

MATHEMATICS – FULL SYLLABUS
MOCK TEST PAPER - 2
CBSE BOARD CLASS – XII (2025-26)

Time Allowed: 3 hours

Maximum Marks: 80

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If $(A - 2B) = \begin{bmatrix} 1 & -2 \\ 3 & 0 \end{bmatrix}$ and $(2A - 3B) = \begin{bmatrix} -2 & 2 \\ 3 & -3 \end{bmatrix}$ then $B = ?$ [1]
 - a) $\begin{bmatrix} -4 & 6 \\ -3 & -3 \end{bmatrix}$
 - b) None of these
 - c) $\begin{bmatrix} 4 & -6 \\ 3 & -3 \end{bmatrix}$
 - d) $\begin{bmatrix} 6 & -4 \\ -3 & 3 \end{bmatrix}$
2. Let $A = \begin{bmatrix} 1 & \sin \theta & 1 \\ -\sin \theta & 1 & \sin \theta \\ -1 & -\sin \theta & 1 \end{bmatrix}$ where $0 \leq \theta \leq 2\pi$. Then [1]
 - a) $\text{Det}(A) = 0$
 - b) $\text{Det}(A) \in [2, 4]$
 - c) $\text{Det}(A) \in (2, 4)$
 - d) $\text{Det}(A) \in (2, \infty)$
3. The system of equations $x + 2y = 5$, $4x + 8y = 20$ has [1]
 - a) None of these
 - b) no solution
 - c) a unique solution
 - d) infinitely many solutions
4. At $x = 2$, $f(x) = [x]$ is [1]
 - a) Continuous but not differentiable
 - b) None of these
 - c) Continuous as well as differentiable
 - d) Differentiable but not continuous
5. The lines l_1 and l_2 intersect. The shortest distance between them is [1]
 - a) infinity
 - b) negative
 - c) positive
 - d) zero

6. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x \sin\left(\frac{dy}{dx}\right)$ is [1]

a) not defined b) 1
c) 2 d) 3

7. The corner points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5), (15, 15), (0, 20). Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the maximum of Z occurs at both the points (15, 15) and (0, 20) is [1]

a) $q = 3p$ b) $q = 2p$
c) $p = q$ d) $p = 2q$

8. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ Then [1]

a) $\vec{a} \perp \vec{b}$ b) none of these
c) $|\vec{a}| = |\vec{b}|$ d) $\vec{a} \parallel \vec{b}$

9. $\int \sqrt[3]{x} dx = ?$ [1]

a) $\frac{4}{3}x^{\frac{4}{3}} + C$ b) $\frac{3}{4}x^{\frac{4}{3}} + C$
c) $\frac{3}{2}x^{\frac{2}{3}} + C$ d) $\frac{4}{3}x^{\frac{3}{4}} + C$

10. Total number of possible matrices of order 3×3 with each entry 2 or 0 is [1]

a) 27 b) 81
c) 9 d) 512

11. Maximize $Z = 5x + 3y$, subject to constraints $x + y \leq 300$, $2x + y \leq 360$, $x \geq 0$, $y \geq 0$. [1]

a) 1020 b) 1050
c) 1040 d) 1030

12. If $\vec{a} \cdot \vec{b} = 0$ and $\vec{a} \times \vec{b} = 0$, then which one of the following is correct? [1]

a) \vec{a} is parallel to \vec{b} b) $\vec{a} = 0$ or $\vec{b} = 0$
c) \vec{a} is perpendicular to \vec{b} d) None of these

13. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then the value of $|\text{adj } A|$ is [1]

a) a^2 b) a^6
c) a^9 d) a^{27}

14. If A and B are events such that $P(A|B) = P(B|A)$, then [1]

a) $A \subset B$ but $A \neq B$ b) $A = B$
c) $A \cap B = \emptyset$ d) $P(A) = P(B)$

15. Consider the following statements in respect of the differential equation $\frac{d^2y}{dx^2} + \cos\left(\frac{dy}{dx}\right) = 0$ [1]

i. The degree of the differential equation is not defined.
ii. The order of the differential equation is 2.

Which of the above statement(s) is/are correct?

15. a) Both (i) and (ii)
b) Only (ii)
c) Only (i)
d) Neither (i) nor (ii)

16. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors, no two of which are collinear and the vector $\vec{a} + \vec{b}$ is collinear with $\vec{c}, \vec{b} + \vec{c}$ is collinear with \vec{a} , then $\vec{a} + \vec{b} + \vec{c} =$ [1]
a) \vec{a}
b) \vec{c}
c) \vec{b}
d) None of these

17. If $y = \sec^{-1} \left(\frac{x^2+1}{x^2-1} \right)$ then $\frac{dy}{dx} = ?$ [1]
a) $\frac{-1}{(1+x^2)}$
b) None of these
c) $\frac{2}{(1+x^2)}$
d) $\frac{-2}{(1+x^2)}$

18. The lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x-1}{-2} = \frac{y-2}{-4} = \frac{z-3}{-6}$ are [1]
a) parallel
b) intersecting
c) skew
d) coincident

19. **Assertion (A):** If x is real, then the minimum value of $x^2 - 8x + 17$ is 1.
Reason (R): If $f''(x) > 0$ at a critical point, then the value of the function at the critical point will be the minimum value of the function.
a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** Every function is invertible.
Reason (R): Only bijective functions are invertible.
a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Find the principal value of $\text{cosec}^{-1}(-2)$. [2]

OR

Find the principal value of $\text{cosec}^{-1}(-\sqrt{2})$.

22. Find the intervals in function $f(x) = 2x^3 - 24x + 107$ is increasing or decreasing. [2]

23. The radius r of a right circular cone is decreasing at the rate of 3 cm/minute and the height h is increasing at the rate of 2 cm/minute. When $r = 9$ cm and $h = 6$ cm, find the rate of change of its volume. [2]

OR

as fast as x - coordinates.

24. Evaluate the integral: $\int_0^1 \log(1 + x) dx$ [2]

25. Find the values of x for which the function , [2]
 $f(x) = kx^3 - 9x^2 + 9x + 3$ is increasing in R

Section C

26. Evaluate: $\int \frac{dx}{(e^x - 1)^2}$. [3]

27. In a school, there are 1000 students, out of which 430 are girls. It is known that out of 430, 10% of the girls study in class XII, what is the probability that a student chosen randomly studies in class XII, given that the chosen student is a girl? [3]

28. Evaluate $\int_0^{\pi} e^{2x} \cdot \sin\left(\frac{\pi}{4} + x\right) dx$. [3]

OR

Evaluate: $\int \frac{3x+1}{\sqrt{5-2x-x^2}} dx$

29. Solve the following differential equation: [3]

$$xdy - (y - x^3)dx = 0$$

OR

Find the particular solution of the differential equation $(1 + x^2) \frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$, given that $y = 0$ when $x = 1$.

30. Solve the Linear Programming Problem graphically: [3]

Minimize $Z = x - 5y + 20$ Subject to

$$x - y \geq 0$$

$$-x + 2y \geq 2$$

$$x \geq 3$$

$$y \leq 4$$

$$x, y \geq 0$$

OR

Solve the Linear Programming Problem graphically:

Maximize $Z = 50x + 30y$ Subject to

$$2x + y \leq 18$$

$$3x + 2y \leq 34$$

$$x, y \geq 0$$

31. Differentiate the function with respect to x : $\tan^{-1}\left(\frac{a+b \tan x}{b-a \tan x}\right)$. [3]

Section D

32. Find the area enclosed by the parabola $y^2 = 4ax$ and the line $y = mx$. [5]

33. Let n be a positive integer. Prove that the relation R on the set Z of all integers numbers defined by $(x, y) \in R \Leftrightarrow x - y$ is divisible by n , is an equivalence relation on Z . [5]

OR

Let $A = R - \{3\}$, $B = R - \{1\}$. If $f : A \rightarrow B$ be defined by $f(x) = \frac{x-2}{x-3} \forall x \in A$. Then, show that f is bijective.

