FULL SYLLABUS

MOCK TEST PAPER - 2

CBSE BOARD CLASS – XI (2025-26)

SUBJECT: MATHEMATICS **MAX. MARKS : 80**

CLASS : XI **DURATION : 3 HRS**

General Instructions:

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's** and 02 Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)**-type questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)**-type questions of 3 marks each.
5. **Section D** has 4 **Long Answer (LA)**-type questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

SECTION – A

Questions 1 to 20 carry 1 mark each.

1. The equation of straight line passing through the point (–1, 2) and making an angle of 135° with the axis of x is

(a) $x - y = 1$ (b) $y + x = -1$ (c) $x + y = 1$ (d) $-x - y = 0$

2. The number of subsets of a set containing n elements is

(a) 2^n (b) $2^n - 1$ (c) $2^n - 1$ (d) n^n

3. The mean of 50 observations is 20 and their standard deviation is 2. The sum of all squares of all the observations is:

(a) 2020 (b) 2200 (c) 20200 (d) 20020

4. Let S = set of all points inside the square, T = the set of points inside the triangle and C = the set of points inside the circle. If the triangle and circle intersect each other and are contained in a square. Then

(a) $S \cap T \cap C = \phi$ (b) $S \cup T \cup C = C$ (c) $S \cup T \cup C = S$ (d) $S \cup T = S \cap C$

5. Conjugate of complex number $i^3 - 4$ is

(a) $i^3 + 4$ (b) $4 - i$ (c) $-4 + i$ (d) $-4 - i$

6. If ${}^nC_{12} = {}^nC_8$, then n is equal to

(a) 20 (b) 12 (c) 6 (d) 30

7. $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{\sec^2 x - 2}{\tan x - 1}$ is equal to

(a) 3 (b) 1 (c) 0 (d) 2

8. Given set A = {1, 2, 3, ..., 10}. Relation R is defined in set A as R = {(a, b) ∈ A × A : a = 2b}. Then range of relation R is

(a) {2, 4, 6, 8, 10} (b) {1, 3, 5, 7, 9}

(c) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5)} (d) {1, 2, 3, 4, 5}

9. The value of $\sin(45° + \theta) - \cos(45° - \theta)$ is

(a) $2\cos\theta$ (b) $2\sin\theta$ (c) 1 (d) 0

10. If $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$, then $\tan(2A + B)$ is equal to

(a) 1 (b) 2 (c) 3 (d) 4

11. The minimum value of $3 \cos x + 4 \sin x + 8$ is

(a) 5 (b) 9 (c) 7 (d) 3

12. In a *G.P,* the 3*rd* is 24 and the 6*th* term is 192, then the 10*th* term is:

(a) 1084 (b) 3290 (c) 3072 (d) 2340

13. Which term of the *G.P* 5, 10, 20, 40,.... is 5120?

(a) 11*th* (b) 10*th* (c) 6*th* (d) 5*th*

14. Suppose that each child born is equally likely to be a boy or a girl. Consider a family with exactly three children. Write how many elements in the sample space are there, which represents all possible genders of the three children.

(a) 7 (b) 3 (c) 8 (d) 2

15. Equation of a circle which passes through (3, 6) and touches the axes is

(a) $x^2 + y^2 + 6x + 6y + 3 = 0$ (b) $x^2 + y^2 - 6x - 6y - 9 = 0$

(c) $x^2 + y^2 - 6x - 6y + 9 = 0$ (d) none of these

16. If M and N are any two events, the probability that at least one of them occurs is

(a) $P(M) + P(N) - 2P(M \cap N)$ (b) $P(M) + P(N) - P(M \cap N)$

(c) $P(M) + P(N) + P(M \cap N)$ (d) $P(M) + P(N) + 2P(M \cap N)$

17. Given the integers $r > 1$, $n > 2$ and coefficients of $(3r)$th and $(r + 2)$th terms in the binomial expansion of $(1 + x)^{2n}$ are equal then

(a) $n = 2r$ (b) $n = 3r$ (c) $n = 2r + 1$ (d) none of these

18. If L, M, N be the feet of the perpendicular segments drawn from a point P(3, 4, 6) on the XY, XZ, YZ planes respectively. What are the coordinates of L, M and N?

(a) L(3, 0, 0), M(0, 0, 3), N(0, 6, 0) (b) L(3, 4, 0), M(3, 0, 6), N(0, 4, 6)

(c) L(0, 0, 0), M(3, 0, 6), N(0, 4, 0) (d) L(3, 4, 6), M(3, 4, 6), N(3, 4, 6)

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

19. **Assertion (A):** For the relation {(1, 2), (2, 3), (1, 5), (3, 4)} domain is {1, 2, 3}.

Reason (R): For a given relation R from set A to set B, given by $R = \{(a, b) \in A \times B : a$ is related to $b\}$ the set of elements $a \in A$ for $(a, b) \in R$ is known as its domain.

20. **Assertion (A):** $\tan(3\pi - \theta) = \tan \theta$

Reason (R): $\tan(\pi - \theta) = -\tan \theta$ and $\tan(2\pi + A) = \tan A$.

SECTION – B

Questions 21 to 26 carry 2 marks each.

21. Find the focus, vertex and directrix of the following parabola : $x^2 = -16y$

OR

Find the eccentricity of the ellipse if its latus rectum is equal to one half of its minor axis.

22. If $y = \dfrac{\cos x}{1+\sin x}$, then find $\dfrac{dy}{dx}$.

23. If U = $\{x : x \leq 10, x \in N\}$, A = $\{x : x \in N, x \text{ is prime}\}$, B = $\{x : x \in N, x \text{ is even}\}$, write $A \cap B'$ in roster form.

24. Prove the following : sin 10° + sin 20° + sin 40° + sin 50° = sin 70° + sin 80°.

25. Find n, if : ${}^{2n-1}P_n : {}^{2n+1}P_{n-1} = 22:7$

OR

Find the number of different signals that can be generated by arranging at least 2 flags in order (one below the other) on a vertical staff, if five different flags are available.

