9. RATIONAL NUMBERS

Rational numbers are those numbers, which can be expressed in the form of a p/q, where p and q are integers and $q \neq 0$. The collection of all rational numbers are represented by Q.

$$\therefore \quad Q = \left\{\frac{p}{q}; p, q \text{ are integers and } q \neq 0\right\}$$

(i). Equivalent Rational Numbers

Two rational numbers are said to be equivalent, if numerator and denominator of both rational numbers are in proportion.

For example; $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{10}{20} = \frac{25}{50} = \frac{48}{96}$ etc.

(ii). Insertion of Rational Number Between Two Numbers

Methods to find rational number(s) between two numbers can be shown by the following flow chart :

Methods to find Rational Number(s) between two numbers

One Rational Number:
Between two numbers x and y, where $y > x$, a rational number is given by $\frac{x+y}{2}$.

More than one or n rational numbers :
Between two numbers x and y, where $y > x$, n rational numbers are given by $(x + d), (x + 2d)$, $(x + 3d), \ldots$, $(x + nd)$ where $d = \frac{y - x}{n + 1}$.

(iii). Terminating Decimals

Every fraction p/q can be expressed as a decimal. If the decimal expression of p/q terminates, i.e., comes to an end, then the decimal so obtained is called a terminating decimal.

(iv). Repeating or Recurring Decimals

A decimal in which a digit or a set of digits repeats periodically, is called a repeating or a recurring decimal.

10. IRRATIONAL NUMBERS

A number, which is not rational is called an irrational number.
In other words, number which cannot be expressed in the form of p/q, where p and q are integers, and $q \neq 0$.
For example: π, e, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ etc.

The set of irrational numbers is denoted by S or Q' or I.

Notes:

1. π is the ratio of circumference to diameter of a circle and it is an irrational number. $\frac{22}{7}$ or 3.142 is just an approximate value of π.

2. An irrational number can neither be expressed as a terminating nor as a repeating decimal.

POLYNOMIALS

1. POLYNOMIAL IN ONE VARIABLE

An algebraic expression of the form

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0,\ a_n \neq 0,$$

where $a_0, a_1, a_2, \dots, a_n$ are constants and n is a non-negative integer, is known as a polynomial in one variable x.

- **Degree of a Polynomial**
  It is the highest power of the variable in the polynomial.

  For example: $2x^3 + 3x + 4$ is a polynomial in x of degree 3.

- **Terms of a polynomial**
  The parts of a polynomial separated by '+' or '−' sign are called terms of the polynomial.

- **Coefficients**

  Numerical factor of term with its sign is called coefficient of that term.

  Each term of a polynomial has a coefficient.

  For example; In polynomial $7x^3 + 5x^2 - 3x + 2$, the coefficient of x is -3.

- **Classification of Polynomials**

  On the basis of number of terms, a polynomial can be classified as :

Polynomial	Definition	Examples
Monomial	A polynomial containing one non-zero term	$5x, 2x^2$
Binomial	A polynomial containing two non-zero terms	$5 + 7x,\ y^{30} + 1$
Trinomial	A polynomial containing three non-zero terms	$8 + 3x + x^2, 7 + 5y + 6y^5$

On the basis of degree of variables, a polynomial can be classified as :

Polynomial	Definition	Examples
Constant polynomial	A polynomial of degree 0	$3, 5, -70$
Linear polynomial	A polynomial of degree 1	$3x + 5, 2y - 9$
Quadratic polynomial	A polynomial of degree 2	$x^2 + 5x + 3, 2y^2 - 7y + 9$
Cubic polynomial	A polynomial of degree 3	$4x^3 + 3x^2 - 7x - 1$
Biquadratic polynomial	A polynomial of degree 4	$x^4 - 3x^3 + 2x^2 - 5x + 7$

- **Zero Polynomial**

  The constant polynomial 0 is called zero polynomial. The degree of a zero polynomial is not defined.

