who like product *A*, product *B* and product *C* respectively.
Number of people who liked product $A = n(A) = 21$,
Number of people who liked product $B = n(B) = 26$,
Number of people who liked product $C = n(C) = 29$,
Number of people who liked product *A* and $B = n(A \cap B) = 14$
Number of people who liked product *C* and $A = n(C \cap A) = 12$
Number of people who liked product *B* and $C = n(B \cap C) = 14$
Number of people who liked all three products *A*, *B* and $C = n(A \cap B \cap C) = 8$
Let us draw a Venn diagram



Number of people who liked product *C* only
$= n(C) - n(C \cap A) - n(B \cap C) + n(A \cap B \cap C) = 29 - 12 - 14 + 8 = 11$
Hence, the number of people who like product *C* only is 11.

33. The diameter of circles (in mm) drawn in a design are given below :

Diameter (in mm)	33 – 36	37 – 40	41 – 44	45 – 48	49 – 52
No. of circles	15	17	21	22	25

Calculate the standard deviation and mean diameter of the circles.
Ans.

Class interval	**Mid Value (x_i)**	**Frequencies (f_i)**	$u_i = \frac{x_i - 42.5}{4}$	f_iu_i	u_i^2	$f_iu_i^2$
33 – 36	34.5	15	–2	–30	4	60
37 – 40	38.5	17	–1	–17	1	17
41 – 44	42.5	21	0	0	0	0
45 – 48	46.5	22	1	22	1	22
49 – 52	50.5	25	2	50	4	100
Total		$\sum f_i = 100$		$\sum f_iu_i =$ **25**		$\sum f_iu_i^2 =$ **199**

Here, we have taken
$u_i = \frac{x_i - a}{h} = \frac{x_i - 42.5}{4}$

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Where, assumed mean is 42.5 and $h = 4$.

Now, mean, $\bar{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$ = 42.5 + 4 × $\frac{25}{100}$ = 42.5 + 1 = 43.5

Variance, $\sigma_2 = h^2\left[\frac{\sum f_i u_i^2}{N} - \left(\frac{\sum f_i u_i}{N}\right)^2\right] = 4^2\left[\frac{199}{100} - \left(\frac{25}{100}\right)^2\right]$

$= 4^2\left[1.99 - \frac{1}{16}\right]$ = 30.84

Standard deviation = $\sqrt{Variance}$

⇒ Standard deviation = $\sqrt{30.84}$ = 5.55

34. Three numbers are in AP their sum is 15. If 1, 3, 9 be added to them respectively they form a GP. Find the numbers.

Ans. Let these numbers be $a - d$, a, $a + d$ (keeping in mind the rule of selection of three numbers in AP).

Given that $(a - d) + a + (a + d) = 15$

⇒ $3a = 15 \Rightarrow a = 5$

∴ The numbers are now $5 - d$, 5, $5 + d$.

Adding 1, 3, 9 in the numbers respectively. We get the new numbers i.e.,

$5 - d + 1$, $5 + 3$, $9 + 5 + d$ are in GP

⇒ $6 - d$, 8, $14 + d$ form a GP

⇒ $\frac{8}{6-d} = \frac{14+d}{8}$

⇒ $64 = (6 - d)(14 + d) \Rightarrow 64 = 84 - 14d + 6d - d^2$

⇒ $d^2 + 8d - 20 = 0 \Rightarrow d^2 + 10d - 2d - 20 = 0 \Rightarrow (d + 10)(d - 2) = 0$

⇒ $d = -10$ or $d = 2$

Therefore, the numbers are 15, 5, –5 or 3, 5, 7.

OR

Find the value of n, so that $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ may be geometric mean between a and b.

Ans. $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ is the G.M. between a and b.

$$\Rightarrow \frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \sqrt{ab}$$

$$\Rightarrow a^{n+1} + b^{n+1} = a^{\left(n+\frac{1}{2}\right)} b^{\frac{1}{2}} + a^{\frac{1}{2}} b^{\left(n+\frac{1}{2}\right)}$$

$$\Rightarrow a^{n+1} - a^{\left(n+\frac{1}{2}\right)} b^{\frac{1}{2}} = a^{\frac{1}{2}} b^{\left(n+\frac{1}{2}\right)} - b^{n+1}$$