SECTION – C

Questions 27 to 31 carry 3 marks each.

26. Prove the following identity: tan 13A – tan 7A – tan 6A = tan 13A tan 7A tan 6A

OR

Prove the following identity: $\dfrac{\sin A+\sin 3A+\sin 5A+\sin 7A}{\cos A+\cos 3A+\cos 5A+\cos 7A} = \tan 4A$

27. If x + iy = $\sqrt{\dfrac{a+ib}{c+id}}$, prove that $(x^2 + y^2)^2 = \dfrac{a^2+b^2}{c^2+d^2}$.

28. Show that the points A(1, 2, 3), B(–1, –2, – 1), C(2, 3, 2) and D(4, 7, 6) are the vertices of a parallelogram ABCD, but it is not a rectangle.

29. Find the domain and the range of the function : $f(x) = \sqrt{x^2-4}$

30. Find $(a + b)^4 - (a - b)^4$. Hence, evaluate $(\sqrt{3}+\sqrt{2})^4 - (\sqrt{3}-\sqrt{2})^4$.

OR

Expand following using binomial expansion $(1 - x + x^2)^4$

31. Solve : $5(2x - 7) - 3(2x + 3) \leq 0$ and $2x + 19 \leq 6x + 47$ and represent the solution on number line.

SECTION – D

Questions 32 to 35 carry 5 marks each.

32. In a class, 18 students took Physics, 23 students took Chemistry and 24 students took Mathematics. Of these 13 took both Chemistry and Mathematics, 12 took both Physics and Chemistry and 11 took both Physics and Mathematics. If 6 students offered all the three subjects, find :

(*i*) Total number of students in the class.
(*ii*) How many took Mathematics but not Chemistry ?
(*iii*) How many took exactly one of the 3 subjects ?

33. Find mean, variance and standard deviation using short cut method.

Height in (cm)	70–75	75–85	80–85	85–90	90–95	96–100	100–105	105–110	110–115
Number of children	3	4	7	7	15	9	6	6	3

34. If p^{th}, q^{th}, r^{th} and sth terms of an A.P. are in G.P, then show that $(p - q)$, $(q - r)$, $(r - s)$ are also in G.P.

OR

Let S be the sum, P the product and R the reciprocals of n terms in a G.P. Prove that $P^2R^n = S^n$.

35. Differentiate $x\sin x$ from the first principle.

OR

Evaluate: $\lim_{x \to 0} \frac{\tan x + 4\tan 2x - 3\tan 3x}{x^2 \tan x}$

SECTION – E(Case Study Based Questions)

Questions 36 to 38 carry 4 marks each.

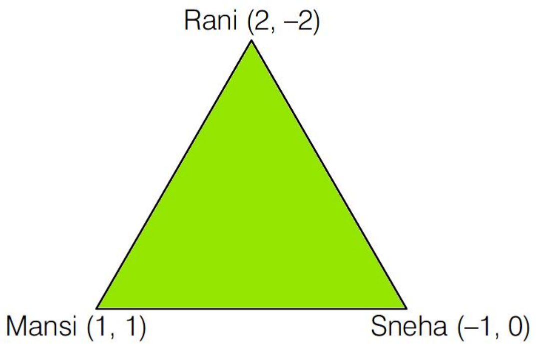
36. Case-Study 1:

One of the four persons John, Rita, Aslam or Gurpreet will be promoted next month. Consequently, the sample space consists of four elementary outcomes S = {John promoted, Rita promoted, Aslam promoted, Gurpreet promoted}. You are told that the chances of John's promotion is same as that of Gurpreet, Rita's chances of promotion are twice as likely as Johns. Aslam's chances are four times that of John.

(a) What is the probability that John got promotion?
(b) What is the probability that Rita got promotion?
(c) What is the probability that Aslam got promotion?
(d) What is the probability that Gurpreet got promotion?

37. Case-Study 2:

One triangular shaped pond is there in a park. Three friends Rani, Mansi, Sneha are sitting at the corners of the triangular park. They are studying in Class XI in an International.
Rani marked her position as (2, –2), Mansi marked as (1, 1) and Sneha marked her position as (–1, 0) as shown in figure given below.



Based on the above information answer the following questions.
(i) Find the equation of lines formed by Rani and Mansi. (1)

(ii) Find the Slope of equation of line formed by Rani and Sneha. (1)
(iii) Find the equation of median of lines through Rani. (1)
(iv) Find the equation of altitude through Mansi. (1)

38. Case-Study 3:

Seema wants a mobile number having 10 digits. It is not just a group of numbers strung out at random. All mobile numbers have 3 things in common. a 2-digit Access code (AC), a 3-digit provider code (PC), and a 5 digit subscriber code (SC). AC code and PC code are fixed, then

(i) How many mobile numbers are possible if no start with 98073 and no other digit can repeat? (1)
(ii) How many AC code are possible if both digit in AC code are different and must be greater than 6? (1)
(iii)How many mobile numbers are possible if AC and PC code are fixed and digits can repeat? (1)
(iv) How many mobile numbers are possible with AC code 98 and PC code 123 and digit used in AC and PC code will not be used in SC code? (1)

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SECTION – A
Questions 1 to 20 carry 1 mark each.

1. The equation of straight line passing through the point (–1, 2) and making an angle of 135° with the axis of x is
(a) x – y = 1 (b) y + x = –1 (c) x + y = 1 (d) –x – y = 0
Ans. (c), slope of line m = tan 135° = tan(180° – 45°) = –tan 45° = –1
equation of line passing through (–1, 2) is y – 2 = –1(x + 1) ⇒ y + x = 1

2. The number of subsets of a set containing n elements is
(a) 2^n (b) $2^n – 1$ (c) $2^n – 1$ (d) n^n
Ans. (a), if a set contains n elements then there are 2^n subsets of the set.

