PRACTICE PAPER 06 (2024-25)

CHAPTER 06 PERMUTATION AND COMBINATIONS

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : XI **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. The number of triangles that are formed by choosing the vertices from a set of 12 points, seven of which lie on the same line is

(a) 105 (b) 15 (c) 175 (d) 185

2. If ${}^nC_{12} = {}^nC_8$, then n is equal to

(a) 20 (b) 12 (c) 6 (d) 30

3. There are 10 points in a plane, out of which 4 points are collinear. The number of triangles formed with vertices at these points is

(a) 20 (b) 120 (c) 116 (d) none of these

4. 10 students are participating in a competition. In how many different ways can the first prize be won? (There are 3 prizes)

(a) 720 (b) 60 (c) 30 (d) 120

5. If ${}^{15}P_r = 2730$, then ${}^5P_r$.

(a) 3 (b) 30 (c) 15 (d) 20

6. Total number of words formed by 2 vowels and 3 consonants taken from 4 vowels and 5 consonants is equal to

(a) 60 (b) 120 (c) 7200 (d) 720

7. All the letters of the word 'EAMCOT' are arranged in different possible ways. The number of such arrangements in which no two vowels are adjacent to each other is

(a) 360 (b) 144 (c) 72 (d) 54

8. The number of different four digit numbers that can be formed with the digits 2, 3, 4, 7 and using each digit only once is

(a) 120 (b) 96 (c) 24 (d) 100

For Q9 and Q10, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

9. **Assertion (A):** If the letters W, I, F, E are arranged in a row in all possible ways and the words (with or without meaning) so formed are written as in a dictionary, then the word WIFE occurs in the 24th position.
Reason (R): The number of ways of arranging four distinct objects taken all at a time is C(4,4).

10. **Assertion (A):** The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^{9}C_{3}$.
Reason (R): The number of ways of choosing any 3 places, from 9 different places is ${}^{9}C_{3}$.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. If ${}^{9}P_{5} + 5.\ {}^{9}P_{4} = {}^{10}P_{r}$, then find r.

12. Find r, if : ${}^{15}C_{r} : {}^{15}C_{r-1} = 11:5$

13. How many words can be formed using all the letters of the word EQUATION so that (*i*) all the vowels are together, (*ii*) consonants occupy the odd places ?

14. A committee of 3 persons is to be constituted from a group of 2 men and 3 women. In how many ways can this be done ? How many of these committees would consist of one man and two women?

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. If ${}^{n}P_{r} = 336$, ${}^{n}C_{r} = 56$. Find n and r and hence find ${}^{n-1}C_{r-1}$.

16. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has at least one boy and one girl ?

17. Find the number of words with or without meaning which can be made using all the letters of the word AGAIN. If these words are written as in a dictionary, what will be the 50th word ?

SECTION – D

Questions 18 carry 5 marks.

18. (a) Find the number of different 8-letter arrangements that can be made from the letters of the word DAUGHTER so that
(*i*) all vowels occur together.
(*ii*) all vowels do not occur together.
(b) Find the number of arrangements of the letters of the word INDEPENDENCE. In how many of these arrangements,
(*i*) do all the vowels always occur together
(*ii*) do all the vowels never occur together
(*iii*) do the words begin with I and end in P ?

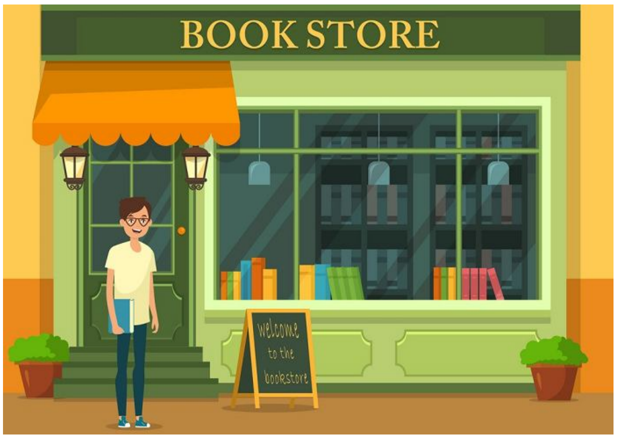
OR

What is the number of ways of choosing 4 cards from a pack of 52 playing cards ? In how many of these (*i*) four cards are of the same suit? (*ii*) four cards belong to four different suits?
(*iii*) are face cards? (*iv*) two are red cards and two are black cards?
(*v*) cards are of the same colour?

