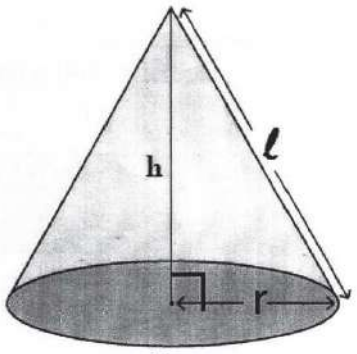
CHAPTER-11

SURFACE AREAS AND VOLUMES

MIND MAP



Cone

Slant height of cone $l = \sqrt{h^2 + r^2}$

Curved Surface area of cone $= \pi r l$

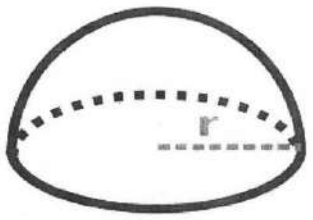
Total Surface area of cone $= \pi r l + \pi r^2$
$= \pi r(l + r)$

Volume of cone $= \frac{1}{3}\pi r^2 h$

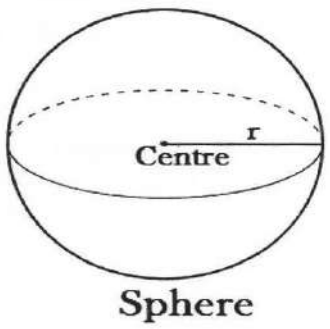
Total Surface area of hemisphere $= 3\pi r^2$

Curved Surface area of hemisphere $= 2\pi r^2$

Volume of hemisphere $= \frac{2}{3}\pi r^3$



Hemisphere



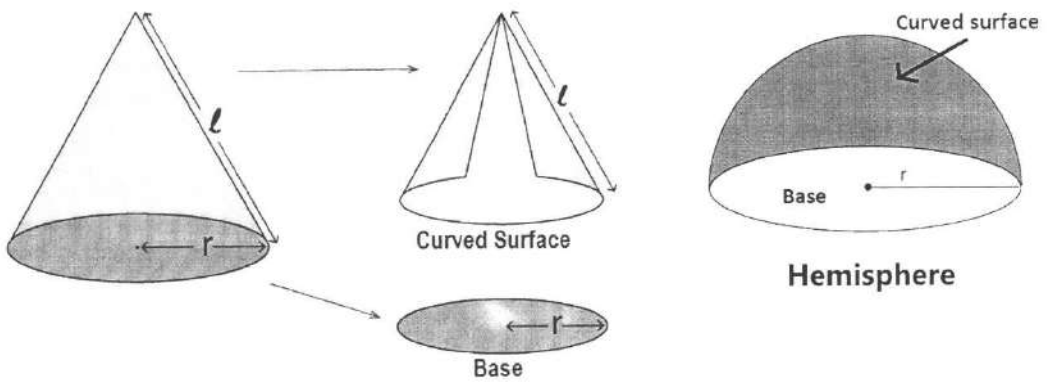
Sphere

Total Surface area of Sphere $= 4\pi r^2$

Volume of Sphere $= \frac{4}{3}\pi r^3$

Keys points

- The surface of a solid object or shape can be flat or non-flat. The non-flat surface is known as curved surface.
- Surface Area is the area of the surface of a solid object or shape.
  (i) **Lateral / Curved Surface Area :-** Area of the curved surface of a solid object or shape.



  (ii) **Total Surface Area :-** Area of all the surfaces, curved and flat both, of a solid object or shape.
- **Volume** is the shape occupied by any solid object. For hollow objects, volume is termed as capacity.
- Surface area is a **two-dimensional** measurement and measured as square-units i.e. m^2, cm^2, whereas volume is a **three-dimensional** measurement and measured as cubic-units i.e. m^3, cm^3.
- Examples of surface area are : Wrapping a gift, painting a wall, covering a bowl with a lid etc.
- Examples of volume are : Water in tank, matchboxes in a packet, soup in a bowl etc.
- A sphere has only curved surface. So, curved surface area of a sphere is also its total surface area.
- The height, slant height and radius of a cone together form a right angled triangle where height and radius are sides but slant height is the hypotenuse.
- $1m^3 = 1000\ l$ (litre)
- $1000\ cm^3 = 1\ l$ (litre)

Very Short Answer Questions (1 Mark)

