MATHEMATICS – FULL SYLLABUS
MOCK TEST PAPER - 1
CBSE BOARD CLASS – XII (2025-26)

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. Matrices A and B will be inverse of each other only if **[1]**

a) AB = 0, BA = I b) AB = BA = I

c) AB = BA = 0 d) AB = BA

2. $\begin{vmatrix} \cos 70^\circ & \sin 20^\circ \\ \sin 70^\circ & \cos 20^\circ \end{vmatrix}$ = ? **[1]**

a) $\cos 50^o$ b) $\sin 50^o$

c) 1 d) 0

3. Adj.(KA) = ______ **[1]**

a) K^{n-1} Adj. A b) None of these

c) K Adj. A d) K^n Adj.A

4. The value of k for which $f(x) = \begin{cases} \frac{3x + 4\tan x}{2}, \text{ when } x \neq 0 \\ k, \text{ when } x = 0 \end{cases}$ is continuous at x = 0, is **[1]**

a) 3 b) 7

c) None of these d) 4

5. If a line makes angles $\alpha, \beta, \gamma, \delta$ with four diagonals of a cube then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\theta$ is equal to **[1]**

a) $\frac{1}{3}$ b) $\frac{2}{3}$

c) $\frac{8}{3}$ d) $\frac{4}{3}$

6. The solution of the DE $x\frac{dy}{dx} = \cot y$ is **[1]**

a) x sec y = C

b) x cos y = C

c) x tan y = C

d) none of these

7. If the constraints in a linear programming problem are changed **[1]**

a) he change in constraints is ignored

b) the problem is to be re-evaluated

c) the objective function has to be modified

d) solution is not defined

8. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined at an angle θ such that $|\vec{a} + \vec{b}| < 1$, then **[1]**

a) $\theta > \frac{2\pi}{3}$

b) $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$

c) $\theta < \frac{\pi}{3}$

d) None of these

9. $\int \frac{dx}{\sqrt{(x-3)^2-1}} =?$ **[1]**

a) $\log|x + \sqrt{x^2 - 6x + 8}| + C$

b) None of these

c) $\log|(x-3) - \sqrt{x^2 - 6x + 8}| + C$

d) $\log|(x-3) + \sqrt{x^2 - 6x + 8}| + C$

10. If $\begin{bmatrix} x-y & 2x-y \\ 2x+z & 3z+w \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 5 & 13 \end{bmatrix}$ then **[1]**

a) z = 4, w = 3

b) z = 1, w = 2

c) z = 3, w = 4

d) z = 2, w = -1

11. The solution set of the inequation 2x + y > 5 is **[1]**

a) None of these

b) open half plane not containing the origin

c) half plane that contains the origin

d) whole xy-plane except the points lying on the line 2x + y = 5

12. Let $\hat{a}, \hat{b}$ be two unit vectors and θ be the angle between them. What is $\sin\left(\frac{\theta}{2}\right)$ equal to? **[1]**

a) $\frac{|\hat{a}-\hat{b}|}{2}$

b) $\frac{|\hat{\mathbf{a}}-\hat{\mathbf{b}}|}{2}$

c) $\frac{|\hat{\mathbf{a}}-\hat{\mathbf{b}}|}{4}$

d) $\frac{|\hat{\mathbf{a}}+\hat{\mathbf{b}}|}{4}$

13. If A and B are square matrices of order 3, such that Det.A = –1, Det.B =3 then, the determinant of 3AB is equal to **[1]**

a) –27

b) -81

c) –9

d) 81

14. If A and B are two events such that P(A) = $\frac{1}{2}$, P(B) = $\frac{1}{3}$, $P(\frac{A}{B}) = \frac{1}{4}$ then $P\left(A' \cap B'\right)$ equals **[1]**

a) $\frac{1}{12}$

b) $\frac{3}{16}$

c) $\frac{1}{4}$

d) $\frac{3}{4}$

15. The solution of the equation (2y - 1) dx - (2x + 3)dy = 0 is: **[1]**

a) $\frac{2x-1}{2y-1} = k$

b) $\frac{2x-1}{2y+3} = k$

c) $\frac{2y+1}{2x-3} = k$

d) $\frac{2x+3}{2y-1} = k$

16. ABCD is a parallelogram with AC and BD as diagonals. Then, $\overrightarrow{AC} - \overrightarrow{BD} =$ **[1]**

a) $3\overrightarrow{AB}$

b) $\overrightarrow{AB}$

c) $4\overrightarrow{AB}$

d) $2\overrightarrow{AB}$

17. If y = $\sin^{-1}\left\{\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right\}$ then $\frac{dy}{dx} = ?$ **[1]**

a) $\frac{1}{2\sqrt{1-x^2}}$

b) None of these

c) $\frac{1}{2(1+x^2)}$

d) $\frac{-1}{2\sqrt{1-x^2}}$

18. If (a_1, b_1, c_1) and (a_2, b_2, c_2) be the direction ratios of two parallel lines then **[1]**

a) $a_1^2 + b_1^2 + c_1^2 = a_2^2 + b_2^2 + c_2^2$

b) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

c) $a_1 = a_2$, $b_1 = b_2$, $c_1 = c_2$

d) $a_1a_2 + b_1b_2 + c_1c_2 = 0$

19. **Assertion (A):** f(x) = $2x^3 - 9x^2 + 12x - 3$ is increasing outside the interval (1, 2). **[1]**

Reason (R): f'(x) < 0 for x ∈ (1, 2).

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The Greatest Integer Function f: R→R, given by f(x) = [x] is one-one. **[1]**

Reason (R): A function f: A →B is said to be injective if f(a) = f(b) ⇒ a = b.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. For the principal values, evaluate $\sin^{-1}\left[\cos\left\{2cosec^{-1}(-2)\right\}\right]$ **[2]**

OR

$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

22. Find the intervals in which the function f given by f(x) = $2x^2 - 3x$ is decreasing. **[2]**

23. Prove that the function does not have maxima or minima: h(x) = $x^3 + x^2 + x + 1$ **[2]**

OR

A circular disc of radius 3 cm is being heated. Due to expansion, its radius increases at the rate of 0.05 cm/s. Find the rate at which its area is increasing when radius is 3.2 cm.

24. Evaluate: $\int \frac{1}{a^2\sin^2 x + b^2\cos^2 x} dx$ **[2]**

25. Find the point of local maxima or local minima and the corresponding local maximum and minimum values of function: f(x) = $x^4 - 62x^2 + 120x + 9$ **[2]**

Section C

26. Evaluate the definite integral $\int_1^2 \frac{5x^2}{x^2+4x+3}$ **[3]**

27. Three distinguishable balls are distributed in three cells. Find the conditional probability that all the three occupy the same cell, given that at least two of them are in the same cell. **[3]**

28. Evaluate: $\int \frac{(2x+1)}{(x+2)(x-3)} dx$. **[3]**

OR

Prove $\int_0^{\frac{\pi}{2}} \sin^3 x dx = \frac{2}{3}$

29. Find the particular solution of the differential equation $2ye^{x/y}dx + (y - 2xe^{x/y})dy = 0$, given that x = 0, when y = 1. **[3]**

OR

Solve the differential equation $(1 + x^2) \frac{dy}{dx} + (1 + y^2) = 0$, given that y = 1, when x = 0

30. Maximize Z = 100x + 170y subject to **[3]**

$3x+2y \leq 3600$

$x+4y \leq 1800$

$x \geq 0, y \geq 0$

OR

Maximise the function Z = 11x + 7y, subject to the constraints: $x \leqslant 3, y \leqslant 2, x \geqslant 0, y \geqslant 0$.

