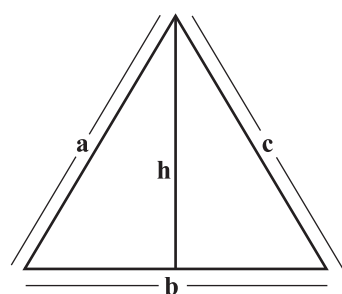# HERON'S FORMULA

Mind Map

Area of triangle (general formula) = $\dfrac{1}{2} \times b \times h$

where b = base

and h = height

Area of triangle (Herons formula) = $\sqrt{s(s-a)(s-b)(s-c)}$

where s is semi-perimeter and $s = \dfrac{a+b+c}{2}$

a, b and c are sides of a triangles

Keys points

- When base and height of a triangle are known, then area of triangle is found using general formula.

- Herons formula is used to find area of triangle when all the three sides of triangle are known.

- All sides of an equilateral triangle are equal.

- An isosceles triangle has two equal sides while a scalene triangle has no side equal.

- The sum of all the sides is called the perimeter.

- $(s - a) + (s - b) + (s - c) = 3s - (a + b + c) = s$

- Herons formula can be used to find the area of any kind of triangle.

- Altitude of an equilateral triangle $= \dfrac{\sqrt{3}a}{2}$

    a - side

- For right angle triangle

    $(\text{Base})^2 + (\text{altitude})^2 = (\text{Hypothenuse})^2$

Very Short Answer Questions (1 mark)

1. The altitude of an equilateral triangle is 9 cm. The area of this triangle is:

    (a) $18\sqrt{3}$ cm^2

    (b) $24\sqrt{3}$ cm^2

    (c) $25\sqrt{3}$ cm^2

    (d) $27\sqrt{3}$ cm^2

2. The sides of a triangle are 3 cm, 4 cm and 5 cm. The area of triangle will be:

    (a) 6 cm^2

    (b) 8 cm^2

    (c) 5 cm^2

    (d) 6 cm^2

3. An isosceles right triangle has area 8 cm^2. The length of the hypotenuse is:

    (a) $\sqrt{24}$ cm

    (b) $\sqrt{32}$ cm

    (c) $\sqrt{48}$ cm

    (d) $\sqrt{16}$ cm

4. The side of an equilateral triangle is 12 cm. The area of this triangle is :

    (a) 72 cm^2

    (b) 36 cm^2

    (c) $12\sqrt{3}$ cm^2

    (d) $36\sqrt{3}$ cm^2

5. The base of a triangular field is three times its height. If area of field is 13.5 hectares then its base is:

    (a) 900 m

    (b) 600 m

    (c) 1200 m

    (d) 300 m

6. The perimeter of an isosceles triangle is 32 cm. The ratio of equal sides to the base is 3 : 2. The sides of triangle are:

    (a) 8 cm, 8 cm, 12 cm

    (b) 8 cm, 8 cm, 8 cm

    (c) 8 cm, 12 cm, 12cm

    (d) 12 cm, 12 cm, 12 cm

7. The height corresponding to the longest side of the triangle whose sides are 42 cm, 34 cm and 20 cm in length is:

    (a) 15 cm

    (b) 36 cm

    (c) 16 cm

    (d) 23 cm

8. For an isosceles triangle having base b and each of equal side a, the perimeter will be _____.

9. Two sides of a triangle are 8 cm and 11 cm and its perimeter is 32 cm. The length of third side will be _____.

10. The formula used to find the area of scalene triangle is called _____.

11. The sides of a triangle are in ratio 2 : 1 : 3 and its perimeter is 24 cm. The length of the longest side of triangle will be _____.

12. The perimeter of an equilateral triangle is 60 cm. Its area will be _____.

13. Find the area of a triangle whose base and altitude are 6 cm and 3 cm respectively.

14. The area of a triangle of base 35 cm is 420 sq. cm. Find its altitude.

15. Find the area of a triangle whose base is 15 cm long and the corresponding height is 9.8 cm.

16. Find the area of an equilateral triangle with side $2\sqrt{3}$ cm.

17. Find the area of an equilateral triangle of side 'a' units.

18. Find the area of an isosceles triangle each of whose equal sides is 13 cm and base is 24 cm.

19. The height of an equilateral triangle is 6 cm. Find its side.

20. Find the semi-perimeter of an equilateral triangle of side 2a units.

Short Answer type-I Question (2 Marks)