3. The mean of 50 observations is 20 and their standard deviation is 2. The sum of all squares of all the observations is:
(a) 2020 (b) 2200 (c) 20200 (d) 20020
Ans. (c) 20200

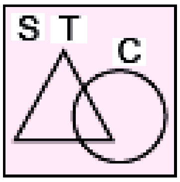
$$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - (\bar{x})^2}$$

$$2 = \sqrt{\frac{\Sigma x_i^2}{50} - (20)^2} \Rightarrow 4 = \frac{\Sigma x_i^2}{50} - 400$$

$$404 = \frac{\Sigma x_i^2}{n}$$

$$404 \times 50 = \Sigma x_i^2 \Rightarrow \Sigma x_i^2 = 20200$$

4. Let S = set of all points inside the square, T = the set of points inside the triangle and C = the set of points inside the circle. If the triangle and circle intersect each other and are contained in a square. Then
(a) S ∩ T ∩ C = ϕ (b) S ∪ T ∪ C = C (c) S ∪ T ∪ C = S (d) S ∪ T = S ∩ C
Ans. (c) S ∪ T ∪ C = S



5. Conjugate of complex number $i^3 – 4$ is

(a) $i^3 + 4$ (b) $4 - i$ (c) $-4 + i$ (d) $-4 - i$

Ans. (c) $-4 + i$

as $i^3 - 4 = -i - 4 = -4 - i$

$\overline{-4 - i} = -4 + i$

6. If ${}^nC_{12} = {}^nC_8$, then n is equal to

(a) 20 (b) 12 (c) 6 (d) 30

Ans. (a), as ${}^nC_{12} = {}^nC_8 \Rightarrow {}^nC_{n-12} = {}^nC_8$

$\Rightarrow n - 12 = 8 \Rightarrow n = 8 + 12 = 20$

7. $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{\sec^2 x - 2}{\tan x - 1}$ is equal to

(a) 3 (b) 1 (c) 0 (d) 2

Ans.

(*d*), $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{\sec^2 x - 2}{\tan x - 1} = \lim\limits_{x \to \frac{\pi}{4}} \dfrac{1 + \tan^2 x - 2}{\tan x - 1}$

$= \lim\limits_{x \to \frac{\pi}{4}} \dfrac{(\tan x + 1)(\tan x - 1)}{(\tan x - 1)}$

$= \lim\limits_{x \to \frac{\pi}{4}} (\tan x + 1) = \lim\limits_{x \to \frac{\pi}{4}} \tan x + 1 = 1 + 1 = 2$

8. Given set A = {1, 2, 3, ..., 10}. Relation R is defined in set A as R = {(a, b) ∈ A × A : a = 2b}. Then range of relation R is

(a) {2, 4, 6, 8, 10} (b) {1, 3, 5, 7, 9}

(c) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5)} (d) {1, 2, 3, 4, 5}

Ans. (d), as R = {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5)}

9. The value of sin (45° + θ) – cos (45° – θ) is

(a) 2 cos θ (b) 2 sin θ (c) 1 (d) 0

Ans. (d), sin (45° + θ) – cos (45° – θ)

= sin (45° + θ) – [sin{90° – (45° – θ)}]

= sin (45° + θ) – sin(45° + θ) = 0

10. If tan A = $\dfrac{1}{2}$ and tan B = $\dfrac{1}{3}$, then tan(2A + B) is equal to

(a) 1 (b) 2 (c) 3 (d) 4

Ans.

(*c*), as $\tan(2A + B) = \dfrac{\tan 2A + \tan B}{1 - \tan 2A \cdot \tan B}$

$$= \frac{\dfrac{2\tan A}{1 - \tan^2 A} + \tan B}{1 - \left(\dfrac{2\tan A}{1 - \tan^2 A}\right)\tan B} = \frac{\dfrac{2 \times \frac{1}{2}}{1 - \frac{1}{4}} + \dfrac{1}{3}}{1 - \left(\dfrac{2 \times \frac{1}{2}}{1 - \frac{1}{4}}\right) \times \dfrac{1}{3}}$$

$$= \frac{\dfrac{4}{3} + \dfrac{1}{3}}{1 - \dfrac{4}{3} \times \dfrac{1}{3}} = \frac{5/3}{5/9} = 3$$

11. The minimum value of 3 cos x + 4 sin x + 8 is

(a) 5 (b) 9 (c) 7 (d) 3

Ans. (d), let y = 3 cos x + 4 sin x + 8

$\Rightarrow$ y = 5 $\left[\frac{3}{5}\cos x+\frac{4}{5}\sin x\right]$ + 8

= 5 cos(x – α) + 8, when cos α = $\frac{3}{5}$

Also, $-1 \le \cos(x-\alpha) \le 1$

$\Rightarrow -5 \le 5\cos(x-\alpha) \le 5 \Rightarrow 3 \le 5\cos(x-\alpha) + 8 \le 13 \Rightarrow 3 \le y \le 13$

So, minimum value is 3.

12. In a *G.P,* the 3*rd* is 24 and the 6*th* term is 192, then the 10*th* term is:

(a) 1084 (b) 3290 (c) 3072 (d) 2340

Ans. *(c)* 3072

Here, $a_3 = 24$ and $a_6 = 192$

So, $ar^2 = 24$ and $ar^5 = 192$

On dividing, we get $r^3 = 8 \Rightarrow r = 2, a = 6$

$a_{10} = ar^9 = (6.\ 2^9) = 3072$

13. Which term of the *G.P* 5, 10, 20, 40,.... is 5120?

(a) 11*th* (b) 10*th* (c) 6*th* (d) 5*th*

Ans. (a) 11th

Given sequence 5, 10, 20, 40, ... is a G.P.