$$\Rightarrow a^{\left(n+\frac{1}{2}\right)}\left[a^{\frac{1}{2}} - b^{\frac{1}{2}}\right] = b^{\left(n+\frac{1}{2}\right)}\left(a^{\frac{1}{2}} - b^{\frac{1}{2}}\right) \Rightarrow a^{\left(n+\frac{1}{2}\right)} = b^{\left(n+\frac{1}{2}\right)} \left[\because a^{\frac{1}{2}} - b^{\frac{1}{2}} \neq 0 \text{ as } a \neq b\right]$$

$$\Rightarrow \left(\frac{a}{b}\right)^{\left(n+\frac{1}{2}\right)} = 1 \Rightarrow \left(\frac{a}{b}\right)^{\left(n+\frac{1}{2}\right)} = \left(\frac{a}{b}\right)^0 \Rightarrow \left(n + \frac{1}{2}\right) = 0 \Rightarrow n = \frac{-1}{2}$$

35. Find the derivative of $\frac{2}{x+1} - \frac{x^2}{3x-1}$

Ans.

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Here $f(x) = \frac{2}{x+1} - \frac{x^2}{3x-1}$

$\therefore\ f'(x) = \frac{d}{dx}\left[\frac{2}{x+1} - \frac{x^2}{3x-1}\right] = \frac{d}{dx}\left(\frac{2}{x+1}\right) - \frac{d}{dx}\left(\frac{x^2}{3x-1}\right)$

$= \frac{(x+1)\frac{d}{dx}(2) - 2\frac{d}{dx}(x+1)}{(x+1)^2} - \frac{(3x-1)\frac{d}{dx}(x^2) - x^2\frac{d}{dx}(3x-1)}{(3x-1)^2}$ [Quotient rule]

$= \frac{(x+1)\times 0 - 2\times 1}{(x+1)^2} - \frac{(3x-1)(2x) - x^2\times 3}{(3x-1)^2} = \frac{-2}{(x+1)^2} - \frac{6x^2 - 2x - 3x^2}{(3x-1)^2} = \frac{-2}{(x+1)^2} - \frac{3x^2-2x}{(3x-1)^2}$

OR

Evaluate: $\lim\limits_{y\to 0} \frac{(x+y)\sec(x+y) - x\sec x}{y}$

Ans.

$\lim\limits_{y\to 0} \frac{(x+y)\sec(x+y) - x\sec x}{y}$

$= \lim\limits_{y\to 0} \frac{x\sec(x+y) - x\sec x + y\sec(x+y)}{y} = \lim\limits_{y\to 0}\left[\frac{x\sec(x+y) - x\sec x}{y} + \frac{y\sec(x+y)}{y}\right]$

$= \lim\limits_{y\to 0} \frac{x}{y}\left(\frac{1}{\cos(x+y)} - \frac{1}{\cos x}\right) + \lim\limits_{y\to 0}\sec(x+y) = \lim\limits_{y\to 0} \frac{x(\cos x - \cos(x+y))}{(y\cos x\cos(x+y))} + \sec x$

$= \lim\limits_{y\to 0} \frac{x\cdot 2\sin\frac{x+x+y}{2}\sin\frac{x+y-x}{2}}{y\cos x\cos(x+y)} + \sec x = \lim\limits_{y\to 0} \frac{x\sin\left(x+\frac{y}{2}\right)}{\cos x\cos(x+y)} \cdot \lim\limits_{\frac{y}{2}\to 0} \frac{\sin\frac{y}{2}}{\frac{y}{2}} + \sec x$

$= \frac{x\sin x}{\cos x\cos x}\cdot 1 + \sec x = x\tan x\sec x + \sec x = \sec x(x\tan x + 1)$

SECTION – E(Case Study Based Questions)

Questions 36 to 38 carry 4 marks each.

36. Case-Study 1:

Two students, Anil and Vijay, appeared in a highly competitive examination. Anil has been preparing part-time while managing a job, which has left him with limited preparation time. On the other hand, Vijay, though dedicated, has struggled with certain key concepts. Based on their preparation and past performance, the probability that Anil will qualify the examination is estimated to be 0.05, and the probability that Vijay will qualify is estimated at 0.10. Additionally, the probability that both students will qualify together, due to their independent preparation and individual strengths, is calculated as 0.02.

