PRACTICE PAPER 04 (2024-25)
CHAPTER 04 COMPLEX NUMBERS AND QUADRATIC EQUATIONS

SUBJECT: MATHEMATICS **MAX. MARKS : 40**
CLASS : XI **DURATION : 1½ hrs**

General Instructions:
(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A
Questions 1 to 10 carry 1 mark each.

1. The value of $(1 + i)^4 - (1 - i)^4$ is:
(a) 8 (b) 4 (c) –8 (d) –4

2. The real part of $\frac{(1+i)^2}{(3-i)}$ is:
(a) 1/3 (b) 1/5 (c) −1/3 (d) none of these

3. If $\left(\frac{1+i}{1-i}\right)^x = 1$, then which of the following is correct?
(a) $x = 2n$ (b) $x = 2n + 1$ (c) $x = 4n$ (d) $x = 4n + 1$

4. If $\frac{1-in}{1+in} = a + ib$ then $a^2 + b^2 =$
(a) 1 (b) –1 (c) 0 (d) none of these

5. $1 + i^2 + i^4 + i^6 + ... + i^{2n}$ is:
(a) positive (b) negative (c) 0 (d) cannot be determined

6. If $a + ib = 3 - 4i$ then *Re* and *Im* part of the complex number are:
(a) 3, 4 (b) 3, –4 (c) 4, 3 (d) 4, –3

7. Conjugate of complex number $i^3 - 4$ is
(a) $i^3 + 4$ (b) $4 - i$ (c) $-4 + i$ (d) $-4 - i$

8. If a + ib = c + id, then
(a) $a^2 + c^2 = 0$ (b) $b^2 + c^2 = 0$ (c) $b^2 + d^2 = 0$ (d) $a^2 + b^2 = c^2 + d^2$

For Q9 and Q10, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

9. **Assertion (A):** Simplest form of i^{35} is – i.
Reason (R): Additive inverse of (1 – i) is equal to – 1 + i.

10. **Assertion (A):** If $z_1 = 2 + 3i$ and $z_2 = 3 - 2i$ then $z_1 - z_2 = -1 + 5i$.
Reason(R): If $z_1 = (a + ib)$ and $z_2 = (c + id)$ then $z_1 - z_2 = (a - c) + i(b - d)$.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Evaluate: $\sqrt{-16}+3\sqrt{-25}+\sqrt{-36}-\sqrt{-625}$

OR

Express $i^{15} - 3i^7 + 2i^{109} + i^{100} - i^{17} + 5i^3$. in the form $(a + ib)$.

12. Express in the standard form $a + ib$: $\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$

13. Solve the quadratic equation: $x^2+x+\frac{1}{\sqrt{2}}=0$

14. Express $\frac{3-i}{5+6i}$ in the form $(a + ib)$.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. If $(x + iy)^3 = u + iv$, then show that, $\frac{u}{x}+\frac{v}{y}=4(x^2-y^2)$.

16. If $a + ib = \frac{(x+i)^2}{2x^2+1}$, prove that $a^2 + b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$.

17. If z is a complex number, such that $|z| = 1$, prove that $\frac{z-1}{z+1}$ is purely imaginary.

SECTION – D

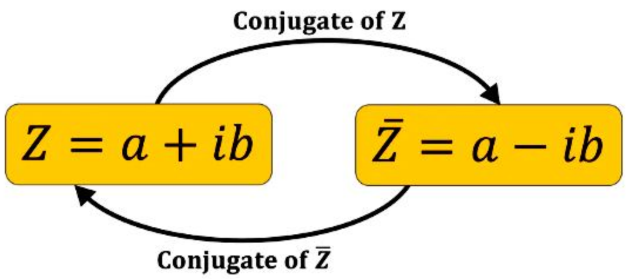
Questions 18 carry 5 marks.

18. Find the magnitude and conjugate of the number $\left(\frac{1}{1-4i}-\frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$

SECTION – E (Case Study Based Questions)

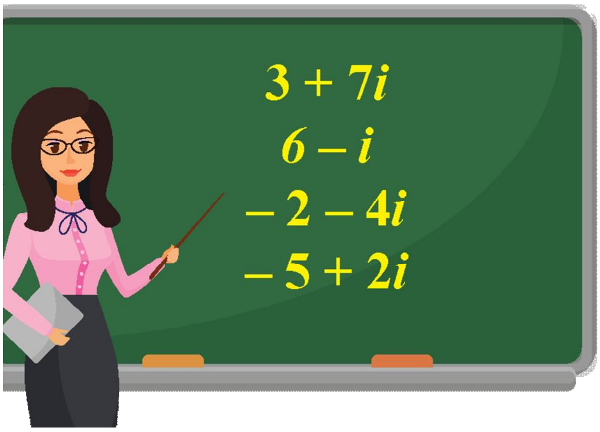
Questions 19 to 20 carry 4 marks each.