Here $a = 5$, $r = 2$ and $an = 5120$

As we know that the general term of a GP is given by $a_n = ar^{n-1}$

$\Rightarrow 5120 = 5.\ 2^{n-1} \Rightarrow 2^{n-1} = 5120/5$

$\Rightarrow 2^{n-1} = 1024 \Rightarrow 2^{n-1} = 2^{10} \Rightarrow n - 1 = 10 \Rightarrow n = 11$

Hence, 11th term is 5120.

14. Suppose that each child born is equally likely to be a boy or a girl. Consider a family with exactly three children. Write how many elements in the sample space are there, which represents all possible genders of the three children.

(a) 7 (b) 3 (c) 8 (d) 2

Ans. (c), there are 8 elements in the sample space.

{BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG}

15. Equation of a circle which passes through (3, 6) and touches the axes is

(a) $x^2 + y^2 + 6x + 6y + 3 = 0$ (b) $x^2 + y^2 - 6x - 6y - 9 = 0$

(c) $x^2 + y^2 - 6x - 6y + 9 = 0$ (d) none of these

Ans. (c), let the circle be situated in Ist quadrant.

Since the circle passes through

(3, 6) and touches the axes.

Let the centre of the circle be (a, a)

So, equation of circle

$(x - a)^2 + (y - a)^2 = r^2$

$(3 - a)^2 + (6 - a)^2 = a^2$

$\Rightarrow 9 + a^2 - 6a + 36 + a^2 - 12a = a^2$

$\Rightarrow a^2 - 18a + 45 = 0 \Rightarrow a^2 - 15a - 3a + 45 = 0$

$\Rightarrow (a - 15)(a - 3) = 0 \Rightarrow a = 3$ or $a = 15$ (rejected)

So equation of circle $(x - 3)^2 + (y - 3)^2 = 3^2$

$\Rightarrow x^2 + y^2 - 6x - 6y + 9 = 0$

16. If M and N are any two events, the probability that at least one of them occurs is

(a) $P(M) + P(N) - 2P(M \cap N)$ (b) $P(M) + P(N) - P(M \cap N)$

(c) $P(M) + P(N) + P(M \cap N)$ (d) $P(M) + P(N) + 2P(M \cap N)$

Ans. (b), M and N are two events,
Probability that at least one of them occurs is
$P(M \cup N) = P(M) + P(N) - P(M \cap N)$

17. Given the integers r > 1, n > 2 and coefficients of (3r)th and (r + 2)th terms in the binomial expansion of $(1 + x)^{2n}$ are equal then

(a) n = 2r (b) n = 3r (c) n = 2r + 1 (d) none of these

Ans. (a), we know (r + 1)th term in the expansion of $(a + b)^n$ is given by
$T_{r+1} = {}^nC_r a^{n-r} b^r$
Coefficient of (r + 1)th term = ${}^nC_r$
According to the given condition, $T_{3r} = T_{r+2}$
i.e. ${}^{2n}C_{(3r-1)} = {}^{2n}C_{(r+1)}$
$\Rightarrow 3r - 1 = r + 1$ or $3r - 1 + r + 1 = 2n$
$\Rightarrow r = 1$ or $4r = 2n$
$\Rightarrow r = 1$ (rejected) or $n = 2n$

18. If L, M, N be the feet of the perpendicular segments drawn from a point P(3, 4, 6) on the XY, XZ, YZ planes respectively. What are the coordinates of L, M and N?

(a) L(3, 0, 0), M(0, 0, 3), N(0, 6, 0) (b) L(3, 4, 0), M(3, 0, 6), N(0, 4, 6)
(c) L(0, 0, 0), M(3, 0, 6), N(0, 4, 0) (d) L(3, 4, 6), M(3, 4, 6), N(3, 4, 6)

Ans. (b), as z-coordinates is zero in XY plane, y-coordinate is zero is XZ-plane, x coordinate is zero in YZ-plane.
L(3, 4, 0), M(3, 0, 6), N(0, 4, 6)

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

19. Assertion (A): For the relation {(1, 2), (2, 3), (1, 5), (3, 4)} domain is {1, 2, 3}.
Reason (R): For a given relation R from set A to set B, given by $R = \{(a, b) \in A \times B : a$ is related to $b\}$ the set of elements $a \in A$ for $(a, b) \in R$ is known as its domain.
Ans. (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

20. Assertion (A): $\tan(3\pi - \theta) = \tan \theta$
Reason (R): $\tan(\pi - \theta) = -\tan \theta$ and $\tan(2\pi + A) = \tan A$.
Ans. (d) Assertion (A) is false but Reason (R) is true.

SECTION – B

Questions 21 to 26 carry 2 marks each.

21. Find the focus, vertex and directrix of the following parabola : $x^2 = -16y$
Ans. $x^2 = -16y$ is of the form $x^2 = 4ay$ with $a = -4$.
Focus is (0, – 4), vertex is (0, 0) and directrix is $y = 4$.

OR

Find the eccentricity of the ellipse if its latus rectum is equal to one half of its minor axis.
Ans.

Latus rectum = $\frac{1}{2}$ (minor axis)

$$\Rightarrow \frac{2b^2}{a} = \frac{1}{2}(2b) \Rightarrow b = \frac{a}{2}$$

Now, apply $b^2 = a^2(1 - e^2)$, we get $e = \frac{\sqrt{3}}{2}$

22. If $y = \frac{\cos x}{1+\sin x}$, then find $\frac{dy}{dx}$.

Ans.

$$y = \frac{\cos x}{1+\sin x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(1+\sin x)(-\sin x) - \cos x(\cos x)}{(1+\sin x)^2}$$

$$= \frac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2}$$

$$= \frac{-\sin x - 1}{(1+\sin x)^2} = \frac{-(1+\sin x)}{(1+\sin x)^2}$$

$$= \frac{-1}{1+\sin x}.$$

23. If U = $\{x : x \le 10, x \in \text{N}\}$, A = $\{x : x \in \text{N}, x \text{ is prime}\}$, B = $\{x : x \in \text{N}, x \text{ is even}\}$, write A ∩ B′ in roster form.