31. If x = a sin 2t (1 + cos 2t) and y = b cos 2t (1 – cos 2t), then find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$. **[3]**

Section D

32. Using integration, find the area of the region in the first quadrant enclosed by the Y-axis, the line y = x and the circle $x^2 + y^2 = 32$. **[5]**

33. Show that the function $f : R \rightarrow \{x \in R : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in R$ is one-one and onto function. **[5]**

OR

Show that the function f: $R_0 \rightarrow R_0$, defined as f (x) = $\frac{1}{x}$, is one-one onto, where R_0 is the set non-zero real numbers. Is the result true, if the domain R_0 is replaced by N with co-domain being same as R_0?

34. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations 2x - 3y + 5z = 11; 3x + 2y - 4z = -5; x + y - 2z = -3 **[5]**

35. Find the image of the point (0, 2, 3) in the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$ **[5]**

OR

Find the coordinates where the line thorough (3, -4, -5) and (2, -3, 1) crosses the plane 2x + y + z = 7.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

There are two antiaircraft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.

(i) How is Bayes' theorem different from conditional probability?

(ii) Write the rule of Total Probability.

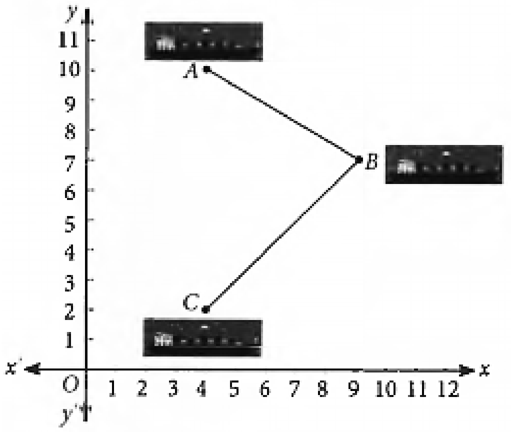
(iii) What is the probability that the shell fired from exactly one of them hit the plane?

OR

If it is know that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B?

37. **Read the text carefully and answer the questions:** **[4]**

A barge is pulled into harbour by two tug boats as shown in the figure.



(i) Find position vector of A.

(ii) Find position vector of B.

(iii) Find the vector $\overrightarrow{AC}$ in terms of $\hat{i}, \hat{j}$.

OR

If $\vec{A} = 4\hat{i} + 3\hat{j}$ and $\vec{B} = 3\hat{i} + 4\hat{j}$, then find $|\vec{A}| + |\vec{B}|$

38. **Read the text carefully and answer the questions:** **[4]**

In an elliptical sport field, the authority wants to design a rectangular soccer field with the maximum possible area. The sport field is given by the graph of $\frac{x^2}{a^2} + \frac{y^2}{b^2}$ = I

(i) If the length and the breadth of the rectangular field be 2x and 2y respectively, then find the area function in terms of x.

(ii) Find the critical point of the function.

Solution

Section A

1.

(b) AB = BA = I

Explanation: Here it is given that A & B are inverse of each other.

$\therefore$ A^{-1} = B ...(i)

Also B^{-1} = A ...(ii)

From definition of inverse matrix, we know that-

AA^{-1} = I

$\therefore$ A^{-1} = B ...from eq (i)

$\Rightarrow AB = AA^{-1}$ = I...(iii)

Similarly, BB^{-1} = I

$\therefore$ B^{-1} = A ...(from eq(ii))

$\Rightarrow$ BA = BB^{-1} = I ...(iv)

So, from (iii) and (iv), we get

AB = BA = I

2.

(d) 0

Explanation: By evaluating given determinant and using sin (90 - A) = cos A. we get value of det. = 0

3. **(a)** K^{n-1} Adj. A

Explanation: Adj. (KA) = K^{n-1} Adj.A, where K is a scalar and A is a n $\times$ n matrix.

4.

(b) 7

Explanation: $\Rightarrow f(x) = \frac{3x+4\tan x}{x}$ is continuous at x = 0

$\Rightarrow f(x) = \lim\limits_{x\to 0} \frac{3x+4\tan x}{x}$

$\Rightarrow f(x) = \lim\limits_{x\to 0} \frac{3x}{x} + \frac{4\tan x}{x}$

$\Rightarrow f(x) = 3 + 4\lim\limits_{x\to 0} \frac{\tan x}{x}$

$\Rightarrow$ f(x) = 3 + 4

$\therefore$ k = 7

5.

(d) $\frac{4}{3}$

Explanation: $\frac{4}{3}$

6.

(b) x cos y = C

Explanation: Given: $x\frac{dy}{dx} = \cot y$

Separating the variable, we obtain

$\frac{dy}{coty} = \frac{dx}{x}$

$\tan y\ dy = \frac{dx}{x}$

Integrating both sides, we obtain

$\int \tan y\, dy = \int \frac{dx}{x}$

log sec y = log x + log c

C=xcos y

7.

(b) the problem is to be re-evaluated

Explanation: The optimisation of the objective function of a LPP is governed by the constraints. Therefore, if the constraints in a linear programming problem are changed, then the problem needs to be re-evaluated.

8.

(b) $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$

Explanation: We know that,

If $\vec{a}$and$\vec{b}$ are two-unit vectors inclined at an angle θ

$|\vec{a} + \vec{b}| = \left|2\cos\frac{\theta}{2}\right|$

According to question,

$|\vec{a} + \vec{b}| < 1$ or -1< $\vec{a} + \vec{b}$<1

⇒$\left|2\cos\frac{\theta}{2}\right| < 1$

$\implies -1 < 2cos\frac{\theta}{2} < 1$

⇒$\frac{-1}{2} < \cos\frac{\theta}{2} < \frac{1}{2}$

⇒$\frac{2\pi}{3} > \frac{\theta}{2} > \frac{\pi}{3}$

⇒$\frac{2\pi}{3} < \theta < \frac{4\pi}{3}$

9.

(d) $\log\left|(x-3) + \sqrt{x^2 - 6x + 8}\right| + C$

Explanation: The given integral is $\int \frac{dx}{\sqrt{(x-3)^2 - 1}}$

since we know that,$\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left|x + \sqrt{x^2 - a^2}\right|$

⇒ $\int \frac{dx}{\sqrt{(x-3)^2 - 1}} = \log\left|x - 3 + \sqrt{x^2 - 6x + 9 - 1}\right|$

⇒ $\int \frac{dx}{\sqrt{(x-3)^2 - 1}} = \log\left|x - 3 + \sqrt{x^2 - 6x + 8}\right|$

10.

