PRACTICE PAPER 02 (2024-25)
CHAPTER 02 RELATIONS AND FUNCTIONS

SUBJECT: MATHEMATICS | **MAX. MARKS : 40**
CLASS : XI | **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. If R is a relation on the set A = {1, 2, 3, 4, 6, 7, 8, 9, 11, 12} given by x R y ⇔ y = 2 x, then R is equal to:
(a) {(2, 1), (4, 2), (8, 2), (9, 3)}
(b) {(2, 1), (4, 2), (6, 3)}
(c) (5, 1), (2, 4), (3, 6)}
(d) none of these

2. Range of the function f (x) = $\frac{x}{x+2}$ is
(a) R (b) R – {2} (c) R – {1} (d) R – {–2}

3. Let S = {x | x is a positive multiple of 3 less than 100}, P = {x | x is a prime number less than 20}. Then n(S) + n(P) is:
(a) 34 (b) 41 (c) 33 (d) 30

4. Range of the function f (x) = $\frac{x+4}{|x+4|}$ is
(a) {4} (b) {–4} (c) {–1, 1} (d) any real number

5. If $[x]^2 - 5[x] + 6 = 0$, where [ ] denote the greatest integer function, then
(a) x ∈ [3, 4) (b) x ∈ [2, 3) (c) x ∈ [2, 3) (d) x ∈ [2, 4)

6. Domain of $\sqrt{a^2 - x^2}$ (a > 0) is
(a) (–a, a) (b) [–a, a] (c) [0, a] (d) (–a, 0]

7. If R = {(x, y) : x, y ∈ Z, $x^2 + y^2 \le 16$} is a relation on Z, then the domain of R is:
(a) {0, 1, 2, 3, 4}
(b) {0, –1, –2, –3, –4}
(c) {–4, –3, –2, –1, 0, 1, 2, 3, 4}
(d) none of these

8. Let n(A) = m and n(B) = n. Then the total number of non-empty relations that can be defined from A to B is
(a) m^n (b) $n^m - 1$ (c) mn – 1 (d) $2^{mn} - 1$

For Q9 and Q10, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

9. **Assertion (A):** If (x – 1, y + 2) = (2, 4), then x = 3 and y = 2.
Reason (R): Two ordered pairs (x, y) and (p, q) equal, if their corresponding elements are equal.

10. Let A = {1, 2, 3, 4, 5, 6}. If R is the relation on A desired by {(a, b) : a, b∈A where b is square of a}
Assertion (A): The relations R in roster form is {(1, 1), (2, 4), (3, 9), (4, 16) (5, 25), (6, 36)}.
Reason (R): The domain and range of R is {1, 2, 3, 4, 5, 6}.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Define a relation R on the set N of natural numbers by
R = {(x, y) ; y = x + 3, x is a prime number less than 8 : x, y∈N}.
Depict this relationship using a roster form.
Write down the domain and the range.

12. In function f = {(1, 1), (0, –2), (3, 0), (2, 4)} be a linear function defined by formula, f(x) = a x + b. Then find ‘a’ and ‘b’.

13. Find the domain of each of the following functions given by : $f(x) = \dfrac{x^3 - x + 3}{x^2 - 1}$

14. Find the range of the following functions given by : $f(x) = \dfrac{|x-4|}{x-4}$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Find the domain and the range of the function : $f(x) = \sqrt{x^2 - 4}$

16. Find the domain and range of the real function $f(x) = \sqrt{9 - x^2}$

17. Let A = {1, 2, 3, 4, 5, 6}. Define a relation R from A to A by R = {(x, y) : y = x – 1}. Write R in roster form. Write down the domain, codomain and range of R.

SECTION – D

Questions 18 carry 5 marks.

18. Let A = {–2, –1, 0, 1, 2} and f : A → Z be given by f (x) = x^2 – 2 x – 3. Find:
(a) the range of f
(b) pre-images of 6, – 3 and 5.