Ans. U = {1, 2, 3, ..., 10} A = {2, 3, 5, 7}

B = {2, 4, 6, 8, 10} B′ = {1, 3, 5, 7, 9}

⇒ A ∩ B′ = {2, 3, 5, 7 } ∩ {1, 3, 5, 7, 9} = {3, 5, 7}.

24. Prove the following : sin 10° + sin 20° + sin 40° + sin 50° = sin 70° + sin 80°.

Ans. (sin 50° + sin 10°) + (sin 40° + sin 20°)

= 2 sin 30° cos 20° + 2 sin 30° cos 10°.

= cos 20° + cos 10°

= cos (90° – 70°) + cos (90° – 80°)

= sin 70° + sin 80°.

25. Find n, if : ${}^{2n-1}P_n : {}^{2n+1}P_{n-1} = 22:7$

Ans.

$$\frac{{}^{2n-1}P_n}{{}^{2n+1}P_{n-1}} \Rightarrow \frac{(2n-1)!}{(n-1)!} \times \frac{(n+2)!}{(2n+1)!} = \frac{22}{7}$$

$$\Rightarrow \frac{(n+2)(n+1)}{2(2n+1)} = \frac{22}{7}$$

⇒ 7 (n + 2) (n + 1) = 44 (2n + 1)

⇒ (n – 10) (7n + 3) = 0 ⇒ n = 10, –3/7

⇒ n = 10 as n ∈ N.

OR

Find the number of different signals that can be generated by arranging at least 2 flags in order (one below the other) on a vertical staff, if five different flags are available.

Ans. A signal can consist of either 2 flags, 3 flags, 4 flags or 5 flags. Now, let us count the possible number of signals consisting of 2 flags, 3 flags, 4 flags and 5 flags separately and then add the respective numbers.

There will be as many 2 flag signals as there are ways of filling in 2 vacant places in succession by the 5 flags available. By Multiplication rule, the number of ways is 5 × 4 = 20.

Similarly, there will be as many 3 flag signals as there are ways of filling in 3 vacant places in succession by the 5 flags.
The number of ways is 5 × 4 × 3 = 60
Continuing the same way, we find that the number of 4 flag signals = 5 × 4 × 3 × 2 = 120 and the number of 5 flag signals = 5 × 4 × 3 × 2 × 1 = 120
Therefore, the required number of signals = 20 + 60 + 120 + 120 = 320

SECTION – C

Questions 27 to 31 carry 3 marks each.

26. Prove the following identity: tan 13A – tan 7A – tan 6A = tan 13A tan 7A tan 6A

Ans. tan 13 A = tan (7A + 6A) = $\frac{\tan 7A + \tan 6A}{1 - \tan 7A \tan 6A}$

⇒ tan 13A (1 – tan 7A tan 6A) = tan 7A + tan 6A.
⇒ tan 13A – tan 7A – tan 6A = tan 13A tan 7A tan 6A

OR

Prove the following identity: $\frac{\sin A + \sin 3A + \sin 5A + \sin 7A}{\cos A + \cos 3A + \cos 5A + \cos 7A} = \tan 4A$

Ans.

$$\frac{(\sin 7A + \sin A) + (\sin 5A + \sin 3A)}{(\cos 7A + \cos A) + (\cos 5A + \cos 3A)}$$

$$= \frac{2\sin 4A \cos 3A + 2\sin 4A \cos A}{2\cos 4A \cos 3A + 2\cos 4A \cos A}$$

$$= \frac{2\sin 4A (\cos 3A + \cos A)}{2\cos 4A(\cos 3A + \cos A)} = \tan 4A$$

27. If x + iy = $\sqrt{\frac{a+ib}{c+id}}$, prove that $(x^2 + y^2)^2 = \frac{a^2 + b^2}{c^2 + d^2}$.

Ans.

$$x + iy = \sqrt{\frac{a+ib}{c+id}} \Rightarrow (x+iy)^2 = \frac{a+ib}{c+id}$$

$$\Rightarrow |x + iy|^2 = \frac{|a+ib|}{|c+id|}$$

$$\Rightarrow \left(\sqrt{x^2 + y^2}\right)^2 = \frac{\sqrt{a^2 + b^2}}{\sqrt{c^2 + d^2}}$$

28. Show that the points A(1, 2, 3), B(–1, –2, – 1), C(2, 3, 2) and D(4, 7, 6) are the vertices of a parallelogram ABCD, but it is not a rectangle.

Ans. A (1, 2, 3), B (–1, –2, – 1), C (2, 3, 2) and D (4, 7, 6) are given vertices.
Coordinates of mid-point of AC are

$$\left(\frac{1+2}{2}, \frac{2+3}{2}, \frac{3+2}{2}\right) = \left(\frac{3}{2}, \frac{5}{2}, \frac{5}{2}\right)$$

Coordinates of mid-point of BD are

$$\left(\frac{-1+4}{2}, \frac{-2+7}{2}, \frac{-1+6}{2}\right) = \left(\frac{3}{2}, \frac{5}{2}, \frac{5}{2}\right)$$

As mid points of AC and BD are same

⇒ A, B, C and D are the vertices of a parallelogram.

It will be a rectangle if AC = BD

AC = $\sqrt{(2-1)^2+(3-2)^2+(2-3)^2} = \sqrt{1+1+1} = \sqrt{3}$

BD = $\sqrt{(4+1)^2+(7+2)^2+(6+1)^2} = \sqrt{25+81+49} = \sqrt{155}$

As, AC ≠ BD

∴ A, B, C and D are vertices of a parallelogram and not rectangle.

29. Find the domain and the range of the function : $f(x) = \sqrt{x^2-4}$

Ans. Given, $f(x) = \sqrt{x^2-4}$; For D_f, $f(x)$ must be a real number.