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Raj works at a book store. While arranging some books on the book shelf, he observed that there are 5 History books, 3 Mathematics books and 4 Science books which are to be arranged on the shelf.



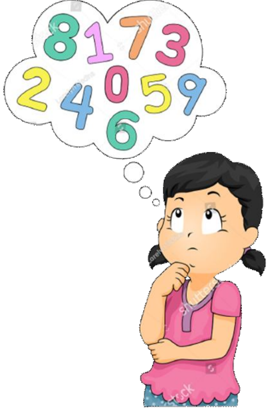
(i) In how many ways can he select either a History book or a Maths book? (1)

(ii) If he selects 2 History books, 1 Maths book and 1 Science book to arrange them, then find the number of ways in which selection can be made. (1)

(iii) Find the number of ways, if the books of same subject are put together. (1)

(iv) Find the number of arrangements, if he selects 3 History books, 2 Maths Books, 2 Science books. (1)

20. Seema wants a mobile number having 10 digits. It is not just a group of numbers strung out at random. All mobile numbers have 3 things in common. a 2-digit Access code (AC), a 3-digit provider code (PC), and a 5 digit subscriber code (SC). AC code and PC code are fixed, then



(i) How many mobile numbers are possible if no start with 98073 and no other digit can repeat? (1)

(ii) How many AC code are possible if both digit in AC code are different and must be greater than 6? (1)

(iii)How many mobile numbers are possible if AC and PC code are fixed and digits can repeat? (1)

(iv) How many mobile numbers are possible with AC code 98 and PC code 123 and digit used in AC and PC code will not be used in SC code? (1)

PRACTICE PAPER 17 (2024-25)
CHAPTER 06 & 07 PERMUTATION AND COMBINATIONS AND BINOMIAL THEOREM (ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**
CLASS : XI **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. 10 students are participating in a competition. In how many different ways can the first prize be won? (There are 3 prizes)
(a) 720 (b) 60 (c) 30 (d) 120
Ans: (a) 720
Out of 10 students, the first three prizes can be won in ${}^{10}P_3 = \frac{10!}{(10-3)^4} = \frac{10!}{7!} = 10\times 9\times 8 = 720\, ways.$

2. If ${}^nC_{12} = {}^nC_8$, then n is equal to
(a) 20 (b) 12 (c) 6 (d) 30
Ans: (a) 20
${}^nC_{12} = {}^nC_8 \Rightarrow {}^nC_{n-12} = {}^nC_8$
$\Rightarrow n - 12 = 8 \Rightarrow n = 8 + 12 = 20$

3. All the letters of the word 'EAMCOT' are arranged in different possible ways. The number of such arrangements in which no two vowels are adjacent to each other is
(a) 360 (b) 144 (c) 72 (d) 54
Ans: (b) 144
In the word EAMCOT, Consonants = 3, vowels = 3
Since no two vowels have to be together. Possible ways may be VMVCVTV i.e. ${}^4P_3$
3 consonants can be arranged in 3! = 6 ways
So, total number of ways = ${}^4P_3 \times 6 = 4 \times 3 \times 2 \times 6 = 144$

4. If ${}^{15}P_r = 2730$, then ${}^5P_r$.
(a) 3 (b) 30 (c) 15 (d) 20
Ans: (d) 20
${}^{15}P_r = 2730 \Rightarrow \quad {}^{15}P_r = 15\times 182$
$\Rightarrow \quad {}^{15}P_r = 15\times 14\times 13 = {}^{15}P_3 \Rightarrow r = 3$
$\Rightarrow \quad {}^5P_3 = 5\times 4 = 20$