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. A teacher asked randomly his two students to draw graph (figure) on the black board. Two students Raman and Prashant draw figure (graphs) (a) and (b) one by one on the board as shown below:

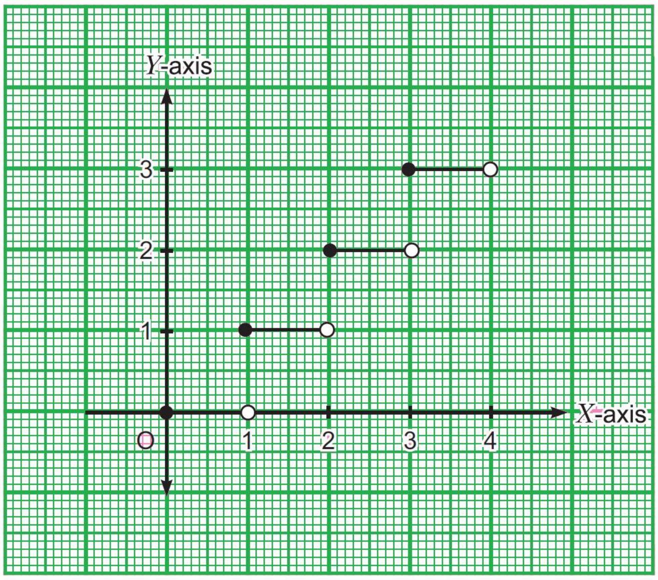


Figure (a)

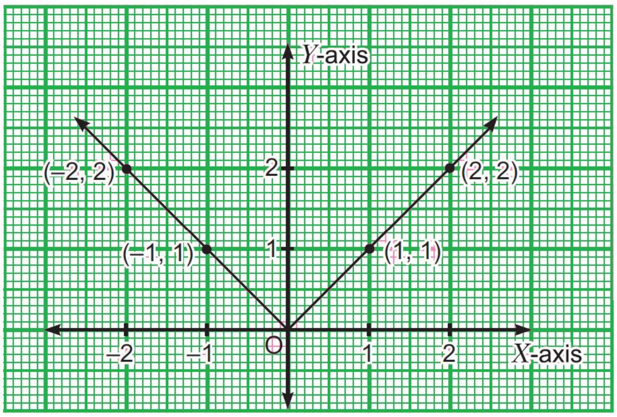


Figure (b)

(i) For the figure (a), write the function in terms of x.
(ii) (a) For the figure (b), write the function in terms of x.

OR

(ii) (b) Write the range of the function f(x) of the figure (a).
(iii) The function f : R → R defined by f(x) = |x| then write the value of f(– 3).

20. Maths teacher started the lesson Relations and Functions in Class XI. He explained the following topics:

Ordered Pairs: The ordered pair of two elements a and b is denoted by (a, b) : a is first element (or first component) and b is second element (or second component).
Two ordered pairs are equal if their corresponding elements are equal. i.e., (a, b) = (c, d) ⇒ a = c and b = d

Cartesian Product of Two Sets: For two non-empty sets A and B, the cartesian product A x B is the set of all ordered pairs of elements from sets A and B.
In symbolic form, it can be written as A x B= {(a, b) : a ∈ A, b ∈ B}

Based on the above topics, answer the following questions.
(i) If (a – 3, b + 7) = (3, 7), then find the value of a and b
(ii) If (x + 6, y – 2) = (0, 6), then find the value of x and y
(iii) If (x + 2, 4) = (5, 2x + y), then find the value of x and y
(iv) Find x and y, if (x + 3, 5) = (6, 2x + y).

PRACTICE PAPER 02 (2024-25)
CHAPTER 02 RELATIONS AND FUNCTIONS (ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**
CLASS : XI **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. If R is a relation on the set A = {1, 2, 3, 4, 6, 7, 8, 9, 11, 12} given by x R y ⇔ y = 2 x, then R is equal to:
(a) {(2, 1), (4, 2), (8, 2), (9, 3)}
(b) {(2, 1), (4, 2), (6, 3)}
(c) (5, 1), (2, 4), (3, 6)}
(d) none of these
Ans. (b) {(2, 1), (4, 2), (6, 3)}
Given, R is a relation on the set
A = {1, 2, 3, 4, 6, 7, 8, 9, 11, 12} given by
x R y ⇔ y = 2x
Here, y = 2 x
if, x = 1, y = 2
x = 2, y = 4
x = 3, y = 6
So, R = {(2, 1), (4, 2), (6, 3)}