(c) z = 3, w = 4

Explanation: By comparing L.H.S and R.H.S

x - y = -1 ...(i)

2x - y = 0 ...(ii)

2x+ z = 5 ...(iii)

3z + w = 13 ...(iv)

Using (i) in equation (ii)

x = -1 + y

ii becomes, -2 + 2y - y = 0

y = 2

x = 1

Putting x in (iii)

2 + z = 5

z = 3

Putting z in (iv)

9 + w = 13

w = 4

11.

(b) open half plane not containing the origin

Explanation: open half plane not containing the origin

On putting x = 0, y = 0 in the given inequality, we get 0 > 5, which is absurd.

Therefore, the solution set of the given inequality does not include the origin.

Thus, the solution set of the given inequality consists of the open half plane not containing the origin.

12. **(a)** $\frac{|\hat{a} - \hat{b}|}{2}$

Explanation: Given $\hat{a}$ and $\hat{b}$ are unit vectors.

Now,

$|\hat{a}+\hat{b}|^2 = (\hat{a}+\hat{b})\cdot(\hat{a}+\hat{b})$
= $|\hat{a}|^2 + |\hat{b}|^2 + 2|\hat{a}||\hat{b}|\cos\theta$
$\Rightarrow |\hat{a}+\hat{b}|^2 = 2 + 2\cos\theta$
= $2(1+\cos\theta)$...(i)
Similarly,
$|\hat{a}+\hat{b}|^2 = 2(1-\cos\theta)$ ....(ii)
From Eq. (ii)
$|\hat{a}-\hat{b}|^2 = 2 \times 2\sin^2\left(\frac{\theta}{2}\right)$
$\sin\left(\frac{\theta}{2}\right) = \frac{|\hat{a}-\hat{b}|}{2}$

13.
(b) -81
Explanation: |3AB| = 3^3|A||B| = 27(-1)(3) = -81

14.
(c) $\frac{1}{4}$
Explanation: Here, P(A) = $\frac{1}{2}$, P(B) = $\frac{1}{3}$ and $P(\frac{A}{B}) = \frac{1}{4}$
$\because P(A/B) = \frac{P(A\cap B)}{P(B)}$
$\Rightarrow$ P(A ∩ B) = $P(A/B)\cdot P(B) = \frac{1}{4}\cdot\frac{1}{3} = \frac{1}{12}$
Now, $P(A'\cap B') = 1 - P(A\cup B)$
$= 1 - [P(A) + P(B) - P(A\cap B)]$
$= 1 - \left[\frac{1}{2} + \frac{1}{3} - \frac{1}{12}\right] = 1 - \left[\frac{6+4-1}{12}\right]$
$= 1 - \frac{9}{12} = \frac{3}{12} = \frac{1}{4}$

15.
(d) $\frac{2x+3}{2y-1} = k$
Explanation: Given that (2y - 1)dx - (2x + 3)dy = 0
$\Rightarrow \frac{(2y-1)}{dy} = \frac{(2x+3)}{dx}$
On integrating both sides, we get
$\frac{1}{2}\log(2x+3) = \frac{1}{2}\log(2y-1) + \log C$
$\Rightarrow \frac{1}{2}$[log (2x + 3) - log(2y - 1)] = log C
$\Rightarrow \frac{1}{2}\log\left(\frac{2x+3}{2y-1}\right)$ = log C
$\Rightarrow \left(\frac{2x+3}{2y-1}\right)^{1/2} = C$
$\Rightarrow \frac{2x+3}{2y-1} = C^2$
$\Rightarrow \frac{2x+3}{2y-1} = k$ where K = C^2

16.
(d) $2\overrightarrow{AB}$
Explanation: Given: ABCD, a parallelogram with diagonals AC and BD.
Then,
$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$
$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD}$
$\Rightarrow \overrightarrow{BD} = \overrightarrow{AD} - \overrightarrow{AB}$
$\therefore \overrightarrow{AC} - \overrightarrow{BD} = \overrightarrow{AB} + \overrightarrow{BC} - (\overrightarrow{AD} - \overrightarrow{AB}) = \vec{AB} + \vec{BC} - \vec{AD} + \vec{AB} = 2\overrightarrow{AB}$ $[\because \overrightarrow{AD} = \overrightarrow{BC}]$

17.
(d) $\frac{-1}{2\sqrt{1-x^2}}$
Explanation: Put x = cos 2θ ,we get
$\Rightarrow y = \sin^{-1}\left(\frac{\sqrt{1+\cos 2\theta}}{2} + \frac{\sqrt{1-\cos 2\theta}}{2}\right)$

$\Rightarrow y=\sin^{-1}\left(\frac{\sqrt{2\cos^2 2\theta}}{2}+\frac{\sqrt{2 \cdot \sin^2 \theta}}{2}\right)$

$\Rightarrow y=\sin^{-1}\left(\frac{\cos 2\theta}{\sqrt{2}}+\frac{\sin 2\theta}{\sqrt{2}}\right)$

$\Rightarrow y=\sin^{-1}\left(\sin\left(\frac{\pi}{4}+2\theta\right)\right.$

$\Rightarrow y=\frac{\pi}{4}+2\theta$

$\Rightarrow \frac{dy}{d\theta}=2$

Put $\theta=\frac{\cos^{-1}x}{2}$,we get

$\Rightarrow \frac{\mathrm{d}\theta}{\mathrm{dx}}=\frac{-1}{4\sqrt{1-\mathrm{x}^2}}$

$\therefore \frac{dy}{dx}=\frac{-1}{2\sqrt{1-x^2}}$

18.

(b) $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

Explanation: We know that if there are two parallel lines then their direction ratios must have a relation $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: Assertion: We have, f(x) = $2x^3 - 9x^2 + 12x - 3$

$\Rightarrow$ f'(x) = $6x^2 - 18x + 12$

For increasing function, f'(x) ≥ 0

$\therefore 6(x^2 - 3x + 2) \geq 0$

$\Rightarrow 6(x - 2)(x - 1) \geq 0$

$\Rightarrow x \leq 1$ and $x \geq 2$

$\therefore$ f(x) is increasing outside the interval (1, 2), therefore it is a true statement.

Reason: Now, f'(x) < 0

$\Rightarrow 6(x - 2)(x - 1) < 0$

$\Rightarrow 1 < x < 2$

$\therefore$ Assertion and Reason are both true but Reason is not the correct explanation of Assertion.

20.

(d) A is false but R is true.

Explanation: Assertion is false because f(1.9) = [1.9] = 1 and f(1.8) = [1.8] = 1. So distinct elements of domain have same image.