19. A conjugate of a complex number is another complex number that has the same real part as the original complex number, and the imaginary part has the same magnitude but opposite sign. If we multiply a complex number with its conjugate, we get a real number.

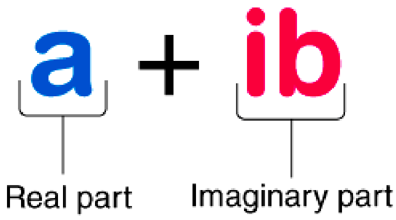


A complex number z is purely real if and only if $\bar{z} = z$ and is purely imaginary if and only if $\bar{z} = -z$.
Based on the above information, answer the following questions.
(a) Find the conjugate of the following : $(6 - 3i)(2 + 5i)$ (1)
(b) Find the multiplicative inverse of the following : $3 + 4i$ (1)
(c) Express $\left(\frac{1}{2} + 2i\right)^3$ in the form $(a + ib)$. (2)

20. Maths Teacher started the Complex numbers Chapter and he explained Complex numbers are the numbers that are expressed in the form of a + ib where, a, b are real numbers and 'i' is an imaginary number called "iota". The value of i = (√-1). For example, 2 + 3i is a complex number, where 2 is a real number (Re) and 3i is an imaginary number (Im).



An imaginary number is usually represented by 'i' or 'j', which is equal to √-1. Therefore, the square of the imaginary number gives a negative value. Since, i = √-1, so, $i^2 = -1$
The complex number is basically the combination of a real number and an imaginary number. The complex number is in the form of **a + ib,** where a = real number and ib = imaginary number. Also, a,b belongs to real numbers and i = √-1.
Hence, a complex number is a simple representation of addition of two numbers, i.e., real number and an imaginary number. One part of it is purely real and the other part is purely imaginary.



Based on the above information, answer the following questions.
(a) Express $(3 + 4i)(6 - 3i)(5 + i)$ in the form $(a + ib)$. (2)
(b) Express $(3i - 7) + (7 - 4i) - (6 + 3i) + i^{23}$ in the form $(a + ib)$. (2)

PRACTICE PAPER 04 (2024-25)
CHAPTER 04 COMPLEX NUMBERS AND QUADRATIC EQUATIONS
(ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**
CLASS : XI **DURATION : 1½ hrs**

General Instructions:
(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A
Questions 1 to 10 carry 1 mark each.

1. The value of $(1 + i)^4 – (1 – i)^4$ is:
(a) 8 (b) 4 (c) –8 (d) –4
Ans. (c) –8
$(1 + i)^4 – (1 – i)^4$
$= ((1 + i)^2)^2 – ((1 – i)^2)^2$
$= (i^2 + 1 + 2i)^2 – (1 + i^2 – 2i)^2$
$= (–1 + 1 + 2i)^2 – (1 – 1 – 2i)^2$
$= (2i)^2 – (– 2i)^2$
$= 4i^2 + 4i^2$
$= 8i^2 = –8$

2. The real part of $\frac{(1+i)^2}{(3-i)}$ is:
(a) 1/3 (b) 1/5 (c) −1/3 (d) none of these
Ans. (d) none of these

3. If $\left(\frac{1+i}{1-i}\right)^x = 1$, then which of the following is correct?
(a) $x = 2n$ (b) $x = 2n + 1$ (c) $x = 4n$ (d) $x = 4n + 1$
Ans. (c) $x = 4n$

4. If $\frac{1-in}{1+in} = a + ib$ then $a^2 + b^2 =$
(a) 1 (b) –1 (c) 0 (d) none of these
Ans. (a) 1

5. $1 + i^2 + i^4 + i^6 + ... + i^{2n}$ is:
(a) positive (b) negative (c) 0 (d) cannot be determined
Ans. (d) cannot be determined
Explanation: $S = 1 + i^2 + i^4 + i^6 + ... + i^{2n}$
$= 1 – 1 + 1 – 1 +...+ (–1)^n$
Obviously, it depends on n.
Hence cannot be determined unless n is known.