2. Range of the function f (x) = $\frac{x}{x+2}$ is
(a) R (b) R – {2} (c) R – {1} (d) R – {–2}
Ans: (c) R – {1}
$y = \frac{x}{x+2} \Rightarrow xy + 2y = x$
$\Rightarrow 2y = x(1-y) \Rightarrow x = \frac{2y}{1-y}$
$\therefore\ y \neq 1$, Range = R – {1}

3. Let S = {x | x is a positive multiple of 3 less than 100}, P = {x | x is a prime number less than 20}. Then n(S) + n(P) is:
(a) 34 (b) 41 (c) 33 (d) 30
Ans. (b) 41
Given, S = {x | x is a positive multiple of 3 less than 100}
So, S = {3, 6, 9, 12 ...., 99}
We can see that it is an A.P. with a = 3, an = 99, d = 3.
Nth term of an A.P. is given by:
an = a + (n – 1)d
⇒ 99 = 3 + (n – 1)3
⇒ 96/3 = n – 1

$\Rightarrow$ n = 33
Therefore, n(S) = 33
Given: P = {x | x is a prime number less than 20}
So, P = {2, 3, 5, 7, 11, 13, 17, 19}
n(P) = 8
Now, n(S) + n(P) = 33 + 8
$\Rightarrow$ n(S) + n(P) = 41

4. Range of the function f (x) = $\frac{x+4}{|x+4|}$ is
(a) {4} (b) {–4} (c) {–1, 1} (d) any real number
Ans: (c) {–1, 1}
$$|x+4| = \begin{cases} x+4, & x \geq -4 \\ -(x+4), & x < -4 \end{cases}$$

5. If $[x]^2 - 5[x] + 6 = 0$, where [ ] denote the greatest integer function, then
(a) x ∈ [3, 4) (b) x ∈ [2, 3) (c) x ∈ [2, 3) (d) x ∈ [2, 4)
Ans: (d) x ∈ [2, 4), we have $[x]^2 - 5[x] + 6 = 0$
$\Rightarrow [x]^2 - 3[x] - 2[x] + 6 = 0$
$\Rightarrow [x]([x] - 3) - 2([x] - 3) = 0$
$\Rightarrow ([x] - 2)([x] - 3) = 0 \Rightarrow [x] - 2 = 0$ or $[x] - 3 = 0$
$\Rightarrow [x] = 2$ or $[x] = 3 \Rightarrow x \in [2, 3)$ or $x \in [3, 4) \Rightarrow x \in [2, 4)$

6. Domain of $\sqrt{a^2 - x^2}$ (a > 0) is
(a) (–a, a) (b) [–a, a] (c) [0, a] (d) (–a, 0]
Ans: (b) [–a, a], let y = $\sqrt{a^2 - x^2}$ the function y is defined if
$a^2 - x^2 \geq 0 \Rightarrow x^2 - a^2 \leq 0$ or $x^2 \leq a^2$
$-a \leq x \leq a$
So, domain of y = [–a, a]

7. If R = {(x, y) : x, y ∈ Z, $x^2 + y^2 \leq 16$} is a relation on Z, then the domain of R is:
(a) {0, 1, 2, 3, 4}
(b) {0, –1, –2, –3, –4}
(c) {–4, –3, –2, –1, 0, 1, 2, 3, 4}
(d) none of these
Ans. (c) {–4, –3, –2, –1, 0, 1, 2, 3, 4}

8. Let n(A) = m and n(B) = n. Then the total number of non-empty relations that can be defined from A to B is
(a) m^n (b) $n^m - 1$ (c) mn – 1 (d) $2^{mn} - 1$
Ans: (d) $2^{mn} - 1$, as n(A) = m, n(B) = n $\Rightarrow$ n(A × B) = mn
So, number of relations = 2^{mn} including void relation f.
Number of non-empty relations = $2^{nm} - 1$

For Q9 and Q10, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.

(d) A is false but R is true.

9. **Assertion (A):** If (x – 1, y + 2) = (2, 4), then x = 3 and y = 2.
Reason (R): Two ordered pairs (x, y) and (p, q) equal, if their corresponding elements are equal.
Ans: (a) Both A and R are true and R is the correct explanation of A.