34. Solve the system of equations [5]

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

35. Find the perpendicular distance of the point $(1, 0, 0)$ from the line $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$. Also, find the coordinates of the foot of the perpendicular and the equation of the perpendicular. [5]

OR

Show that the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ intersect. Also, find the equation of the plane containing them.

Section E

36. **Read the text carefully and answer the questions:**

[4]

For an audition of a reality singing competition, interested candidates were asked to apply under one of the two musical genres-folk or classical and under one of the two age categories-below 18 or 18 and above.

The following information is known about the 2000 application received:

- i. 960 of the total applications were the folk genre.
- ii. 192 of the folk applications were for the below 18 category.
- iii. 104 of the classical applications were for the 18 and above category.

- (i) What is the probability that an application selected at random is for the 18 and above category provided it is under the classical genre? Show your work.
- (ii) An application selected at random is found to be under the below 18 category. Find the probability that it is under the folk genre. Show your work.
- (iii) If $P(A) = 0.4$, $P(B) = 0.8$ and $P(B|A) = 0.6$, then $P(A \cup B)$ is equal to

OR

If A and B are two independent events with

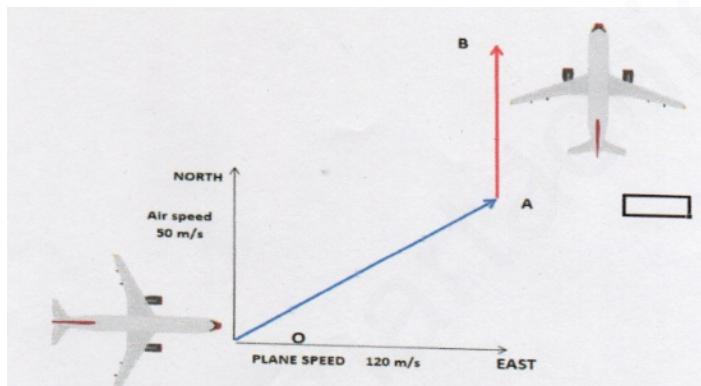
$P(A) = \frac{3}{5}$ and $P(B) = \frac{4}{9}$, then find $P(A' \cap B')$.

37. **Read the text carefully and answer the questions:**

[4]

A plane started from airport O with a velocity of 120 m/s towards east. Air is blowing at a velocity of 50 m/s towards the north As shown in the figure.

The plane travelled 1 hr in OA direction with the resultant velocity. From A and B travelled 1 hr with keeping velocity of 120 m/s and finally landed at B.



- (i) What is the resultant velocity from O to A?
- (ii) What is the direction of travel of plane O to A with east?
- (iii) What is the total displacement from O to A?

OR

What is the resultant velocity from A to B?

38. **Read the text carefully and answer the questions:**

[4]

The temperature of a person during an intestinal illness is given by $f(x) = -0.1x^2 + mx + 98.6$, $0 \leq x < 12$, m being a constant, where $f(x)$ is the temperature in $^{\circ}\text{F}$ at x days.



- (i) Is the function differentiable in the interval $(0, 12)$? Justify your answer.
- (ii) If 6 is the critical point of the function, then find the value of the constant m .

Solution

Section A

1. (a) $\begin{bmatrix} -4 & 6 \\ -3 & -3 \end{bmatrix}$

Explanation: $(A - 2B) = \begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix}$

Multiplying equation by 2

$$2A - 4B = \begin{pmatrix} 2 & -4 \\ 6 & 0 \end{pmatrix} \dots(i)$$

$$2A - 3B = \begin{pmatrix} -2 & 2 \\ 3 & -3 \end{pmatrix} \dots(ii)$$

(ii) - (i)

$$\begin{aligned} B &= \begin{pmatrix} -2 & 2 \\ 3 & -3 \end{pmatrix} - \begin{pmatrix} 2 & -4 \\ 6 & 0 \end{pmatrix} \\ &= \begin{pmatrix} -4 & 6 \\ -3 & -3 \end{pmatrix} \end{aligned}$$

2.

(b) $\text{Det}(A) \in [2, 4]$

Explanation: $A = \begin{bmatrix} 1 & \sin \theta & 1 \\ -\sin \theta & 1 & \sin \theta \\ -1 & -\sin \theta & 1 \end{bmatrix}$

$$|A| = 1 (1 \times 1 - \sin \theta \times (-\sin \theta)) - \sin \theta (-\sin \theta + \sin \theta) + 1 [(-\sin \theta) \times (-\sin \theta) - (-1) \times 1]$$

$$|A| = 1 + \sin^2 \theta + \sin^2 \theta + 1$$

$$|A| = 2 + 2 \sin^2 \theta$$

$$|A| = 2(1 + \sin 2\theta)$$

Now, $0 \leq \theta \leq 2\pi$

$$\Rightarrow \sin 0 \leq \sin \theta \leq \sin 2\pi$$

$$\Rightarrow 0 \leq \sin^2 \theta \leq 1$$

$$\Rightarrow 1 + 0 \leq 1 + \sin^2 \theta \leq 1 + 1$$

$$\Rightarrow 2 \leq 2(1 + \sin^2 \theta) \leq 4$$

$$\therefore \text{Det}(A) \in [2, 4]$$

3.

(d) infinitely many solutions

Explanation: $x + 2y = 5$,

$$4x + 8y = 20$$

$$\Rightarrow A = \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 5 \\ 20 \end{bmatrix}$$

$$|A| = 8 - 8 = 0$$

$$\text{adj}A = \begin{bmatrix} 8 & -2 \\ -4 & 1 \end{bmatrix}$$

$$\text{now } (\text{adj } A)B = \begin{bmatrix} 8 & -2 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 20 \end{bmatrix} = \begin{bmatrix} 40 - 40 \\ -20 + 20 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow (\text{adj } A)B = 0$$

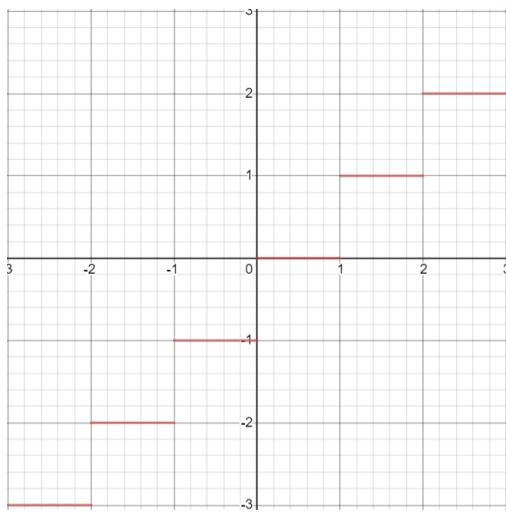
Since, $|A|=0$ and $(\text{adj } A)B=0$

So, the pair of equation have infinitely many solutions

4.

(b) None of these

Explanation: Let us see that graph of the floor function, we get



We can see that $f(x) = [x]$ is neither continuous and non differentiable at $x = 2$.

5.

(d) zero

Explanation: Since the lines intersect. Hence they have a common point in them. Hence the distance will be zero.

6. **(a) not defined**

Explanation: In general terms for a polynomial the degree is the highest power.

Degree of differential equation is defined as the highest integer power of highest order derivative in the equation

Here the differential equation is $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x \sin\left(\frac{dy}{dx}\right)$

Now for degree to exist the given differential equation must be a polynomial in some differentials.