5. Total number of words formed by 2 vowels and 3 consonants taken from 4 vowels and 5 consonants is equal to
(a) 60 (b) 120 (c) 7200 (d) 720
Ans: (c) 7200

total number of words $= {}^4C_2 \times {}^5C_3 \times (2 + 3)!$

$= \frac{4!}{2!2!} \times \frac{5!}{3!2!} \times 5!$

$= \frac{4 \times 3}{2 \times 1} \times \frac{5 \times 4}{2 \times 1} \times 5 \times 4 \times 3 \times 2 \times 1 = 60 \times 120 = 7200$

6. The number of triangles that are formed by choosing the vertices from a set of 12 points, seven of which lie on the same line is

(a) 105 (b) 15 (c) 175 (d) 185

Ans: (d) 185

Required number of triangle $= {}^{12}C_3 - {}^7C_3 = 220 - 35 = 185$

7. The coefficient of x^n in the expansion of $(1 + x)^{2n}$ and $(1 + x)^{2n-1}$ are in the ratio

(a) 1 : 2 (b) 1 : 3 (c) 3 : 1 (d) 2 : 1

Ans: (d) 2 : 1

coefficient of x^n in the expansion of $(1+x)^{2n} = {}^{2n}C_n$

According to given condition

${}^{2n}C_n : {}^{2n-1}C_n = ?$

i.e. $$\frac{{}^{2n}C_n}{{}^{2n-1}C_n} = \frac{\frac{(2n)!}{n!n!}}{\frac{(2n-1)!}{n!(2n-1-n)!}} = \frac{2n}{n} = \frac{2}{1}$$

i.e. ${}^{2n}C_n : {}^{2n-1}C_n = 2 : 1$

8. The total number of terms in the expansion of $(x + a)^{51} - (x - a)^{51}$ after simplification is

(a) 102 (b) 25 (c) 26 (d) none of these

Ans: (c) 26

Total number of terms in the expansion of $(x + a)^{51}$ is 52.

So, $(x + a)^{51} - (x - a)^{51}$ contains 26 terms (out of 52 terms 26 terms will be cancelled)

[or 1 to 51 there are 26 odd numbers]

For Q9 and Q10, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

9. **Assertion (A):** The term independent of x in the expansion of $\left(x + \frac{1}{x} + 2\right)^m$ is $\frac{(4m)!}{(2m!)^2}$

Reason (R): The coefficient of x6 in the expansion of $(1 + x)^n$ is ${}^nC_6$.

Ans: (d) A is false but R is true.

10. **Assertion (A):** If the letters W, I, F, E are arranged in a row in all possible ways and the words (with or without meaning) so formed are written as in a dictionary, then the word WIFE occurs in the 24th position.

Reason (R): The number of ways of arranging four distinct objects taken all at a time is C(4,4).

Ans: (c) A is true but R is false.

Number of ways of arranging four distinct objects in a line is ${}^4P_4 = 4! = 24$. Hence, Reason is false.

Again, when W, I, F, E are arranged in all possible ways, then number of words formed is 4! = 24 and WIFE occurs last of all as its letters are against alphabetical order.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. A committee of 3 persons is to be constituted from a group of 2 men and 3 women. In how many ways can this be done ? How many of these committees would consist of one man and two women?
Ans: Given 2 men and 3 women.
A committee of 3 persons can be selected in $^5C_3 = 10$ ways
A committee consisting of 1 man and 2 women = $^2C_1 \times {}^3C_2 = 2 \times 3 = 6$ ways

12. If the coefficients or $(r-5)^{th}$ and $(2r-1)^{th}$ terms in the expansion of $(1+x)^{34}$ are equal, find r.
Ans: The coefficients of $(r-5)^{th}$ and $(2r-1)^{th}$ terms of the expansion $(1+x)^{34}$ are $^{34}C_{r-6}$ and $^{34}C_{2r-2}$, respectively. Since they are equal so $^{34}C_{r-6} = {}^{34}C_{2r-2}$
Therefore, either $r-6 = 2r-2$ or $r-6 = 34-(2r-2)$
[Using the fact that if $^nC_r = {}^nC_p$, then either $r = p$ or $r = n-p$]
So, we get $r = -4$ or $r = 14$. r being a natural number, $r = -4$ is not possible. So, $r = 14$.