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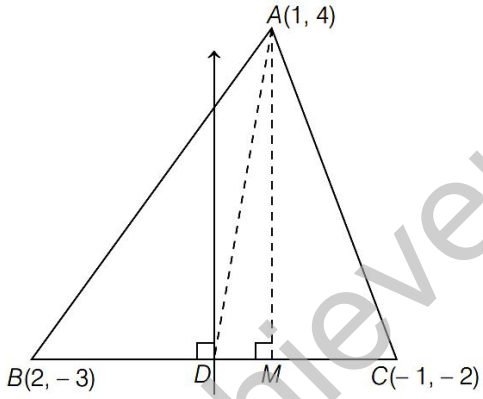
(a) Find the probability that at least one of them will qualify the exam.
(b) Find the probability that at least one of them will not qualify the exam.
(c) Find the probability that both Anil and Vijay will not qualify the exam.
(d) Find the probability that only one of them will qualify the exam.
Ans. (a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.05 + 0.10 - 0.02 = 0.13$
(b) Probability of at least one of them does not qualify
$= P(A' \cup B') = P((A \cap B)') = 1 - P(A \cap B) = 1 - 0.02 = 0.98$
(c) Probability that both Anil and Vijay will not qualify the exam
$= P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B) = 1 - 0.13 = 0.87$
(d) Probability that only one of them will qualify the exam
$= P((A - B) \cup (B - A)) = P(A - B) + P(B - A) = P(A \cup B) - P(A \cap B) = 0.13 - 0.02 = 0.11$

37. Case-Study 2:
One day the mathematics teacher drew a triangle ΔABC while revising straight lines. He marked vertices A(1, 4), B (2, − 3) and C(− 1, − 2) as shown in the given below figure. AD is the median and AM is the altitude through A.



Based on the above information answer the following questions.
(i) Find the slope of BC. (1)
(ii) Find the equation of median through A. (1)
(iii) Find the equation of the altitude through A. (1)
(iv) Find the equation of right bisector of side BC. (1)
Ans:
(i) Slope of BC = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-(-3)}{-1-2} = \frac{-2+3}{-3} = -\frac{1}{3}$
(ii) Since D is the mid-point of BC.
∴ Coordinates of D are $\left(\frac{2-1}{2}, \frac{-3-2}{2}\right) = \left(\frac{1}{2}, -\frac{5}{2}\right)$
∴ Slope of AD = $\frac{-\frac{5}{2}-4}{\frac{1}{2}-1} = \frac{-\frac{13}{2}}{-\frac{1}{2}} = 13$
∴ Equation of the median AD is $y - 4 = 13(x - 1)$
$\Rightarrow 13x - y - 9 = 0$
(iii) Since AM is the altitude through A.
∴ Slope of AM = − 1/slope of BC = 3
∴ Equation of the altitude through A is given by $y - 4 = 3(x - 1)$
$\Rightarrow y - 4 = 3x - 3 \Rightarrow 3x - y + 1 = 0$
(iv) Equation of the right bisector of BC is a line which passes through D and having slope is 3.
$\therefore y - \left(-\frac{5}{2}\right) = 3\left(x - \frac{1}{2}\right) \Rightarrow y + \frac{5}{2} = 3x - \frac{3}{2}$
$\Rightarrow 3x - y - 4 = 0$

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38. Case-Study 3:

Raj works at a book store. While arranging some books on the book shelf, he observed that there are 5 History books, 3 Mathematics books and 4 Science books which are to be arranged on the shelf.

(i) In how many ways can he select either a History book or a Maths book? (1)

(ii) If he selects 2 History books, 1 Maths book and 1 Science book to arrange them, then find the number of ways in which selection can be made. (1)

(iii) Find the number of ways, if the books of same subject are put together. (1)

(iv) Find the number of arrangements, if he selects 3 History books, 2 Maths Books, 2 Science books. (1)

Ans: (i) A History book can be selected in 5 ways and a Maths book can be selected in 3 ways.

Required number of ways = 5 + 3 = 8 [Using addition Principle]

(ii) Now, 2 History books can be chosen in ${}^5P_2$ ways, 1 Maths book can be chosen in ${}^3P_1$ ways and 1 Science book can be chosen in ${}^4P_1$ ways.

∴ Required number of ways = ${}^5P_2 \times {}^3P_1 \times {}^4P_1 = 240$.

(iii) Number of ways of arranging History books = 5!

Number of ways of arranging Maths books = 3!

Number of ways of arranging Science books = 4!

∴ Required number of way if the books of same subject are put together = 3!· 5!· 3! · 4!. = 103680

(iv) Number of ways of choosing 3 History books = ${}^5P_3$

Number of ways of choosing 2 Maths books = ${}^3P_2$

and number of ways of choosing 2 Science books = ${}^4P_2$

∴ Total number of ways = ${}^5P_3 \times {}^3P_2 \times {}^4P_2 = 4320$

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