21. Find the area of an equilateral triangle whose sides are 4 cm each.

22. If sum of two sides of a triangle is 17 cm and its perimeter is 30 cm, find the length of third side.

23. If each side of a triangle is double then its perimeter of triangle increased by how much?

24. If area of a triangle is 50 cm^2 and one of its sides is 10 cm then find the length of corresponding altitude.

25. The sides of a triangle are 11 cm, 60 cm and 61 cm. Find the altitude to the smallest side.

26. The ratio between the sides of a triangle are 3 : 5 : 7 and its perimeter is 300 cm. Find the sides of triangle.

27. Find the area of isosceles triangle whose non equal side is of 12 cm and its corresponding altitude is 7.5 cm.

28. a, b and c are the sides of a triangle. If $(s - a) = 5$ cm, $(s - b) = 10$ cm and $(s - c) = 1$ cm, then find the value of a, b and c.

29. The area of an equilateral triangle is $36\sqrt{3}$ cm^2. Find the side of the equilateral triangle.

Short Answer Type-II Questions (3 Marks)

30. The cost of levelling a right angled triangular park is ₹2 per km^2. The cost of levelling the whole part is ₹2700. If horizontal side of the park is 45 km long then find the length of longest side of the park.

31. Find the area of a triangle if $(s - a) = 35$, $(s - b) = 30$ and $(s - c) = 25$ where a, b, c are sides of triangle and s is its semi-perimeter. (take $\sqrt{105} = 10.25$)

32. The sides of a triangular field are 51 m, 37 m and 20 m. Find the number of flower beds that can be prepared if each bed is to occupy 9 m^2 of area.

33. Using Heron's formula, show that the area of an equilateral triangle is $\dfrac{\sqrt{3}}{4} x^2$, where x is the side.

34. The sides of a triangle are $x, x + 1, 2x - 1$ and its area is $x\sqrt{10}$ sq. units. Find the value of x.

35. The perimeter of a triangle is 50 cm. One side of a triangle is 4 cm longer than the smaller side and the third side is 6 cm less than twice the smaller side. Find the area of the triangle.

36. Find the area of shaded region in the figure.

How may triangular flower beds of 6 m^2 can be made from this area?

[use $\sqrt{105} = 10.25$]
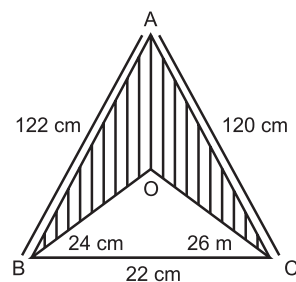
37. The sides of a triangular sheet are 5 cm, 12 cm and 13 cm. Find the cost of painting on the sheet at the rate of ₹30 per cm^2.

38. One side of a right angled triangle is 20 cm and the difference in lengths of its hypotenuses and other side is 8 cm. Find the other side and area of the triangle.

39. The perimeter of a triangle is x cm and its sides are p, q and r cm. What will be the area of triangle? Use the Heron's formula.

40. If every side of a equilateral triangle is doubled, then find the percentage increase in the area of the triangle.

Long Answer Questions (5 Marks)

41. Find the ratio between the area of triangle ΔABC and ΔDEF.
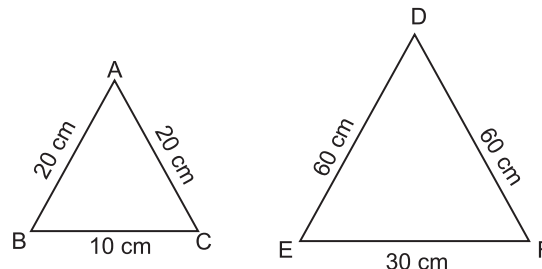
42. While selling clothes for making flags, a shopkeeper claims to sell each piece of cloth in the shape of an equilateral triangle of each side 12 cm while actually he was selling the same in the shape of an isosceles triangle with side 12 cm, 10 cm and 10 cm. How much cloth was he saving in selling each flag?

43. A piece of land is in the shape as given in the figure, has been cut along diagonal AC. The two pieces of land has been distributed between Ram and Sohan. Who will get larger piece of land in terms of area and how much? [Use $\sqrt{10} = 3.15$]
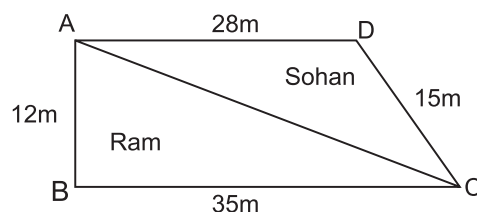
44. A triangular hoarding of dimensions 11 m, 6 m and 15 m is used for commercial activities. The hoarding yield an earning of ₹5000 per m^2 per month.