13. Find r, if : $^{15}C_r : {}^{15}C_{r-1} = 11:5$
Ans: $\dfrac{^{15}C_r}{^{15}C_{r-1}} = \dfrac{11}{5}$

$$\frac{15!}{r!(15-r)!} \times \frac{(r-1)!(16-r)!}{15!} = \frac{11}{5}$$

$$\frac{16-r}{r} = \frac{11}{5}$$

$\Rightarrow 80 - 5r = 11r \Rightarrow r = 5$

14. Using Binomial Theorem, indicate which number is larger $(1.1)^{10000}$ or 1000.
Ans: $(1.1)^{10000} = (1+0.1)^{10000}$
$= {}^{10000}C_0 + {}^{10000}C_1(0.1) + {}^{10000}C_2(0.1)^2 + ... + {}^{10000}C_{10000}(0.1)^{10000}$
$= 1 + 1000 + (+\text{ve terms}) > 1000$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Evaluate: $(\sqrt{3}+\sqrt{2})^6 + (\sqrt{3}-\sqrt{2})^6$
Ans:
Consider $(\sqrt{3}+\sqrt{2})^6 + (\sqrt{3}-\sqrt{2})^6)$

We know $(x+y)^n + (x-y)^n$
$= 2\,[{}^nC_0 x^n + {}^nC_2 x^{n-2}y^2 + {}^nC_4 x^{n-4}y^4 + ...]$

Here $x = \sqrt{3}$, $y = \sqrt{2}$, $n = 6$

$\therefore \quad (\sqrt{3}+\sqrt{2})^6 + (\sqrt{3}-\sqrt{2})^6$

$$= 2[{}^6C_0(\sqrt{3})^6 + {}^6C_2(\sqrt{3})^4(\sqrt{2})^2 + {}^6C_4(\sqrt{3})^2 (\sqrt{2})^4 + {}^6C_6(\sqrt{2})^6]$$

$$= 2[1 \times 27 + 15 \times 9 \times 2 + 15 \times 3 \times 4 + 1 \times 8]$$

$$= 2[27 + 270 + 180 + 8] = 970$$

16. Simplify: $(x+\sqrt{x-1})^6+(\sqrt{x}-\sqrt{x-1})^6$

Ans: Use $(x + y)^n + (x - y)^n$

$= 2[{}^nC_0\, x^n + {}^nC_2\, x^{n-2}\, y^2 + {}^nC_4\, x^{n-4}\, y^4 + ...]$

$= 2[{}^6C_0\, x^6 + {}^6C_2\, x^4(x - 1) + {}^6C_4\, x^2\, (x - 1)^2\, {}^6C_6(x - 1)^3]$

$= 2(x^6 + 15x^5 - 29x^3 + 12x^2 + 3x - 1)$

17. By using binomial theorem show that : $6^n - 5n - 1$ is divisible by 25, $n \in$ N.

Ans: $6^n - 5n - 1 = (1 + 5)^n - 5n - 1$

$= [1 + 5n + {}^nC_2 \,.\, 5^2 + {}^nC_3 \,.\, 5^3 + ...5^n]-5n - 1$

$= 25[{}^nC_2 + 5.{}^nC_3 + ...5^{n-2}]$

which is divisible by 25.

SECTION – D

Questions 18 carry 5 marks.

18. What is the number of ways of choosing 4 cards from a pack of 52 playing cards ? In how many of these (*i*) four cards are of the same suit? (*ii*) four cards belong to four different suits? (*iii*) are face cards? (*iv*) two are red cards and two are black cards? (*v*) cards are of the same colour?

Ans: There will be as many ways of choosing 4 cards from 52 cards as there are combinations of 52 different things, taken 4 at a time. Therefore The required number of ways

$= {}^{52}C_4 = \dfrac{52!}{4!48!} = 270725$

(*i*) There are four suits : diamond, club, spade, heart and there are 13 cards of each suit. Therefore, there are ${}^{13}C_4$ ways of choosing 4 diamonds, similarly, there are ${}^{13}C_4$ ways of choosing 4 clubs, ${}^{13}C_4$ ways of choosing 4 spades and ${}^{13}C_4$ ways of choosing 4 hearts. Therefore

The required number of ways = ${}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4$.