Here differentials mean $\frac{dy}{dx}$ or $\frac{d^2y}{dx^2}$ or $\dots \frac{d^ny}{dx^n}$

The given differential equation is not polynomial because of the term $\sin\frac{dy}{dx}$ and hence degree of such a differential equation is not defined.

7. **(a) $q = 3p$**

Explanation: Since Z occurs maximum at $(15, 15)$ and $(0, 20)$, therefore, $15p + 15q = 0p + 20q \Rightarrow q = 3p$.

8. **(a) $\vec{a} \perp \vec{b}$**

Explanation: Here $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$

$$\Rightarrow |\vec{a} + \vec{b}|^2 = |\vec{a} - \vec{b}|^2$$

$$\Rightarrow (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b})$$

$$\Rightarrow |a|^2 + 2\vec{a} \cdot \vec{b} + |b|^2 = |a|^2 - 2\vec{a} \cdot \vec{b} + |b|^2$$

$$\Rightarrow 2\vec{a} \cdot \vec{b} = -2\vec{a} \cdot \vec{b}$$

$$\Rightarrow 4\vec{a} \cdot \vec{b} = 0$$

$$\Rightarrow \vec{a} \cdot \vec{b} = 0$$

$$\Rightarrow \vec{a} \perp \vec{b}$$

9.

(b) $\frac{3}{4}x^{\frac{4}{3}} + C$

Explanation: Given integral is $\int \sqrt[3]{x} dx$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \sqrt[3]{x} dx = \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + C$$

$$= \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + C$$

$$= \frac{3}{4}x^{\frac{4}{3}} + C$$

10.

(d) 512

Explanation: Since each element a_{ij} can be filled in two ways (with either '2' or '0'), total number of possible matrices is

$$8 \times 8 \times 8 = 512$$

11. (a) 1020

Explanation: Here, Maximize $Z = 5x + 3y$, subject to constraints $x + y \leq 300$, $2x + y \leq 360$, $x \geq 0$, $y \geq 0$.

Corner points	$Z = 5x + 3y$
P(0, 300)	900
Q(180, 0)	900
R(60, 240)	1020.....(Max.)
S(0, 0)	0

Hence, the maximum value is 1020

12.

(b) $\vec{a} = 0$ or $\vec{b} = 0$

Explanation: Given that, $\vec{a} \cdot \vec{b} = 0$,

i.e. \vec{a} and \vec{b} are perpendicular to each other and $\vec{a} \times \vec{b} = 0$

i.e. \vec{a} and \vec{b} are parallel to each other. So, both conditions are possible iff $\vec{a} = 0$ and $\vec{b} = 0$

13.

(b) a^6

Explanation: $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$

$$|A| = a^3$$

$$|\text{adj } A| = |A|^{3-1} = |A|$$

$$|\text{adj } A| = (a^3)^2 = a^6$$

14.

(d) $P(A) = P(B)$

Explanation: It is given that : $P(A | B) = P(B | A)$

$$\Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{P(B \cap A)}{P(A)}$$

$$\Rightarrow \frac{1}{P(B)} = \frac{1}{P(A)} \Rightarrow P(A) = P(B)$$

15. (a) Both (i) and (ii)

Explanation: Both (i) and (ii)

16.

(d) None of these

Explanation: Given that $\vec{a} + \vec{b}$ is collinear with \vec{c}

$$\therefore \vec{a} + \vec{b} = x\vec{c} \dots(i)$$

where x is scalar and $x \neq 0$

$\vec{b} + \vec{c}$ is collinear with \vec{a}

$$\vec{b} + \vec{c} = y\vec{a} \dots(ii)$$

y is scalar and $y \neq 0$

Subtracting (ii) from (i) we get

$$\vec{a} - \vec{c} = x\vec{c} - y\vec{a}$$

$$\vec{a} + y\vec{a} = x\vec{c} + \vec{c}$$

$$\vec{a}(1 + y) = (1 + x)\vec{c}$$

As given

\vec{a}, \vec{c} are not collinear. (no two vectors are collinear)

$$\therefore 1 + y = 0 \text{ and } 1 + x = 0$$

$$y = -1 \text{ and } x = -1$$

Putting value of x in equation (i)

$$\vec{a} + \vec{b} = -\vec{c}$$

$$\vec{a} + \vec{b} + \vec{c} = 0$$

17.

(d) $\frac{-2}{(1+x^2)}$

Explanation: Given that $y = \sec^{-1} \left(\frac{x^2+1}{x^2-1} \right)$

$$\Rightarrow \sec y = \frac{x^2+1}{x^2-1}$$

Since $\tan^2 x = \sec^2 x - 1$, thus

$$\tan^2 y = \left(\frac{x^2+1}{x^2-1} \right)^2 - 1 = \frac{4x^2}{(x^2-1)^2}$$

$$\text{Hence, } \tan y = -\frac{2x}{1-x^2} \text{ or } y = \tan^{-1} \left(-\frac{2x}{1-x^2} \right)$$

Let $x = \tan \theta \Rightarrow \theta = \tan^{-1} x$

$$\text{Hence, } y = \tan^{-1} \left(-\frac{2 \tan \theta}{1 - \tan^2 \theta} \right)$$

Using $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$, we obtain

$$y = \tan^{-1}(-\tan 2\theta)$$

Using $-\tan x = \tan(-x)$, we obtain

$$y = \tan^{-1}(\tan(-2\theta)) = -2\theta = -2\tan^{-1} x$$

Differentiating with respect to x, we obtain

$$\frac{dy}{dx} = \frac{-2}{1+x^2}$$

18.

(d) coincident

Explanation: The equation of the given lines are

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{3} \dots \text{(i)}$$

$$\frac{x-1}{-2} = \frac{y-2}{-4} = \frac{z-3}{-6}$$

$$= \frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3} \dots \text{(ii)}$$

Thus, the two lines are parallel to the vector $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ and pass through the points (0, 0, 0) and (1, 2, 3).

Now,

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = (\hat{i} + 2\hat{j} + 3\hat{k}) \times (\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= \vec{0} [\because \vec{a} \times \vec{a} = \vec{0}]$$

So, here the distance between the given two parallel lines is 0, the given lines are coincident.

19. (a) Both A and R are true and R is the correct explanation of A.

Explanation: Let $f(x) = x^2 - 8x + 17$

$$\therefore f'(x) = 2x - 8$$

$$\text{So, } f'(x) = 0, \text{ gives } x = 4$$

Here $x = 4$ is the critical number

$$\text{Now, } f''(x) = 2 > 0, \forall x$$

So, $x = 4$ is the point of local minima.

\therefore Minimum value of $f(x)$ at $x = 4$,

$$f(4) = 4 \times 4 - 8 \times 4 + 17 = 1$$

Hence, we can say that both Assertion and Reason are true and Reason is the correct explanation of the Assertion.

20.

(d) A is false but R is true.

Explanation: Assertion is false because every function is not invertible. The function which is one-one and onto i.e. bijective functions are invertible so reason is true.

Section B

21. $\text{cosec}^{-1} x$ represents an angle in $[-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}]$ whose cosent is x.

$$\text{Let } x = \text{cosec}^{-1}(-2)$$

$$\Rightarrow \operatorname{cosec} x = -2 = \operatorname{cosec} \left(-\frac{\pi}{6} \right)$$

$$\Rightarrow x = -\frac{\pi}{6}$$

\therefore Principal value of $\operatorname{cosec}^{-1}(-2)$ is $-\frac{\pi}{6}$.