1. If the volume of a sphere is numerically equal to its surface area. Then radius of sphere is:

(a) 1 unit (b) 3 unit

(c) 2 unit (d) 6 unit

2. The surface area of a solid hemisphere having radius r is:

(a) $2\pi r^2$ (b) $3\pi r^2$

(c) $4\pi r^2$ (d) $\frac{2}{3}\pi r^3$

3. The height of cone of diameter 10 cm and slant height 13 cm is:

(a) 12 cm (b) 13 cm

(c) $\sqrt{69}$ cm (d) $\sqrt{194}$ cm

4. The radius of a sphere is 2r. Its volume will be:

(a) $\frac{32}{3}\pi r^3$ (b) $4\pi r^3$

(c) $\frac{4}{3}\pi r^3$ (d) $\frac{8\pi^3}{3}$

5. If the radius of a sphere is increased by 10% then its volume will be increased by:

(a) 11.1% (b) 22.1%

(c) 33.1% (d) 44.1%

6. The radius of the sphere is 7 cm. The surface area of the sphere is _____.

7. If the radius of a sphere is doubled then the ratio of their volume is _____.

8. The diameter of sphere whose surface area is 55.44 m^2 is _____.

9. Total surface area of a cone whose radius is $\frac{p}{2}$ and slant height is l is _____.

10. Volume of a cone $= \frac{1}{3}$ (radius)$^2 \times$ _____.

11. The radius of a hemispherical balloon increases from 6 cm to 12 cm as air is being pumped into it. Find the radius of the surface area of the balloon in the two cases.

12. The radius of a sphere is 21 cm. What is the surface area of the sphere?

13. The slant height of a cone is 20 cm and its diameter is 24 cm. The height of the cone is:

(a) 12 cm (b) 16 cm

(c) 10 cm (d) 14 cm

14. If the radius of a sphere is halved then its surface-area will become

(a) half (b) one-third

(c) double (d) one-fourth

15. Find the curved surface area of a cone having slant height 29 cm and base radius 21 cm.

16. Find the volume of a sphere having surface area 616 m^2.

Short Answer type-I Questions (2 Marks)

17. A conical pit of diameter 7 m is 25 m deep. Find its capacity in kilolitres.

18. The diameter of a hemispherical bowl is 21 cm. What is the quantity of milk (in litres) it can hold?

19. A right angled triangle having sides 6 cm, 8 cm and 10 cm is rotated about the side 8 cm. Find the total surface area of the solid so obtained.

20. A birthday cap is in the form of a cone of radius 10 cm and height 15 cm. How much area of a sheet is required to make 10 such caps? (Use $\pi = 3.14$)

21. The total surface area of a sphere and hemisphere is equal. Find the ratio of their volumes.

22. What is the volume of the largest cone that can be inscribed completely in a hollow hemispehere of radius 7 cm?

23. Which is false? Correct the false statement.

(a) Volume of the hollow sphere = $\frac{4}{3}\pi\left(R^3 - r^3\right)$

(b) Volume of a hemisphere = $\frac{2}{3}\pi r^3$

(c) Total surface area of a hemisphere = $3\pi r^2$

(d) Curved surface area of a hemisphere = πr^2

24. A cone is 8.4 cm high and the radius of its base is 2.1 cm. It is melted and recast into a sphere. Find the radius of the sphere.

25. Find the area of canvas required for a conical tent of height 24 m and base radius 7 m.

26. Find the ratio of total surface area of a sphere and a hemisphere of same radius.

27. If the radius and slant height of a cone are $\frac{r}{2}$ and $2l$ then find its total surface area.

28. A cone and a hemisphere have equal base and equal volumes. Find the ratio of their heights.

Short Answer type-II Questions (3 Marks)

29. A hemispherical bowl is to be painted from inside at the rate of ₹ 20 per 100 m^2. The total cost of painting is ₹ 30.80. Find the inner surface area of the bowl.

30. The radius of a sphere is 10 cm. If the radius is increased by 1 cm then prove that volume of the sphere is increased by 33.1%.

31. The diameter of a hemisphere is decreased by 30%. What will be the percentage change in its total surface area?

32. The volume of a sphere is 4851 cm^3. How much should its radius be reduced so that its volume becomes $\frac{4312}{3}$ cm^3?

33. The volumes of the two spheres are in the ratio 64 : 27. Find the ratio of their surface areas.

34. Twenty Seven solid iron spheres each of radius r and surface area S are melted to form sphere with surface area S'. Find the

(i) radius R of the new sphere.