➢ **Value of a Polynomial**

If $p(x)$ is a polynomial, then value of $p(x)$ at $x = a$ is given by $p(a)$.

2. ZERO OF A POLYNOMIAL

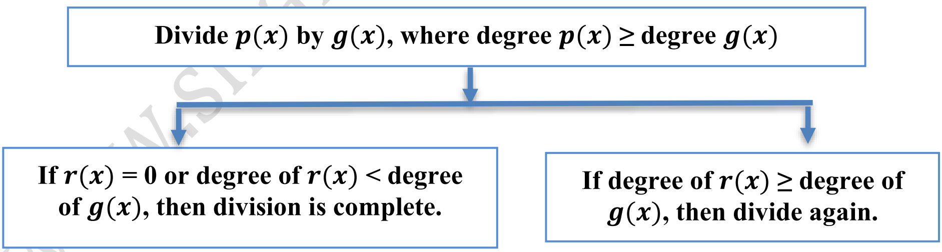
A real number a is called a zero of a polynomial $p(x)$, if $p(a) = 0$.

Note:

1. '0' may be a zero of a polynomial.
2. Every linear polynomial in one variable has a unique zero.
3. A non-zero constant polynomial has no zero.
4. Every real number is a zero of the zero polynomial.
5. A polynomials can have more than one zero.

3. DIVISION OF A POLYNOMIAL

If $p(x)$ and $g(x)$ are any two polynomials, such that degree of $p(x) \geq$ degree of $g(x)$ and $g(x) \neq 0$, then we find polynomials $q(x)$ and $r(x)$, such that $p(x) = g(x)q(x) + r(x)$, where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$.



4. **REMAINDER THEOREM**

Let $p(x)$ be a polynomial of degree greater than or equal to 1 and a be any real number. If we divide $p(x)$ by $(x - a)$, then the remainder is given by $p(a)$.

5. **FACTOR THEOREM**

Let $p(x)$ be a polynomial of degree greater than or equal to 1 and a be any real number then,

(i) If $p(a) = 0$, then $(x - a)$ is factor of $p(x)$.

(ii) If $(x - a)$ is factor of $p(x)$, then $p(a) = 0$.

6. **METHODS OF FACTORISATION**

By splitting the middle term

To factorise a polynomial $ax^2 + bx + c$, write b as a sum of two integers, whose product is ac.

Using the Factor Theorem

To factorise a polynomial $ax^2 + bx + c$, first make coefficient of x^2 equal to 1 i.e., $ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = a\ g(x)$ (say). Then, find all possible factors of $\frac{c}{a}$. By trial method, find the values of a and b for which $g(x) = 0$. Then, $(x - a)$ and $(x - b)$ are the factors of $g(x)$ and the given polynomial as well.

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

1. Linear equation in two variables

$a_1x + b_1y + c_1 = 0$ … (1)
$a_2x + b_2y + c_2 = 0$ … (2)

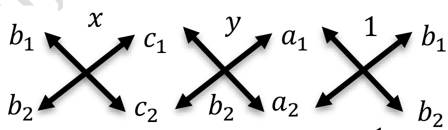
2. Substitution Method

$a_1x + b_1y + c_1 = 0$ … (1)
$a_2x + b_2y + c_2 = 0$ … (2)
Substituting the value of x from eq. (1) in eq. (2) and finding y and vice versa.

3. Elimination Method

$a_1x + b_1y = c_1$ … (1)
$a_2x + b_2y = c_2$ … (2)
Multiplying eq. (1) by a_2 and eq. (2) by a_1 making the coefficient of x same and hence eliminating x and finding y.

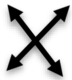
4. Cross multiplication method



$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - a_1c_2} = \frac{1}{a_1b_2 - b_1a_2}$$

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - b_1a_2}$$

$$y = \frac{c_1a_2 - a_1c_2}{a_1b_2 - b_1a_2}$$

Intersecting lines $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$  (intersect at 1 point) one solution

Coincident $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$  Infinite solutions

Parallel lines $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$  (Non-Intersecting) No solution