OR

$$\text{Let } \operatorname{cosec}^{-1}(-\sqrt{2}) = y. \text{ Then, } \operatorname{cosec} y = -\sqrt{2} = -\operatorname{cosec} \left(\frac{\pi}{4} \right) = \operatorname{cosec} \left(-\frac{\pi}{4} \right).$$

We know that the range of the principal value branch of

$$\operatorname{cosec}^{-1} \text{ is } \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] - \{0\} \text{ and } \operatorname{cosec} \left(-\frac{\pi}{4} \right) = -\sqrt{2}.$$

Therefore, the principal value of $\operatorname{cosec}^{-1}(-\sqrt{2})$ is $-\frac{\pi}{4}$.

22. Given: $f(x) = 2x^3 - 24x + 107$

$$\Rightarrow f'(x) = \frac{d}{dx}(2x^3 - 24x + 107)$$

$$\Rightarrow f'(x) = 6x^2 - 24$$

For $f(x)$ lets find critical point, for this we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow 6x^2 - 24 = 0$$

$$\Rightarrow 6(x^2 - 4) = 0$$

$$\Rightarrow (x - 2)(x + 2) = 0$$

$$\Rightarrow x = -2, 2$$

clearly, $f'(x) > 0$ if $x < -2$ and $x > 2$

and $f'(x) < 0$ if $-2 < x < 2$

Thus, the function $f(x)$ increases on $(-\infty, -2) \cup (2, \infty)$ and $f(x)$ is decreasing on interval $x \in (-2, 2)$.

23. We know that Volume of right circular cone = $\frac{\pi r^2 h}{3}$

$$\frac{\partial V}{\partial t} = \frac{\pi}{3} \left(2rh \frac{\partial r}{\partial t} + r^2 \frac{\partial h}{\partial t} \right)$$

$$\frac{\partial V}{\partial t} = \frac{\pi}{3} (108 \times -3 + 81 \times 2)$$

$$\frac{\partial V}{\partial t} = \frac{\pi}{3} (-162) = -54\pi \text{ cm}^2/\text{min}$$

Therefore Volume is decreasing at rate $54\pi \text{ cm}^2/\text{min.}$

OR

Given curve is,

$$6y = x^3 + 2$$

$$\Rightarrow 6 \frac{dy}{dt} = 3x^2 \cdot \frac{dx}{dt} \dots (i)$$

$$\text{Given: } \frac{dy}{dt} = 2 \cdot \frac{dx}{dt} \dots (ii)$$

$$\text{from (i) and (ii), } 2 \left(2 \frac{dx}{dt} \right) = x^2 \cdot \frac{dx}{dt}$$

$$\Rightarrow x = \pm 2$$

when $x = 2$, $y = \frac{5}{3}$; when $x = -2$, $y = -1$

Therefore, Points are $(2, \frac{5}{3})$ and $(-2, -1)$

24. Let $I = \int_0^1 \log(1+x) dx$, then

$$I = \int_0^1 \log(1+x) \times 1 dx$$

$$= [\log(1+x)x]_0^1 - \int_0^1 \frac{x}{1+x} dx$$

$$= [\log(1+x)x]_0^1 - \int_0^1 \left(1 - \frac{1}{1+x} \right) dx$$

$$= [x \log(1+x)]_0^1 - [x - \log(1+x)]_0^1$$

$$= \log 2 - 1 + \log 2$$

$$= 2 \log 2 - 1$$

$$= \log 4 - \log e$$

$$= \log \frac{4}{e}$$

25. we have, $f(x) = kx^3 - 9x^2 + 9x + 3$

$$\Rightarrow f'(x) = 3kx^2 - 18x + 9$$

Since $f(x)$ is increasing on R , therefore, $f'(x) > 0 \forall x \in R$

$$\begin{aligned}
&\Rightarrow 3kx^2 - 18x + 9 > 0, \forall x \in R \\
&\Rightarrow kx^2 - 6x + 3 > 0, \forall x \in R \\
&\Rightarrow k > 0 \text{ and } 36 - 12k < 0 [\because ax^2 + bx + c > 0, \forall x \in R \Rightarrow a > 0 \text{ and discriminant} < 0] \\
&\Rightarrow k > 3
\end{aligned}$$

Hence, $f(x)$ is increasing on R , if $k > 3$.

Section C

26. Putting $t = e^x - 1$

$$e^x = t + 1$$

$$dt = e^x dx$$

$$\frac{dt}{e^x} = dx$$

$$\frac{dt}{t+1} = dx$$

$$\text{Putting above by have by partial fractions. } \frac{1}{(1+t)t^2} = \frac{A}{t+1} + \frac{Bt+C}{t^2} \dots (1)$$

$$A(t^2) + (Bt + C)(t + 1) = 1$$

$$\text{Put } t + 1 = 0$$

$$t = -1$$

$$A = 1$$

Equating coefficients

$$A + B = 0$$

$$1 + B = 0$$

$$B = -1$$

$$C = 1$$

From equation (1), we get,

$$\begin{aligned}
\frac{1}{(1+t)t^2} &= \frac{1}{t+1} + \frac{-t+1}{t^2} \\
\int \frac{1}{(1+t)t^2} dt &= \int \frac{1}{t+1} dt - \int \frac{t}{t^2} dt + \int \frac{1}{t^2} dt \\
&= \log|t+1| - \int \frac{1}{t} dt + \int \frac{1}{t^2} dt \\
&= \log|t+1| - \log|t| - \frac{1}{t} + c \\
\int \frac{1}{(e^x-1)^2} dx &= \log|e^x| - \log|e^x - 1| - \frac{1}{e^x-1} + c
\end{aligned}$$

27. Let 'A' be the event that the chosen student studies in class XII and B be the event that the chosen student is a girl.

There are 430 girls out of 1000 students

$$\text{So, } P(B) = P(\text{Chosen student is girl}) = \frac{430}{1000} = \frac{43}{100}$$

Since, 10% of the girls studies in class XII

So, total number of girls studies in class XII

$$= \frac{10}{100} \times 430 = 43$$

Then, $P(A \cap B) = P(\text{Chosen student is a girl of class XII})$

$$= \frac{43}{1000}$$

\therefore Required probability = $P(A / B)$

$$\begin{aligned}
&= \frac{P(A \cap B)}{P(B)} \quad [\because P(A / B) = \frac{P(A \cap B)}{P(B)}] \\
&= \frac{43/1000}{43/100} = \frac{1}{10}
\end{aligned}$$

28. According to the question, $I = \int_0^{\pi} e^{2x} \cdot \sin\left(\frac{\pi}{4} + x\right) dx \dots (i)$

$$\text{Consider, } I_1 = \int_I e^{2x} \sin\left(\frac{\pi}{4} + x\right) dx \dots \dots \dots (ii)$$

By using integration by parts, we get

$$\begin{aligned}
&= \sin\left(\frac{\pi}{4} + x\right) \int e^{2x} dx - \int \left\{ \frac{d}{dx} \sin\left(\frac{\pi}{4} + x\right) \int e^{2x} dx \right\} dx \\
&= \sin\left(\frac{\pi}{4} + x\right) \frac{e^{2x}}{2} - \int \cos\left(\frac{\pi}{4} + x\right) \frac{e^{2x}}{2} dx \\
&= \frac{e^{2x}}{2} \sin\left(\frac{\pi}{4} + x\right) - \frac{1}{2} \int_I e^{2x} \cos\left(\frac{\pi}{4} + x\right) dx
\end{aligned}$$