⇒ $\sqrt{x^2-4}$ must be a real number. ⇒ $x^2 - 4 \geq 0 \Rightarrow (x+2)(x-2) \geq 0$

⇒ Either $x \leq -2$ or $x \geq 2$. ⇒ $D_f = (-\infty, -2] \cup [2, \infty)$.

For R_f, let $y = \sqrt{x^2-4}$ ... (*i*)

As square root of a real number is always non-negative, $y \geq 0$.

On squaring (*i*), we get $y^2 = x^2 - 4 \Rightarrow x^2 = y^2 + 4$ but $x^2 \geq 0\ \forall\ x \in D_f$.

⇒ $y^2 + 4 \geq 0 \Rightarrow y^2 \geq -4$, which is true $\forall\ y \in R$,

Also, $y \geq 0$. ⇒ $R_f = [0, \infty)$.

30. Find $(a+b)^4 - (a-b)^4$. Hence, evaluate $(\sqrt{3}+\sqrt{2})^4 - (\sqrt{3}-\sqrt{2})^4$.

Ans. Consider, $(a+b)^4 - (a-b)^4$

$= ({}^4C_0\, a^4 + {}^4C_1\, a^3b + {}^4C_2\, a^2b^2 + {}^4C_3\, ab^3 + {}^4C_4\, b^4) - ({}^4C_0\, a^4 - {}^4C_1\, a^3b + {}^4C_2\, a^2b^2 - {}^4C_3\, ab^3 + {}^4C_4\, b^4)$

$= 2[{}^4C_1\, a^3b + {}^4C_3\, ab^3]$

$\therefore\ (\sqrt{3}+\sqrt{2})^4 - (\sqrt{3}-\sqrt{2})^4$

$= 2[{}^4C_1 (\sqrt{3})^3(\sqrt{2}) + {}^4C_3(\sqrt{3})(\sqrt{2})^3]$

$= 2\left[4 \times 3\sqrt{3} \times \sqrt{2} + 4 \times \sqrt{3} \times 2\sqrt{2}\right]$

$= 2[20\sqrt{6}] = 40\sqrt{6}$.

OR

Expand following using binomial expansion $(1 - x + x^2)^4$

Ans.

$(1-x+x^2)^4$

$= [(1-x)+x^2]^4$

$= {}^4C_0 (1-x)^4 + {}^4C_1 (1-x)^3 (x^2) + {}^4C_2 (1-x)^2 (x^2)^2 + {}^4C_3 (1-x)(x^2)^3 + {}^4C_4 (x^2)^4$

$= (1-4x+6x^2-4x^3+x^4) + 4(1-3x+3x^2-x^3)x^2 + 6(1-2x+x^2)x^4 + 4(1-x)x^6 + x^8$

$= 1-4x+10x^2-16x^3+19x^4-16x^5+10x^6-4x^7+x^8$

31. Solve : $5(2x-7) - 3(2x+3) \leq 0$ and $2x + 19 \leq 6x + 47$ and represent the solution on number line.

Ans. Consider the inequation

$5(2x - 7) - 3(2x + 3) \leq 0$

⇒ $10x - 35 - 6x - 9 \leq 0 \Rightarrow 4x \leq 44 \Rightarrow x \leq 11$ …(*i*)

Consider the inequation,

$2x + 19 \leq 6x + 47$

⇒ $19 - 47 \leq 6x - 2x$

⇒ $-28 \leq 4x \Rightarrow -7 \leq x$

⇒ $x \geq -7$ …(*ii*)

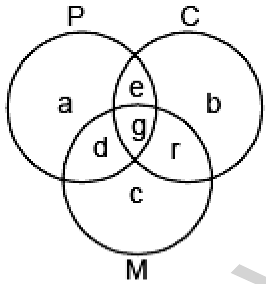
From (*i*) and (*ii*), we get

Solution as $-7 \le x \le 11$
Representation on number line

SECTION – D

Questions 32 to 35 carry 5 marks each.

32. In a class, 18 students took Physics, 23 students took Chemistry and 24 students took Mathematics. Of these 13 took both Chemistry and Mathematics, 12 took both Physics and Chemistry and 11 took both Physics and Mathematics. If 6 students offered all the three subjects, find :
(*i*) Total number of students in the class.
(*ii*) How many took Mathematics but not Chemistry ?
(*iii*) How many took exactly one of the 3 subjects ?
Ans. P : Physics; C : Chemistry M : Mathematics.



$a + d + e + g = 18; f + g = 13$
$b + e + f + g = 23; e + g = 12$
$c + d + f + g = 24$; $d + g = 11; g = 6$
$g = 6, f = 7, e = 6, d = 5, a = 1, b = 4, c = 6.$
(*i*) Total number of students in class = $a + b + c + d + e + f + g = 35$
(*ii*) Mathematics but not Chemistry = $d + c = 11$
(*iii*) Exactly one of the three subjects = $a + b + c = 11$

33. Find mean, variance and standard deviation using short cut method.

Height in (cm)	70–75	75–85	80–85	85–90	90–95	96–100	100–105	105–110	110–115
Number of children	3	4	7	7	15	9	6	6	3

Ans.

C.I.	x_i	f_i	$d_i = \frac{x_i - 92.5}{5}$	$f_i d_i$	$f_i d_i^2$
70 – 75	72.5	3	–4	–12	48
75 – 80	77.5	4	–3	–12	36
80 – 85	82.5	7	–2	–14	28
85 – 90	87.5	7	–1	–7	7
90 – 95	92.5	15	0	0	0
95 – 100	97.5	9	1	9	9
100 – 105	102.5	6	2	12	24
105 – 110	107.5	6	3	18	54
110 – 115	112.5	3	4	12	48
Total		$\sum f_i = 60$		$\sum f_i d_i = 6$	$\sum f_i d_i^2 = 254$

From the table, A = 92.5, $h = 5$

(*i*) Mean = A + $\frac{\sum f_i d_i}{\sum f_i} \times h = 92.5 + \frac{6}{60} \times 5$ = 92.5 + 0.5 = 93.0

(*ii*) Standard deviation =

$$h\sqrt{\frac{\Sigma f_i d_i^2}{\Sigma f_i} - \left(\frac{\Sigma f_i d_i}{\Sigma f_i}\right)^2} = 5\sqrt{\frac{254}{60} - \left(\frac{6}{60}\right)^2} = 5\sqrt{4.23 - 0.01} = 5\sqrt{4.22}$$

= 5 × 2.05 = 10.25

(*iii*) Variance = (Standard Deviation)^2 = $(5\sqrt{4.22})^2$ = 25 × 4.22 = 105.5

34. If p^{th}, q^{th}, r^{th} and sth terms of an A.P. are in G.P, then show that $(p - q)$, $(q - r)$, $(r - s)$ are also in G.P.