Therefore Greatest Integer function f: R$\rightarrow$R, given by f(x) = [x] is not one-one.

The reason is true because by definition a function f: A$\rightarrow$ B is said to be injective if distinct elements of domain has distinct images i.e. f(a) = f(b) $\Rightarrow$ a = b.

Section B

21. First of all we need to find the principal value for $\text{cosec}^{-1}(-2)$

Let,

$\text{cosec}^{-1}(-2) = Y$

$\Rightarrow$ cosec y = –2

$\Rightarrow$ –cosec y = 2

$\Rightarrow -\text{cosec}\,\frac{\pi}{6} = 2$

As we know that $\text{cosec}(-\theta) = -\text{cosec}\theta$

$\therefore -\text{cosec}\,\frac{\pi}{6} = \text{cosec}\left(\frac{-\pi}{6}\right)$

The range of principal value of cosec^{-1} (-2) is $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]-\{0\}$ and

$\text{cosec}\left(\frac{-\pi}{6}\right) = -2$

Thus,the princi value of $\text{cosec}^{-1}(-2)$ is $\frac{-\pi}{6}$.

$\therefore$ Now, the question changes to

$\text{Sin}^{-1}[\cos\frac{-\pi}{6}]$

$\text{Cos}(-\theta) = \cos(\theta)$

∴ we can write the above expression as

$\text{Sin}^{-1}[\cos\frac{\pi}{6}]$

Let,

$\text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right) = Y$

$\Rightarrow \sin y = \frac{\sqrt{3}}{2}$

$\Rightarrow Y = \frac{\pi}{3}$

The range of principal value of $\sin^{-1}$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ and $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

Therefore, the principal value of $\text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is $\frac{\pi}{3}$

Hence, the principal value of the given equation is $\frac{\pi}{3}$

OR

Let $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = y$

$\Rightarrow \cos y = \frac{\sqrt{3}}{2}$

$\Rightarrow \cos y = \cos\frac{\pi}{6}$

$\Rightarrow y = \frac{\pi}{6}$

Since, the principal value branch of $\cos^{-1}$ is $[0, \pi]$.

Therefore, principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is $\frac{\pi}{6}$.

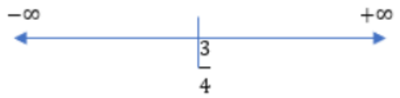
22. It is given that function $f(x) = 2x^2 - 3x$

$\Rightarrow f'(x) = 4x - 3$

If $f'(x) = 0$, then we get,

$x = \frac{3}{4}$

So, the point $x = \frac{3}{4}$, divides the real line into two disjoint intervals, $\left(-\infty, \frac{3}{4}\right)$ and $\left(\frac{3}{4}, \infty\right)$



Now, in interval $\left(-\infty, \frac{3}{4}\right)$, $f'(x) = 4x - 3 < 0$

Therefore, the given function (f) is strictly decreasing in interval $\left(-\infty, \frac{3}{4}\right)$

23. $h(x) = x^3 + x^2 + x + 1$

$\Rightarrow h'(x) = 3x^2 + 2x + 1$

$h(x) = 0$

$\Rightarrow 3x^2 + 2x + 1 = 0$

$\Rightarrow x = \frac{-2 \pm 2\sqrt{2}i}{6}$

$\Rightarrow \frac{-1 \pm \sqrt{2}i}{3}, \notin R$

Therefore, there does not exist $c \in R$ such that $h'(c) = 0$, i.e, there are no real critical points.

Hence, function h does not have maxima or minima.

OR

Given , $\frac{dr}{dt} = 0.05 cm/sec$

$A = \pi r^2$

Now $\frac{dA}{dt} = 2\pi r.\frac{dr}{dt}$

$= 2\pi(3.2) \times 0.05$(given r=3.2cm)

$= 0.320\pi cm^2/s$

24. Let $I = \int \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x} dx$

Dividing the numerator and denominator of the given integrand by $\cos^2 x$, we obtain

$I = \int \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x} dx = \int \frac{\sec^2 x}{a^2 \tan^2 x + b^2} dx$

Putting $\tan x = t$ and $\sec^2 x\, dx = dt$, we get

$I=\int \frac{dt}{a^2t^2+b^2}=\frac{1}{a^2}\int \frac{dt}{t^2+(b/a)^2}=\frac{1}{a^2}\times\frac{1}{b/a}\tan^{-1}\left(\frac{t}{b/a}\right)+C$

$\Rightarrow I=\frac{1}{ab}\tan^{-1}\left(\frac{at}{b}\right)+C=\frac{1}{ab}\tan^{-1}\left(\frac{a\tan x}{b}\right)+C$

25. We have Local max. value is 68 at x = 1 and local min. values are −1647 at x = -6 and -316 at x = 5

Also F'(x) = $4x^3 - 124x + 120 = 0$

$\Rightarrow 4(x^3 - 31x + 30) = 0$

For x = 1, the given equation becomes, 0

$\therefore$ x-1 is a factor,

4(x - 1)(x + 6)(x - 5) = 0

⇒ x = 1, -6, 5

$F''(1) < 0$, 1 is the point of max.

$F''(-6)$ and $f''(5) > 0$, -6 and 5 are point of min.

F(1) = 68

F(-6) = -1647

F(5) = -316

Section C

26. According to the question, $I=\int_1^2 \frac{5x^2}{x^2+4x+3}dx$

$I=5\int_1^2\left(1+\frac{-4x-3}{x^2+4x+3}\right)dx$

$=5\int_1^2 dx-5\int_1^2\frac{4x+3}{x^2+4x+3}dx$

$\Rightarrow \quad I=5[x]_1^2-5\int_1^2\frac{4x+3}{(x+3)(x+1)}dx$

Using partial fraction,

let $\frac{4x+3}{(x+3)(x+1)}=\frac{A}{x+3}+\frac{B}{x+1}$

$\Rightarrow \quad \frac{4x+3}{(x+3)(x+1)}=\frac{A(x+1)+B(x+3)}{(x+3)(x+1)}$

$\Rightarrow 4x+3=A(x+1)+B(x+3)$

Comparing the coefficients of like from both sides,

$\Rightarrow A+B=4\Rightarrow A=4-B$

and A + 3B = 3$\Rightarrow 4-B+3B=3$

$\Rightarrow \quad B=-\frac{1}{2}$, then $A=4+\frac{1}{2}=\frac{9}{2}$

Now, from Equation (i), we get

$I=5(2-1)-5\int_1^2\left(\frac{9/2}{x+3}+\frac{-1/2}{x+1}\right)dx$

$=5-5\left[\frac{9}{2}\log|x+3|-\frac{1}{2}\log|x+1|\right]_1^2$

$=5-5\left[\left(\frac{9}{2}\log 5-\frac{1}{2}\log 3\right)-\left(\frac{9}{2}\log 4-\frac{1}{2}\log 2\right)\right]$

$=5-5\left[\frac{9}{2}(\log 5-\log 4)-\frac{1}{2}(\log 3-\log 4)\right]$

$=5-5\left[\frac{9}{2}\log\frac{5}{4}-\frac{1}{2}\log\frac{3}{2}\right]$

$=5-\frac{45}{2}\log\frac{5}{4}+\frac{5}{2}\log\frac{3}{2}$

27. Since each ball can be placed in a cell in three ways. Therefore, three distinct balls can be placed in three cells in 3 $\times$ 3 $\times$ 3 = 27 ways. Consider the following events:

E = All balls are in the same cell, F = At least two balls are in the same cell.