$= 4 \times \dfrac{13!}{4!9!} = 2860$

(*ii*) There are 13 cards in each suit. Therefore, there are ${}^{13}C_1$ ways of choosing 1 card from 13 cards of diamond, ${}^{13}C_1$ ways of choosing 1 card from 13 cards of hearts, ${}^{13}C_1$ ways of choosing 1 card from 13 cards of clubs, ${}^{13}C_1$ ways of choosing 1 card from 13 cards of spades. Hence, by multiplication principle, the required number of ways.

$= {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 = 13^4$

(*iii*) There are 12 face cards and 4 are to be selected out of these 12 cards. This can be done in ${}^{12}C_4$ ways. Therefore, the required number of ways = $\dfrac{12!}{4!8!}$ = 495.

(*iv*) There are 26 red cards and 26 black cards. Therefore, the required number of ways

$= {}^{26}C_2 \times {}^{26}C_2$

$= \left(\dfrac{26!}{2!24!}\right)^2 = (325)^2 = 105625$

(v) 4 red cards can be selected out of 26 red cards in ${}^{26}C_4$ ways. 4 black cards can be selected out of 26 black cards in ${}^{26}C_4$ ways.

Therefore, the required number of ways

$= {}^{26}C_4 + {}^{26}C_4$

$= 2 \times \dfrac{26!}{2!22!} = 29900$

OR

(a) Find the number of different 8-letter arrangements that can be made from the letters of the word DAUGHTER so that

(*i*) all vowels occur together.

(*ii*) all vowels do not occur together.

(b) Find the number of arrangements of the letters of the word INDEPENDENCE. In how many of these arrangements, (*i*) do all the vowels always occur together

(*ii*) do all the vowels never occur together

(*iii*) do the words begin with I and end in P ?

Ans: (a) (*i*) There are 8 different letters in the word DAUGHTER, in which there are 3 vowels, namely, A, U and E. Since the vowels have to occur together, we can for the time being, assume them as a single object (AUE). This single object together with 5 remaining letters (objects) will be counted as 6 objects. Then we count permutations of these 6 objects taken all at a time. This number would be ${}^6P_6 = 6!$. Corresponding to each of these permutations, we shall have 3! permutations of the three vowels A, U, E taken all at a time. Hence, by the multiplication principle the required number of permutations = $6! \times 3! = 4320$.

(*ii*) If we have to count those permutations in which all vowels are never together, we first have to find all possible arrangements of 8 letters taken all at a time, which can be done in 8! ways. Then, we have to subtract from this number, the number of permutations in which the vowels are always together. Therefore, the required number = $8! – 6! \times 3! = 6!(7 \times 8 – 6)$

$= 50 \times 6! = 50 \times 720 = 36000$

(b) There are 12 letters, of which N appears 3 times, E appears 4 times and D appears 2 times and the rest are all different.

Therefore, the required number of arrangements = $\frac{12!}{3!2!4!} = 1663200$.

(*i*) There are 5 vowels in the given word, which are 4 Es and 1 I. Since, they have to always occur together, we treat them as a single object $\boxed{EEEEI}$ for the time being. This single object together with 7 remaining objects will account for 8 objects. These 8 objects, in which there are 3 Ns and 2 Ds, can be rearranged in $\frac{8!}{3!2!}$ ways. Corresponding to each of these arrangements, the 5 vowels E, E, E, E and I can be rearranged in $\frac{5!}{4!}$ ways. Therefore, by multiplication principle the required number of arrangements = $\frac{8!}{3!2!} \times \frac{5!}{4!} = 16800$

(*ii*) The required number of arrangements = the total number of arrangements (without any restriction) – the number of arrangements where all the vowels occur together.