10. Let A = {1, 2, 3, 4, 5, 6}. If R is the relation on A desired by {(a, b) : a, b∈A where b is square of a}
Assertion (A): The relations R in roster form is {(1, 1), (2, 4), (3, 9), (4, 16) (5, 25), (6, 36)}.
Reason (R): The domain and range of R is {1, 2, 3, 4, 5, 6}.
Ans: (c) A is true but R is false.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Define a relation R on the set N of natural numbers by
R = {(x, y) ; y = x + 3, x is a prime number less than 8 : x, y∈N}.
Depict this relationship using a roster form.
Write down the domain and the range.
Ans. Given N = Set of all natural numbers and
R = {(x ,y) : y = x + 3, x is a prime number less than 8 ; x, y∈N}
= {(x, y) : y = x + 3, x∈ {2, 3, 5, 7} ; x, y∈N}.
The given relation in roster form can be written as
R = {(2, 5), (3, 6), (5, 8) (7, 10)}.
Hence, domain of R = {2, 3, 5, 8} and range of R = {5, 6, 8, 10}.

12. In function f = {(1, 1), (0, –2), (3, 0), (2, 4)} be a linear function defined by formula, f(x) = a x + b. Then find ‘a’ and ‘b’.
Ans. Here, f(x) = ax + b
f = {(1, 1)} (0, –2), (3, 0), (2, 4)}
and f (x) = a x + b
then, f (0) = –2
⇒ –2 = a × 0 + b ⇒ b = –2
and f (1) = 1
⇒ 1 = a × 1 + b
⇒ 1 = a + b
⇒ 1 = a –2 ⇒ a = 3

13. Find the domain of each of the following functions given by : $f(x)=\dfrac{x^3-x+3}{x^2-1}$

Ans: $f(x)=\dfrac{x^3-x+3}{x^2-1}$,
For domain, $x^2 - 1 \neq 0 \Rightarrow x \neq \pm 1$
∴ Domain = R – {–1, 1}

14. Find the range of the following functions given by : $f(x)=\dfrac{|x-4|}{x-4}$

Ans: $y=\dfrac{(x-4)}{(x-4)} or \dfrac{-(x-4)}{(x-4)}$, i.e. 1 or –1
∴ range {–1, 1}

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Find the domain and the range of the function : $f(x) = \sqrt{x^2 - 4}$

Ans: Given, $f(x) = \sqrt{x^2 - 4}$; For D_f., $f(x)$ must be a real number.

$\Rightarrow \sqrt{x^2 - 4}$ must be a real number. $\Rightarrow x^2 - 4 \geq 0 \Rightarrow (x + 2)(x - 2) \geq 0$

$\Rightarrow$ Either $x \leq -2$ or $x \geq 2$. $\Rightarrow D_f = (-\infty, -2] \cup [2, \infty)$.

For R_f, let $y = \sqrt{x^2 - 4}$ ... (i)

As square root of a real number is always non-negative, $y \geq 0$.

On squaring (i), we get $y^2 = x^2 - 4 \Rightarrow x^2 = y^2 + 4$ but $x^2 \geq 0\ \forall\ x \in D_f$.

$\Rightarrow y^2 + 4 \geq 0 \Rightarrow y^2 \geq -4$, which is true $\forall\ y \in R$,

Also, $y \geq 0$. $\Rightarrow R_f = [0, \infty)$.

16. Find the domain and range of the real function $f(x) = \sqrt{9 - x^2}$

Ans: Given function is $f(x) = \sqrt{9 - x^2}$

For domain of 'f', $9 - x^2 \geq 0$

$\Rightarrow 9 \geq x^2 \Rightarrow x^2 \leq 9 \Rightarrow -3 \leq x \leq 3$

$\therefore$ Domain is $\{x \in R \mid -3 \leq x \leq 3\}$, i.e. $[-3, 3]$

For range : $f(x) = \sqrt{9 - x^2} \Rightarrow y = \sqrt{9 - x^2}$

$\sqrt{9 - x^2}$ is always +ve

$\Rightarrow y$ is always +ve.

$\Rightarrow y^2 = 9 - x^2 \Rightarrow x^2 = 9 - y^2$

$\Rightarrow x = \sqrt{9 - y^2}$

For x to exist $9 - y^2 \geq 0 \Rightarrow y^2 \leq 9 \Rightarrow -3 \leq y \leq 3$

As $y \geq 0$

$\therefore$ Range = $[0, 3]$

17. Let A = {1, 2, 3, 4, 5, 6}. Define a relation R from A to A by R = {(x, y) : y = x – 1}. Write R in roster form. Write down the domain, codomain and range of R.