All balls can be placed in the same cell in three ways.

$\therefore P(E)=\frac{3}{27}$

Now, we have,

P(F) = P (At least two balls are in the same cell) = 1 - F (Balls are placed in distinct cells)

$\Rightarrow P(F)=1-\frac{3!}{27}=1-\frac{6}{27}=\frac{21}{27}$

Clearly, E $\subset$ F

$\therefore E\cap F=E$

$\Rightarrow P(E\cap F)=P(E)=\frac{3}{27}$

Hence, required probability is given by,

P(E/F) = $\frac{P(E\cap F)}{P(F)}=\frac{3/27}{21/27}=\frac{1}{7}$

28. Let, $\mathrm{I}=\int \frac{(2x+1)}{(x+2)(x+3)} dx$

Using partial fractions we have,

$\frac{2x+1}{(x+2)(x-3)}=\frac{A}{x+2}+\frac{B}{x-3}$ .... (i)

Which implies 2x + 1 = A(x - 3) + B(x + 2)

Now put x - 3 = 0, x = 3

2 × 3 + 1 = A(0) + B (3 + 2)

So $B=\frac{1}{5}$

Now put x + 2 = 0, x = -2

-4 + 1 = A(-2 - 3) + B(0)

So $A=\frac{3}{5}$

From equation (1), we get,

$\frac{2x+1}{(x+2)(x-3)}=\frac{3}{5} \times \frac{1}{x+2}+\frac{7}{5} \times \frac{1}{x-3}$

$\int \frac{2x+1}{(x+2)(x-3)} dx=\frac{3}{5} \int \frac{1}{x+2} dx+\frac{7}{5} \int \frac{1}{x-3} dx$

$=\frac{3}{5} \log |x+2|+\frac{7}{5} \log |x-3|+c$

OR

Given integral is: $\int_0^{\frac{\pi}{2}} \sin^3 x dx$

To prove: $\int_0^{\frac{\pi}{2}} \sin^3 x dx=\frac{2}{3}$

Let $I=\int_0^{\frac{\pi}{2}} \sin^3 x dx$ ...(i)

$=\int_0^{\frac{\pi}{2}} \sin x \cdot \sin^2 x dx$

$=\int_0^{\frac{\pi}{2}} \sin x \cdot\left(1-\cos^2 x\right) dx$

$\Rightarrow \mathrm{I}=\int_0^{\frac{\pi}{2}} \sin \mathrm{x} \mathrm{dx}-\int_0^{\frac{\pi}{2}} \sin \mathrm{x} \cdot \cos^2 \mathrm{x} d\mathrm{x}$

$\Rightarrow \mathrm{I}=[-\cos \mathrm{x}]_0^{\pi / 2}-\mathrm{I}_1$ ...(ii)

Now, we solve I_1:

$\Rightarrow \mathrm{I}_1=\int_0^{\frac{\pi}{2}} \sin \mathrm{x} \cdot \cos^2 \mathrm{xdx}$

Let cos x = t ⇒ -sin x dx = dt ⇒ sinx dx = -dt

When x = 0 then t = 1 and when x = $\frac{\pi}{2}$ then t = 0

$\Rightarrow \mathrm{I}_1=\int_1^0 \mathrm{t}^2(-\mathrm{dt})$

$=-\int_1^0 t^2(dt)$

$=-\left[\frac{t^3}{3}\right]_1^0$

$=-\left\{-\frac{1}{3}\right\}$

$\Rightarrow \mathrm{I}_1=\frac{1}{3}$

Using this value in equation (ii)

$\Rightarrow \mathrm{I}=[-\cos \mathrm{x}]_0^{\pi / 2}-\frac{1}{3}$

$\Rightarrow \mathrm{I}=-\left\{\cos \frac{\pi}{2}-\cos 0\right\}-\frac{1}{3}$

$\Rightarrow \mathrm{I}=1-\frac{1}{3}$

$\Rightarrow \mathrm{I}=\frac{2}{3}$

Hence Proved.

29. According to the question,

Given differential equation is,

$2y e^{x/y} dx+\left(y-2x e^{x/y}\right) dy=0$

$\Rightarrow \frac{dx}{dy}=\frac{2x e^{x/y}-y}{2y^{x/y}}$ ...(i)

Let $F(x, y)=\frac{\left(2x e^{\frac{x}{y}}-y\right)}{\left(2y e^{\frac{x}{y}}\right)}$

On replacing x by λx and y by λy both sides, we get

$$F(\lambda x, \lambda y) = \frac{\left(2\lambda x e^{\frac{\lambda x}{\lambda y}} - \lambda y\right)}{\left(2aye^{\frac{\partial x}{2y}}\right)}$$

$$\Rightarrow \quad F(\lambda x, \lambda y) = \frac{\lambda\left(2xe^{x/y} - y\right)}{\lambda\left(2ye^{x/y}\right)} = \lambda^0[F(x, y)]$$

Thus, F (x, y) is a homogeneous function of degree zero.

Therefore, the given differential equation is a homogeneous differential equation.

put $x = vy$,

$\Rightarrow \quad \frac{dx}{dy} = v + y\frac{dv}{dy}$

$v + y\frac{dv}{dy} = \frac{2ve^v - 1}{2e^v}$

$\Rightarrow \quad y\frac{dv}{dy} = \frac{2\nu e^v - 1}{2e^v} - v$

$= \frac{2ve^v - 1 - 2ve^v}{2e^v}$

$\Rightarrow \quad 2e^v dv = \frac{-dy}{y}$

On integrating both sides, we get

$\int 2e^v dv = -\int \frac{dy}{y}$

$\Rightarrow \quad 2e^v = -\log|y| + C$

$\Rightarrow \quad 2e^{x/y} + \log|y| = C \ \left[\text{put } v = \frac{x}{y}\right]$ ...(ii)

On substituting x = 0 and y = 1 in Eq. (ii), we get

$2e^0 + \log|1| = C \Rightarrow C = 2$

On substituting the value of C in Eq. (ii), we get

$2e^{x/y} + \log|y| = 2$

which is the required particular solution of the given differential equation.