Calculate the total earning by the hoarding in a month. [Use $\sqrt{2} = 1.41$]

45. In the given kite ABCD is a square having diagonal 48 cm. How much paper of each colour is required to make this kite?

[Use [$\sqrt{21} = 4.58$]

IX – Mathematics

1. (d) $27\sqrt{3}$ cm^2

2. (a) 6 cm^2

3. (b) $\sqrt{32}$ cm

4. (d) 12 cm

5. (a) 900 m

6. (c) 8 cm, 12 cm, 12 cm

7. (c) 16 cm

8. $2a + b$ units

9. 13 cm

10. Heron's formula

11. 12 cm

12. $100\sqrt{3}$ cm^2

13. 9 cm^2

14. 24 cm

15. 73.5 cm^2

16. $3\sqrt{3}$ cm^2

17. $\dfrac{\sqrt{3}}{4}a^2$ units^2

18. 50 cm

19. $4\sqrt{3}$ cm

20. 3a units

21. $4\sqrt{3}$ cm^2

22. 13 cm

23. 4 times

24. 10 cm

25. 60 cm

26. 60 cm, 100 cm, 140 cm

27. 45 cm^2

28. $a = 11$ cm, $b = 6$ cm, $c = 15$ cm

29. 12 cm

30. 75 km

31. $s = 90$

 Area = 1537.5

32. No of flower beds

$$= \frac{\text{Area of field}}{\text{Area of 1 Flower Bed}} = 34$$

34. $S = \dfrac{4x}{2} = 2x$;

 Area =

$$\sqrt{2x(2x-x)(2x-x-1)(2x-2x+1)}$$

$$\Rightarrow x\sqrt{10} = x\sqrt{2(x-1)}$$

$$\Rightarrow x = 6$$

35. Let the length of smallest side = x m

 $\therefore$ Other two sides will be $x + 4$

 and $2x - 6$

 $\therefore$ Perimeter of triangle = $x + x + 4$

 $\qquad\qquad\qquad\qquad\qquad + 2x - 6$

 $\Rightarrow x = 13$

 Area of $\Delta = 109.6$ m^2

36. Area = 1074 m^2,

No. of flower beds = 179

37. ₹ 900

38. Let given side 'a' = 20, hypotenuse = b

other side = c

By Pythagoras theorem

$a^2 = b^2 - c^2$

$\Rightarrow a^2 = (b - c)(b + c)$

$\Rightarrow 20^2 = 8 \times (b + c)$

$\Rightarrow b + c = 50$

So $a = 20$, $b = 29$, $c = 21$

39. $\sqrt{\dfrac{x}{2}\left(\dfrac{x}{2} - p\right)\left(\dfrac{x}{2} - q\right)\left(\dfrac{x}{2} - r\right)}$

40. 300% [Hint: Increase in area of triangle

= Area of new triangle – Area of given triangle

Percentage Increase = $\dfrac{\text{Increased Area}}{\text{Initial Area}} \times 100$]

41. 1 : 9

42. Area of equilateral part = 62.352 cm^2

Area of isosceles part = 48 cm^2

$\left[\text{Difference} = 14.352 \text{ cm}^2\right]$

43. Ram, 210 m^2

44. ₹1,41,000

40. Area of yellow part = area of blue part

= 288 cm^2

Required blue / yellow paper = 576 cm^2

Required black paper = 229 cm^2

CHAPTER-10

HERON'S FORMULA

PRACTICE TEST

Time: 1 hr   **M.M: 20**

1. If $(s - a) = 4$ cm, $(s - b) = 20$ cm, $(s - c) = 11$ cm. Find s. (1)

2. The sides of triangle are 35 cm, 54 cm and 61 cm. Find the length of its longest altitude. (1)

3. Find the area of isosceles triangle whose equal sides are of length 15 cm each and the third side is 12 cm. (2)

4. If each side of triangle is doubled, then find the ratio of area of new triangle thus formed and the given triangle. (2)

5. A triangular park ABC has sides 120m, 80m and 50m. A gardner has planted some trees inside the park leaving 5 m width along each side of park. Find the area in which he planted the trees. (3)

6. The sides of a triangle are in the ratio 25 : 17 : 12 and its perimeter is 540 cm. Find the area of the triangle. (3)

7. The length of sides of a triangle are 7 cm, 12 cm and 13 cm. Find the length of perpendicular from opposite vertex to the side whose length is 12 cm. (3)

8. The cost of fencing an equilateral triangular field at ₹5 per metre is ₹1920. Find its area & all sides. (5)