Ans. Here

$a_p = a + (p - 1)d$ ...(*i*)

$a_q = a + (q - 1)d$ ...(*ii*)

$a_r = a + (r - 1)d$ ...(*iii*)

$a_s = a + (s - 1)d$ ...(*iv*)

Given that a_p, a_q, a_r and as are in G.P.

$$\frac{a_q}{a_p} = \frac{a_r}{a_q} = \frac{a_q - a_r}{a_p - a_q} = \frac{q - r}{p - q} \quad ...(v)$$

Similarly,

$$\frac{a_r}{a_q} = \frac{a_s}{a_r} = \frac{a_r - a_s}{a_q - a_r} = \frac{r - s}{q - r} \quad ...(vi)$$

Hence, by (*v*) and (*vi*)

$$\frac{q - r}{p - q} = \frac{r - s}{q - r},$$

i.e., $p - q$, $q -$ i and $r - s$ are in G.P.

OR

Let S be the sum, P the product and R the reciprocals of *n* terms in a G.P. Prove that $P^2R^n = S^n$.

Ans.

Let GP, be $a, ar, ar^2, ..., ar^{n-1}$

$$S = a + ar + ... + ar^{n-1} = \frac{a(1 - r^n)}{1 - r} \quad ...(i)$$

$P = a \cdot (ar) \cdot (ar^2) ... (ar^{n-1})$

$= a^n \cdot r^{1 + 2 + ... + (n-1)} = a^n \cdot r^{(n-1)n/2}$ ...(*ii*)

$$R = \frac{1}{a} + \frac{1}{ar} + ... + \frac{1}{ar^{n-1}} = \frac{\frac{1}{a}\left[\left(\frac{1}{r}\right)^n - 1\right]}{\frac{1}{r} - 1} = \frac{(1 - r^n)}{ar^{n-1}(1 - r)} \quad ...(iii)$$

$$\text{LHS} = P^2R^n = [a^n \cdot r^{(n-1)n/2}]^2 \cdot \left[\frac{(1 - r^n)}{ar^{n-1}(1 - r)}\right]^n$$

$$= \frac{a^n(1 - r^n)^n}{(1 - r)^n} \quad ...(iv)$$

$$\text{RHS} = S^n = \left[\frac{a(1 - r^n)}{1 - r}\right]^n = \frac{a^n(1 - r^n)^n}{(1 - r)^n} \quad ...(v)$$

From (*iv*) and (*v*), $P^2R^n = S^n$

35. Differentiate $x\sin x$ from the first principle.

Ans.

Let $y = x \sin x$.

Let δy be an increment in y, corresponding to an increment δx in x.

Then, $y + \delta y = (x + \delta x) \sin (x + \delta x)$

$\Rightarrow \quad \delta y = (x + \delta x) \sin (x + \delta x) - x \sin x$

$$\Rightarrow \quad \frac{\delta y}{\delta x} = \frac{(x + \delta x) \sin (x + \delta x) - x \sin x}{\delta x}$$

$$\Rightarrow \quad \frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} \frac{\{(x + \delta x) \sin (x + \delta x) - x \sin x\}}{\delta x}$$

$$= \lim_{\delta x \to 0} \left[\frac{x\{\sin (x + \delta x) - \sin x\}}{\delta x} + \sin (x + \delta x) \right]$$

$$= \lim_{\delta x \to 0} \frac{x\{\sin (x + \delta x) - \sin x\}}{\delta x} + \lim_{\delta x \to 0} \sin (x + \delta x) = \lim_{\delta x \to 0} \left\{ \frac{2x \cos \left(x + \frac{\delta x}{2} \right) \sin \left(\frac{\delta x}{2} \right)}{\delta x} \right\} + \sin x$$

$$= \left\{ x \cdot \lim_{\delta x \to 0} \cos \left(x + \frac{\delta x}{2} \right) \cdot \lim_{\delta x \to 0} \frac{\sin (\delta x/2)}{(\delta x/2)} \right\} + \sin x = (x \cdot \cos x \cdot 1) + \sin x = (x \cos x + \sin x).$$

OR

Evaluate: $\lim_{x \to 0} \frac{\tan x + 4 \tan 2x - 3 \tan 3x}{x^2 \tan x}$

Ans.

$$\lim_{x \to 0} \frac{\tan x + 4 \tan 2x - 3 \tan 3x}{x^2 \tan x}$$

$$= \lim_{x \to 0} \frac{\tan x + 4 \frac{2 \tan x}{1 - \tan^2 x} - 3 \left(\frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} \right)}{x^2 \tan x} = \lim_{x \to 0} \frac{1 + \frac{8}{1 - \tan^2 x} - 3 \left(\frac{3 - \tan^2 x}{1 - 3 \tan^2 x} \right)}{x^2}$$

$$= \lim_{x \to 0} \frac{(1 - \tan^2 x)(1 - 3 \tan^2 x) + 8(1 - 3 \tan^2 x) - 3(3 - \tan^2 x)(1 - \tan^2 x)}{x^2 (1 - \tan^2 x)(1 - 3 \tan^2 x)}$$

$$= \lim_{x \to 0} \frac{1 - 4 \tan^2 x + 3 \tan^4 x + 8 - 24 \tan^2 x - 9 + 12 \tan^2 x - 3 \tan^4 x}{x^2 (1 - \tan^2 x)(1 - 3 \tan^2 x)}$$

$$= \lim_{x \to 0} \frac{-16 \tan^2 x}{x^2 (1 - \tan^2 x)(1 - 3 \tan^2 x)} = -16 \lim_{x \to 0} \left(\frac{\tan x}{x} \right)^2 \times \frac{1}{1 - \tan^2 x} \times \frac{1}{1 - 3 \tan^2 x} = -16$$

SECTION – E(Case Study Based Questions)

Questions 36 to 38 carry 4 marks each.