By using integration by parts for second integral, we get

$$= \frac{e^{2x}}{2} \sin\left(\frac{\pi}{4} + x\right) - \frac{1}{2} \left[\cos\left(\frac{\pi}{4} + x\right) \frac{e^{2x}}{2} - \int -\sin\left(\frac{\pi}{4} + x\right) \frac{e^{2x}}{2} dx \right]$$

$$\begin{aligned}
&= \frac{e^{2x}}{2} \sin\left(\frac{\pi}{4} + x\right) - \frac{e^{2x}}{4} \cos\left(\frac{\pi}{4} + x\right) - \frac{1}{4} \int e^{2x} \sin\left(\frac{\pi}{4} + x\right) dx \\
\Rightarrow I_1 &= \frac{e^{2x}}{4} \left\{ 2 \sin\left(\frac{\pi}{4} + x\right) - \cos\left(\frac{\pi}{4} + x\right) \right\} - \frac{1}{4} I_1 \quad [\text{From eq.(ii)}] \\
\Rightarrow I_1 + \frac{1}{4} I_1 &= \frac{e^{2x}}{4} \left\{ 2 \sin\left(\frac{\pi}{4} + x\right) - \cos\left(\frac{\pi}{4} + x\right) \right\} \\
\Rightarrow \frac{5}{4} I_1 &= \frac{e^{2x}}{4} \left\{ 2 \sin\left(\frac{\pi}{4} + x\right) - \cos\left(\frac{\pi}{4} + x\right) \right\} \\
\Rightarrow I_1 &= \frac{e^{2x}}{5} \left\{ 2 \sin\left(\frac{\pi}{4} + x\right) - \cos\left(\frac{\pi}{4} + x\right) \right\}
\end{aligned}$$

From (i) and (ii), we get

$$\begin{aligned}
\therefore I &= [I_1]_0^\pi \\
&= \left[\frac{e^{2x}}{5} \left\{ 2 \sin\left(\frac{\pi}{4} + x\right) - \cos\left(\frac{\pi}{4} + x\right) \right\} \right]_0^\pi \\
&= \frac{1}{5} \left[e^{2\pi} \left\{ 2 \sin\left(\frac{\pi}{4} + \pi\right) - \cos\left(\frac{\pi}{4} + \pi\right) \right\} - e^0 \left\{ 2 \sin\left(\frac{\pi}{4} + 0\right) - \cos\left(\frac{\pi}{4} + 0\right) \right\} \right] \\
&= \frac{1}{5} \left[e^{2\pi} \left\{ -2 \sin\frac{\pi}{4} + \cos\frac{\pi}{4} \right\} - e^0 \left\{ 2 \sin\frac{\pi}{4} - \cos\frac{\pi}{4} \right\} \right] \\
&= \frac{1}{5} \left[e^{2\pi} \left\{ -2 \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right\} - 1 \left\{ 2 \times \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \right\} \right] \\
&= \frac{1}{5} \left[e^{2\pi} \left\{ -\frac{1}{\sqrt{2}} \right\} - \frac{1}{\sqrt{2}} \right] \\
&= -\frac{1}{5\sqrt{2}} [e^{2\pi} + 1] \\
\therefore I &= -\frac{1}{5\sqrt{2}} [e^{2\pi} + 1] \text{ sq units.}
\end{aligned}$$

OR

Let the given integral be,

$$I = \frac{3x+1}{\sqrt{5-2x-x^2}} dx$$

$$\text{Let } 3x+1 = \lambda \frac{d}{dx} (5-2x-x^2) + \mu$$

$$= \lambda(-2-2x) + \mu$$

$$3x+1 = (-2\lambda)x + 2\lambda + \mu$$

Comparing the coefficients of like powers of x,

$$-2\lambda = 3 \Rightarrow \lambda = -\frac{3}{2}$$

$$-2\lambda + \mu = 1$$

$$\Rightarrow -2\left(-\frac{3}{2}\right) + \mu = 1$$

$$\mu = -2$$

$$\text{So, } I = \int \frac{-\frac{3}{2}(-2-2x)-2}{\sqrt{5-2x-x^2}} dx$$

$$= -\frac{3}{2} \int \frac{(-2-2x)}{\sqrt{5-2x-x^2}} dx - 2 \int \frac{1}{\sqrt{-[x^2+2x-5]}} dx$$

$$I = -\frac{3}{2} \int \frac{(-2-2x)}{\sqrt{5-2x-x^2}} dx - 2 \int \frac{1}{\sqrt{-[x^2+2x+(1)^2-(1)^2-5]}} dx$$

$$I = -\frac{3}{2} \int \frac{(-2-2x)}{\sqrt{5-2x-x^2}} dx - 2 \int \frac{1}{\sqrt{-[(x+1)^2-(\sqrt{6})^2]}} dx$$

$$I = -\frac{3}{2} \int \frac{(-2-2x)}{\sqrt{5-2x-x^2}} dx - 2 \int \frac{1}{\sqrt{(\sqrt{6})^2-(x+1)^2}} dx$$

$$I = -\frac{3}{2} \times 2\sqrt{5-2x-x^2} - 2 \sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right) + c \quad [\text{since, } \int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + c, \int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c]$$

$$I = -3\sqrt{5-2x-x^2} - 2 \sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right) + c$$

$$29. xdy - (y - x^3)dx = 0$$

This can be written as

$$xdy = (y - x^3)dx$$

Divide throughout by x,

$$\frac{dy}{dx} = \frac{y}{x} - x^2$$

$$\frac{dy}{dx} - \frac{y}{x} = -x^2$$

This is a linear differential equation of the form,

$$\frac{dy}{dx} + Py = Q$$

The integrating factor I.F is

$$e^{\int P dx} = e^{\int \frac{-1}{x} dx} \quad dx = e^{-\log x} = e^{\log(\frac{1}{x})} = \frac{1}{x}$$

The required solution is

$$ye^{\int P dx} = \int Q e^{\int P dx} \cdot dx + c$$

$$y \cdot \left(\frac{1}{x}\right) = - \int x^2 \times \frac{1}{x} dx + c$$

$$\frac{y}{x} = - \int x dx + c$$

$$\frac{y}{x} = \frac{-x^2}{2} + c$$

$$\frac{y}{x} + \frac{x^2}{2} = c$$

$$2y + x^3 = 2cx$$

$\Rightarrow x^3 - 2cx + 2y = 0$ is the required solution.

OR

$$(1 + x^2) \frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$$

Divide both sides by $1 + x^2$

$$\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{(1+x^2) \cdot (1+x^2)}$$

$$\frac{dy}{dx} + \left(\frac{2x}{1+x^2}\right)y = \frac{1}{(1+x^2)^2}$$

Comparing with $\frac{dy}{dx} + Py = Q$,

$$P = \frac{2x}{1+x^2} \quad \& \quad Q = \frac{1}{(1+x^2)^2}$$

Finding Integrating factor:

$$IF = e^{\int P dx}$$

$$IF = e^{\int \frac{2x}{1+x^2} dx}$$

$$\text{Let } 1 + x^2 = t$$

Diff. w.r.t. x

$$2x = \frac{dt}{dx}$$

$$dx = \frac{dt}{2x}$$

$$\text{Thus, IF} = e^{\int \frac{2x}{t} \frac{dt}{2x}}$$

$$IF = e^{\int \frac{dt}{t}}$$

$$IF = e^{\log |t|}$$

$$IF = t$$

$$IF = 1 + x^2$$

Solution of the differential equation:

$$y \times \text{I.F.} = \int Q \times I.F. dx$$

Putting values,

$$y \times (1 + x^2) = \int \frac{1}{(1+x^2)^2} (1 + x^2) dx$$

$$y (1 + x^2) = \int \frac{1}{(1+x^2)} dx$$

$$y (1 + x^2) = \tan^{-1} x + C \dots (1)$$

Putting that $y = 0$ and $x = 1$,

$$0(1 + 1^2) = \tan^{-1}(1) + C$$

$$0 = \frac{\pi}{4} + C$$

$$C = -\frac{\pi}{4}$$

Putting value of C in eq(1),

$$y(1 + x^2) = \tan^{-1} x - \frac{\pi}{4}$$

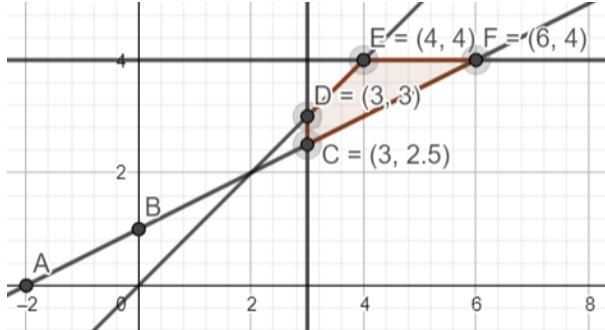
30. First, we will convert the given inequations into equations, we obtain the following equations:

$$x - y = 0, -x + 2y = 2, x = 3, y = 4, x = 0 \text{ and } y = 0$$

Region represented by $x - y \geq 0$ or $x \geq y$ The line $x - y = 0$ or $x = y$ passes through the origin. The region to the right of line $x = y$ will satisfy the given inequation. Check by taking an example like if we take a point (4,3) to the right of the line $x = y$. Here $x \geq y$. So, it satisfies the given inequation. Take a point (4,5) to the left of the line $x = y$. Here, $x \leq y$. That means it does not satisfy the

given inequation. Region represented by $-x + 2y \geq 2$ The line $-x + 2y = 2$ meets the coordinate axes at A(-2,0) and B(0,1). respectively. By joining these points we obtain the line $-x + 2y = 2$. Clearly (0,0) does not satisfy the inequation $-x + 2y \geq 2$. So, the region in xy plane which does not contain the origin represents the solution set of the inequation $-x + 2y \geq 2$ The line $x = 3$ is the line that passes through the point (3,0) and is parallel to Y-axis. $x \geq 3$ is the region to the right of line $x = 3$ The line $y = 4$ is the line that passes through the point (0,4) and is parallel to X-axis. $y \leq 4$ is the region below the line $y = 4$ Region represented by $x \geq 0$ and $y \geq 0$:

since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations $x \geq 0$ and $y \geq 0$ The feasible region determined by subject to the constraints are $x - y \geq 0$, $-x + 2y \geq 2$, $x \geq 3$, $y \leq 4$, non negative $x \geq 0$ and $y \geq 0$ are as follows.



The corner points of the feasible region are

$$C\left(3, \frac{5}{2}\right), D(3,3), E(4,4) \text{ and } F(6,4)$$

The values of objective function at the corner points are as follows:

$$\text{Corner point: } z = x - 5y + 20$$

$$C\left(3, \frac{5}{2}\right) : 3 - 5 \times \frac{5}{2} + 20 = \frac{21}{2}$$

$$D(3,3) : 3 - 5 \times 3 + 20 = 8$$

$$E(4,4) : 4 - 5 \times 4 + 20 = 4$$

$$F(6,4) : 6 - 5 \times 4 + 20 = 6$$

Therefore, the minimum value of objective function Z is 4 at the point E(4,4) . Hence, $x = 4$ and $y = 4$ is the optimal solution of the given LPP.

Thus, the optimal value of objective function Z is 4.

OR

First, we will convert the given inequations into equations, we obtain the following equations:

$$2x + y = 18, 3x + 2y = 34$$

Region represented by $2x + y \geq 18$:

The line $2x + y = 18$ meets the coordinate axes at A(9,0) and B(0,18) respectively. By joining these points we obtain the line $2x + y = 18$ Clearly (0,0) does not satisfy the inequation $2x + y \geq 18$. So, the region in xy plane which does not contain the origin represents the solution set of the inequation $2x + y \geq 18$.

Region represented by $3x + 2y \leq 34$:

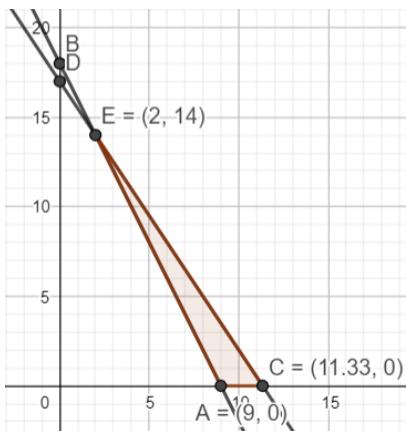
The line $3x + 2y = 34$ meets the coordinate axes at

$$C\left(\frac{34}{3}, 0\right) \text{ and } D(0,17) \text{ respectively.}$$

By joining these points we obtain the line $3x + 2y = 34$ Clearly (0,0) satisfies the inequation $3x + 2y \leq 34$. So, the region containing the origin represents the solution set of the inequation $3x + 2y \leq 34$

The corner points of the feasible region are A(9,0)

$$C\left(\frac{34}{3}, 0\right) \text{ and } E(2,14) \text{ and feasible region is bounded}$$



The values of Z objective function at these corner points are as follows.

Corner point	$Z = 50x + 30y$
A(9, 0)	$50 \times 9 + 3 \times 0 = 450$
$C\left(\frac{34}{3}, 0\right)$	$50 \times \frac{34}{3} + 30 \times 0 = \frac{1700}{3}$
E(2, 14)	$50 \times 2 + 30 \times 14 = 520$

Therefore, the maximum value of objective function Z is

$\frac{1700}{3}$ at the point $\left(\frac{34}{3}, 0\right)$. Hence, $x = \frac{34}{3}$ and $y = 0$ is the optimal solution of the given LPP.

Thus, the optimal value of objective function Z is $\frac{1700}{3}$.

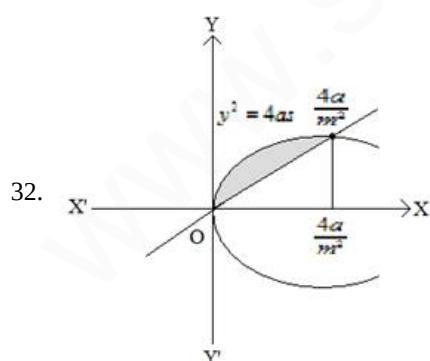
$$\begin{aligned}
 31. \text{ Let, } y &= \tan^{-1} \left[\frac{a+b \tan x}{b-a \tan x} \right] \\
 \Rightarrow y &= \tan^{-1} \left[\frac{\frac{a+b \tan x}{b}}{\frac{b-a \tan x}{b}} \right] \\
 \Rightarrow y &= \tan^{-1} \left[\frac{\frac{a}{b} + \tan x}{1 - \frac{a}{b} \tan x} \right] \\
 \Rightarrow y &= \tan^{-1} \left[\frac{\tan(\tan^{-1} \frac{a}{b}) + \tan x}{1 - \tan(\tan^{-1} \frac{a}{b}) \times \tan x} \right] \\
 \Rightarrow y &= \tan^{-1} \left[\tan \left(\tan^{-1} \frac{a}{b} + x \right) \right] \\
 \Rightarrow y &= \tan^{-1} \left(\frac{a}{b} \right) + x
 \end{aligned}$$

Differentiate it with respect to x,

$$\begin{aligned}
 \frac{dy}{dx} &= 0 + 1 \\
 \therefore \frac{dy}{dx} &= 1
 \end{aligned}$$

Hence the derivative is equal to 1 for the given function.