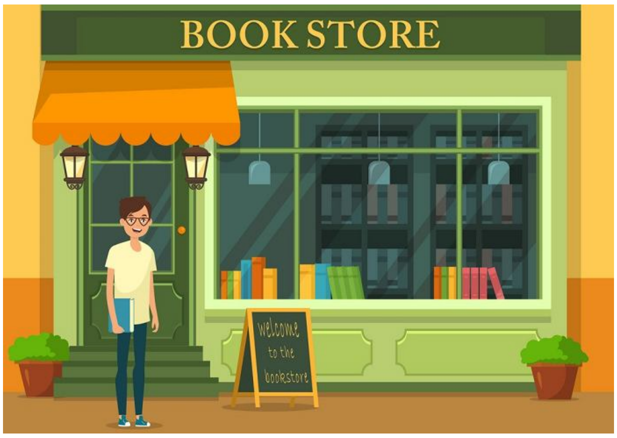
$= 1663200 – 16800 = 1646400$

(*iii*) Let us fix I and P at the extreme ends (I at the left end and P at the right end). We are left with 10 letters. Hence, the required number of arrangements = $\frac{10!}{3!2!4!} = 12600$

SECTION – E (Case Study Based Questions)

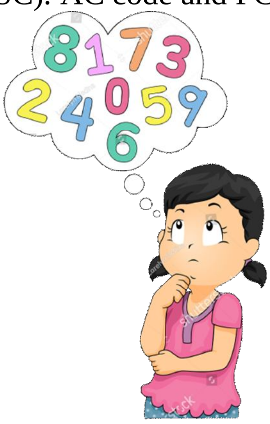
Questions 19 to 20 carry 4 marks each.

19. Raj works at a book store. While arranging some books on the book shelf, he observed that there are 5 History books, 3 Mathematics books and 4 Science books which are to be arranged on the shelf.



(i) In how many ways can he select either a History book or a Maths book? (1)
(ii) If he selects 2 History books, 1 Maths book and 1 Science book to arrange them, then find the number of ways in which selection can be made. (1)
(iii) Find the number of ways, if the books of same subject are put together. (1)
(iv) Find the number of arrangements, if he selects 3 History books, 2 Maths Books, 2 Science books. (1)
Ans: (i) A History book can be selected in 5 ways and a Maths book can be selected in 3 ways.
Required number of ways = 5 + 3 = 8 [Using addition Principle]
(ii) Now, 2 History books can be chosen in $^5P_2$ ways, 1 Maths book can be chosen in $^3P_1$ ways and 1 Science book can be chosen in $^4P_1$ ways.
∴ Required number of ways = $^5P_2 \times {}^3P_1 \times {}^4P_1 = 240$.
(iii) Number of ways of arranging History books = 5!
Number of ways of arranging Maths books = 3!
Number of ways of arranging Science books = 4!
∴ Required number of way if the books of same subject are put together = 3!· 5!· 3! · 4!. = 103680
(iv) Number of ways of choosing 3 History books = $^5P_3$
Number of ways of choosing 2 Maths books = $^3P_2$
and number of ways of choosing 2 Science books = $^4P_2$
∴ Total number of ways = $^5P_3 \times {}^3P_2 \times {}^4P_2 = 4320$

20. Seema wants a mobile number having 10 digits. It is not just a group of numbers strung out at random. All mobile numbers have 3 things in common. a 2-digit Access code (AC), a 3-digit provider code (PC), and a 5 digit subscriber code (SC). AC code and PC code are fixed, then



(i) How many mobile numbers are possible if no start with 98073 and no other digit can repeat? (1)
(ii) How many AC code are possible if both digit in AC code are different and must be greater than 6? (1)
(iii)How many mobile numbers are possible if AC and PC code are fixed and digits can repeat? (1)
(iv) How many mobile numbers are possible with AC code 98 and PC code 123 and digit used in AC and PC code will not be used in SC code? (1)
Ans: (i) 98073 V IV III II I
The digits which can be used are, 6, 5, 4, 2, 1
Number of ways to fill the 5 places = 5! = 120
(ii) Digits which can be used in AC code are 7, 8, 9
Total AC code = $^3P_2 = 3! = 6$
(iii) If AC and PC are fixed then only 5 digits is to be filled if digits can repeat then total ways = $100000 = 10^5$
(iv) Total ways = 5 × 5 × 5 × 5 × 5 = 3125