(ii) Ratio of S and S'.

35. The diameter of a metallic ball is 4.2 cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm^3.

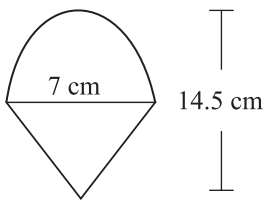
36. The base radius of a cone and radius of a Hemisphere is 12 cm. The height of the cone is 16 cm. Which of these has greater curved surface area?

37. Gautam has some balls of radius 2.1 cm. The total volume of the balls is 582.12 cm^3. How many balls does Gautam has?

Long Answer type Questions (5 Marks)

38. A right circular cone is 5.4 cm high and radius of its base is 2 cm. It is melted and recast into another right circular cone with radius of base as 1.5 cm. Find the equal height of the new cone. Prove that both the cones have equal volume.

39. A toy in the form of a cone mounted on a hemisphere of diameter 7 cm. The total height of the toy is 14.5 cm. Find the volume and the total surface are of the toy.



(Take $\pi = \frac{22}{7}$)

40. The slant height of a cone is 14 cm and its curved surface area is 308 cm^2. Find the volume and total surface area of the cone.

41. If h, c and v respectively, are the height, the curved surface and volume of the cone then prove that $3pvh^3 - c^2h^2 + 9v^2 = 0$

42. The curved surface area of a cone is two-third of its total surface area. If total surface area of the cone is 231 cm^2 then find the volume of the cone.

43. A hemispherical steel bowl is 0.25 cm thick and its inner radius is 5 cm. Find the cost of coating the complete bowl at the rate of 0.85 per cm^2. (Use $\pi = 3.14$)

Chapter - 11
Surface Area and Volumes
Answers

1. (b) 3 units

2. (b) $3\pi r^2$

3. (a) 12 cm

4. (a) $\frac{32}{3}\pi r^3$

5. 33.1%

6. 616 cm^2

7. 1 : 8

8. 2.1 m

9. $\frac{\pi p^2}{4} + \pi pl$

10. height

11. 1 : 4

12. 5544 cm^2

13. (b) 16 cm

14. (d) one-forth

15. 1914 cm^2

16. 1437.3 cm^3

17. $r = \frac{7}{2}$ m, $h = 25$ m

$$\text{capacity} = \frac{1925}{6}\text{m}^3$$
$$= 320.83 \text{ kl}$$

18. Volume = 2425.5 cm^3

$$\text{Quantity of milk} = \frac{2425.5}{1000}$$
$$= 2.43 \text{ litre}$$

19. $h = 8$ cm, $l = 10$ cm, $r = 6$ cm

Total surface area $= \pi r(l + r)$

$$= \frac{22}{7} \times 6 \times 16 = 301.7 \text{ cm}^2$$

20. $1570\sqrt{13}$ cm^2

21. $3\sqrt{3} : 4$

22. $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$

$$= \frac{22 \times 49}{3} = \frac{1078}{3} \text{ cm}^3$$

23. (d) Curved surface area of a hemisphere = πr^2

24. 2.1 cm

25. 550 m^2

26. 4 : 3

27. $\pi r\left(1 + \frac{r}{4}\right)$

28. 2 : 1

29. 154 m^2

30. $r_1 = 10$ cm $\Rightarrow$ $v_1 = \frac{4}{3}\pi(10)^3$

$r_2 = 11$ cm $\Rightarrow$ $v_2 = \frac{4}{3}\pi(11)^3$

Increase in volume of sphere

$$= \left(\frac{v_2 - v_1}{v_1} \times 100\right)\% = \frac{331}{1000} \times 100\%$$
$$= 3.31\%$$

31. $r = 0.7r$

New diameter of hemisphere = 0.7d ; radius = $0.7\frac{d}{2}$

Decrease in total surface area = $\left[\frac{3\pi\left(\frac{d}{2}\right)^2 - 3\pi\left(\frac{0.7d}{2}\right)^2}{3\pi\left(\frac{d}{2}\right)^2} \times 100\right]\%$

$= 0.51 \times 100 = 51\%$

32. $V_1 = \frac{4}{3}\pi R^3 = 4851 \Rightarrow R = \frac{21}{2} = 10.5$ cm

$V_2 = \frac{4}{3}\pi r^3 = \frac{4312}{3} \Rightarrow r = 7$ cm

Decrease in radius = 10.5 – 7 = 3.5 cm

33. $\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27} \Rightarrow \left(\frac{r_1}{r_2}\right) = \frac{4}{3}$