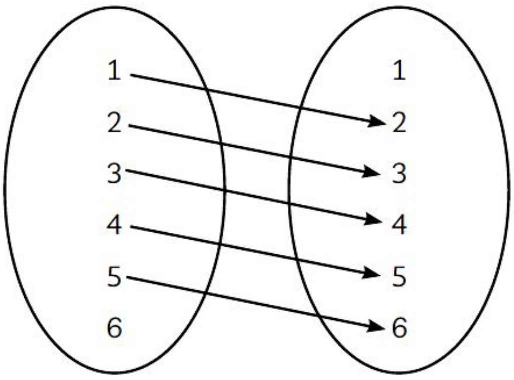
Ans. Given A = {1, 2, 3, 4, 5, 6} and

R = {(x, y) : y = x – 1}.

The given relation in roster form can be written as,

R = {(1, 0), (2, 1),(3, 2), (4, 3), (5, 4), (6, 5)}

The given relation can be represented with the following arrow diagram:



So, domain of R = {1, 2, 3, 4, 5, 6}, co-domain of R = {0, 1, 2, 3, 4, 5} and range of R = {2, 3, 4, 5, 6}.

SECTION – D

Questions 18 carry 5 marks.

18. Let A = {–2, –1, 0, 1, 2} and f : A → Z be given by $f(x) = x^2 - 2x - 3$. Find:

(a) the range of f

(b) pre-images of 6, – 3 and 5.

Ans. (a) We have, $f(x) = x^2 - 2x - 3$,

$\therefore f(-2) = (-2)^2 - 2(-2) - 3 = 5$,

$f(-1) = (-1)^2 - 2(-1) - 3 = 0$, $f(0) = -3$,

$f(1) = 1^2 - 2 \times 1 - 3 = -4$ and

$f(2) = 2^2 - 2 \times 2 - 3 = -3$.

So, range (f) = $\{f(-2), f(-1), f(0), f(1), f(2)\}$

$= \{5, 0, -3, -4, -3\}$.

(b) Let x be a pre-image of 6. Then,

$f(x) = 6$

$\Rightarrow x^2 - 2x - 3 = 6$

$\Rightarrow x^2 - 2x - 9 = 0$

$\Rightarrow x = 1 \pm \sqrt{10}$

Since, $x = 1 \pm \sqrt{10} \notin$ -A.

So, there is no pre-image of 6.

Let x be a pre-image of –3.

Then, $f(x) = -3$

$\Rightarrow x^2 - 2x - 3 = -3$

$\Rightarrow x^2 - 2x = 0$

$\Rightarrow x = 0, 2$.

Clearly, $0, 2 \in A$.

So, 0 and 2 are pre-images of –3.

Let x be a pre-image of 5. Then, $f(x) = 5$

$\Rightarrow x^2 - 2x - 3 = 5$

$\Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow (x - 4)(x + 2) = 0$

$\Rightarrow x = 4, -2$.

Since, $-2 \in A$ but $4 \notin A$.

So, –2 is a pre-image of 5.

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. A teacher asked randomly his two students to draw graph (figure) on the black board. Two students Raman and Prashant draw figure (graphs) (a) and (b) one by one on the board as shown below:

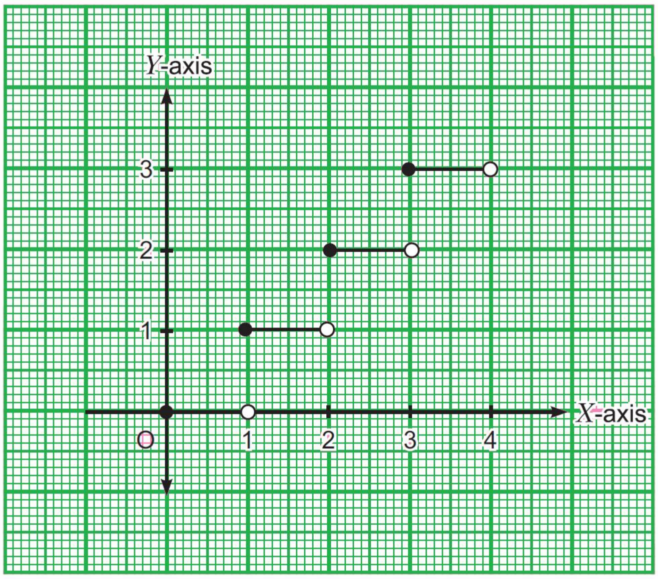


Figure (a)