36. Case-Study 1:

One of the four persons John, Rita, Aslam or Gurpreet will be promoted next month. Consequently, the sample space consists of four elementary outcomes S = {John promoted, Rita promoted, Aslam promoted, Gurpreet promoted}. You are told that the chances of John's promotion is same as that of Gurpreet, Rita's chances of promotion are twice as likely as Johns. Aslam's chances are four times that of John.

(a) What is the probability that John got promotion?
(b) What is the probability that Rita got promotion?
(c) What is the probability that Aslam got promotion?
(d) What is the probability that Gurpreet got promotion?
Ans. Let Event:
J = John promoted
R = Rita promoted
A = Aslam promoted
G = Gurpreet promoted
Given sample space, S = {John promoted, Rita promoted, Aslam promoted, Gurpreet promoted}
i.e., $S = \{J, R, A, G\}$
It is given that, chances of John's promotion is same as that of Gurpreet.

$$P(T) = P(G)$$

Rita's chances of promotion are twice as likely as John.

$$P(R) = 2P(T)$$

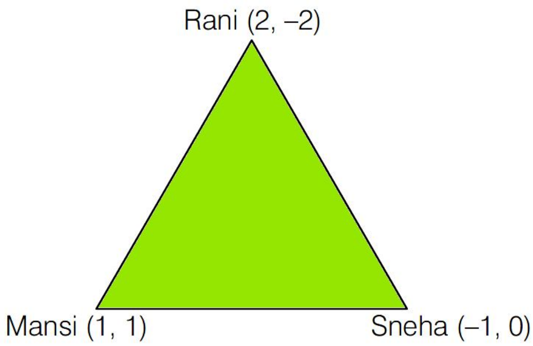
and Aslam's chances of promotion are four times that of John.

$$P(A) = 4P(D)$$

(a) Now, $P(C) + P(R) + P(A) + P(C) = 1$
$\Rightarrow P(J) + 2P(J) + 4P(J) + P(J) = 1$
$\Rightarrow 8P(J) = 1$
$\Rightarrow P(T) = P(\text{John Promoted}) = 1/8$
(b) $P(\text{Rita promoted}) = P(R)$
$= 2P(J) = 2 \times 1/8 = 1/4$
(c) $P(\text{Aslam promoted})$
$= 4P(J)$
$= 4 \times 1/8 = 1/2$
(d) $P(\text{Gurpreet promoted})$
$= P(G) = P(J) = 1/8$

37. Case-Study 2:

One triangular shaped pond is there in a park. Three friends Rani, Mansi, Sneha are sitting at the corners of the triangular park. They are studying in Class XI in an International.
Rani marked her position as (2, –2), Mansi marked as (1, 1) and Sneha marked her position as (–1, 0) as shown in figure given below.



Based on the above information answer the following questions.
(i) Find the equation of lines formed by Rani and Mansi. (1)
(ii) Find the Slope of equation of line formed by Rani and Sneha. (1)
(iii) Find the equation of median of lines through Rani. (1)
(iv) Find the equation of altitude through Mansi. (1)

Ans: (i) The equation of line AB is $y-1=\frac{-2-1}{2-1}(x-1)\left[\because y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)\right]$

$\Rightarrow$ y −1 = − 3x + 3 $\Rightarrow$ 3x + y = 4
(ii) Slope of equation of line AC is
(iii) Let D be the mid-point of BC.

Coordinates of D are $\left(\frac{1-1}{2},\frac{0+1}{2}\right)=\left(0,\frac{1}{2}\right)$

$\therefore$ Equation of AD is $y+2=\frac{\frac{1}{2}+2}{0-2}(x-2)\Rightarrow y+2=\frac{-5}{4}(x-2)$

$\Rightarrow$ 4y + 8 = − 5x + 10
$\Rightarrow$ 5x + 4y = 2
(iv) Slope of AC = − 2/3
$\therefore$ Slope of BE = 3/2 [$\because$ BE $\perp$ AC]

Equation of altitude through B is $y-1=\frac{3}{2}(x-1)\Rightarrow$ 3x − 2y =1

38. Case-Study 3:

Seema wants a mobile number having 10 digits. It is not just a group of numbers strung out at random. All mobile numbers have 3 things in common. a 2-digit Access code (AC), a 3-digit provider code (PC), and a 5 digit subscriber code (SC). AC code and PC code are fixed, then

(i) How many mobile numbers are possible if no start with 98073 and no other digit can repeat? (1)
(ii) How many AC code are possible if both digit in AC code are different and must be greater than 6? (1)
(iii)How many mobile numbers are possible if AC and PC code are fixed and digits can repeat? (1)
(iv) How many mobile numbers are possible with AC code 98 and PC code 123 and digit used in AC and PC code will not be used in SC code? (1)

Ans: (i) 98073 V IV III II I
The digits which can be used are, 6, 5, 4, 2, 1
Number of ways to fill the 5 places = 5! = 120
(ii) Digits which can be used in AC code are 7, 8, 9
Total AC code = $^3P_2$ = 3! = 6
(iii) If AC and PC are fixed then only 5 digits is to be filled if digits can repeat then total ways = 100000 = 10^5
(iv) Total ways = 5 × 5 × 5 × 5 × 5 = 3125