Ratio of surface area $= \frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = 16:9$

34. (1) $R = 3r$ (2) $S : S' = 1 : 9$

35. 345.39 g

36. CSA of hemisphere = 288 π cm^2
l = 20 cm,
CSA of cone = 240 π cm^2
So, hemisphere has more CSA.

37. 15 balls

38. Volume of new cone = Volume of old cone

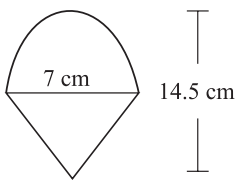
$\Rightarrow \pi (1.5)^2 \times h = \pi (2)^2 \times 5.4$

$\Rightarrow h = 9.6$ cm

Volume difference $= \frac{1\pi}{3}(5.4 \times 2 \times 2 - 1.5 \times 1.5 \times 9.6) = 0$

39. Volume of toy = CSA of (cone + hemisphere) $= 231\ cm^3$

Total surface area of toy = $204.05\ cm^2$



40. radius $r = 7$ cm

TSA = $462\ cm^2$

Volume = $\frac{1078}{\sqrt{3}}\ cm^3$

41. Let r - radius and l - Slant height

$l = \sqrt{r^2 + h^2}$, $v = \frac{1}{3}\pi r^2 h$, $c = \pi rl$

$\therefore\ 3\pi vh^3 - c^2h^2 + 9v^2$

$= 3\pi \times \frac{1}{3}\pi r^2 h \times h^3 - (\pi rl)^2 h^2 + 9\left(\frac{1}{3}\pi r^2 h\right)^2$

$= \pi^2 r^2 h^4 - \pi^2 r^2 l^2 h^2 + \pi^2 r^4 h^2$

$= \pi^2 r^2 h^4 - \pi^2 r^2 h^2 (r^2 + h^2) + \pi^2 r^2 h^2$

$= \pi^2 r^2 h^4 - \pi^2 r^4 h^2 - \pi^2 r^2 h^4 + \pi^2 r^4 h^2 = 0$

42. CSA = $\frac{2}{3}$ TSA $\Rightarrow$ CSA = $154\ cm^2$

Also CSA + πr^2 = TSA $\Rightarrow \pi r^2 = 77\ cm^2$

$\Rightarrow r^2 = \frac{49}{2}\ cm^2$

Now CSA = $154\ cm^2 \Rightarrow l = 7\sqrt{2}$ cm

Volume of cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \frac{49}{2} \times 7\sqrt{2} = \frac{539\sqrt{2}}{3}\ cm^3$

43. Inner radius $r = 5$ cm; Outer radius $R = 5.25$ cm

TSA of bowl = Outer SA + Inner SA + Area of thickness

$= 2\pi R^2 + 2\pi r^2 + (\pi R^2 - \pi r^2)$

$= 2p(R^2 + r^2) + \pi(R^2 - r^2)$

$= 256.7\ cm^2$ (approx)

Cost of coating = ₹ 218.19

CHAPTER-11
SURFACE AREA AND VOLUMES
PRACTICE TEST

Time: 1 hr **M.M: 20**

1. The volume of a sphere is 310.4 cm^3. Find its radius. (1)

2. Three spheres of radii 3 cm, 4 cm, 5 cm are melted to form a new sphere. Find the radius of the new sphere. (1)

3. The ratio of radius and slant height of a cone is 4:7. If the curved surface area of cone is 192 cm^2 then find its radius. (2)

4. A semicircular sheet of paper of diameter 14 cm is bend to form a conical cup. Find the capacity of the cup. (2)

5. The seed of a corn has dimensions 1.8 cm × 0.8 cm × 0.2 cm. The height of the corn-tube is 13.7 cm and its radius is 4.2 cm. Assuring that the corn seeds are of same size, find the number of seeds on the corn-tube. (3)

6. The radius of a sphere is 5 cm. If the radius is increased by 20% then how much percentage increase will be in the volume? (3)

7. The surface area of the cone is double of the other and slant height of the second cone is double of the first cone. Find the ratio of their radii. (3)

8. Ajay kept ice-cream in a hemi spherical bowl of 28 cm diameter. He filled ice-cream in ice-cream cones of 8 cm diameter and height 6 cm. A hemisphere of diameter 8 cm is also kept over the cone. In how many cups the ice-cream is filled? (5)