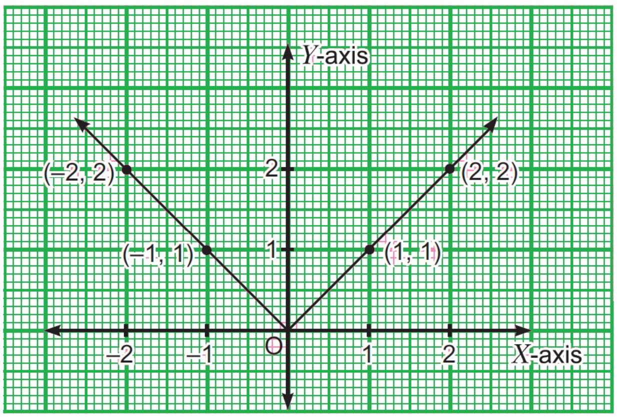


Figure (b)

(i) For the figure (a), write the function in terms of x.

(ii) (a) For the figure (b), write the function in terms of x.

OR

(ii) (b) Write the range of the function f(x) of the figure (a).

(iii) The function $f : R \rightarrow R$ defined by $f(x) = |x|$ then write the value of $f(-3)$.

Ans: (i) From the given figure, we have

when, $0 \leq x < 1$ & $y = 0$

when $1 \leq x < 2$ & $y = 1$

$2 \leq x < 3$ & $y = 2$

So, it is greatest integer function.

$\therefore f(x) = [x] \leq x$, where x is an integer.

(ii) (a) From the given figure (b), we have

For $x = 1, -1$ & $y = 1$

$x = 2, -2$ & $y = 2$

$\therefore y = |x|$

So, it is a modulus function.

$\therefore f(x) = |x|$

OR

(ii) (b) For the figure (a), we have

$f(x) = [x] \leq x$, where x is an integer

$\therefore$ Its range is the set of integers.

(iii) We have $f(x) = |x|$

$\Rightarrow f(-x) = |-3| = 3$

20. Maths teacher started the lesson Relations and Functions in Class XI. He explained the following topics:

Ordered Pairs: The ordered pair of two elements a and b is denoted by (a, b) : a is first element (or first component) and b is second element (or second component).

Two ordered pairs are equal if their corresponding elements are equal. i.e., $(a, b) = (c, d) \Rightarrow a = c$ and $b = d$

Cartesian Product of Two Sets: For two non-empty sets A and B, the cartesian product A x B is the set of all ordered pairs of elements from sets A and B.

In symbolic form, it can be written as $A \times B = \{(a, b) : a \in A, b \in B\}$

Based on the above topics, answer the following questions.

(i) If $(a - 3, b + 7) = (3, 7)$, then find the value of a and b

(ii) If $(x + 6, y - 2) = (0, 6)$, then find the value of x and y

(iii) If $(x + 2, 4) = (5, 2x + y)$, then find the value of x and y

(iv) Find x and y, if $(x + 3, 5) = (6, 2x + y)$.

Ans:

(i) We know that, two ordered pairs are equal, if their corresponding elements are equal.

$(a - 3, b + 7) = (3, 7)$

$\Rightarrow a - 3 = 3$ and $b + 7 = 7$ [equating corresponding elements]

$\Rightarrow a = 3 + 3$ and $b = 7 - 7 \Rightarrow a = 6$ and $b = 0$

(ii) $(x + 6, y - 2) = (0, 6)$

$\Rightarrow x + 6 = 0 \Rightarrow x = -6$ and $y - 2 = 6 \Rightarrow y = 6 + 2 = 8$

(iii) $(x + 2, 4) = (5, 2x + y)$

$\Rightarrow x + 2 = 5 \Rightarrow x = 5 - 2 = 3$ and $4 = 2x + y \Rightarrow 4 = 2 \times 3 + y \Rightarrow y = 4 - 6 = -2$

(iv) $x + 3 = 6$, $2x + y = 5 \Rightarrow x = 3$, $y = 1$