Section D



$$y^2 = 4ax \dots\dots(1)$$

$$y = mx \dots\dots(2)$$

Using (2) in (1), we get,

$$(mx)^2 = 4ax$$

$$\Rightarrow m^2 x^2 = 4ax$$

$$x(m^2 x - 4a) = 0$$

$$\Rightarrow x = 0, \frac{4a}{m^2}$$

From (2),

When $x = 0, y = m(0) = 0$

$$\text{When } x = \frac{4a}{m^2}, y = m \times \frac{4a}{m^2} = \frac{4a}{m}$$

\therefore points of intersection are $(0, 0)$ and $(\frac{4a}{m^2}, \frac{4a}{m})$

$$\text{Area} = \int_0^{4a/m^2} \sqrt{4ax} dx - \int_0^{4a/m^2} mx dx$$

$$= \sqrt{4a} \int_0^{4a/m^2} \sqrt{x} dx - m \int_0^{4a/m^2} x dx$$

$$= \sqrt{4a} \left[\frac{2}{3} x^{\frac{3}{2}} \right]_0^{4a/m^2} - m \left[\frac{x^2}{2} \right]_0^{4a/m^2}$$

$$= \sqrt{4a} \left[\frac{2}{3} \left(\frac{4a}{m^2} \right)^{\frac{3}{2}} - 0 \right] - \frac{m}{2} \left[\left(\frac{4a}{m^2} \right)^2 - 0 \right]$$

$$= \frac{2}{3m^3} (4a)^2 - \frac{1}{2m^3} (4a)^2$$

$$= \frac{(4a)^2}{m^3} \left[\frac{2}{3} - \frac{1}{2} \right]$$

$$= \frac{8a^2}{3m^3} \text{ sq unit.}$$

33. We observe the following properties of relation R.

Reflexivity: For any $a \in \mathbb{N}$

$$a - a = 0 = 0 \times n$$

$\Rightarrow a - a$ is divisible by n

$$\Rightarrow (a, a) \in R$$

Thus, $(a, a) \in R$ for all $a \in \mathbb{Z}$. So, R is reflexive on \mathbb{Z}

Symmetry: Let $(a, b) \in R$. Then,

$$(a, b) \in R$$

$\Rightarrow (a - b)$ is divisible by n

$$\Rightarrow (a - b) = np \text{ for some } p \in \mathbb{Z}$$

$$\Rightarrow b - a = n(-p)$$

$\Rightarrow b - a$ is divisible by n $[\because p \in \mathbb{Z} \Rightarrow -p \in \mathbb{Z}]$

$$\Rightarrow (b, a) \in R$$

Thus, $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in \mathbb{Z}$.

So, R is symmetric on \mathbb{Z} .

Transitivity: Let $a, b, c \in \mathbb{Z}$ such that $(a, b) \in R$ and $(b, c) \in R$. Then,

$$(a, b) \in R$$

$\Rightarrow (a - b)$ is divisible by n

$$\Rightarrow a - b = np \text{ for some } p \in \mathbb{Z}$$

and, $(b, c) \in R$

$\Rightarrow (b - c)$ is divisible by n

$$\Rightarrow b - c = nq \text{ for some } q \in \mathbb{Z}$$

$\therefore (a, b) \in R$ and $(b, c) \in R$

$$\Rightarrow a - b = np \text{ and } b - c = nq$$

$$\Rightarrow (a - b) + (b - c) = np + nq$$

$$\Rightarrow a - c = n(p + q)$$

$\Rightarrow a - c$ is divisible by n $[\because p, q \in \mathbb{Z} \Rightarrow p + q \in \mathbb{Z}]$

$$\Rightarrow (a, c) \in R$$

Thus, $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in \mathbb{Z}$.

OR

Given that, $A = \mathbb{R} - \{3\}$, $B = \mathbb{R} - \{1\}$.

$$f: A \rightarrow B \text{ is defined by } f(x) = \frac{x-2}{x-3} \quad \forall x \in A$$

For injectivity

$$\text{Let } f(x_1) = f(x_2) \Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$$

$$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$$

$$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 3x_2 - 2x_1 + 6$$

$$\Rightarrow -3x_1 - 2x_2 = -3x_2 - 2x_1$$

$$\Rightarrow -x_1 = -x_2 \Rightarrow x_1 = x_2$$

So, $f(x)$ is an injective function

For surjectivity

$$\text{Let } y = \frac{x-2}{x-3} \Rightarrow x - 2 = xy - 3y$$

$$\Rightarrow x(1-y) = 2 - 3y \Rightarrow x = \frac{2-3y}{1-y}$$

$$\Rightarrow x = \frac{3y-2}{y-1} \in A, \forall y \in B \text{ [codomain]}$$

So, $f(x)$ is surjective function.

Hence, $f(x)$ is a bijective function.

34. Let $\frac{1}{x} = u, \frac{1}{y} = v$ and $\frac{1}{z} = w$

$$2u + 3v + 10w = 4$$

$$4u - 6v + 5w = 1$$

$$6u + 9v - 20w = 2$$

$$A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X = \begin{bmatrix} u \\ v \\ w \end{bmatrix}, B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$\text{Now, } |A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix}$$

$$= 2[120 - 45] - 3[-80 - 30] + 10[36 + 36]$$

$$= 150 + 330 + 720 = 1200 \neq 0$$

$\Rightarrow A$ is non-singular and hence A^{-1} exists.

$$\text{Now, } A_{11} = 75, A_{12} = 110, A_{13} = 72$$

$$A_{21} = 150, A_{22} = -100, A_{23} = 0$$

$$A_{31} = 75, A_{32} = 30, A_{33} = -2$$

$$\therefore adj A = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} (adj A) = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \frac{1}{1200} \begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

$$\begin{bmatrix} y \\ v \\ w \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

$$u = \frac{1}{2}, v = \frac{1}{3}, w = \frac{1}{5}$$

$$\frac{1}{x} = \frac{1}{2}, \frac{1}{y} = \frac{1}{3}, \frac{1}{z} = \frac{1}{5}$$

$$x = 2, y = 3, z = 5$$

35. Suppose the point $(1, 0, 0)$ be P and the point through which the line passes be $Q(1, -1, -10)$. The line is parallel to the vector

$$\vec{b} = 2\hat{i} - 3\hat{j} + 8\hat{k}$$

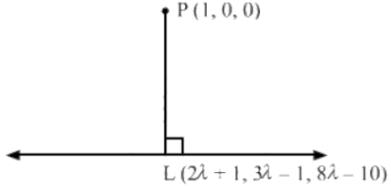
Now,

$$\vec{PQ} = 0\hat{i} - \hat{j} - 10\hat{k}$$

$$\therefore \vec{b} \times \vec{PQ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 8 \\ 0 & -1 & -10 \end{vmatrix}$$

$$\begin{aligned}
&= 38\hat{i} + 20\hat{j} - 2\hat{k} \\
&\Rightarrow |\vec{b} \times \vec{PQ}| = \sqrt{38^2 + 20^2 + 2^2} \\
&= \sqrt{1444 + 400 + 4} \\
&= \sqrt{1848} \\
d &= \frac{|\vec{b} \times \vec{PQ}|}{|\vec{b}|} \\
&= \frac{\sqrt{1848}}{\sqrt{77}} \\
&= \sqrt{24} \\
&= 2\sqrt{6}
\end{aligned}$$

Suppose L be the foot of the perpendicular drawn from the point P(1,0,0) to the given line-



The coordinates of a general point on the line

$$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$$

$$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} = \lambda$$

$$\Rightarrow x = 2\lambda + 1$$

$$y = -3\lambda - 1$$

$$z = 8\lambda - 10$$

Suppose the coordinates of L be

$$(2\lambda + 1, -3\lambda - 1, 8\lambda - 10)$$

Since, The direction ratios of PL are proportional to,

$$2\lambda + 1 - 1, -3\lambda - 1 - 0, 8\lambda - 10 - 0, \text{ i.e., } 2\lambda, -3\lambda - 1, 8\lambda - 10$$

Since, The direction ratios of the given line are proportional to 2, -3, 8, but PL is perpendicular to the given line.