OR

The given differential equation is,

$(1 + x^2)\ \frac{dy}{dx} + (1 + y^2) = 0$, y = 1, when x = 0

$\Rightarrow (1 + x^2)\ \frac{dy}{dx} = -(1 + y^2)$

$\Rightarrow \frac{1}{1+y^2}dy = -\frac{1}{(1+x^2)}dx$

Integrating both sides, we get

$\int \frac{1}{1+y^2}dy = -\int \frac{1}{(1+x^2)}dx$

$\Rightarrow \tan^{-1} y = -\tan^{-1} x + C$

$\Rightarrow \tan^{-1} y + \tan^{-1} x = C$ ...(i)

Given: x = 0, y = 1

Substituting the values of x and y in (i), we get

$\frac{\pi}{4} + 0 = C$

$\Rightarrow C = \frac{\pi}{4}$

Substituting the value of C in (i), we get

$\tan^{-1} y + \tan^{-1} x = \frac{\pi}{4}$

$\Rightarrow \tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{x+y}{1-xy}\right) = \frac{\pi}{4}$

$\Rightarrow \frac{x+y}{1-xy} = 1$

$\Rightarrow x + y = 1 - xy$

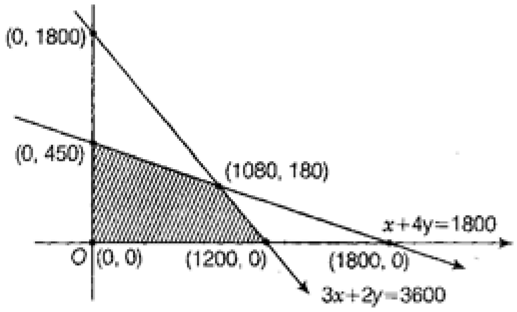
Hence, x + y = 1 - xy is the required solution.

30. Maximise Z= 100x + 170y subject to

$3x + 2y \leqslant 3600, x + 4y \leqslant 1800, x \geqslant 0, y \geqslant 0$

From the shaded feasible region it is clear that the coordinates of corner points are (0,0),(1200,0),(1080,180) and (1080,180) and (0,450).

On solving x + 4y = 1800 and 3x + 2y = 3600 we get x = 1080 and y = 180.

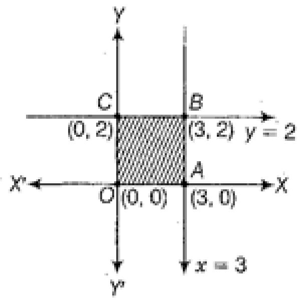


Corner Points	Corresponding value of Z = 100x + 170y
(0, 0)	0
(1200, 0)	$1200 \times 100 = 12000$
(1080, 180)	$100 \times 1080 + 170 \times 180 = 138600 \ (\max imum)$
(0, 450)	$0 + 170 \times 450 = 76500$

Hence, the maximum is 138600.

OR

Maximise Z = 11x + 7y, subject to the constraints $x \leqslant 3, y \leqslant 2, x \geqslant 0, y \geqslant 0$.



The shaded region as shown in the figure as OABC is bounded and the coordinates of corner points are (0, 0), (3, 0), (3, 2), and (0, 2), respectively.

Corner Points	Corresponding value of Z
(0, 0)	0
(3, 0)	33
(3, 2)	47 (Maximum)
(0, 2)	14

Hence, Z is maximise at (3, 2) and its maximum value is 47.

31. We know that, x = a sin 2t (1 + cos 2t) and y = b cos 2t (1 – cos 2t)

$\Rightarrow \frac{dy}{dt}$ = a[-2sin 2t sin 2t + 2cos 2t (1 + cos 2t)]....(I)

and $\frac{dx}{dt}$ = b[2sin 2t cos 2t - 2sin 2t (1 - cos 2t)]...(ii)

$\therefore \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{b[2 \sin 2t \cos 2t - 2 \sin 2t(1-\cos 2t)]}{a[-2 \sin^2 2t + 2 \cos 2t(1+\cos 2t)]}$. [Using (i) and (ii)]

$\Rightarrow \left.\frac{dy}{dx}\right]_{at \ t=\pi/4} = \frac{b\left[2 \sin \frac{\pi}{2} \cos \frac{\pi}{2} - 2 \sin \frac{\pi}{2}\left(1-\cos \frac{\pi}{2}\right)\right]}{a\left[-2 \sin^2\left(\frac{\pi}{2}\right) + 2 \cos \frac{\pi}{2}\left(1+\cos \frac{\pi}{2}\right)\right]} = \frac{b}{a} \cdot \frac{(0-1)}{(-1-0)} = \frac{b}{a}$

Section D

32. According to the question ,

Given, equation of circle is $x^2 + y^2 = 32$ ......(i)

Given ,equation of line is y = x ....(ii)

Consider $x^2 + y^2 = 32$,

$\Rightarrow x^2 + y^2 = (4\sqrt{2})^2$

Given circle has centre at (0, 0) and

radius of circle is = $4\sqrt{2}$

To find the point of intersection ,

On substituting y = x in Eq. (i), we get

$2x^2 = 32 \Rightarrow x^2 = 16 \Rightarrow x = \pm 4$

When x = 4 , then y = 4

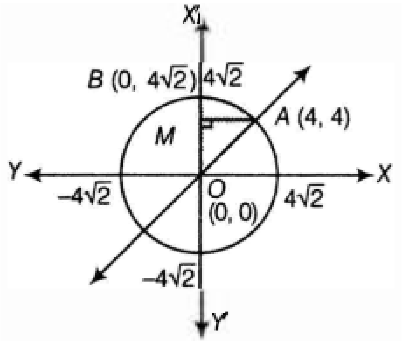
When x = -4 , then y = -4

Thus, the points of intersection are (4, 4) and (-4, - 4)

So, given line and the circle intersect in the first quadrant at point A(4, 4) and

The circle cut the Y-axis at point B $(0,4\sqrt{2})$.

Now, let us sketch the graph of given curves, we get



Let us draw AM perpendicular to Y-axis.

Required area = Area of shaded region OABO

$= \int_0^4 x_{(\text{line})} dy + \int_4^{4\sqrt{2}} x_{(\text{circle})} dy$

$\because x^2 + y^2 = 32 \Rightarrow x = \pm\sqrt{32 - y^2}$,but we need area of region enclosed in the first quadrant only,so x = $\sqrt{32 - y^2}$

$= \int_0^4 y dy + \int_4^{4\sqrt{2}} \sqrt{32 - y^{-2}} dy$

$= \left[\frac{y^2}{2}\right]_0^4 + \int_4^{4\sqrt{2}} \sqrt{(4\sqrt{2})^2 - y^2} dy$

$= \frac{1}{2}(16 - 0) + \left[\frac{y}{2}\sqrt{32 - y^2} + \frac{32}{2}\sin^{-1}\left(\frac{y}{4\sqrt{2}}\right)\right]_4^{4\sqrt{2}}$

$= 8 + \left[16\sin^{-1}(1) - \left\{2 \times 4 + 16\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right\}\right]$

$= 8 + \left[16 \cdot \frac{\pi}{2} - 8 - 16 \cdot \frac{\pi}{4}\right]$

$= 16\left(\frac{\pi}{2} - \frac{\pi}{4}\right)$

$= 16 \cdot \frac{\pi}{4}$

$= 4\pi$ sq units

33. f is one-one: For any x, y $\in$ R - {+1}, we have $f(x) = f(y)$

$\Rightarrow \frac{x}{1+|x|} = \frac{y}{|y|+1}$

$\Rightarrow$ xy + x = xy + y

$\Rightarrow$ x = y

Therefore, f is one-one function.