6. If $a + ib = 3 - 4i$ then *Re* and *Im* part of the complex number are:

(a) 3, 4 (b) 3, –4 (c) 4, 3 (d) 4, –3

Ans. *(b) 3, –4*

Explanation: Here, complex number is

$a + ib = 3 - 4i$

A general complex number can be written as

Re + $i(Im)$.

So, Re = 3

And Im = –4

7. Conjugate of complex number $i^3 - 4$ is

(a) $i^3 + 4$ (b) $4 - i$ (c) $-4 + i$ (d) $-4 - i$

Ans: (c) $-4 + i$

$i^3 - 4 = -i - 4 = -4 - i$

$\therefore \overline{-4 - i} = -4 + i$

8. If a + ib = c + id, then

(a) $a^2 + c^2 = 0$ (b) $b^2 + c^2 = 0$ (c) $b^2 + d^2 = 0$ (d) $a^2 + b^2 = c^2 + d^2$

Ans: (d) $a^2 + b^2 = c^2 + d^2$

a + ib = c + id

$\Rightarrow |a + ib| = |c + id|$

$\Rightarrow \sqrt{a^2 + b^2} = \sqrt{c^2 + d^2}$

On squaring both sides, we get $a^2 + b^2 = c^2 + d^2$

For Q9 and Q10, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

9. **Assertion (A):** Simplest form of i^{35} is $-i$.

Reason (R): Additive inverse of $(1 - i)$ is equal to $-1 + i$.

Ans: (d) A is false but R is true.

10. **Assertion (A):** If $z_1 = 2 + 3i$ and $z_2 = 3 - 2i$ then $z_1 - z_2 = -1 + 5i$.

Reason(R): If $z_1 = (a + ib)$ and $z_2 = (c + id)$ then $z_1 - z_2 = (a - c) + i(b - d)$.

Ans: (a) Both A and R are true and R is the correct explanation of A.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Evaluate: $\sqrt{-16} + 3\sqrt{-25} + \sqrt{-36} - \sqrt{-625}$

Ans: $\sqrt{-16} + 3\sqrt{-25} + \sqrt{-36} - \sqrt{-625}$

$= i\sqrt{16} + 3i\sqrt{25} + i\sqrt{36} - i\sqrt{625}$

$= 4i + 15i + 6i - 25i$

$= 25i - 25i = 0$

OR

Express $i^{15} - 3i^7 + 2i^{109} + i^{100} - i^{17} + 5i^3$. in the form $(a + ib)$.

Ans: $i^{15} - 3i^7 + 2i^{109} + i^{100} - i^{17} + 5i^3$

$= (i^2)^7.i - 3(i^2)^3\, i + 2(i^2)^{54}.\, i + (i^2)^{50} - (i^2)^8.\, i + 5(i^2).i$

$= (-1)^7 . i - 3(-1)^3 i + 2(-1)^{54} . i + (-1)^{50} - (-1)^8 . i + 5(-1)i$
$= -i + 3i + 2i + 1 - i - 5i = 1 - 2i$

12. Express in the standard form $a + ib$: $\left[i^{18} + \left(\frac{1}{i}\right)^{25}\right]^3$

Ans:

We have $\left[i^{18} + \left(\frac{1}{i}\right)^{25}\right]^3$

$= \left[i^{4\times4+2} + (-i)^{25}\right]^3 = \left[i^2 + (-1)^{25} i^{25}\right]^3 = \left[-1 - i^{4\times6+1}\right]^3 = [-1-i]^3$

$= -\left[1 + 3i + 3i^2 + i^3\right] = -\left[1 + 3i - 3 - i\right] = -[-2 + 2i] = 2 - 2i.$

13. Solve the quadratic equation: $x^2 + x + \frac{1}{\sqrt{2}} = 0$

Ans:

Given equation is $x^2 + x + \frac{1}{\sqrt{2}} = 0$

It can be written as $\sqrt{2}x^2 + \sqrt{2}x + 1 = 0$

$\therefore \text{D} = b^2 - 4ac = \left(\sqrt{2}\right)^2 - 4 \times \sqrt{2} \times 1 = 2 - 4\sqrt{2} = -2\left(2\sqrt{2} - 1\right)$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-\sqrt{2} \pm \sqrt{-2\left(2\sqrt{2}-1\right)}}{2 \times \sqrt{2}}$

$\therefore x = \frac{-1 \pm \sqrt{(2\sqrt{2}-1)}i}{2}$

14. Express $\frac{3-i}{5+6i}$ in the form $(a + ib)$.

Ans:

$\frac{(3-i)(5-6i)}{25+36} = \frac{15 - 18i - 5i + 6i^2}{61} = \frac{9-23i}{61}$

$= \frac{9}{61} - \frac{23}{61}i$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. If $(x + iy)^3 = u + iv$, then show that, $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$.