$$\therefore 2(2\lambda) - 3(-3\lambda - 1) + 8(8\lambda - 10) = 0$$

$\Rightarrow \lambda = 1$ Substituting $\lambda = 1$ in $(2\lambda + 1, -3\lambda - 1, 8\lambda - 10)$ we get the coordinates of L as (3, -4, -2). Equation of the line PL is given by

$$\frac{x-1}{3-1} = \frac{y-0}{-4-0} = \frac{z-0}{-2-0}$$

$$= \frac{x-1}{1} = \frac{y}{-2} = \frac{z}{-1}$$

$$\Rightarrow \vec{r} = \hat{i} + \lambda(\hat{i} - 2\hat{j} - \hat{k})$$

OR

$$\text{Given lines are } \vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$$

$$\text{and } \vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$$

On comparing both equations of lines with

$$\vec{r} = \vec{a} + \lambda\vec{b} \text{ respectively, we get,}$$

$$\vec{a}_1 = \hat{i} + \hat{j} - \hat{k}, \vec{b}_1 = 3\hat{i} - \hat{j}$$

$$\text{and } \vec{a}_2 = 4\hat{i} - \hat{k}, \vec{b}_2 = 2\hat{i} + 3\hat{k}$$

$$\text{Now } \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix}$$

$$= \hat{i}(-3 - 0) - \hat{j}(9 - 0) + \hat{k}(0 + 2)$$

$$= -3\hat{i} - 9\hat{j} + 2\hat{k}$$

$$\text{and } \vec{a}_2 - \vec{a}_1 = (4\hat{i} - \hat{k}) - (\hat{i} + \hat{j} - \hat{k}) = 3\hat{i} - \hat{j}$$

$$\text{Now, } (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (3\hat{i} - \hat{j}) \cdot (-3\hat{i} - 9\hat{j} + 2\hat{k})$$

$$= -9 + 9 = 0$$

Hence, given lines are coplanar.

Now, cartesian equations of given lines are

$$\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$$

and $\frac{x-4}{2} = \frac{y-0}{0} = \frac{z+1}{3}$

Then, equation of plane containing them is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ x - 1 & y - 1 & z + 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix} = 0$$

$$(x - 1)(-3 - 0) - (y - 1)(9 - 0) + (z + 1)(0 + 2) = 0$$

$$-3x + 3 - 9y + 9 + 2z + 2 = 0$$

$$3x + 9y - 2z = 14$$

Section E

36. Read the text carefully and answer the questions:

For an audition of a reality singing competition, interested candidates were asked to apply under one of the two musical genres-folk or classical and under one of the two age categories-below 18 or 18 and above.

The following information is known about the 2000 application received:

- 960 of the total applications were the folk genre.
- 192 of the folk applications were for the below 18 category.
- 104 of the classical applications were for the 18 and above category.

(i) According to given information, we construct the following table.

Given, total applications = 2000

	Folk Genre	Classical Genre
	960 (given)	$2000 - 960 = 1040$
Below 18	192 (given)	$1040 - 104 = 936$
18 or Above 18	$960 - 192 = 768$	104 (given)

Let E_1 = Event that application for folk genre

E_2 = Event that application for classical genre

A = Event that application for below 18

B = Event that application for 18 or above 18

$$\therefore P(E_2) = \frac{1040}{2000}$$

$$\text{and } P(B \cap E_2) = \frac{104}{2000}$$

$$\text{Required Probability} = \frac{P(B \cap E_2)}{P(E_2)}$$

$$= \frac{\frac{104}{2000}}{\frac{1040}{2000}} = \frac{1}{10}$$

$$(ii) \text{ Required probability} = P\left(\frac{\text{folk}}{\text{below 18}}\right)$$

$$= P\left(\frac{E_1}{A}\right)$$

$$= \frac{P(E_1 \cap A)}{P(A)}$$

$$\text{Now, } P(E_1 \cap A) = \frac{192}{2000}$$

$$\text{and } P(A) = \frac{192+936}{2000} = \frac{1128}{2000}$$

$$\therefore \text{Required probability} = \frac{\frac{192}{2000}}{\frac{1128}{2000}} = \frac{192}{1128} = \frac{8}{47}$$

(iii) Here,

$$P(A) = 0.4, P(B) = 0.8 \text{ and } P(B|A) = 0.6$$

$$\therefore P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$\Rightarrow P(B \cap A) = P(B|A) \cdot P(A)$$

$$= 0.6 \times 0.4 = 0.24$$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\begin{aligned}
 &= 0.4 + 0.8 - 0.24 \\
 &= 1.2 - 0.24 = 0.96
 \end{aligned}$$

OR

Since, A and B are independent events, A' and B' are also independent. Therefore,

$$P(A' \cap B') = P(A') \cdot P(B')$$

$$= (1 - P(A))(1 - P(B))$$

$$= \left(1 - \frac{3}{5}\right) \left(1 - \frac{4}{9}\right)$$

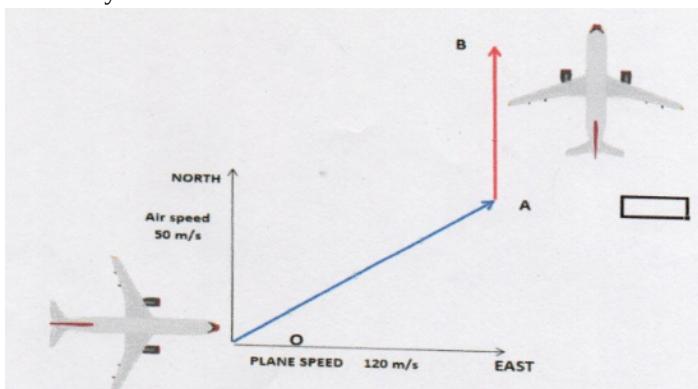
$$= \frac{2}{5} \cdot \frac{5}{9}$$

$$= \frac{2}{9}$$

37. Read the text carefully and answer the questions:

A plane started from airport O with a velocity of 120 m/s towards east. Air is blowing at a velocity of 50 m/s towards the north As shown in the figure.

The plane travelled 1 hr in OA direction with the resultant velocity. From A and B travelled 1 hr with keeping velocity of 120 m/s and finally landed at B.



(i) Resultant velocity from O to A

$$\begin{aligned}
 &= \sqrt{(V_{\text{Plane}})^2 + (V_{\text{wind}})^2} \\
 &= \sqrt{(120)^2 + (50)^2} \\
 &= \sqrt{14400 + 2500} \\
 &= \sqrt{16900} \\
 &= 130 \text{ m/s}
 \end{aligned}$$

$$(ii) \tan \theta = \frac{V_{\text{wind}}}{V_{\text{aeroplane}}}$$

$$\tan \theta = \frac{50}{120}$$

$$\tan \theta = \frac{5}{12}$$

$$\theta = \tan^{-1}\left(\frac{5}{12}\right)$$

(iii) Displacement from O to A = Resultant velocity \times time

$$\begin{aligned}
 |\vec{OA}| &= |\vec{V}| \times t \\
 &= 130 \times \frac{18}{5} \times 1 \\
 &= 468 \text{ km}
 \end{aligned}$$

OR

Since, from A to B both Aeroplane and wind have velocity in North direction.

So,

$$\vec{V}_{\text{plane, A to B}} = 120 + 50$$

$$= 170 \text{ m/s}$$

38. Read the text carefully and answer the questions:

The temperature of a person during an intestinal illness is given by $f(x) = -0.1x^2 + mx + 98.6$, $0 \leq x < 12$, m being a constant, where $f(x)$ is the temperature in $^{\circ}\text{F}$ at x days.



(i) $f(x) = -0.1x^2 + mx + 98.6$, being a polynomial function, is differentiable everywhere, hence, differentiable in $(0, 12)$.

(ii) $f(x) = -0.2x + m$

At Critical point

$$0 = -0.2 \times 6 + m$$

$$m = 1.2$$