If f is one-one, let y = R - {1}, then f(x) = y

$\Rightarrow \frac{x}{x+1} = y$

$\Rightarrow x = \frac{y}{1-y}$

It is cleat that x $\in$ R for all y = R - {1}, also x = $\neq$ -1

Because x = -1

$\Rightarrow \frac{y}{1-y} = -1$

$\Rightarrow$ y = -1 + y

which is not possible.

Thus for each R - {1} there exists $x = \frac{y}{1-y} \in$ R - {1} such that

$f(x) = \frac{x}{x+1} = \frac{\frac{y}{1-y}}{\frac{y}{1-y}+1} = y$

Therefore f is onto function.

OR

We observe the following properties of f.

Injectivity: Let x, y $\in R_0$ such that f(x) = f(y). Then,

$f(x) = f(y) \Rightarrow \frac{1}{x} = \frac{1}{y} \Rightarrow x = y$

So, f : $R_0 \rightarrow R_0$ is one-one.

Surjectivity: Let y be an arbitrary element of R_0 (co-domain) such that f(x) = y. Then,

$f(x) = y \Rightarrow \frac{1}{x} = y \Rightarrow x = \frac{1}{y}$

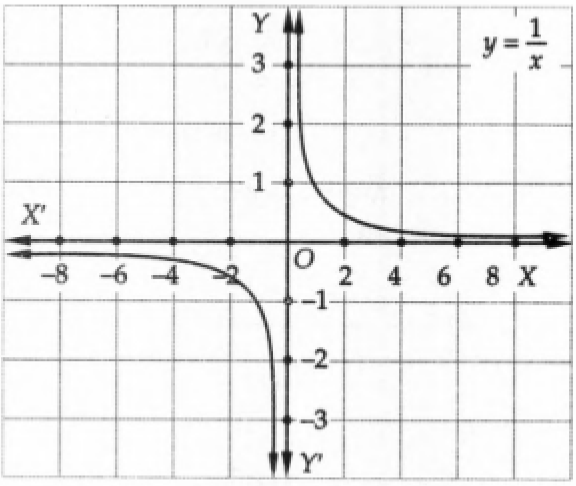
Clearly, $x = \frac{1}{y} \in R_0$ (domain) for all y $\in R_0$ (co-domain).

Thus, for each y $\in R_0$ (co-domain) there exits $x = \frac{1}{y} \in R_0$ (domain) such that $f(x) = \frac{1}{x} = y$

So, f : $R_0 \rightarrow R_0$ is onto.

Hence, f: $R_0 \rightarrow R_0$ is one-one onto.

This is also evident from the graph of f(x) as shown in fig.



Let us now consider f : N $\rightarrow R_0$ given by $f(x) = \frac{1}{x}$

For any x, y $\in$ N, we find that

f(x) = f(y) $\Rightarrow \frac{1}{x} = \frac{1}{y} \Rightarrow x = y$

So, f: N $\rightarrow R_0$ is one-one.

We find that $\frac{2}{3}, \frac{3}{5}$ etc. in co-domain R_0 do not have their pre-image in domain N. So, f: N $\rightarrow R_0$ is not onto.

Thus, f: N $\rightarrow R_0$ is one-one but not onto.

34. Given: Matrix $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$

$\Rightarrow |A| = 2(-4 + 4) - (-3)(-6 + 4) + 5(3 - 2) = 0 - 6 + 5 = -1 \neq 0$

$\therefore A^{-1}$ exists and $A^{-1} = \frac{1}{|A|}$ (adj. A)...(i)

Now, $A_{11} = 0$, $A_{12} = 2$, $A_{13} = 1$ and $A_{21} = -1$, $A_{22} = -9$, $A_{23} = -5$ and $A_{31} = 2$, $A_{32} = 23$, $A_{33} = 13$

$\therefore$ adj. A = $\begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}' = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$

From eq. (i),

$A^{-1} = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$

Now, Matrix form of given equations is AX = B

$\Rightarrow \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

Here A = $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

Therefore, solution is unique and X = $A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 0-5+6 \\ -22-45+69 \\ -11-25+39 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Therefore, x = 1, y = 2 and z = 3

35. We have,

$\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3} = \lambda$

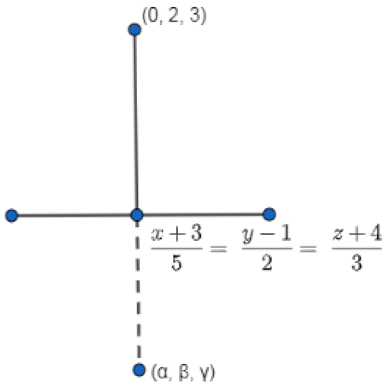
Therefore, the foot of the perpendicular is $(5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$

The direction ratios of the perpendicular is

$(5\lambda - 3 - 0) : (2\lambda + 1 - 2) : (3\lambda - 4 - 3)$

$\Rightarrow (5\lambda - 3) : (2\lambda - 1) : (3\lambda - 7)$

Direction ratio of the line is 5 : 2 : 3



From the direction ratio of the line and direction ratio of its perpendicular, we have

$5(5\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 7) = 0$

$\Rightarrow 25\lambda - 15 + 4\lambda - 2 + 9\lambda - 21 = 0$

$\Rightarrow 38\lambda = 38$

$\Rightarrow \lambda = 1$

Therefore, the foot of the perpendicular is (2, 3, -1)

The foot of the perpendicular is the mid-point of the line joining (0, 2, 3) and (a, β, y)

Therefore, we have

$\frac{\alpha+0}{2} = 2 \Rightarrow \alpha = 4$

$\frac{\beta+2}{2} = 3 \Rightarrow \beta = 4$

$\frac{\gamma+3}{2} = -1 \Rightarrow \gamma = -5$

Thus, the imgae is (4, 4, -5)_

OR

Given points are A(3, -4, -5) and B(2, -3, 1)

Direction ratios of AB are (3 - 2, -4 + 3, - 5 - 1) and (1, -1, -6)

Equation of line AB is,

$\frac{x-2}{1} = \frac{y+3}{-1} = \frac{z-1}{-6} = \lambda$

Any point on the line is $(\lambda + 2, -\lambda - 3, -6\lambda + 1)$

This point lie on the plane 2x + y + z = 7

Therefore $2(\lambda + 2) - \lambda - 3 - 6\lambda + 1 = 7$

$-5\lambda + 2 = 7$

$-5\lambda = 5$

$\lambda = -1$

Required point of intersection of line and plane is

(-1 + 2, 1 - 3, 6 + 1)

i.e (-1 + 2, 1 - 3, 6 + 1)
i.e (1, -2, 7)

Section E

36. **Read the text carefully and answer the questions:**

There are two antiaircraft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.