Ans: Given, $(x + iy)^3 = u + iv$
$\Rightarrow x^3 + 3x^2 iy + 3x(iy)^2 + (iy)^3 = u + iv$
$\Rightarrow (x^3 - 3xy^2) + i(3x^2 y - y^3) = u + iv$
Equating real and imaginary parts on both sides, we get,
$x^3 - 3xy^2 = u$ and $3x^2 y - y^3 = v$

$\Rightarrow \frac{u}{x} = x^2 - 3y^2$ and $\frac{v}{y} = 3x^2 - y^2$

$\therefore \frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$.

16. If $a + ib = \frac{(x+i)^2}{2x^2+1}$, prove that $a^2 + b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$.

Ans:

Consider, $a + ib = \frac{(x+i)^2}{2x^2+1}$,

$\Rightarrow \quad |a+ib| = \left|\frac{(x+i)^2}{2x^2+1}\right|$

$\Rightarrow \quad |a+ib| = \frac{|x+i|^2}{|2x^2+1|} \quad [\because |z^2| = |z|^2]$

$\Rightarrow \sqrt{a^2+b^2} = \frac{(\sqrt{x^2+1^2})^2}{2x^2+1}$

17. If z is a complex number, such that $|z| = 1$, prove that $\frac{z-1}{z+1}$ is purely imaginary.

Ans:

Let $z = x + iy$ then $|z| = 1 \Rightarrow \sqrt{x^2+y^2} = 1$

$\Rightarrow x^2 + y^2 = 1 \quad ...(i)$

Consider, $\frac{z-1}{z+1} = \frac{x+iy-1}{x+iy+1} = \frac{(x-1)+iy}{(x+1)+iy}$

$= \frac{\{(x-1)+iy\}\{(x+1)-iy\}}{(x+1)^2+y^2}$

$= \frac{(x^2-1)-(x-1)yi+(x+1)yi+y^2}{(x+1)^2+y^2}$

$= \frac{(x^2+y^2-1)+(-xy+y+xy+y)i}{(x+1)^2+y^2}$

Using (i), we get

$= 0 + \frac{2y}{(x+1)^2+y^2}\, i$, which is purely imaginary.

SECTION – D

Questions 18 carry 5 marks.

18. Find the magnitude and conjugate of the number $\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$

Ans:

Consider, $\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$

$= \left\{\frac{1+i-2+8i}{(1-4i)(1+i)}\right\}\left\{\frac{(3-4i)(5-i)}{(5+i)(5-i)}\right\}$

$$= \left\{\frac{-1+9i}{1+i-4i-4i^2}\right\}\left\{\frac{15-3i-20i+4i^2}{25-i^2}\right\}$$

$$= \left\{\frac{-1+9i}{5-3i}\right\}\left\{\frac{11-23i}{26}\right\}$$

$$= \left\{\frac{(-1+9i)\,(5+3i)}{25-9i^2}\right\}\left\{\frac{11-23i}{26}\right\}$$

$$= \left\{\frac{-5-3i+45i+27i^2}{25+9}\right\}\left\{\frac{11-23i}{26}\right\}$$

$$= \left\{\frac{-32+42i}{34}\right\}\left\{\frac{11-23i}{26}\right\}$$

$$= \left\{\frac{-16+21i}{17}\right\}\left\{\frac{11-23i}{26}\right\}$$

$$= \frac{-176+368i+231i-483i^2}{442}$$

$$= \frac{307+599i}{442} = \frac{307}{442}+\frac{599}{442}i$$

$$\text{Magnitude} = \sqrt{\left(\frac{307}{442}\right)^2+\left(\frac{599}{442}\right)^2}$$

$$= \sqrt{\frac{94249+358801}{(442)^2}}$$

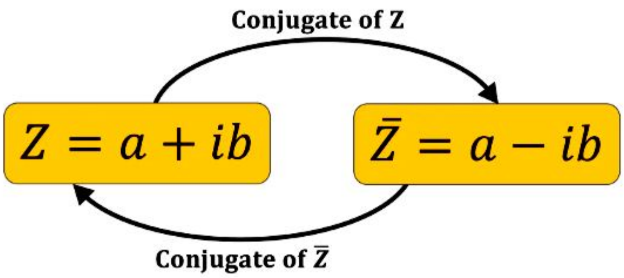
$$= \sqrt{\frac{453050}{442}} = \sqrt{\frac{226525}{221}}$$

$$\text{Conjugate} = \frac{307}{442}-\frac{599}{442}i$$

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. A conjugate of a complex number is another complex number that has the same real part as the original complex number, and the imaginary part has the same magnitude but opposite sign. If we multiply a complex number with its conjugate, we get a real number.



A complex number z is purely real if and only if $\bar{z}$ = z and is purely imaginary if and only if $\bar{z}$ = -z.
Based on the above information, answer the following questions.
(a) Find the conjugate of the following : (6 – 3i) (2 + 5i) (1)

(b) Find the multiplicative inverse of the following : 3 + 4*i* (1)

(c) Express $\left(\frac{1}{2}+2i\right)^3$ in the form (*a* + *ib*). (2)

Ans: (a)

$$\overline{(6-3i)(2+5i)} = \overline{(12+30i-6i-15i^2)}$$

$$= \overline{27+24i} = 27-24i$$

(b) Multiplicative inverse of (3 + 4*i*) = $\frac{1}{3+4i}$

$$= \frac{3-4i}{9+16} = \frac{3-4i}{25} = \frac{3}{25} - \frac{4}{25}i$$

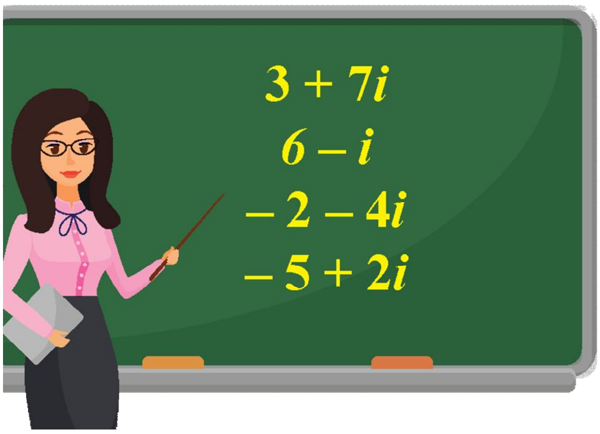
(c)

$$\left(\frac{1}{2}+2i\right)^3 = \left(\frac{1}{2}+2i\right)\left(\frac{1}{4}-4+2i\right)$$

$$= \left(\frac{1}{2}+2i\right)\left(-\frac{15}{4}+2i\right)$$

$$= -\frac{15}{8}+i-\frac{15i}{2}-4 = -\frac{47}{8}-\frac{13}{2}i$$

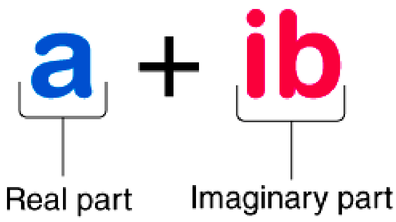
20. Maths Teacher started the Complex numbers Chapter and he explained Complex numbers are the numbers that are expressed in the form of a + ib where, a, b are real numbers and 'i' is an imaginary number called "iota". The value of i = (√-1). For example, 2 + 3i is a complex number, where 2 is a real number (Re) and 3i is an imaginary number (Im).



An imaginary number is usually represented by 'i' or 'j', which is equal to √-1. Therefore, the square of the imaginary number gives a negative value. Since, i = √-1, so, i^2 = -1

The complex number is basically the combination of a real number and an imaginary number. The complex number is in the form of **a + ib,** where a = real number and ib = imaginary number. Also, a,b belongs to real numbers and i = √-1.

Hence, a complex number is a simple representation of addition of two numbers, i.e., real number and an imaginary number. One part of it is purely real and the other part is purely imaginary.



Based on the above information, answer the following questions.

(a) Express (3 + 4*i*) (6 – 3*i*) (5 + *i*) in the form (*a* + *ib*). (2)

(b) Express $(3i - 7) + (7 - 4i) - (6 + 3i) + i^{23}$ in the form $(a + ib)$. (2)

Ans: (a) $(3 + 4i)(6 - 3i)(5 + i)$

$= (3 + 4i)(30 + 6i - 15i - 3i^2)$

$= (3 + 4i)(33 - 9i) = 99 - 27i + 132i - 36i^2$

$= 135 + 105i$

(b) $3i - 7 + 7 - 4i - 6 - 3i + (i^2)^{11}.\ i$

$= -6 - 4i + (-1)^{11}.\ i$

$= -6 - 5i$