(i) Bayes' theorem defines the probability of an event based on the prior knowledge of the conditions related to the event whereas in case of the condition probability, we find the reverse probabilities using Bayes' theorem.

(ii) Consider on event E which occurs via two different events A and B. The probability of E is given by the value of total probability as:

P(E) = P(A ∩ E) + P(B ∩ E)

P(E) = P(A) P$\left(\frac{E}{A}\right)$ + P(B)$\left(\frac{E}{B}\right)$

(iii) Let P be the event that the shell fired from A hits the plane and Q be the event that the shell fired from B hits the plane. The following four hypotheses are possible before the trial, with the guns operating independently:

E_1 = PQ, E_2 = $\bar{P}\bar{Q}$, E_3 = $\bar{P}$Q, E_4 = P$\bar{Q}$

Let E = The shell fired from exactly one of them hits the plane.

P(E_1) = 0.3 × 0.2 = 0.06,

P(E_2) = 0.7 × 0.8 = 0.56,

P(E_3) = 0.7 × 0.2 = 0.14,

P(E_4) = 0.3 × 0.8 = 0.24

P$\left(\frac{E}{E_1}\right)$ = 0, P$\left(\frac{E}{E_2}\right)$+0, P$\left(\frac{E}{E_3}\right)$ = 1, P$\left(\frac{E}{E_4}\right)$ = 1

P(E) = P(E_1) · P$\left(\frac{E}{E_1}\right)$ + P(E_2) · P$\left(\frac{E}{E_2}\right)$ + P(E_3).

P$\left(\frac{E}{E_3}\right)$ + P(E_4) · P$\left(\frac{E}{E_4}\right)$

= 0.14 + 0.24 = 0.38

OR

By Bayes' Theorem,

$$P\left(\frac{E_3}{E}\right) = \frac{P(E_3)\cdot P\left(\frac{E}{E_3}\right)}{P(E_1)\cdot P\left(\frac{E}{E_1}\right)+P(E_2)\cdot P\left(\frac{E}{E_2}\right)+P(E_3)P\left(\frac{E}{E_3}\right)+P(E_4)\cdot P\left(\frac{E}{E_4}\right)}$$

$= \frac{0.14}{0.38} = \frac{7}{19}$

NOTE: The four hypotheses form the partition of the sample space and it can be seen that the sum of their probabilities is 1. The hypotheses E_1 and E_2 are actually eliminated as P$\left(\frac{E}{E_1}\right)$ = P$\left(\frac{E}{E_2}\right)$ = 0

Alternative way of writing the solution:

i. P(Shell fired from exactly one of them hits the plane)

= P[(Shell from A hits the plane and Shell from B does not hit the plane) or (Shell from A does not hit the plane and Shell from B hits the plane)]

= 0.3 × 0.8 + 0.7 × 0.2 = 0.38

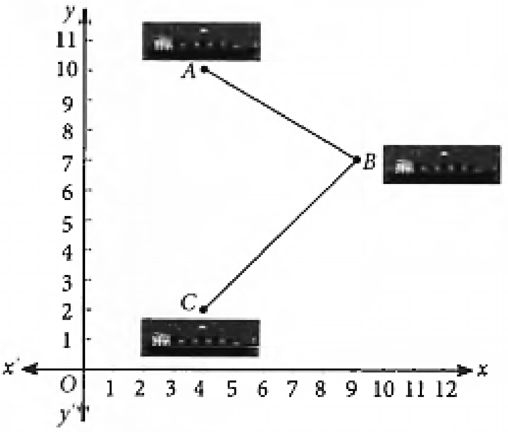
ii. $\frac{\text{P(Shell fired from B hit the plane} \cap \text{Exactly one of them hit the plane)}}{\text{P(Exactly one of them hit the plane)}}$

$= \frac{P(\text{Shell from only } B \text{ hit the plane})}{P(\text{Exactly one of them hit the plane})}$

$= \frac{0.14}{0.38} = \frac{7}{19}$

37. **Read the text carefully and answer the questions:**

A barge is pulled into harbour by two tug boats as shown in the figure.



(i) $4\hat{i} + 10\hat{j}$ Here, (4, 10) are the coordinates of A.

$\therefore$ P.V. of A = $4\hat{i} + 10\hat{j}$

(ii) Here, (9, 7) are the coordinates of B.

$\therefore$ P.V. of B = $9\hat{i} + 7\hat{j}$

(iii) Here, P.V. of A = $4\hat{i} + 10\hat{j}$ and P.V. of C = $4\hat{i} + 2\hat{j}$

$\therefore \overrightarrow{AC} = (4 - 4)\hat{i} + (2 - 10)\hat{j} = -8\hat{j}$

OR

We have, $\vec{A} = 4\hat{i} + 3\hat{j}$; and $\vec{B} = 3\hat{i} + 4\hat{j}$

$|\vec{A}| = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25}$ = 5

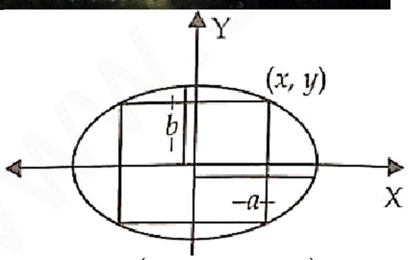
and $|\vec{B}| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25}$ = 5

Thus, $|\vec{A}| + |\vec{B}|$ = 5 + 5 = 10

38. **Read the text carefully and answer the questions:**

In an elliptical sport field, the authority wants to design a rectangular soccer field with the maximum possible area. The sport field is given by the graph of $\frac{x^2}{a^2} + \frac{y^2}{b^2}$ = I

(i)



Let (x, y) = $\left(x, \frac{b}{a}\sqrt{a^2 - x^2}\right)$ be the upper right vertex of the rectangle.

The area function A = 2x $\times\ 2\frac{b}{a}\sqrt{a^2 - x^2}$

= $\frac{4b}{a}x\sqrt{a^2 - x^2}$, x $\in$ (0, a).

(ii) $\frac{dA}{dx} = \frac{4b}{a}\left[x \times \frac{-x}{\sqrt{a^2-x^2}} + \sqrt{a^2 - x^2}\right]$

$= \frac{4b}{a} \times \frac{a^2-2x^2}{\sqrt{a^2-x^2}}$

$= -\frac{4b}{a} \times \frac{2\left(x+\frac{a}{\sqrt{2}}\right)\left(x-\frac{a}{\sqrt{2}}\right)}{\sqrt{a^2-x^2}}$

$\frac{dA}{dx} = 0 \Rightarrow x = \frac{a}{\sqrt{2}}$.

$x = \frac{a}{\sqrt{2}}$ is the critical point.